Unit - 5

Graph theory

100% 1- Tree { BST } {

2-3 Q 2- Graph

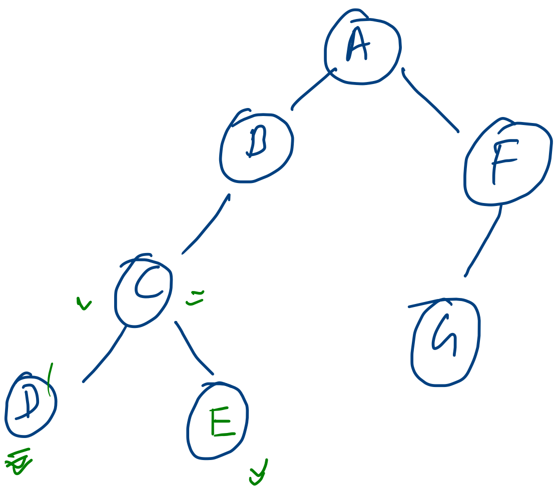
maxim ← 100% {3- Recurrance Relation & generating func

4- Combitor^c { 2 marks }

Inorder → left root right

Preorder → root left right

Postorder → left right root



Inorder → D C E B A G F

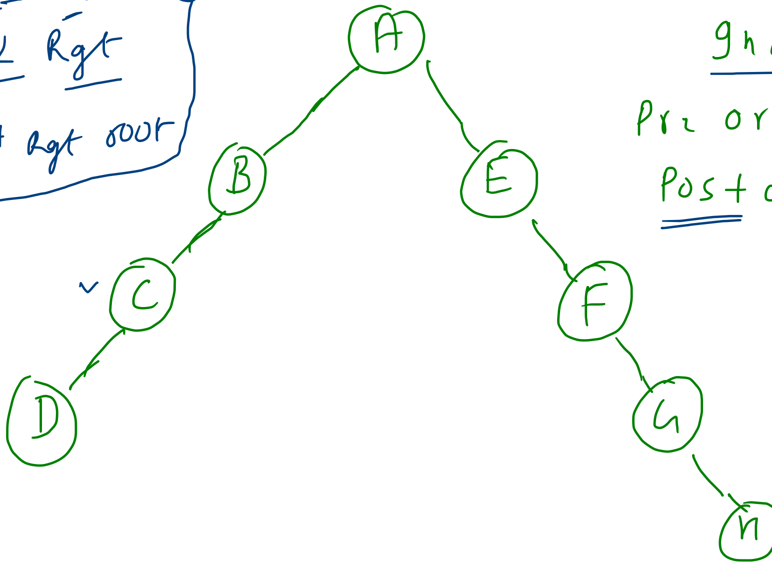
Preorder → A B C D E F G

Postorder → D E C B G F A

Io - L R Rgt

Pre → Rt L Rgt

posto → Left Rgt root



Inorder — D C B A ~~H G F E~~ E F G H

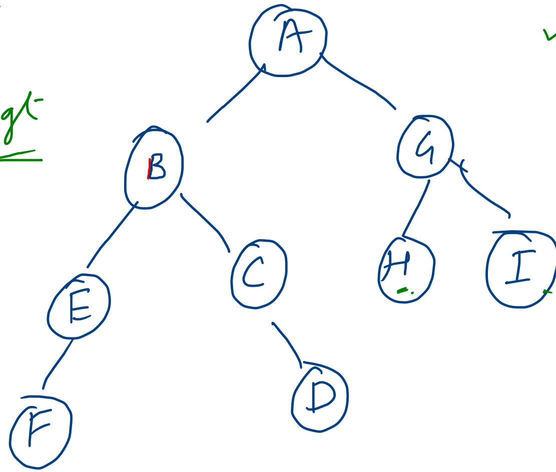
Pre order — A B C D E F G H

Postorder — D C B H G F E A

ino left root rgt

Pre → Root left rgt

post left rgt root



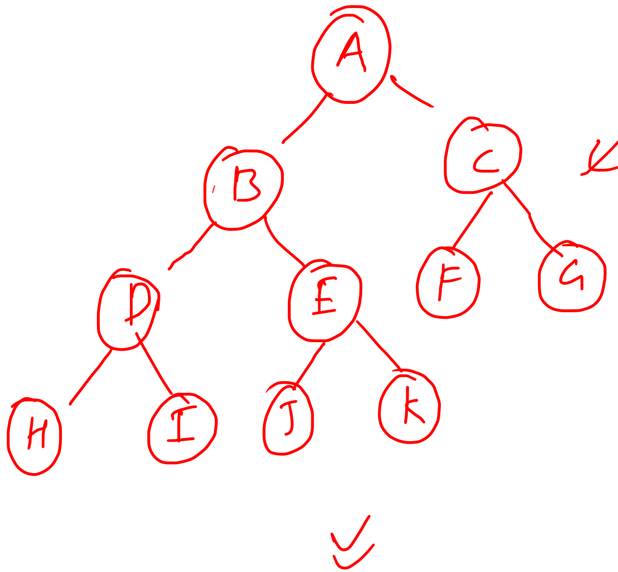
Inorder — F E B C D A H G I

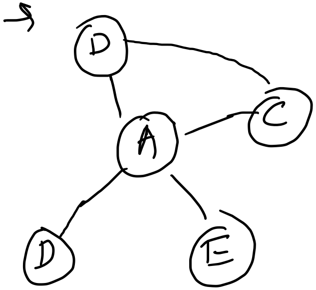
Preorder — A B E F C D G H I

Post order — F E D C B H I G A

root left Rgt Preorder — A B D H I E J K C F G

left root Rgt Inorder → H D I B J E K A F C G



Graph → 

Structure (V, E)

where

$V \rightarrow$ set of vertex ($v_1, v_2, v_3 \cdots v_n$)

$E \rightarrow$ set of edge ($e_1, e_2, e_3 \cdots e_n$)

A = { }

empty

A

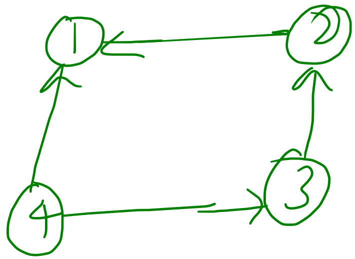
V = { non empty }

E = { }

V is non empty set

Directed graph

→ ordered pair

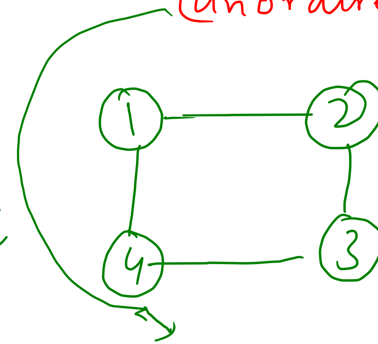


$R = \{(1,2)\}$

$E = \{(2,1)\ (3,2)\ (4,3)\ (4,1)\}$

Undirected graph

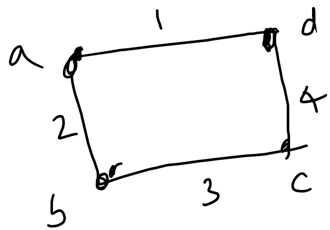
→ (unordered pair)



E (4,1) (1,2) (4,3) (2,3)

(1,4) (2,1) (3,4) (3,2)

1) Adjacent vertex → if two vertex are joined by the same edge they are called adjacent vertex.



$V = \{a, b, c, d\}$

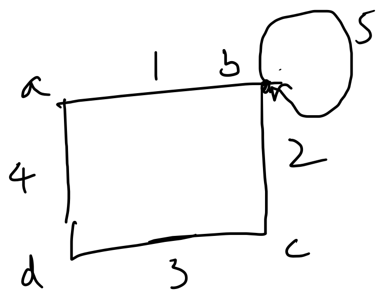
$E = \{1, 2, 3, 4\}$

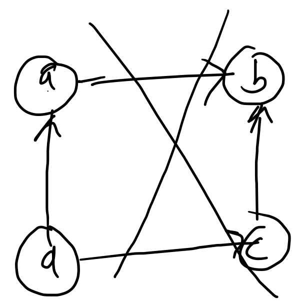
(a, d) (a, b) (c, d) (c, b) → adjacent vertex

(a, c) (b, d) → Not adjacent vertex.

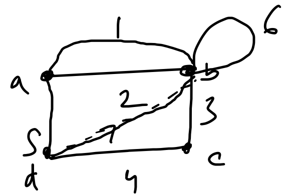
2) <u>Adjacent edge</u> — If two edges are incident on same vertex then they are called adjacent edge.

3) <u>Self loop</u> → Edge having same vertex (v_i, v_i) as it end vertices called self loop.



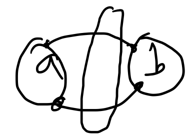


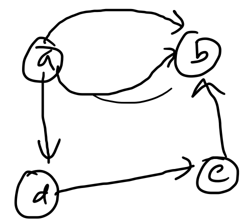
4) Parallel edge → When more than one edge associated with a given pair of such edge are called parallel edge



1, 2 → Parallel edges

6 → self loop.

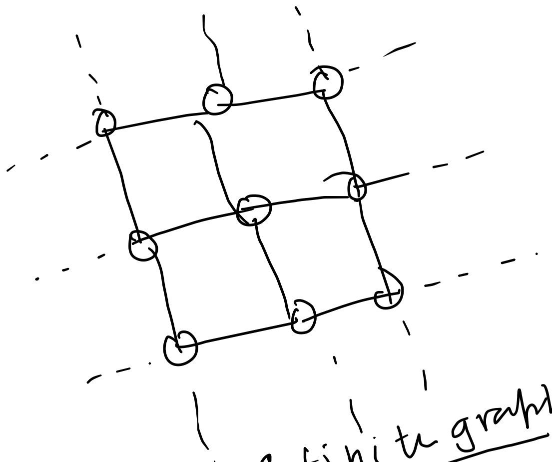




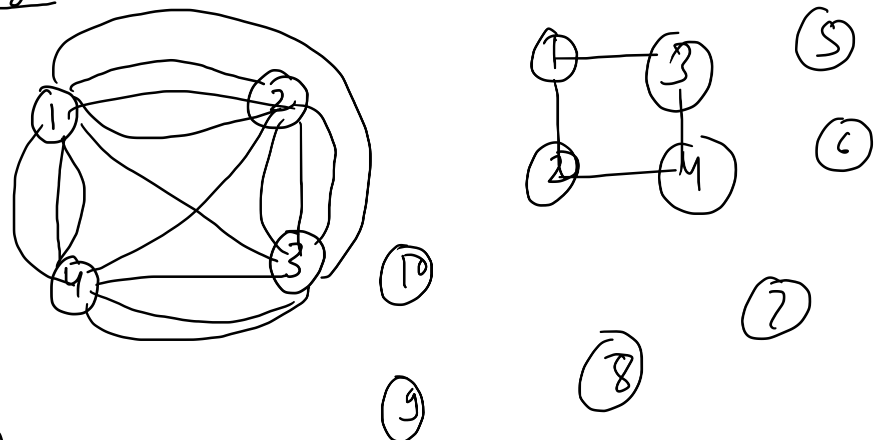
	self loop	‖el edge
Graph	✓	✓
multigraph	✗	✓
Pseudograph	✓	✗
simple graph	✗	✗

→ 95%

5) Finite graph → A graph G(V, E) is said to be finite graph if no of vertices as well as no of edges are finite.



