Drift Diffusion Models of children's interactions in a Repeated Prisoner's Dilemma Game (Team 2): Final Report

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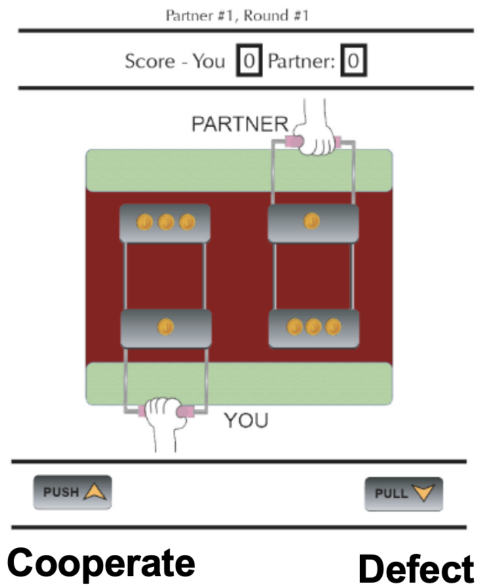
1. Introduction

The goal of this project is to gain a deeper understanding of the data from a Repeated Prisoner's Dilemma Game played by children. Understanding this data will provide insights about the behavior and decision making of children with different characteristics. Our goal was to find the relationship between a child's aggression, and their decisions in the repeated prisoner's dilemma game. We expect to see children with higher levels of aggression defect more often in the game. We used a few different methods to analyze trends in the data, focusing on the decision between aggression level and decisions to defect or cooperate.

1.1 Game Overview

The experiment set up each child against an automated opponent, where they would play three repeated prisoner's dilemma games, with each game having ten rounds to it. Because the partners are automated, for every round they made the same decision (to defect or cooperate), across all the subjects. This allows us to look at differences how the children cooperated or defected in responses to the same partener. Worth noting is that the children think they are playing against other real children - so we expect them to treat their partner as if they are playing against another child.

The children (aged 12-17) play against three automated opponents for 10 rounds each. In each round, they must choose between giving 1 coin to themselves (defect) or 3 coins to their partner (cooperate). Their goal is to collect as many coins as they can. Also worth noting is that the children can trade in these coins for prizes at the end of the game, incentivizing the children to win coins.



The three simulated opponents each have their own strategy: tit-for-tat, cooperative, or defective. The tit-for-tat partner cooperates first then mimics what their partner chose last round. The cooperative partner cooperates 8 out of 10 rounds and the defective partner defects 8 out of 10 rounds. The game is designed to provoke either forgiving for unforgiving behavior from the children.

1.3 Data Overview

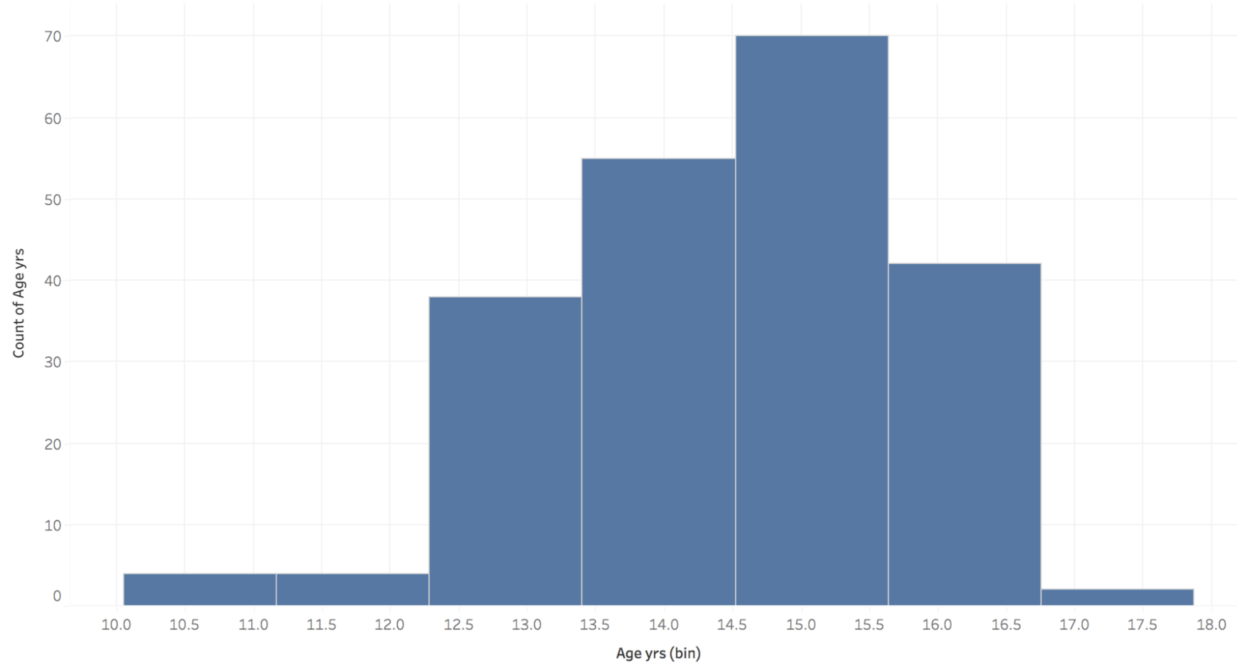
We have two datasets with about 200 children each including some characteristics and their and their partner's decisions through playing against the three simulated opponents. Several characteristics related to the child's behavior are recorded. These characteristics are represented as numerical data (taken from a scale). Some of these characteristics include hyperactivity, bullying, and aggression. Some of these characteristics have a feature corresponding to child reported behavior, and a corresponding feature for parent reported behavior.

