CSE 241 Algorithms and Data Structures

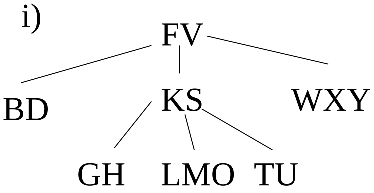
B-tree Practice Problems

1. Suppose that you have an application in which you want to use B-trees. Suppose that the computer you will be using has disk blocks holding 4096 bytes, the key is 4 bytes long, each child pointer (which is a disk block id) is 4 bytes, the parent is 4 bytes long and the data record reference (which is a disk block id along with a offset within the block) is 8 bytes.

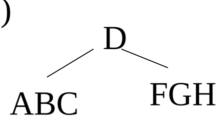
   You have an application in which you want to store 1,000,000 items in your B-tree. What value would you select for t? (Show how you derived it.) What is the maximum number of disk pages that will be brought into main memory during a search? Remember that the root is kept in main memory at all times.

2. Which of the following are legal B-trees for when the minimum branching factor $t = 3$? For those that are not legal, give one or two sentence very clearly explaining what property was violated.

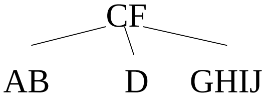
i)



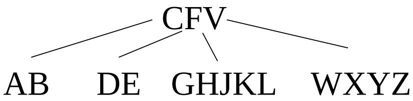
ii)



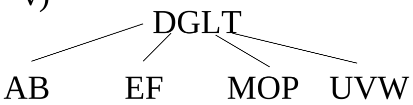
iii)



iv)

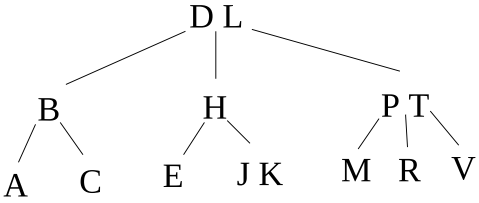


v)



3. Show the B-tree that results when inserting R,Y,F,X,A,M,C,D,E,T,H,V,L,W,G (in that order) branching factor of <u>$t = 3$</u>. You need only draw the trees just before and after each split.

4. Show the B-tree the results when deleting A, then deleting V and then deleting P from the following B-tree with a minimum branching factor of <u>$t = 2$</u>.



5. Show a way to represent the original B-tree from problem 4 as a red-black tree. You can indicate the color of each node by circling it with red or black or just by putting a "r" or "b" next to it.

   The solutions are on the back (except 5 which is on the web page since it didn't fit). Also, there are some more practice problems on the course webpage under Homeworks. I strongly recommend that you solve these BEFORE looking at the solutions.

SOLUTIONS:

1. We want to select t so that a full node uses as much of a disk block as possible. In a full node there are $2t-1$ keys (4 bytes each), $2t-1$ data record references (8 bytes each), $2t$ child pointers (4 bytes each), a parent pointer (4 bytes), the number of keys (4 bytes) and the leaf bit (which we'll go ahead and assume takes 4 bytes though 1 bit would do). Hence we want to pick t as large as we can so that $12(2t-1)+4(2t)+12 = 32t \leq 4096$. Solving for t yields that we need $t = 128$. In class, we argued that the number of disk pages that must be read (d-1 using the notation from class) is at most $log_t(n+1)/2$. Since $\log_{128}(n+1)/2 = \log_{128} \approx 2.7$ and the number of levels below the root must be an integer, at most 2 disk pages will need to be brought into main memory during a search.

2. (i): Not legal since the height is not balanced. More specifically, both the node with "BD" and "KS" are at the same level but "BD" is a leaf and "KS" is not.