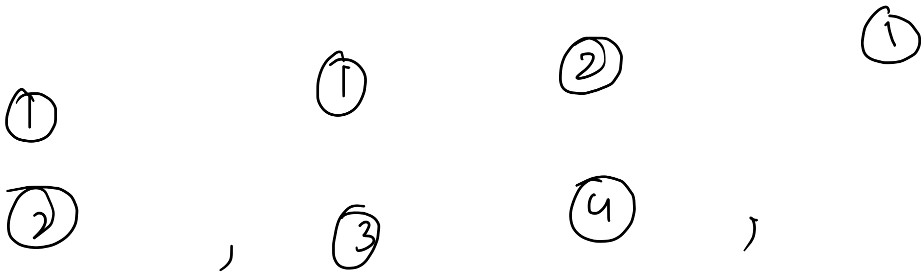
Infinite graph



(6) Null graph → A graph G(V, E) is said to be Null graph when vertex set is non Empty set while edge set is empty.

$$V = \{\text{non Empty}\}$$

$$E = \{\ \}$$



(7) Trivial graph → A Null graph is said to be trivial graph it consists only one vertex.

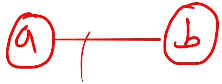
(a)

graph

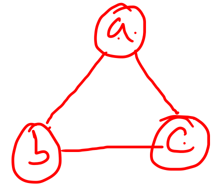
(8) complete graph → A graph G (V, E) is said to be complete graph when there is an edge between every pair of vertex and denoted by Kn   n ∈ no of vertex.

K_1

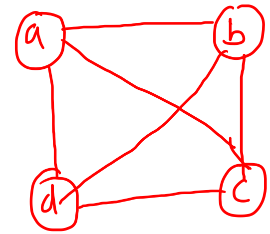
K_2



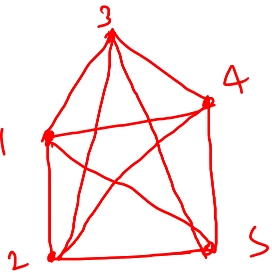
K_3



K_4



K_5



Properties →

K_n

Degree of vertex $\rightarrow (n-1)$

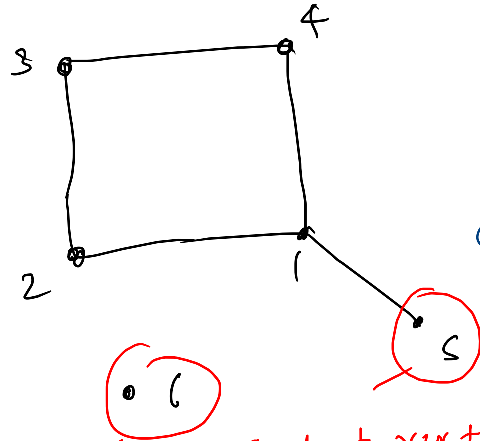
$(n-1) + (n-1) \; \cdots \; n$

Total no edge $\rightarrow \dfrac{n(n-1)}{2}$

$\Rightarrow {}^{n}C_2 \Rightarrow \dfrac{n(n-1)}{2}$

Hand shaking theorem

$$\sum_{i=1}^{m} d(v_i) = 2|E|$$



pendent vertex

isolated vertex

$d(1) = 3$ (o)

$d(2) = 2$

$d(3) = 2$

$d(4) = 2$

$\boxed{d(5) = 1}$ (o)

$\boxed{d(6) = 0}$

10 ✓

PT → Total no of odd degree vertices is even

$$\underbrace{\sum_{i=1}^{n} d(v_i)}_{\substack{2|E| \\ \text{even}}} = \underbrace{\sum_{\text{even}} d(v_i)}_{2 * \delta} + \underbrace{\sum_{\text{odd}} d(v_i)}_{2 \times E}$$

Even

* A graph contains 21 edges. and 3 vertices of degree 4. and all the other vertices of degree 2. findout total no of vertex. ?

$$|E| = 21$$

No of vertex = N

$$\sum_{i=1}^{n} d(v_i) = 2|E|$$

$$= 42$$

$$3 * 4 + 2(N-3) = 42$$

$$12 + 2N - 6 = 42$$

$$2N = 42 - 12 + 6$$

$$2N = 36 \quad N = 18$$ ✓

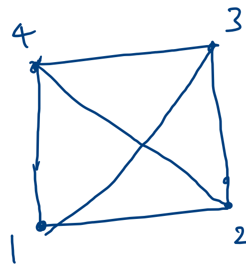
A Graph G has 24 edges and degree of each vertex is 4 and find out total no of vertex?

$$x \times 4 = 24 \times 2$$

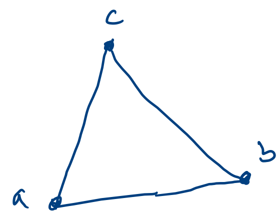
$$x = 12$$

Regular Graph → A graph in which all the vertices are of equal no of degree is called a regular graph. and denoted by '**k - regular**' where is degree of vertices.

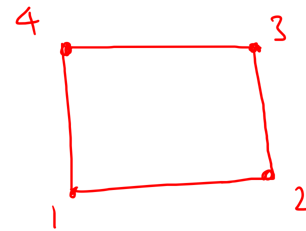
all the complete graph are (n-1) regular where n is no of vertex.



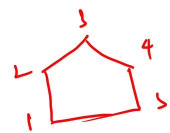
3 - Regular



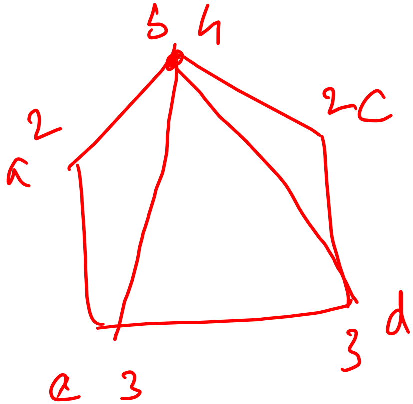
2 - Regular



2 - Regular



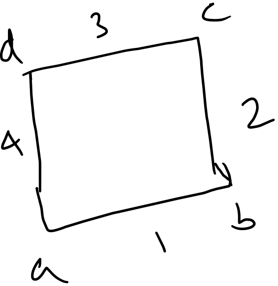
2 - Regular

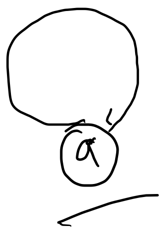


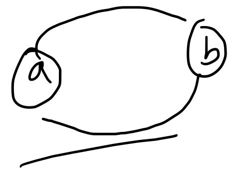
Not a regular graph

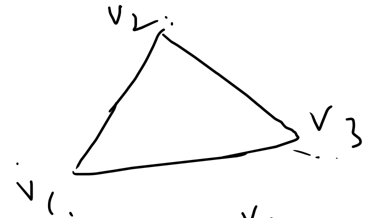
cyclic graph →

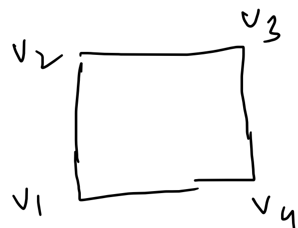
A graph consists of n vertices ($n \geq 3$) $v_1, v_2, v_3 \ldots v_n$ and edges (v_1, v_2) $(v_2, v_3) \ldots (v_{n-1}, v_n)$ and (v_n, v_1) is called a cyclic graph (C_n)

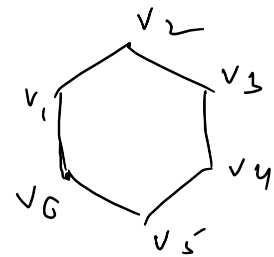








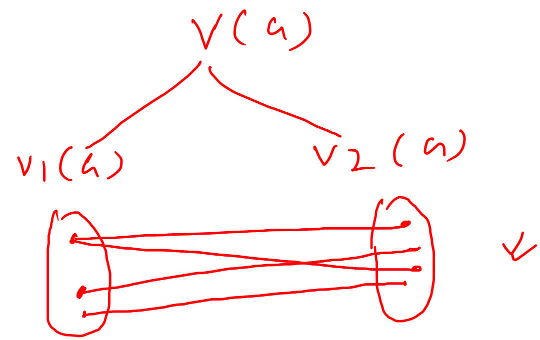


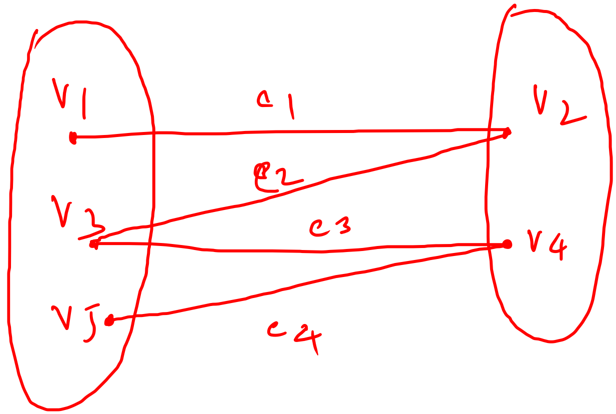


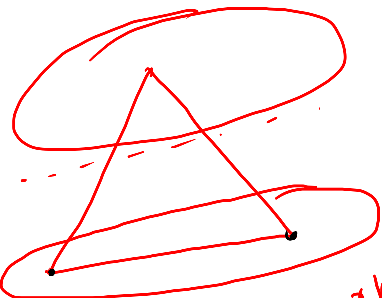
eg

Bipartite Graph —

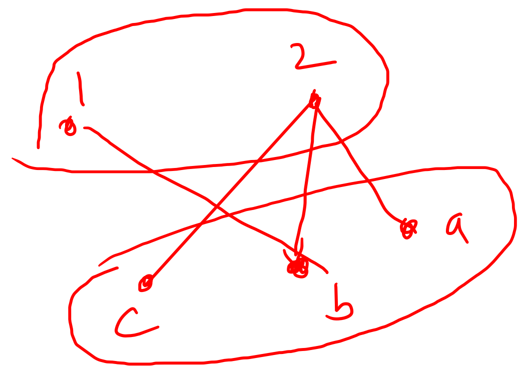
A Graph $G(V, E)$ is called Bi-partite if its vertex set $V(G)$ can be partitioned in to two non empty disjoint subset $V_1(G)$ & $V_2(G)$ in such a way that each edge $e \in E(G)$ has its one end point in $V_1(G)$ and another end point in $V_2(G)$, the partition $V = V_1 \cup V_2$ is called bi-partite Graph of G.

