

\* Decrease print   Increase print

ip: n = 3

op: 3 2 1 1 2 3

#1: function printDec(n) {

    if (n == 0) {  
        return;

        process.stdout.write(n + " "); // 5  
        printDec(n - 1); // 4 3 2 1

} // Recursion

function printInc(n) {

    if (n == 0) {  
        return;

    printInc(n - 1); // 1 2 3 4

    process.stdout.write(n + " "); // 5

} // Recursion

function printDI(n) {

    printDec(n); // 5 4 3 2 1

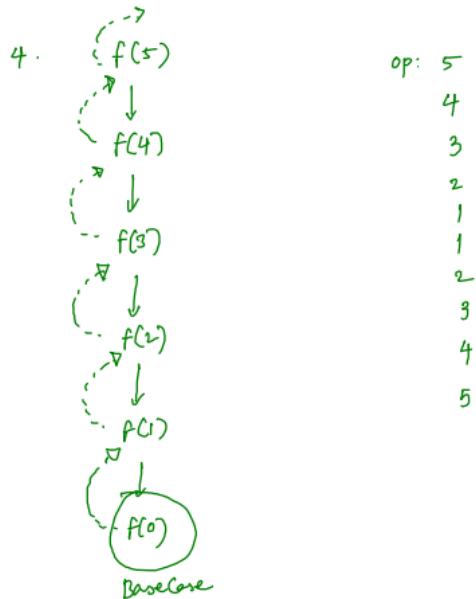
    printInc(n); // 1 2 3 4 5

} // not Recursion

(not calling itself)

#2:

```
3. function printDI(n) {  
    if(n == 0) {  
        return;  
    }  
    console.log(n); // 5  
    printDI(n-1); // 4 3 2 1 1 2 3 4  
    console.log(n); // 5
```

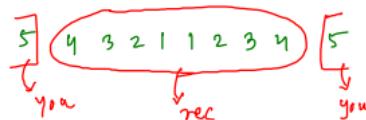


1. faith  $\Rightarrow$

printDI(n)  
 $\rightarrow$  will print  $n \rightarrow 1$  and  $1 \rightarrow n$

2. printDI(5)

$\Rightarrow$  5 4 3 2 1 1 2 3 4 5  
printDI(4)  $\uparrow$   
 $\Rightarrow$  4 3 2 1 1 2 3 4



## \* Factorial :

ip:  $n=5$ , op:  $5! = 5 \times 4 \times 3 \times 2 \times 1$   $0! = 1$

```
3. function factorial(n) {  
    if(n == 0) {  
        return 1;  
    }  
    const smallAns = factorial(n-1); // 4!  
    return n * smallAns;  
}
```

4.  $f(5)$  5  $\times$  4! = 120

$4 \times 3$  = 24  $\times$   $f(4)$

$3 \times 2$  = 6  $\times$   $f(3)$

$2 \times 1$  = 2  $\times$   $f(2)$

$1 \times 1$  = 1  $\times$   $f(1)$

1  $\downarrow$  f(0)

Basecase

1. faith

$\Rightarrow \text{factorial}(n)$

$\Rightarrow n \times n-1 \times n-2 \times \dots \times 2 \times 1$

2. factorial(5)

$\Rightarrow 5 \times 4 \times 3 \times 2 \times 1$

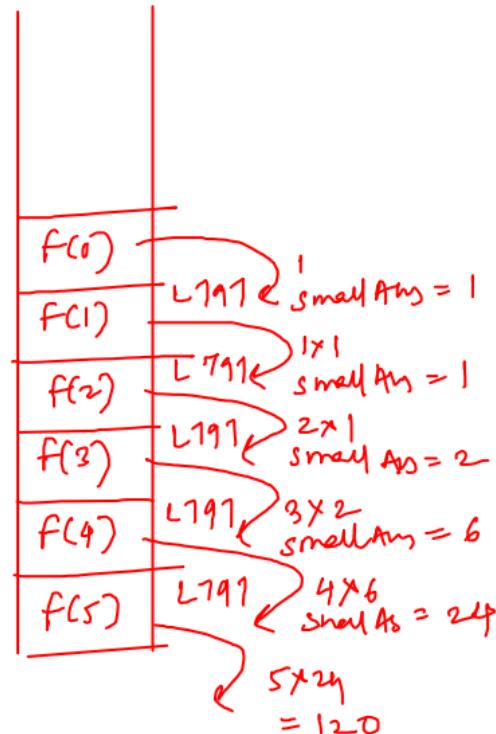
factorial(4)

