UNIT - 4

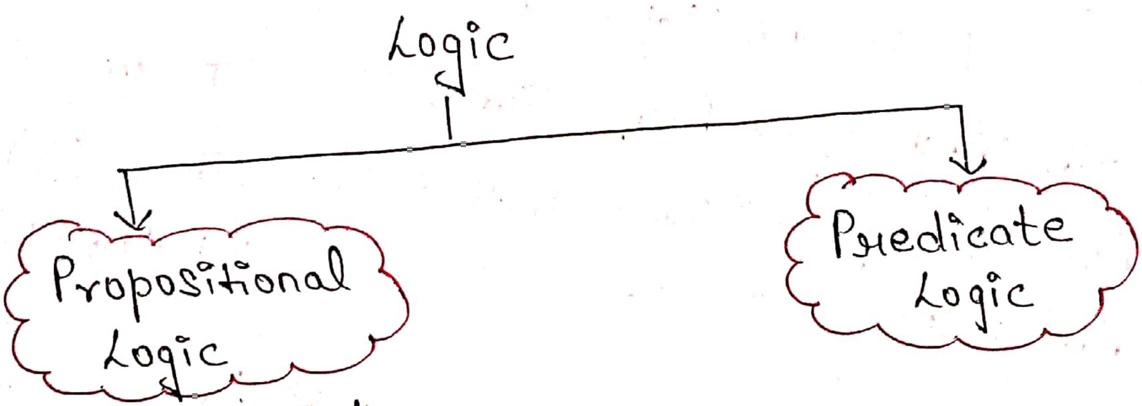
Lecture - 1

Logics and Proofs

Logic: Any 'formal system' can be considered a logic if it has:

a) a well-defined syntax;
b) a well-defined semantics; and
c) a well-defined proof-theory.

- **Syntax:** Syntax of a logic defines the syntactically acceptable objects of the language, which are properly called well-formed formulae (wff).
- **Semantics:** Semantics of a logic associate each formula with a meaning.
- **Proof Theory:** The proof-theory is concerned with manipulating formulae according to certain rules.



Propositional Logic:

1) A proposition is:
   a) A declarative sentence
   b) Either True or false, but not both.

Examples:

1.) Washington D.C, is the capital of the USA. (True).

2) Toronto is the Capital of Canada. (False)

3) $1+1=2$. (True)

4) $2+2=3$ (False)

* All the following sentences are declarative so propositions.

Examples:

1) What time is it?
2) Read this carefully

} Not propositions bcoz not declarative sentences.

3) $x+1=2$
4) $x+y=z$.

} Not propositions bcoz neither true nor false.
But can be turned into a propositions if we assign values to the variables.

2) Propositional variables: p, q, r, s, ...
Conventional letters or variables used to represent propositions.

p : John Major is prime Minister.

3) Truth Value of a Proposition: True (T) or False (F).

a) For True proposition — truth value is True (T)

b) For False " — " " False (F).

4) Propositional Calculus or Propositional Logic:
The area of Logic that deals with propositions

5) Compound Propositions:
New propositions are formed from existing propositions (combining one or more propositions) using Logical operators.

Logical Operators : (also known as Connectives)

1.) Negation operator ($\neg$ or $-$) (not)
2.) Conjunction ($\wedge$, and)
3.) Disjunction ($\vee$, or)
4.) Conditional statement ($\rightarrow$) (if-then)
5.) Biconditional Statement. ($\leftrightarrow$) (if and only if)(iff)

① Negation Operator : ($\neg$ or $-$)

a) Let p be a proposition. The negation of p, denoted by $\neg p$ or $\overline{p}$, is the statement {" It is not the case that p."}

b) The proposition $\neg p$ is read "not p".

c) Truth value of the negation of p, $\overline{p}$, is the opposite of the truth value of p.

d) Truth Table :

The Truth Table for Negation of a proposition.

p	$\neg p$
T	F
F	T

e) Example :

1.) Mohan's PC runs Linux.

