# Previous approach

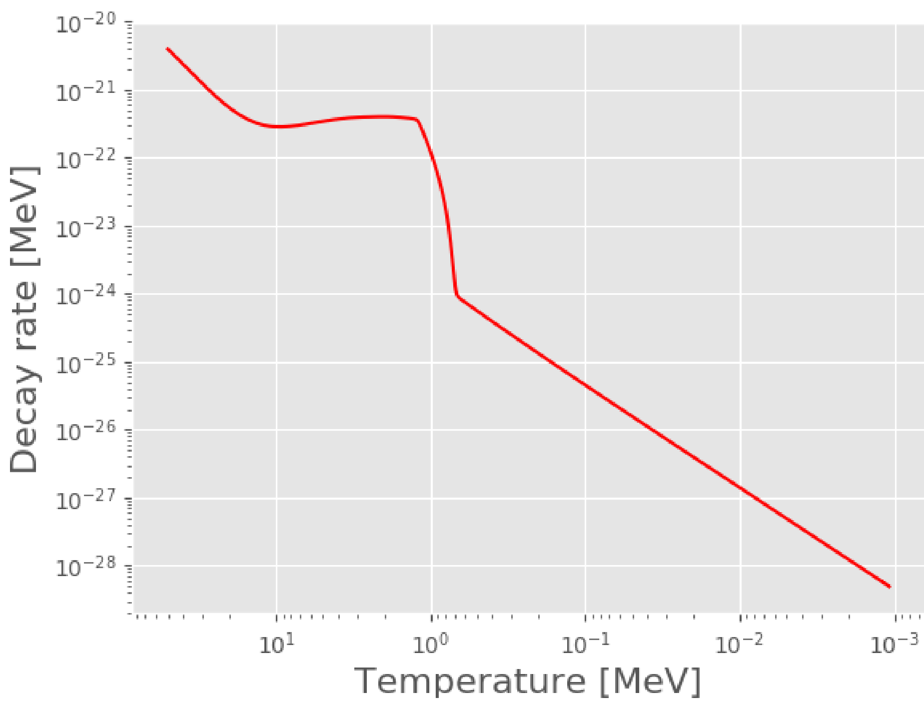
System consists of photons, 3 active neutrinos and 1 HNL. In this toy model, the only interaction is the decay of HNL into 3 active neutrinos by:

$$N \to \bar{\nu}_e + \nu_e + \nu_\tau$$

Original idea was that the decay width of this channel can be estimated numerically by using the Boltzmann equation

$$\frac{\partial n}{\partial t} + 3Hn = -\Gamma n \ ,$$

which gave the following result




I've found some reasons why this computation doesn't work. Some are listed below, others we can discuss later.

## Current approach

The code calculates the collision integral after using the Adams-Moulton method as

$$I_{code} = \frac{f_{prediction} - f_{previous}}{\Delta ln(a)} \ ,$$

which according to the Boltzmann equation is equal to:

$$I_{code} = \frac{df}{dln(a)} = \frac{1}{H} I_{coll}$$

Now, in the case of only decay as above (so no creation of HNL out of 3 active neutrinos), $I_{coll}$ should be

$$\begin{aligned} I_{coll} &= -\frac{1}{64\pi^3 E_N p_N} \int dp_2 dp_3 \frac{p_2 p_3}{E_2 E_3} S f_N (1-f_2)(1-f_3)(1-f_4) D(p_1, p_2, p_3, p_4) \\ &= -f_N \Gamma = -f_N \frac{1}{\gamma} \Gamma_{CM} = -f_N \frac{m_N}{E_N} \Gamma_{CM} \ , \end{aligned}$$

