

# Fast Elaboration for Dependent Type Theories

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EUTypes WG Meeting, Krakow, 24 February 2019

# Motivation, overview

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Current goals:

- Considering elaboration from ground-up, with performance as priority.
- Benchmarking a prototype against Coq and Agda.

## Elaboration

Computing (explicit, well-typed) core from (implicit, incomplete) source language. Includes type checking, unification, desugaring, tactics, etc.

Minimal example for filling holes:

```
id : (A : Set) → A → A
```

```
id A x = x
```

```
id' : (A : Set) → A → A
```

```
id' A x = id _ x
```

Output:

```
id : (A : Set) → A → A
```

```
id A x = x
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```
id' : (A : Set) → A → A
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id' A x = id A x
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Two core computational tasks in elaboration:

- ①  $\beta\eta$ -conversion checking.
- ② Generating solutions for holes (metavariables).

# Solving metas in the standard way

1: Source:

```
id : (A : Set) → A → A
```

```
id A x = x
```

```
id' : (A : Set) → A → A
```

```
id' A x = id _ x
```

2: Plug hole with fresh meta:

```
α = λ A x. ?
```

```
id : (A : Set) → A → A
```

```
id A x = x
```

```
id' : (A : Set) → A → A
```

```
id' A x = id (α A x) x
```

3: Solve meta:

```
α = λ A x. A
```

```
id : (A : Set) → A → A
```

```
id A x = x
```

```
id' : (A : Set) → A → A
```

```
id' A x = id (α A x) x
```

4: Unfold meta in output:

```
id : (A : Set) → A → A
```

```
id A x = x
```

```
id' : (A : Set) → A → A
```

```
id' A x = id A x
```

## Problems with the standard way

Metas are essentially unscoped: solutions can't refer to other definitions and meta solutions. Hence: everything must be unfolded.

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Input:

```
id' : {A : Set} → A → A
id' = id id id id
```

Output:

```
id' : {A : Set} → A → A
id' = λ {A} →
  (id {((A → A) → A → A) → (A → A) → A → A})
  (id {(A → A) → A → A})
  (id {A → A})
  (id {A})
```

## A better elaboration output

```
id' : {A : Set} → A → A
id' {A} =
  let α : Set = A
      β : Set = α → α
      γ : Set = β → β
      δ : Set = γ → γ
  in (id {δ}) (id {γ}) (id {β}) (id {α})
```

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In dependent TT, the size of solved metas often *dominates* the elaboration output. Hence, poor meta solutions imply poor elaboration output, and also cause slowdowns whenever we need to compute with these solutions during further elaboration.



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Example: elaborating length-indexed vector expressions with implicit length indices is quadratic in Agda and Coq: each `cons` contains a unary natural number index with the size of the vector tail.

(Can hash consing help? Not really: overheads and failure to handle beta redexes.)

## Scoping for metavariables

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- ① Full precision: metas are elaborated into `let`-definitions in arbitrary local scopes.
  - ▶ Dependently typed upgrade of Krishnawami and Dunfield's mixed-prefix bidirectional checkers.
  - ▶ Allows fast `let`-generalization.
  - ▶ More efficient, better output.
  - ▶ Challenging to implement.

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  - ▶ Allows fast `let`-generalization.
  - ▶ More efficient, better output.
  - ▶ Challenging to implement.
- ❷ Limited precision: metas only have top-level scope, and are elaborated into top-level mutual (unordered) definition blocks.
  - ▶ Easy to implement.
  - ▶ Less efficient and captures less sharing.
  - ▶ Implemented in prototype.

## Evaluators for elaboration

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Recall the two computational tasks: **conversion checking**, **meta solution generation**.

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Solution: a “glued” evaluator, which computes two different semantic values at the same time.

- ① *Glued values*: fully unfolded values, which also carry local values around.
- ② *Local values*: these are computed to some head normal form while **not** unfolding some class of definitions.

# Minimal glued evaluator in Haskell

Glues call-by-need and call-by-name machines together.

```
data Tm  = Var Int | App Tm Tm | Lam Tm
data Val = VNe Int [Val] [Cl] | VLam [Maybe Val] [Maybe Cl] Tm
data Cl  = Cl [Maybe Cl] Tm
```

```
eval :: [Maybe Val] → [Maybe Cl] → Tm → Val
```

```
eval vs cs t = case t of
```

```
  Var i → case vs !! i of
```

```
    Just v  -> v
```

```
    Nothing -> VNe (length vs - i - 1) [] []
```

```
  App t u → case (eval vs cs t, eval vs cs u) of
```

```
    (VLam vs' cs' t', u') → eval (u':vs') (Cl cs u :cs') t'
```

```
    (VNe i vs' cs' , u') → VNe i (u':vs') (Cl cs u :cs')
```

```
  Lam t → VLam vs cs t
```

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In principle, one could glue together any number of different evaluators, each optimized for a specific task. Gluing just two machines seems to strike a good balance of complexity and constant overheads.

## Takeaways so far

Kernel should consist of core syntax **and** a carefully chosen semantic domain (in our case, a particular environment machine). (Early example: Coquand (1996), since then: mini-TT, cubicaltt).

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Computing in the presence of metas is critically important, so metas should be in the kernel as well.

We get a larger kernel than in the Coq-style, but benefits seem to be significant.



# Prototype implementation

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