Negation : It is not the case that Mohan's PC runs Linux

(or)

Mohan's PC does not run Linux.

2) p : Vandana's smartphone has at least 32GB of memory.

Negation of p : $\bar{p}$: It is not the case that Vandana's smartphone has at least 32GB of memory.

or

Vandana's smartphone does not have at least 32GB of memory.

or

Vandana's smartphone has less than 32GB of memory.

f) The negation operator constructs a new proposition from a single existing proposition.

2) Conjunction ($\wedge$)

a) Let p and q be propositions. The conjunction of p and q, denoted by $p \wedge q$, is the proposition. {"p and q".}

b) The conjunction $p \wedge q$ is true when both p and q are true and is false otherwise.

c) Truth Table :

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

d) Example :

p : Today is friday

q : It is raining today

$p \land q$: Today is friday and it is raining today.

"p and q"

$p \land q$: True — on rainy friday

false — otherwise

- Any day that is not a friday
- fridays when it does not rain.

(3) Disjunction : (Inclusive OR) ($\lor$)

a) Let p and q be propositions. The disjunction of p and q, denoted by $p \lor q$, is the proposition.

{"p or q".}

b) The disjunction $p \lor q$ is false when both p and q are false and is true otherwise.

c) Truth Table :

p	q	$p \lor q$
T	T	T
T	F	T
F	T	T
F	F	F

d.) Example :

p : Today is friday

q : It is raining today.

$p \vee q$ = "p or q"

$p \vee q$: "Today is friday or it is raining today."

True : a) Today is friday
b) It is raining today
c) It is a rainy friday

False : Today is not friday and it does not rain.

4) Exclusive OR : ($\oplus$) or ($\bar{\vee}$) {Exclusive Disjunction}

a) Let p and q be propositions. The Exclusive OR of 'p' & 'q', denoted by '$p \oplus q$', is the proposition
{" p or q (but not both)"} or {p either q}

b) The Exclusive OR '$p \oplus q$' is true when exactly one of p and q is true and is false otherwise.

c) Truth Table :

p	q	$p \oplus q$ or $p \bar{\vee} q$
T	T	F
T	F	T
F	T	T
F	F	F

Ex: You want to drink tea or coffee?
drink either tea or coffee but not both.

2) $p \bar{\vee} q$: Today is friday either it is raining today.

5) NAND operator : ($\uparrow$)

a) Let p and q be propositions. The NAND of 'p' & 'q' denoted by '$\neg(p \wedge q)$' or '$(\overline{p \wedge q})$', is the proposition
{"It is not the case that p and q"}

b) The NAND $(\overline{p \wedge q})$ is false when both p and q are true and is true otherwise.

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c) Truth Table :

p	q	$p \wedge q$	$\overline{p \wedge q}$ or $(p \uparrow q)$
T	T	T	F
T	F	F	T
F	T	F	T
F	F	F	T

Ex: $p \wedge q$: Today is friday and it is raining today

$\overline{p \wedge q}$: It is not the case that today is friday and it is raining today.

6) NOR operator : $(\downarrow)$

a) Let p and q be propositions. The NOR of 'p' & 'q', is the proposition denoted by '$\neg(p \vee q)$' or '$(\overline{p \vee q})$',

{"It is not the case that p or q."}

b) The NOR $(\overline{p \vee q})$ is true when both p and q are false and is true otherwise.

c) Truth Table :

p	q	$p \vee q$	$\overline{p \vee q}$ or $(p \downarrow q)$
T	T	T	F
T	F	T	F
F	T	T	F
F	F	F	T

Ex: $p \vee q$: Today is friday or it is raining today.

$\overline{p \vee q}$: It is not the case that today is friday or it is raining today.

