APPLICATIONS OF PARTIAL DIFFERENTIAL EQUATIONS

QUESTION BANK

Classify the following Partial Differential Equations

1. $2\frac{\partial^2 z}{\partial x^2} - 3\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} - 3\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$. (2015,2 Marks)
2. $u_{xx} + 3u_{xy} + u_{yy} = 0$(2014,2 Marks)
3. $\frac{\partial^2 z}{\partial x^2} + 3\frac{\partial^2 z}{\partial x \partial y} + 2\frac{\partial^2 z}{\partial y^2} = 0$. (2015,2 Marks)
4. $yu_{xx} + (x+y)u_{xy} + xu_{yy} = 0$(2014,2 Marks)
5. $f_{xx} + 2f_{xy} + 4f_{yy} = 0$(2014,2 Marks)
6. Show that the equation $\frac{\partial^2 z}{\partial x^2} + 2x\frac{\partial^2 z}{\partial x \partial y} + (1-y^2)\frac{\partial^2 z}{\partial y^2} = 0$ is elliptic for all values of x and y in the region $x^2 + y^2 < 1$, parabolic on boundary and hyperbolic outside the region.
7. $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{1}{c^2}\frac{\partial u}{\partial t}$. (2011,2 Marks)
8. Classify the following differential equations as to type in second quadrant of x y plane

   $\sqrt{y^2 + x^2}\, u_{xx} + 2(x-y)u_{xy} + \sqrt{y^2 + x^2}\, u_{yy} = 0$ (2014, 2 Marks)
9. $x^2\frac{\partial^2 z}{\partial t^2} - 3\frac{\partial^2 z}{\partial x \partial t} + x\frac{\partial^2 z}{\partial x^2} + 17\frac{\partial z}{\partial t} = 100z$.

Using the method of separation of variables solve from question from 1 to 15.

1. $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$ (UPTU 2005,9,12,14, 5 Marks)
2. $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u,\ where\ u(x,0) = 6e^{-3x}$. (UPTU 2006,10,11,15)
3. $3\frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y} = 0$,given $u(x,0) = 4e^{-x}$
4. $\frac{\partial z}{\partial x} + \frac{\partial^2 z}{\partial y} = 0$, z(x, 0) = 0, z(x, π) = 0, z(0,y) = 4sin 3y. (UPTU 2010, 12, 14, 10 Marks)
5. $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial y}$ + 2u, u (0, y) = 0 and $\frac{\partial}{\partial x}u(0,y) = 1 + e^{-3y}$. (UPTU 2009, 10,13,5 Marks)
6. $\frac{\partial u}{\partial x} = 4\frac{\partial u}{\partial y}$ where u(0,y)= $8e^{-3y}$. (UPTU 2008)
7. $x\frac{\partial^2 \mathrm{u}}{\partial \mathrm{x}^2} + 2\mathrm{y}\ \mathrm{u} = 0$. (UPTU 2015,5 Marks)
8. $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$. (UPTU 2015,5 Marks)

9. $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=3u$,given $u(0,y)=4e^{-y}-e^{-5y}$ (UPTU 2010,10 Marks)

10. $\frac{\partial^2 z}{\partial x^2}=e^{-x}\cos y$ given that z = 0 when x = 0 and $\frac{\partial z}{\partial x}=0$,when y = 0. (UPTU 2012,5 Marks)

11. $\frac{\partial u}{\partial x}=\frac{\partial u}{\partial y}$ (UPTU 2012,2 Marks)

12. $y^3\frac{\partial u}{\partial x}+x^2\frac{\partial u}{\partial y}=0.$ (UPTU 2011,5 Marks)

13. $\frac{\partial^2 \mathrm{u}}{\partial \mathrm{x}^2}-\frac{\partial \mathrm{u}}{\partial \mathrm{y}}=0.$ (UPTU 2007,10 Marks)

14. $\frac{\partial^2 u}{\partial x\partial t}=e^{-t}\cos x$ given that u = 0 when t = 0 and $\frac{\partial u}{\partial t}=0$,when x = 0. (UPTU 2009,5 Marks)

15. $4\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=3u$,given $u(0,y)=3e^{-y}-e^{-5y}$.

