

# Division Algorithms: Signed- Magnitude Data

# Division Algorithms: Outline

- Division: Shift – subtract Methodology
  - 3 ways to represent signed number
    - Signed Magnitude
    - 1's Complement
    - 2's Complement
  - $\text{Dividend} = \text{divisor} * \text{quotient} + \text{remainder}$
- Division By Hand: Long Division
- Hardware Algorithm: Restoring Method

# Division algorithms--overview

## Division with a paper and pencil

We repeatedly

- Comparison
- Shifting
- Subtraction

$$\begin{array}{r} 8 \\ 3 \overline{) 26} \\ \underline{24} \phantom{0} \\ 2 \phantom{0} \end{array}$$

## Division on computers: only deals with 0s and 1s

the same procedures, only easier than the decimals

## How it is done in binary—an example

Dividend: 111011 (59)

Divisor: 11 (3)

Quotient: 10011 (19)

Remainder: 10 (2)

$$\begin{array}{r} 10011 \text{ r } 10 \\ 11 \overline{) 111011} \\ \underline{-11} \phantom{000} \\ 101 \phantom{00} \\ \underline{-11} \phantom{0} \\ 101 \phantom{0} \\ \phantom{1}11 \\ \underline{\phantom{1}11} \\ 10 \end{array}$$

# Division Algorithm: Long Division

1. Determine Quotient Sign bit
2. Compare divisor with partial remainder
3. Subtract divisor from partial remainder if partial remainder > divisor
4. Shift divisor to the right
5. Repeat 2–4 until visit least significant bit of partial remainder
6. Combine the sign result from number 1 with the quotient and remainder

Signed-Magnitude

Dividend = -221 = 111011101

Divisor = 3 = 011

$$1. \quad Q_s = \text{Divisor}_s \text{ XOR } \text{Dividend}_s = 1$$

$$\text{Remainder}_s = \text{Dividend}_s = 1$$

2. – 4.

$$\begin{array}{r}
 \phantom{11} \quad \quad \quad 1 \ 0 \ 0 \ 1 \ 0 \ 0 \ 1 \\
 11 \overline{) \phantom{0} 1 \ 1 \ 0 \ 1 \ 1 \ 1 \ 0 \ 1} \\
 \phantom{11} \quad \quad 1 \ 1 \\
 \hline
 \phantom{11} \quad \quad \phantom{0} 0 \ 0 \ 1 \ 1 \\
 \phantom{11} \quad \quad \phantom{0} \phantom{0} 1 \ 1 \\
 \hline
 \phantom{11} \quad \quad \phantom{0} \phantom{0} \phantom{0} 1 \ 0 \ 1 \\
 \phantom{11} \quad \quad \phantom{0} \phantom{0} \phantom{0} \phantom{0} 1 \ 1 \\
 \hline
 \phantom{11} \quad \quad \phantom{0} \phantom{0} \phantom{0} \phantom{0} \phantom{0} 1 \ 0
 \end{array}$$

