Vortex precession in a 2D parabolic trap

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1 Introduction

In this example we simulate the precession of a single quantum vortex in a harmonic trap, and compare with the analytical result found by Fetter.

As simple model of this process, we start with the damped GPE

$$i\hbar\partial_t\psi = (1 - i\gamma)(L - \mu)\psi$$

where the GP operator is defined as

$$L\psi \equiv \left(-\frac{\hbar^2\nabla^2}{2m} + V(\mathbf{r}) + g|\psi|^2\right)\psi$$

and as usual the S-wave interaction parameter is $g = 4\pi\hbar^2 a/m$ for S-wave scattering lenght a.

The trap is chosen to be cylindrically symmetric

$$V(\mathbf{r}) = \frac{m\omega_\perp^2}{2}(x^2 + y^2)$$

The number $\gamma \ll 1$ describes irreversable interactons between condensate and noncondensate atoms, inducing condensate growth.

2 Loading the package

First, we load some useful packages, setting defaults for `Plots`.

```
using Plots, LaTeXStrings
gr(titlefontsize=12,size=(500,300),transpose=true,colorbar=false)
```

```
Plots.GRBackend()
```

Now load `FourierGPE`

```
using FourierGPE
```

In this example, we work in oscillator units. The units of length and time are $a_\perp = \sqrt{\hbar/m\omega_\perp}$ and $1/\omega_\perp$ respectively.

3 Initialize simulation

Initialize default sim with domain and grid parameters

```
L = (20.0,20.0)
N = (128,128)
sim = Sim(L,N)
@unpack_Sim sim;
```

4 Declare the potential

```
import FourierGPE.V
V(x,y,t)::Float64 = 0.5*(x^2 + y^2)
```

```
V (generic function with 3 methods)
```

5 Thomas-Fermi initial state

```
ψ0(x,y,μ,g) = sqrt(μ/g)*sqrt(max(1.0-V(x,y,0.0)/μ,0.0)+im*0.0)
x,y = X

μ = 25.0
ψi = ψ0.(x,y',μ,g)
ϕi = kspace(ψi,sim)
@pack_Sim! sim;
```

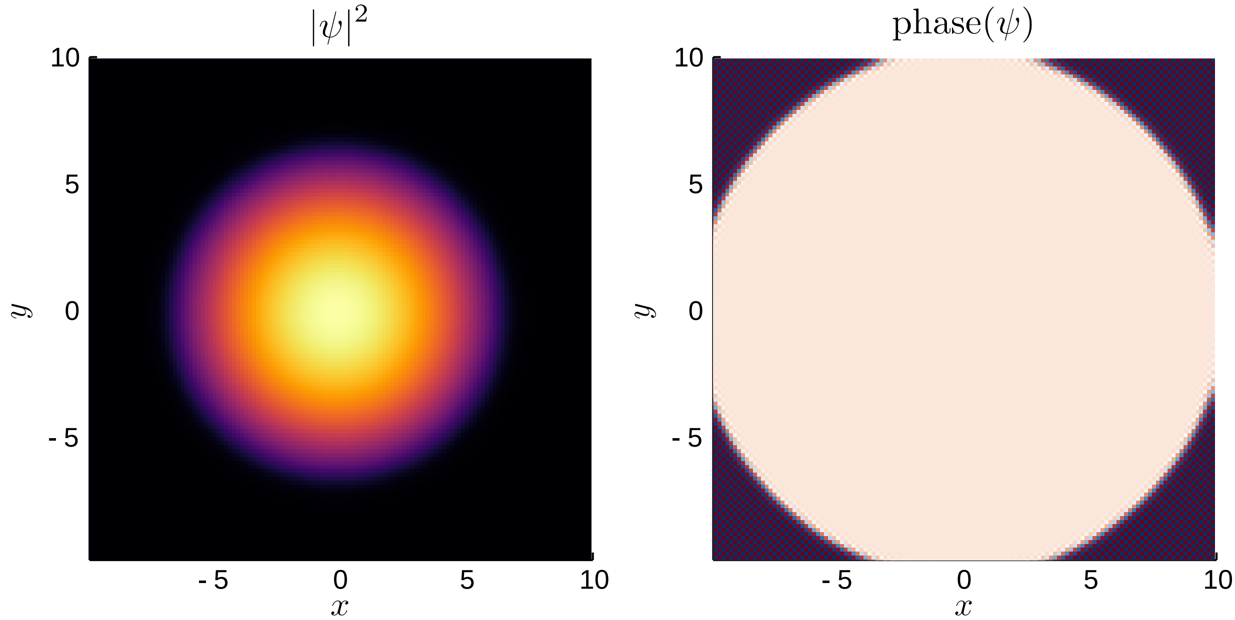
6 Imaginary-time evolution in k space

```
@time sol = runsim(sim);
```

```
2.700594 seconds (4.53 M allocations: 289.054 MiB, 2.28% gc time)
```

