Kalman Filter and Its Application

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***Abstract*—Kalman filter is a minimum-variance estimation for dynamic systems and has attracted much attention with the increasing demands of target tracking. Various algorithms of Kalman filter was proposed for deriving optimal state estimation in the last thirty years. This paper briefly surveys the recent developments about Kalman filter (KF), Extended Kalman filter (EKF) and Unscented Kalman filter (UKF). The basic theories of Kalman filter are introduced, and the merits and demerits of them are analyzed and compared. Finally relevant conclusions and development trends are given.**

***Keywords*-Kalman filter; Extended Kalman filter; Unscented Kalman filter**

I. Introduction

Kalman filter is an algorithm that uses a series of data observed over time, which contains noise and other inaccuracies, to estimates unknown variables with more accuracy. It was proposed by R. E. Kalman in 1960 [1], and became a standard approach for optimal estimation. Because it has merits of real time, fast, efficient, and strong anti-interference, Kalman filter has been widely applied in the fields of orbit calculation, target tracking and navigation, such as calculations of spacecraft orbit, tracking of maneuvering target and positioning of GPS. Further more, it also plays an important role in the fields of integrated navigation and dynamic positioning, sensor data fusion, microeconomics, and especially in the field of digital image processing and the current hot research fields like pattern recognition, image segmentation and image edge detection.

II. Filting Methods and Applications

Kalman filter mainly includes Kalman filter (KF), Extended Kalman filter (EKF) and Unscented Kalman filter (UKF).

A. Kalman Filter

Kalman filter is a linear optimal status estimation method, which is known as one of the most famous Bayesian filter theories [2]. Status equation is a linear representation of w_k, u_{k-1} and x_{k-1}. Observation equation is a linear representation of x_k and v_k. A dynamic model is presented with status equation and observation equation through the reliable estimation corrected by measurements [3]. Kalman filter (systematic) status equation is defined as follow:

$$x_k = Ax_{k-1} + Bu_{k-1} + w_k \tag{1}$$

Observation equation is defined as follow:

$$z_k = Hx_k + v_k \tag{2}$$

In the above formulas: x_k, z_k, A, H, w_k, u_{k-1}, v_k is the status vector, the observation vector, the status transition matrix, the observation matrix, the system noise vector, the system control vector, the observation noise vector, respectively. w_k and v_k are assumed to satisfy positive definite, symmetric and uncorrelated, zero mean Gaussian white noise vector; k is a subscript; w_k and v_k are satisfied:

$$E(w) = 0, cov(w) = E(ww^T) = Q \tag{3}$$

$$E(v) = 0, cov(v) = E(vv^T) = R, E(wv^T) = 0 \tag{4}$$

$\widehat{x}_k^- \in R^n$ is defined as the prior status estimation derived from status transition equation at the moment of k-1, $\widehat{x}_k$ is defined as the posterior status estimation combines the measurements at the moment of k. The deviations are shown in equation (5) and equation (6):

$$e_k^- = x_k - \hat{x}_k^- \tag{5}$$

$$e_k = x_k - \hat{x}_k \tag{6}$$

The priori and posterior estimation deviation covariance equations are defined as equation (7) and equation (8):

$$P_k^- = E[e_k^- e_k^{-T}] \tag{7}$$

$$P_k = E[e_k e_k^T] \tag{8}$$

We should find a status estimation equation $\widehat{x}_k$ that calculate the posterior status estimation in order to get the Kalman filter equations. It requires the calculation formula of $\widehat{x}_k$ is the linear combination of a priori estimation and weighted difference between true measurements and measured forecast value. The following prediction and update equations from the Kalman filter theory are obtained. Prediction equations are defined as follows:

$$\widehat{x}_k^- = A\widehat{x}_{k-1} + Bu_{k-1} \tag{9}$$

$$P_k^- = AP_{k-1}A^T + Q \tag{10}$$

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Update equations are defined as follows:

$$K_k = P_k^- H^T (H P_k^- H^T + R)^{-1} \tag{11}$$

$$\widehat{x}_{k-1} = \widehat{x}_k^- + K_k (z_k - H \widehat{x}_k^-) \tag{12}$$

$$P_k = (I - K_k H) P_k^- \tag{13}$$

Where K_k, $\widehat{x}_k$, P_k, I is the Kalman gain matrix, the optimum filter value, the filter deviation matrix, the unit matrix, respectively. Kesari Verma applied KF to optical flow for multiple objects tracking. He concluded that multiple objects can be tracked simultaneously under moderate lighting changes, similar background shapes. However, KF does not show good performance for dim light video dataset, low resolution videos, stationary object and change in velocity [4]. Sandy Mahfouz proposed a method that combines machine learning with Kalman filter to estimate instantaneous positions of a moving target. The application of his method can obtain the accelerations of targets and accurate estimation of position [5]. H. A. Patel applied KF to the tracking of single moving object in the field of security automated surveillance systems, and completed experiments successfully on the video datasets [6].

B. Extended Kalman Filter

Kalman filter is only suitable for linear systems, and requires that the observation equation is linear. Most of practical applications are nonlinear systems; therefore the research of nonlinear filter is very important. One of the most classic algorithms in the field of nonlinear filter is Extended Kalman filter. The basic idea of EKF is to focus on the value of first-order nonlinear Taylor expansion around the status of the estimated, then transform the nonlinear system into a linear equation [7]. EKF algorithm is commonly used in nonlinear filter systems, and the calculation is easy to be implemented. However, Taylor expansion belongs to linear process, so only if the system status and observation equations are close to linear and continuous, the results of EKF are relatively close to the true value. In addition, the filtering result is affected by the status and measurement noise. The covariance matrix of the system status and observation noise remain unchanged in the process of EKF. If the noise covariance matrix of status and observation are not estimated accurately enough, the cumulative error may lead to the divergence of filter [8]. Considering the following nonlinear system:

$$x(k) = f(x(k-1), w(k-1)) \tag{14}$$

$$y(k) = h(x(k), v(k)) \tag{15}$$

$x(k)$ is the n-dimensional status vector of the system; $y(k)$ is the m-dimensional observation vector at the moment of k; the process noise $w(k-1)$ and the measurement noise $v(k)$ are the same as equation(3) and equation (4); function f and h, which are the transition matrix and observation matrix, are non-linear functions, respectively. Prediction equations are defined as follows:

$$A = \frac{df}{dx}|x = \widehat{x}_{k-1} \tag{16}$$

$$x_k^- = f(\widehat{x}_{k-1}) \tag{17}$$

$$P_k^- = A P_k A^T + Q \tag{18}$$

Update equations are defined as follows:

$$A = \frac{dh}{dx}|x = \widehat{x}_{k-1}^- \tag{19}$$

$$K_k = P_k^- H^T (H P_k^- H^T + R)^{-1} \tag{20}$$

$$\widehat{x}_k = \widehat{x}_k^- + K_k (y_k - h(\widehat{x}_k^-)) \tag{21}$$

$$P_k = (I - K_k H) P_k^- \tag{22}$$

Roland Hostettler applied EKF to vehicle tracking based on road surface vibration measurements. His experiments indicate that EKF is feasible and further development is needed [9]. Jian-fan Zhang improved EKF algorithm, and applied it to road vehicle tracking. The improved algorithm has a good performance for road vehicles in motion and better adaptability for the uncertainty of noise [10]. Tamer Mekky Ahmed Habib applied EKF to spacecraft orbit estimation and control according to GPS measurements. His experiments show that EKF could deal nonlinear systems simply and converge fast for the error resulted from estimation [11]. Ling Fan proposed EKF based real-time dynamic state and parameter estimation using phasor measurement unit data. The EKF based estimation can estimate two dynamic states along with four unknown parameters [12]. Firstly, EKF is premised on the basis of first-order Taylor series expansion for nonlinear equations. The results of EKF would different from the true value greatly if EKF were used in the situation of nonlinear systems or expansion point deviated from the true value greatly. Secondly, EKF assumes that the status and observation noise are independent in white noise processing, but the characteristics of noise may not meet the characteristics of white noise. Because the status and observation noise may change after a first-order Taylor series expansion, the assumption for noise will be also inconsistent with reality. Finally, EKF needs to re-calculate the Jacobian matrix of current time observation equation during the observation time of each EKF process. For the reason that the calculation of matrix is very complex, so it is difficult to solve the Jacobian matrix in some systems.