1.3.1: Distribution of data

We created histograms for notable variables (for example age), to get a sense of how our data set is distributed. The average and median age of children in the study was 16. These histograms are shown below:

- **Age of children in study:**

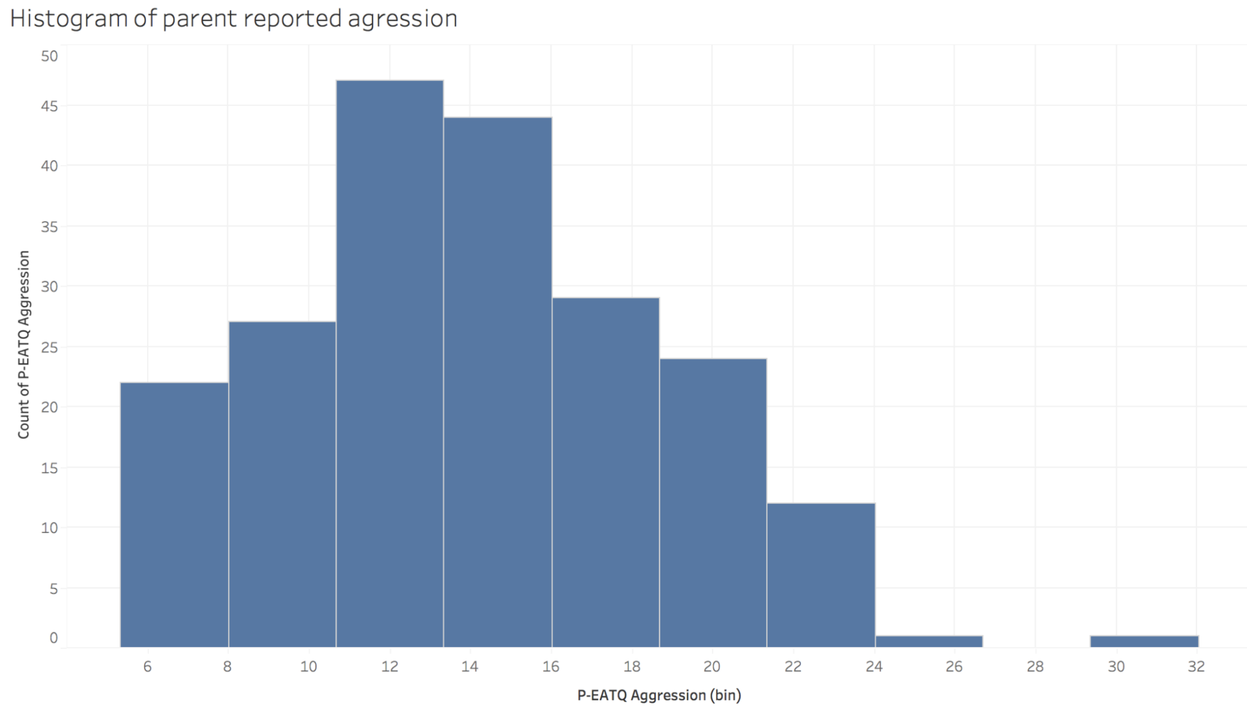
Age histogram



The trend of count of Age yrs for Age yrs (bin).

- **(Parent reported) child aggression**

Aggression is reported on a scale, where a higher score corresponds to more aggression.

