# MODULE 2

## Introduction :

Enhancement is the process of manipulating an image so that the result is more suitable than the original for a specific application. The word specific indicates that the enhancement techniques are application oriented.

## Intensity Transformation & Spatial filtering :

The term spatial domain refers to the image plane itself, & processing of image is based on direct manipulation of pixels in an image.

The two spatial domain processing methods are.

1) Intensity transformation.
2) Spatial filtering.

Intensity transformation is applied to each an every pixels of an image, for the purpose of contrast manipulation & image thresholding.

Spatial filtering deals with performing operations such as image sharpening, by working in a neighborhood of every pixel in an image. high pass filtering

Spatial domain processes can be denoted by the expression.

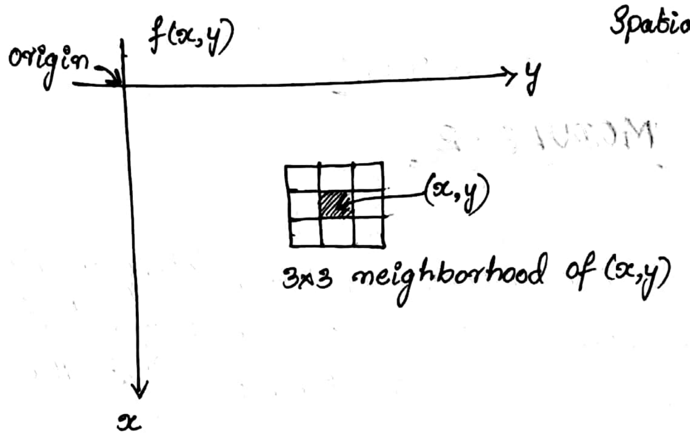
$$g(x,y) = T[f(x,y)]$$

where, $f(x,y) \longrightarrow$ original image (i/p)

$g(x,y) \longrightarrow$ processed image (o/p)

$T \longrightarrow$ operator on $f$ defined over a neighborhood of point $(x,y)$

Spatial domain




The point (x,y) is an arbitrary location in the image & is the center of neighborhood. By moving the origin of the neighborhood from pixel to pixel & applying the operator T to the pixels in the neighborhood to yield the o/p at that location. Thus for any (x,y), the o/p image 'g' at those Co-ordinates is equal to the result of applying T to the neighborhood with origin at (x,y) in 'f'.

**Note:** Zeros needs to be padded at the borders of the image, it depends on the size of the neighborhood

Neighborhood is also called as Spatial filter / Spatial mask / Kernel / template / window. The process which uses the Spatial filter is called as Spatial filtering.

## Intensity Transformation :

For a 1x1 neighborhood, g depends on the value of 'f' at a Single point (x,y) & T is now an intensity Transformation function [Gray level transformation / mapping]

Mathematically, $s = T(r)$

where, s & r are Intsity values of g & f respectively at any point (x,y).

eg: Contrast adjustment, sometimes also called as Contrast Stretching, where 'T' acts as a Thresholding function.

Let i/p image intensities be 'r' & o/p image intensities be 'S'. These values are related by an expression $S = T(r)$, where $T$ is the transformation that maps pixel values r into a pixel value S.

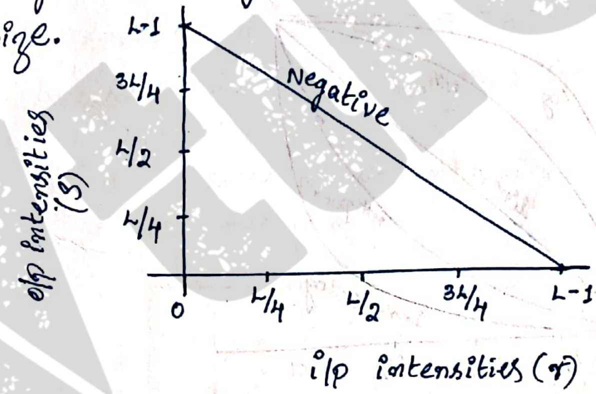
## Basic Intensity (Gray level) Transformation functions.

### Image Negative:

The negative of an image with grey level in the range $[0, L-1]$ is obtained by using the expression.

$$\boxed{S = L-1-r}$$

Reverting the intensity levels of an image in this manner produces the equivalent of a photographic negative. This type of processing is practically suited for enhancing white or gray details embedded in dark regions of an image especially when the black areas are dominent in size.




eg Digital Mammogram

Cancer detection.

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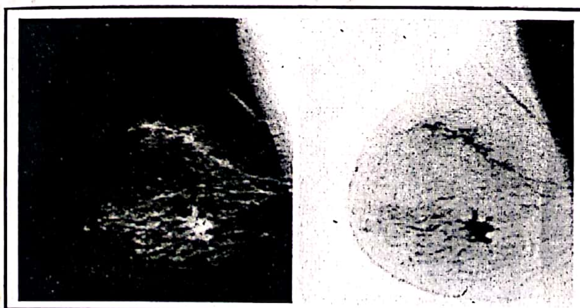

a b

**FIGURE 3.4** (a) Original digital mammogram. (b) Negative image obtained using the negative transformation in Eq. (3.2-1). (Courtesy of G.E. Medical Systems.)

## Log Transformations:

The general form of the log transformation is

$$\boxed{s = c\log(1+r)}$$

where c is a constant & $r \geq 0$ (Assumption)

The transformation maps a narrow range of gray level values in the i/p image into a wider range of o/p gray levels. The opposite is true for higher values of i/p gray levels.

We would use this transformation to expand the values of dark pixels in an image while compressing the higher level values. The opposite is true for inverse log transformation.

The log transformation function has an important characteristic that it compresses the dynamic range of images with large variations in pixel values.

Eg: Fourier Spectrum.

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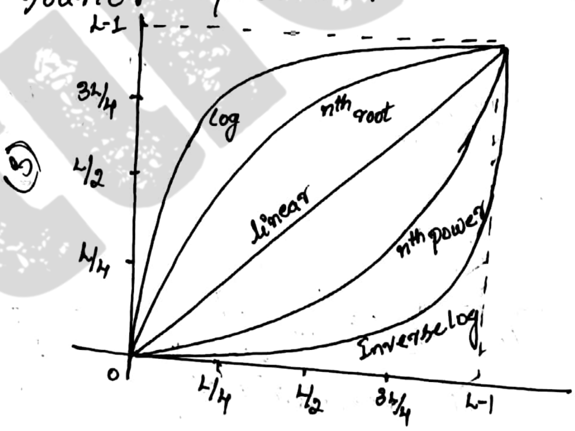


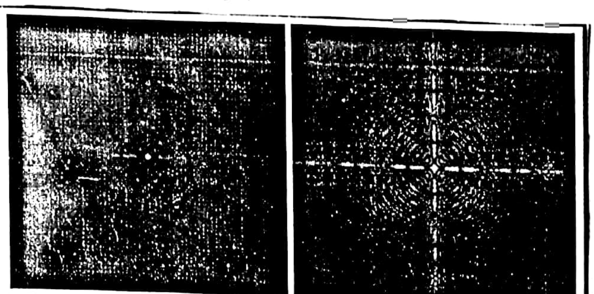

a b
**FIGURE 3.5** (a) Fourier spectrum. (b) Result of applying the log transformation given in Eq. (3.2-2) with c = 1.

## Power law (Gamma) Transformations.

The general form of the power law transformation is $\boxed{s = cr^{\gamma}}$

where, $c$ & $\gamma$ are positive constants.

Power law curves with fractional values of $\gamma$ map a narrow range of dark i/p values into a wider range of o/p values, with the opposite being true for higher values of i/p gray levels. we may get various curves by varying the values of $\gamma$.

Sometimes, $s = c(r+\epsilon)^{\gamma}$

where $\epsilon \rightarrow$ offset [It is a measurable o/p when i/p is zero]

offsets are an issue of display calibration.

when $c = \gamma = 1$, $\underline{\underline{s = r}}$ [linear]

A variety of devices used for image capture, printing & display respond according to a power law. The process used to correct this power law response phenomenon is called Gamma Correction..

Eg: CRT devices have intensity to vltg response i.e a power function.

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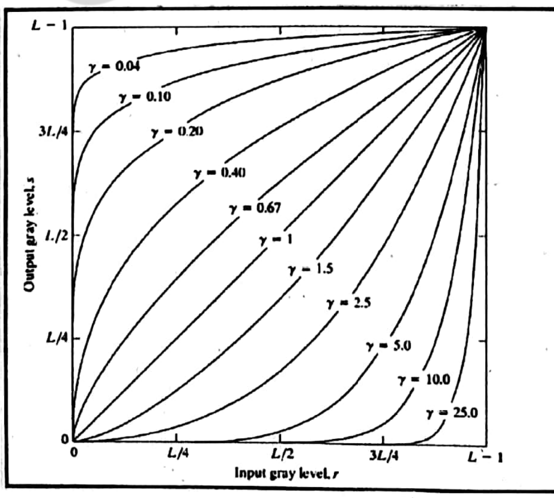



**FIGURE 3.6** Plots of the equation $s = cr^{\gamma}$ for various values of $\gamma$ ($c = 1$ in all cases).

Gamma Correction is important if displaying an image accurately on a Computer Screen is of concern. Images that are not Corrected properly can look either bleached out or too dark.

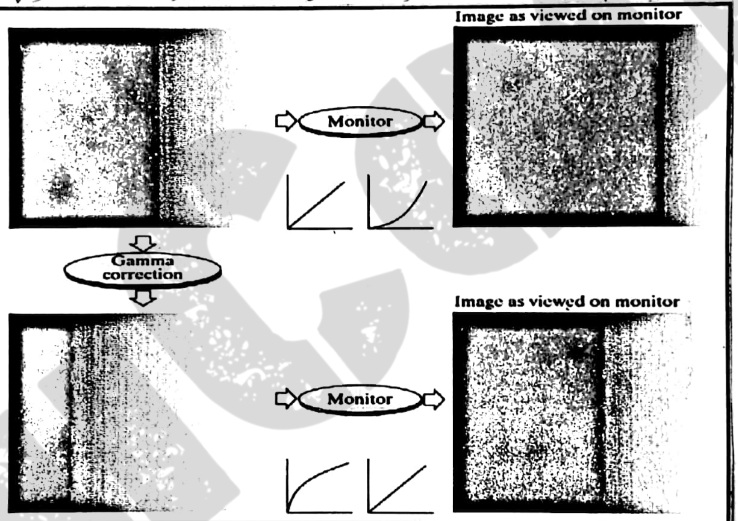
Color phenomenon also uses this concept of gamma Correction. It is becoming more popular due to use of Images over the Internet.

It is important in general purpose contrast manipulation. To make an Image black we use $\gamma > 1$ & $\gamma < 1$ for white image.

** Example (X-Ray, MRI's, CRT Monitors, 3D Imaging)

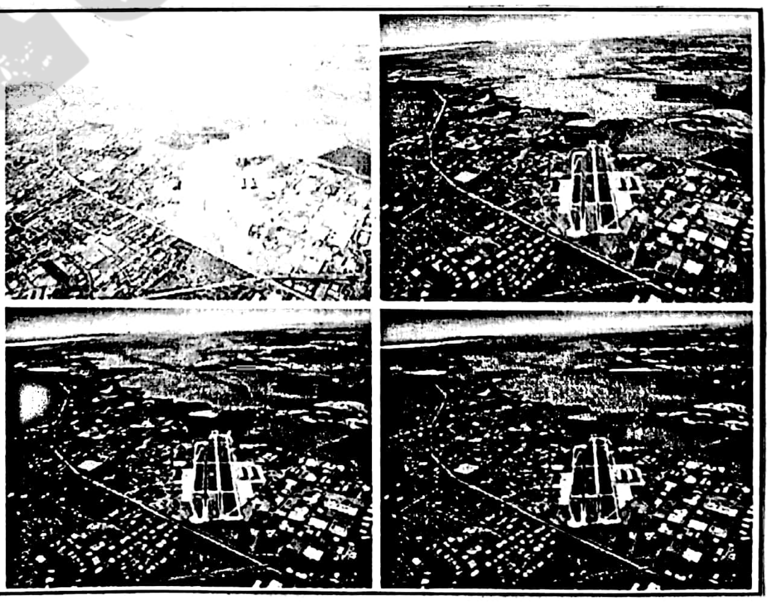
a b
c d

**FIGURE 3.7**
(a) Linear-wedge gray-scale image.
(b) Response of monitor to linear wedge.
(c) Gamma-corrected wedge.
(d) Output of monitor.




a b
c d

**FIGURE 3.9**
(a) Aerial image. (b)–(d) Results of applying the transformation in Eq. (3.2-3) with $c = 1$ and $\gamma = 3.0, 4.0$, and $5.0$, respectively. (Original image for this example courtesy of NASA.)



## Piecewise - linear Transformation function.

The principal advantage of piecewise linear function is that these functions can be arbitrarily complex. But their specification requires considerably more user i/p.

Also practical implementation of some important transformations can be formulated only as piecewise function.

### Contrast Stretching:

It is the simplest piecewise linear transformation fun$^n$.

Low contrast images can result from poor illumination, lack of dynamic range in the imaging sensor. or wrong setting of lens aperture during image acquisition.

The idea behind contrast stretching is to increase the dynamic range of gray levels in the image being processed.

∴ Contrast Stretching is a process of expanding the range of intensity levels in an image so that it spans the full intensity range of the recording medium or display devices.

Consider the locations of points $(r_1, s_1)$ & $(r_2, s_2)$ which control the shape of transformation fun$^n$.

1) If $r_1 = s_1$ & $r_2 = s_2$, the transformation is a linear function that produces no change in intensity level.

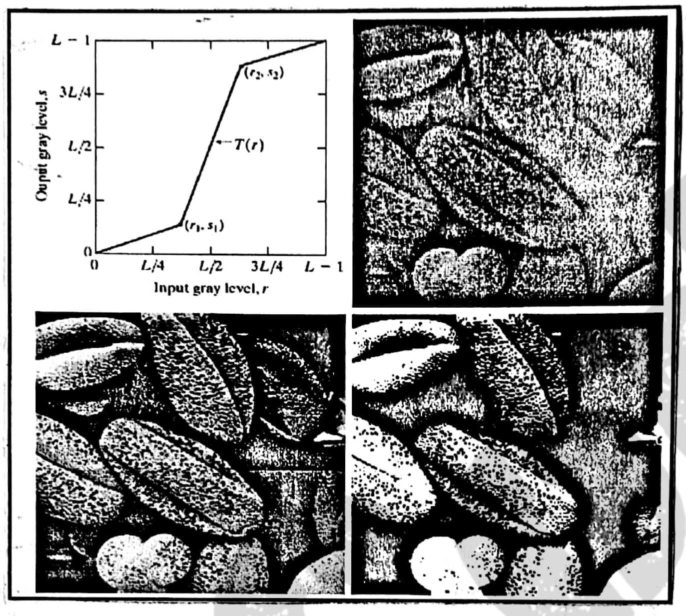
2) If $r_1 = r_2$, $s_1 = 0$ & $s_2 = L-1$, The transformation becomes a threshold function. that creates binary image.

3) Intermediate values of $(r_1, s_1)$ & $(r_2, s_2)$ produce various degrees of spread in the gray value of the o/p image thus affecting its contrast.

Generally $r_1 \le r_2$ & $s_1 \le s_2$ so that the function is single valued & monotonically increasing. This condition is essential to avoid artifacts in the processed image.

$$s = \frac{r - r_{min}}{r_{max} - r_{min}} \times 255$$

** Example




→ fig (b) . low contrast image

→ fig (c) Contrast stretching is obtained by setting.

$(r_1, s_1) = (r_{min}, 0)$ & $(r_2, s_2) = (r_{max}, L-1)$

→ fig (d) Result of thresholding function.

$(r_1, s_1) = (m, 0)$ & $(r_2, s_2) = (m, L-1)$

where m is the mean intensity level in the image.

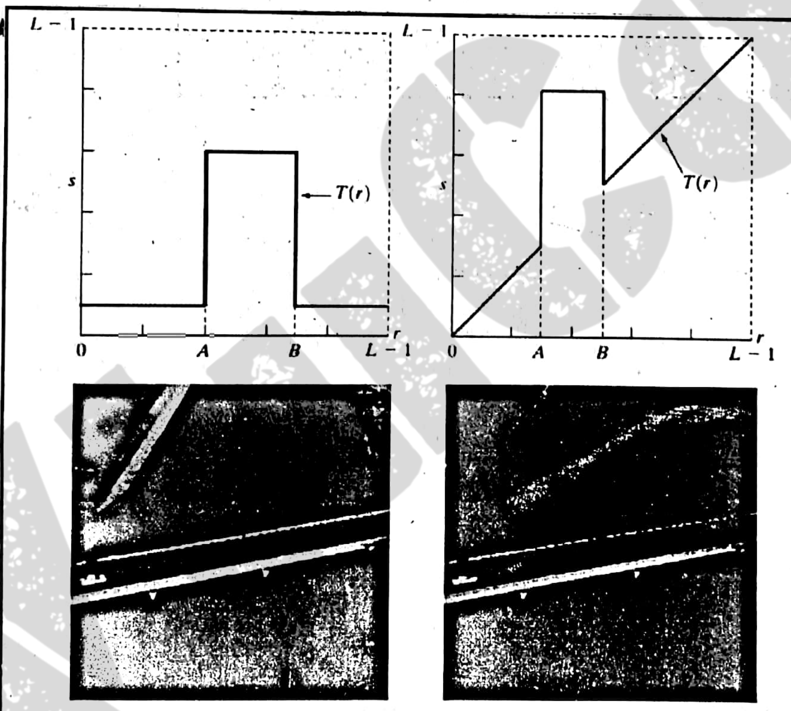
# Intensity (Gray) level Slicing :

Highlighting a specific range of gray levels in an image is often desirable. for example when enhancing features such as masses of water in satellite image & enhancing flaws in X-ray images.