$\Rightarrow 4 \times 3 \times 2 \times 1$

$5! = 5 \times 4!$

```
792 function factorial_recursive(n) {  
793     if (n == 0) {  
794         return 1;  
795     }  
796  
797     const smallAns = factorial_recursive(n - 1);  
798     return n * smallAns;  
799 }
```

(post-way)  
(getting the answer)



# pre-order way :

```
function factorial (curr-num, curr-fact, n) {  
    if (curr-num == n) {  
        } return curr-fact  
    }  
  
    return factorial (curr-num + 1, curr-fact * (curr-num + 1), n)  
}
```

1. factorial (curr-num, curr-fact, n)

⇒ curr-fact will store  
the (curr-num) !

2. factorial (1, 1, 5)

factorial (2, 2, 5)

factorial (3, 6, 5)

factorial (4, 24, 5)

factorial (5, 120, 5)

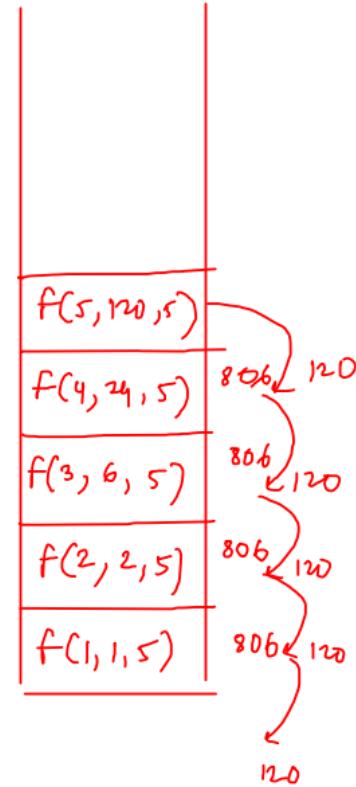
\* Keep building your answer as recursion  
progress using some additional variables in  
the function parameters.

```
801 function fact(curr_num, curr_fact, n) {  
802     if (curr_num == n) {  
803         return curr_fact;  
804     }  
805  
806     return fact(curr_num + 1, curr_fact * (curr_num + 1), n);  
807 }
```

fact(1, 1, 5)

9:30 - 9:45 pm

BREAK



#  
double  
pascal's  
triangle

	0	1	2	3	4	5
0	1					
1	1	1				
2		2	1			
3		3	3	1		
4		1	4	6	4	1
5	1	5	10	10	5	1
0	1	1	=	>	4	5

$$\text{arr}[5][2] = \text{arr}[4][1] + \text{arr}[4][2]$$
$$= 4 + 6 = 10$$

no. of cols = row + 1

row = 0, no. of cols = 1

row = 1, no. of cols = 2

row = 2, no. of cols = 3

arr[row][0] = 1

arr[row][no. of cols - 1] = 1

arr[row][col] = arr[row-1][col-1] + arr[row-1][col];

for (row = 0; row < n; row++) {

    for (col = 0; col <= row; col++) {

        if (col == 0 || col == row) {

            arr[row][col] = 1;

        } else {

            arr[row][col] = arr[row-1][col-1] + arr[row-1][col];

        }

}

\*  $x^n$  :

$$\text{ip: } x = 2, n = 4 \quad \text{op: } 2^4 = 16$$

3. function power(x, n) {

```
if (n == 0) {  
    return 1;  
}
```

```
const smallAns = power(x, n - 1) //  $x^3$ 
```

```
return x + smallAns
```

4.  $f(2, 4) \xrightarrow{\text{at } x} \downarrow \xrightarrow{\text{at } n} 2 \times 8 = 16$

$$\begin{array}{c} \xrightarrow{\text{at } x} \\ \xrightarrow{\text{at } n} f(2, 3) \end{array}$$

$$\begin{array}{c} \xrightarrow{\text{at } x} \\ \xrightarrow{\text{at } n} f(2, 2) \end{array}$$

$$\begin{array}{c} \xrightarrow{\text{at } x} \\ \xrightarrow{\text{at } n} f(2, 1) \end{array}$$

$$\begin{array}{c} \xrightarrow{\text{at } x} \\ \xrightarrow{\text{at } n} f(2, 0) \end{array}$$