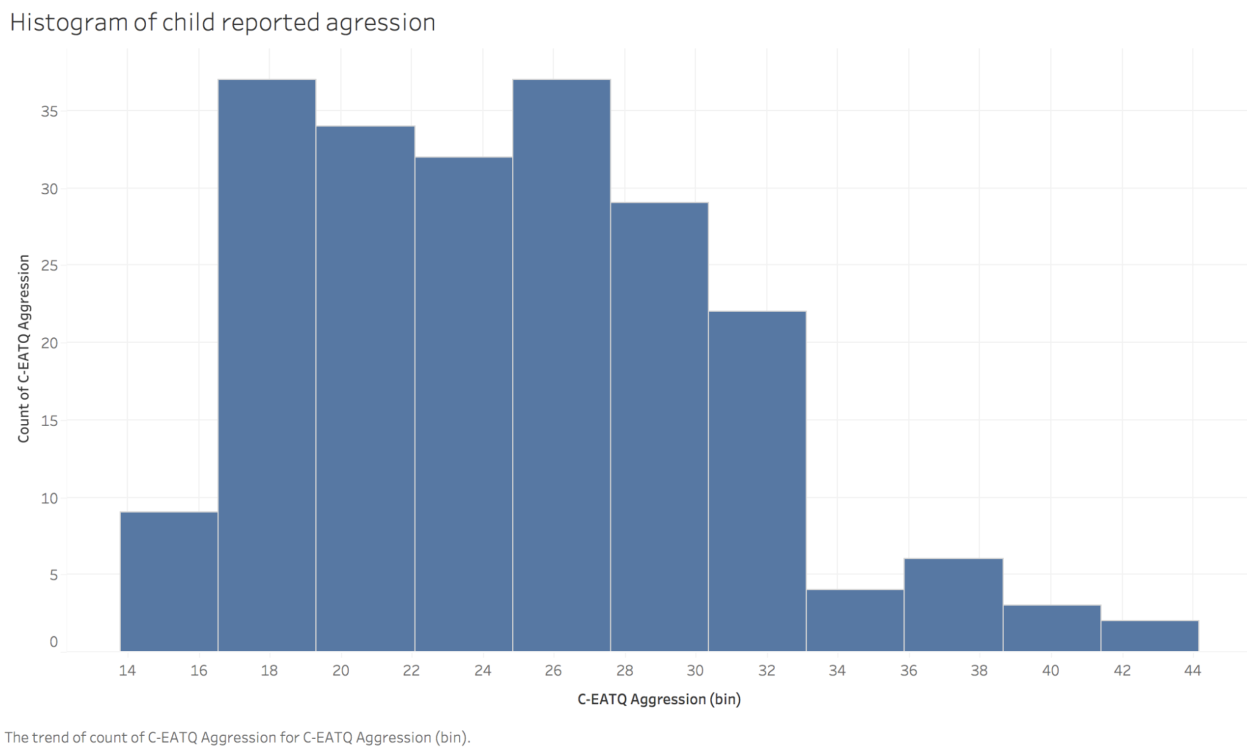


The trend of count of P-EATQ Aggression for P-EATQ Aggression (bin).

We can see that parent reported aggression (so child's aggression as reported by parents), is also skewed left, so there are more children with lower levels of parent reported aggression in our data set. Having a set of children with more widely distributed aggression may reveal some more patterns in the data.

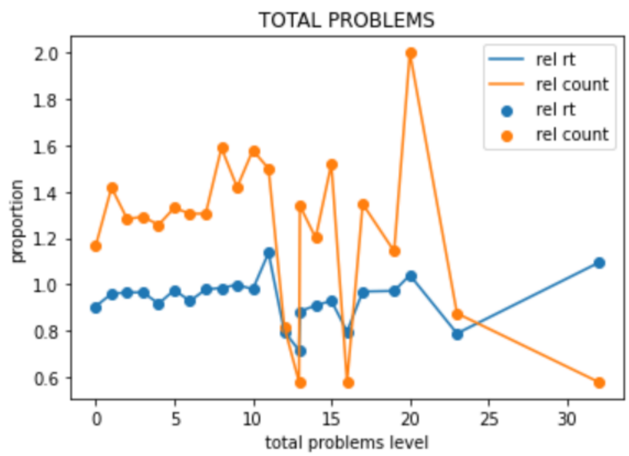
- **(Child reported) aggression**:

As we can see below, child reported aggression is skewed to the right, so there are more children with lower levels of reported aggression than those with higher levels of reported aggression.



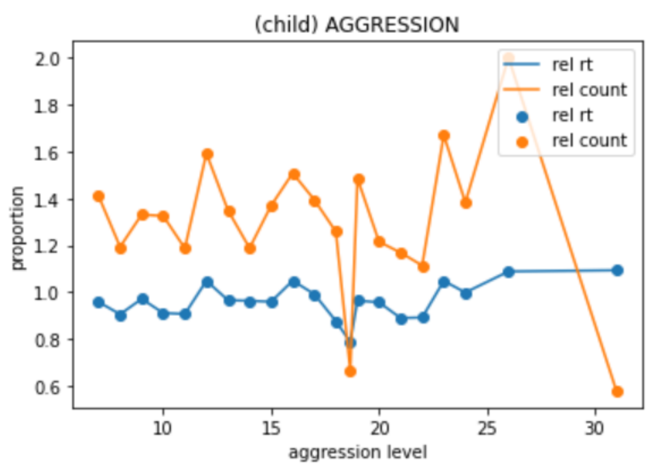
2. Data Exploration

Now that we have a general understanding of how our data is distributed, we explore our data in search for patterns. In particular, we are looking for a relationship between aggression and decisions to defect or cooperate.