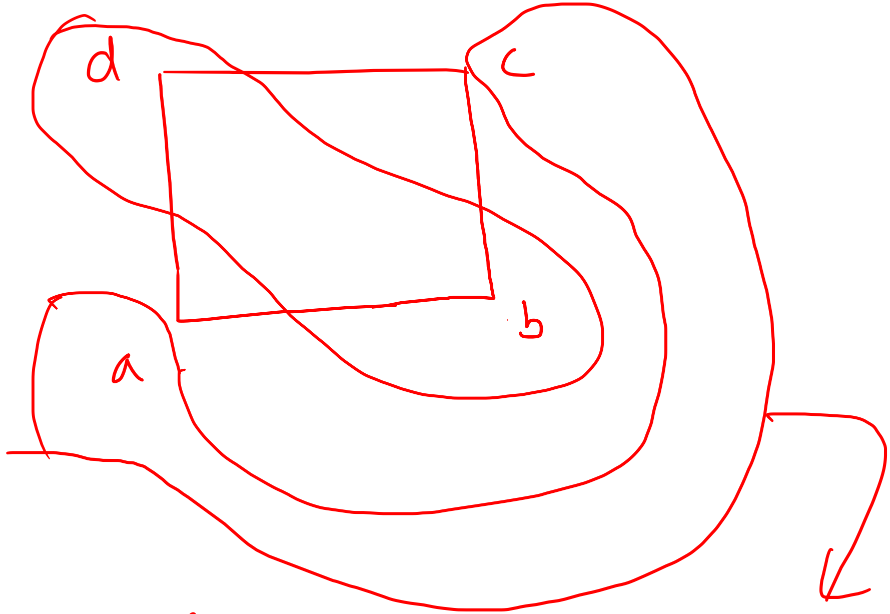




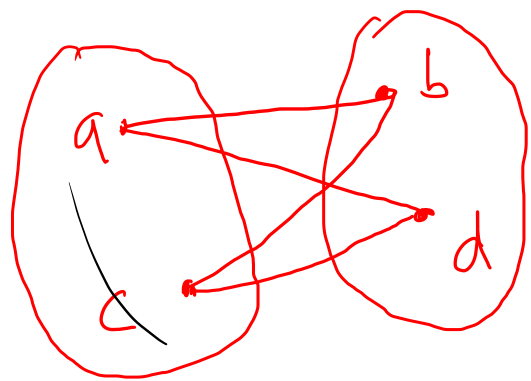


Not a bipartite Graph





✓ bipartite graph



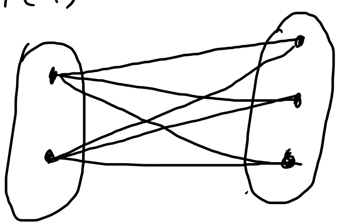
graph

Complete bi-partite graph – A bi-partite graph is said to be complete bi-partite graph if and only if each vertex of subset $V_1(G)$ is connected every other vertex of $V_2(G)$ by an edge $\in E(G)$. and denoted by $\underline{K_{m,n}}$

where m is vertex of group 1 and n is vertex of group 2

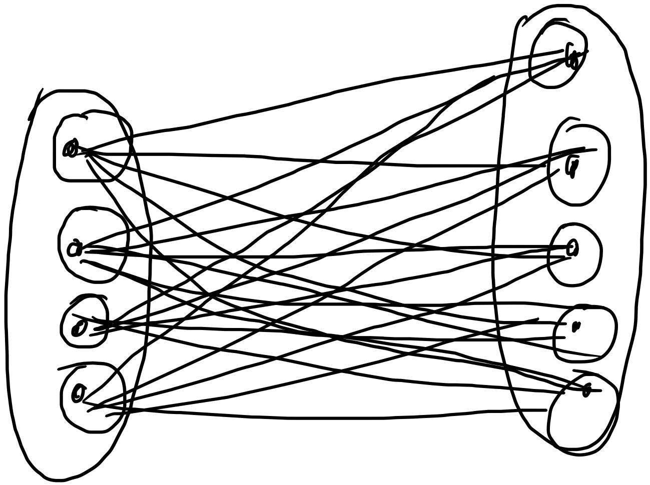
V

$V_1(G)$ $V_2(G)$

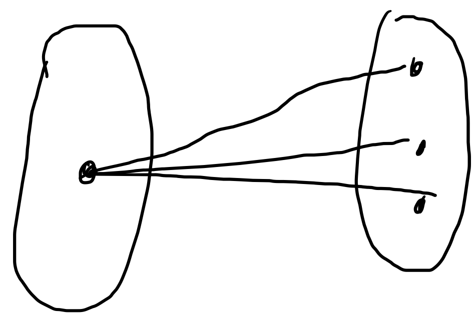


$\rightarrow$ $K_{2,3}$

$K_{4,5}$



K_{13}



i) **Walk** - A walk is defined as a finite alternative sequence of vertices and edges beginning and ending with vertex such that each edge is incident with vertices preceding and following it

open walk

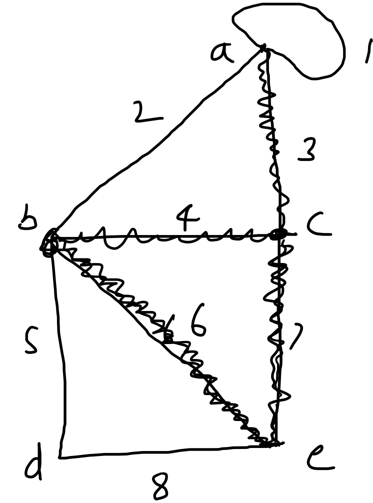
→ No edge appears more than on in a walk however a vertex may appear more than once.

ii) **Closed walk** - When a walk beginning and ending at the same vertex. it is called closed walk.

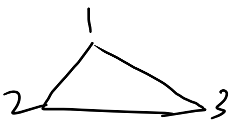
iii) **Path** → An open walk in which no vertex appears more than one is called path.

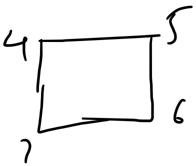
open walk → OW; closed walk → CW

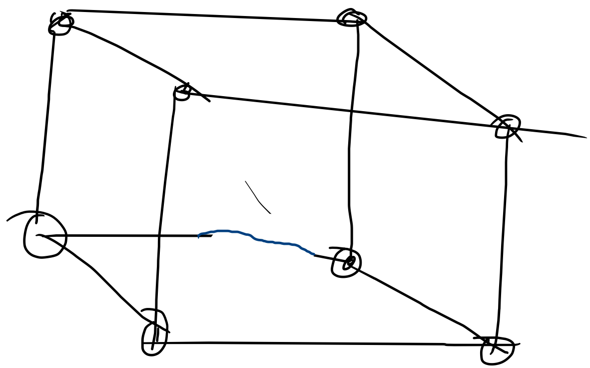
	OW	CW	Path
1) a 2 b 5 d 8 e	✓	✗	✓
2) a 2 b 6 e 8 d 5 b 4 c	✓	✗	✗
3) a 3 c 7 e 6 b 4 c 7 e 6 b	✗	✗	✗
4) a 1 a 3 c 7 e	✓	✗	✗
5) a 3 c 4 b 2 a	✓	✓	✗
6) a 3 c 7 e 6 b 4 c	✓	✗	✗



Connected graph → A graph is said to be connected when there a 'path' b/w every pair of vertex



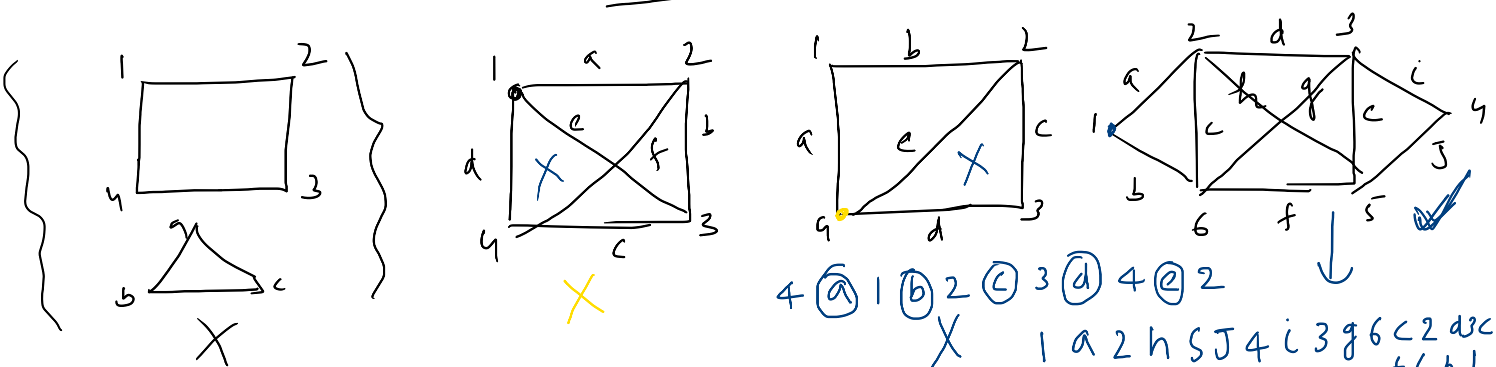




v.v.i.v
gmb

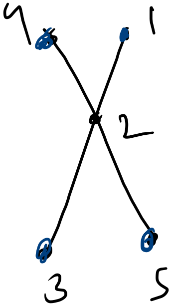
Euler circuit / cycle — A closed walk which visit "every edge of the graph exactly once".

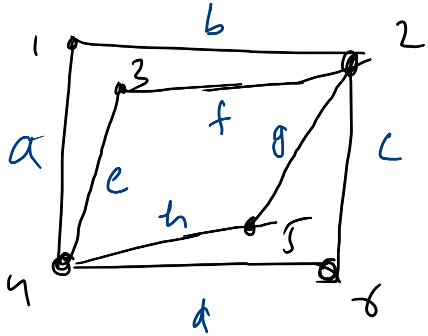
Euler graph → A graph 'G' which contains a euler circuit is called euler graph.