C. Unscented Kalman Filter

Unscented Kalman filter was proposed by Julier and Uhlmann, and is another large class of methods that applied the sampling strategy close to nonlinear distribution [13]. UKF uses linear Kalman filter framework based on Unscented Transform (UT), and definites sampling strategy instead of random sampling strategy. The number of sampling points in UKF (generally defined as Sigma-points) is small, and depends on the sampling strategy. The most commonly used sampling strategy is $2n+1$ symmetrically-Sigma sampling strategy [14]. The basic idea of UT is: under the premise that the sampling mean is x and covariance is Px, we select a set of points (Sigma points set) and apply the non-linear transform to each Sigma sampling points, then we can obtain the nonlinear transformed set of points y and Py, which means statistics points of the Sigma after the transform [15]. The processing steps of UKF include: (1) Initializing the status vector and status estimation error covariance. (2) Selecting the sampling points according to the status vector and error covariance of the prior moment, and calculating the weighted value. (3) Calculating the mean and covariance propagation through the equation of status, and finishing time update by the selected sampling points. (4) Finishing measurement update through a nonlinear observation equation by the selected sampling points. (5) Updating Kalman filter coefficients. Status and observation equations are the same as equation (14) and equation (15). Sigma points selected are defined as follows:

$$x_0 = \widehat{x}_k, x_i = \widehat{x}_k + (\sqrt{(n+\lambda)P_k})_k^T, (i = 1, ..., n) \quad (23)$$

$$x_{i+n} = \widehat{x}_k - (\sqrt{(n+\lambda)P_k})_k^T, (i = 1, ..., n) \quad (24)$$

$$W_0^m = \lambda/(\lambda+n), W_i^m = 1/2(\lambda+n), (i = 1, ..., 2n) \quad (25)$$

$$W_0^c = W_0^m + (1-\alpha^2+\beta), W_i^c = 1/2(\lambda+n), (i = 1, ..., 2n) \quad (26)$$

$$\lambda = \alpha^2(k+n) - n \quad (27)$$

Where n is the dimension of status vector; $(\sqrt{(n+\lambda)P_k})_i^T$ is the square root of the matrix; α indicates the dispersion degree of selected points, k is usually selected as zero; β is used to describe the distribution of status variables, W_i^m and W_i^c represent the weighted value of the first point when solving the mean and covariance. Time update of UKF is defined as follows:

$$\varepsilon_i = f(x_i) \quad (28)$$

$$\widehat{x}_{k+1/k} = \sum W_i^m \varepsilon_i \quad (29)$$

$$P_{k+1/k} = \sum W_i^c (\varepsilon_i - \widehat{x}_{k+1/k})(\varepsilon_i - \widehat{x}_{k+1/k})^T \quad (30)$$

Measurement update of UKF is defined as follows:

$$Z_i = h(\varepsilon_i) \quad (31)$$

$$\widehat{z}_{k+1/k} = \sum W_i^m Z_i \quad (32)$$

$$P_{zz} = \sum W_i^c (Z_i - \widehat{z}_{k+1/k})(Z_i - \widehat{z}_{k+1/k})^T \quad (33)$$

$$P_{xz} = \sum W_i^c (\varepsilon_i - \widehat{x}_{k+1/k})(\varepsilon_i - \widehat{z}_{k+1/k})^T \quad (34)$$