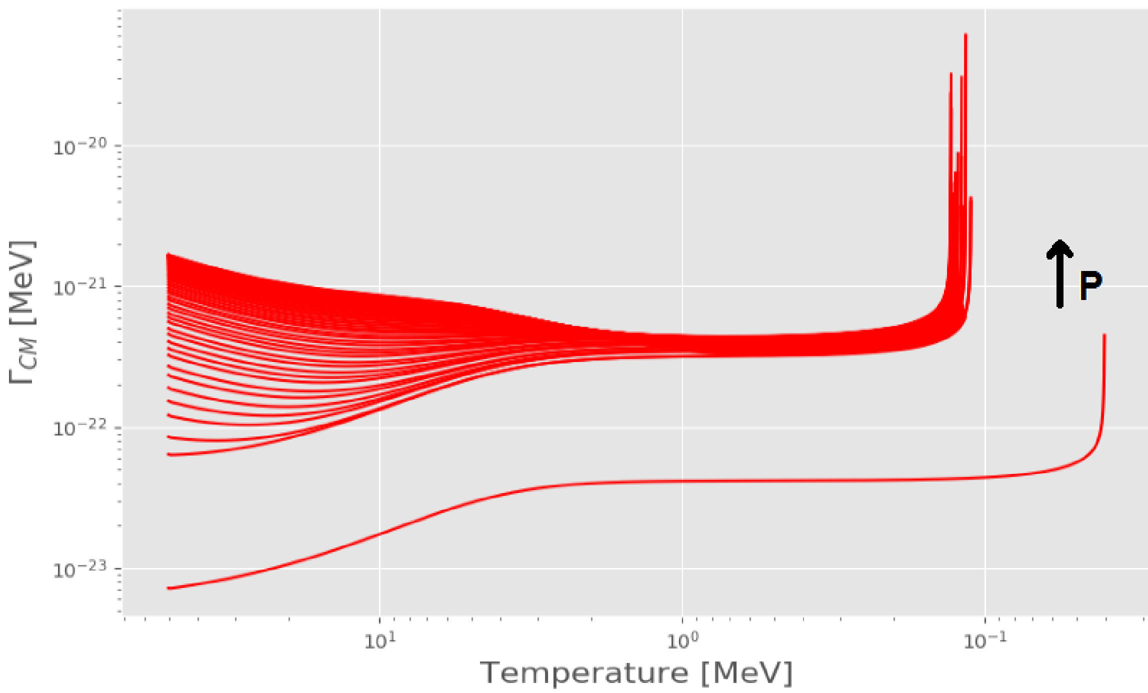
with $\Gamma_{CM}$ the decay width in the center-of-mass frame. Therefore, this quantity can be computed as

$$\Gamma_{CM} = -H \frac{E_N}{m_N} \frac{1}{f_N} I_{code}$$

When I was reproducing the previous plot, I noticed some strange behaviour with the collision integral: it would go to 0 at $T \sim 0.4$ MeV. This doesn't make any sense, considering that the HNLs should keep on decaying and the collision integral should depend only on the momentum of the decaying particle *as defined on the grid*. Eventually, I tracked it down to the D-function, which vanishes around this temperature.

The collision integral in the code is computed in comoving coordinates, and for some reason we also say that the momenta $p_N, p_2$, $p_3$ and $p_4$ decrease as $a^{-1}$, while I think we should keep the grids of these momenta fixed. When an HNL with $p_N = 0$ decays, then the *initial* momenta of its decay products don't depend on the scale factor. Keeping this in mind, then saying that the grid over which we integrate decreases as $a^{-1}$ doesn't make much sense.

So I calculated the collision integral by saying that the grids of $p_N$, $p_2$ and $p_3$ are fixed and $p_4$ is obtained by momentum conservation. This approach gets rid of the factor $M^5/x^5$ in front of the integral; rest stays the same (although I am still not so sure how you defined the distribution function in the code?). Using the formula above for each HNL momentum, I then obtain the following plot:

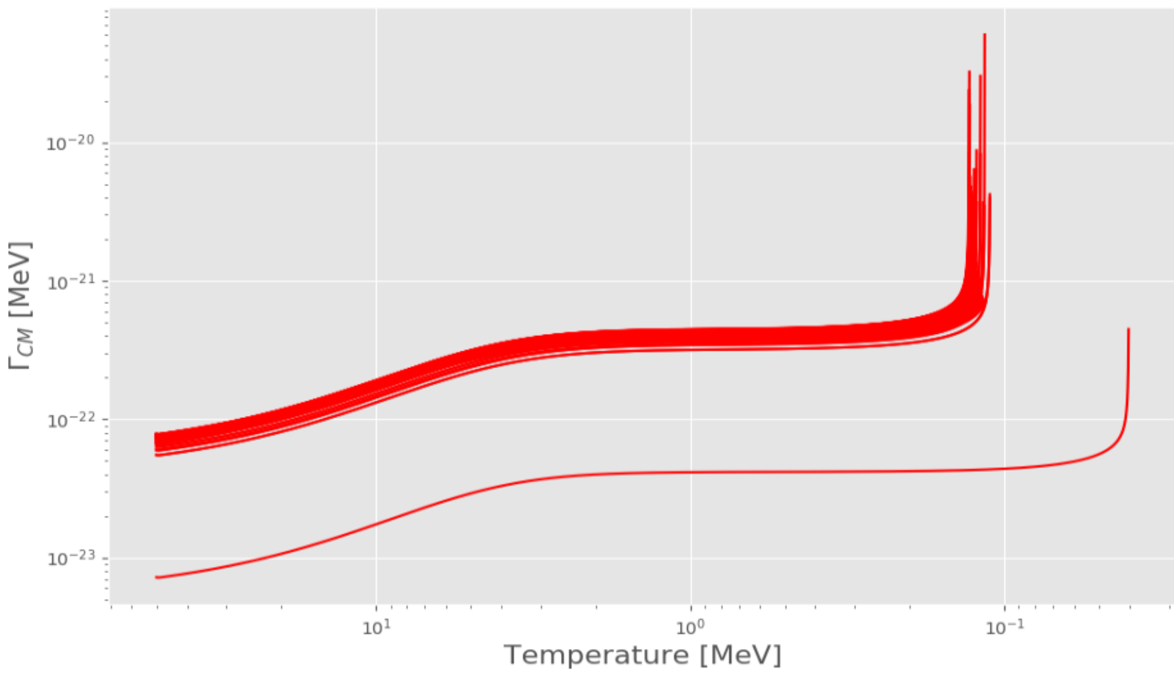



Taking the mean of the curves at the plateau gives a lifetime of around 1.5 second. Reason for the divergence is that all HNLs have decayed at that time. The outlier is the one with HNL momentum $p_N = 0$.

I tried some other methods to compute $\Gamma$, but this seems the one with the smallest deviations. I don't think this is the most robust method, but it's a progress compared to what we had before.

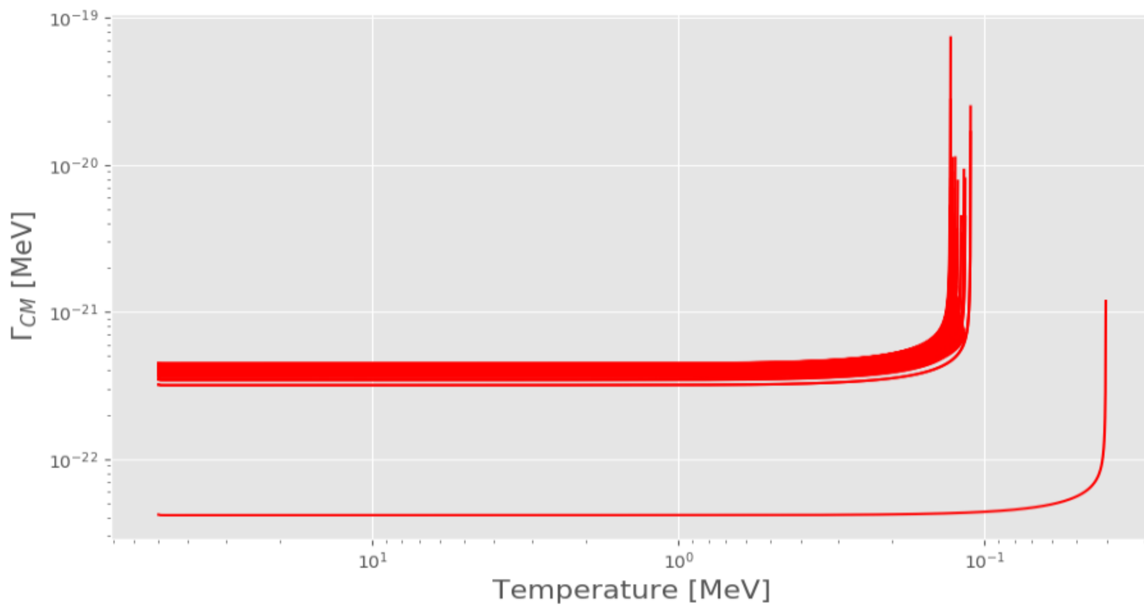
# Update 1

Forgot to change input of distribution interpolation and got rid of scale factor in calculation of energy (was there for no reason). Result looks better.