4 a 1 b 2 c 3 d 4 e 2
X

1 a 2 h 5 J 4 i 3 g 6 c 2 d 3 c 5 f 6 b 1

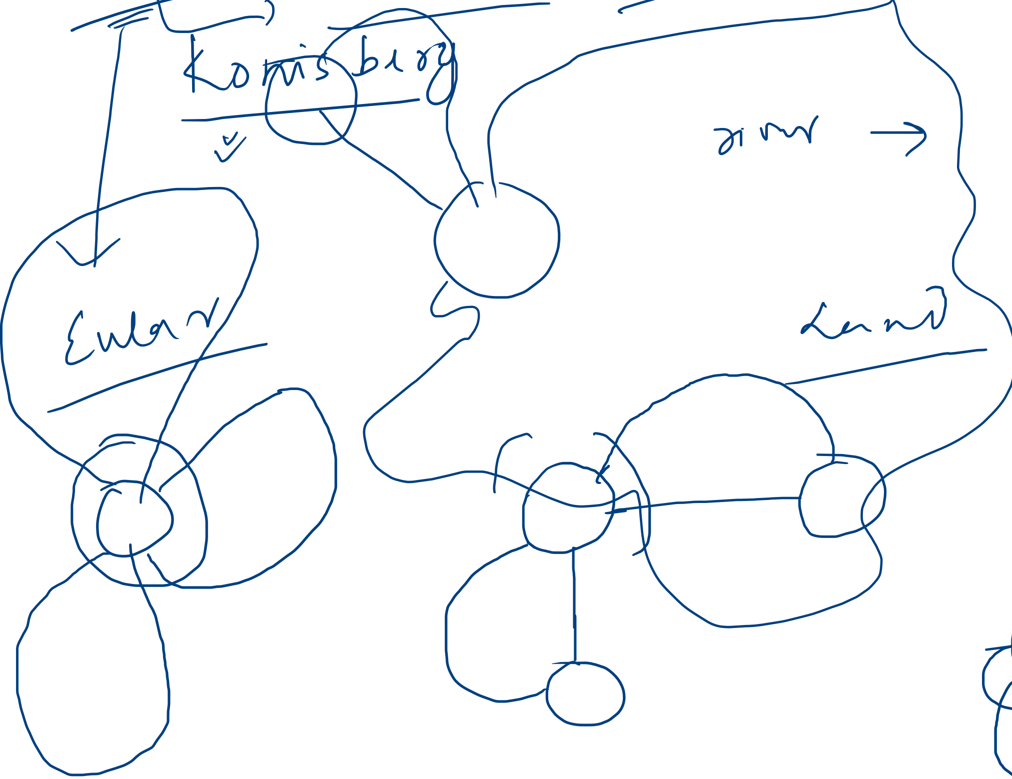


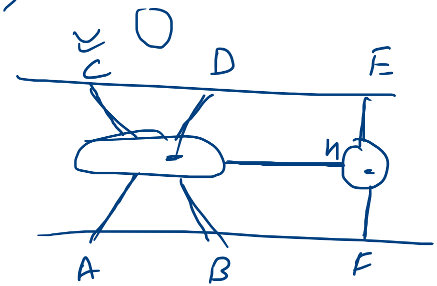


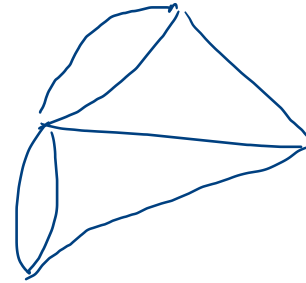
4 a 1 b 2 c 6 d 4 h 5 g 2 f 3

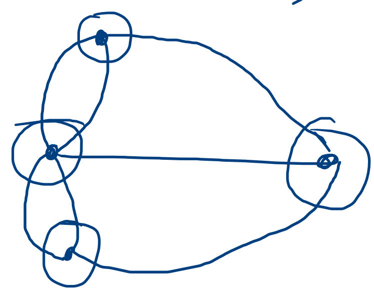
e 4

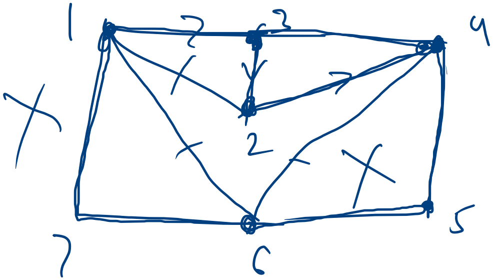
1736 1840 → Kant Land →

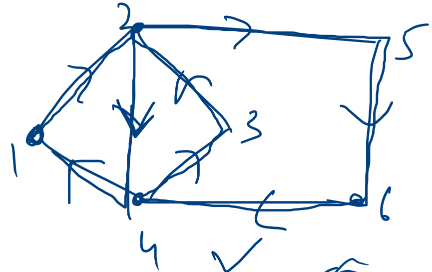


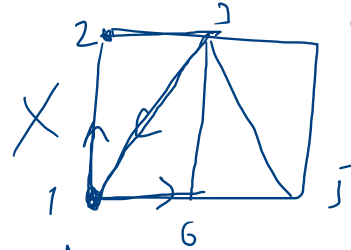


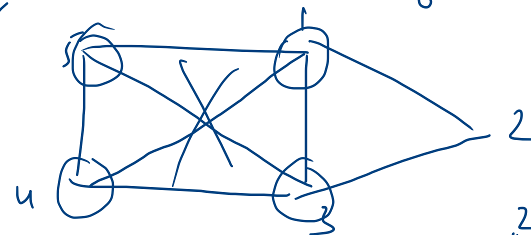


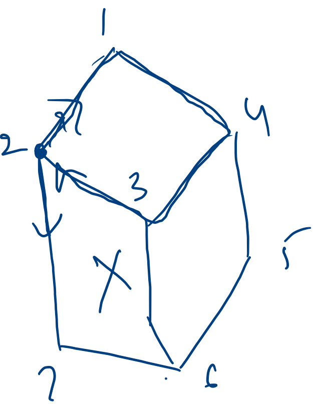


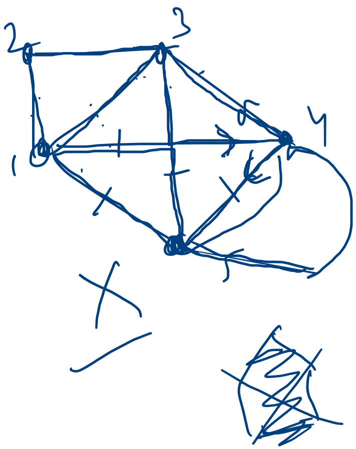


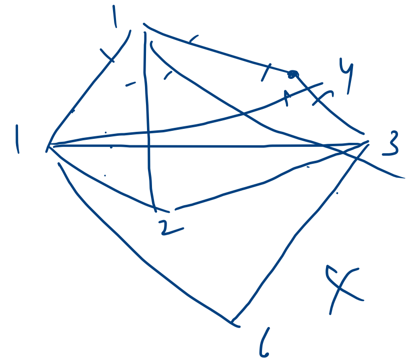


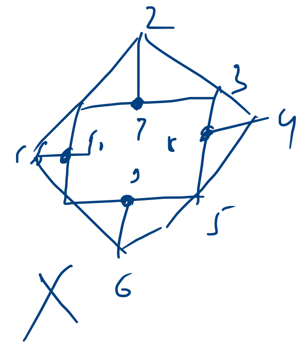




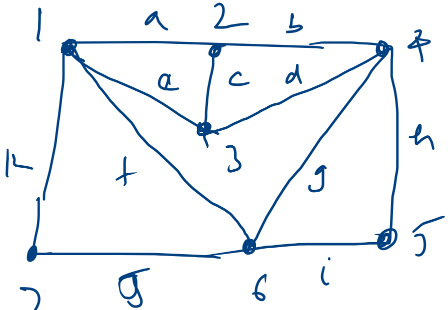








Hamiltonian Graph → G in a connected graph is defined as a closed walk that traverse every vertex of G exactly once, except and of course starting and ending vertex., that is called hamiltonian circuit and such graph is called Hamiltonian graph.

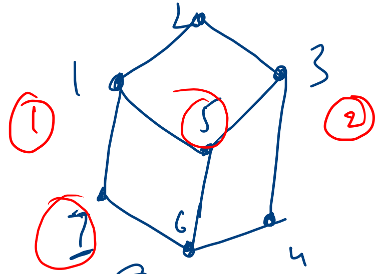


✓

1 a 2 c 3 d 4 h 5 i ~~6~~ 5 7

k 1

E ✗
H ✓

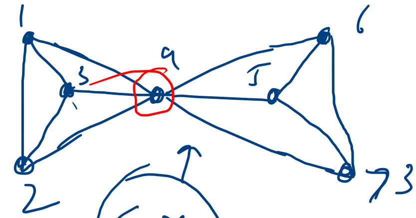


E ✗

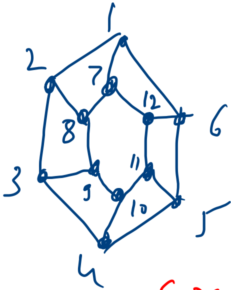
1 2 3 4 6 5 1

E ✗
H ✗

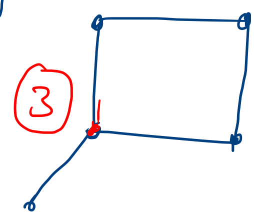
7 ✗



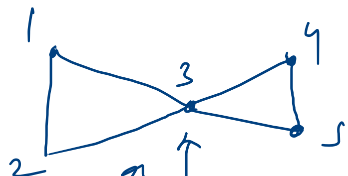
E ✗
H ✗



E ✗

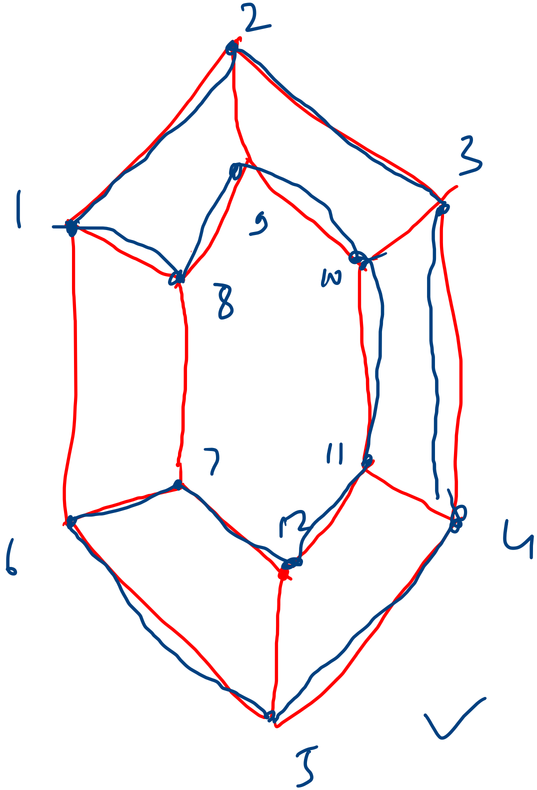


E ✗
H ✗



E ✓
H ✗

articulation point



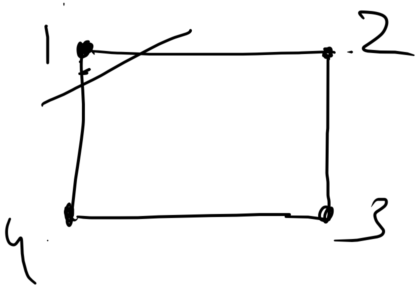
H V

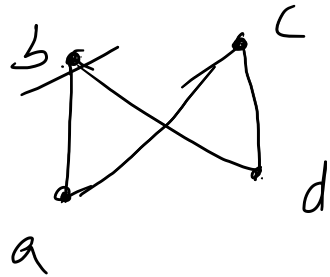
E X

Isomorphism – Two graph are said to be Isomorphic if they are perhaps the same graph just drawn differently with different name i.e they have identical behaviour for any graph-theoretic properties.

or

Two graph G_1 and G_2 are said to be isomorphic, if there is one to one correspondence b/w vertices and edges, and incidence relationship is preserved.





i) vertex same (no)

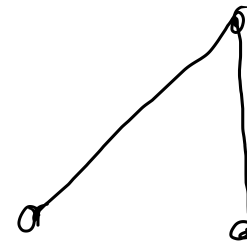
ii) edge same (no)