There are two ways of doing this;

1) One method is to display a high value for all gray level in the range of interest. and a low (black) value for all the other gray level.

2) Second method is to brighten the desired ranges of gray levels but preserve the background & gray level tonalities in the image.




a b
c d

**FIGURE 3.11** (a) This transformation highlights range [A, B] of gray levels and reduc all others to a constant level. (b) This transformation highlights range [A, B] but preserves all other levels. (c) An image. (d) Result of using the transformation in (a).

Exc

## Bit plane Slicing:

Sometimes it is important to highlight the contribution made to the total image appearence by specific bits. Suppose that each pixel is represented by 8-bits. Imagine that an image is composed of eight 1-bit planes ranging from bit plane '0' for the least significant bit to bit plane '7' for the most significant bit.

In terms of 8-bit bytes, plane '0' contains all the lower order bits in the image & plane 7 contains all the higher order bits.

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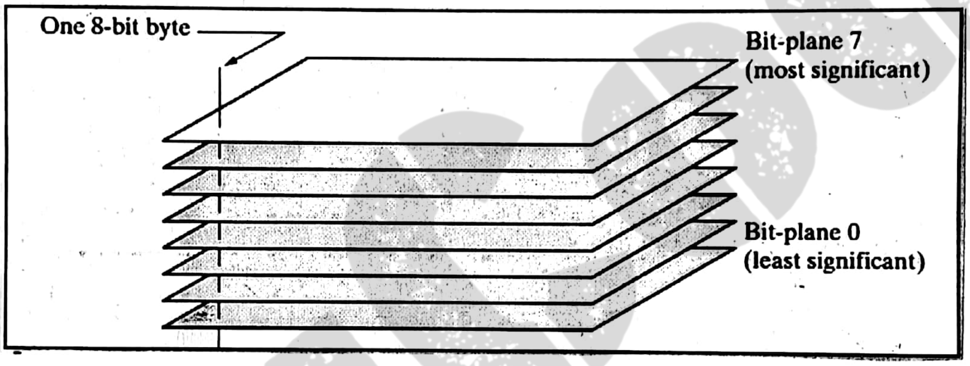


High order bits contain the majority of visually significant data & contribute to more subtle details in the image. Separating a digital image into its bit planes is useful for analyzing the relative importance played by each bit of the image.

It helps in determining the adequacy of the number of bits used to quantize each pixel. It is also useful for image compression.

To reconstruct image from its ~~by using~~ bit planes, it is required to multiply the pixels of the $n^{th}$ plane by the constant $2^{n-1}$

eg: if $n=8$ bit plane 8 is multiplied by 128
if $n=7$ bit plane 7 is multiplied by 64

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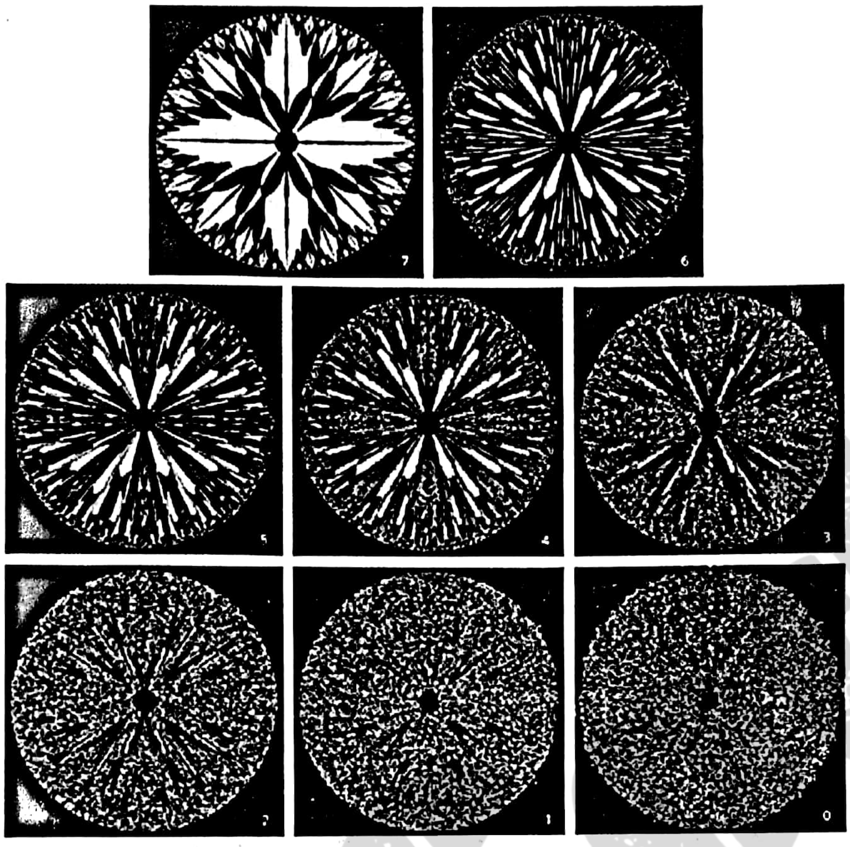


**FIGURE 3.14** The eight bit planes of the image in Fig. 3.13. The number at the bottom. right of each image identifies the bit plane.

## Histogram Processing:

The Histogram of a digital image with gray levels in the range [0, L-1] is a discrete function of the form

$$h(r_k) = n_k$$

where, $r_k$ is $k^{th}$ gray level

$n_k$ is the number of pixels in the image having the level $r_k$

A normalized histogram is given by the equation.

$$P(r_k) = \frac{n_k}{MN} \quad \text{for} \quad k = 0, 1, 2 \cdots L-1$$

where, $p(r_k)$ gives the estimate of the probability of occurance of gray level $r_k$.

MN is the total number of pixels in an image.

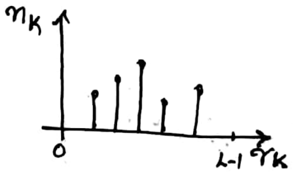
The sum of all components of a normalized histogram is equal to '1'.

$$\therefore \; h(r_k) = n_k$$

$$\text{or } h(r_k) = p(r_k) = \frac{n_k}{MN}$$

hence histograms are simply plots of $[h(r_k) = n_k$ versus $r_k]$ or $[p(r_k) = n_k/MN$ versus $r_k]$

eg:

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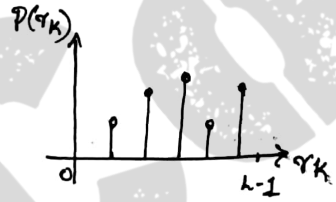


- Histograms manipulation can be used for image enhancement in Spatial domain. Although histograms are simple to calculate in Software & also economic for hardware implementations. thus it is very popular in real time image processing.

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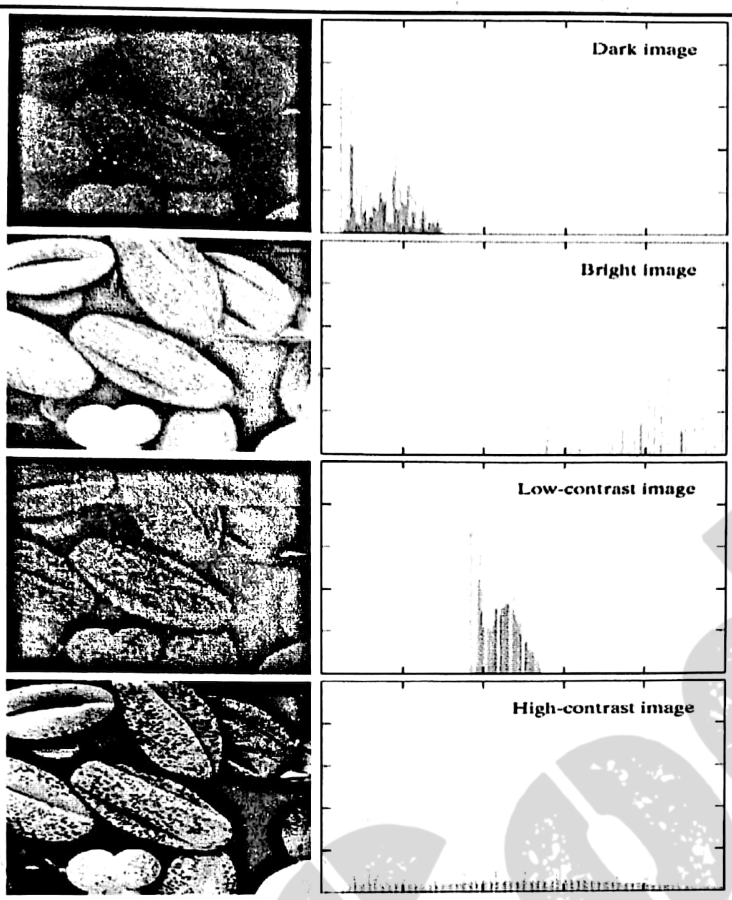


a b

**FIGURE 3.15** Four basic image types: dark, light, low contrast, high contrast, and their corresponding histograms. (Original image courtesy of Dr. Roger Heady, Research School of Biological Sciences, Australian National University, Canberra, Australia.)

In the dark image the components of the histogram are concentrated on the low (dark) side of the gray scale. In case of bright image the histogram components are shifted towards the high side of the gray scale.

The histogram of a low contrast image will be narrow & will be centered towards the middle of the gray scale.

The components of the histogram in the high contrast image cover a broad range of the gray scale. The net effect of this will be an image that shows a great deal of gray levels details & has high dynamic range.

# Histogram Equalization

Histogram equalization is a common technique for enhancing the appearance of images. Suppose we have an image which is predominantly dark. Then its histogram would be skewed towards the lower end of the gray scale & all the image detail are compressed into the dark end of the histogram.

If we could 'stretch out' the gray levels at the dark end to produce a more uniformly distributed histogram then the image would become much clearer.

Let there be a continuous function with $r$ being gray levels of the image to be enhanced. with range $[0, L-1]$ with $r=0$ representing black & $r=L-1$ representing white.

The transformation function is give by.

$$s = T(r) \quad , \quad 0 \le r \le L-1$$

It produces a level 's' for every pixel value $r$ in the original image.

The transformation function is assumed to fulfill following conditions

a) $T(r)$ is a monotonically increasing function in the interval $0 \le r \le L-1$

b) $0 \le T(r) \le L-1$ for $0 \le r \le L-1$

In some formulations, $r = T^{-1}(s)$ $\quad 0 \le s \le L-1$

in which case we change condition (a) to

(a') $T(r)$ is a strictly monotonically increasing function in the interval $0 \le r \le L-1$

- Monotonically increasing condition preserves the increasing order from black to white in the o/p image.

→ The second condition guarantees that the o/p gray levels will be in same range as the i/p levels.

→ Third condition guarantees that the mapping from $s$ back to $r$ will be one to one, thus preventing ambiguities.

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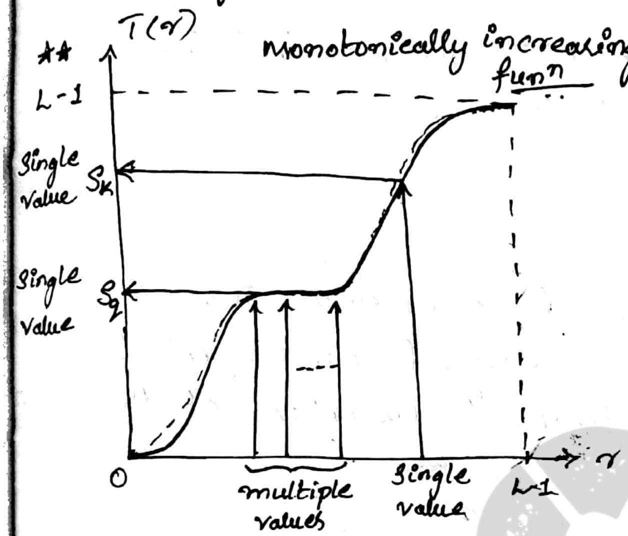
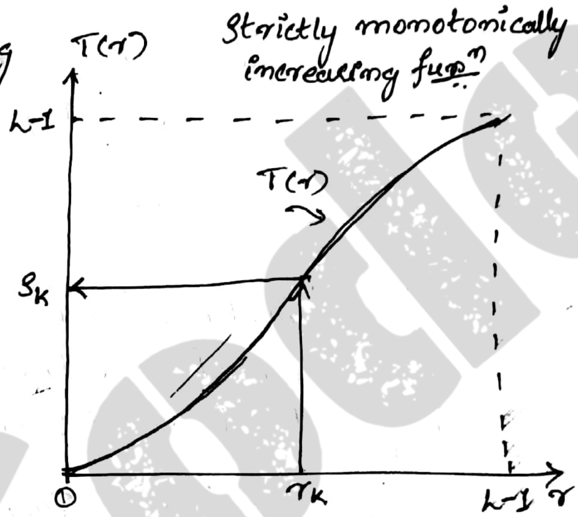



- The gray levels of the image may be viewed as random variables in the interval [0, L-1]. A fundamental descriptor of a random variable is its probability density function [PDF].

Let $P_r(r)$ & $P_s(s)$ denote the PDF of random variables $r$ & $s$ respectively. Basic results from an elementary probability theory states that if $P_r(r)$ & $T(r)$ are known & $T(r)$ is continuous & differentiable over the range of values of interest.

then $$P_s(s) = P_r(r)\left|\frac{dr}{ds}\right| \longrightarrow (1)$$

Thus the PDF of the transformed variable $s$ is the determined by the gray levels PDF of the i/p image & by the chosen transformations function.

A transformation function of a particular importance in image processing.

$$S = T(r) = (L-1)\int_0^r P_r(w)\, dw \longrightarrow ②$$

where, 'w' is a dummy variable of integration.
The above eqⁿ is the cumulative distribution function of "r".

using this definition of T we see that the derivative of S with respect to 'r' is

$$\frac{ds}{dr} = P_r(r)$$

from Leibniz's rule in basic calculus that the derivative of a definite integral with respect to its upper limit is the integrand evaluated at the limit.

ie $$\frac{dS}{dr} = \frac{dT(r)}{dr} \longrightarrow ③$$

from eqⁿ ②

$$\frac{ds}{dr} = (L-1)\frac{d}{dr}\left[\int_0^r P_r(w)\, dw\right] \longrightarrow 3ⓐ$$

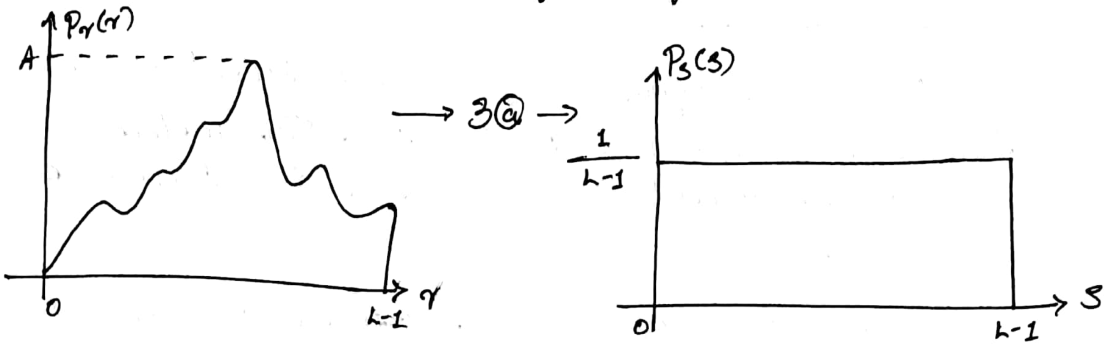
$$\frac{ds}{dr} = (L-1)\, P_r(r) \longrightarrow ④ \qquad \because P_r(r) = \frac{d}{dr}\left(\int_0^r P_r(w)\, dw\right)$$

Substituting eqⁿ ④ in ① we get

$$P_s(s) = P_r(r)\left|\frac{1}{(L-1)\, P_r(r)}\right|$$

$$P_s(s) = \frac{1}{L-1} \qquad 0 \le s \le L-1$$

$P_s(s)$ is a uniform probability density function




$P_s(s)$ always uniform, independent of $P_r(r)$

The discrete form of the transform is.

$$S_K = T(r_K) = (L-1)\sum_{j=0}^{K} P_r(r_j)$$

$$S_K = \frac{(L-1)}{MN}\sum_{j=0}^{K} n_j \qquad K = 0, 1, 2, \text{----} L-1$$

Given an image the process of histogram equalization consists simple of implementing the transformation. funⁿ which is based on information that can be extracted. directly from the given image, without the need for further parameter specification.

