Previous approach

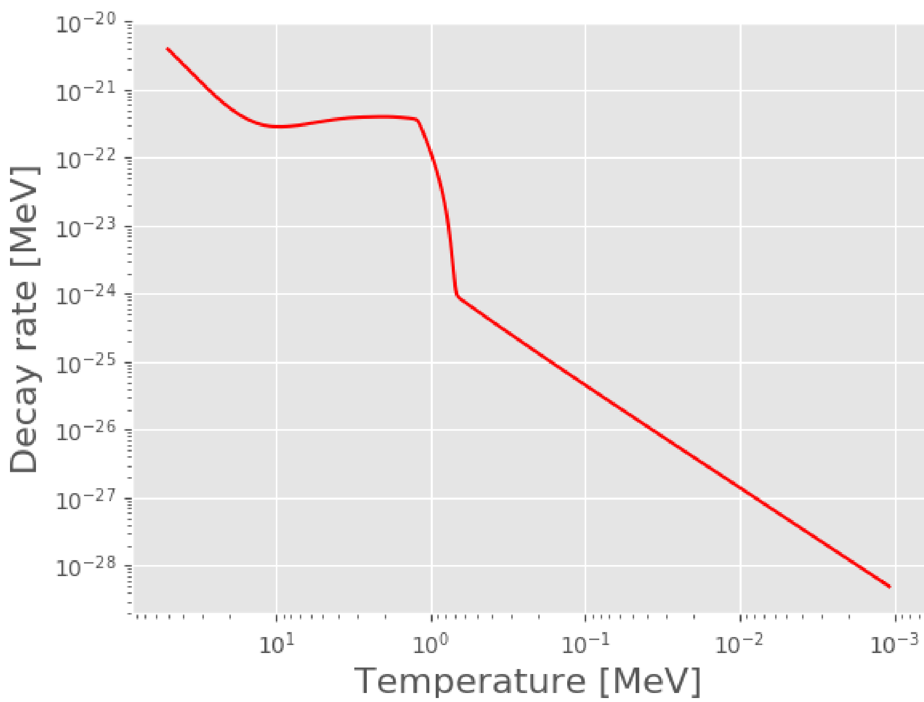
System consists of photons, 3 active neutrinos and 1 HNL. In this toy model, the only interactions is the decay of HNL into 3 active neutrinos by:

$$N \to \bar{\nu}_e + \nu_e + \nu_\tau$$

Original idea was that the decay width of this channel can be estimated numerically by using the Boltzmann equation

$$\frac{\partial n}{\partial t} + 3Hn = -\Gamma n \ ,$$

which gave the following result



I've found some reasons why this computation doesn't work. Some are listed below, others we can discuss later.

Current approach

The code calculates the collision integral after using the Adams-Moulton method as

$$I_{code} = \frac{f_{prediction} - f_{previous}}{\Delta ln(a)} \ ,$$

which according to the Boltzmann equation is equal to:

$$I_{code} = \frac{df}{dln(a)} = \frac{1}{H} I_{coll}$$

Now, in the case of only decay as above (so no creation of HNL out of 3 active neutrinos), I_{coll} should be

$$\begin{aligned} I_{coll} &= -\frac{1}{64\pi^3 E_N p_N} \int dp_2 dp_3 \frac{p_2 p_3}{E_2 E_3} S f_N (1-f_2)(1-f_3)(1-f_4) D(p_1, p_2, p_3, p_4) \\ &= -f_N \Gamma = -f_N \frac{1}{\gamma} \Gamma_{CM} = -f_N \frac{m_N}{E_N} \Gamma_{CM} \ , \end{aligned}$$