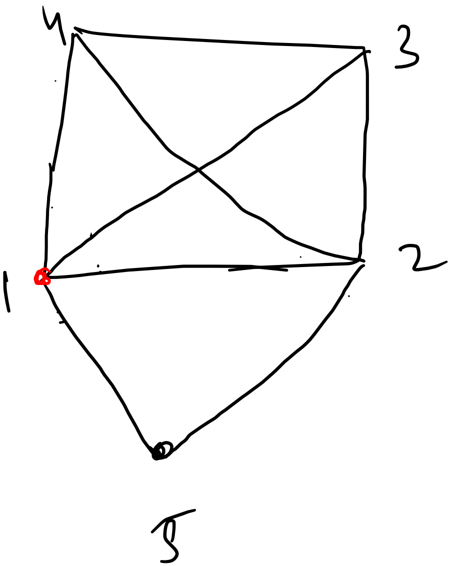
iii) degree same (total)

iv) cycle same (min & max)

v) degree arrangement same

vi) adjacency vertex & edges (same kind)





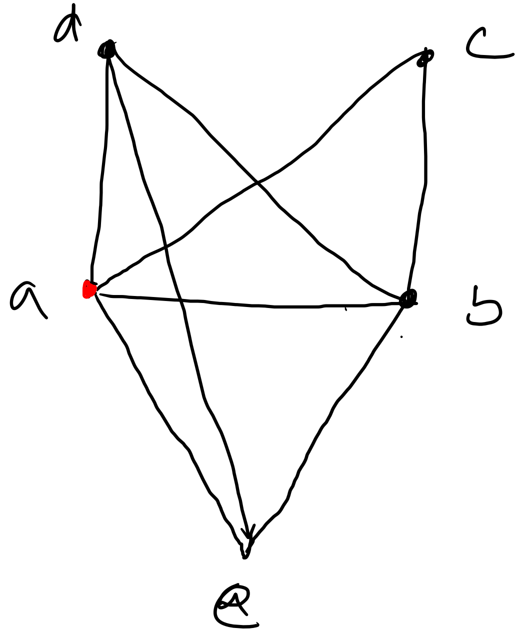
X

$d(2) - 1$

$d(4) \to 2$

$d(3) \to 2$

16

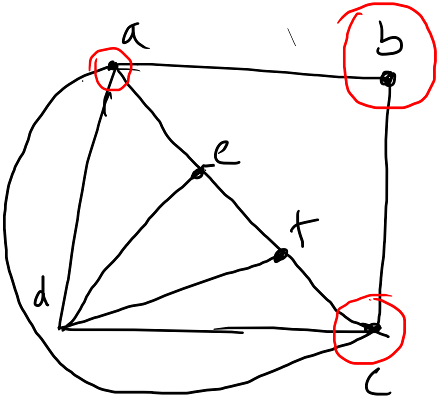


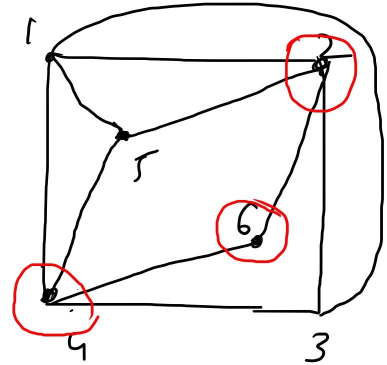
$d(2) = 1$

$d(4) - 2$

$d(3) \to 2$

16





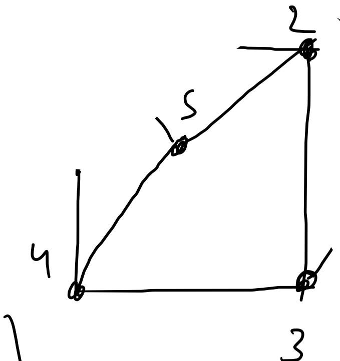
$|V| = 6$

$|E| = 10$

$d(2) = 1$

$d(3) = 2$

$d(4) = 3$



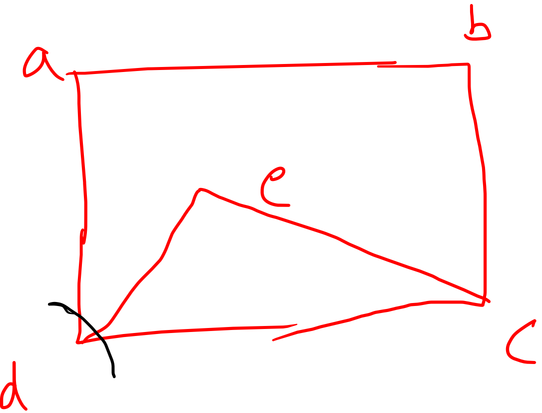
$|V| = 6$

$|E| = 10$

$d(2) = 1$

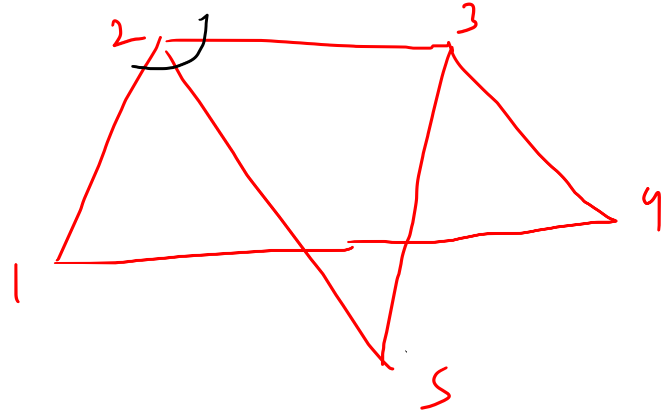
$d(3) = 2$

$d(4) = 3$



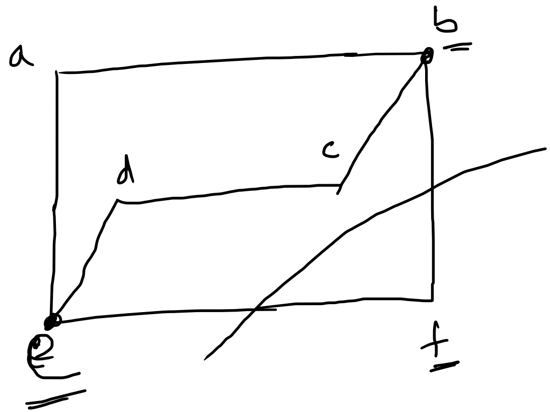
$|E| = 6$

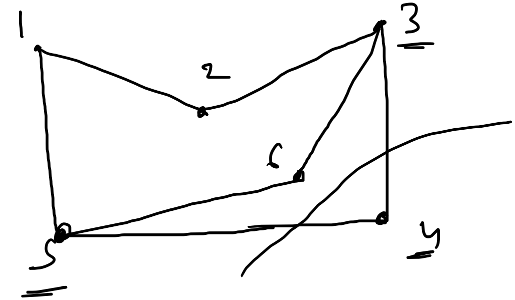
$|V| = 5$



$|E| = 6$

$|V| = 5$





f, 4 delu

$|E| = 7$

$|V| = 6$

$d(2) = 4$

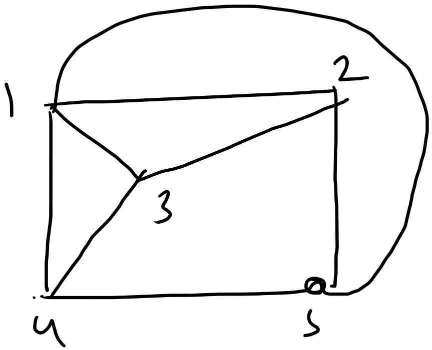
$d(3) = 2$

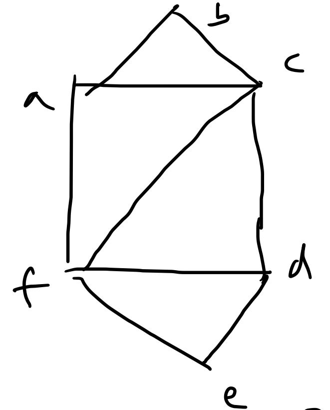
$|E| = 7$

$|V| = 6$

$d(2) = 4$

$d(3) = 2$





total no vertex - ?

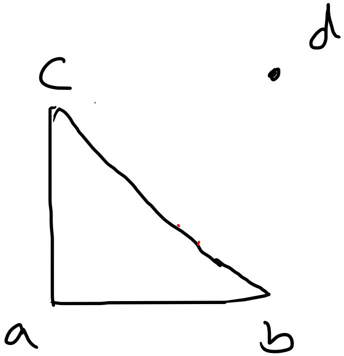
Not isomorphic

Complement of graph → complement of a simple graph G is a graph on the same set of vertex as of G, such that there will be an edge between two vertex (u, v) in $\bar{G}$ iff there is no edge between (u, v) in G.

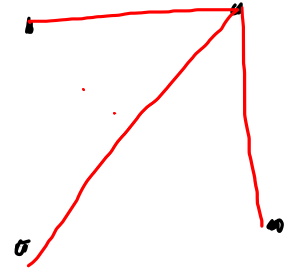
(u, v) are adjacent in $\bar{G}$, iff they are not adjacent in G

$$G(V, E)$$

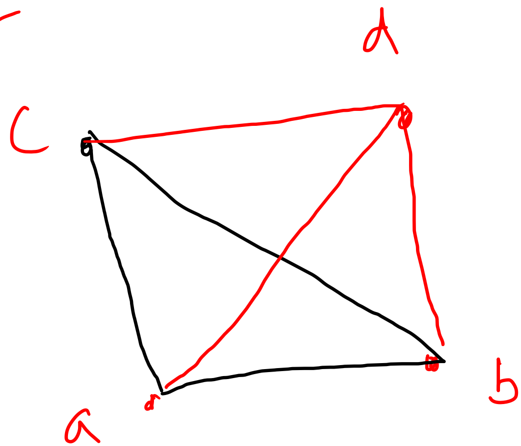
$$G(\bar{V}, \bar{E}) \longrightarrow \text{complement of graph}$$