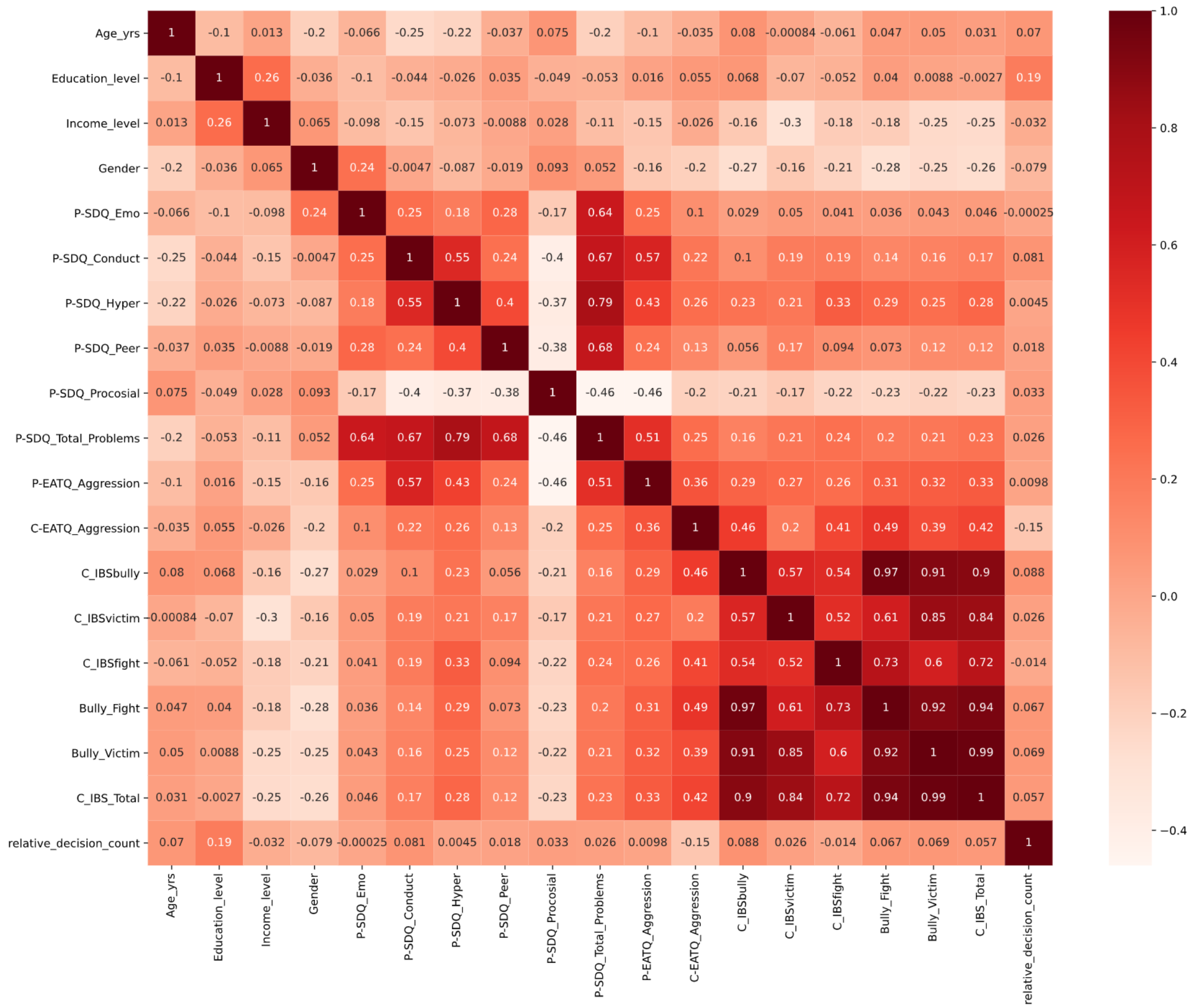
$$\text{relative rate} = \frac{cooperation\ rate}{defection\ rate}$$

$$\text{relative count} = \frac{cooperation\ count}{defection\ count}$$



The plots above both illustrate the lack of clear trends in the data. Giving that we don't see any clear patterns when looking for very general trends in cooperation vs. aggression, we decided to try and further classify types of decisions (i.e. strategies) rather than just looking at total cooperation or defection across all ten rounds.

We also wanted to understand relationships that existed between other variables in our dataset. To do this, we created the correlation matrix below. Darker colors correspond to variables with higher correlation, where a score of 1 corresponds to variables that are perfectly correlated (in this case this is only true for variables that are the same).