BaseCase

1. faith

$\Rightarrow \text{power}(x, n)$

$\Rightarrow x^n$

2.  $\text{power}(2, 4) \Rightarrow 2^4 \Rightarrow 16$

$\text{power}(2, 3) \Rightarrow 2^3 \Rightarrow 8$

$$2^4 = \underbrace{(2 * 2^3)}_{\text{you}} \xrightarrow{\text{rec}}$$

# pre-order way:

```
function power(x, n, curr-power) {  
    if (n == 0) {  
        return curr-power;  
    }  
    return power(x, n-1, curr-power * x);  
}
```

$x \rightarrow 0$   
 $x^{th}$

n-calls  
 $\Rightarrow O(n)$

power(2, 5) 1  
↓  
\*2

power(2, 4 2)

↓  
power(2, 3, 4)  
↓  
\*2

power(2, 2, 8)  
↓  
\*2

power(2, 1, 16)

↓  
power(2, 0, 16)  
Base case  
→ ans

\* optimized  $x^n$ :

```
function power(x, n) {
    if (n == 0)
        return 1;
    else
        smallAns = power(x, n/2);
        if (n%2 == 1) {
            return smallAns * smallAns + x;
        }
        return smallAns * smallAns;
}
```

$$2^{15} = 2^7 \star 2^7 \quad \text{extra when } n \text{ is odd}$$

$$F(2, 10) \quad \begin{matrix} 32+32 \\ \downarrow \\ 64 \end{matrix} \quad \approx 1024$$

$$\begin{matrix} 32+32 \\ \downarrow \\ 64 \end{matrix} \quad \begin{matrix} f(2, 5) \\ \downarrow \\ 32+32 \end{matrix}$$

$$10 \text{ calls (power)} \rightarrow O(n)$$

$$4 \text{ calls (optimized power)} \rightarrow O(\log n)$$

$$\begin{matrix} 32+32 \\ \downarrow \\ 64 \end{matrix} \quad \begin{matrix} f(2, 2) \\ \downarrow \\ 32+32 \end{matrix}$$

$$\begin{matrix} 32+32 \\ \downarrow \\ 64 \end{matrix} \quad \begin{matrix} f(2, 1) \\ \downarrow \\ 1 \end{matrix}$$

BaseCase

1. faith

$$\Rightarrow \text{power}(x, n)$$

$$\Rightarrow x^n$$

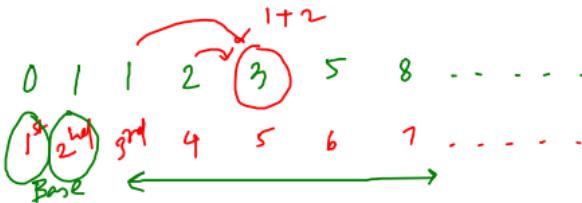
2.  $\text{power}(2, 10) \Rightarrow 1024$   
 $\text{power}(2, 9) \Rightarrow 512$

$$2^{10} = \begin{matrix} 2^9 & \star & 2^9 \\ \downarrow & & \downarrow \\ \text{you} & & \text{rec} \end{matrix}$$

$$2^5 = \begin{matrix} 2^3 & \star & 2^2 \\ \downarrow & & \downarrow \\ \text{rec} & & \text{you} \end{matrix}$$

$$2^{10} = \begin{matrix} 2^5 & \star & 2^5 \\ \downarrow & & \downarrow \\ \text{rec} & & \text{rec} \\ \downarrow & & \downarrow \\ \text{you} & & \cancel{\text{you}} \end{matrix}$$