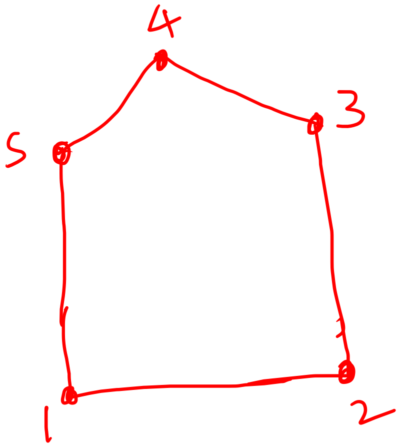


$G(V, E)$

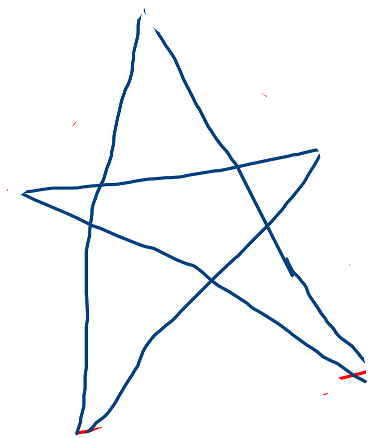


$\bar{G}(\bar{V}, \bar{E})$

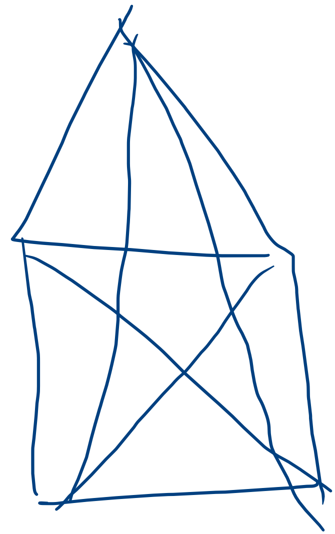


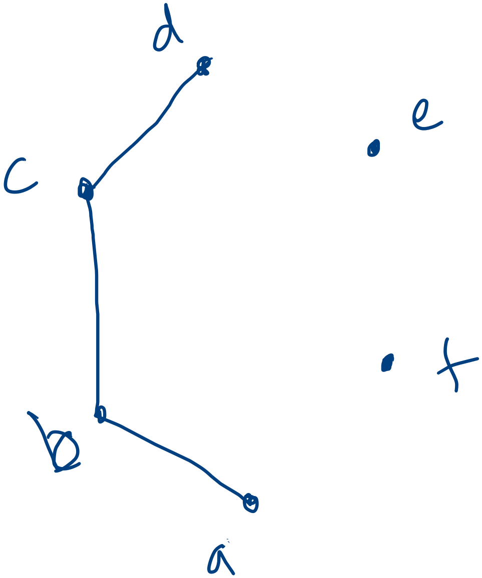


$\cup$

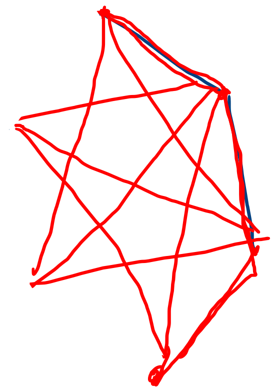


$\Rightarrow$





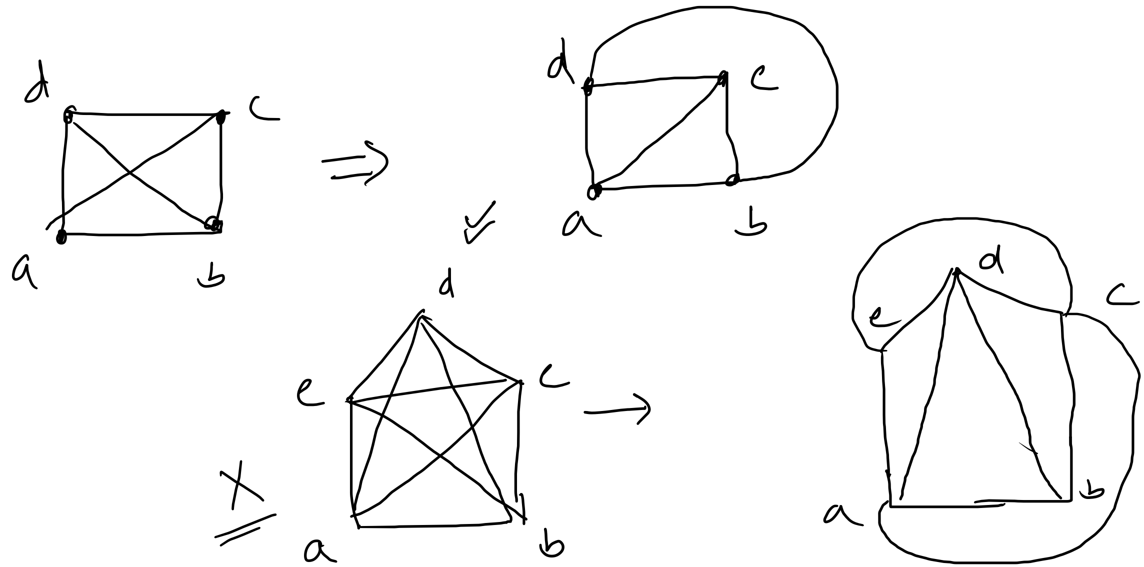
$G(V, \bar{E})$



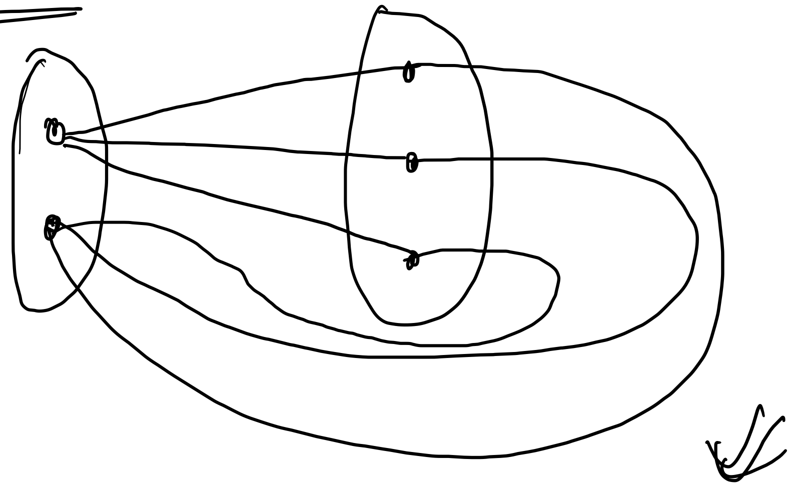
$\bar{G}(\bar{V}, \bar{E})$

Planar Graph

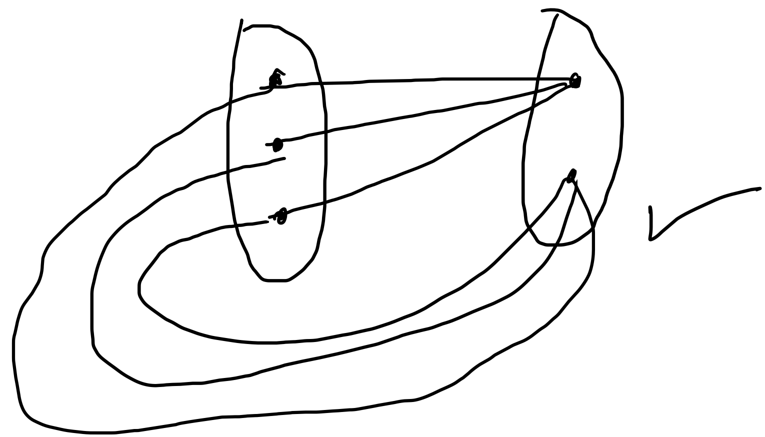
— A Graph that can be embedded in a single plane such that its edges intersect only at end points.



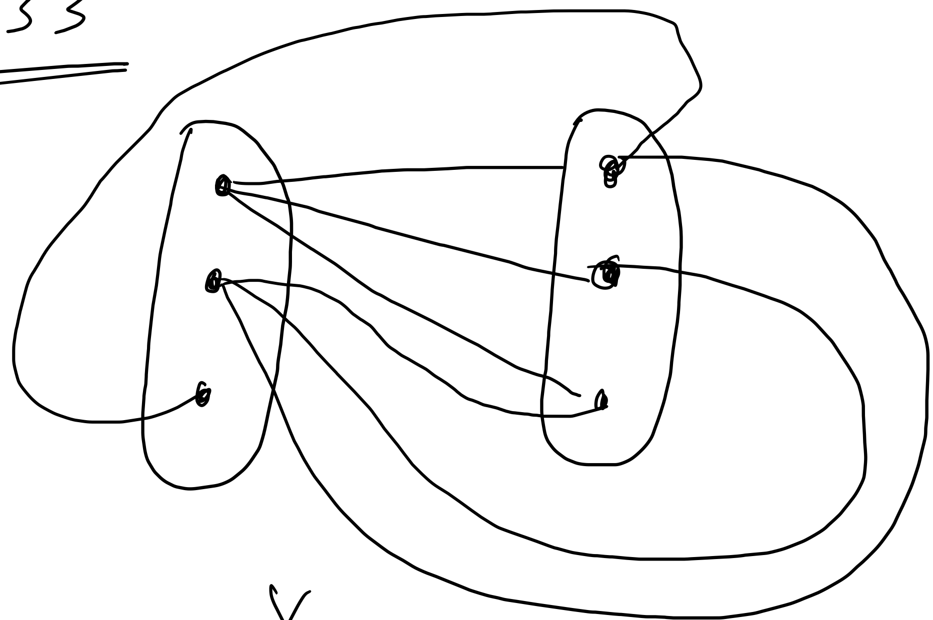
K_{23}



K_{32}



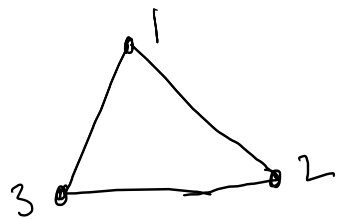
K_{33}



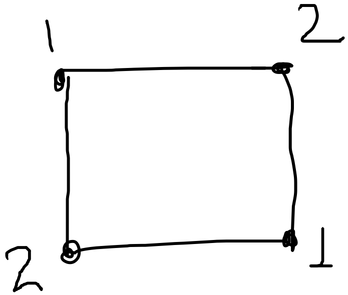
X

Imp coloring of graph. → cromatic number of graph

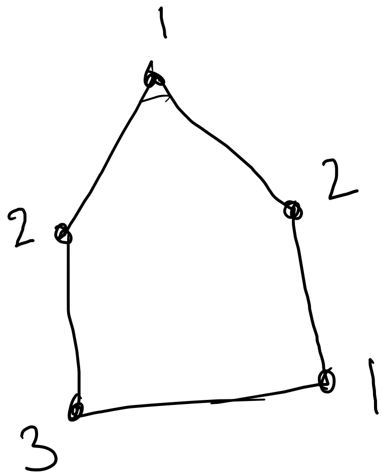
→ minimum no of color required to color every vertex of a graph such that no two adjucent vertex have same color.



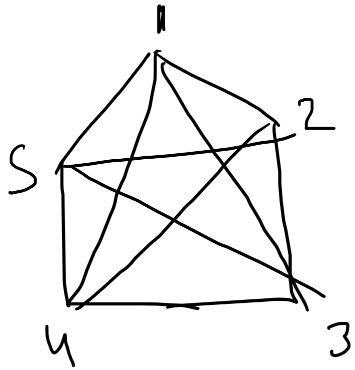
$\chi(G) = 3$



$$\chi(G) = 2$$



$$\chi(G) = 3$$



$\chi(G) = 5$

K_{10}

$\chi(G) = 10$

imp

Recurrence Relation

fib

0 1 1 2 3 5 8 13 ......