Equalization automatically determines a transformation function that seeks to produce an o/p image that has a uniform histogram. It is a good approach when automatic enhancement is needed.

## Histogram Matching (Specification)

Histogram equalization is best suited when automatic enhancement is desired. This method is simple to implement & results from this techniques are predictable.

But in some application, uniform histogram is not the best approach. In particular, It is useful some times to be able to specify the shape of the histogram that we wish the processed image to have.

The method used to generate a processed image that has a specified histogram is called Histogram matching (or) Histogram Specification.

Let $P_r(r)$ & $P_z(z)$ denote their corresponding continuous probability density function. where $r$ & $z$ denote the intensity levels of the i/p & o/p images. $P_z(z)$ is the specified PDF.

Let $s$ be a random variable with the property.

$$s = T(r) = L-1 \int_0^r P_r(w) \cdot dw \longrightarrow (1)$$

where, as before, $w$ is a dummy variable of integration. Suppose that we define a random variable $z$ with the property.

$$G(z) = L-1 \int_0^z P_z(t) \cdot dt = s \longrightarrow (2)$$

$$\therefore G(z) = T(r)$$

$$z = G^{-1}[T(r)] = G^{-1}(s) \longrightarrow (3)$$

Equation (1) & (3) shows that an image where intensity levels have a specified probability density function can be obtained from a given image by using the following procedure:

1) Obtain $P_r(r)$ from the i/p image & use eqⁿ ① to obtain the value of 'S'.

2) use the specified PDF in eqⁿ ② to obtain the transformation function $G(z)$.

3) Obtain the inverse transformation $z = G^{-1}(S)$

4) Obtain the o/p image by first equalizing the i/p image using eqⁿ ①; the pixel values in this image are the S values. For each pixel with values S in the equalized image, perform the inverse mapping $z = G^{-1}(S)$ to obtain the corresponding pixel in the o/p image. When all pixels have been thus processed, the PDF of the o/p image will be equal to the specified PDF

Assuming continuous intensity values, suppose that an image has the intensity PDF $P_r(r) = \frac{2r}{(L-1)^2}$ for $0 \le r \le (L-1)$ & $P_r(r) = 0$ for other values of 'r'.

Find the Transformation function that will produce an image whose intensity PDF is $P_z(z) = \frac{3z^2}{(L-1)^3}$ for $0 \le z \le (L-1)$ & $P_z(z) = 0$ for other values of z.

WKT

Histogram Equalization

$$S = T(r) = (L-1)\int_0^r P_r(w)\,dw = \frac{2}{L-1}\int_0^r w\,dw = \frac{r^2}{(L-1)}$$

we are interested in an image with a specified histogram so we find next.

$$G(z) = (L-1)\int_0^z P_z(w)\cdot dw = \frac{3}{(L-1)^2}\int_0^z w^2\cdot dw = \frac{z^3}{(L-1)^2}$$

finally we require that $G(z) = s$, but $G(z) = \frac{z^3}{(L-1)^2}$

So $\frac{z^3}{(L-1)^2} = s$ & we have $\underline{z = \left[(L-1)^2 s\right]^{1/3}}$

we can generate the $z$'s directly from the intensities '$r$' of the i/p image.

$$z = \left[(L-1)^2 s\right]^{1/3} = \left[(L-1)^2 \frac{r^2}{(L-1)}\right]^{1/3} = \left[(L-1) r^2\right]^{1/3}$$

from the above eq$^n$ intermediate step of equalizing the image can be skipped; all we need to obtain the transformation function $T(r)$ that maps $r$ to $s$. Then the two steps can be obtained into a single transformation from $r$ to $z$.

The discrete formulation of the histogram equalization transformation is given by.

$$S_K = T(r_K) = (L-1)\sum_{j=0}^{K} P_r(r_j) \longrightarrow (4)$$

$$S_K = T(r_K) = \frac{L-1}{MN}\sum_{j=0}^{K} n_j \;, \quad K = 0,1,2,\dots L-1$$

Similarly, given a specific value of $S_K$, the discrete formulation involves computing the transformation function

$$G(z_q) = (L-1)\sum_{i=0}^{q} P_z(z_i) \longrightarrow (5)$$

for a value of $q$, so that $G(z_q) = S_K$.

$$\underline{z_q = G^{-1}(S_K)}$$

In other words, this operation gives a value of z for each value of S; thus it performs a mapping from S to Z.

## Local histogram processing:

Histogram equalization & Histogram matching methods are global. These techniques are suitable for overall enhancement of an image.

But there are some applications in which it is necessary to enhance details over small areas in an image. The number of pixels in these areas may have negligible influence on the computation of a global transformation whose shape does not necessarily guarantee the desired local enhancement.

The solution is to device transformation functions based on the intensity distribution in a neighborhood of every pixel in the image.

Histogram processing are easily adapted to local enhancement. The procedure is to define a neighborhood & move its center from pixel to pixel.

At each location, the histogram equalization or histogram specification transformation function is obtained. This function is then used to map the intensity of the pixel centered in the neighborhood.

The neighborhood center is then moved to an adjacent pixel location & the process is repeated.

# Histogram Statistics for Image Enhancement

1) Mean of $n^{th}$ moment of $r$

$$\mu_n(r) = \sum_{i=0}^{L-1} (r_i - m)^n \, p(r_i)$$

$m \rightarrow$ mean (avg intensity) value of $r$

$$m = \sum_{i=0}^{L-1} P(r_i)\, r_i$$

Eg: For 2nd moment

$$\mu_2(r) = \sum_{i=0}^{L-1} (r_i - m)^2 \, P(r_i)$$

Sample mean,

$$m = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y)$$

Sample Variance,

$$\sigma^2 = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} [f(x,y) - m]^2$$

1) Consider the two images subsets, $S_1$ & $S_2$ shown in the following fig. For $V=\{1\}$, determine whether these two subsets are (a) 4-adjacent (b) 8-adjacent or (c) m-adjacent.

$S_1$ $S_2$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 | 0 | 1 (q) | 1 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 (p) | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 |

we have to choose these pixels as p & q.

(a) $S_1$ & $S_2$ are not 4-Connected because $q$ is not in the set $N_4(p)$.

(b) $S_1$ & $S_2$ are 8-Connected because $q$ is in the set $N_8(p)$

(c) $S_1$ & $S_2$ are m-Connected because.

(a) $q$ is in $N_D(p)$

(b) the set $N_4(p) \cap N_4(q)$ is empty.

2) (a) ~~take~~ Let $V=\{0,1\}$ & Compute the lengths of the shortest 4, 8 & m-path betn p & q. If a particular path doesnot exist between these two points. Explain why.

(b) Repeat for $V=\{1,2\}$

| | | | | |
|---|---|---|---|---|
| | 3 | 1 | 2 | 1 (q) |
| | 2 | 2 | 0 | 2 |
| | 1 | 2 | 1 | 1 |
| (p) | 1 | 0 | 1 | 2 |

(a) when $V = \{0, 1\}$ 4-path does not exist between p & q because it is impossible to get from p to q by travelling along points that are both 4-adjacent & also have values from V.

ie

| | | | | |
|---|---|---|---|---|
| | 3 | 1 | 2 | 1 (q) |
| | 2 | 2 | 0 | 2 |
| | 1 | 2 | 1 | 1 |
| (p) | 1 | 0 | 1 | 2 |

⟶ 4 & 8 path

- - - - - -> m path.

One possibility for the 8 path (it is not unique)

$V = \{1, 2\}$

| | | | | |
|---|---|---|---|---|
| | 3 | 1 | 2 | 1 (q) |
| | 2 | 2 | 0 | 2 |
| | 1 | 2 | 1 | 1 |
| (p) | 1 | 0 | 1 | 2 |

Length = 4

| | | | | |
|---|---|---|---|---|
| | 3 | 1 | 2 | 1 (q) |
| | 2 | 2 | 0 | 2 |
| | 1 | 2 | 1 | 1 |
| (p) | 1 | 0 | 1 | 2 |

4-path (not unique)

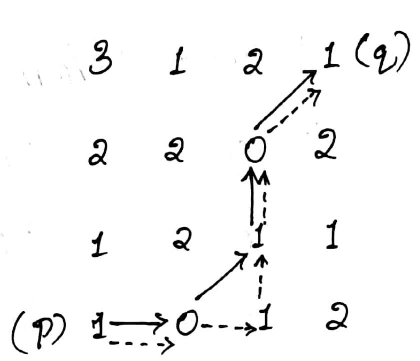
Length = 6

| | | | | |
|---|---|---|---|---|
| | 3 | 1 | 2 | 1 (q) |
| | 2 | 2 | 0 | 2 |
| | 1 | 2 | 1 | 1 |
| (p) | 1 | 0 | 1 | 2 |

m-path

Length = 6

for $V = \{0, 1\}$ 3




8 path, length = 4

m path, length = 5

## Question paper Problems

① Perform Histogram equalization of the image

$$\begin{bmatrix} 4 & 4 & 4 & 4 & 4 \\ 3 & 4 & 5 & 4 & 3 \\ 3 & 5 & 5 & 5 & 3 \\ 3 & 4 & 5 & 4 & 3 \\ 4 & 4 & 4 & 4 & 4 \end{bmatrix}$$

$M \times N = 5 \times 5$

**Soln:** Intensity levels are integers in the range $[0, L-1]$
$= [0, 7]$

$$S_k = T(r_k) = (L-1) \sum_{j=0}^{k} P_r(r_j)$$

$$= \frac{(L-1)}{MN} \sum_{j=0}^{k} n_j \qquad k = 0, 1, 2, \ldots 7$$

$$P_r(r_k) = \frac{n_k}{MN}$$

| $r_K$ | $n_K$ | $P_r(r_K) = n_K/MN$ | $S_K$ | Histogram equalized values |
|---|---|---|---|---|
| $r_0 = 0$ | 0 | 0 | $S_0 = 0$ | 0 |
| $r_1 = 1$ | 0 | 0 | $S_1 = 0$ | 0 |
| $r_2 = 2$ | 0 | 0 | $S_2 = 0$ | 0 |
| $r_3 = 3$ | 6 | 0.24 | $S_3 = 1.68$ | 2 |
| $r_4 = 4$ | 14 | 0.56 | $S_4 = 5.6$ | 6 |
| $r_5 = 5$ | 5 | 0.2 | $S_5 = 7$ | 7 |
| $r_6 = 6$ | 0 | 0 | $S_6 = 7$ | 7 |
| $r_7 = 7$ | 0 | 0 | $S_7 = 7$ | 7 |

$K=0, \quad S_0 = (7)\sum_{j=0}^{0} 0 = \underline{\underline{0}}$

$K=1, \quad S_1 = 7\sum_{j=0}^{1} P_r(r_j) = 0+0 = \underline{\underline{0}}$

$K=2, \quad S_2 = 7\sum_{j=0}^{2} P_r(r_j) = 0+0+0 = \underline{\underline{0}}$

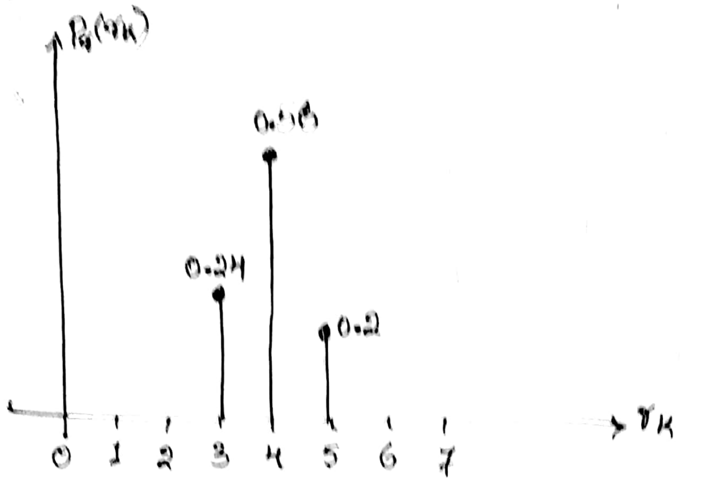
$K=3, \quad S_3 = 7\sum_{j=0}^{3} P_r(r_j) = 0+0+0+0.24 = \underline{\underline{1.68}}$

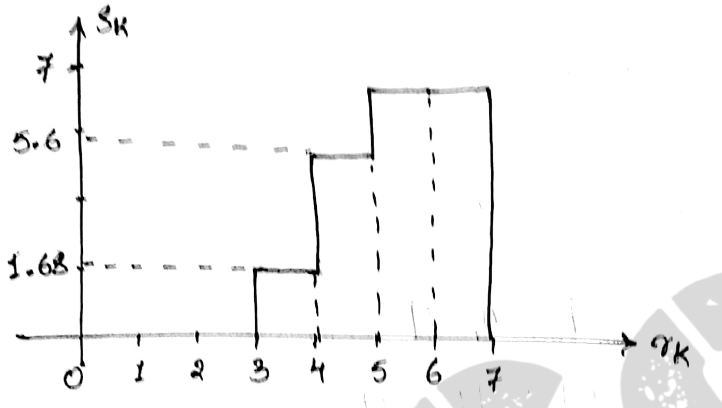
$K=4, \quad S_4 = 7\sum_{j=0}^{4} P_r(r_j) = 0+0+0+0.24+0.56 = \underline{\underline{5.6}}$

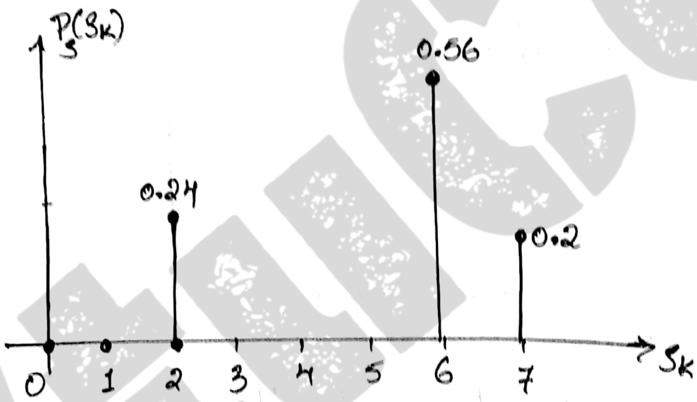
$K=5, \quad S_5 = 7\sum_{j=0}^{5} P_r(r_j) = 0+0+0+0.24+0.56+0.2 = \underline{\underline{7}}$

$K=6, \quad S_6 = \underline{\underline{7}}$

$K=7, \quad S_7 = \underline{\underline{7}}$




② Let $p$ & $q$ be the pixels at co-ordinates $(10, 12)$ & $(15, 20)$ respectively. Find out which distance measure gives the minimum distance betn the pixels.

