MSBD 5006 Due: 6th Oct.

Assignment #1 Solution

Instructor: Shiqing LING Name: XXX ID 1234567

Problem 1.

(a)

ACF:

```
> round(c( acf(A, lag.max=24, type="correlation", plot=FALSE)$acf ), 4)
 [1]  1.0000  0.2246 -0.0022 -0.0762 -0.0297 -0.0224 -0.0507 -0.0504 -0.0849
[10] -0.0792 -0.0045  0.0757  0.2675  0.0177 -0.0516 -0.0922 -0.0244 -0.0417
[19] -0.0911 -0.0802 -0.0509 -0.0576 -0.0487  0.0526  0.2473
```

PACF:

```
> round(c( acf(A,lag.max=24, type="partial", plot=FALSE)$acf ), 4)
 [1]  0.2246 -0.0554 -0.0668  0.0032 -0.0205 -0.0502 -0.0318 -0.0752 -0.0565
[10]  0.0164  0.0604  0.2396 -0.1062 -0.0291 -0.0570 -0.0037 -0.0473 -0.0697
[19] -0.0353 -0.0011 -0.0481 -0.0614  0.0373  0.1678
```

(b)

```
> Box.test(A,lag=12,type="Ljung")

        Box-Ljung test

data:  A
X-squared = 69.652, df = 12, p-value = 3.72e-10
```

After L-B test, p-value $= 3.72e^{-10} < 0.05$, can't accept H_0.

Thus: The first 12 lags of ACF are not all zero.

Problem 2

(a)

```
> round(c( acf(A, lag.max=12, type="correlation", plot=FALSE)$acf ), 4)
 [1]  1.0000  0.1367 -0.0624 -0.0334 -0.0591 -0.0091  0.0028 -0.0111 -0.0782
[10] -0.0271  0.0430 -0.0356  0.0053
```

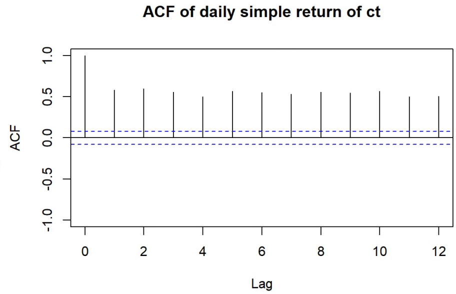
(b)

```
> Box.test(A,lag=12,type="Ljung")

        Box-Ljung test

data:  A
X-squared = 16.812, df = 12, p-value = 0.1568
```

After Ljung-Box test, p-value $= 0.1568 > 0.05$, accept H_0.

Thus: the first 12 lags of ACF are zero.

Problem 3



(a)

```
> round(c( acf(c_t, lag.max=12, type="correlation", plot=FALSE)$acf ), 4)
 [1] 1.0000 0.5834 0.5964 0.5579 0.5008 0.5663 0.5515 0.5299 0.5555 0.5458 0.5636 0.4976 0.5030
> round(c( acf(c_t, lag.max=12, type="partial", plot=FALSE)$acf ), 4)
 [1]  0.5834  0.3881  0.2117  0.0702  0.2221  0.1592  0.0699  0.1176  0.1139  0.1212 -0.0483  0.0122
> Box.test(c_t,lag=12,type="Ljung")

        Box-Ljung test

data:  c_t
X-squared = 2186.6, df = 12, p-value < 2.2e-16
```

$p = 2.2e-16 < 0.05$, can't accept H_0.

Thus: the first 12 lags of ACF are not all zero.

