

<https://www.mbacrystalball.com/blog/2015/10/09/set-theory-tutorial/>

Question: In a class of 100 students, 35 like science and 45 like math. 10 like both. How many like either of them and how many like neither?

olution:

Total number of students, $n(\mu) = 100$

Number of science students, $n(S) = 35$

Number of math students, $n(M) = 45$

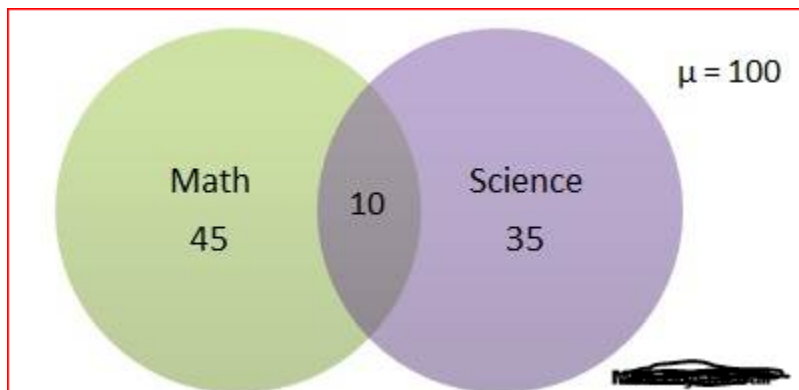
Number of students who like both, $n(M \cap S) = 10$

Number of students who like either of them,

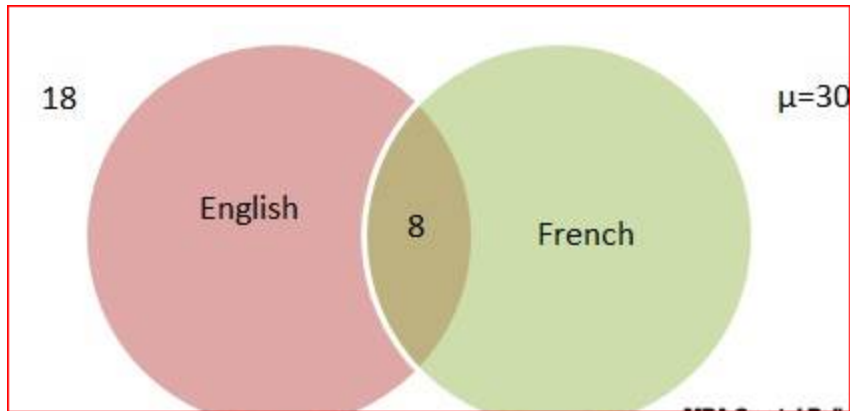
$$n(M \cup S) = n(M) + n(S) - n(M \cap S)$$

$$\rightarrow 45 + 35 - 10 = 70$$

$$\text{Number of students who like neither} = n(\mu) - n(M \cup S) = 100 - 70 = 30$$



Problem 1: There are 30 students in a class. Among them, 8 students are learning both English and French. A total of 18 students are learning English. If every student is learning at least one language, how many students are learning French in total?



Every student is learning at least one language. Hence there is no one who fall in the category 'neither'.

So in this case, $n(E \cup F) = n(\mu)$.

It is mentioned in the problem that a total of 18 are learning English. This DOES NOT mean that 18 are learning ONLY English. Only when the word 'only' is mentioned in the problem should we consider it so.

Now, 18 are learning English and 8 are learning both. This means that $18 - 8 = 10$ are learning ONLY English.

$$n(\mu) = 30, n(E) = 10$$

$$n(E \cup F) = n(E) + n(F) - n(E \cap F)$$

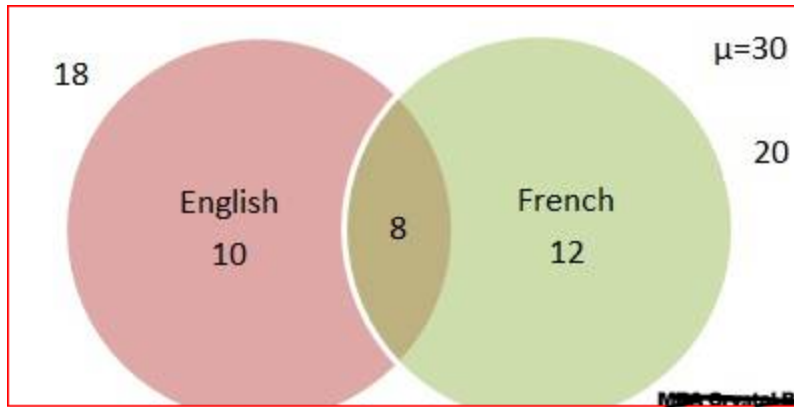
$$30 = 18 + n(F) - 8$$

$$n(F) = 20$$

Therefore, total number of students learning French = 20.

Note: The question was only about the total number of students learning French and not about those learning ONLY French, which would have been a different answer, 12.

Finally, the Venn diagram looks like this.



Problem 2: Among a group of students, 50 played cricket, 50 played hockey and 40 played volley ball. 15 played both cricket and hockey, 20 played both hockey and volley ball, 15 played cricket and volley ball and 10 played all three. If every student played at least one game, find the number of students and how many played only cricket, only hockey and only volley ball?