Quotient = 11001001 = -73

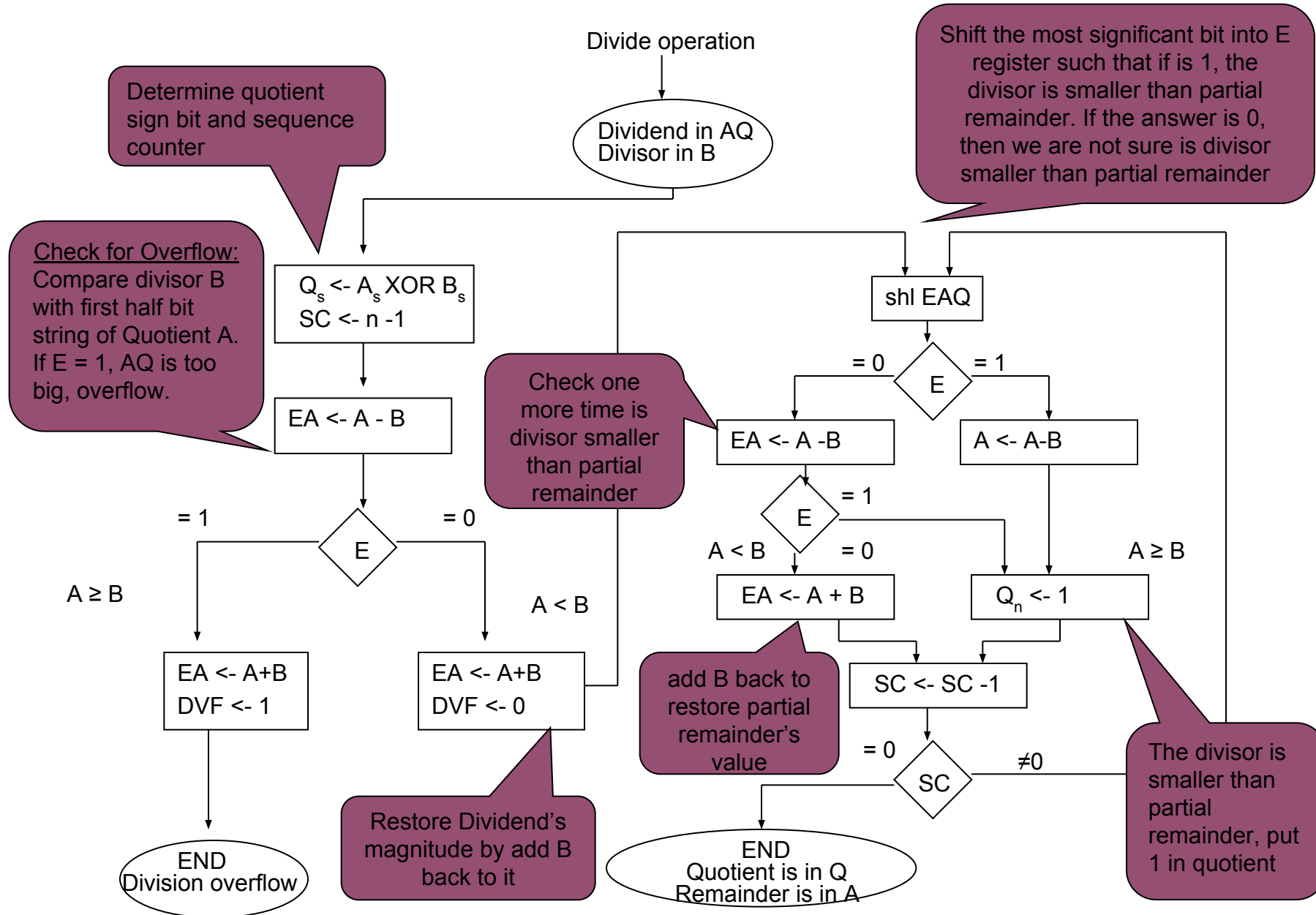
Remainder = 110 = -2

$$-221 = -73 * 3 + -2$$

# Division Algorithm: Restoring Method

- Division Overflow
    - Constraint by register size
    - Dividend = A Q; Divisor = B
      - $A > B$  overflow
    - DVF: divide-overflow flip-flop
  - Divide by Zero
    - Take care when check overflow
1. Determine Sign for quotient and remainder
  2. Check for Overflow
  3. Shift dividend to left
  4. Check is partial remainder larger than divider
  5. If larger, subtract divisor from partial remainder and put 1 for quotient
  6. If smaller, do nothing
  7. Repeat 3–6 for N times such that  $N = \text{number of how many bit pattern a register can hold} - 1$
  8. Combine sign with quotient and remainder

# Division Algorithm: Restoring Method



# Example

- Ques: Perform Division algorithm on 448/17
- Sol: step 1: sign check, divisor n dividend both hv same sign.
- Step 2: 448- -> 0111000000
  - $17 \rightarrow 10001$
  - $AQ \rightarrow \text{dividend} \rightarrow 448 \rightarrow 0111000000$
  - $A \rightarrow 01110, \quad Q \rightarrow 00000$
  - $B \rightarrow 10001$
  - $B'+1 \rightarrow 01111$
- Step 3:  $EA \rightarrow A-B \rightarrow A+B'+1 \rightarrow 01110+01111=11101$  CARRY BIT?
- HERE  $E=0$
- STEP 4:



- Shl EAQ= 0 01110 00000

- 0 11100 00000

|                      | E | A     | Q     | SC  |
|----------------------|---|-------|-------|-----|
| INITIAL              | 0 | 01110 | 00000 | 101 |
| Shl EAQ              | 0 | 11100 | 00000 |     |
| E=0, EA $\oplus$ A-B |   | 01111 |       |     |
|                      | 1 | 01011 | 00001 | 100 |
| Shl EAQ              | 0 | 10110 | 00010 |     |
| E=0, EA $\oplus$ A-B |   | 01111 |       |     |
|                      | 1 | 00101 |       |     |
| E=1, Qn $\oplus$ 1   |   |       | 00011 | 011 |
| Shl EAQ              | 0 | 01010 | 00110 |     |
| E=0, EA $\oplus$ A-B |   | 01111 |       |     |
|                      | 0 | 11001 |       |     |
| E=0, EA $\oplus$ A+B |   | 10001 |       |     |
|                      | 1 | 01010 | 00110 | 010 |
| Shl EAQ              | 0 | 10100 | 01100 |     |
| E=0, EA $\oplus$ A-B |   | 01111 |       |     |
|                      | 1 | 00011 |       |     |
| E=1,                 |   |       | 01101 | 001 |
| Shl EAQ              | 0 | 00110 | 11010 |     |