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7) Conditional Statements : ($\rightarrow$)

a) Let p and q be propositions, The conditional statement
$p \rightarrow q$ is the proposition
{"If p, then q"}

b) $p \rightarrow q$ is false when p is true and q is false, and true otherwise.

c) In the conditional statement $p \rightarrow q$,
p — called the hypothesis (or antecedent or premise)
q — called the conclusion (or consequence)

d) Also called an Implication.

e) Truth Table :

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

f) The following ways to express the conditional statement $p \rightarrow q$:

1) "If p, then q"
2) "If p, q"
3) "p is sufficient for q"
4) "q if p"
5) "q when p"
6) "a necessary condition for p is q"
7) "q unless ¬p"
8) "p implies q"
9) "p only if q"

10) "a sufficient condition for q is p"
11) "q whenever p"
12) "q is necessary for p"
13) "q follows from p"

g) Example :

1) If $\underbrace{\text{Maria learns discrete Mathematics}}_{p}$, then $\underbrace{\text{she will find a good job.}}_{q}$ If p, then q

p : Maria learns discrete Mathematics

q : She will find a good job.

Represent in different ways :

a) Maria will find a good job when she learns discrete Mathematics (q when p)

b) For Maria to get a good job, it is sufficient for her to learn discrete mathematics (sufficient condition for q is p)

c) Maria will find a good job unless she does not learn discrete Mathematics (q unless not p)

Note:

{p only if q ≠ q only if p}

Ex: p : You go

q : I go.

$p \to q$: If You go, then I go.

$\equiv$

p only if q : You go only if I go.

q only if p : I go only if You go.

not same.

8) <u>BiConditional</u> <u>Statement</u>: ($\longleftrightarrow$)

a) Let p and q be propositions. The biconditional statement $p \leftrightarrow q$ is the proposition
{"p if and only if q"}

b) $p \leftrightarrow q$ is true when p and q have the same truth values, and is false otherwise.

c) Also Known as bi-implications.

d) $p \leftrightarrow q$ is true when both the conditional statements $p \rightarrow q$ and $q \rightarrow p$ are true and is false otherwise.
i.e why we use – "if and only if"

e) The other ways to express $p \leftrightarrow q$:
1) "p is necessary and sufficient for q"
2) "If p then q, and conversely"
3) "p iff q"
"if and only if"

f) Truth Table:

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

p	q	$p \rightarrow q$	$q \rightarrow p$	A
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

$A = (p \rightarrow q) \wedge (q \rightarrow p)$

Note:

$$\boxed{p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)}$$

Both Have same truth value so they are equivalent.

$$\boxed{p \rightarrow q \equiv \neg p \vee q}$$

(or)

$$\boxed{p \rightarrow q \Leftrightarrow \neg p \vee q}$$

9) Example :

p : You can take the flight

q : You buy a ticket

$p \longleftrightarrow q$: You can take the flight if and only if you buy a ticket.

— This stmt is true
- If you buy a ticket and take the flight
- If you do not buy a ticket and you cannot take the flight.

Other forms of Biconditional stmt are :

1.) You can take flight is necessary and sufficient for you buy a ticket {type - p is necessary and sufficient for q}

2.) If you can take flight then you buy a ticket and if you buy a ticket then you can take flight. {type - If p then q and If q then p}

3.) You can take flight iff you buy a ticket. {type - "p iff q"}

Q⇒ Determine if the following stmts are true or false:

1.) "2+2 =4 if and only if 1+1 = 2".

2.) "1+1 =2 if and only if 2+3 =4".

3.) "1+1 =3 if and only if humans can fly"

1.) True (True, True)

2.) False (True, False)

3.) True (false, false)

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Converse, Inverse and Contrapositive :

1) For conditional statement $p \to q$

a) Converse : $q \to p$
b) Inverse : $\neg p \to \neg q$
c) Contrapositive : $\neg q \to \neg p$

New Conditional Statements formed from conditional statement $p \to q$.

① Contrapositive :

a) Contrapositive of $p \to q$ is $\neg q \to \neg p$

b) Any Conditional and its contrapositive are logically equivalent. (have the same truth ~~value~~ table)

– when two compound propositions always have the same truth value we call them equivalence.

c) Proof by truth table :

p	q	$p \to q$	$\neg q$	$\neg p$	$\neg q \to \neg p$
T	T	T	F	F	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

Same truth table so logically Equivalent.