Soln let $(x, y) = (10, 12)$

$(s, t) = (15, 20)$

Euclidean distance

$$D_e(p,q) = \left[(x-s)^2 + (y-t)^2\right]^{1/2}$$

$$D_e(p,q) = \left[(10-15)^2 + (12-20)^2\right]^{1/2}$$

$$D_e(p,q) = \left[(-5)^2 + (-8)^2\right]^{1/2}$$

$$D_e(p,q) = \left[25 + 64\right]^{1/2}$$

$$D_e(p,q) = 9.43 \simeq 9$$

$D_4$ - distance [city block distance]

$$D_4(p,q) = |x-s| + |y-t|$$

$$D_4(p,q) = |10-15| + |12-20|$$

$$D_4(p,q) = 5 + 8$$

$$D_4(p,q) = 13$$

$D_8$ distance [Chess board distance]

$$D_8(p,q) = \max(|x-s|, |y-t|)$$

$$D_8(p,q) = \max(|10-15|, |12-20|)$$

$$D_8(p,q) = \max(5, 8)$$

$$D_8(p,q) = 8$$ (min distance)

③ find $D_8$ & $D_m$ for the following 2-D section with $V=\{0,1\}$ & $V=\{1,2\}$ betn $p$ & $q$. 7

$$\begin{array}{cccccc} & 5 & 4 & 3 & 1 & 1 \ (q)\ (s,t) \\ & 5 & 4 & 0 & 2 & 0 \\ & 3 & 2 & 0 & 2 & 4 \\ & 2 & 1 & 1 & 3 & 5 \\ (p)\ (x,y) & 1 & 3 & 5 & 1 & 3 \end{array}$$

Soln for $V=\{0,1\}$

$$D_8 = \max(|x-s|, |y-t|)$$

WKT, The above image is of size 5x5

$(x,y) = (4,1)$ 5

$(s,t) = (1,4)$ 5

$$V=\{0,1\} \qquad \begin{array}{cccccc} & 5 & 4 & 3 & 1 & 1 \ (q) \\ & 5 & 4 & 0 & 2 & 0 \\ & 3 & 2 & 0 & 2 & 4 \\ & 2 & 1 & 1 & 3 & 5 \\ (p) & 1 & 3 & 5 & 1 & 3 \end{array}$$

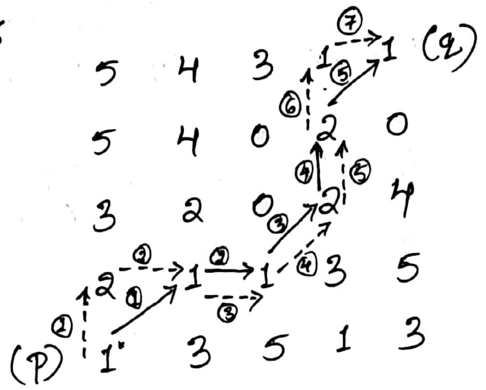
8 path length is $\underline{5}$

m path length is $\underline{6} = D_m$

$$D_8 = \max(|4-1|, |1-4|) = (|3|, |-3|)$$

$$\boxed{D_8 = 3}$$

for $V = \{1, 2\}$




8-path length = 5

$D_m$ = m-path length = 7

**Note**

----> m-path

——> 8-path

(4) Perform histogram equalization for a 3-bit image of size 64×64 using the following data

| $r_k$ | $n_k$ | $P_r(r_k) = n_k/MN$ |
|---|---|---|
| $r_0 = 0$ | 790 | 0.19 |
| $r_1 = 1$ | 1023 | 0.25 |
| $r_2 = 2$ | 850 | 0.21 |
| $r_3 = 3$ | 656 | 0.16 |
| $r_4 = 4$ | 329 | 0.08 |
| $r_5 = 5$ | 245 | 0.06 |
| $r_6 = 6$ | 122 | 0.03 |
| $r_7 = 7$ | 81 | 0.02 |

Soln:
$$S_K = T(r_K) = (L-1)\sum_{j=0}^{K} P_r(r_j)$$

$S_0 = 1.33$ $S_3 = 5.67$ $S_6 = 6.86$

$S_1 = 3.08$ $S_4 = 6.23$ $S_7 = 7.00$

$S_2 = 4.55$ $S_5 = 6.65$

| $r_K$ | $S_K$ | histogram equalized values |
|---|---|---|
| 0 | 1.33 | 1 |
| 1 | 3.08 | 3 |
| 2 | 4.55 | 5 |
| 3 | 5.67 | 6 |
| 4 | 6.23 | 6 |
| 5 | 6.65 | 7 |
| 6 | 6.86 | 7 |
| 7 | 7.00 | 7 |

$P_s(S_K) = n_K / MN$

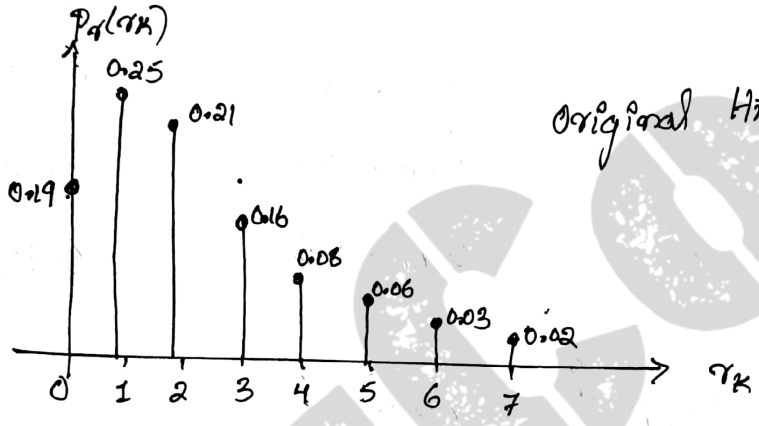
$r_0 \rightarrow 1 \rightarrow 790/4096 = 0.19$

$r_1 \rightarrow 3 \rightarrow 1023/4096 = 0.25$

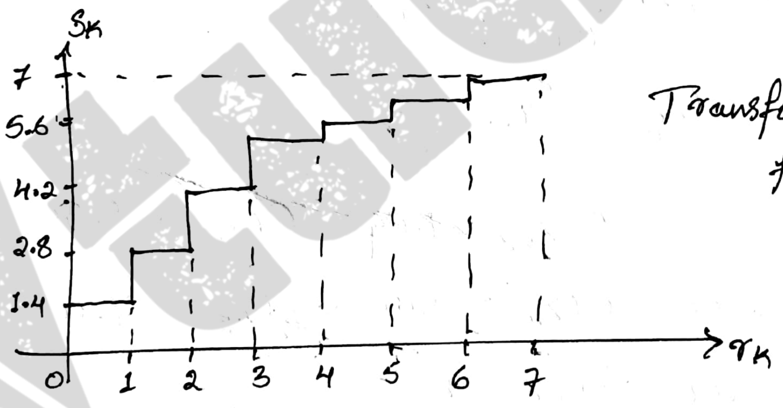
$r_2 \rightarrow 5 \rightarrow 850/4096 = 0.21$

$(r_3+r_4) \rightarrow 6 \rightarrow \frac{656+829}{4096} = 0.24$

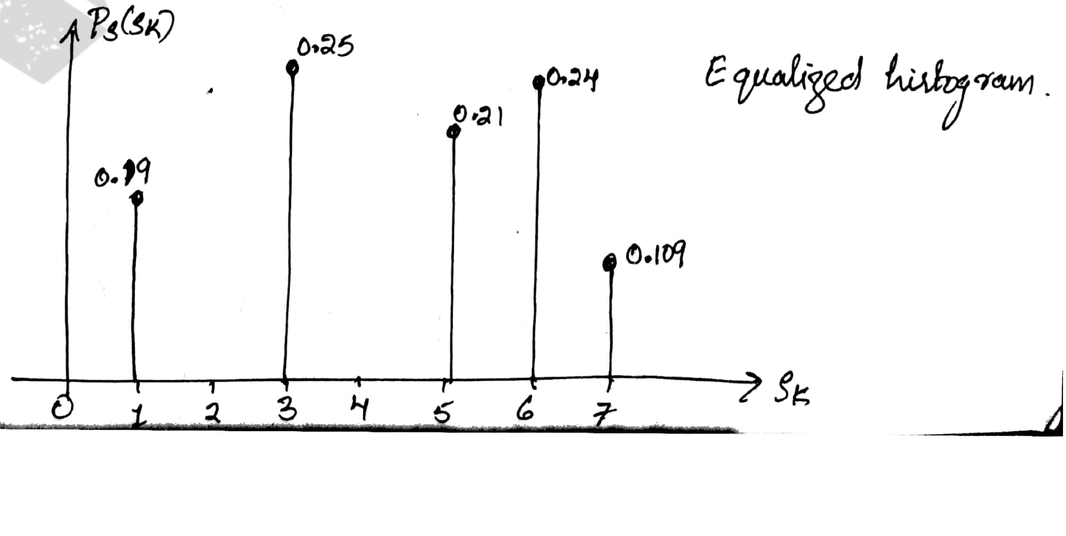
$(r_5+r_6+r_7) \rightarrow 7 \rightarrow \frac{245+122+81}{4096} = 0.109$




Original Histogram




Transformation function




Equalized histogram.

Histogram matching (continuation) 10

| $z_q$ | Specified $P_z(z_q)$ | ~~Actual~~ $G(z_q)$ | Actual Value $P_z(z_k)$ |
|---|---|---|---|
| | | quantization | 0.0 |
| $z_0 = 0$ | 0.00 | $0. \rightarrow 0$ | |
| $z_1 = 1$ | 0.00 | $0 \rightarrow 0$ | 0.0 |
| $z_2 = 2$ | 0.00 | $0 \rightarrow 0$ | 0.0 |
| $z_3 = 3$ | 0.15 | $1.05 \rightarrow 1$ | 0.19 |
| $z_4 = 4$ | 0.20 | $2.45 \rightarrow 3$ | 0.25 |
| $z_5 = 5$ | 0.30 | $4.55 \rightarrow 5$ | 0.21 |
| $z_6 = 6$ | 0.20 | $5.95 \rightarrow 6$ | 0.24 |
| $z_7 = 7$ | 0.15 | $7.00 \rightarrow 7$ | 0.109 |

$$G(z_q) = (L-1)\sum_{i=0}^{q} P_z(z_i)$$

$$G(z_q) = S_k.$$

$G(z_0) = 0$ $\quad G(z_3) = 1.05$ $\quad G(z_6) = 5.95$

$G(z_1) = 0.00$ $\quad G(z_4) = 2.45$ $\quad G(z_7) = 7.00$

$G(z_2) = 0.00$ $\quad G(z_5) = 4.55$

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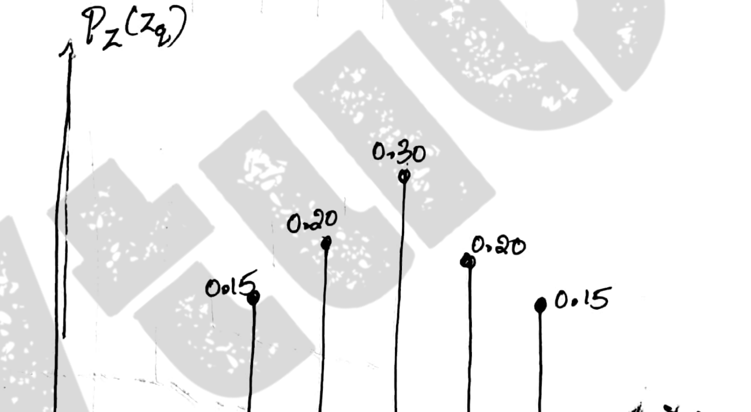
| $Z_q$ | $G(Z_q) = S_k$ |
|---|---|
| $Z_0 = 0$ | 0 |
| $Z_1 = 1$ | 0 |
| $Z_2 = 2$ | 0 |
| $Z_3 = 3 \leftarrow$ | 1 |
| $Z_4 = 4 \leftarrow$ | 3 |
| $Z_5 = 5 \leftarrow$ | 5 |
| $Z_6 = 6 \leftarrow$ | 6 |
| $Z_7 = 7 \leftarrow$ | 7 |

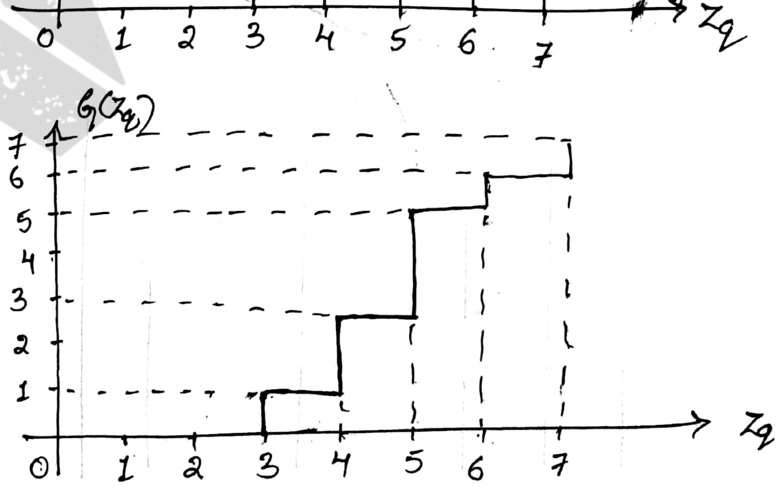
$\therefore$

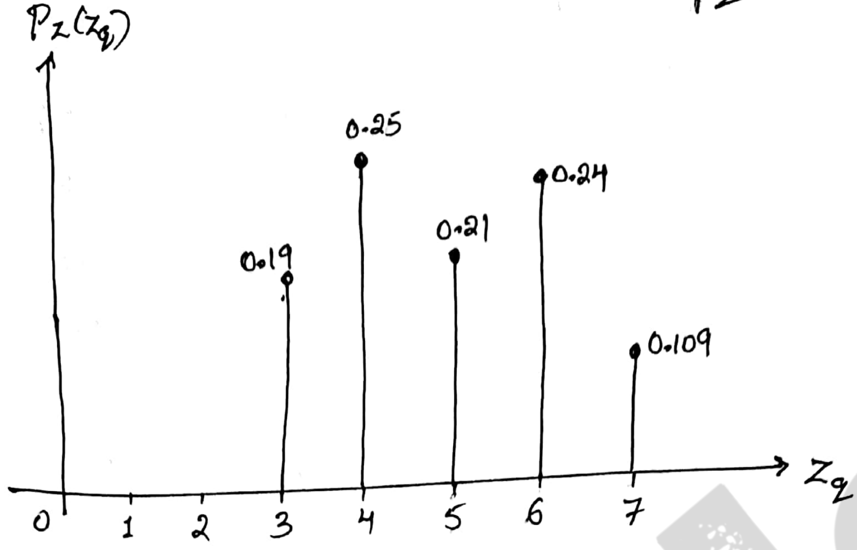
| $S_k$ | $\rightarrow$ | $Z_q$ |
|---|---|---|
| 1 | $\rightarrow$ | 3 |
| 3 | $\rightarrow$ | 4 |
| 5 | $\rightarrow$ | 5 |
| 6 | $\rightarrow$ | 6 |
| 7 | $\rightarrow$ | 7 |

$\therefore G(Z_q) = S_k$

$\therefore Z_q = G^{-1}(S_k)$






Calculation of $P_z(z_q)$ is same as $P_s(s_k)$

## Fundamentals of Spatial filtering:

Spatial filtering is one of the principal tools used in the field of image processing for a broad spectrum of applications. The name filter is borrowed from frequency domain processing. where filtering refers to passing or rejecting certain frequency components.

eg: Low pass filtering, ⟶ Smoothening of image.

Spatial filters are also called Spatial masks, Kernels, templates, & windows

## The mechanics of Spatial filtering:

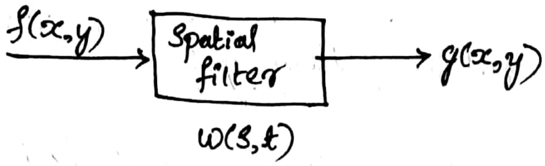
Spatial filter consists of 1) neighborhood

2) A predefined operation that is performed on the image pixels encompassed by the neighborhood.

A filtered image is generated by assigning the results of filtering (new pixel value) to a corresponding location as the center of the filter visits each pixel in the input image.

If the operations performed on the image pixels is linear, then the filter is called a linear Spatial filter. Otherwise nonlinear.

Let us consider any point $(x,y)$ in the image, the response $g(x,y)$ of the filter is the sum of products of the filter co-efficients & the image pixels.




$$g(x,y) = f(x,y) * w(s,t)$$

$$g(x,y) = w(-1,-1), f(x-1,y-1) + w(-1,0)f(x-1,y) + \cdots$$
$$+ w(0,0)f(x,y) + \cdots + w(1,1)f(x+1,y+1)$$

The center co-efficient of the filter $w(0,0)$, aligns with the pixels at $(x,y)$ for a mask of size $m \times n$

<u>assume</u> $m = 2a+1$, $n = 2b+1$ where $a$ & $b$ are positive constants.

Smallest filter size is $3 \times 3$ (odd value is preferable)

In general, linear spatial filtering of an image of size $M \times N$ with a filter of size $m \times n$ is given by the expression

$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t)$$

where $x$ & $y$ are varied so that each pixel in $w$ visits every pixel in $f$.

The closely related operations for spatial filtering are

1) <u>Correlation</u>: It is a process of moving a filter mask over the image & computing the sum of products at each location.

2) <u>Convolution</u>: The process is same as above but, the filter must be rotated by 180° first & then perform the operation.

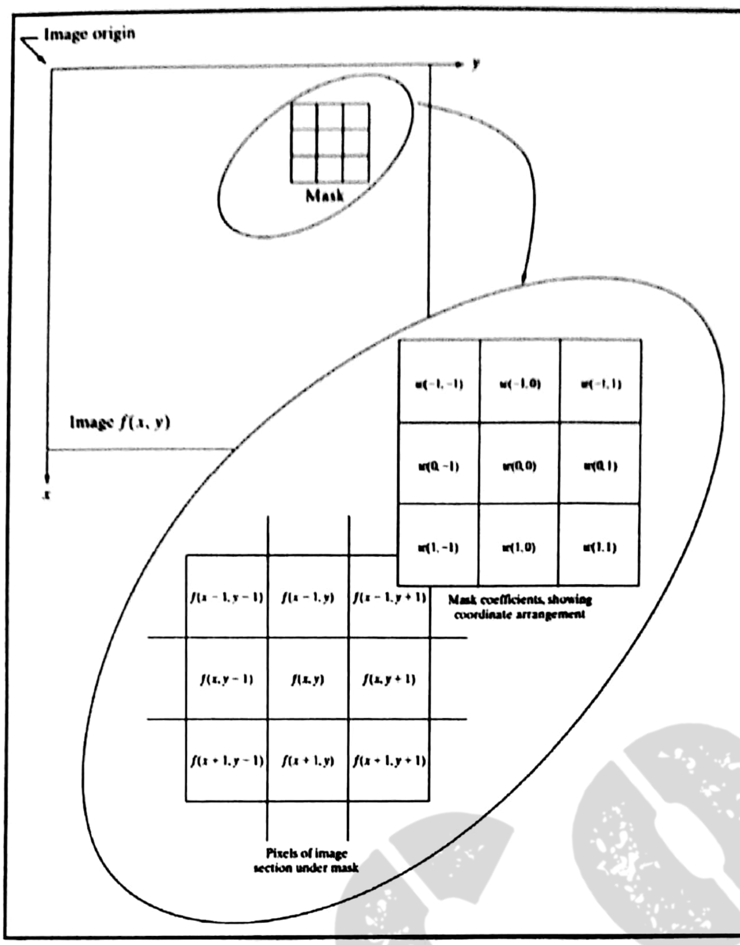



**FIGURE 3.32** The mechanics of spatial filtering. The magnified drawing shows a 3 × 3 mask and the image section directly under it; the image section is shown displaced out from under the mask for ease of readability.

