# Curs nr. 1

1.1.1. (e) $F = \big(p \wedge (q \vee r)\big) \Leftrightarrow \big((p \wedge q) \vee (p \wedge r)\big)$

| $p$ | $q$ | $r$ | $q \vee r$ | $p \wedge q$ | $p \wedge r$ | $p \wedge (q \vee r)$ | $(p\wedge q) \vee (p \wedge r)$ | $F$ |
|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 |
| 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 1 |
| 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 1 |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 | 0 | 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

1.2.1.

$$Z = \{x \,/\, x \in \mathbb{N},\ 0 \le x < 10\} = \{0,1,2,3,4,5,6,7,8,9\}$$

$$\{1,2,3\} = \{x \in \mathbb{R} \,/\, (x-1)(x-2)(x-3) = 0\}$$

În teoria naivă a mulțimilor apar paradoxuri $\rightsquigarrow$ axiomatizarea teoriei!

ZF Zermelo Fraenkel

ZFC -||- + choice

Gödel Bernays J. Neuman

Ierarhia universurilor lui Grothendieck.

$$0,5 = \frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \dots = \frac{-1}{-2} = \frac{-2}{-4} = \dots$$

$$\emptyset = \{x \in \mathbb{N} \mid x+1=0\} = \{x \in \mathbb{R} \mid x^2 < 0\}$$

$$= \{x \in \mathbb{Z} \mid 2x = 1\} = \{x \mid x \neq x\} = \dots$$

$$A \subset B \iff (A \subseteq B \text{ și } A \neq B)$$

1.28. Dem.

$p := $ "$x \in A$", $q := $ "$x \in B$", $r := $ "$x \in C$"

De arătat

a) $p \Rightarrow p$

b) $\big((p \Rightarrow q) \wedge (q \Rightarrow r)\big) \Rightarrow (p \Rightarrow r)$

Se verif. cu tabelul de adev.

c) $(p \Leftrightarrow q) \Leftrightarrow \big((p \Rightarrow q) \wedge (q \Rightarrow p)\big)$

(d) $s$: "$x \in \emptyset$" – falsă întotdeauna

De verif $s \Rightarrow p$ – întotdeauna adev.

(e) $\emptyset, \emptyset'$ mulțimi vide amândouă

Cf. (d) $\emptyset \subseteq \emptyset'$ (1)

$\emptyset'$ vidă $\xRightarrow{d}$ $\emptyset' \subseteq A$ pt. orice mulțime $A$

deci $\emptyset' \subseteq \emptyset$ (2)

Din (1), (2) $\Rightarrow$ $\emptyset = \emptyset'$. $\boxtimes$

1.2.11. $p =$ "$x \in A$", $q =$ "$x \in B$", $r =$ "$x \in C$"

De arătat!

(a) $(p \vee (q \vee r)) \Leftrightarrow ((p \vee q) \vee r)$ la fel pt. $\wedge$

(b) $(p \vee q) \Leftrightarrow (q \vee p)$ —— u ——

(d) $p \vee p \Leftrightarrow p \Leftrightarrow p \wedge p$

(f) $\neg(p \vee q) \Leftrightarrow (\neg p \wedge \neg q)$ $\quad \neg(p \wedge q) \Leftrightarrow (\neg p \vee \neg q)$

$\{a,b\} = \{b,a\}$

$(a,b) \neq (b,a)$

De ex. def.

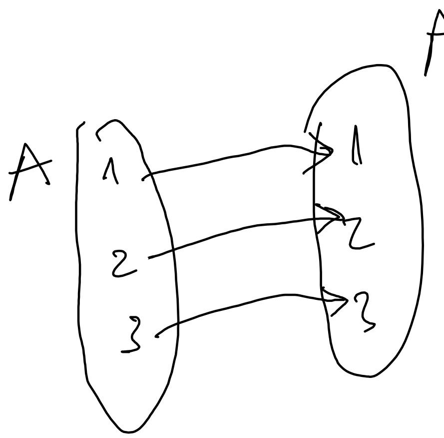
$(a,b) = \{a, \{a,b\}\} \neq \{b, \{b,a\}\} = (b,a)$

Alternativ

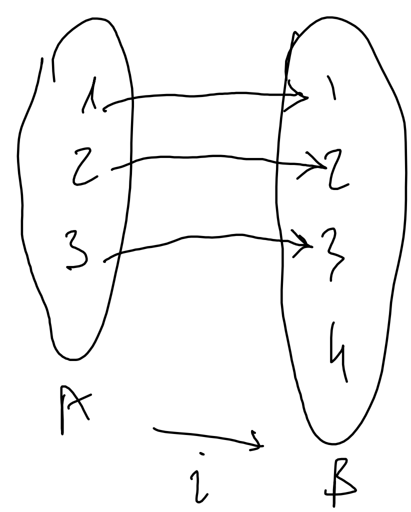
$(a,b) = \{a, \{b\}\} \neq \{b, \{a\}\} = (b,a)$

$f: \mathbb{R} \to \mathbb{R}, \ f(x) = \frac{1}{x}$ nu este bine def.

pt. că $\frac{1}{0}$ nu se definește.