Example :

$p \to q$: "If you get 100% in this course, you will get an A+"

Contrapositive : $\neg q \to \neg p$: "If You do not get an A+ in this course, You did not get 100%."

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2) Converse:

a) Converse of $p \to q$ is $q \to p$

b) Any Conditional and its converse are not logically equivalent.

c) Example:

$p \to q$: "If you get 100% in this course, you will get an A+"

$q \to p$: If you get an A+ in this course, you scored 100%.

Both are not equivalent

3) Inverse:

a) Inverse of $p \to q$ is $\neg p \to \neg q$

b) Any Conditional and its Inverse are not logically equivalent.

c) Example:

$\neg p \to \neg q$: "If you do not get 100% in this course then you will not get an A+"

Truth Table of Contrapositive, Converse and Inverse:

p	q	$\neg p$	$\neg q$	$\neg q \to \neg p$	$q \to p$	$\neg p \to \neg q$
T	T	F	F	T	T	T
$\boxed{T}$	$\boxed{F}$	F	T	$\boxed{F}$	T	T
$\boxed{F}$	$\boxed{T}$	T	F	T	$\boxed{F}$	F
F	F	T	T	T	T	T

Same as $p \to q$

Same

Note

1.) Conditional Stmt and its Contrapositive are logically equivalent.

$$p \to q \equiv \neg q \to \neg p$$

2.) Converse and Inverse are logically equivalent.

$$q \to p \equiv \neg p \to \neg q$$

Example: write the contrapositive, converse and Inverse of Conditional Stmt
" The hometown wins whenever it is raining"

It is conditional stmt of type q whenever p.

p : "It is raining"

q : The home town wins".

1) Contrapositive : $\neg q \to \neg p$
If the hometown does not wins then it is not raining.

2.) Converse : $q \to p$
If the hometown wins then it is raining.

3) Inverse : $\neg p \to \neg q$
If it is not raining then the home town does not wins.

Q ⇒ Determine which of the following stmts are true.

1) If $1+1=2$ then $2+2=5$

2.) If $1+1=3$, then $2+2=4$

3) If $1+1=3$, then $2+2=5$

4) If monkeys can fly, then $1+3=3$

1) False (True, False)

2) True (F, T)
3) True (F, F)
4) True (F, F).

Precedence of Logical Operators:

Operator	Precedence
$\neg$ (not)	1
$\wedge$	2
$\vee$	3
$\rightarrow$	4
$\leftrightarrow$	5

} Associativity (Left to Right)

Truth Table of Compound Propositions:

Q⇒ Construct the truth table of the compound proposition

$(p \vee \neg q) \rightarrow (p \wedge q)$

p	q	$\neg q$	$p \vee \neg q$	$p \wedge q$	$(p \vee \neg q) \rightarrow (p \wedge q)$
T	T	F	T	T	T
T	F	T	T	F	F
F	T	F	F	F	T
F	F	T	T	F	F

2) $(p \rightarrow q) \leftrightarrow (\neg p \vee \neg q)$
3) $(\neg p \rightarrow q) \wedge (p \leftrightarrow q)$
4) $(q \rightarrow (\neg p \vee q)) \rightarrow p$

2) $(p \to q) \leftrightarrow (\neg p \vee \neg q)$

p	q	$p \to q$	$\neg p$	$\neg q$	$\neg p \vee \neg q$	$(p \to q) \leftrightarrow (\neg p \vee \neg q)$
T	T	T	F	F	F	F
T	F	F	F	T	T	F
F	T	T	T	F	T	T
F	F	T	T	T	T	T

3) $(\neg p \to q) \wedge (p \leftrightarrow q)$

p	q	$\neg p$	$\neg p \to q$	$p \leftrightarrow q$	$(\neg p \to q) \wedge (p \leftrightarrow q)$
T	T	F	T	T	T
T	F	F	T	F	F
F	T	T	T	F	F
F	F	T	F	T	F

