Bright soliton

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1 Prelude

First, let's load some packages and set plot defaults.

```
using Pkg, Plots, LaTeXStrings
gr(legend=false,titlefontsize=12,size=(500,300),colorbar=false,grid=false)
using FourierGPE
```

2 Gross-Pitaevskii equation

We are going to solve the Gross-Pitaevskii equation

$$i\hbar\frac{\partial\psi(x,t)}{\partial t} = \left(-\frac{\hbar^2\partial_x^2}{2m} + V(x,t) + g|\psi|^2\right)\psi$$

for particular initial and boundary conditions.

2.1 Bright soliton

The bright soliton provides a good test of any numerical simulation of the Gross-Pitaevskii equation as it involves a delicate balance between kinetic dispersion and the attractive non-linearity. An initial state with finite momentum also tests the periodicity of the fft method since the soliton will eventually wrap around the domain.

The bright soliton wavefunction with wavenumber k describing its collective motion is

$$\psi_s(x) = \sqrt{\frac{N_s}{2\xi_s}}\mathrm{sech}(x/\xi_s)e^{ikx}$$

where the soliton scale ξ_s for N_s particles is given by

$$\xi_s \equiv \frac{2}{|g|N_s}$$

3 Simulation

3.1 Potential function

```
import FourierGPE.V
V(x,t) = 0.0 |> complex
```

```
V (generic function with 3 methods)
```

3.2 Units

In any numerical calculations we should have a clear understanding of our choice of physical units.

In length unit ξ_s, and time unit

$$t_s \equiv \frac{m\xi_s^2}{\hbar},$$

and rescaled wavefunction $\bar{\psi} = \psi\sqrt{\xi_s}$, our dimensionless form of the equation of motion is

$$i\frac{\partial\bar{\psi}(\bar{x},\bar{t})}{\partial\bar{t}} = \left(-\frac{\bar{\partial}_x^2}{2} + \bar{g}|\bar{\psi}|^2\right)\bar{\psi}$$

where the dimensionless interaction parameter is

$$\bar{g} \equiv \frac{m\xi_s}{\hbar^2}g < 0$$

4 Initialize the simulation

Create the `sim` struct holding all parameters, with predefined grids.

```
L = (60.0,)
N = (512,)
sim = Sim(L,N)
@unpack_Sim sim;
```

5 Parameters

```
μ = 25.0
g = -0.01
γ = 0.0
Ns = 200
ξs = 2/abs(g)/Ns
us = 20
tf = 1π |> Float64
Nt = 150
t = LinRange(0.,tf,Nt);
```

5.1 Initial condition

We initialize the bright soliton with dimensionless velocity u_s as

```
x = X[1]
ψs(x) = sqrt(Ns/2ξs)*sech(x/ξs)*exp(im*us*x)
ψi = ψs.(x)
ϕi = kspace(ψi,sim)

@pack_Sim! sim; #finally, pack everything up for simulation.
```

5.2 Evolve in k-space

Now we have everything we need to evolve in k-space

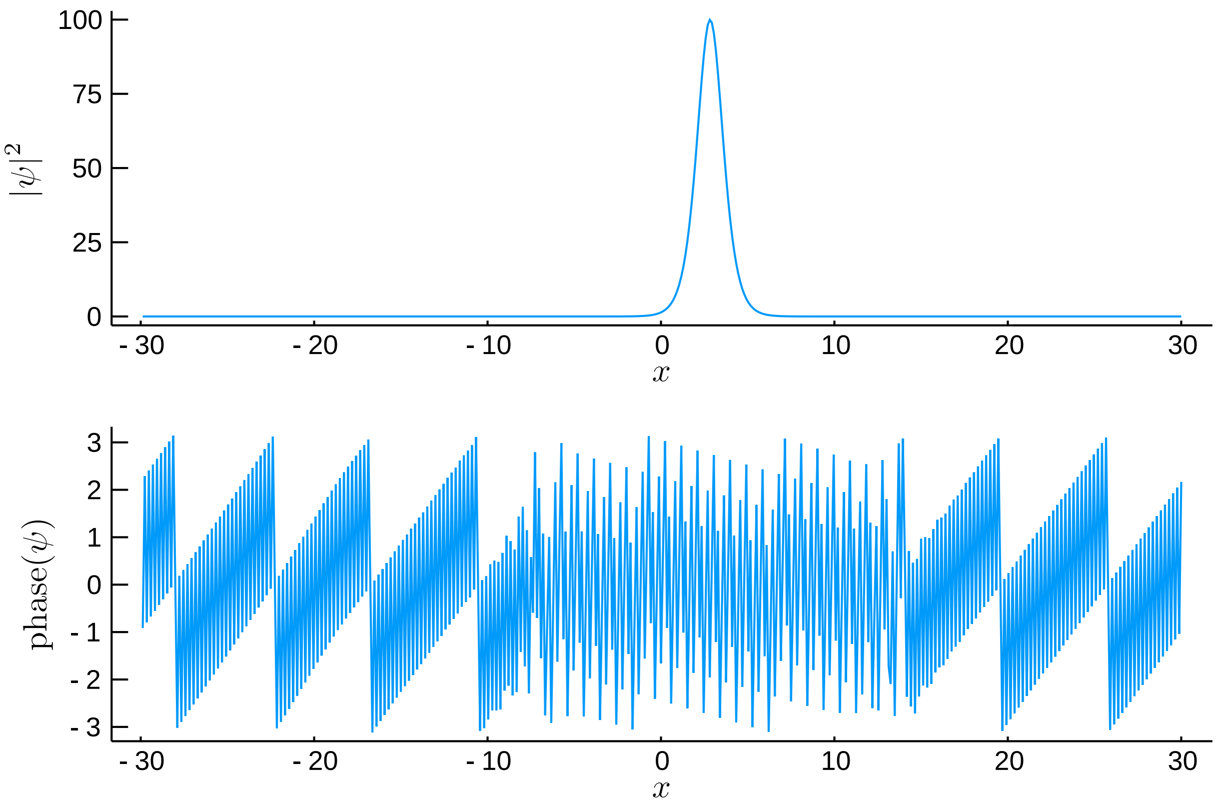
```
@time sol = runsim(sim);
```

```
1.031067 seconds (6.37 M allocations: 422.479 MiB, 4.67% gc time)
```

5.3 Plot the solution

(check the phase!)

```
showpsi(x,xspace(sol[end],sim))
```



or we can make an animation, found in the media folder.

```
anim = @animate for i=1:Nt-8
    ψ = xspace(sol[i],sim)
    y=abs2.(ψ)
```

```
    plot(x,y,fill=(0,
0.2),size=(600,150),legend=false,grid=false,xticks=false,yticks=false,axis=false)
end

gif(anim, "./media/brightsoliton.gif", fps = 25)
```