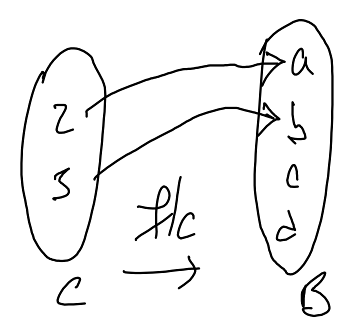



$1_A = id_A$




f este ca în 1.3.3. a) și

$C = \{2,3\}$



$$X = \{1,2\} \subseteq \{1,2,3\} = A$$

$$f(X) = \{f(1), f(2)\} = \{a, \cancel{a}\} = \{a\}$$

$$Y = \{a, c\} \subseteq B$$

$$f^{-1}(Y) = \{x \in A \mid f(x) \in \{a,c\}\} = \{1,2\}$$

$$f^{-1}(\{c,d\}) = \emptyset .$$

Compunerea :

$$\begin{tikzcd} A \arrow[r, "f"] \arrow[rr, bend right, "\exists\, g\circ f"'] & B \arrow[r, "g"] & C \end{tikzcd} \qquad \begin{tikzcd} A \arrow[r, "f"] \arrow[dr, "\exists\, g\circ f"'] & B \arrow[d, "g"] \\ & C \end{tikzcd}$$

$$\begin{array}{ccccc} x & \longmapsto & f(x) & \longmapsto & g(f(x)) \\ \in & & \in & & \in \\ A & & B & & C \end{array}$$

$$B \xrightarrow{g} C \qquad A \xrightarrow{f} B$$

dacă $C \neq A$

$$\nexists\, f \circ g$$

1.3.7.

$$\begin{tikzcd}
A \arrow[r, "f"] \arrow[rrr, bend left=50, "(h\circ g)\circ f"] \arrow[rr, bend right, "g\circ f"'] \arrow[rrr, bend right=50, "h\circ(g\circ f)"'] & B \arrow[r, "g"] \arrow[rr, bend left, "h\circ g"] & C \arrow[r, "h"] & D
\end{tikzcd}$$

$(h\circ g)\circ f : A \longrightarrow D$

$h\circ(g\circ f) : A \longrightarrow D$

Fie $x\in A$ : $\left[(h\circ g)\circ f\right](x) = (h\circ g)(f(x)) = h(g(f(x)))$

$$\left[h\circ(g\circ f)\right](x) = h((g\circ f)(x)) = h(g(f(x)))$$

Deci $(h\circ g)\circ f = h\circ(g\circ f)$.

$$\begin{tikzcd}
A \arrow[d, "1_A"'] \arrow[dr, "f\circ 1_A"] & \\
A \arrow[r, "f"] \arrow[dr, bend right, "1_B\circ f"'] & B \arrow[d, "1_B"] \\
 & B
\end{tikzcd}$$

$f\circ 1_A : A \longrightarrow B$

$f : A \longrightarrow B$

$1_B\circ f : A \longrightarrow B$

$$\boxed{\begin{array}{l} \text{Obs.} \\ A \neq B \\ \Downarrow \\ \nexists\ f\circ 1_B \end{array}}$$

Fie $x\in A$ $\quad (f\circ 1_A)(x) = f(1_A(x)) = f(x)$

$$(1_B\circ f)(x) = 1_B(f(x)) = f(x)$$

Deci $\quad f\circ 1_A = f = 1_B\circ f$ $\quad\boxtimes$

$$\begin{tikzcd}
B \arrow[d, "f'"'] \arrow[dr, "f\circ f'"] \arrow[dr, bend left=60, "1_B"] & \\
A \arrow[r, "f"] \arrow[dr, "f'\circ f"] \arrow[dr, bend right=60, "1_A"'] & B \arrow[d, "f'"] \\
& A
\end{tikzcd}$$

1.3.9. <u>Dem</u>. Presup. că $\exists f', f'' : B \to A$ a.î.

$$f' \circ f = 1_A = f'' \circ f \quad \text{și} \quad f \circ f' = 1_B = f \circ f''$$

$$f' = f' \circ 1_B = f' \circ (f \circ f'') \overset{\text{asoc}}{=} (f' \circ f) \circ f'' = 1_A \circ f'' = f''$$

$\Rightarrow$ unicit.

$(f^{-1})^{-1} = f$ pt. că formulele care definesc inversa sunt simetrice în $f$ și $f'$. $\boxed{X}$

$$\boxed{\ln y = x \iff e^x = y}$$

1.3.11.

$$\begin{tikzcd} A \arrow[r, shift left, "f"] \arrow[rr, bend left=50, "g\circ f"] & B \arrow[l, shift left, "f^{-1}"] \arrow[r, shift left, "g"] & C \arrow[l, shift left, "g^{-1}"] \arrow[ll, bend left=50, "f^{-1}\circ g^{-1}"] \end{tikzcd}$$

**obs** Dacă $A \neq C$

atunci

$\not\exists\ g^{-1}\circ f^{-1}$.

$$\left.\begin{aligned} (f^{-1}\circ g^{-1})\circ(g\circ f) &= f^{-1}\circ(g^{-1}\circ g)\circ f = f^{-1}\circ 1_B\circ f \\ &= f^{-1}\circ f = 1_A \\ (g\circ f)\circ(f^{-1}\circ g^{-1}) &= g\circ(f\circ f^{-1})\circ g^{-1} = g\circ 1_B\circ g^{-1} \\ &= g\circ g^{-1} = 1_C \end{aligned}\right\} \Rightarrow$$

$$f^{-1}\circ g^{-1} = (g\circ f)^{-1}. \quad \boxtimes.$$