## Image Enhancement in the frequency domain :-

Image filtering can be done in frequency domain by means of Fourier Transformation. Before going to develop image processing algorithms using fourier transform, it is required to look into its basics.

★ Fourier Series States that any periodic function can be expressed as the sum of Sines and/or Cosines of different frequencies, each multiplied by a different Co-efficient.

Even functions that are not periodic can be expressed as the integral of sines and/or cosines multiplied by a weighing function. This formulation is called Fourier transform

Both representations share the important characteristic that a function, expressed in either a fourier series or transform, can be reconstructed completely via an inverse process, with no loss of information.

Important mathematical equations: [2Dimensional]

★ Discrete fourier transform of $f(x,y)$

$$F(u,v) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y)\, e^{-j2\pi(ux/M + vy/N)}, \quad \begin{matrix} 0 \le u \le M-1 \\ 0 \le v \le N-1 \end{matrix}$$

★ Inverse discrete fourier transform of $F(u,v)$

$$f(x,y) = \frac{1}{MN} \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} F(u,v)\, e^{j2\pi(ux/M + vy/N)}, \quad \begin{matrix} 0 \le x \le M-1 \\ 0 \le y \le N-1 \end{matrix}$$

where $f(x,y)$ is a digital image of size $M \times N$

★ Polar form representation

$$F(u,v) = |F(u,v)|\, e^{j\phi(u,v)}$$

where $$|F(u,v)| = \left[ R^2(u,v) + I^2(u,v) \right]^{1/2}$$
is called fourier spectrum.

$$\phi(u,v) = \arctan\left[\frac{I(u,v)}{R(u,v)}\right]$$

is called phase angle.

* Power Spectrum,

$$P(u,v) = |F(u,v)|^2$$

$$P(u,v) = R^2(u,v) + I^2(u,v)$$

where R & I are the real & imaginary parts of $F(u,v)$.

* The fourier transform of a real function is Conjugate symmetric ~~is~~. i.e $F^*(u,v) = F(-u,-v)$ if $f(x,y)$ is real.

if $f(x,y)$ is imaginary, its fourier transform is conjugate antisymmetric: $F^*(-u,-v) = -F(u,v)$

$$\therefore \quad |F(u,v)| = |F(-u,-v)|$$

* Phase angle exhibits odd symmetry about the origin

$$\phi(u,v) = -\phi(-u,-v)$$

* WKT, $F(u,v) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y)\, e^{-j2\pi(ux/M + vy/N)}$

let $u=0$ & $v=0$

$F(0,0) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y)$ which indicates that the zero frequency term is proportional to the avg value of $f(x,y)$

i.e $F(0,0) = MN \cdot \underbrace{\frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y)}$

$F(0,0) = MN\, \bar{f}(x,y)$ where $\bar{f}$ denotes avg of '$f$'

& $|F(0,0)| = MN\, |\bar{f}(x,y)|$

Because frequency component $u$ & $v$ are zero at the origin, $F(0,0)$ is called the DC Component of the transform.

* Circular Convolution is preferable for image Processing. because here we are dealing with images of Same Size.

$$f(x,y) * h(x,y) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} f(m,n)\, h(x-m,\, y-n)$$

$$, x = 0,1,2,\ldots M-1$$
$$\& \; y = 0,1,2,\ldots N-1$$

Also $f(x,y) * h(x,y) \Leftrightarrow F(u,v) H(u,v)$

Conversely $f(x,y) \cdot h(x,y) \Leftrightarrow F(u,v) * H(u,v)$

Zero padding is an important task while performing Circular convolution [To make length of the sequences equal]

Let $f(x,y)$ & $h(x,y)$ be two image arrays of sizes $A \times B$ & $C \times D$ pixels, respectively. Wraparound error in their Circular convolution can be avoided by padding these functions with zeros.

$$f_p(x,y) = \begin{cases} f(x,y) & 0 \le x \le A-1 \;\; \& \;\; 0 \le y \le B-1 \\ 0 & A \le x \le P \;\; \text{or} \;\; B \le y \le Q \end{cases}$$

$$\& \; h_p(x,y) = \begin{cases} h(x,y) & 0 \le x \le C-1 \;\; \& \;\; 0 \le y \le D-1 \\ 0 & C \le x \le P \;\; \text{or} \;\; D \le y \le Q \end{cases}$$

with $P \ge A+C-1$

$Q \ge B+D-1$

Resulting padded images are of size $P \times Q$. if both arrays are of same size $M \times N$ then we require

$$P \geq 2M-1$$
&
$$Q \geq 2N-1$$

Translation to center of the frequency rectangle. [M/2, N/2]

ie $f(x,y)(-1)^{x+y} \Longleftrightarrow F(u-M/2, v-N/2)$

& $f(x-M/2, y-N/2) \Longleftrightarrow F(u,v)(-1)^{u+v}$

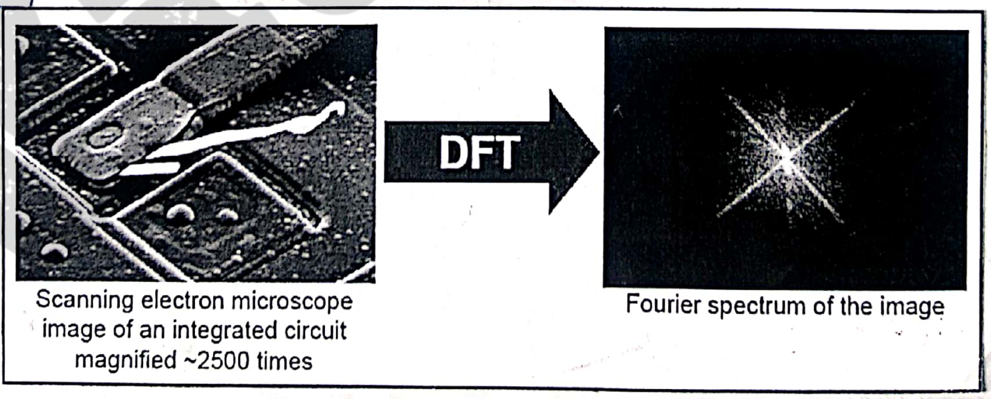
## The Basics of filtering in the frequency domain.

Filtering techniques in the frequency domain are based on modifying the fourier transform to achieve a specific objective & then computing the IDFT to get back to the image domain.

WKT $F(u,v)$ has Magnitude $|F(u,v)|$ & phase $\angle F(u,v)$ components

Analysis of phase angle is not very useful. However, The Magnitude (Spectrum) provides some useful guidelines as to gross characteristics of the image from which the spectrum was generated.

Example




Scanning electron microscope image of an integrated circuit magnified ~2500 times

Fourier spectrum of the image

In fig (a) we note two principal features: Strong edges that run approximately at $\pm 45^0$ & two white oxide protrusions resulting from thermally induced failure.

In fig (b) The fourier Spectrum Shows prominent Components along the $\pm 45^0$ directions that correspond to the edges just mentioned. Looking Carefully along the vertical axis, we see a vertical component that is off-axis slightly to the left. This component was Caused by the edges of the oxide protrusions.

Note how the angle of the frequency Component with respect to the vertical axis Corresponds to the inclination of the long white element, & note also the zeros in the vertical frequency component, Corresponding to the narrow vertical Span of the oxide protusions.

## frequency domain filtering fundamentals:

filtering in the frequency domain Consists of modifying the fourier transform of an image & then computing the inverse transform to obtain the processed result.

Basic filtering equation is $\boxed{g(x,y) = \mathcal{F}^{-1}\{H(u,v).F(u,v)\}}$

where $\mathcal{F}^{-1} \longrightarrow$ IDFT, $F(u,v) \longrightarrow$ DFT of $f(x,y)$ &
$H(u,v)$ is a filter function [filter transfer function]
$g(x,y)$ is filtered image (o/p).

Here $H(u,v)$, $F(u,v)$ & $g(x,y)$ are arrays of Same Size $M \times N$

The filter function $H(u,v)$ modifies $F(u,v)$ to yield $g(x,y)$. The filter function is symmetric about their center, which requires that $F(u,v)$ be centered also. This can be accomplished by multiplying the i/p image by $(-1)^{x+y}$ prior to computing its transform.

Low frequencies in the transform are related to slowly varying intensity components in an image, such as walls of a room or a cloudless sky in an outdoor scene.

High frequencies are caused by sharp transitions in intensity, such as edges & noise.

A low pass filter that attenuates high frequencies while passing low frequencies would blur an image, while a high pass filter would enhance sharp detail, but cause a reduction in contrast of the image.

In fig ©, Adding a small constant to the filter does not affect sharpening appreciably. but it does prevent elimination of the DC term & thus preserves tonality.

Result of removal of dc term.

**

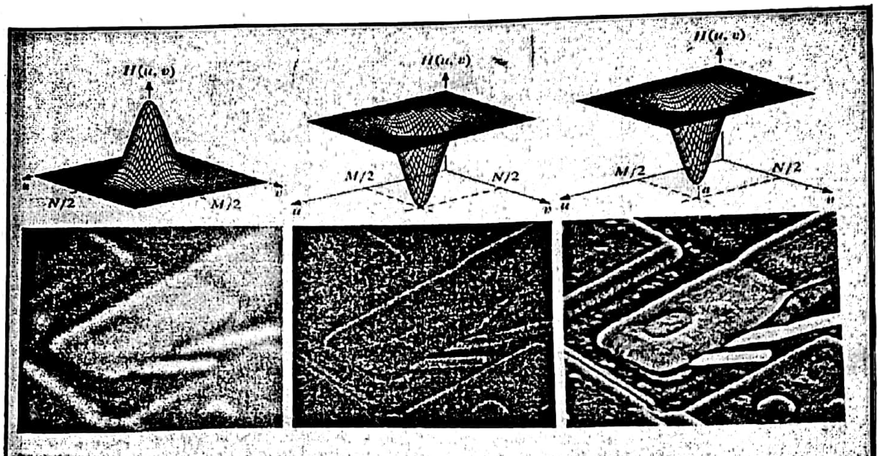
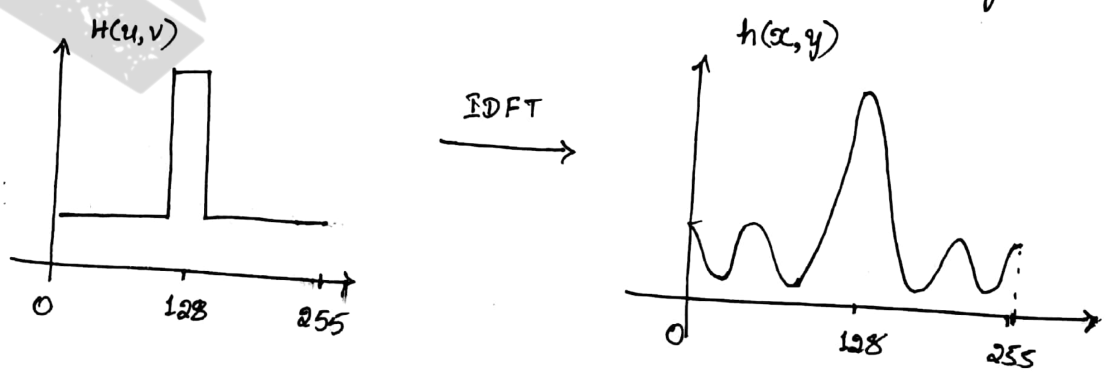


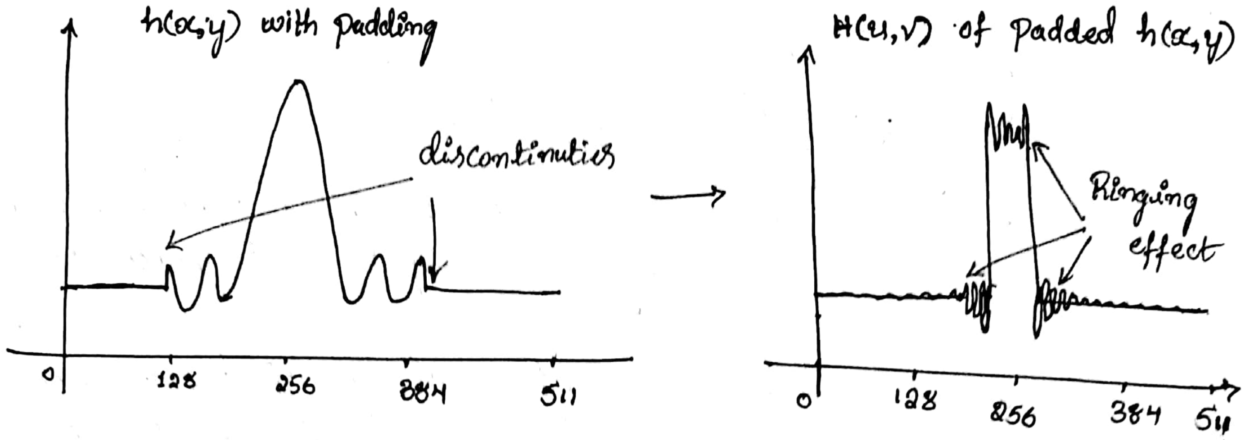

FIGURE 4.31 Top row: frequency domain filters. Bottom row: corresponding filtered images obtained using Eq. (4.7-1). We used $a = 0.85$ in (c) to obtain (f) (the height of the filter itself is 1). Compare (f) with Fig. 4.29(a).

- As we seen earlier, zero padding is very important in image processing, but the question is, whether to pad zeros to an image in spatial domain or in the frequency domain.

however padding is done in spatial domain, which leads to relationship betn spatial padding & filters pecified directly in the frequency domain.

To handle padding of a frequency domain filter is to
→ Construct the filter to be of the same size as the image.
→ Compute the IDFT of the filter to obtain spatial filter $h(x,y)$
→ Pad $h(x,y)$ with zeros. in spatial domain
→ Compute its DFT to return to the frequency domain.






The discontinuties in the spatial filter created ringing in its frequency domain representation.

We know that, the DFT is a complex array, we can represent in in terms of real & imaginary parts.

$$F(u,v) = R(u,v) + jI(u,v)$$

then, $g(x,y) = \mathcal{F}^{-1}\{H(u,v).R(u,v) + jH(u,v).I(u,v)\}$

## Summary of steps for filtering in the frequency domain.

1) Given an input image $f(x,y)$ of size $M \times N$, obtain the padding parameters $P$ & $Q$. Typically $P = 2M$ & $Q = 2N$
2) Form a padded image $f_p(x,y)$ of size $P \times Q$ by appending the necessary number of zeros to $f(x,y)$
3) Multiply $f_p(x,y)$ by $(-1)^{x+y}$ to center its transform (To remove DC component)
4) Compute the DFT, $F(u,v)$ of the image from step 3.
5) Generate a real, symmetric filter function, $H(u,v)$ of size $P \times Q$ with center at co-ordinates $(P/2, Q/2)$.
   Form the product $G(u,v) = H(u,v)F(u,v)$ using array multiplication.

6) Obtain the processed image:

$$g_p(x,y) = \{ real [\mathcal{J}^{-1}[G(u,v)]] \} (-1)^{x+y}$$

where the real part is selected in order to ignore parasitic complex components resulting from computational inaccuracies.

7) Obtain the final processed result $g(x,y)$ by extracting the $M \times N$ region from the top, left quadrant of $g_p(x,y)$.

a b c
d e f
g h

**FIGURE 4.36** (a) An $M \times N$ image, $f$. (b) Padded image, $f_p$ of size $P \times Q$. (c) Result of multiplying $f_p$ by $(-1)^{x+y}$. (d) Spectrum of $F_p$. (e) Centered Gaussian lowpass filter, $H$, of size $P \times Q$. (f) Spectrum of the product $HF_p$. (g) $g_p$, the product of $(-1)^{x+y}$ and the real part of the IDFT of $HF_p$. (h) Final result, $g$, obtained by cropping the first $M$ rows and $N$ columns of $g_p$.

Correspondance between filtering in the Spatial & frequency domain:

The frequency domain filtering operation is given by:

$G(u,v) = H(u,v) \cdot F(u,v)$

In order to find its equivalent representation in spatial domain.

$$f(x,y) \xrightarrow{} \boxed{h(x,y)} \longrightarrow g(x,y)$$

i/p image — filter — filtered (o/p) image.