3. Models

Initially, one of our main goals was to make a drift diffusion model using children's reaction time to see how aggression played a role in their tendency to fall into a state of cooperation or defection. However, we determined that the reaction times were too large to be a good fit for a drift diffusion model. Drift diffusion models work best on split-second decisions, and most of the decisions we observed in the data seemed to be more thought out and strategic.

Thus, we shifted our focus to looking for other patterns in the data that could further explain the relationship between children's decision making, behavior, and their characteristics, especially aggression.

3.1. Research Questions

The main questions that guided us were:

1.) *Are children with higher aggressive traits more less likely to return to cooperation over the three rounds after partner defection (less likely to forgive)?*

2.) *Does the partner's deviation from their initial pattern (as pre-programmed) cause a change in state (e.g., preference) in the child?*

3.) *Can we predict these changes in state (positive and negative) based on the child's traits?*

4.) *How much is a child's behavior influenced by how aggressive they are?*

In the next sections we will talk about the steps we took to uncover answers to these questions in the data.

3.2. Regressions

We used regressions to try and formalize the correlation between aggression and cooperation. In particular we look at the relationship between various parameters and the relative cooperation rate whose calculation is shown below. We performed two multivariate regressions: one using several features of the dataset, and one only using features chosen by a Lasso model.

3.2.1. Multivariate Linear Regression using all variables

Initially, we created a regression model using all the characteristics as dependent variables. In this model, we looked at relative cooperation rate (defined earlier) as a function of parameters such as parent's education level, parent's income, aggression, age, etc. We are mainly interested in the beta coefficients on aggression - representing the relationship between aggression level and relative cooperation rate. Because the variables do not have the same scale - so one point increase on the aggression scale is not equivalent to one point increase on the relative cooperation rate scale - we are not interested in the actual value of Beta, we are just interested in its sign (so if it is positive or negative, representing a positive or negative correlation respectively), and its corresponding p-value. Generally, for a beta coefficient to be statistically significant, we look for a p-value less than or equal to 0.05.

This regression model is defined below.

$$y = \beta_0(Intercept) + \beta_1(Education_level) + \beta_2(Income) + \beta_3(gender) + \beta_4(P-SDQ\ Emo) + \beta_5(P-SDQ\ Conduct) + \beta_6(P-SDQ\ Hyper) + \beta_7(P-SDQ\ Peer) + \beta_8(P-SDQ\ Prosocial) + \beta_9(P-SDQ\ Total\ problems) + \beta_{10}(P-EATQ\ Agression) + \beta 11(C-EATQ\ Agression) + \beta 12(C\ IBSbully) + \beta 13(C\ IBSvictim) + \beta 14(C\ IBSfight) + \beta 15(Bully\ Fight) + \beta 16(Bully\ Victim) + \beta 17(C\ IBS\ Total) + error$$

where y = relative cooperation count = average (number of times cooperated ÷ number of times defected)

When we ran this regression, we got the following beta values. However, these beta values and corresponding p-values are not accurate as there is a high level of multicollinearity in these variables. Multicollinearity occurs when variables in a regression are highly correlated with each other - as is the case with many variables. This high degree of collinearity can be seen by the correlation matrix. The book: “Understanding Regression Analysis” states: “Multicollinearity is a problem because it undermines the statistical significance of an independent variable”(Allen, 176).

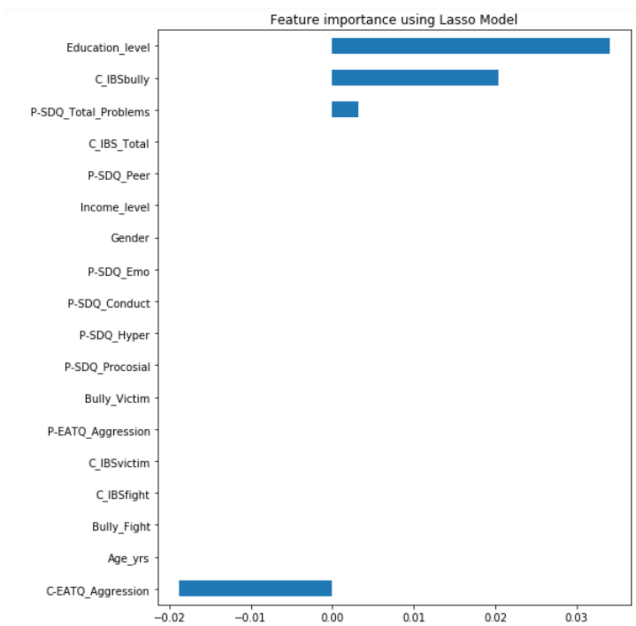
	Coef.	Std.Err.	t	P>|t|
Age_yrs	0.072592	0.022100	3.284666	0.001219
Education_level	0.090292	0.026106	3.458642	0.000672
Income_level	-0.014347	0.024141	-0.594306	0.553026
Gender	-0.055056	0.084831	-0.649007	0.517130
P-SDQ_Emo	1.326604	1.363360	0.973041	0.331790
P-SDQ_Conduct	1.438208	1.371492	1.048645	0.295695
P-SDQ_Hyper	1.313318	1.365909	0.961497	0.337544
P-SDQ_Peer	1.340353	1.367058	0.980465	0.328123
P-SDQ_Procosial	0.034424	0.023055	1.493145	0.137085
P-SDQ_Total_Problems	-1.319919	1.365234	-0.966808	0.334889
P-EATQ_Aggression	-0.003146	0.011871	-0.265029	0.791279
C-EATQ_Aggression	-0.026123	0.008416	-3.103979	0.002206
C_IBSbully	0.020343	0.016699	1.218230	0.224671
C_IBSvictim	-0.016691	0.013431	-1.242736	0.215521
C_IBSfight	-0.005691	0.024193	-0.235220	0.814295
Bully_Fight	0.014652	0.014969	0.978836	0.328926
Bully_Victim	0.003652	0.014885	0.245360	0.806447
C_IBS_Total	-0.002039	0.010982	-0.185621	0.852943

