$$\frac{\left[\sin\frac{(j-i)\pi}{2n} * \sin\left[(j+i)\frac{\pi}{2n}\right] * \cos\frac{i\pi}{2n}\cos\frac{j\pi}{2n}\right]^2}{\sin^2\left[\frac{i\pi}{2n}\right] + \sin^2\left[\frac{j\pi}{2n}\right]}$$

$(i+j) \to$ even $\to p(i+1, j) - p(i, j) > 0$

$i > 0,\ j > i$

$\sin(a+b) = \sin A \cos B + \cos A \sin B.$

$(i+j) \to$ odd $\to p(i+1, j) - p(i+1, j)$

$p(i,j) - p(i+1,j)$ $\quad$ $\xrightarrow{\text{even}}$ $\sin \frac{?}{?}\left[\frac{\alpha\pi}{2n}\right]$

$i+j$: odd. $\quad$ $\xrightarrow{\text{odd}}$ $\sin\left[\frac{\alpha\pi}{2n}\right]$

$\boxed{\cos \frac{j\pi}{2n}}^2$ out. $\boxed{\text{Common}}$

$$\left[\sin(j-i)\alpha \sin((j+i)\alpha) \cos(i\alpha)\right]^2 \cdot \left[\sin^2(i+1)\alpha + \sin^2 j\alpha\right]$$

$$- \left[\sin(j-i-1)\alpha \sin(j+i+1)\alpha \cos(i+1)\alpha\right]^2$$

$$\left[\sin^2(i\alpha) + \sin^2 j\alpha\right]$$

---

den: $\sin^2(i+1)\alpha \longrightarrow \sin^2 i\alpha$

Common: $\dfrac{\cos^2 j\alpha}{\sin^2 i\alpha + \sin^2 j\alpha} \geq \boxed{\dfrac{\cos^2 j\alpha}{2\sin^2 j\alpha.}}$

since $j \geq i$

$$\left[\sin(j-i)\alpha \sin(j+i)\alpha \cos(i\alpha)\right] + \sin(j-i-1)\alpha \sin(j+i+1)\alpha \cos(i+1)\alpha\right]$$

$$\left[\qquad - \qquad\right] \Rightarrow \beta.$$

$$\geq (\sin(j-i)\alpha \sin(j+i)\alpha \cos(i\alpha)) \cdot \beta$$

$$= (\sin^2 j\alpha \cos^2 i\alpha - \cos^2 j\alpha \sin^2 i\alpha) \cos i\alpha \cdot \beta$$

$$= (\cos^2 i\alpha - \cos^2 (j\alpha)) \cos i\alpha \cdot \beta.$$

exp

$\cancel{\sin[(j-i)\alpha \quad \sin(i+i}$

$$\left[\sin(j-i)\alpha \sin(j+i)\alpha \cos i\alpha - \sin(j-i-1)\alpha \sin(j+i+1)\alpha \cos(i+1)\alpha\right]$$

$$\cancel{\left[\sin(j-i)\alpha \sin(j+i)\alpha - \sin(j-i-1)\alpha \sin(j+i+1)\alpha\right]} \; (\cancel{\cos i\alpha})$$

$$\left[\cos^2 i\alpha - \cos^2 j\alpha\right] \cos i\alpha - \cancel{\cos^2(i+1)\alpha}$$

$$- \left[\cos^2(i+1)\alpha - \cos^2 j\alpha\right] \cos(i+1)\alpha$$

$$\left[\cos^3 i\alpha - \cos^3(i+1)\alpha\right] + \cos^2 j\alpha \left[\cos(i+1)\alpha - \cos(i)\alpha\right]$$

$$\geq \left[\cos(i\alpha) - \cos(i+1)\alpha\right] \left(\cos i\alpha - \cos j\alpha\right)$$

Finally $\dfrac{\cos^4 i\alpha}{2\sin^2 j\alpha} \cos i\alpha \left(\cos i\alpha - \cos (i+1)\alpha\right)$

$$(\cos i\alpha + \cos j\alpha)(\cos i\alpha - \cos j\alpha)^2$$

$\boxed{(i+j) \text{ is odd}}$

$j > i$

$$\frac{\cos^4 j\alpha}{2\sin^2 j\alpha}\ \cos^2 i\alpha\left[\cos^2 i\alpha - 2\cos i\alpha\cos j\alpha + \cos^2 j\alpha\right]$$

$$(\cos i\alpha - \cos(i+1)\alpha)$$

$$\rightarrow \underbrace{\cos i\alpha - \cos i\alpha \cos\alpha}\ + \sin i\alpha \sin\alpha$$

$$\cos i\alpha\left[\frac{\alpha^2}{2!} - \frac{x^4}{4!}\right] + \sin i\alpha\left[\alpha - \frac{\alpha^3}{3!}\right]$$

$$\geq \left[(\cos i\alpha)\frac{\alpha^2}{3} + \frac{\sin i\alpha \cdot \alpha}{2}\right]$$

$$\cos^2 i\alpha\left(\quad\right) - \left(\quad\right)$$

$$+\frac{\cos^2 i\alpha}{4}\int_{i+1}^{n}\frac{\cos^4 j\alpha}{\sin^2 j\alpha}.$$

$$-\cos i\alpha\int_{i+1}^{n}\frac{2\cos^5 j\alpha}{\sin^2 j\alpha}$$

$$+\int_{i+1}^{n}\frac{\cos^6 j\alpha}{\sin^2 j\alpha}$$

$$\frac{1}{2j^2\alpha^2}\left[\cancel{n-\frac{i^2\alpha^2}{2}-\alpha(1+}\right.$$

$$\left.1-\frac{i^2\alpha^2}{2}-1+\frac{j^2\alpha^2}{2}\right]^2$$

$$\left(\cancel{\frac{\alpha^2}{3}}+\frac{i\alpha^2}{2}\right)$$

$$=\frac{1}{2j^2\alpha^2}\,\frac{i\alpha^2}{2}\,\cancel{\left[\right.}\,\frac{\alpha^4}{2}\left[j^2-i^2\right]^2$$

$$=\frac{\alpha^4}{8}\,\frac{i}{j^2}\left[j^2-i^2\right]^2$$

$$\geq\frac{\alpha^4}{8}\,\frac{i}{\cancel{j^2}}\,(j-i)^2\cancel{j^2}$$

$$\geq\frac{(j-i)^2 i\cdot\alpha^4}{8}$$

$$\sum_{i+1}^{n} j=\frac{n(n+1)}{2}$$

$$\frac{\text{For } i:}{\text{Sum}(j)}\quad\frac{\alpha^4}{8}\,i\left[\frac{n^2}{2}-\frac{i^2}{2}\right]\approx\boxed{O(1)}$$

$$\text{Sum}(i)\approx\alpha^4\cdot n^3\approx\boxed{\frac{1}{n}}$$