Circular convolution.

$$g(x,y) = h(x,y) \circledast f(x,y)$$

if $f(x,y) = \delta(x,y)$

$$g(x,y) = h(x,y) * \delta(x,y)$$

$$\boxed{g(x,y) = h(x,y)}$$

$$\delta(x,y) \xleftrightarrow{DFT} 1$$

$$DFT\{f(x,y) = \delta(x,y)\} \Rightarrow \{F(u,v) = 1\}$$

$$\therefore \quad G(u,v) = H(u,v) \cdot 1$$

& $\quad h(x,y) \xleftrightarrow{DFT} H(u,v)$

Then the filtered o/p image is.

$$g(x,y) = \mathcal{J}^{-1}\{G(u,v)\}$$

$$\boxed{g(x,y) = \mathcal{J}^{-1}\{H(u,v)\} = h(x,y)}$$

$h(x,y)$ is called impulse response because it is obtained as a response to the impulse input. Also, because all quantities in a discrete implementations are finite, such filters are called finite impulse response [FIR] filters.

In practice, because of speed & ease of implementation in the hardware &/or in software issues, we prefer spatial (convolution) filtering. However, filtering concepts are more intutive in the frequency domain.

In order to extract the advantages of both the domains, it is required to Specify a filter in frequency domain,

→ Compute its IDFT & then use the resulting, full size Spatial filter as a guide for constructing smaller spatial filter masks.

In order to understand this, let us consider a

1D Gaussian filter $H(u) = Ae^{-u^2/2\sigma^2}$

where, $\sigma \rightarrow$ Standard deviation.

in Spatial domain $h(x) = \sqrt{2\pi}\,\sigma A e^{-2\pi^2\sigma^2 x^2}$

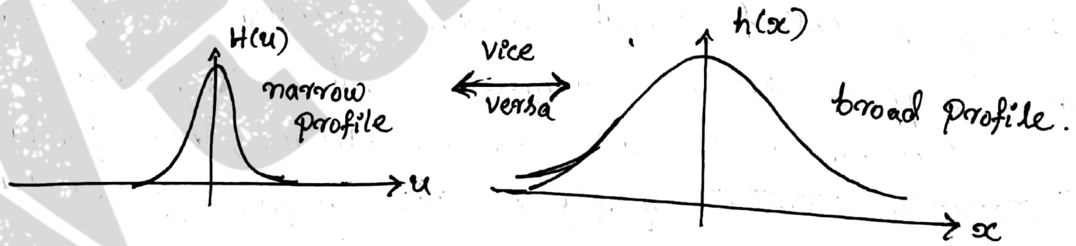
} DFT pairs, Gaussian & real functions.

☆ The above DFT pairs behaves reciprocally.

when $H(u)$ has a broad profile (large value of $\sigma$) $h(x)$ has narrow profile & vice versa.

☆ As $\sigma$ approaches infinity, $H(u)$ is a constant function. & $h(x)$ tends towards an impulse.

ie

## Image Smoothing using Frequency Domain Filters.

Smoothing filters are used for blurring & for noise reduction. Blurring is used in preprocessing tasks, such as removal of small details from an image prior to object extraction. & bridging of small gaps in lines or curves.

The o/p of a smoothing, linear spatial filter is simply the average of the pixels contained in the neighborhood of the filter mask.

These filters are called averaging filters, also referred to a low pass filter.

**Box filter:**

$$\frac{1}{9} \times \begin{array}{|c|c|c|} \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array}$$

3x3 Smoothing filter.

$$R = \frac{1}{9} \sum_{i=1}^{9} z_i$$

**weighted average filter:**

$$\frac{1}{16} \times \begin{array}{|c|c|c|} \hline 1 & 2 & 1 \\ \hline 2 & 4 & 2 \\ \hline 1 & 2 & 1 \\ \hline \end{array}$$

$$g(x,y) = \frac{\sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t)\, f(x+s, y+t)}{\sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t)}, \quad \begin{array}{l} x = 0,1,2,\text{---}M-1 \\ y = 0,1,2,\text{---}N-1 \end{array}$$

Smoothing (blurring) is achieved in the frequency domain by high frequency attenuation. [i.e low pass filtering]

Here we consider 3 types of Low pass filters

a) Ideal low pass filtering.

b) Butterworth filter.

c) Gaussian filter.

Ideal low pass filter is a very sharp filter, & Gaussian filter is very smooth filter. The butterworth filter has a parameter called the <u>filter order</u>. for higher order values, the butterworth filter approaches the ideal. filter.

For lower order values, the butterworth filter is like a gaussian filter. hence it provides a transition between two "extremes"

Let $H(u,v)$ is a discrete function of size $P \times Q$

<u>Ideal Lowpass filters</u>:

A 2D low pass filter that passes all frequencies within a circle of radius $D_0$ from the origin & "cuts off" all frequencies outside this circle is called an ideal low pass filter [ILPF]

$$H[u,v] = \begin{cases} 1, & \text{if } D(u,v) \le D_0 \\ 0, & \text{if } D(u,v) > D_0 \end{cases}$$

where $D_0 \longrightarrow$ Positive constant & $D(u,v)$ is the distance between a point $(u,v)$ in the frequency domain & the center of the frequency rectangle.

$$D(u,v) = \left[(u - P/2)^2 + (v - Q/2)^2\right]^{1/2}$$

where, P & Q are zero padded sizes.

★★

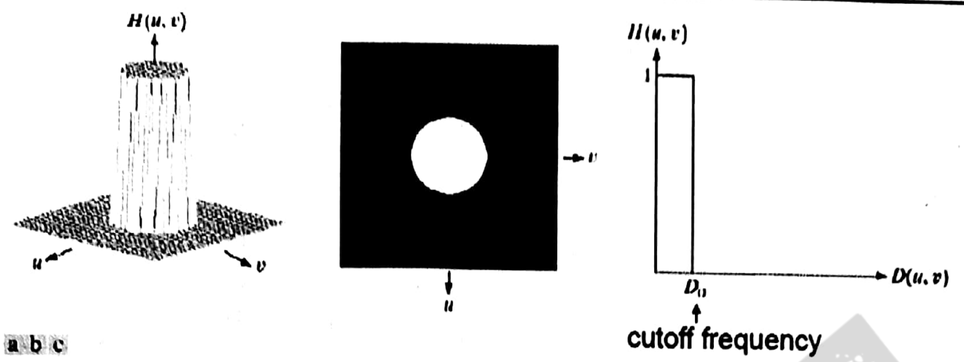



**FIGURE 4.10** (a) Perspective plot of an ideal lowpass filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross section.

The ideal low pass filter is radially symmetric about the origin, which means that the filter is completely defined by a radial cross section.

The point of transition betn $H(u,v) = 1$ & $H(u,v) = 0$ is called cutoff frequency. $[D_0]$. But ideal LPF cannot be designed using electronic components. hence these are "nonphysical" filters.

The total image power $P_T$. is given by

$$P_T = \sum_{u=0}^{P-1} \sum_{v=0}^{Q-1} P(u,v)$$

where $P(u,v) = |F(u,v)|^2 \longrightarrow$ Power Spectrum.

If DFT has been centered, a circle of radius $D_0$ with origin at the center of the frequency rectangle encloses "$\alpha$" percent of the power.

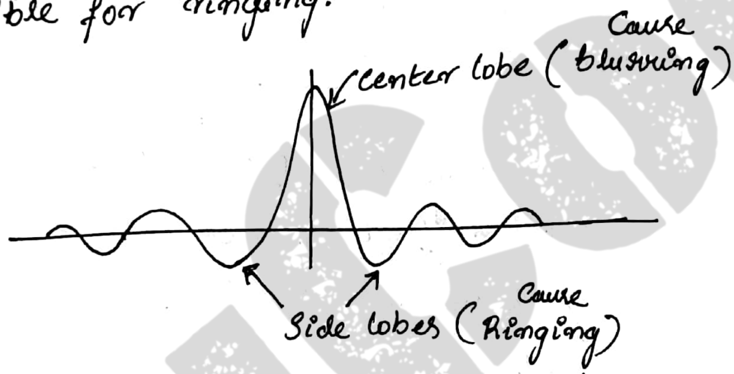
$$\alpha = 100 \left[ \sum_u \sum_v \frac{P(u,v)}{P_T} \right]$$ & summation is taken over values of $(u,v)$ that lie inside the circle or on its boundary.

We know that, the box like filter function in frequency domain has the shape of Sinc function in Spatial domain.

In Spatial domain, filtering is done by convolving $h(x,y)$ with the image $f(x,y)$. but $f(x,y)$ is assumed as $\delta(x,y)$, i.e discrete impulse whose strength is proportional to the intensity of the image at that location.

Convolving a Sinc function with an impulse copies the sinc at the location of the impulse.

The center lobe of the sinc is principal cause of blurring, while the outer, smaller lobes are mainly responsible for ringing.

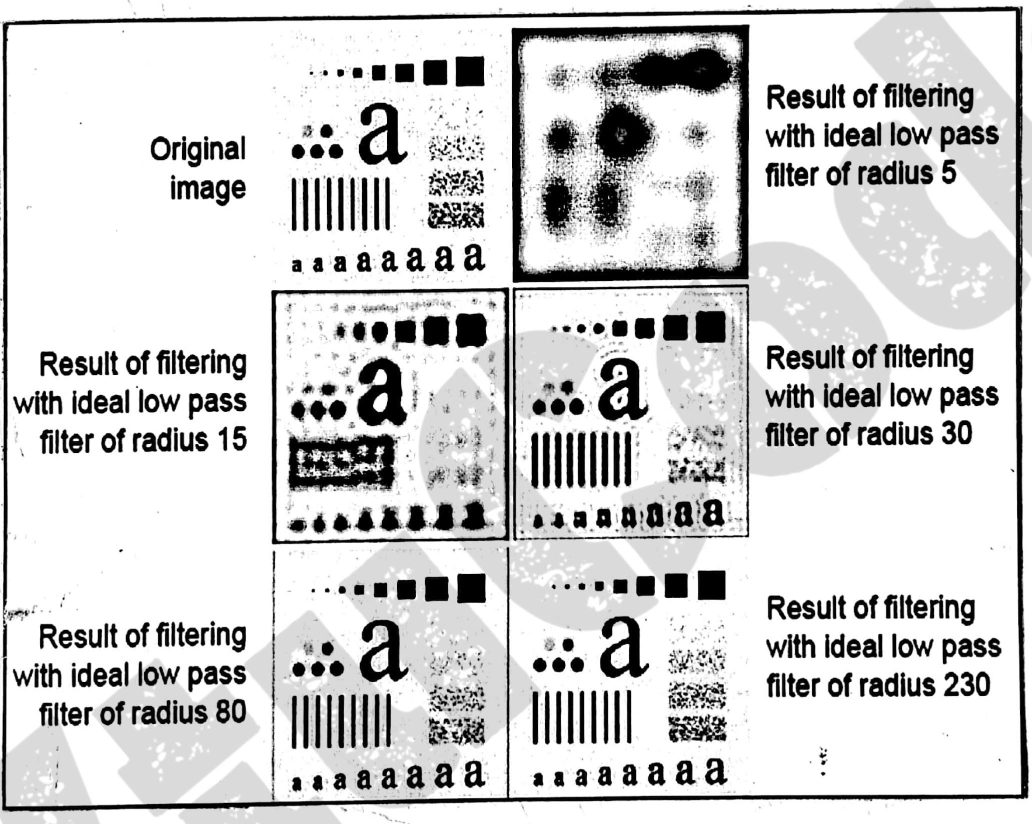



We know that, when $H(u,v)$ has broader profile, then $h(x,y)$ approaches smaller profile & vice versa.

i.e As $D_0$ of $H(u,v)$ becomes larger, the more the sinc approaches an impulse, which in the limit, cause no blurring at all when convolved with the image.

a b

**FIGURE 4.11** (a) An image of size 500 × 500 pixels and (b) its Fourier spectrum. The superimposed circles have radii values of 5, 15, 30, 80, and 230, which enclose 92.0, 94.6, 96.4, 98.0, and 99.5% of the image power, respectively.

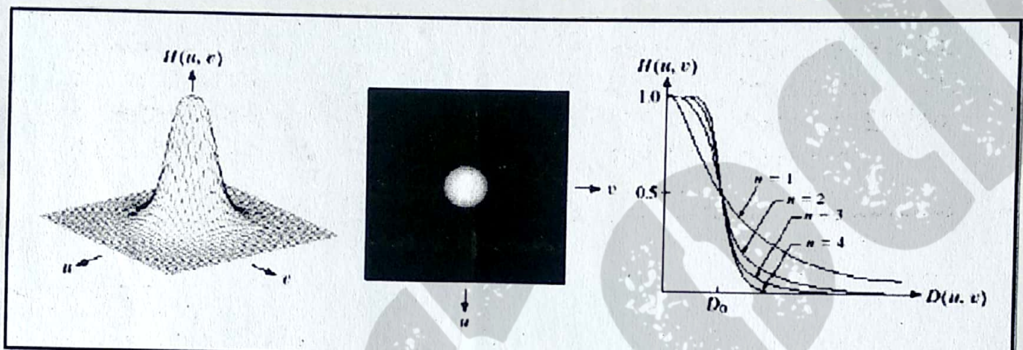


## Butterworth Lowpass filters. : [BLPF]

The Transfer function of a butterworth lowpass filter of order $n$ & with cutoff frequency at a distance $D_0$ from the origin is defined as,

$$H(u,v) = \frac{1}{1 + \left[\frac{D(u,v)}{D_0}\right]^{2n}}$$

where $D(u,v) = \left[(u - P/2)^2 + (v - Q/2)^2\right]^{1/2}$




The BLPF Transfer function does not have a sharp discontinuity that gives a clear cutoff between passed & filtered frequencies.

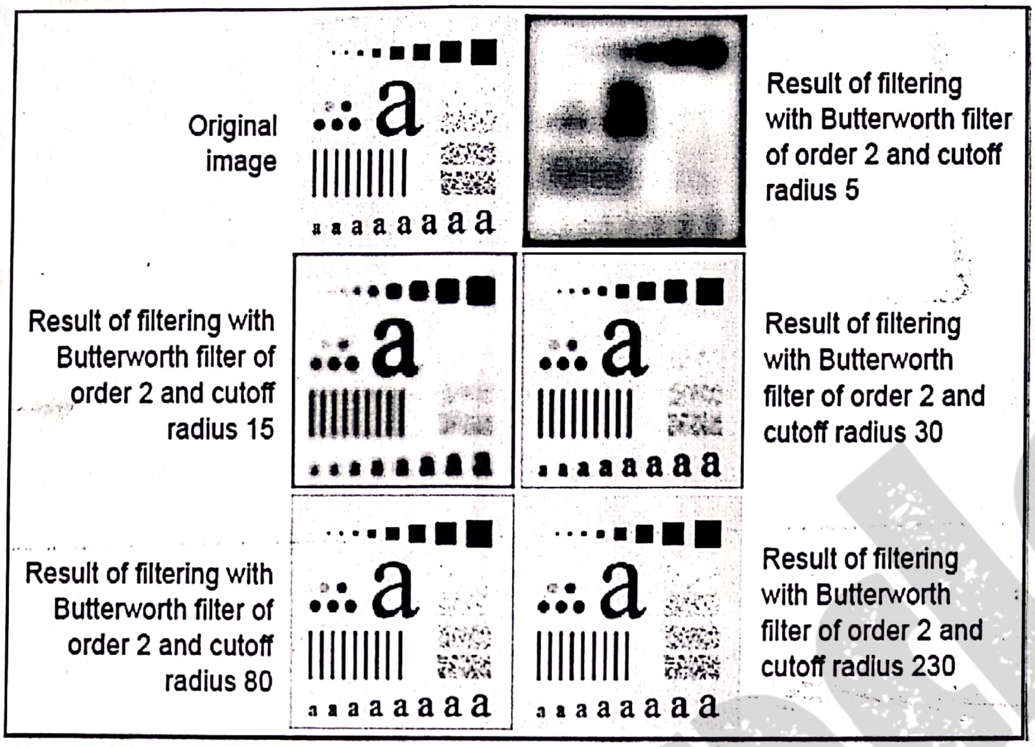
For defining cut off frequency, $H(u,v)$ is down to a certain fraction of its maximum value

50% from its max$^m$ value when $D(u,v) = D_0$

Another value used is $1/\sqrt{2}$ of the max$^m$ value of $H(u,v)$

ie $$H[u,v] = \frac{1}{1 + [\sqrt{2}-1]\left[\frac{D(u,v)}{D_0}\right]^{2n}}$$

$$H[u,v] = \frac{1}{1.414\left[\frac{D(u,v)}{D_0}\right]^{2n}}$$



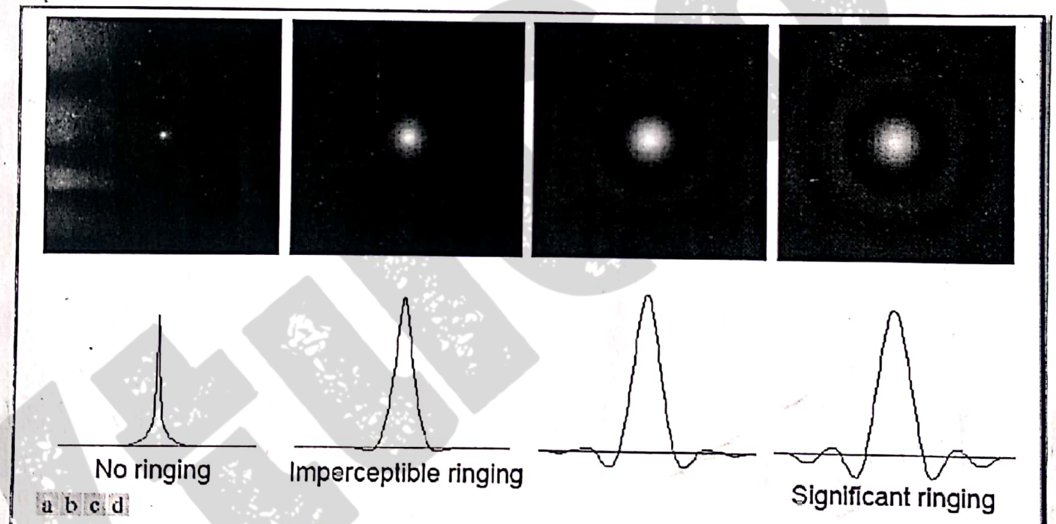




**FIGURE 4.16** (a)–(d) Spatial representation of BLPFs of order 1, 2, 5, and 20, and corresponding gray-level profiles through the center of the filters (all filters have a cutoff frequency of 5). Note that ringing increases as a function of filter order.