As explained above, these values are not accurate. Rather, these values are listed to serve as a contrast between the values obtained from only using the Lasso model, which we will talk about next.

3.2.2. Lasso Model for Variable Selection

To address the problem of multicollinearity in our regression, we set up another regression using the lasso model. Lasso regression is a type of regression that uses shrinkage to reduce the number of parameters. Using the lasso model, we found four features that best estimated the relative cooperation rate:

Variable	Beta Coefficient	p - value
Parent Education	0.1638	3.173060e-15
C_IBSbully	0.0166	2.489705e-01
P-SDQ_Total_Problems	0.0116	1.854501e-01
Child reported aggression	-0.0024	7.220199e-01



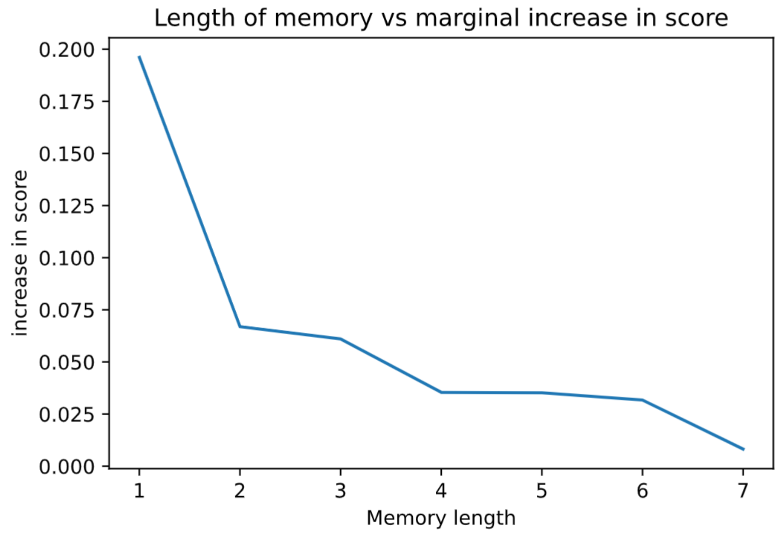
The feature importance graph shows the Lasso calculation of feature importance. This is a way of deciding which variables should be included in the regression. As we can from the graph above, where a larger bar indicates higher feature importance in predicting relative cooperation count. As we can see when we only include Lasso selected variables in our regression, we get much lower p-values, supporting the idea that multicollinearity caused the statistical significance of these variables to be underpredicted while all variables were included in the regression. We do see that child reported aggression has a significant negative correlation with cooperation. This is what we would expect - for more aggressive kids to be less likely to cooperate. The beta coefficient tells us that 1 additional aggressiveness point on the child reported aggression scale corresponds to a decrease in cooperation proportion by 0.0024.

3.3. Random Forest Time Series Predictor Model

While regressions are valuable in finding relationships between aggregated attributes, such as the total number of times a child defects, we needed to create a model that could use the dataset as a time series to understand the children's decision making processes, based not only on characteristics but on previous events in their game. We chose the random forest model because the binary decisions of the game could be easily modeled by a decision tree, but a single decision tree is prone to overfitting. Random forests contain many decision trees that are each trained on a random subset of the training data, so they are all slightly different. Also, a random forest can use a random subset of features when splitting nodes. To make a prediction, the random forest outputs the average of all the decisions by its decision trees. The random forest is more robust to overfitting, which was a primary concern of ours because of the small size of the dataset. The random forest is also more robust to overfitting than a more complex model, such as an LSTM neural network.

