★ Matrix multiplication :

$$\begin{array}{c|ccc} & 0 & 1 & 2 \\ \hline 0 & 1 & 2 & 3 \\ 1 & 4 & 5 & 6 \end{array} \quad * \quad \begin{array}{c|cc} & 0 & 1 \\ \hline 0 & 10 & 11 \\ 1 & 20 & 21 \\ 2 & 30 & 31 \end{array} \quad = \quad \begin{array}{c|cc} & 0 & 1 \\ \hline 0 & 1\times10 + 2\times20 + 3\times30 = 140 & 1\times11 + 2\times21 + 3\times31 = 146 \\ 1 & 4\times10 + 5\times20 + 6\times30 = 320 & 4\times11 + 5\times21 + 6\times31 = 335 \end{array}$$

[A] 2×3 (B) 3×2 (C) (2×2)

$$\begin{array}{c|cc} & 0 & 1 \\ \hline 0 & 140 & 146 \\ 1 & 320 & 325 \end{array}$$

1. $A(m_1 \times n_1)$ and $B(m_2 \times n_2)$

   if and only if $n_1 == m_2$

   ⇒ no. of cols in A (n_1) = no. of rows in B (m_2)

2. the resultant matrix 'C' will be $(m_1 \times n_2)$

   $(2\times3) * (3\times2) \Rightarrow (2\times2)$

C[0][0]

⇒ multiply 0th row of A with 0th col of B

C[0][1]

⇒ multiply 0th row of A with 1st col of B

★ TC : O(rowsA * colsB * colsA)

⇒ $O(N^3)$ ★ SC : $O(N^2)$

Cols in A

$$
\begin{array}{c|ccc}
 & 0 & 1 & 2 \\
\hline
0 & a_{00} & a_{01} & a_{02} \\
1 & a_{10} & a_{11} & a_{12}
\end{array}
\qquad
\begin{array}{c|cccc}
 & 0 & 1 & 2 & 3 \\
\hline
0 & b_{00} & b_{01} & b_{02} & b_{03} \\
1 & b_{10} & b_{11} & b_{12} & b_{13} \\
2 & b_{20} & b_{21} & b_{22} & b_{23}
\end{array}
=
\begin{array}{c|cccc}
 & 0 & 1 & 2 & 3 \\
\hline
0 & c_{00} & c_{01} & c_{02} & c_{03} \\
1 & c_{10} & c_{11} & c_{12} & c_{13}
\end{array}
$$

(2×3) rows in B (3×4) (2×4)

$$c_{00} = a_{0\overbrace{0 \cdot b_0}^{0}0} + a_{0\overbrace{1 \cdot b_1}^{1}0} + a_{0\overbrace{2 \cdot b_2}^{2}0}$$

$$c_{01} = a_{0\overbrace{0 \cdot b_0}^{0}1} + a_{0\overbrace{1 \cdot b_1}^{1}1} + a_{0\overbrace{2 \cdot b_2}^{2}1}$$

$$c_{02} = a_{0\overbrace{0 \cdot b_0}^{0}2} + a_{0\overbrace{1 \cdot b_1}^{1}2} + a_{0\overbrace{2 \cdot b_2}^{2}2}$$

res[r][c] = sum(a[r][?] * b[?][c])

= a[r][0] * b[0][c]

+ a[r][1] * b[1][c]

+ a[r][2] * b[2][c]

$$= \sum_{k=0}^{\text{cols}(A)-1 \text{ (or) } \text{rows}(B)-1} A[r][k] * B[k][c]$$

* Spiral traversal :

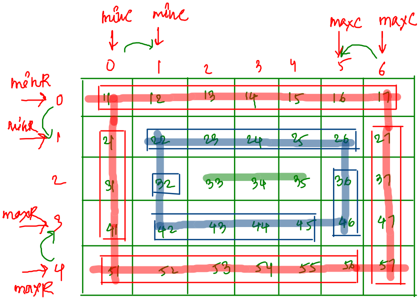
	0	1	2	3	4	5	6
0	11	12	13	14	15	16	17
1	21	22	23	24	25	26	27
2	31	32	33	34	35	36	37
3	41	42	43	44	45	46	47
4	51	52	53	54	55	56	57

op : 11 12 13 14 15 16 17 27 37 47 57
56 55 54 53 52 51 41 31 21 22 23
24 25 26 36 46 45 44 43 42 32
33 34 35

* TC : how many times each element is visited ? $\Rightarrow$ 1

total tc = $(n \times m) \times 1$

$\Rightarrow O(n \times m)$



① top wall → minR
② right wall → maxC
③ bottom wall → maxR
④ left wall → minC

① Top wall ((0,0) (0,1) (0,2) (0,3) (0,4) (0,5) (0,6))

(r = minR, c = minC to maxC)

```
for(let c= minC ; c<=maxC ; c++) {
    psw( mat [minR] [c] + " ");
}
minR++;
```

(once the Job is completed move to next wall)

② Right wall ((1,6) (2,6) (3,6) (4,6))

[r = minR to maxR, c = maxC]

```
for (let r = minR ; r<= maxR ; r++) {
    psw( mat [r] [maxC] + " ");
}
maxC--;
```

③ Bottom wall

[r = maxR, c = maxC to minC]

```
for ( let c= maxC ; c>= minC; c--) {
    psw ( mat [maxR] [c] + " ");
}
maxR--;
```