16. Write two dimensional wave equation. (UPTU 2012,2 Marks)

17. For obtaining the displacement of a finite string of length L that is fixed at both ends and released from rest with initial displacement f(x), write all the conditions used in it. (UPTU 2014 CO 2 Marks)

18. Find the displacement of the string of length L that is fixed at both ends and released from rest with an initial displacement f(x) (UPTU 2015,5 Marks)

19. Find the deflection of the vibrating string which is fixed at the ends x= 0 and x = 2 and the motion is started by displacing the string into the form $sin^3\left(\frac{\pi x}{l}\right)$ and releasing it with zero initial velocity.

(UPTU 2010,12,14,10 Marks)

20. Find the deflection of vibrating string of unit length, whose end points are fixed and if initial velocity is zero and initial deflaction is given by $u(x,0)=\begin{cases} 1, & 0\le x\le\frac{1}{2} \\ -1, & \frac{1}{2}\le x\le 1\end{cases}$ (UPTU 2012,14,10 Marks)

21. A transversely vibrating string of length l is strecthced between two points A and B. the initial displacement of each point of string is zero and the initial velocity at a distance x from A is kx(l-x), k being constant. Find the form of the string at any subssequent time. (UPTU 2011CO,10 Marks)

22. Find the deflection of vibrating string (length $l=\pi$, ends fixed and c^2=1) corresponding to zero initial velocity and initial deflection $f(x)=k(\sin x-\sin 2x)$.

23. A string is stretched and fastened to two points at a distance l apart. Motion is started by displacing the string in the fom y = a sin $(\frac{\pi x}{l})$ from which it is eleased at time t= 0. Show that the displcement of any point at a distance x from one end at time t is y(x,t) = a sin $\frac{\pi x}{l}$ cos $\frac{\pi ct}{l}$. (UPTU 2004, 09,11,12,10 Marks)

24. Solve completely the equation $\frac{\partial^2 y}{\partial t^2}=c^2\frac{\partial^2 y}{\partial x^2}$, representing the vibrations of string of length l ,fixed at both ends, given that y(0,t)=0,y(x,0)=f (x) and $\frac{\partial}{\partial t}y(x,0)=0.$ (UPTU 2005,10Marks)

25. A String is stretched between the fixed points (0,0) and (l,0) and released at rest from the initial deflection given by $\begin{cases} \frac{2kx}{l}, & 0 \le x \le \frac{l}{2} \\ \frac{2k(l-x)}{l}, & \frac{l}{2} \le x \le l \end{cases}$ Find the deflection of the string at any time t.

26. A taunt string of length l has its ends x = 0 and x = l fixed. The point where x = l/3 is drawn a side a small distance 'b' and released from rest in that position. The displacement y(x,t) satisfies $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$.determine the displacement.

27. Find the deflection u(x,y,t) of the tightly stretched rectngular membrane with sides a and b having wave velocity c =1.if the initial velocity is zero and the initial deflection is $f(x,y) = a\, sin\left(\frac{2\pi x}{a}\right) sin\left(\frac{3\pi y}{b}\right)$.

28. Find the deflection u(x,y,t) of the tightly stretched rectngular membrane with a = b = c = 1.if the initial velocity is zero and the initial deflection is $f(x,y) = A\, sin\pi x\, sin2\pi y$.

29. Find the deflection u(x,y,t) of the tightly stretched rectengular membrane with sides a and b whose boundary is fixed, given that it starts starts from rest $u(x,y,0) = xy(1-x)(2-y)$

30. Solve the one-dimensional heat equation $\frac{\partial^2 y}{\partial x^2} = \frac{1}{c^2}\frac{\partial u}{\partial t}$, under the suitable conditions. (UPTU 2007,11)

31. Find the steady state temperature didistribution in a plate of length of 20 where ends are kept at 40 °C and 100 °C respectively. (UPTU 2015,2 Marks)

32. Find the steady state temperature distribution in a rod of length 20 cm, whose ends are kept at zero degree temperature. (UPTU 2014,2 Marks)

33. Find the steady state temperature didistribution in a plate of length of 2 meter where ends are kept at 30 °C and 70 °C respectively. (UPTU 2015,2 Marks)

34. Find the steady state temperature didistribution in a plate of length of 20 cm ,where ends are kept at 0 °C and 60 °C respectively. (UPTU 2014,2 Marks)