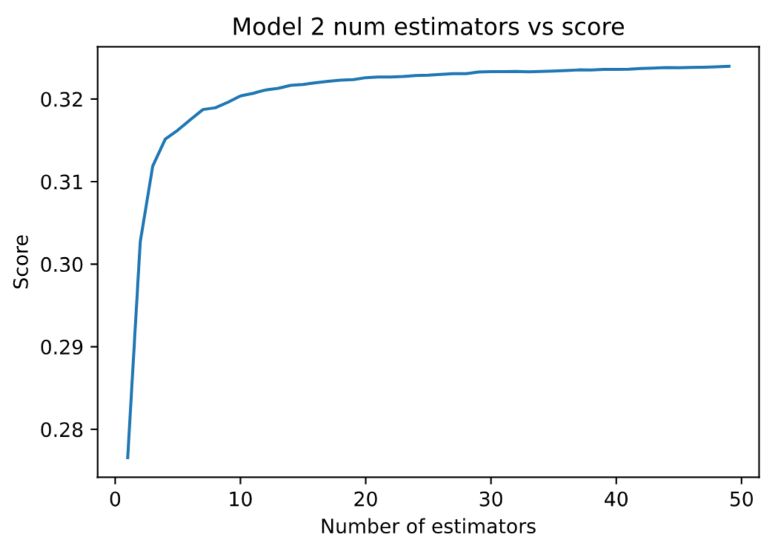
3.3.1. Design

We built a time series predictor model using a random forest to predict a child's decision using the three previous partner decisions and the child's aggressiveness. We wanted to use this model to focus on aggressiveness in relation to a child's behavior. We decided to use three previous partner decisions by testing the accuracy of the model using different numbers of partner decisions. We called the number of partner decisions the model consumed the length of its memory, as it would use its memory of these previous moves to make its next decision. Using a longer memory would boost the accuracy of the model, but it would shrink the amount of data we could use to train it. If the length of the memory was x, we could not use the first x child decisions as there were fewer than x partner decisions before them. Thus, we had to balance a longer memory and more accuracy with having more training data and better resilience to overfitting. We discovered that as we increased the memory of the model, the marginal increase in accuracy shrank:

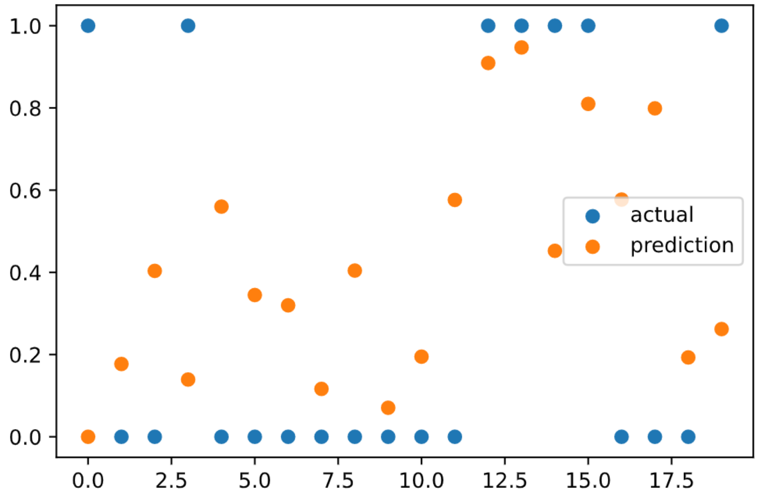


This matches up with what we would expect! The child of course can remember all of their partner's previous moves in these short trials and they all influence their decision making, but the immediately preceding moves will have a greater effect on their decision than their partner's moves several turns ago. We decided on using a memory length of three for our model as this seemed like a good balance between the tradeoffs.

We also had to decide how many individual decision trees should be in our random forest. To determine how this affected our accuray, we produced the following chart:



There was a sharp increase in accuracy as we increased to 10 estimators (individual decision trees) and after that the marginal increase in accuracy was not much. However, since the dataset was so small and training a model was very fast, we decided to use 50 decision trees. Our final model had an average accuracy of 0.32, which is not high but shows definitive evidence of correlation. To visualize the results of our random forest time series predictor, we created this graph which shows a sample of the actual vs predicted decisions. The closer the blue and orange dots, the more accurate the prediction.