4) $(q \to (\neg p \vee q)) \to p$

p	q	$\neg p$	$\neg p \vee q$	$q \to (\neg p \vee q)$	$(q \to (\neg p \vee q) \to p$
T	T	F	T	T	T
T	F	F	F	T	T
F	T	T	T	T	F
F	F	T	T	T	F

Q=) Prove that

$(p \to \neg q) \wedge (p \to \neg r) \equiv \neg (p \wedge (q \vee r))$

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$A = (p \to \neg q) \wedge (p \to \neg r)$ $B = \neg(p \wedge (q \vee r))$

	q	r	$\neg q$	$\neg r$	$p \to \neg q$	$p \to \neg r$	A	$q \vee r$	$p \wedge (q \vee r)$	B
T	T	T	F	F	F	F	F	T	T	F
T	T	F	F	T	F	T	F	T	T	F
T	F	T	T	F	T	F	F	T	T	F
T	F	F	T	T	T	T	T	F	F	T
F	T	T	F	F	T	T	T	T	F	T
F	T	F	F	T	T	T	T	T	F	T
F	F	T	T	F	T	T	T	T	F	T
F	F	F	T	T	T	T	T	F	F	T

Same truth value so logically equivalent proved

HW Express, in other 13 ways the following Conditional stmts :

1.) The home town wins whenever it is raining.

2.) If you get 90% marks in final exams then I will give you 'A' grade.

Q3.) Two propositions are given:

p : At least 10 inches of rain fell today in Mizorum.

q : Today is friday.

Express into $\neg p$, $\neg q$, $P \wedge Q$, $P \vee Q$, $P \oplus q$

Q4⇒ Prove that (Using truth table)

a) $P \to (Q \to R) \equiv (P \wedge Q) \to R$

b) $\neg(P \vee Q) \vee (\neg P \wedge Q) \equiv \neg P$

c) $(\neg P \wedge (\neg Q \wedge R)) \vee (Q \wedge R) \vee (P \wedge R) \equiv R$

Q5⇒ Find the truth table

a) $(P \to Q \wedge \neg Q) \to \neg P$ b) $(P \to \neg Q) \wedge (P \to \neg R)$

Well formed formula :

A well formed formula is one which should not be ambigous. It can be obtained by following Rules:

1.) If statement 'A' is well formed formula then negation of 'A' is also well formed formula.

2.) A statement variable standing alone is a well formed formula

3.) If 'A' and 'B' are well formed formulae then $(A \wedge B)$, $(A \vee B)$, $(A \rightarrow B)$, $(A \leftrightarrow B)$ are also well formed formulae.

4.) A string of symbol having statement variables connectives and parenthesis is a well formed formula if it can be obtained by finite applications of previous pre Rules.

Ex: $\neg P \vee Q$ is not well formed formula — Since stmt is ambigous (Connectives used but parenthesis not used)

$\neg(P \vee Q)$ $\quad$ $(\neg P \vee Q)$

Q⇒ Determine which of the following are well formed formula and if not then represent it as well formed formulaes:

1.) $(P \rightarrow (P \vee Q))$ Well formed

2.) $((P \rightarrow (\neg P)) \rightarrow \neg P)$ Well formed

3.) $((\neg Q \wedge P) \wedge Q)$ Well formed

4.) $((P \rightarrow (Q \rightarrow R)) \rightarrow ((P \rightarrow Q) \rightarrow (P \rightarrow R))$ Not well formed.

$((P \rightarrow (Q \rightarrow R)) \rightarrow ((P \rightarrow Q) \rightarrow (P \rightarrow R)))$ – well formed.

Tautology, Contradiction, Contingency :

1) Tautology :

A compound proposition that is always true, no matter what the truth values of the propositions that occurs in it, is called a tautology.

Ex: compound proposition $(P \vee \neg P)$.

2) Contradiction :

A compound proposition that is always false, no matter what the truth values of the propositions that occur in it, is called a contradiction.

Ex: compound proposition $(P \wedge \neg P)$.

3) Contingency :

A compound proposition that is neither a tautology or a Contradiction is Called a Contingency.

