2.2.3 - Rejection Sampling of a Hemisphere

If we choose points randomly in a unit cube, the chance that our points are distributed in the hemisphere is the volume of the unit hemisphere which is:

$V_{unitSphere} = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(\frac{1}{2})^3 = \frac{\pi}{6}$ since $r = \frac{1}{2}$

$V_{unitHemisphere} = \frac{\pi}{12}$

Thus the probability that a random point in the unit cube is inside the hemisphere is:

$p(x) = \frac{\pi}{12}$

In order to find the number of loop necessary to have a sample, we can see this problem as a geometric distribution whose probability is $p(x)$ that would mean that the esperance of loop necessary to have a sample is:

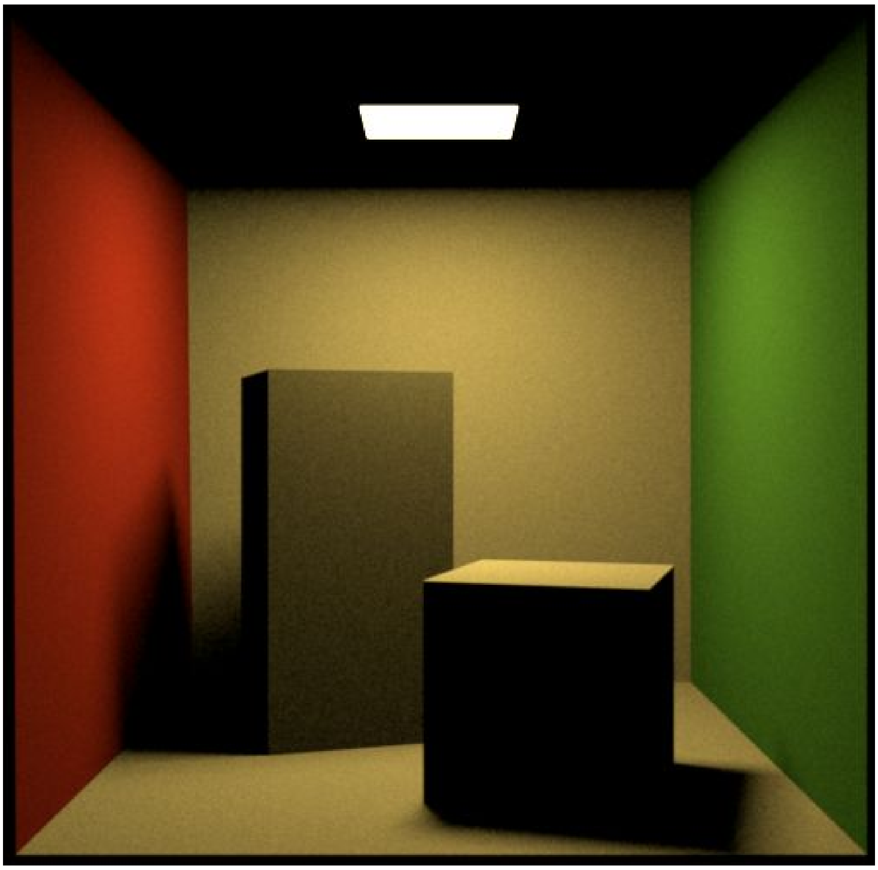
$E[X] = \frac{1}{p(x)} = \frac{12}{\pi} = 3.82 \simeq 4$

We need, on average, 4 loops iteration to have a sample.

2.3.1 - Rejection Sampling of a Hemisphere

Here is a screenshot of our implementation of the diffuse brdf.

Here, 16384 samples per pixel were calculate for a running time of ~1h30, the exposure was increased to match the reference image from the handout.



2.3.2 - Phong Sampling

Here are screenshots of our implementation of the phong brdf, again we have 16000 samples per pixel and a runtime of ~1h30.