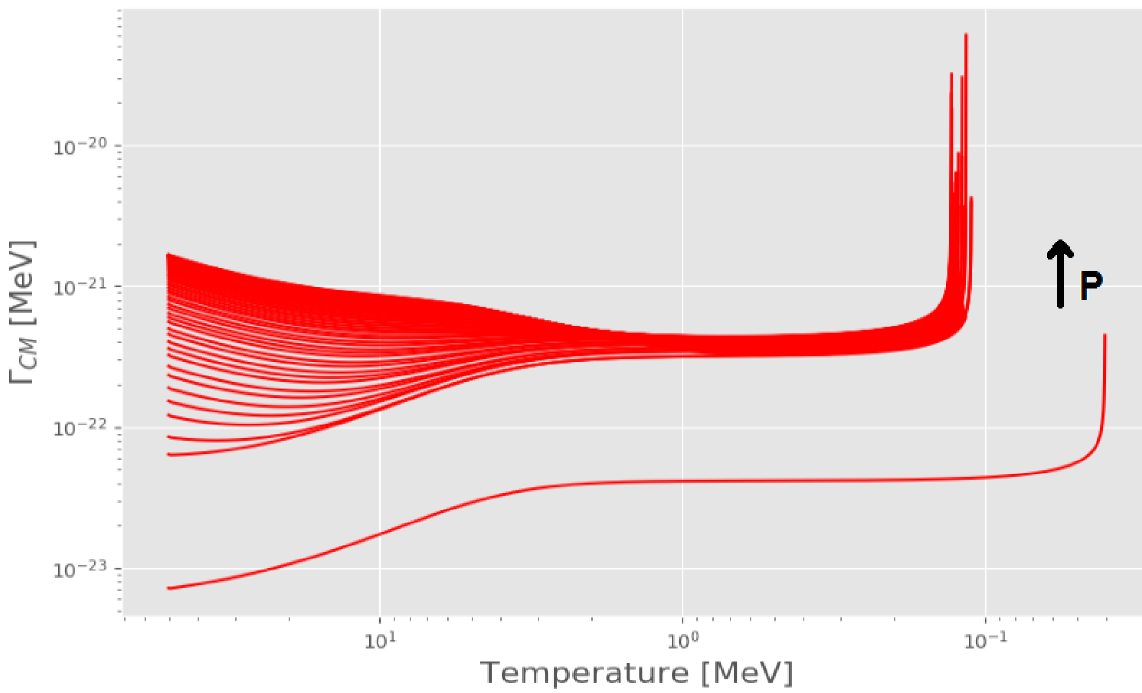
with Γ_{CM} the decay width in the center-of-mass frame. Therefore, this quantity can be computed as

$$\Gamma_{CM} = H \frac{E_N}{m_N} \frac{1}{f_N} I_{code}$$

When I was reproducing the previous plot, I noticed some strange behaviour with the collision integral: it would go to 0 at $T \sim 0.4$ MeV. This doesn't make any sense, considering that the HNLs should keep on decaying and the collision integral should depend only on the momentum of the decaying particle *as defined on the grid*. Eventually, I tracked it down to the D-function, which vanishes around this temperature.

The collision integral in the code is computed in comoving coordinates, and for some reason we also say that the momenta p_N, p_2, p_3 and p_4 decrease as a^{-1}, while I think we should keep the grids of these momenta fixed. When an HNL with $p_N = 0$ decays, then the *initial* momenta of its decay products don't depend on the scale factor. Keeping this in mind, then saying that the grid over which we integrate decreases as a^{-1} doesn't make much sense.

So I calculated the collision integral by saying that the grids of p_N, p_2 and p_3 are fixed and p_4 is obtained by momentum conservation. This approach gets rid of the factor M^5/x^5 in front of the integral; rest stays the same (although I am still not so sure how you defined the distribution function in the code?). Using the formula above for each HNL momentum, I then obtain the following plot:



Taking the mean of the curves at the plateau gives a lifetime of around 1.5 second. Reason for the divergence is that all HNLs have decayed at that time. The outlier is the one with HNL momentum $p_N = 0$.

I tried some other methods to compute Γ, but this seems the one with the smallest deviations. I don't think this is the most robust method, but it's a progress compared to what we had before.