Ex: Truth table of AND, OR, NAND, NOR etc.

4) Satisfiable :

A compound proposition is satisfiable if there is an assignment of truth values to the variables in compound proposition that makes compound proposition true.

Ex: Truth table of AND, OR, NAND, NOR etc.

[All stmt values except Contradiction is Satisfiable]

Examples of a Tautology and a Contradiction					
P	$\neg P$	$P \wedge \neg P$		$P \vee \neg P$	
T	F	F	Contradiction	T	Tautology
F	T	F		T	

Q⇒ Determine whether the stmt $[(P \to Q) \wedge (Q \to R)] \to (P \to R)$ is Contradiction or tautology with the help of truth value.

Soln : Truth Table

$$[(P \to Q) \wedge (Q \to R)] \to (P \to R)$$
$$\downarrow$$
$$Z \to W$$

P	Q	R	$\underbrace{P \to Q}_{X}$	$\underbrace{Q \to R}_{Y}$	$\underbrace{X \wedge Y}_{Z}$	$\underbrace{P \to R}_{W}$	$Z \to W$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	T	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

Since, all Resultant values are true so, it is tautology

Logical Equivalence : (≡ , ⇔)

a) The Compound propositions 'p' and 'q' are called logically equivalent if $p \leftrightarrow q$ is a tautology.

b) The notation $p \equiv q$ denotes that p and q are logically equivalent.

c) Compound propositions that have the same truth values in all possible cases are called logically equivalent.

Example : show that $\neg p \vee q$ and $p \to q$ are logically equivalent.

Truth Tables for $\neg p \vee q$ and $p \to q$				
p	q	$\neg p$	$\neg p \vee q$	$p \to q$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

Same truth value so logically Equivalent

2) Prove that

$p \vee (q \wedge r)$ and $(p \vee q) \wedge (p \vee r)$ are logically equivalent.

or

$$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$$

p	q	r	$q \wedge r$	$p \vee (q \wedge r)$	$(p \vee q)$	$(p \vee r)$	$(p \vee q) \wedge (p \vee r)$
T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	F	F	T	F	F
F	F	T	F	F	F	T	F
F	F	F	F	F	F	F	F

Same truth value so logically Equivalent.

Laws of Logical Equivalence :

Equivalence	Name
1) $P \wedge T \equiv P$ $P \vee F \equiv P$	Identity Laws
2.) $P \vee T \equiv T$ $P \wedge F \equiv F$	Domination Laws
3) $P \vee P \equiv P$ $P \wedge P \equiv P$	Idempotent Laws
4) $\neg(\neg P) \equiv P$	Double Negation
5) $P \vee Q \equiv Q \vee P$ $P \wedge Q \equiv Q \wedge P$	Commutative Law
6.) $(P \vee Q) \vee R \equiv P \vee (Q \vee R)$ $(P \wedge Q) \wedge R \equiv P \wedge (Q \wedge R)$	Associative Law
7) $P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$ $P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$	Distributive. Law
8.) $\neg(P \wedge Q) \equiv \neg P \vee \neg Q$ $\neg(P \vee Q) \equiv \neg P \wedge \neg Q$	De-Morgan's Law
9.) $P \vee (P \wedge Q) \equiv P$ $P \wedge (P \vee Q) \equiv P$	Absorption Law
10.) $P \vee \neg P \equiv T$ $P \wedge \neg P \equiv F$	Negation Law

'T' denotes the compound proposition that is always true
'F' " " " " " " " false

Logical Equivalences Involving Conditional Stmts :

1) $p \rightarrow q \equiv \neg p \vee q$
2) $p \rightarrow q \equiv \neg q \rightarrow \neg p$
3) $p \vee q \equiv \neg p \rightarrow q$
4) $p \wedge q \equiv \neg (p \rightarrow \neg q)$
5) $\neg (p \rightarrow q) \equiv p \wedge \neg q$
6) $(p \rightarrow q) \wedge (p \rightarrow r) \equiv p \rightarrow (q \wedge r)$
7) $(p \rightarrow r) \wedge (q \rightarrow r) \equiv (p \vee q) \rightarrow r$
8) $(p \rightarrow q) \vee (p \rightarrow r) \equiv p \rightarrow (q \vee r)$
9) $(p \rightarrow r) \vee (q \rightarrow r) \equiv (p \wedge q) \rightarrow r$

Logical Equivalences Involving Biconditional Statements :

1) $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$
2) $p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$
3) $p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$
4) $\neg (p \leftrightarrow q) \equiv p \leftrightarrow \neg q$.