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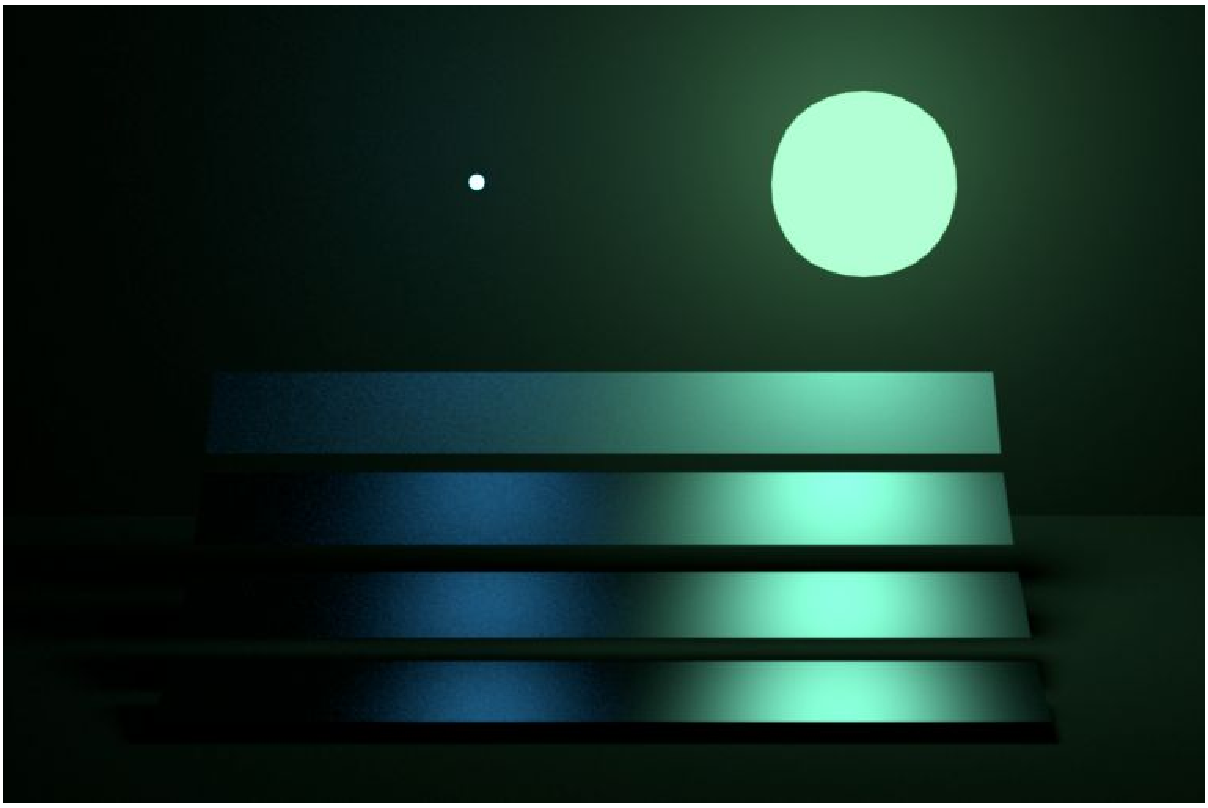
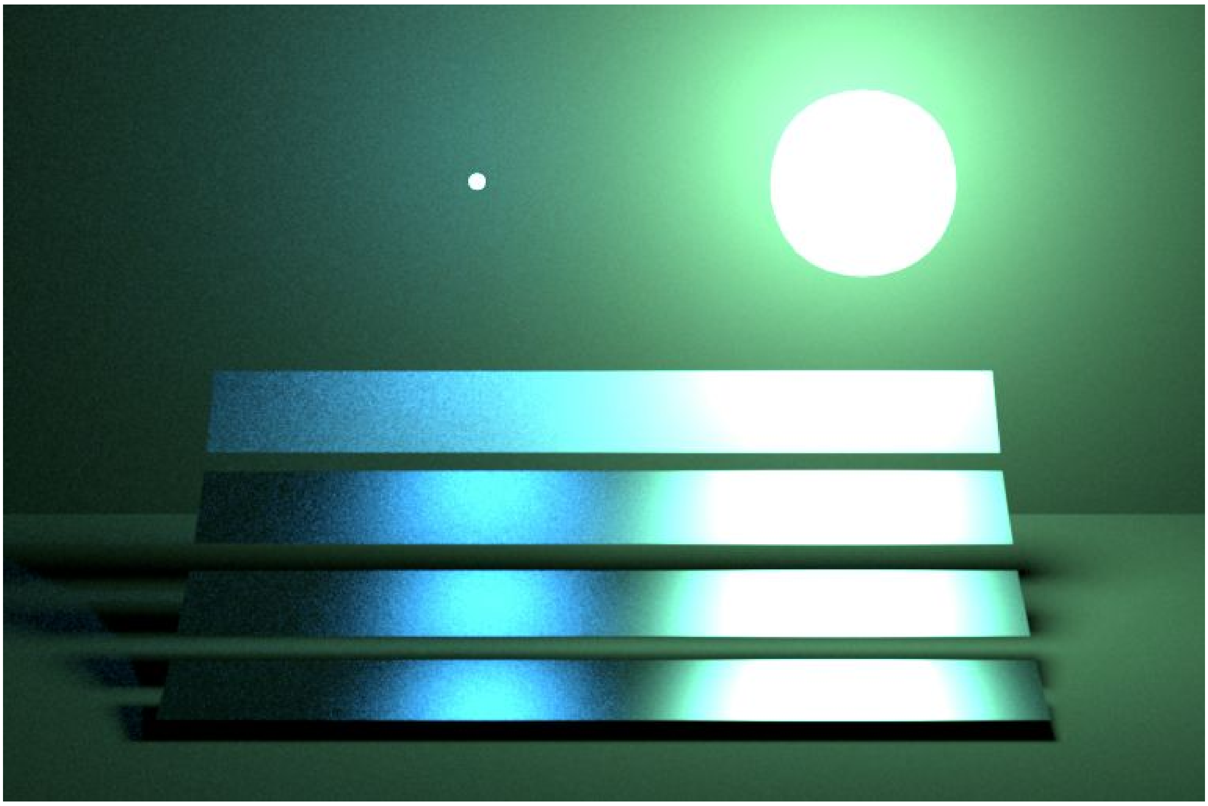
Jason Racine - 244270
Alexandre Veuthey - 224295