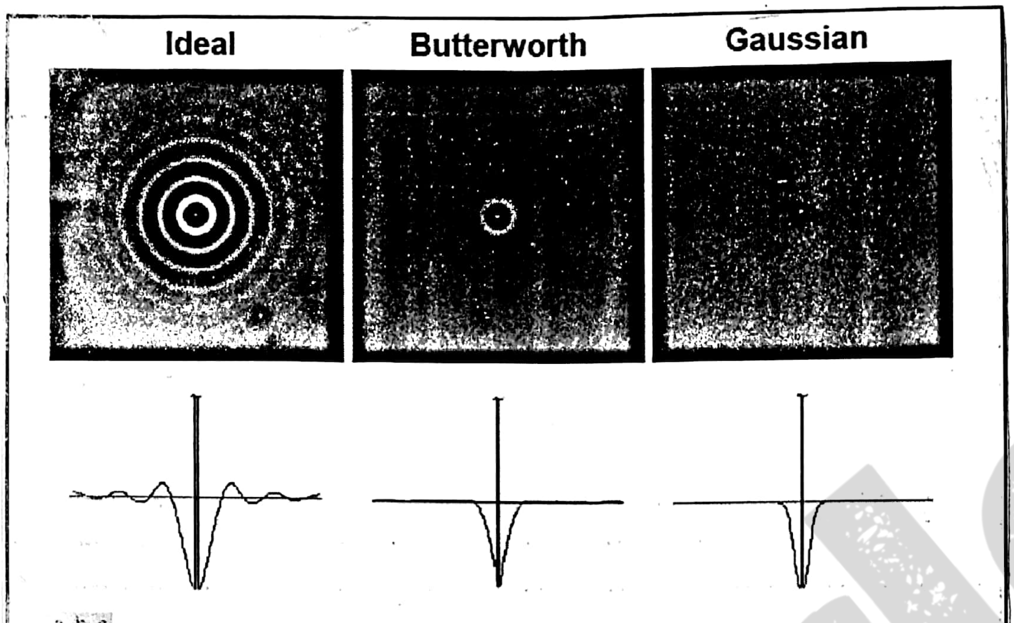



**FIGURE 4.23** Spatial representations of typical (a) ideal, (b) Butterworth, and (c) Gaussian frequency domain highpass filters, and corresponding gray-level profiles.

Ringing effect in HPF

## Gaussian low pass filter: [GLPF]

The Transfer function of 2D Gaussian LPF is given $H(u,v) = e^{-D^2(u,v)/2\sigma^2}$

where $D(u,v) = \left[(u - P/2)^2 + (v - Q/2)^2\right]^{1/2}$

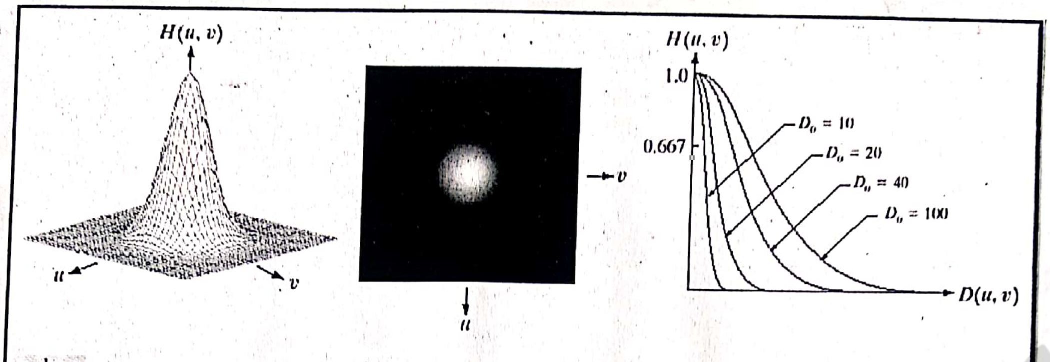
$\sigma$ is the measure of $H(u,v)$ spread about the Center

By letting $\sigma = D_0$, we can express the filter using the notation of the other filters

$$H(u,v) = e^{-D^2(u,v)/2D_0^2}$$

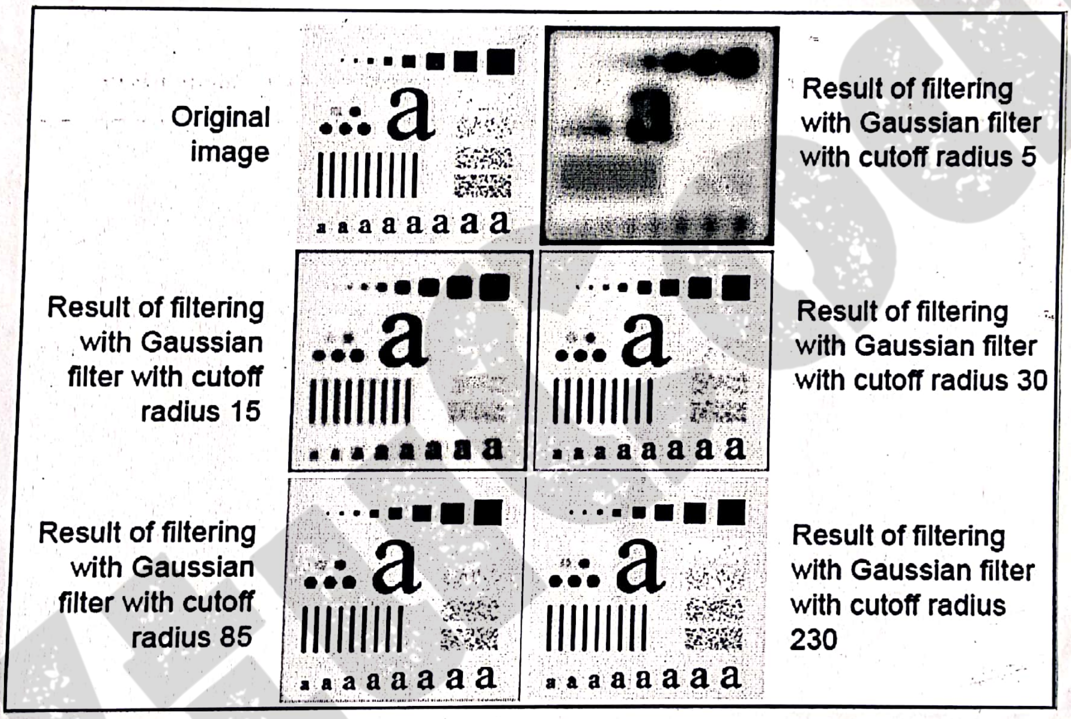
where $D_0$ is the cut off frequency, when $D(u,v) = D_0$ then GLPF is down to 0.607 of its maximum value.

WKT both $H(u,v)$ & its IDFT $h(x,y)$ are Gaussian.
hence no ringing effect.



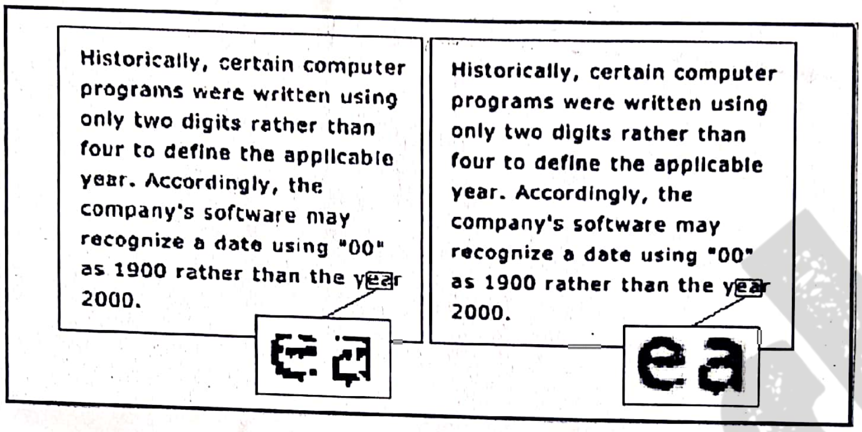

a b c

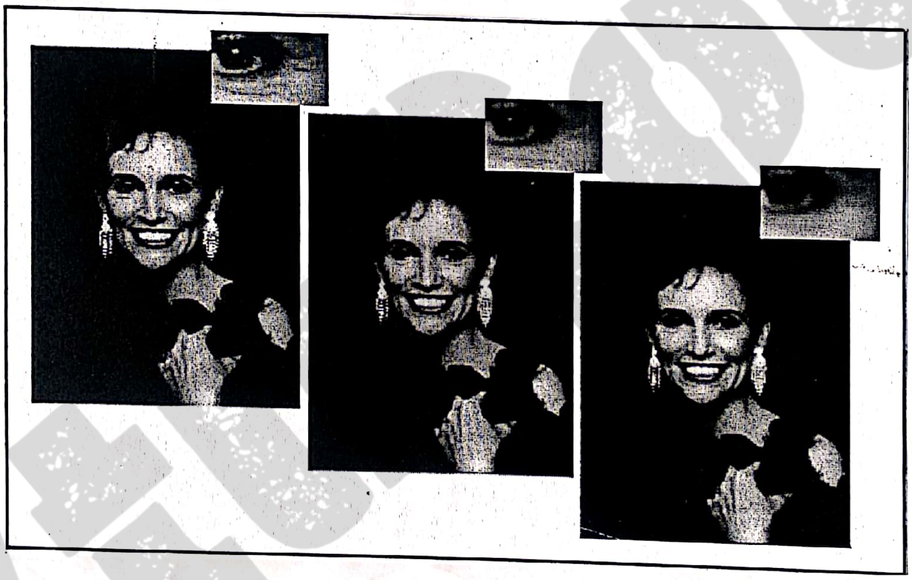
**FIGURE 4.17** (a) Perspective plot of a GLPF transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections for various values of $D_0$.



In cases where tight control of the transition between low and high frequencies about the cut off frequency are needed, then the BLPF presents a more suitable choice.

★★



Additional examples of low pass filtering.

# Image Sharpening using frequency domain filters.

Image Sharpening can be achieved in the frequency domain by highpass filtering, which attenuates the low frequency components without disturbing high-frequency information in the fourier transform.

Let $H(u,v)$ be radially Symmetric, & discrete function of size $P \times Q$. i.e $u = 0, 1, 2, --- P-1$ & $v = 0, 1, 2, ---- Q-1$

A high pass filter is obtained from a given LPF using the equation $H_{HP}(u,v) = 1 - H_{LP}(u,v)$

where, $H_{LP}(u,v) \longrightarrow$ Transfer function of the LPF.

## Ideal Highpass filters: [IHPF]

A 2D ideal high pass filter is defined as.

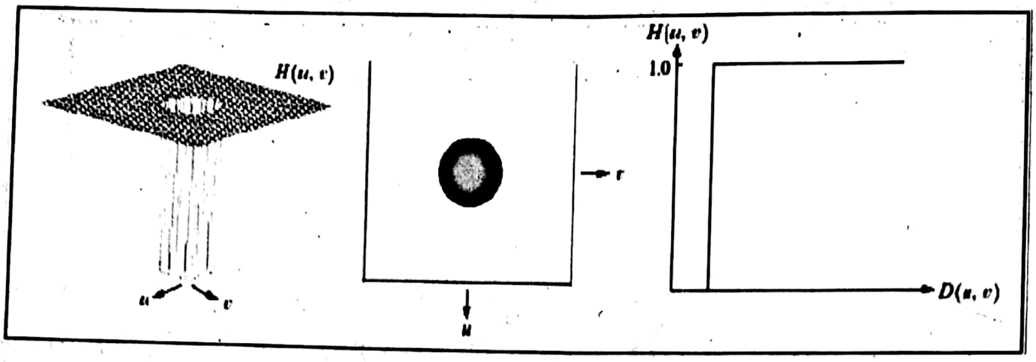
$$H(u,v) = \begin{cases} 0, & \text{if } D(u,v) \leq D_0 \\ 1, & \text{if } D(u,v) > D_0 \end{cases}$$

where, $D(u,v) = \left[ (u - P/2)^2 + (v - Q/2)^2 \right]^{1/2}$

$D_0 \longrightarrow$ cut off frequency.

The IHPF is the opposite of the ILPF in the sense that it sets to zero all frequencies inside a circle of radius $D_0$ while passing without attenuation, all frequencies outside the circle.

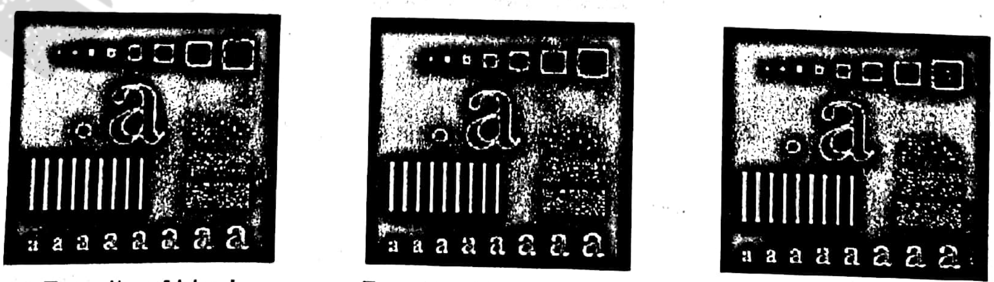
As in the case of ILPF, the IHPF is not physically realizable.




As $D_0$ value increases, more sharper the o/p image.

Ringing effect

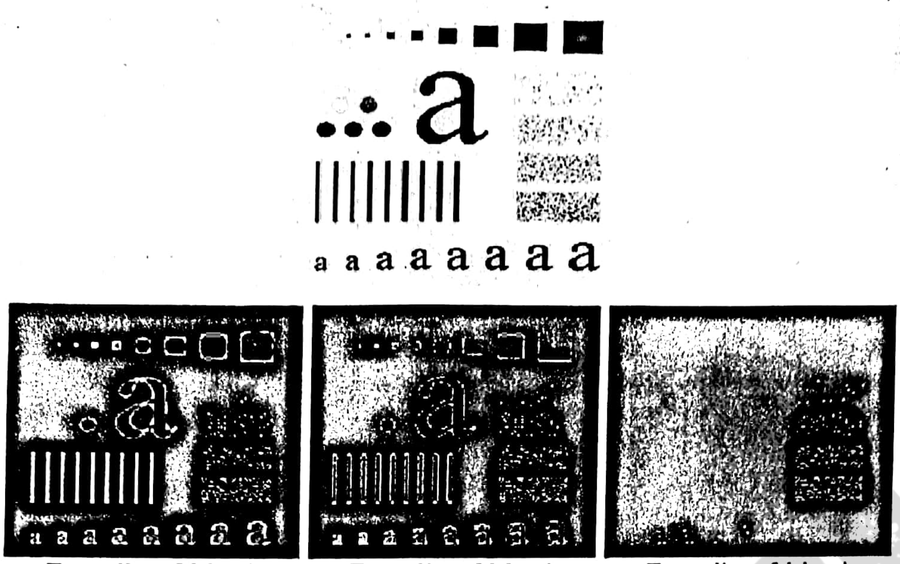
Result
Comparison of all the high pass filters



Results of ideal high pass filtering with $D_0 = 15$

Results of Butterworth high pass filtering of order 2 with $D_0 = 15$

Results of Gaussian high pass filtering with $D_0 = 15$



Results of ideal high pass filtering with $D_0 = 15$

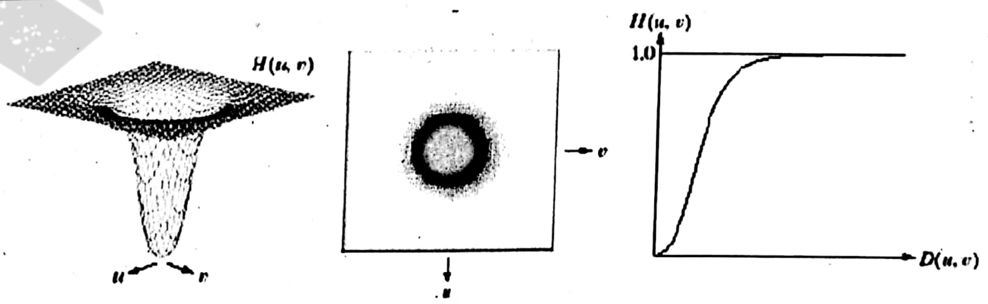
Results of ideal high pass filtering with $D_0 = 30$

Results of ideal high pass filtering with $D_0 = 80$

## Butterworth Highpass filters : [BHPF]

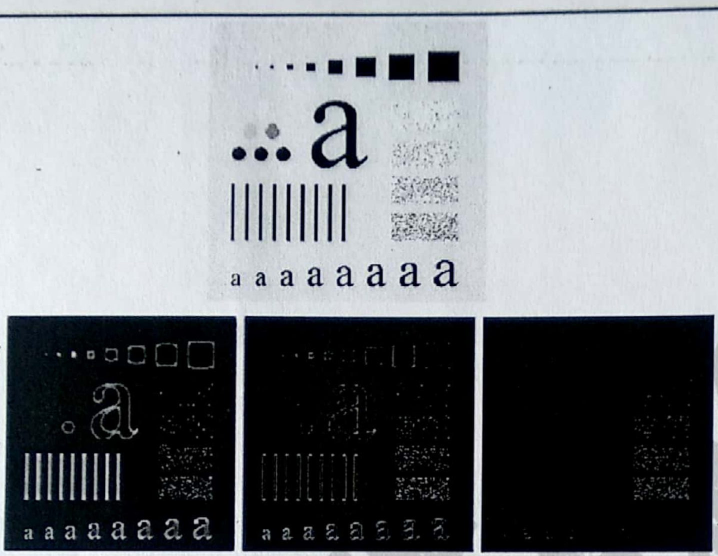
The Transfer function of a 2D butterworth high pass filter of order n & cutoff frequency $D_0$ is defined as.

$$H(u,v) = \frac{1}{1 + \left[\frac{D_0}{D(u,v)}\right]^{2n}}$$



As with low pass filters, we can expect butterworth highpass filters to be have smoother than IHPF.

unlike IHPF, the BHPF has smoother transition from stop band to pass band.