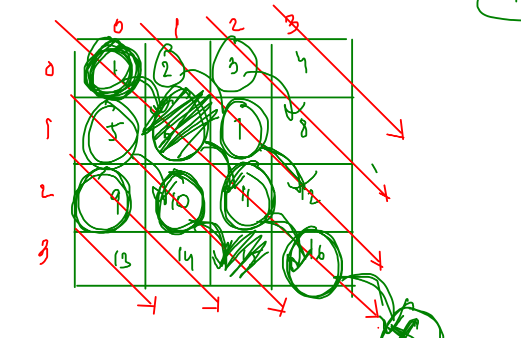
④ Left wall

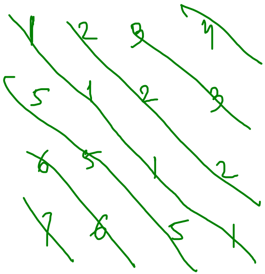
[r = maxR to minR , c = minC]

```
for ( r= maxR ; r>= minR ; r--) {
    psw( mat [r] [minC] + " ");
}
minC++;
```

★ Repeat this process until all the elements are printed.

* Toeplitz matrix :





All elements in a same diagonal must be equal.

m(0)(0) == m(1)(1) == m(2)(2) == m(3)(3)

m(1)(0) == m(2)(1) == m(3)(2)

m(r)(c) → next diagonal element → m(r+1)(c+1)

m(0)(0) → m(1)(1)

m(2)(0) → m(3)(1)

m(2)(1) → m(3)(2)

```
for(let r=0; r<rows; r++){
    for(let c=0; c<cols; c++){
                  (r+1 < rows && c+1 < cols)
        if (m(r)(c) != m(r+1)(c+1)) {
            return false;
        }
    }
}
return true;
```

* TC : O(n×m)

* Absolute diagonal difference :

$$
\begin{array}{c|cccc}
 & 0 & 1 & 2 & 3 \\
\hline
0 & 1 & 2 & 3 & 4 \\
1 & 5 & 6 & 7 & 8 \\
2 & 9 & 10 & 11 & 12 \\
3 & 13 & 14 & 15 & 16
\end{array}
$$

anti diagonal

diagonal

$4+7+10+13$
$= 34$

$= 1+6+11+16$
$= 34$

abs(34 − 34)
= 0

* TC : O(N)

(0,0), (1,1), (2,2), (3,3) → diag
(0,3), (1,2), (2,1), (3,0) → adiag

$i = \cancel{0}\ \cancel{1}\ \cancel{2}\ 3$
$j = \cancel{0}\ \cancel{1}\ \cancel{2}\ 3$ } diag i=0 i++ ; j=0 j++

$i = \cancel{0}\ \cancel{1}\ \cancel{2}\ 3$
$j = \cancel{3}\ \cancel{2}\ \cancel{1}\ 0$ } adiag i=0 i++; ; j = cols-1 j--;

---

	diag	adiag
r=0	(0,0)	(0,3)
	(1,1)	(1,2)
r=4	(2,2)	(2,1)
	(3,3)	(3,0)

diag (0,0)
col = row (1,1)

adiag

col = n − row − 1

r = 0, c = 4 − 0 − 1 = 3
r = 1, c = 4 − 1 − 1 = 2
r = 2, c = 4 − 2 − 1 = 1
r = 3, c = 4 − 3 − 1 = 0

★ Alternate matrix :

	0	1	2	3
0	10	1	2	3
1	7	6	13	9
2	14	11	12	16
3	5	4	15	8

4 x 4 Chess Board

(r, c)

$\downarrow$

$r+c$

even $\rightarrow$ Black

odd $\rightarrow$ white

Black	white
(0, 0) = 0	(0, 1) = 1
(0, 2) = 2	(0, 3) = 3
(1, 1) = 2	(1, 0) = 1
(1, 3) = 4	(1, 2) = 3
(2, 0) = 2	(2, 1) = 3
(2, 2) = 4	(2, 3) = 5
(3, 1) = 4	(3, 0) = 3
(3, 3) = 6	(3, 2) = 5

bsum = 10 + 2 + 6 + 9 + 14 + 12 + 4 + 8

= 75

wsum = 1 + 3 + 7 + 13 + 11 + 16 + 5 + 15

= 71

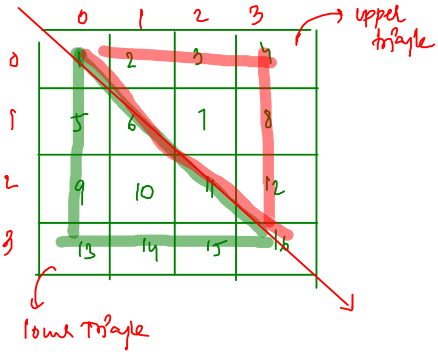
★ TC : O (n × m)

→ Iterate on all elements

→ Based of r+c is even → add to bsum

odd → add to wsum

* sum of upper and lower triangles :



lower		upper	
(0, 0)	0>=0	(0, 0)	0<=0
(1, 0)	1>=0	(0, 1)	0<=1
(1, 1)	1>=1	(0, 2)	0<=2
(2, 0)	2>=0	(0, 3)	0<=3
(2, 1)	2>=1	(1, 1)	1<=1
(2, 2)	2>=2	(1, 2)	1<=2
(3, 0)	3>=0	(1, 3)	1<=3
(3, 1)	3>=1	(2, 2)	2<=2
(3, 2)	3>=2	(2, 3)	2<=3
(3, 3)	3>=3	(3, 3)	3<=3
$r>=c$		$r<=c$	

(r, c)

r>=c lower

r<=c upper

* TC : O(N * M)

→ same as previous problem