$n \geq 1$

$a_0 = 0$
$a_1 = 1$

$$a_n = a_{n-1} + a_{n-2}$$

$n=2$ $\quad a_2 = a_{2-1} + a_{2-2}$

$\Rightarrow a_1 + a_0$

$a_2 = 1 + 0$

$a_2 = 1$

$n=3$

$a_3 = a_2 + a_1$

$= 1 + 1$

$a_3 = 2$

$n=4$

$a_4 = a_3 + a_2$

$= 2 + 1$

$3, 6, 12, 24, 48, 96 \ldots\ldots$

$\boxed{a_0 = 3}$

$a_n = a_{n-1} + a_{n-}$

$n \geq 1$

$\boxed{a_n = 2 a_{n-1}}$

$n = 1 \quad a_1 = 2 a_{1-1}$

a_0

$a_1 = 6$

$n = 2$

$a_2 = 2 a_1$

6

$a_2 = 12$

Recurrance Relation

Homogeneous | non-homogeneous

i) when root are different $(\alpha_1, \alpha_2) \Rightarrow \boxed{a_r = A_1 \alpha_1^r + A_2 \alpha_2^r}$

ii) when root are same $(\alpha_1 = \alpha_2) \Rightarrow \boxed{a_r = A_1 \alpha^r + A_2 r \alpha^r}$

iii) when three roots are same $(\alpha_1 = \alpha_2 = \alpha_3) \Rightarrow a_r = A_1 \alpha^r + A_2 r \alpha^r + A_3 r^2 \alpha^r$

iv) when some root are same & some are different

$(\underline{\alpha_1}, \underline{\alpha_2 = \alpha_3})$ $a_r = A_1 \alpha_1^r + [A_2 \alpha_2^r + A_3 r \alpha_2^r]$

Non-homogeneous

Case 1 When we have constant term $\{2a_{r+2} - a_{r+1} - 6a_r = 4,\ 5,\ 6\}$

put $a_r = A \quad \forall r$

Case 2 Polynomial in r

r^2+1

$$\boxed{a_r^P = d_0 + d_1 r}$$

$$\boxed{a_r^P = d_0 + d_1 r + d_2 r^2}$$

r^3

$$\boxed{a_r^P = d_0 + d_1 r + d_2 r^2 + d_3 r^3}$$

Case 3 $\alpha^r \quad (\alpha \in I$

$$\boxed{a_r^P = P \cdot \alpha^r}$$

Homogeneous eqn

homogeneous $a_r - 5a_{r-1} + 6a_{r-2} = 0$

non homogeneous

$a_r - 5a_{r-1} + 6a_{r-2} = 6r$

$6r^2$

$6r^2 + 2$

$6 \cdot 2^r$

$a_r - 5a_{r-1} + 6a_{r-2} = 5$

Que (4) find the soln

$$a_r - 5a_{r-1} + 6a_{r-2} = 0 \quad ; \quad \boxed{a_0 = 2} \ \& \ \boxed{a_1 = 5}$$

→ characteristics eqn

$$\frac{a^r}{a^{r-2}} \quad \frac{a^r}{\frac{a^r}{a^2}} \quad a^2$$

$$5\left\{\frac{\frac{a^r}{a}}{\frac{a^r}{a^2}}\right\} \quad \frac{a^2}{a}$$

$$\frac{a_r}{a_{r-2}} - \frac{5a_{r-1}}{a_{r-2}} + \frac{6\cancel{a_{r-2}}}{\cancel{a_{r-2}}} = 0$$

$$\boxed{\alpha^2 - 5\alpha + 6 = 0}$$

$$\alpha^2 - 3\alpha - 2\alpha + 6 = 0$$

$$\alpha(\alpha - 3) - 2(\alpha - 3) = 0 \qquad \boxed{(\alpha - 2)(\alpha - 3) = 0}$$

$$\boxed{a_0 = 2 \quad a_1 = 5}$$

$$\boxed{\alpha = 2, 3}$$

$$\boxed{\alpha_1 = 2, \quad \alpha_2 = 3}$$

$$\boxed{a_r = A_1 \alpha_1^r + A_2 \alpha_2^r}$$

$$\boxed{a_r \Rightarrow A_1 2^r + A_2 3^r} \quad \text{---- (a)}$$

Put $r = 0$ in eq (a)

$$a_0 = A_1 2^{\cancel{0}\,1} + A_2 3^{\cancel{0}\,1}$$

$$\boxed{2 = A_1 + A_2} \quad \text{-------- (1)}$$

Put $r = 1$ in eq (a)

$$a_1 = A_1 2^1 + A_2 3^1$$

$$\boxed{5 = 2A_1 + 3A_2}$$

---- (2)

Sol eq (1) + (2)

$$\boxed{A_2 = 1}$$

$$\boxed{A_1 = 1}$$

$$a_r = 1 * 2^r + 1 * 3^r \Rightarrow \boxed{a_r = 3^r + 2^r} \quad r \geq 0$$

Ans

Exam 2

$$a_r - 6a_{r-1} + 9a_{r-2} = 0 \qquad a_0 = 1 \quad a_1 = 2$$

Step 1

Char n — divide a_{r-2}

$$\alpha^2 - 6\alpha + 9 = 0$$

$$\alpha^2 - 3\alpha - 3\alpha + 9 = 0$$

$$\alpha(\alpha - 3) - 3(\alpha - 3) = 0$$

$$(\alpha - 3)(\alpha - 3) = 0$$

root are same $\rightarrow$ $\boxed{\alpha_1 = \alpha_2 = 3}$

$$\boxed{a_r = A_1 \alpha^r + A_2 r \alpha^r}$$

$$a_r = A_1 3^r + A_2 r 3^r$$

$a_0 = 1$

$a_1 = 2$

$r = 0$

$$a_0 = A_1 + 0$$

$$\boxed{A_1 = 1}$$

$r = 1$

$$a_1 = 1 \times 3^1 + A_2 \times 1 \times 3^1$$

$$2 = 3 + 3A_2$$

$$A_2 = -1/3$$

$$\boxed{a_r = 3^r - \frac{r}{3} 3^r}$$

Answer

Q3 $a_r - 3a_{r-1} + 3a_{r-2} - a_{r-3} = 0$

$$\left\{\begin{aligned} a_0 &= 1 \\ a_1 &= 3 \\ a_2 &= 5 \end{aligned}\right.$$

Char^n divide a_{r-3}

$$\alpha^3 - 3\alpha^2 + 3\alpha - 1 = 0$$

$$(\alpha - 1)^3 = 0$$

$$\boxed{\alpha_1 = \alpha_2 = \alpha_3 = 1}$$

$$\boxed{a_r = A_1 \alpha^r + A_2 r \alpha^r + A_3 r^2 \alpha^r}$$

$$\boxed{a_r = A_1 + A_2 r + A_3 r^2}$$

put $r = 0, 1, 2$ respectively

Ques 4

$$a_r - 5a_{r-1} + 8a_{r-2} - 4a_{r-3} = 0$$

$a_0 = 1$
$a_1 = 2$
$a_2 = 3$

divide $\rightarrow$ a_{r-3}

charn $\quad \alpha^3 - 5\alpha^2 + 8\alpha - 4 = 0$

$$(\alpha-1)(\alpha-2)^2 = 0$$

$$\boxed{\alpha_1 = 1, \ \underbrace{\alpha_2 = \alpha_3}_{} = 2}$$

$$\boxed{a_r = A_1\alpha_1^r + \left[A_2\alpha_2^r + A_3 r\alpha_2^r\right]}$$

99.9 %

Qus

Non-homogenous -

$$a_{r+2} - a_{r+1} - 6a_r = 4$$

$$a_r = a_r^{h} + a_r^{p}$$

homogenous

Particular

Q

$a_{r+2} - a_{r+1} - 6a_r = 4$

$\left\{ \begin{array}{l} A_0 = 1 \\ A_1 = 2 \end{array} \right\}$

$$\boxed{a_r = a_r^h + a_r^p}$$

$a_r^h \Rightarrow a_{r+2} - a_{r+1} - 6a_r = 0$

Char$^n \rightarrow \alpha^2 - \alpha - 6 = 0$

$\alpha - 3\alpha + 2\alpha - 6 = 0$

$\alpha(\alpha - 3) + 2(\alpha - 3) = 0$

$(\alpha + 2)(\alpha - 3) = 0$

$\alpha = -2, 3$

$$a_r^h = A_1 \alpha_1^r + A_2 \alpha_2^r$$

$$a_r^h \Rightarrow A_1 (-2)^r + A_2 (3)^r$$

Put $r = 0$

$$a_0 = A_1 + A_2 \Rightarrow \boxed{A_1 + A_2 = 1} \quad \cdots (1)$$

Put $r = 0$

$$a_1 = -2A_1 + 3A_2 \Rightarrow -2A_1 + 3A_2 = 2 \quad - (11)$$

$$A_2 = \frac{4}{5}$$

$$A_1 = 1/5$$

$$\boxed{a_r^h = \frac{-2^r}{5} + \frac{4 \cdot 3^r}{5}}$$

$\underline{a_r^p}$ $\quad \underline{a_{r+2}} - \underline{a_{r+1}} - 6\underline{a_r} = 4$

4 as a constant term so

put $a_r = A \ \forall \ r$

$$A - A - 6A = 4$$

$$\boxed{A = -\frac{2}{3}}$$

$a_r = a_r^h + a_r^p \Rightarrow \frac{1}{5}\left((-2)^r + 4\cdot 3^r\right) - \frac{2}{3}$ Ans

Type 2

$$a_{r+2} - 4a_r = r$$

$a_0 = 0$
$a_1 = 1$

$$\boxed{a_r = a_r^h + a_r^p}$$

$a_r^h \Rightarrow a_{r+2} - 4a_r = 0$

Char^n $\alpha^2 - 4 = 0$

$$\boxed{\alpha = \pm 2}$$

$a_r^h = A_1 \cdot 2^r + A_2 (-2)^r \Rightarrow \boxed{a_r^h = \frac{1}{4}\left[2^r - (-2)^r\right]}$

Put $r = 0$ $\quad a_0 = A_1 + A_2 \Rightarrow \boxed{A_1 + A_2 = 0}$ --- ①

$r = 1$ $\quad a_1 = 2A_1 - 2A_2 \Rightarrow 2A_1 - 2A_2 = 1$ --- 2

$A_1 = 1/4, \; A_2 = -1/4$

$a_r^P \Rightarrow \quad a_{r+2} - 4a_r = r$

Put $\quad a_r^P = d_0 + d_1 r \quad \cdots \quad (1)$

$$[d_0 + d_1(r+2)] - 4[d_0 + d_1 r] = r$$

$$d_0 + d_1 r + 2d_1 - 4d_0 - 4d_1 r = r$$

$$\underbrace{d_0 + 2d_1 - 4d_0} + \underbrace{d_1 r - 4d_1 r} = r$$

$$-3d_0 + 2d_1 + r[-3d_1] = r + 0$$

equate coefficient at both side

$-3d_1 = 1 \Rightarrow d_1 = -\frac{1}{3}$

$-3d_0 + 2d_1 = 0 \Rightarrow d_0 = -\frac{2}{9}$

$$a_r^p = d_0 + d_1 r$$

$$a_r^p = -\frac{2}{9} - \frac{1}{3} r$$

$$a_r = a_r^h + a_r^p$$

$$\boxed{a_r \Rightarrow \frac{1}{4}\left(2^r - (-2)^r\right) - \left(\frac{2}{9} + \frac{1}{3} r\right)}$$

