$\Rightarrow I = [0, 2)$

Seminarul 7

① Determinați mulț. de convergență a seriei de puteri

a) $\sum_{n=1}^{\infty} \frac{1}{n^2} \cdot (x-1)^n$

$\sum_{n=1}^{\infty} a_n (x - x_0)^n$

$a_n = \frac{1}{n^2}, \; x_0 = 1,$

$r = \lim_{n \to \infty} \left| \frac{a_n}{a_{n+1}} \right| = \lim_{n \to \infty} \frac{(n+1)^2}{n^2} = 1$

$\Rightarrow (1-1, 1+1) \subseteq I \subseteq [1-1, 1+1]$

pt. $x = 2 \Rightarrow \sum_{n=1}^{\infty} \frac{1}{n^2}$ conv. $(p = 2 > 1) \Rightarrow 2 \in I$

pt. $x = 0 \Rightarrow \sum_{n=1}^{\infty} \frac{1}{n^2}(-1)^n$ conv. $\Rightarrow 0 \in I$

$\Rightarrow I = [0, 2]$

$(n-3)!!$
$(2n-1)!$

b) $\sum_{n=0}^{\infty} \left(\frac{\pi}{2} - \operatorname{arctg} n\right) x^n$

$a_n = \frac{\pi}{2} - \operatorname{arctg} n, \quad x_0 = 0$

$$R = \lim_{n\to\infty} \frac{\frac{\pi}{2} - \operatorname{arctg} n}{\frac{\pi}{2} - \operatorname{arctg}(n+1)} = \lim_{x\to\infty} \frac{\frac{\pi}{2} - \operatorname{arctg} x}{\frac{\pi}{2} - \operatorname{arctg}(x+1)} \frac{0}{0}$$

$$= \lim_{x\to\infty} \frac{-\frac{1}{1+x^2}}{-\frac{1}{1+(1+x)^2}} = 1 \Rightarrow (-1, 1) \subseteq I \subseteq [-1, 1]$$

pt. $x = -1 \Rightarrow \sum_{n=0}^{\infty} \underbrace{\left(\frac{\pi}{2} - \operatorname{arct} n\right)}_{a_n \searrow 0} (-1)^n$ conv $\Rightarrow -1 \in I$

pt. $x = 1 \Rightarrow \sum_{n=0}^{\infty} \left(\frac{\pi}{2} - \operatorname{arctg} n\right)$ s.t.p $\sim \sum_{n=1}^{\infty} \frac{1}{n}$ div

$\exists \lim_{n\to\infty} \frac{\frac{\pi}{2} - \operatorname{arctg} \frac{1}{n}}{\frac{1}{n}} \in (0, +\infty)$

$\Rightarrow I = [-1, 1)$