## Update 2

The reason why the curves above are increasing in the beginning is because of the Pauli blocking factors $(1-f_{\nu_\tau})(1-f_{\nu_e})(1-f_{\bar{\nu_e}})$. Due to the expansion, the active neutrino densities decrease and the Pauli blocking factors approach 1 and as a result the curves will be flat. As a check I put these factors equal to 1 from the beginning and I got:




Only thing (issue?) left is the difference between curves with different momenta.

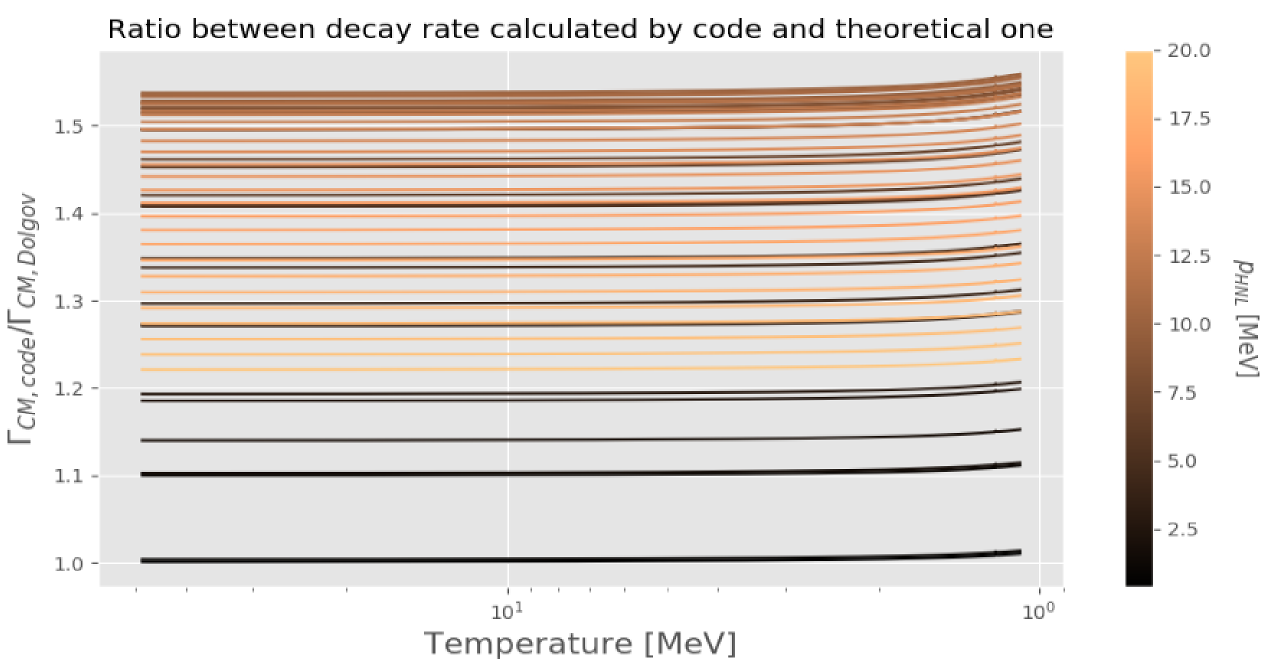
# Update 3

Comparison with Dolgov et al. is made. Here the following decays are included:

$$N \to \overline{\nu_{e/\mu/\tau}} + \nu_{e/\mu/\tau} + \nu_\tau$$
$$N \to e^+ + e^- + \nu_\tau$$

Theoretical decay width in the CM without Pauli blocking factors is given by:

$$\Gamma_{CM} = \frac{(1 + \tilde{g_L}^2 + g_R^2) G_F^2 m_N^5 (sin^2(2\theta))/4}{192\pi^3}$$




Note: I got rid of the curve with $p_{HNL} = 0$ and considered only the flat part of the curves.