Q3

$$a_r + 5a_{r-1} + 6a_{r-2} = 3r^2$$

$a_0 = 0$
$a_1 = 1$

$$a_r = a_r^{h} + a_r^{p}$$

$a_r^h \Rightarrow \quad a_r + 5a_{r-1} + 6a_{r-2} = 0$

Char^n $\quad \alpha^2 + 5\alpha + 6 = 0$

$$\alpha_1 = -2 \quad \alpha_2 = -3$$

$$a_r^h = A_1(-2)^r + A_2(-3)^r$$

$A_1 = +1$
$A_2 = -1$

$$\boxed{a_r^h = (-2)^r - (-3)^r}$$

$a_r^P \Rightarrow \quad a_r + 5a_{r-1} + 6a_{r-2} = 3r^2$ --- ①

put $\boxed{a_r = d_0 + d_1 r + d_2 r^2}$ in eqn ①

$$d_0 + d_1 r + d_2 r^2 + 5[d_0 + d_1(r-1) + d_2(r-1)^2] + 6[d_0 + d_1(r-2) + d_2(r-2)^2] = 3r^2$$

$$d_0 + d_1 r + d_2 r^2 + 5[d_0 + d_1 r - d_1 + d_2 r^2 + d_2 - 2d_2 r] + 6[d_0 + d_1 r - 2d_1 + d_2 r^2 + 4d_2 - 4d_2 r] = 3r^2$$

$$d_0 + d_1 r + d_2 r^2 + 5d_0 + 5d_1 r - 5d_1 + 5d_2 r^2 + 5d_2 - 10d_2 r + 6d_0 + 6d_1 r - 12d_1 + 6d_2 r^2 + 24d_2 - 24d_2 r = 3r^2$$

$$12d_0 - 17d_1 + 29d_2 + r(12d_1 - 34d_2) + 12d_2 r^2 = 3r^2$$

Compare cofficient

$$12 d_2 = 3 \quad \text{---} \quad \text{I}$$

$$\boxed{d_2 = 1/4}$$

$$12 d_1 - 34 d_2 = 0 \quad \text{---} \quad \text{II}$$

$$\boxed{d_1 = \frac{17}{24}}$$

$$12 d_0 - 17 d_1 + 29 d_2 = 0 \quad \text{---} \quad \text{III}$$

$$\boxed{d_0 = \frac{\cancel{15}}{\cancel{24}}}$$

$$d_0 = \frac{115}{288}$$

$$12 d_0 - 17 d_1 + 29 d_2 = 0$$

$$12 d_0 - 17 \times \frac{17}{24} + \frac{29}{4} = 0$$

$$12 d_0 = \frac{17 \times 17}{24} - \frac{29}{4}$$

$$= \frac{289 - 29 \times 6}{24}$$

$$d_0 = \frac{115}{24 \times 12}$$

$$\boxed{a_r^p = \frac{115}{288} + \frac{17}{24} r + \frac{1}{4} r^2}$$

$$a_r = a_r^h + a_r^p$$

$$\boxed{a_r \Rightarrow 2^r - (-3)^r + \frac{115}{288} + \frac{17}{24} r + \frac{1}{4} r^2}$$

Q: $a_r - 7a_{r-1} + 10a_{r-2} = 6 + 8r$

Sol

$$\boxed{a_r = a_r^h + a_r^p}$$

$a_r^h = \alpha^2 - 7\alpha + 10 = 0$

$$\boxed{a_r^h = A_1 2^r + A_2 5^r}$$

a_r^p $a_r = d_0 + d_1 r$

$d_0 + d_1 r - 7(d_0 + d_1(r-1)) + 10(d_0 + d_1(r-2)) = 6 + 8r$

✓

Type Q

$$a_r + 5a_{r-1} + 6a_{r-2} = 42 \cdot 4^r \; ---$$

a_r^{h}

$$\alpha^2 + 5\alpha + 6 = 0$$

$$\alpha = -2, -3$$

$$\boxed{a_r^{h} = A_1(-2)^r + A_2(-3)^r}$$

a_r^{p}

$$\text{Let } a_r = \underline{P \cdot \alpha^r} \Rightarrow P \cdot 4^r$$

$$a_r + 5a_{r-1} + 6a_{r-2} = 42 \cdot 4^r$$

$$P \cdot 4^r + 5 \cdot P \cdot 4^{r-1} + 6 \cdot P \cdot 4^{r-2} = 42 \cdot 4^r$$

$$p \cdot 4^r + \frac{5 \cdot p \cdot 4^r}{4} + \frac{6 p 4^r}{4^2} = 42 \cdot 4^4$$

$$4^r \left[p + \frac{5p}{4} + \frac{6p}{4^2} \right] = 42 \cdot 4$$

compare coeffs

$$p + \frac{5p}{4} + \frac{6p}{16} = 42$$

$$\boxed{p = 16}$$

$$a_r^p = 16\, 4^r$$

$$a_r = a_r^{h} + a_r^{p}$$

$$\boxed{a_r = A_1(-2)^r + A_2(-3)^r + 16.4^r}$$

Q

$$a_r - 5a_{r-1} + 6a_{r-2} = 2^r + r \quad r \geq 2$$

$$a_r = a_r^h + a_r^p$$

$$a_r^h \Rightarrow \alpha^2 - 5\alpha + 6 = 0$$

$$\alpha = 2, 3$$

$$\boxed{a_r^h = A_1 2^r + A_2 3^r}$$

$$a_r^p = ?$$

$$a_r = \underline{p_1 \cdot r} \cdot 2^r + \underline{p_2} + p_3 r$$

$$a_r - 5a_{r-1} + 6a_{r-2} = 2^r + r$$

$$(p_1 r 2^r + p_2 + p_3 r) - 5\left(p_1 (r-1) 2^{r-1} + p_2 + p_3(r-1)\right)$$
$$+ 6\left(p_1 (r-2) 2^{r-2} + p_2 + p_3(r-2)\right)$$
$$= 2^r + r$$

$$p_1 r 2^r + p_2 + p_3 r - 5\left(\frac{p_1}{2}(r \cdot 2^r - 2^r) + p_2 + p_3 r - p_3\right)$$
$$+ 6\left(\frac{p_1}{4}(r \cdot 2^r - 2 \cdot 2^r) + p_2 + p_3 r - 2p_3\right) = \underline{\cancel{2^r}} + r$$

Q

$$a_r - 4a_{r-1} + 4a_{r-2} = (r+1)2^r$$

$$\boxed{a_r = (P_0 + P_1 r) r^2 2^r}$$

Generating func

$$A(z) = a_0 + a_1 z + a_2 z^2 + a_3 z^3 - - -$$

$$+ a_r z^{\cancel{r}} - - - a_n z^n$$

$r < n$

$$\boxed{A(z) = \sum_{i=0}^{n} a_i z^i}$$

$$a_0 z^0 + a_1 z^1 + a_2 z^2 - - -$$

$$(1 - az)^{-1} \Rightarrow$$

$$\Rightarrow 1 + (-1)(-az) + \frac{(-1)(-2)}{\lfloor 2}(-az)^2$$

$$+ \frac{\cancel{(-1)(-2)(-3)}}{\cancel{\lfloor 3}}(-az)^3 \; - - -$$

$$\Rightarrow 1 + az + a^2 z^2 + a^3 z^3 \; - - -$$

$$\Rightarrow \sum_{i=0}^{n} a^i \cdot z^i$$

Calculate generating function

$a_n - 7a_{n-1} + 100\, a_{n-2} = 0$ --- (1)

for all $n > 1$ and $a_0 = 3$ $a_1 = 3$

what will be generating func

$$\boxed{g(n) = \sum_{n=0}^{\infty} a_n x^n}$$

$\Rightarrow \sum_{n=2}^{\infty} a_{n-2}\, x^{n-2}$

multiply eq (1) with x^n in both side

$a_0 x^0 + a_1 x^1$

$a_0 + a_1 x$

$$a_n x^n - 7a_{n-1} x^n + 10\, a_{n-2} \cdot x^n = 0 \cdot x^n$$

$$\underline{a_n x^n} - 7\, a_{n-1}\, x^n + 10\, a_{n-2} \cdot x^n = 0$$

$$\sum_{n=2}^{\infty} a_n x^n - 7 \sum_{n=2}^{\infty} a_{n-1} x^n + 10 \sum_{n=2}^{\infty} a_{n-2} \cdot x^n = 0$$

$$\left[\sum_{n=0}^{\infty} a_n x^n - a_0 - a_1 x\right] - 7 \cdot x \left[\sum_{n=1}^{\infty} a_{n-1} x^{n-1} - a_0\right] + 10 \cdot x^2 \left[\sum_{n=2}^{\infty} a_{n-2} x^{n-2}\right] = 0$$

$$\left(g(x) - a_0 - a_1 x\right) - 7 \cdot x\left[g(x) - a_0\right]$$

$$+ 10 x^2 \left[g(x)\right] = 0$$

$$a_0 = 3 \quad a_1 = 3$$

$$g(x) - \underline{a_0} - \underline{a_1 x} - 7x\, g(x) + \underline{7 x a_0}$$

$$+ 10 x^2 g(x) = 0$$

$$g(x)\left[1 - 7x + 10 x^2\right] = a_0 + a_1 x - 7 x a_0$$

$$= 3 + 3x - 21x$$

$$= 3 - 18x$$

$$g(x) = \frac{3 - 18x}{10x^2 - 7x + 1}$$

$$g(x) = \frac{3 - 18x}{(5x-1)(2x-1)}$$

$$g(x) = \frac{3-18x}{(5x-1)(2x-1)} \Rightarrow \frac{A}{(5x-1)} + \frac{B}{(2x-1)}$$

$$\frac{3-18x}{\cancel{(5x-1)(2x-1)}} = \frac{A(2x-1) + B(5x-1)}{\cancel{(5(x)-1)(2x-1)}}$$

$$3 - 18x = A(2x-1) + B(5x-1)$$

Put $x = \frac{1}{5}$

$$3 - \frac{18}{5} = A\left(\frac{2}{5} - 1\right)$$

$$\frac{15-18}{\cancel{5}} = A\left(\frac{2-5}{\cancel{5}}\right)$$

$$-3 = A(-3)$$

$$A = 1$$

Put $x = \frac{1}{2}$

$$3 - \frac{18}{2} = 0 + B\left(\frac{5}{2} - 1\right)$$

$$\frac{6-18}{\cancel{2}} = B\left(\frac{5-2}{\cancel{2}}\right)$$

$$-12 = 3B$$

$$B = -4$$

$$\Rightarrow \frac{1}{(5x-1)} - \frac{4}{(2x-1)}$$

$$\Rightarrow \frac{4}{(1-2x)} - \frac{1}{(1-5x)}$$

$$\Rightarrow 4(1-2x)^{-1} - (1-5x)^{-1}$$

$$4\left(\sum_{n=0}^{\infty} 2^n x^n\right) - \sum_{n=0}^{\infty} 5^n x^n \quad // \text{Ans}$$