35. Find the temperature in a bar of lentgh 2 ,whose ends are kept at zero temperature and the initial temperature is $sin\frac{\pi x}{2} + 3sin\frac{5\pi x}{2}$. (UPTU 2007,08,11,15,10 Marks)

36. Determine the solution of one dimensional heat equation $\frac{\partial u}{\partial t} = c^2\frac{\partial^2 u}{\partial x^2}$ subject to the conditions u(0,t)=0, u(l,t)=0, u(x,0) = x , l being the length of the bar. (UPTU 2006,15,5 Marks)

37. Solve $\frac{\partial u}{\partial t} = a^2\frac{\partial^2 u}{\partial x^2}$ under the condition u(0,t)=0, u(l,t)=0,t > 0. And initial condition u(x,0)=x(10-x), l being the length of the bar. (UPTU 2014,5Marks)

38. Find the temperature distribution in a rod of length 'a' which is perfectly insulated including the ends and the initial temperature distribution is x(a-x) (UPTU 2015,10 Marks)

39. Find the temperature in a bar of lentgh l ,whose ends are kept at zero temperature and the initial temperature is $\mu_{0.}$ (UPTU 2015,5 Marks)

40. Find the temperature in a bar of lentgh π ,whose ends are kept at zero temperature and the initial temperature is $100 \cos x$ (UPTU 2015,5 Marks)

41. Obtain one dimensional heat equation given that u (0, t) = u (l, t) = 0 and the initial condition u(x,0) = x, 0 < x < l, where l is the length of the bar. (UPTU 2013, 5 Marks)

42. Solve $\frac{\partial u}{\partial t}$ = α $\frac{\partial^2 y}{\partial x^2}$, α constant, subject to the boundary condition u (0, t) = u(π, t) = 0 and the initial condition u(x,0) = sin 2x. (UPTU 2012)

43. An insulated rod of length l has its ends A and B maintained at 0^0c and 100^0c respectively untill steady state conditions prevail. If B is Suddenly reduced to 0^0c and maintained at 0 ^{0}c, find the temperature at a distance x at any time t. (UPTU 2011)

44. A homogeneous rod of conducting material of length l has its ends kept at zero temperature. The temperature at the Centre is T and falls uniformly to zero at the two ends. Find the temperature distribution. (UPTU 2012, 10 Marks)

45. The temperature distribution in a bar of length π which is perfectly insulated at ends x = 0 and x = π is governed by the partial differential equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, assuming the initial temperature distributions as u(x, 0) = f(x) = $\cos^2$x. Find the temperature distribution at any instant of time t.

(UPTU 2010, 10 Marks)

46. The temperature distribution in a bar of length π which is perfectly insulated at ends x = 0 and x = π, governed by the partial differential equation $\frac{\partial u}{\partial t} = k^2 \frac{\partial^2 u}{\partial x^2}$, assuming the initial temperature as u(x,0) = f(x) = cos mx. Find the temperature distribution at any instant of time, k and m being constant. (UPTU, 2011, 10 Marks)

47. Find the temperature u(x ,t) in a slab , whose ends x=0, and x = l are kept at temperature zero and where initial temperature f(x) is given by $f(x) = \begin{cases} A, 0 < x < \frac{l}{2} \\ 0, l/2 < x < l \end{cases}$ (UPTU, 2010, 10 Marks)

48. Find the temperature distribution in a rod of length 2 meter, whose ends are fixed at temperature zero and the initial temperature distribution f(x) = 100 x. (UPTU 2012,5 Marks)

49. A rod of length l with insulated sides is initially at a uniform temperature u_0. Its ends are suddenly cooled at zero temperature and are kept at that temperature. Find the temperature function u(x, t)

(UPTU, 2010 CO, 10 Marks)

50. Find the temperature distribution in a rod of length L, Whose end points are mentioned at temperature zero and the initial temperature distribution is f(x) (UPTU 2011, 5 Marks)

51. Solve the equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ with boundary condition $u(x, 0) = 3\ sin\pi x, u(0, t) = 0, u(l, t) = 0.$

The temperature distribution in a bar of length π which is perfectly insulated at ends x = 0 and x = π, governed by the partial differential equation $\frac{\partial u}{\partial t} = k^2\ \frac{\partial^2 y}{\partial x^2}$. Assuming the initial temperature as u(x,0)= Cos x. Find the temperature distribution at any instant of time, where K is constant.