## \* Fibonacci:



$$n^{\text{th}} \text{fib} = n-1^{\text{th}} \text{fib} + n-2^{\text{th}} \text{fib}$$

$$6^{\text{th}} \text{fib} = 5^{\text{th}} \text{fib} + 4^{\text{th}} \text{fib}$$

$$= 3 + 2 = 5$$

1. faith

$\text{fib}(n) \Rightarrow n^{\text{th}}$  fibonacci  
number

2. logic

$$\text{fib}(6) = 5$$

$$\text{fib}(5) = 3$$

$$\text{fib}(4) = 2$$

$$\Rightarrow \text{fib}(6) = (\text{fib}(5)) + (\text{fib}(4))$$

rec      you      rec

```
function fib(n) {
    if (n == 1) {
        return 0;
    } else if (n == 2) {
        return 1;
    } else {
        return fib(n-1) + fib(n-2);
    }
}
```

## \* $\text{fib}(2)$

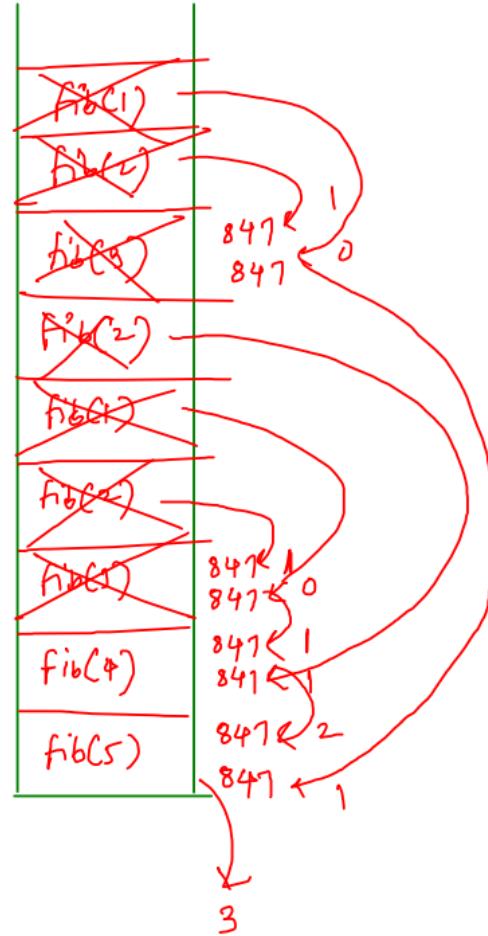
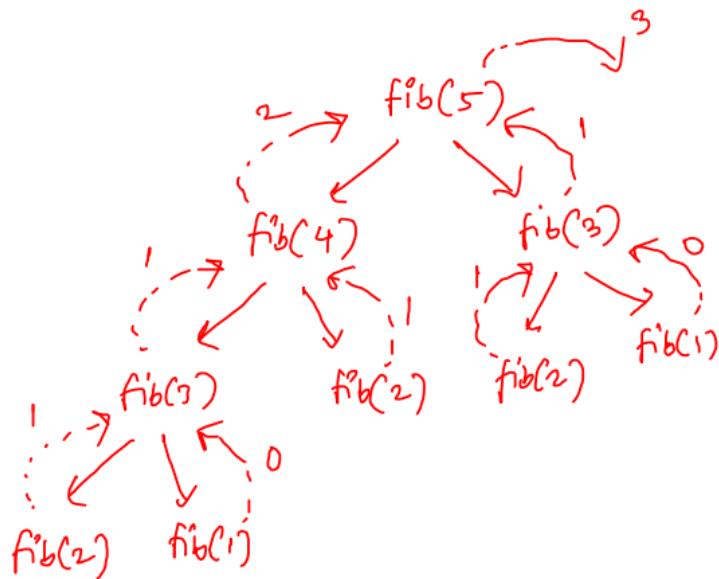
$$= \underline{\text{fib}(1)} + \underline{\text{fib}(0)}$$

$$= 0 + (\cancel{\text{fib}(-1)}) + (\cancel{\text{fib}(-2)})$$

$$= 0 + \text{fib}(-2) + \text{fib}(-3) + \text{fib}(-2) + \text{fib}(-1)$$

⋮

```
843 function fib(n) {  
844     if (n == 1) return 0;  
845     if (n == 2) return 1;  
846  
847     return fib(n - 1) + fib(n - 2);  
848 }
```



```
843     function fib(n) {  
844         if (n == 1) return 0;  
845         if (n == 2) return 1;  
846  
847         return fib(n - 1) + fib(n - 2);  
848     }
```