Let's pull out the final state and verify that it is indeed the ground state of the harmonic trap:

```
ϕg = sol[end]
ψg = xspace(ϕg,sim)
showpsi(x,y,ψg)
```



7 Time dynamics: precession of an off-axis vortex

We can use `VortexDistributions` to imprint a vortex off axis and test vortex precession rate according to the GPE.

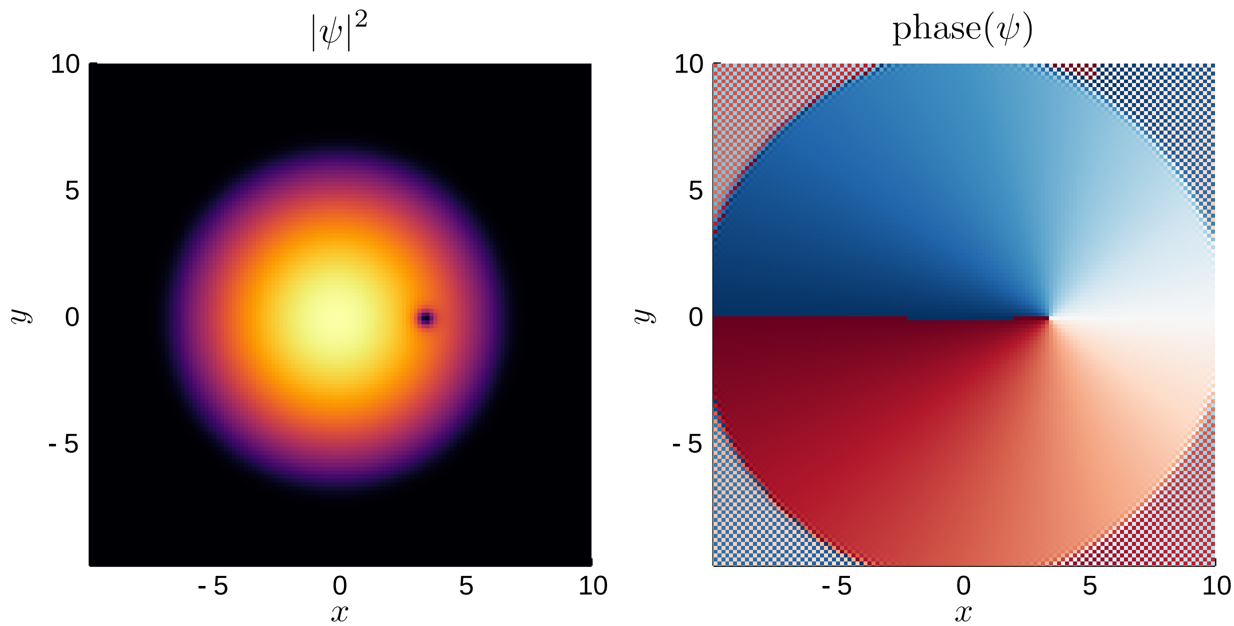
```
using VortexDistributions
```

8 Initial state

We imprint a vortex inside the Thomas-Fermi radius

```
healing(x,y,μ,g) = 1/sqrt(g*abs2(ψ0(x,y,μ,g)))
Rtf = sqrt(2*μ)
rv = 0.5*Rtf
xv,yv,cv = rv, 0.0, 1
pv1 = PointVortex(xv,yv,cv) # coordinates and charge for vortex
ξv = healing(xv,yv,μ,g) # local healing length at the vortex
v1 = ScalarVortex(ξv,pv1) # define scalar GPE vortex with local healing length

ψ1 = Torus(copy(ψg),x,y) # methods in VortexDistributions require type conversion
vortex!(ψ1,v1) # phase/density imprint vortex
ψv = ψ1.ψ #pull out the new wavefunction
showpsi(x,y,ψv)
```