Results of Butterworth high pass filtering of order 2 with $D_0$ = 30

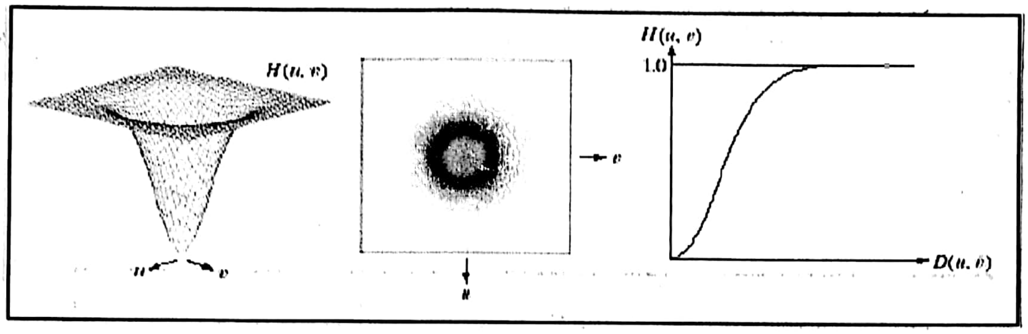
From the example shown above, we can obsere that ringing effect reduces as Do increases, which in turn low frequency component attenuates more. As a result we can get sharper images at the o/p end.

## Gaussian Highpass filters: [GHPF]

The transfer function of the GHPF with cutoff frequency $D_0$ is given by.

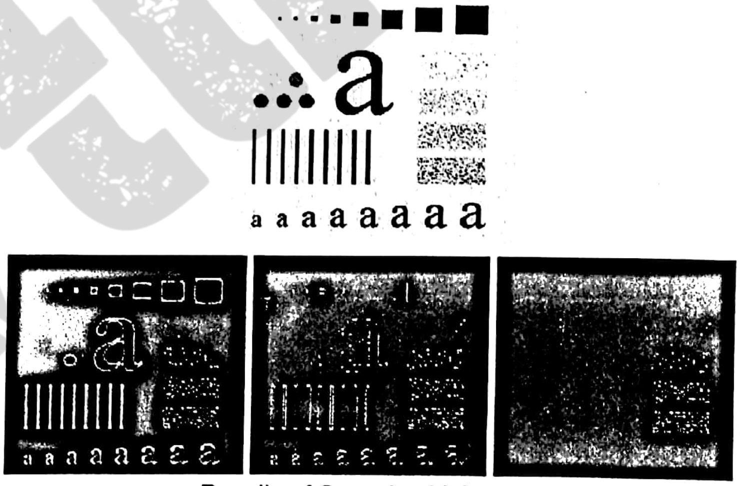
$$H(u,v) = 1 - e^{-D^2(u,v)/2D_0^2}$$

wher $D(u,v) = \left[(u - P/2)^2 + (v - Q/2)^2\right]^{1/2}$




The results obtained are more gradual than with the IHPF & BHPF. Even the filtering of the smaller objects & thin bars is cleaner with the gaussian filter.

The high pass filtered image lost its gray tones because the DC term was reduced to '0'. The net result is that dark tones typically predominate in highpass filtered images. Thus requiring additional processing to enhance details of interest. A simple approach is to threshold the filtered image.


Results of Gaussian high pass filtering with $D_0 = 15$

Results of Gaussian high pass filtering with $D_0 = 30$

Results of Gaussian high pass filtering with $D_0 = 80$

## The Laplacian in the frequency domain :

Laplacian can be implemented in frequency domain using the filter.

$$H(u,v) = -4\pi^2(u^2+v^2)$$

or with respect to the center of the frequency rectangle using the filter

$$H(u,v) = -4\pi^2\left[(u-P/2)^2 + (v-Q/2)^2\right]$$

$$\underline{H(u,v) = -4\pi^2 D^2(u,v)}$$

The laplacian image is obtained as;

$$\nabla^2 f(x,y) = \mathfrak{J}^{-1}\{H(u,v) \cdot F(u,v)\}$$

where $F(u,v) \longrightarrow$ DFT of $f(x,y)$

filtered (enhanced) image, $\underline{g(x,y) = f(x,y) + C\nabla^2 f(x,y)}$

here $\underline{C=-1}$ because $H(u,v)$ is negative.

### In Spatial domain

$$\nabla^2 f(x,y) = f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1) - 4f(x,y)$$

The filter mask used to implement above equation is

| | | |
|---|---|---|
| 0 | 1 | 0 |
| 1 | -4 | 1 |
| 0 | 1 | 0 |

$f(x,y)$ & $\nabla^2 f(x,y)$ had comparable values. However $\nabla^2 f(x,y)$ introduces DFT scaling factor that can be several orders of magnitude larger than the maximum value of 'f'.

Thus difference between f and its laplacian must be brought into comparable ranges. The easiest way to handle this problem is to normalize the values of $f(x,y)$ to the range $[0, 1]$ & divide $\nabla^2 f(x,y)$ by its max$^m$ value, which will bring it to the approximate range $[-1, 1]$

Because laplacian has negative values.

Consider, $g(x,y) = f(x,y) + C\nabla^2 f(x,y)$

in the frequency domain.

$$\mathfrak{I}\{g(x,y)\} = \mathfrak{I}\{f(x,y)\} + C.\mathfrak{I}\{\nabla^2 f(x,y)\}$$

$$\mathfrak{I}\{g(x,y)\} = F(u,v) + C.\mathfrak{I}\{\mathfrak{I}^{-1}[H(u,v).F(u,v)]\}$$

$\underline{C=-1}$ & $\mathfrak{I}\mathfrak{I}^{-1} = 1$

$$\mathfrak{I}\{g(x,y)\} = \{F(u,v) - H(u,v)F(u,v)\}$$

$$g(x,y) = \mathfrak{I}^{-1}\{F(u,v) - H(u,v)F(u,v)\}$$

$$g(x,y) = \mathfrak{I}^{-1}\{[1 - H(u,v)]F(u,v)\}$$

$$g(x,y) = \mathfrak{I}^{-1}\{[1 + 4\pi^2 D^2(u,v)]F(u,v)\}$$

## Unsharp masking, highboost filtering, & high Frequency Emphasis filtering:

A process that has been used for many years by the printing & publishing industry to sharpen images consists of subtracting an unsharp (Smoothed) version of an image from the original image. This process is called "unsharp masking".

The steps are:

1) Blur the original image
2) Subtract the blurred image from the original. The resulting difference is called the mask.
3) Add the mask to the original.

Let $\bar{f}(x,y)$ be the blurred image, then unsharp masking is expressed as:

$$g_{mask}(x,y) = f(x,y) - \bar{f}(x,y)$$

Then we add a weighted portion of the mask back to the original image.

$$g(x,y) = f(x,y) + K * g_{mask}(x,y) \longrightarrow (1)$$

where $K \geq 0$ in general,

if $K=1$, then the process is unsharp masking

if $K>1$, the process is referred as highboost filtering

if $K<1$, de-emphasizes the contribution of the unsharp mask.

We know that blurring is nothing but low pass filtering, then $\bar{f}(x,y) = f_{LP}(x,y)$

$$\therefore \quad g_{mask}(x,y) = f(x,y) - f_{LP}(x,y)$$

$$f_{LP}(x,y) = \mathfrak{I}^{-1}\left[H_{LP}(u,v)\,F(u,v)\right]$$

where $f_{LP}(x,y) \longrightarrow$ Smoothed image

$H_{LP}(u,v) \longrightarrow$ LPF

$F(u,v) \longrightarrow$ DFT of $f(x,y)$

Eq$^n$ 1 can be expressed in frequency domain computations,

ie $\quad g(x,y) = f(x,y) + K * g_{mask}(x,y)$

$$g(x,y) = f(x,y) + K*\left[f(x,y) - f_{LP}(x,y)\right]$$

$$g(x,y) = f(x,y) + K*\left[f(x,y) - \mathfrak{I}^{-1}\left[H_{LP}(u,v)F(u,v)\right]\right]$$

By taking Transformation on both sides we get

$$\mathfrak{I}\{g(x,y)\} = \mathfrak{I}\{f(x,y)\} + K*\left[\mathfrak{I}\{f(x,y)\} - \mathfrak{I}\mathfrak{I}^{-1}\left[H_{LP}(u,v)F(u,v)\right]\right]$$

$$\mathfrak{I}\{g(x,y)\} = F(u,v) + K*\left[F(u,v) - H_{LP}(u,v)F(u,v)\right]$$

$$g(x,y) = \mathfrak{I}^{-1}\left\{\left[1 + K*\underbrace{\left[1 - H_{LP}(u,v)\right]}\right]F(u,v)\right\}$$

$$g(x,y) = \mathfrak{I}^{-1}\left\{\left[1 + K*H_{HP}(u,v)\right]F(u,v)\right\}$$

The term $[1 + K*H_{HP}(u,v)]$ is called high frequency emphasis filter

High pass filter eliminates the DC term. Thus reducing the avg intensity in the filtered image to '0'.

The high frequency emphasis filter does not have this problem because of the '1' that is added to the high pass filter.

The constant 'K' gives control over the proportion of high frequencies that influence the final result.

The general equation is,

$$\boxed{g(x,y) = \mathfrak{J}^{-1}\{[K_1 + K_2 * H_{HP}(u,v)]F(u,v)\}}$$

where $K_1 \geq 0$ gives controls of the offset from origin.
& $K_2 \geq 0$ controls the contribution of high frequencies.

## Homomorphic filtering :

An image $f(x,y)$ can be expressed in terms of illumination $i(x,y)$ & reflectance $r(x,y)$ components.

$$f(x,y) = i(x,y)\, r(x,y) \longrightarrow (1)$$

This equation cannot be used directly to operate in frequency domain. because fourier transform of a product is not the product of the transforms. ie

$$\mathfrak{J}\{f(x,y)\} \neq \mathfrak{J}[i(x,y)] \cdot \mathfrak{J}[r(x,y)]$$

To avoid this drawback let us consider.

$$Z(x,y) = \ln f(x,y)$$
$$= \ln[i(x,y)] + \ln[r(x,y)]$$

$$\Im\{z(x,y)\} = \Im\{\ln f(x,y)\}$$

$$\Im\{z(x,y)\} = \Im\{\ln i(x,y)\} + \Im\{\ln r(x,y)\}$$

$$Z(u,v) = F_i(u,v) + F_r(u,v) \longrightarrow (2)$$

where $F_i(u,v)$ & $F_r(u,v)$ are the fourier transforms of $\ln i(x,y)$ & $\ln r(x,y)$ respectively.

we can filter $Z(u,v)$ using filter $H(u,v)$

$$S(u,v) = H(u,v)\,Z(u,v)$$

$$= H(u,v)\left[F_i(u,v) + F_r(u,v)\right]$$

$$S(u,v) = H(u,v)\,F_i(u,v) + H(u,v)\,F_r(u,v) \longrightarrow (3)$$

In Spatial domain,

$$s(x,y) = \Im^{-1}\{S(u,v)\}$$

$$s(x,y) = \Im^{-1}\{H(u,v)\,F_i(u,v)\} + \Im^{-1}\{H(u,v)\,F_r(u,v)\}$$

define $i'(x,y) = \Im^{-1}\{H(u,v)\,F_i(u,v)\}$

& $r'(x,y) = \Im^{-1}\{H(u,v)\,F_r(u,v)\}$

$$s(x,y) = i'(x,y) + r'(x,y) \longrightarrow (4)$$

Since $z(x,y)$ is the modified image formed by taking natural logarithm of the i/p image $(f(x,y))$ we reverse the process by taking the exponential of the filtered result to form the o/p image.

$$g(x,y) = e^{s(x,y)}$$

$$g(x,y) = e^{i'(x,y) + r'(x,y)}$$

$$g(x,y) = e^{i'(x,y)} \, e^{r'(x,y)}$$

$$\boxed{g(x,y) = i_0(x,y) \, r_0(x,y)}$$

where $i_0(x,y) = e^{i'(x,y)}$ & $r_0(x,y) = e^{r'(x,y)}$ } illumination & reflectance components of the o/p image.

<u>Summary of Steps in Homomorphic filtering:</u>

$$f(x,y) \rightarrow \boxed{\ln} \xrightarrow{z(x,y)} \boxed{DFT} \xrightarrow{Z(u,v)} \boxed{H(u,v)} \xrightarrow{S(u,v)} \boxed{IDFT} \xrightarrow{s(x,y)} \boxed{exp} \rightarrow g(x,y)$$

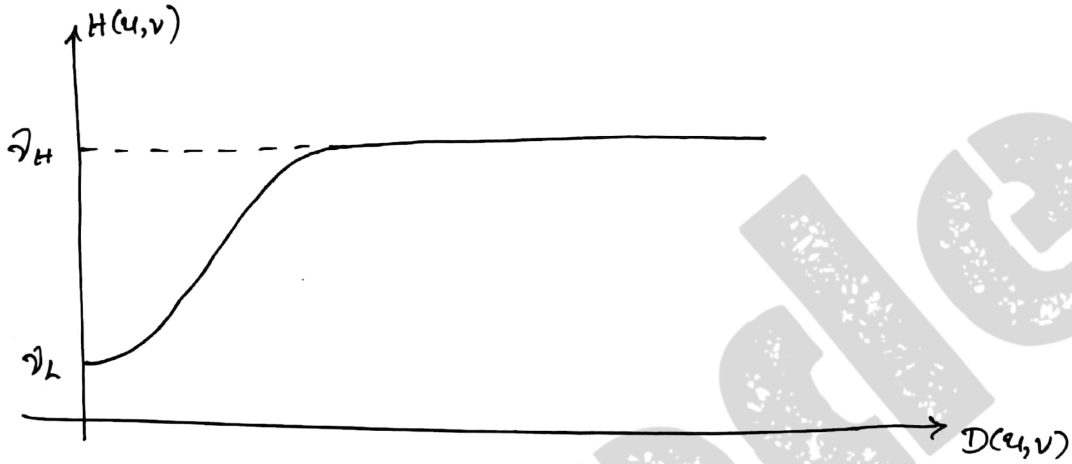
It is also called as <u>homomorphic System.</u>

The Illumination component of an Image generally is characterized by slow spatial variations, while the reflectance component tends to vary abruptly, particularly at the junction of dissimilar objects.

These Characteristics lead to associating the <u>low frequencies</u> of the fourier transform of the logarithm of an image with <u>illumination</u> & the <u>high frequencies</u> with <u>reflectance</u>

A good deal of control can be gained over the illumination & reflectance components with a homomorphic filter. This control requires specification of a filter function $H(u,v)$ the affects the low & high frequency components of the fourier transform in different controllable ways.

ie




Radial cross section of a circularly symmetric homomorphic filter function.

When, $\gamma_L < 1$ & $\gamma_H > 1$, the filter tend to attenuate the contribution made by low frequencies (illumination) & amplify the contibution made by high frequencies (reflectance)

The net result is simultaneous dynamic range compression & contrast enhancement.

Then filter function is given by.

$$H(u,v) = (\gamma_H - \gamma_L)\left[1 - e^{-c\left[\frac{D^2(u,v)}{D_0^2}\right]}\right] + \gamma_L$$