Filter update of UKF is defined as follows:

$$K_{k+1} = P_{xz} P_{zz}^{-1} \quad (35)$$

$$\widehat{x}_{k+1} = \widehat{x}_{k+1/k} + K_k(y_{k+1} - \widehat{z}_{k+1/k}) \quad (36)$$

$$P_{k+1/k+1} = P_{k+1/k} - K_{k+1} P_{zz} K_{k+1}^T \quad (37)$$

Haitao Zhang applied UKF to target tracking, and analyzed the performance of this algorithm. Compared with EKF, UKF is easier to be implemented, and avoids calculating Jacobian and Hessian matrices [16]. Honglei Yan applied UKF to flying target tracking for updating target prediction matrix. The next state of the target prediction matrix is used to be seen as the trajectory prediction value. The results of his data processing show that UKF brings a great convenience for real-time processing platform at the coordinate of camera view field [17]. Sepehr Valipour C. Paramanand proposed a formulation of UKF for depth estimation, and recovered the three-dimensions structure of a scene from motion blur/optical defocus [18]. Yihuan Zhao proposed an improved UKF algorithm based on electronic technology. Simulation results of variably accelerated motion target tracing under three dimensional coordinate shows that the improved algorithm achieves good precision and robustness [19]. Qichuan Ding applied adaptive UKF to visual tracking, and improved tracking real-time and accuracy [20]. Seok-Han Lee combined particle filter with UKF for real-time camera tracking, and concluded that UKF-based tracking achieves successful camera tracking and feature mapping in an augmented reality environment [21].

III. Discussion

A. Kalman Filter

The characteristics of Kalman filter are shown as following [22,23]: (1) Kalman filter is a tool of estimating variables that can be used to estimate the status of linear systems. (2), Kalman filter is a filter of minimum variance estimation. But KF also has limitations: it can only be applied to linear systems, and requires the noise is Gaussian white noise. However, almost all of the systems are non-linear systems in practice; therefore, the application of the standard Kalman filter is limited.

B. Extended Kalman Filter

The algorithm of EKF has fortes; for instance, EKF is a common non-linear filter method that the calculation is simple and easy to be implemented. The limitations of EKF [24,25] include: (1) Because Taylor expansion is linear process, the estimated value of EKF can relatively close to the true value only when the system status and observation equations are nearly linear and continuous. (2) The performance of EKF is relied on status and observation noise. If both of the noise covariance matrices are estimated not accurately enough, errors would be accumulated, as a result EKF gets divergence.

C. Unscented Kalman Filter

The calculation of UKF is similar to EKF algorithm, but the performance of UKF is better than EKF. Using certainty sampling strategy in UKF algorithm reduces computation complexity, and avoids the divergence phenomenon when EKF deals with higher-order unreasonably. Through analyzing, UKF algorithm has the following characteristics [26,27,28,29]: (1) Approximating the probability of density distribution of the nonlinear function, rather than approximating the nonlinear function. (2) The accuracy of nonlinear function statistics is up to at least two orders [30], and UKF can get higher accuracy when using a special sampling strategy such as four-order of Gaussian distribution sampling strategy and Skewness sampling strategy. (3) There is no need to calculate the Jacobian matrix of the derivative. (4) It can handle the cases of non-additive noise and discrete systems and extend the range of application. (5) The calculation of UKF is as much as EKF. (6) Unscented Kalman filter is close to the true value of the status system from the statistical characteristics, so we can get a better solution of nonlinear problems with higher accuracy and faster convergence.

IV. Conclusion

In this paper, the merits and demerits of KF, EKF, UKF algorithm are analyzed and summarized through comparison, also their applications are discussed. Various filtering methods that existed in the current have demerits like the complex structure of algorithms, lack real-time and reliability. We should strengthen the combination of theory and practical application of Kalman filter to improve its practicability.

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