(b)

```
> round(c( acf(z_t, lag.max=12, type="correlation", plot=FALSE)$acf ), 4)
 [1]  1.0000 -0.5155  0.0618  0.0222 -0.1472  0.0962  0.0084 -0.0564  0.0423 -0.0331  0.1005 -0.0856  0.0417
```

(c)

$$\phi_0 = 0.3257 \times (1 - 0.9783) = 0.0071$$

$$r_t = 0.0071 + 0.9783 r_{t-1} + a_t - 0.8073 a_{t-1} - 0.1397 a_{t-4} + 0.1801 a_{t-5}$$

```
> out=arima(c_t,order=c(1,0,5), fixed=c(NA,NA,0,0,NA,NA, NA))
> summary(out)

Call:
arima(x = c_t, order = c(1, 0, 5), fixed = c(NA, NA, 0, 0, NA, NA, NA))

Coefficients:
         ar1      ma1  ma2  ma3      ma4     ma5  intercept
      0.9783  -0.8073    0    0  -0.1397  0.1801     0.3257
s.e.  0.0094   0.0303    0    0   0.0404  0.0410     0.0756

sigma^2 estimated as 0.0341:  log likelihood = 161.66,  aic = -311.32

Training set error measures:
                        ME      RMSE      MAE  MPE MAPE      MASE         ACF1
Training set -0.0001542783 0.1846677 0.131648 -Inf  Inf 0.8361818 -0.008221604
```

Problem 4

(a)

$$\phi_0 = (1 - 0.4172 - 0.2003 + 0.1648) \times 0.0168 = 0.0092$$

$$r_t = 0.0092 + 0.4172 r_{t-1} + 0.2003 r_{t-2} - 0.1648 r_{t-3} + a_t$$

```
> out=arima(da,order=c(3,0,0))
> out

Call:
arima(x = da, order = c(3, 0, 0))

Coefficients:
         ar1     ar2      ar3  intercept
      0.4172  0.2003  -0.1648     0.0168
s.e.  0.0636  0.0679   0.0642     0.0011

sigma^2 estimated as 9.313e-05:  log likelihood = 769.83,  aic = -1529.67
```

(b) $r_t - 0.4172 r_{t-1} - 0.2003 r_{t-2} + 0.1648 r_{t-3} = 0.0092 + a_t$.

B	B^2	B^3
$0.534 - \alpha$	$-0.534\alpha + \beta$	0.534β
$\shortparallel$		
-0.4172		

$$1 - 0.4172 B - 0.2003 B^2 + 0.1648 B^3 = 0.$$

$$\left(1 + \frac{1}{1.871} B\right)\left(1 - \alpha B + \beta B^2\right) = 0 \quad \begin{cases} \alpha = 0.9512 \\ \beta = 0.3086 \end{cases}$$

$$(1 + 0.534 B)(1 - 0.9512 B + 0.3086 B^2) = 0 \Longrightarrow 0.9512^2 + 4 \times (-0.3086) < 0. \text{ cycle exists.}$$

```
> roots=polyroot(c(1, -0.4172, -0.2003, 0.1648))
> roots
[1]  1.543209+0.928256i -1.871006+0.000000i  1.543209-0.928256i
> Mod(roots)
[1] 1.800876 1.871006 1.800876
> k=2*pi/acos(1.543209/1.800876)
> k
[1] 11.60266
```

$k = 11.60$

(c)

Prediction:	0.0140	0.0161	0.0167	0.0171
Standard error:	0.0097	0.0105	0.0111	0.0111

Problem 5

(1)

```
> summary(out)

Call:
arima(x = A, order = c(0, 0, 1))

Coefficients:
         ma1  intercept
      0.1593     0.0109
s.e.  0.0499     0.0029

sigma^2 estimated as 0.0027:  log likelihood = 683.03,  aic = -1360.07

Training set error measures:
                       ME       RMSE        MAE      MPE     MAPE      MASE         ACF1
Training set 2.290569e-05 0.05195749 0.03978375 -280.807 808.6392 0.7566435 -0.009978982
```

$r_t = 0.0109 + a_t + 0.1593\, a_{t-1}$

(2)

```
> Box.test(out$residuals,lag=12,type="Ljung")

        Box-Ljung test

data:  out$residuals
X-squared = 8.2257, df = 12, p-value = 0.7672

> pv=1-pchisq(8.2257,11) #Compute p-value using 11 degrees of freedom
> pv
[1] 0.6929546
```

According to the L-B test for the residuals

p-value $= 0.6929 > 0.05$.

The residuals are not significant

So this is an adequate model.

(3) Prediction: 0.0090 0.0109 0.0109 0.0109