ans

E - discard

A- remainder- 00110

Q – Quotient- 11010

+ve/-ve---- -ve result 2's complement

-ve/ +ve---- -ve result 2's complement

+ve/ +ve - result

# Division Algorithm: Restoring Method

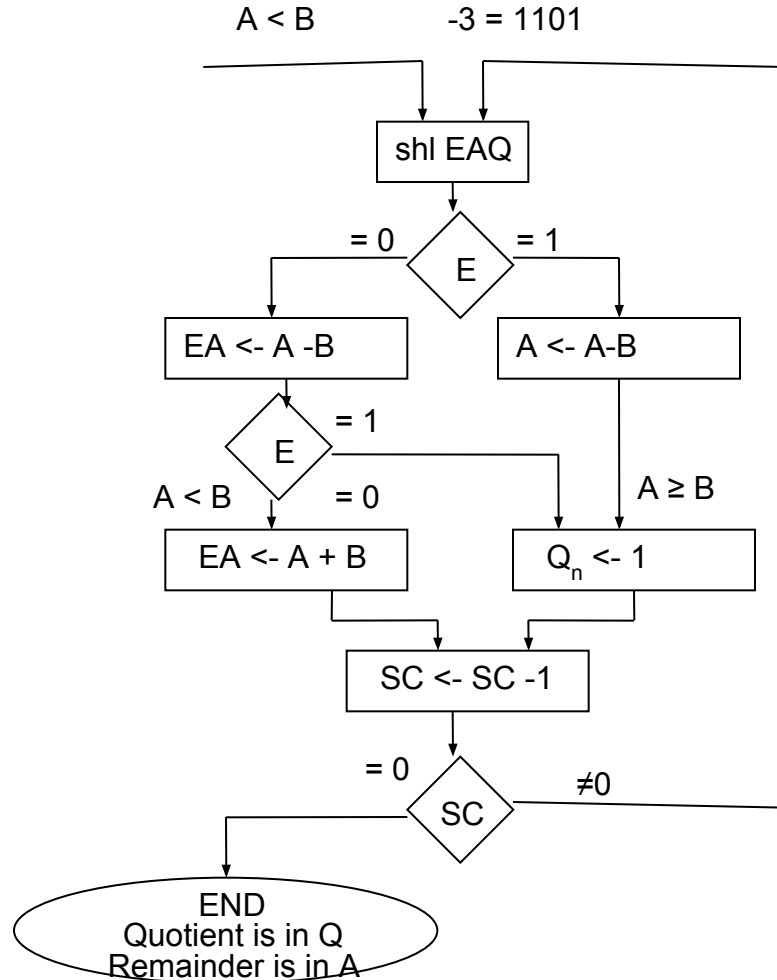
$$-15 / 3 = -5 \dots 0$$

$$15 = 0000 \ 1111$$

$$-15 = 1111 \ 0001$$

$$3 = 0011$$

$$-3 = 1101$$



1.  $Q_s = 1 \text{ XOR } 0 = 1 \dots \text{negative}$

| E | A       | Q       |
|---|---------|---------|
| 0 | 0 0 0 0 | 1 1 1 1 |
|   | 1 1 0 1 |         |

2. Check for Overflow

|   |         |         |
|---|---------|---------|
| 0 | 1 1 0 1 | 1 1 1 1 |
|   | 0 0 1 1 |         |

E = 1 not overflow;  
restore value

|   |         |         |
|---|---------|---------|
| 1 | 0 0 0 0 | 1 1 1 1 |
| 0 | 0 0 0 1 | 1 1 1 0 |
|   | 1 1 0 1 |         |

3&4. Shift to the Left

Check is partial  
remainder big enough

|   |         |         |
|---|---------|---------|
| 0 | 1 1 1 0 | 1 1 1 0 |
|   | 0 0 1 1 |         |

6. E = 0. Restore original  
partial remainder

(S3)

|   |         |         |
|---|---------|---------|
| 1 | 0 0 0 1 | 1 1 1 0 |
| 0 | 0 0 1 1 | 1 1 0 0 |
|   | 1 1 0 1 |         |

3&4. Shift to the Left  
Check is partial  
remainder big enough

(S2)

|   |         |         |
|---|---------|---------|
| 1 | 0 0 0 0 | 1 1 0 1 |
| 0 | 0 0 0 1 | 1 0 1 0 |
|   | 1 1 0 1 |         |

E = 1;  $Q_n = 1$

|   |         |         |
|---|---------|---------|
| 0 | 1 1 1 0 | 1 0 1 0 |
|   | 0 0 1 1 |         |

3&4. Shift to the Left  
Check is partial  
remainder big enough  
6. E = 0. Restore original  
partial remainder

(S1)

|   |         |         |
|---|---------|---------|
| 1 | 0 0 0 1 | 1 0 1 0 |
| 0 | 0 0 1 1 | 0 1 0 0 |
|   | 1 1 0 1 |         |

3&4. Shift to the Left  
Check is partial  
remainder big enough

|   |         |         |
|---|---------|---------|
| 1 | 0 0 0 0 | 0 1 0 1 |
|---|---------|---------|

remainder

quotient