(UPTU, 2011)

52. Find the general solution of two-dimensional Laplace's equation using method of separation of variables. (UPTU 2011, 5 Marks)

53. Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$ = 0 subject to the conditions u (0, y) = u (a, y) = u(x, 0) = 0 and u(x, b)= x.

(UPTU 2015, 5 marks)

54. Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$ = 0 subject to the conditions u(x,0) =0= u(x,1) , u(∞,y) = 0 and u(0,y)= u_0.

(UPTU 2012, 5 marks)

55. Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 y}{\partial x^2}$ = 0 subject to the conditions u(0,y) = u(a, y) = u(x,π) = 0 and u(x,0)= $sin^2 x$.

(UPTU 2011, 10 marks)

56. Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$ = 0 subject to the conditions u(0,y) = u(π,y) = u(x ,b) = 0 and u(x,0)= $x(a - x)$

57. Solve the following Laplace Equation: $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$ = 0 ,$0 \leq x \leq a, 0 \leq y \leq b$ with the boundary conditions $u_x(0, y) = u_x(a, y) = u_y(x, 0) = 0, u_y(x, b) = f(x)$ (UPTU 2011,CO,10 Marks)

58. Solve the following Laplace Equation :$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$ = 0 in a rectangle with u(0,y) = u(π,y) = u(x,b)=0,and u(x,0) = $u_{0.}$, Lim u(x, y) = 0 as y tends to infinity, 0 < x < π. (UPTU 2012, 5 marks)

59. Solve the following Laplace Equation :$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ in a rectangle with u(0,y) = u(a,y) = u(x,b) = 0 and u(x,0)=f(x) along x axis. (UPTU 2008, 5 Marks)

60. Solve the following Laplace Equation :$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ in a rectangle in the xy plane $0 \le x \le a$ and $0 \le y \le b$ satisfy the following boundary conditions with u(x,0) = u(x, b) = u(0,y) = 0 and u(a, y)=f(y) parallel y axis.

(UPTU 2009, 5 Marks)

61. Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ subject to the conditions u(0,y) = u(l, y) = u(x,0) = 0 and u(x,a)=sin $\frac{n\pi}{l}x$.

(UPTU 2003, 04,09,12,10 Marks)

62. A rectangular plate with insulated surfaces is 10 cm wide and so long compared to its width that it may be considered infinite in length without introducing an appreciable error, if the temperature along the short edge y = 0 is given by

$$u(x,y) = \begin{cases} 20x, & 0 < x < 5 \\ 20(10-x), & 5 < x < 10 \end{cases}$$

And the two long edges x = 0 and x = 10 as well as other short edges are kept at zero temperature. Find the temperature u at any point P.

63. Write down the Telegraph Equations. (UPTU 2014, 2 Marks)

64. Determine the electromotive force e(x, t) in a transmission line of length b, t seconds after the ends were suddenly grounded. Assume that R and G are negligible and the initial conditions are i(x, 0) = i_0 and $e(x,0) = e_1 sin\frac{\pi x}{b} + e_1 sin\frac{5\pi}{b}$. Here R and G denote the terms for the effect of leakage and resistance respectively.

(UPTU 2012, 5 Marks)

65. In a telephone wire of length l, a steady voltage distribution of 20 volts at the source end and 12 volts at the terminal end is maintained at time t = 0, the terminal is grounded, assuming L= 0,G = 0, determine the voltage and current ,where symbols have their usual meanings.

66. Assuming the resistance of wire (R) and conductance to ground (C) are negligible, find the voltage v(x, t) and current i(x , t) in a transmission line of length l, t seconds after the ends are suddenly grounded. The initial conditions are v(x, 0) = v_0 sin ($\frac{\pi x}{l}$) and i(x, 0) = i_0. (UPTU 2008, 10 Marks)

67. Solve $\frac{\partial^2 v}{\partial x^2} = LC\frac{\partial^2 v}{\partial t^2}$ assuming that the initial voltage is $V_0 sin\left(\frac{\pi x}{l}\right), v_t(x, 0) = 0\ and\ v = 0$, at the ends x = 0 and x = l. (UPTU 2007,10 Marks)