9 Precession frequency

In the Thomas-Fermi regime, precession frequency is given analytically in terms of the healing length ξ by (see e.g. Fetter JLTP 2010):

$$\Omega_m = \frac{3}{2R_{\mathrm{TF}}^2} \log\left(\frac{R_{\mathrm{TF}}}{\xi\sqrt{2}}\right) \tag{1}$$

$$\Omega_v = \frac{\Omega_m}{1 - r_v^2/R_{\mathrm{TF}}^2} \tag{2}$$

For our parameters this is

```
ξ = 1/sqrt(μ)
Ωm = 3*log(Rtf/ξ/sqrt(2))/2/Rtf^2
Ωv = Ωm/(1-rv^2/Rtf^2)
```

```
0.12875503299472799
```

or a vortex precession period of

```
Tv = 2*π/Ωv
```

```
48.7995316457793
```

10 Set simulation parameters

Let's evolve for one period of Hamiltonian dynamics, as predicted by the Thomas-Fermi analysis:

```
γ = 0.0
tf = Tv
t = LinRange(ti,tf,Nt)
```

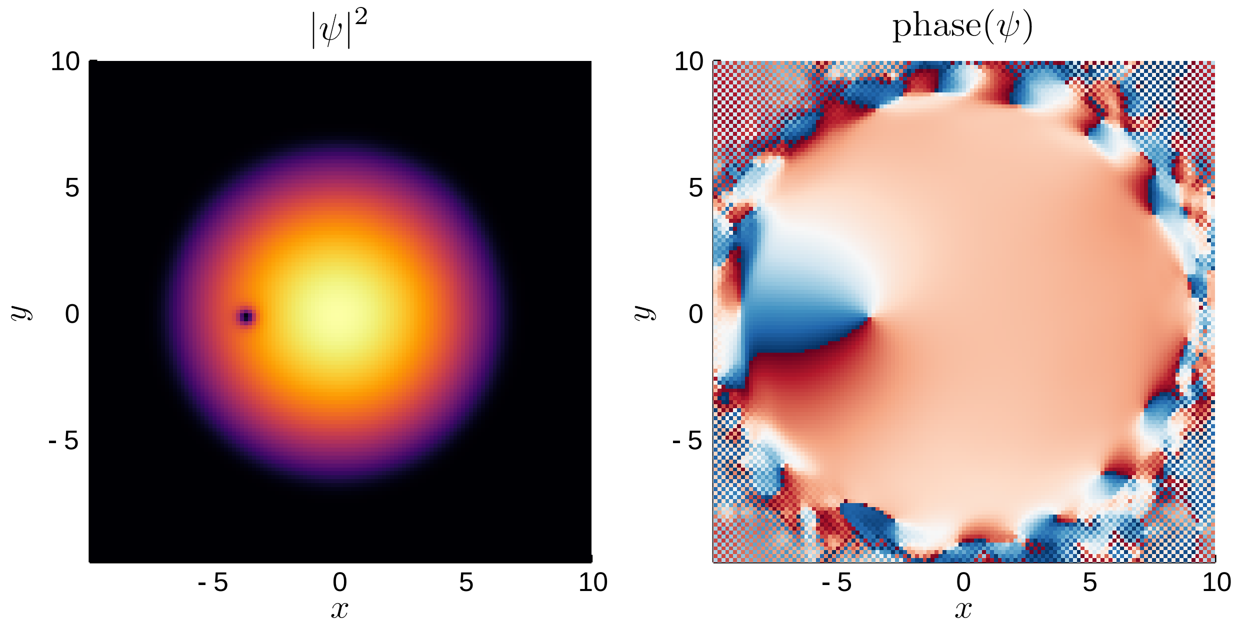
```
ϕi = kspace(ψv,sim)
@pack_Sim! sim; # write over previous sim  and reuse
```

11 Evolve in k space

```
solv = runsim(sim);
```

Pull out a state after some evolution

```
ϕf = solv[100]
ψf = xspace(ϕf,sim)
showpsi(x,y,ψf)
```



We can trim the last few frames to show one orbit

```
anim = @animate for i=1:Nt-6
    ψ = xspace(solv[i],sim)
    showpsi(x,y,ψ)
end;
```

and save the animation to the media folder

```
gif(anim,"./media/vortex.gif",fps=30)
```

The simulation gives a precession frequency within about $\sim 10\%$ of the analytical result.

This last simulation was in real time ($\gamma = 0$), and we see there is very little additional exciation imposed by imprinting and evolving the vortex. This provides a sanity check of both the time dynamics, and the vortex imprinting procedure. The latter used the numerically exact vortex core for the specified local healing length. The vortex phase is the ideal phase for a vortex in an infinite system with constant density specified at the core, and will cause some additional superfluid motion as the BEC responds to this initial state.