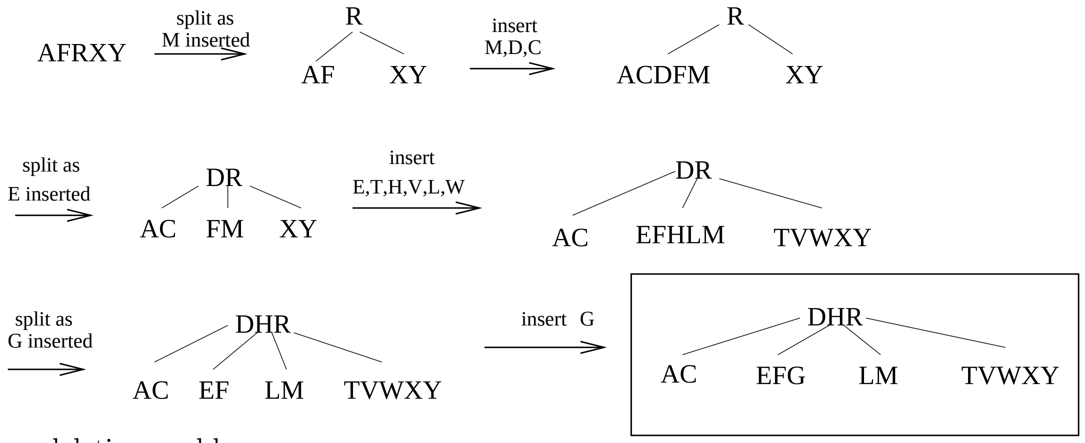
   (ii): This is legal. Remember, that the root can have just a single key.

   (iii): Not legal – the key "D" has less than the minimum allowable size of 2 keys.

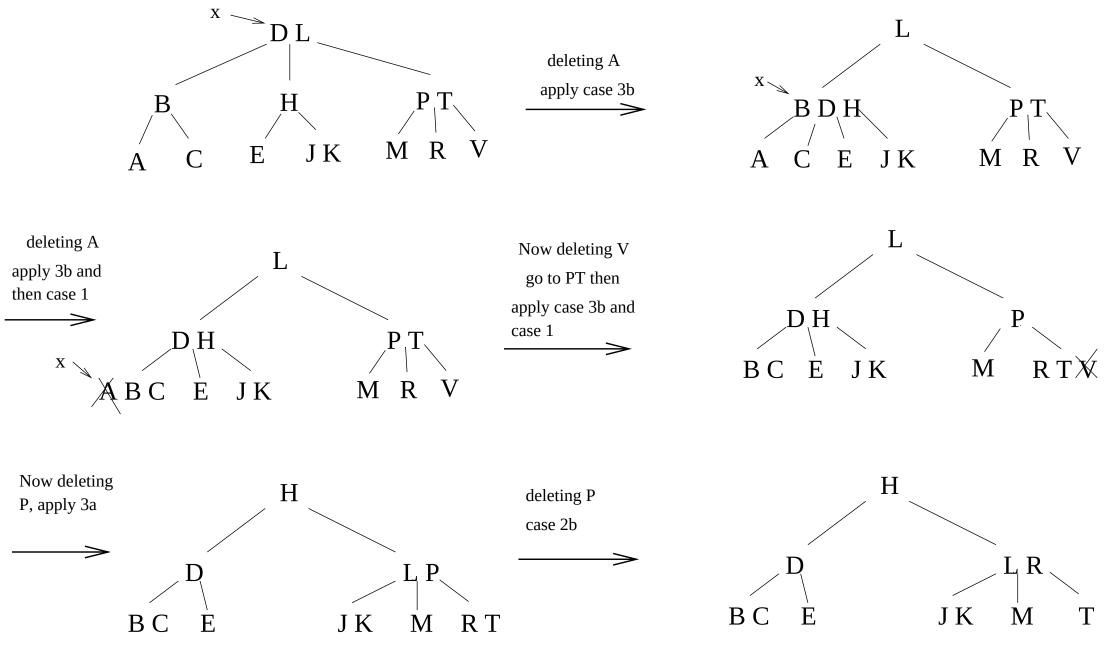
   (iv): This is legal.

   (v): Not legal – there's no leaf node corresponding to the keys between G and L.

3. B-tree insertion problem



4. B-tree deletion problem


x
D L
B
H
P T
A
C
E
J K
M
R
V
deleting A
apply case 3b
L
x
B D H
P T
A
C
E
J K
M
R
V
deleting A
apply 3b and
then case 1
L
D H
P T
x
A B C
E
J K
M
R
V
Now deleting V
go to PT then
apply case 3b and
case 1
L
D H
P
B C
E
J K
M
R T V
Now deleting
P, apply 3a
H
D
L P
B C
E
J K
M
R T
deleting P
case 2b
H
D
L R
B C
E
J K
M
T