Q⇒ Prove that $\neg (P \rightarrow Q) \equiv P \wedge \neg Q$ by using formulae.

L.H.S $= \neg (P \rightarrow Q) = \neg (\neg P \vee Q)$ (Conditional Stmt)

$= \neg (\neg P) \wedge \neg Q$ (De-Morgan's Law)

$= P \wedge \neg Q$ (Double Negation)

$=$ R.H.S proved.

Q⇒ Prove that $\neg(P \vee (\neg P \wedge Q)) \equiv \neg P \wedge \neg Q$

$L.H.S = \neg(P \vee (\neg P \wedge Q))$

$= (\neg P \wedge \underbrace{\neg(\neg P \wedge Q)})$ [De-Morgan's Law]

$= (\neg P \wedge (\neg(\neg P) \vee \neg Q))$ [De-Morgan's Law

$= (\neg P \wedge (P \vee \neg Q))$ [Double Negation]

$= \underbrace{(\neg P \wedge P)}_{F} \vee (\neg P \wedge \neg Q)$ [Distributive Law]

$= F \vee (\neg P \wedge \neg Q)$ [Negation Law]

$= (\neg P \wedge \neg Q) \vee F$ [commutative Law]

$= \neg P \wedge \neg Q$ [Identity Law]

$= R.H.S$ proved.

Q⇒ $(P \wedge Q) \rightarrow (P \vee Q)$. Prove that it is tautology using Equivalence formulae.

$= \underbrace{(P \wedge Q)}_{A} \rightarrow \underbrace{(P \vee Q)}_{B}$

$A \longrightarrow B$

$= \neg(P \wedge Q) \vee (P \vee Q)$ [conditional operation]

$= (\neg P \vee \neg Q) \vee (P \vee Q)$ [De-Morgan's Law]

$= (\neg P \vee P) \vee (\neg Q \vee Q)$ [Associative Law]

$= T \vee T$ [Negation Law]

$= T$

Hence, proved given Statement is tautology.

Note:

The above examples can also be done using truth tables.

Exercise :

1.) Prove that above stmt is tautology using logical Equivalence

$[(P \vee Q) \wedge ((P \rightarrow r) \wedge (Q \rightarrow r))] \rightarrow r$

2.) Using Logical Equivalence, prove that is tautology

a) $((P \rightarrow Q) \wedge \neg Q) \rightarrow \neg P$.

3.) Prove that

1) $P \rightarrow (Q \rightarrow R) \equiv (P \wedge Q) \rightarrow R$

2) $\neg(P \vee Q) \vee (\neg P \wedge Q) \Leftrightarrow \neg P$

3) $(\neg P \wedge (\neg Q \wedge R)) \vee (Q \wedge R) \vee (P \wedge R) \equiv R$

using Logical Equivalence.

4.) $[(P \vee Q) \wedge (P \rightarrow R) \wedge (Q \rightarrow R)] \rightarrow R$ is a tautology with the help of truth table.

5.) Using truth table prove that :

a) $P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$

b) $(P \rightarrow R) \vee (Q \rightarrow R) \equiv (P \wedge Q) \rightarrow R$

c) $(P \leftrightarrow Q) \equiv (P \wedge Q) \vee (\neg P \vee \neg Q)$.

6.) Find the contrapositive, converse and Inverse of the following compound proposition.

a) "You can buy ice-cream only if it is hot outside"

b.) "If the triangle is equilateral then it is equi-triangular".