On the top, the scene is displayed with a normal exposure and on the bottom, with high exposure to show the details of the reflexion on the panes.





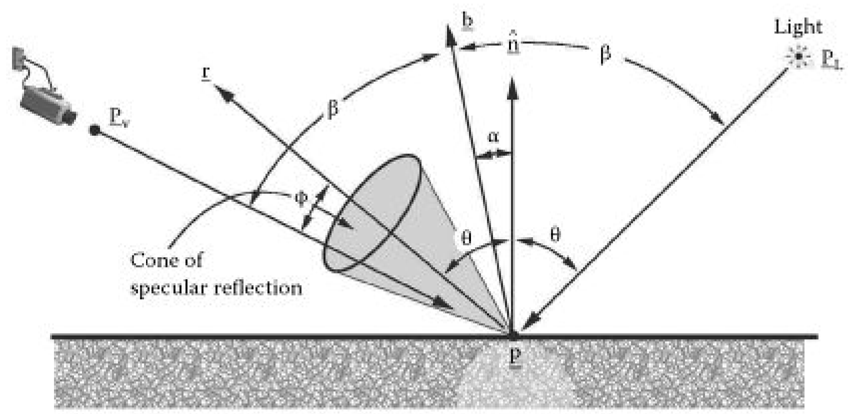
2.3.2a - probability density function
To find the combined pdf used for our implementation, we linearly combine the two pdf given by the “importance sampling” booklet given with the lab to have the following formula:
$pdf_{combined} = \alpha * pdf_{diffuse} + \beta * pdf_{specular}$
We have to find the parameters α and β such that our combined pdf stay unbiased, that is, the sum of α an β should equal to 1.
Since in the *sample* method, the random vector will be chosen depending on the weight of the ks and kd, we did the same for the combined pdf so that $\alpha = \frac{kd}{kd+ks}$ and $\beta = \frac{ks}{kd+ks}$.
The combined pdf is unbiased because, if we take the integral of the combined pdf, it stays equal to 1 since $\alpha + \beta = 1$

$$\int pdf_{combined} = \alpha \int pdf_{diffuse} + \beta \int pdf_{specular} = \alpha + \beta = 1$$

2.3.2b - Reflexion angle calculation
We introduce a vector w_b which is a bisection between w_i and w_o.
We know that $\angle(w_i, n) = \angle(n, w_s)$
We also know that $\angle(w_i, w_b) = \angle(w_b, w_o)$ since w_b is the bisection of w_i and w_o
Finally, we know that $\angle(w_b, n) = \angle(w_b, w_i) - \angle(n, w_i)$

Then, $\angle(w_o, w_s) = \angle(w_o, w_b) + \angle(w_b, n) - \angle(n, w_s)$
Knowing that $\angle(w_o, w_b) = \angle(w_b, w_i) = \angle(w_b, n) + \angle(n, w_i) = \angle(w_b, n) + \angle(n, w_s)$ we can reduce our previous equation to have
$\angle(w_o, w_s) = \angle(w_b, n) + \angle(n, w_s) + \angle(w_b, n) - \angle(n, w_s) = 2 * \angle(w_b, n)$
Which is two times the angle between the normal and the bisection between w_i and w_o

Here is a visual representation taken from the book “Practical algorithms for 3D computer graphics, Fergusson” page 118



r is w_s, P_v is w_o, b is w_b and P_l is w_i