Solution:

$$n(C) = 50, n(H) = 50, n(V) = 40$$

$$n(C \cap H) = 15$$

$$n(H \cap V) = 20$$

$$n(C \cap V) = 15$$

$$n(C \cap H \cap V) = 10$$

No. of students who played at least one game

$$n(C \cup H \cup V) = n(C) + n(H) + n(V) - n(C \cap H) - n(H \cap V) - n(C \cap V) + n(C \cap H \cap V)$$

$$= 50 + 50 + 40 - 15 - 20 - 15 + 10$$

Total number of students = 100.

Let a denote the number of people who played cricket and volleyball only.

Let b denote the number of people who played cricket and hockey only.

Let c denote the number of people who played hockey and volleyball only.

Let d denote the number of people who played all three games.

$$\text{Accordingly, } d = n(C \cap H \cap V) = 10$$

$$\text{Now, } n(C \cap V) = a + d = 15$$

$$n(C \cap H) = b + d = 15$$

$$n(H \cap V) = c + d = 20$$

Therefore, $a = 15 - 10 = 5$ [cricket and volleyball only]

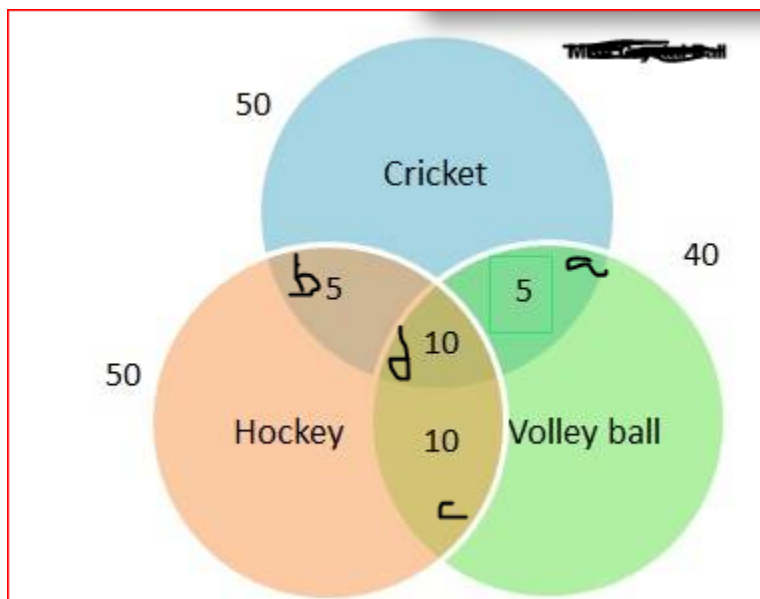
$b = 15 - 10 = 5$ [cricket and hockey only]

$c = 20 - 10 = 10$ [hockey and volleyball only]

No. of students who played only cricket = $n(C) - [a + b + d] = 50 - (5 + 5 + 10) = 30$

No. of students who played only hockey = $n(H) - [b + c + d] = 50 - (5 + 10 + 10) = 25$

No. of students who played only volley ball = $n(V) - [a + c + d] = 40 - (10 + 5 + 10) = 15$



: In a class, there are 100 students, 35 like drawing and 45 like music. 10 like both. Find out how many of them like either of them or neither of them?

Solution:

Total number of students, $n(\mu) = 100$

Number of drawing students, $n(d) = 35$

Number of music students, $n(m) = 45$

Number of students who like both, $n(d \cap m) = 10$

Number of students who like either of them,

$$n(d \cup m) = n(d) + n(m) - n(d \cap m)$$

$$\rightarrow 45 + 35 - 10 = 70$$

$$\text{Number of students who like neither} = n(\mu) - n(d \cup m) = 100 - 70 = 30$$

QIn a set $\{1,2,3,4,5,6,7,8,9,10,11,12,13\}$ is a universal set, what are all the possible subset of the universal set

here the given set be $\{1,2,3,4,5,6,7,8,9,10,11,12,13\}$

the total number of elements are $(n)=13$

so, the possible subsets for the above universal set be $=n!=13!$

$$13 \times 12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 6227020800$$

Theorem. Let A , B , and C denote sets of elements of some domain. Then

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

.

Proof. Let x

be arbitrary, and suppose x is in $A \cap (B \cup C)$. Then x is in A , and either x is in B or x is in C . In the first case, x is in A and B , and hence in $A \cap B$. In the second case, x is in A and C , and hence $A \cap C$. Either way, we have that x is in $(A \cap B) \cup (A \cap C)$

.

Conversely, suppose x

is in $(A \cap B) \cup (A \cap C)$

. There are now two cases.

First, suppose x

is in $A \cap B$. Then x is in both A and B . Since x is in B , it is also in $B \cup C$, and so x is in $A \cap (B \cup C)$

.

The second case is similar: suppose x

is in $A \cap C$. Then x is in both A and C , and so also in $B \cup C$. Hence, in this case also, x is in $A \cap (B \cup C)$, as required.