
MODULE *crdt_fixed*

EXTENDS *TLC, FiniteSets, Naturals, Sequences*
 CONSTANTS *MAX_TIMESTAMP, KEYS, VALUES, N_NODES*
 VARIABLES *timestamp, values, deliverQueues*

vars $\triangleq \langle \text{timestamp}, \text{values}, \text{deliverQueues} \rangle$
nodeIds $\triangleq 1 \dots N_NODES$

DeliverSet(*n, T, t, k, v*) \triangleq
 $\text{values}' = [$
 $\quad \text{values EXCEPT } ![n] = \{ \langle tp, kp, vp \rangle \in \text{values}[n] : tp \notin T \} \cup \{ \langle t, k, v \rangle \}$
 $\quad]$

DeliverDelete(*n, T*) \triangleq
 $\text{values}' = [$
 $\quad \text{values EXCEPT } ![n] = \{ \langle t, k, v \rangle \in \text{values}[n] : t \notin T \}$
 $\quad]$

Deliver(*n, command, payload*) \triangleq
 $\vee \text{command} = \text{"set"}$
 $\quad \wedge \text{DeliverSet}(n, \text{payload}[1], \text{payload}[2], \text{payload}[3], \text{payload}[4])$
 $\vee \text{command} = \text{"delete"}$
 $\quad \wedge \text{DeliverDelete}(n, \text{payload})$

Broadcast(*n, command, payload*) \triangleq
 $\wedge \text{Deliver}(n, \text{command}, \text{payload})$
 $\wedge \text{deliverQueues}' = [$
 $\quad i \in \text{nodeIds} \mapsto$
 $\quad \text{IF } i = n \text{ THEN}$
 $\quad \quad \text{deliverQueues}[i]$
 $\quad \text{ELSE}$
 $\quad \quad \text{Append}(\text{deliverQueues}[i], \langle \text{command}, \text{payload} \rangle)$
 $\quad]$

RequestSet(*n, k, v*) \triangleq
 $\text{LET } \text{matches} \triangleq \{ \langle t, kp, vp \rangle \in \text{values}[n] : k = kp \} \text{ IN}$
 $\text{LET } T \triangleq \{ t : \langle t, kp, vp \rangle \in \text{matches} \} \text{ IN}$
 $\quad \wedge \text{timestamp}' = \text{timestamp} + 1$
 $\quad \wedge \text{Broadcast}(n, \text{"set"}, \langle T, \text{timestamp}, k, v \rangle)$

RequestDelete(*n, k*) \triangleq
 $\text{LET } \text{matches} \triangleq \{ \langle t, kp, v \rangle \in \text{values}[n] : k = kp \} \text{ IN}$
 $\text{LET } T \triangleq \{ t : \langle t, kp, v \rangle \in \text{matches} \} \text{ IN}$
 $\quad \wedge T \neq \{ \}$
 $\quad \wedge \text{Broadcast}(n, \text{"delete"}, T)$

RequestSetOnNode \triangleq

$$\begin{aligned}
& \wedge \text{timestamp} < \text{MAX_TIMESTAMP} \\
& \wedge \exists \langle n, k, v \rangle \in \text{nodeIds} \times \text{KEYS} \times \text{VALUES} : \text{RequestSet}(n, k, v) \\
\text{RequestDeleteOnNode} & \triangleq \\
& \wedge \exists \langle n, k \rangle \in \text{nodeIds} \times \text{KEYS} : \text{RequestDelete}(n, k) \\
& \wedge \text{UNCHANGED } \text{timestamp} \\
\text{DeliverOnNode} & \triangleq \\
& \exists n \in \text{nodeIds} : \\
& \quad \wedge \text{Len}(\text{deliverQueues}[n]) > 0 \\
& \quad \wedge \exists \langle \text{command}, \text{payload} \rangle \in \{\text{Head}(\text{deliverQueues}[n])\} : \\
& \quad \quad \text{Deliver}(n, \text{command}, \text{payload}) \\
& \quad \wedge \text{deliverQueues}' = [\text{deliverQueues} \text{ EXCEPT } ![n] = \text{Tail}(\text{deliverQueues}[n])] \\
& \quad \wedge \text{UNCHANGED } \text{timestamp} \\
\text{DeliverQueuesIsEmpty} & \triangleq \\
& \forall n \in \text{nodeIds} : \text{Len}(\text{deliverQueues}[n]) = 0 \\
\text{Terminating} & \triangleq \\
& \wedge \text{DeliverQueuesIsEmpty} \\
& \wedge \text{UNCHANGED } \text{vars} \\
\text{Init} & \triangleq \\
& \wedge \text{values} = [i \in \text{nodeIds} \mapsto \{\}] \\
& \wedge \text{deliverQueues} = [i \in \text{nodeIds} \mapsto \langle \rangle] \\
& \wedge \text{timestamp} = 1 \\
\text{Next} & \triangleq \\
& \vee \text{RequestSetOnNode} \\
& \vee \text{RequestDeleteOnNode} \\
& \vee \text{DeliverOnNode} \\
& \vee \text{Terminating} \\
\text{Spec} & \triangleq \text{Init} \wedge \Box[\text{Next}]_{\text{vars}} \wedge \text{WF}_{\text{vars}}(\text{DeliverOnNode}) \\
\text{AllValuesEqual} & \triangleq \\
& \forall \langle n1, n2 \rangle \in \text{nodeIds} \times \text{nodeIds} : \\
& \quad \text{values}[n1] = \text{values}[n2] \\
\text{EventuallyConsistent} & \triangleq \Diamond \Box \text{AllValuesEqual}
\end{aligned}$$
