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MODULE *crdt*

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EXTENDS *TLC*, *FiniteSets*, *Naturals*, *Sequences*  
 CONSTANTS *MAX\_TIMESTAMP*, *KEYS*, *VALUES*, *N\_NODES*  
 VARIABLES *timestamp*, *values*, *deliverQueues*

*vars*  $\triangleq$   $\langle \text{timestamp}, \text{values}, \text{deliverQueues} \rangle$   
*nodeIds*  $\triangleq$   $1 \dots N\_NODES$

*DeliverSet*(*n*, *t*, *k*, *v*)  $\triangleq$   
 LET *previous*  $\triangleq$   $\{ \langle tp, kp, vp \rangle \in \text{values}[n] : kp = k \}$  IN  
 IF *previous* =  $\{ \}$   $\vee \forall \langle tp, kp, vp \rangle \in \text{previous} : tp < t$  THEN  
     *values'* = [*values* EXCEPT ![*n*] = (*values*[*n*] \ *previous*)  $\cup$   $\{ \langle t, k, v \rangle \}$ ]  
 ELSE  
     UNCHANGED *values*

*DeliverDelete*(*n*, *t*)  $\triangleq$   
     *values'* = [*values* EXCEPT ![*n*] =  $\{ \langle tp, k, v \rangle \in \text{values}[n] : tp \neq t \}$ ]

*Deliver*(*n*, *command*, *payload*)  $\triangleq$   
      $\vee \text{command} = \text{"set"}$   
          $\wedge \text{DeliverSet}(n, \text{payload}[1], \text{payload}[2], \text{payload}[3])$   
      $\vee \text{command} = \text{"delete"}$   
          $\wedge \text{DeliverDelete}(n, \text{payload})$

*Broadcast*(*n*, *command*, *payload*)  $\triangleq$   
      $\wedge \text{Deliver}(n, \text{command}, \text{payload})$   
      $\wedge \text{deliverQueues}' = [$   
          $i \in \text{nodeIds} \mapsto$   
         IF  $i = n$  THEN  
              $\text{deliverQueues}[i]$   
         ELSE  
              $\text{Append}(\text{deliverQueues}[i], \langle \text{command}, \text{payload} \rangle)$   
      $]$

*RequestSet*(*n*, *k*, *v*)  $\triangleq$   
      $\wedge \text{timestamp}' = \text{timestamp} + 1$   
      $\wedge \text{Broadcast}(n, \text{"set"}, \langle \text{timestamp}, k, v \rangle)$

*RequestDelete*(*n*, *k*)  $\triangleq$   
      $\exists \langle t, kp, v \rangle \in \text{values}[n] :$   
          $\wedge kp = k$   
          $\wedge \text{Broadcast}(n, \text{"delete"}, t)$

*RequestSetOnNode*  $\triangleq$   
      $\wedge \text{timestamp} < \text{MAX\_TIMESTAMP}$   
      $\wedge \exists \langle n, k, v \rangle \in \text{nodeIds} \times \text{KEYS} \times \text{VALUES} : \text{RequestSet}(n, k, v)$

*RequestDeleteOnNode*  $\triangleq$

$$\wedge \exists \langle n, k \rangle \in \text{nodeIds} \times \text{KEYS} : \text{RequestDelete}(n, k) \\ \wedge \text{UNCHANGED } \text{timestamp}$$

$$\text{DeliverOnNode} \triangleq \\ \exists n \in \text{nodeIds} : \\ \wedge \text{Len}(\text{deliverQueues}[n]) > 0 \\ \wedge \exists \langle \text{command}, \text{payload} \rangle \in \{\text{Head}(\text{deliverQueues}[n])\} : \\ \quad \text{Deliver}(n, \text{command}, \text{payload}) \\ \wedge \text{deliverQueues}' = [\text{deliverQueues} \text{ EXCEPT } ![n] = \text{Tail}(\text{deliverQueues}[n])] \\ \wedge \text{UNCHANGED } \text{timestamp}$$

$$\text{DeliverQueuesIsEmpty} \triangleq \\ \forall n \in \text{nodeIds} : \text{Len}(\text{deliverQueues}[n]) = 0$$

$$\text{Terminating} \triangleq \\ \wedge \text{DeliverQueuesIsEmpty} \\ \wedge \text{UNCHANGED } \text{vars}$$

$$\text{Init} \triangleq \\ \wedge \text{values} = [i \in \text{nodeIds} \mapsto \{\}] \\ \wedge \text{deliverQueues} = [i \in \text{nodeIds} \mapsto \langle \rangle] \\ \wedge \text{timestamp} = 1$$

$$\text{Next} \triangleq \\ \vee \text{RequestSetOnNode} \\ \vee \text{RequestDeleteOnNode} \\ \vee \text{DeliverOnNode} \\ \vee \text{Terminating}$$

$$\text{Spec} \triangleq \text{Init} \wedge \Box[\text{Next}]_{\text{vars}} \wedge \text{WF}_{\text{vars}}(\text{DeliverOnNode})$$

$$\text{AllValuesEqual} \triangleq \\ \forall \langle n1, n2 \rangle \in \text{nodeIds} \times \text{nodeIds} : \\ \quad \text{values}[n1] = \text{values}[n2]$$

$$\text{EventuallyConsistent} \triangleq \Diamond \Box \text{AllValuesEqual}$$