Double Sum

$\rho(i-1, j) - \rho(i,j) > 0$

or $\underbrace{\rho(i,j) - \rho(i+1,j)}_{=d} > 0$

$\alpha = \pi/2n$

Sum/any term goes from 0 to $\pi/2$. Hence sin ↑ & cos ↓.
(sin/cos)

$$\therefore d = \frac{(\sin (j-i)\alpha \cdot \sin (j+i)\alpha \cos i\alpha \cos j\alpha)^2}{\sin^2 i\alpha + \sin^2 j\alpha} - \frac{(\sin (j-i-1)\alpha \sin (j+i+1)\alpha \cos (i+1)\alpha \cos j\alpha)^2}{\sin^2 (i+1)\alpha + \sin^2 j\alpha}$$

$\sin^2 i\alpha < \sin^2 (i+1)\alpha$

$\frac{1}{\sin^2 i\alpha} > \frac{1}{\sin^2 (i+1)\alpha}$

$\frac{-1}{\sin^2 i\alpha} < \frac{-1}{\sin^2 (i+1)\alpha}$

$\therefore$ In 2nd term's den, replace $\sin^2 (i+1)\alpha$ by $\sin^2 \alpha$.

Hence

$$d \geq \frac{\cos^2 j\alpha}{\sin^2 i\alpha + \sin^2 j\alpha} \underbrace{\left[\sin^2 (j-i)\alpha \sin^2 (j+i)\alpha \cos^2 i\alpha - \sin^2 (j-i-1)\alpha \sin^2 (j+i+1)\alpha \cos^2 (i+1)\alpha \right]}$$

$A^2 - B^2 = (A-B)(A+B)$

Also $j \geq i+1$

$\therefore \sin^2 j\alpha \geq \sin^2 i\alpha$

And hence

$$d \geq \frac{\cos^2 j\alpha}{2\sin^2 j\alpha} \cdot (A^2 - B^2)$$

Consider A

$A = \sin(j-i)\alpha \cdot \sin(j+i)\alpha \cos i\alpha$

$= (\sin j\alpha \cos i\alpha - \sin i\alpha \cos j\alpha)(\sin j\alpha \cos i\alpha + \cos j\alpha \sin i\alpha) \cos i\alpha$

$\{\text{Expanding } \sin(A \pm B)\}$

$= (\sin^2 j\alpha \cos^2 i\alpha - \sin^2 i\alpha \cos^2 j\alpha) \cos i\alpha$

$\geq (\sin^2 j\alpha \cos^2 i\alpha - \sin^2 j\alpha \cos^2 j\alpha) \cos i\alpha$

$\geq \sin^2 j\alpha \cos i\alpha \cdot [\cos^2 i\alpha - \cos^2 j\alpha]$

Similarly,

$B = (\sin^2 j\alpha \cos^2 (i+1)\alpha - \sin^2 (i+1)\alpha \cos^2 j\alpha) \cos (i+1)\alpha$

$\geq \sin^2 j\alpha \cos (i+1)\alpha [\cos^2 (i+1)\alpha - \cos^2 j\alpha]$

$A+B \geq$ sth. ✓

$A-B = ?$

if $x \geq 2$

$y \geq 1$

$x-y \geq 1$?

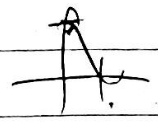
No.

$\therefore A-B = \sin^2 j\alpha [\cos^3 i\alpha - \cos^3 (i+1)\alpha] - \cos^2 j\alpha \left[\begin{array}{l}\sin^2 i\alpha \cos i\alpha \\ - \sin^2 (i+1)\alpha \cos (i+1)\alpha\end{array}\right]$

$A+B \geq \sin^2 j\alpha \left[(\cos^3 i\alpha + \cos^3 (i+1)\alpha) - \cos^2 j\alpha (\cos i\alpha + \cos (i+1)\alpha) \right]$

$\geq \sin^2 j\alpha [\cos i\alpha + \cos (i+1)\alpha] \left[\begin{array}{l}\cos^2 i\alpha - \cos i\alpha \cos (i+1)\alpha + \cos^2 (i+1)\alpha \\ - \cos^2 j\alpha\end{array}\right]$

~~$\cos i\alpha \geq \cos (i+1)\alpha$~~



$\cos i\alpha > \cos (i+1)\alpha$

$\cos i\alpha \geq \cos j\alpha \rightarrow \cos (i+1)\alpha \geq \cos j\alpha$

$\cos i\alpha \cos (i+1)\alpha \geq \cos^2 (i+1)\alpha$ &

$\cos^2 i\alpha \geq \cos i\alpha \cos (i+1)\alpha \qquad \cos^2 (i+1)\alpha \geq \cos^2 j\alpha$

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$$A+B \geq \sin^2 j\alpha\,[\cos i\alpha + \cos(i+1)\alpha]\,[\cos^2(i+1)\alpha - \cos^2 j\alpha]$$
$$\geq \sin^2 j\alpha\,[\cos i\alpha + \cos(i+1)\alpha]\,[\cos^2(i+1)\alpha - \cos^2 j\alpha]$$
$$\geq 2\sin^2 j\alpha\,\cos(i+1)\alpha\,[\cos^2(i+1)\alpha - \cos^2 j\alpha]$$

2^{nd} term of $A-B$:

$\sin^2(i+1)\alpha \geq \sin^2 i\alpha$ $\qquad$ $\sin i\alpha, \cos i\alpha > 0$

$\sin^2(i+1)\alpha\cos(i+1)\alpha \geq \sin^2 i\alpha \cos(i+1)\alpha$ $\qquad$ $\cos i\alpha > \cos(i+1)\alpha$

$$\cos^2 j\alpha\,[\sin^2(i+1)\alpha\cos(i+1)\alpha - \sin^2 i\alpha\cos i\alpha]$$

$$A-B = \sin^2 j\alpha\,\underbrace{[\cos^3 i\alpha - \cos^3(i+1)\alpha]}_{>0} + \cos^2 j\alpha\begin{bmatrix}\sin^2(i+1)\alpha\cos(i+1)\alpha \\ -\sin^2 i\alpha\cos i\alpha\end{bmatrix}$$

$$\geq \quad >0 \quad + \cos^2 j\alpha\,\underbrace{\begin{bmatrix}\sin^2(i+1)\alpha\cos(i+1)\alpha \\ -\sin^2 i\alpha\cos i\alpha\end{bmatrix}}_{?\ +ve\ or\ -ve}$$

$$\therefore d \geq \frac{\cos^2 j\alpha}{2\sin^2 j\alpha}\cdot 2\sin^2 j\alpha\cos(i+1)\alpha\,[\cos^2(i+1)\alpha - \cos^2 j\alpha]\,.\,(A-B)$$

$$d = \cos^2 j\alpha\cos(i+1)\alpha\,[\cos^2(i+1)\alpha - \cos^2 j\alpha]\,[A-B]$$