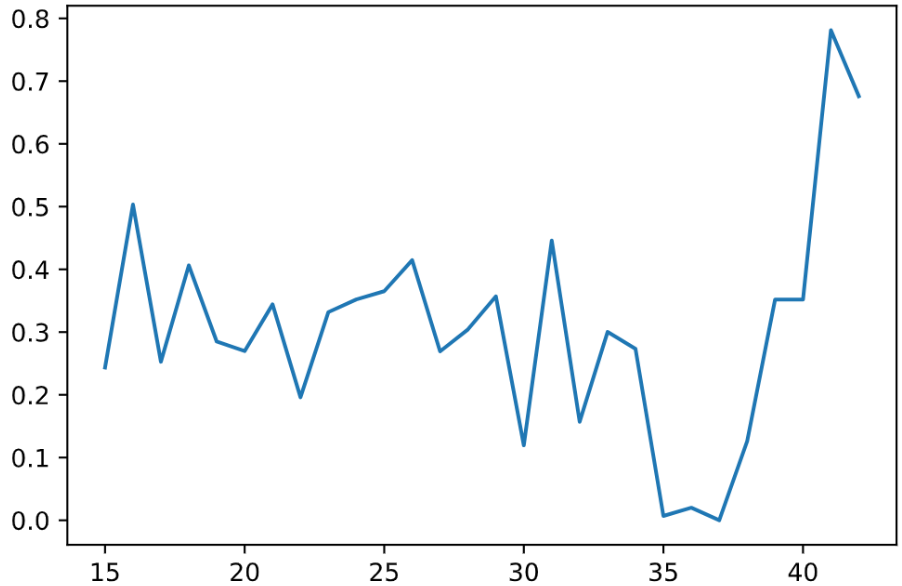


The prediction of our model is a likelihood of cooperation/defection. A result closer to 1 means the model thought the child would be more likely to cooperate, and closer to 0 means more likely to defect. While there is a lot of variation in the predictions, a general trend for the orange dots to move in the same direction as the blue dots is visible. With our trained model, we moved to the next phase of our exploration.

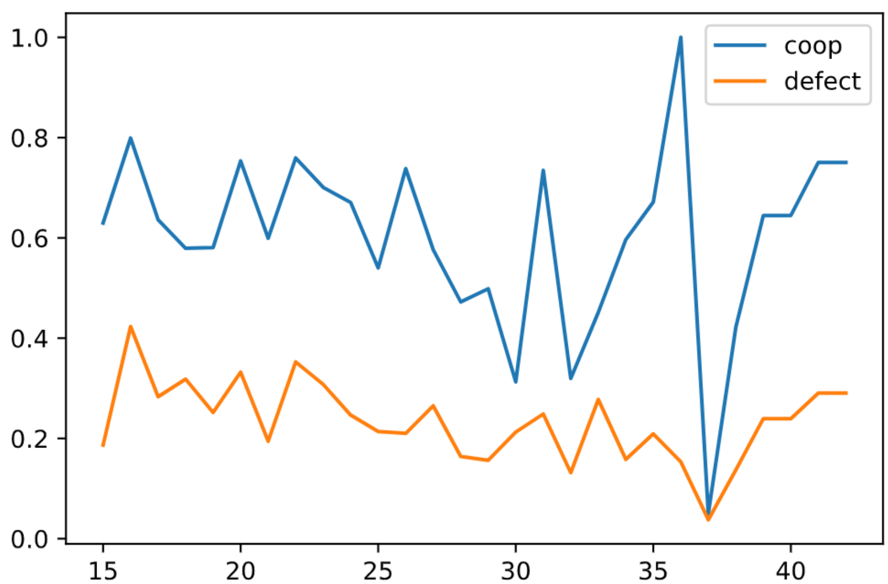
3.3.2. Experiment Design

The purpose of the random forest time series predictor was not only to see if we *could* predict the results, but more importantly to make structured queries to the trained model to understand more about the relationship between children's behavior and their aggressiveness. A trained model would be like having a virtual child who could have any aggressiveness level we give them, and we could see how they respond to various sequences of three partner moves that could be constructed to provoke various responses. For instance, we could have the partner defect, defect, and cooperate to see if children with different levels of aggression would be more likely to forgive and cooperate, or defect because they hold a grudge because of their partner's first two defections.

To run our experiments, we constructed various sequences of three partner moves and tested each sequence on our trained model across the whole range of aggressiveness values. We were hoping to find trends across the aggressiveness ranges. Unfortunately, there were no clear trends that we could find. This graph shows the likeness of cooperation (y-axis) across different aggressiveness levels (x-axis) in response to the partner moves "defect, defect, cooperate". This experiment was intended to measure a child's tendency to forgive.



We had hoped to see a general downward slope in this chart, as we expected that more aggressive children would be less likely to cooperate again after this sequence of partner moves than less aggressive children. We hypothesized that the irregular shape of this graph was because of our small training dataset. With only a few children or less at each distinct aggressiveness value, other differences between the children that were not quantified probably outweighed their similar aggression index. The drop in graph at aggressiveness level 35 and the sudden spike at the end were also present in other experiments we ran. Here the blue line shows the response to three corporations and the orange line shows the response to three defections:



As expected, the orange line is significantly lower than the blue line, showing that children across the range of aggressiveness values are less likely to cooperate after three defections. However, this graph again does now show any trend as aggressiveness increases, other than the same drop after 35 followed by the spike.

3.3.3. Future Potential for This Model

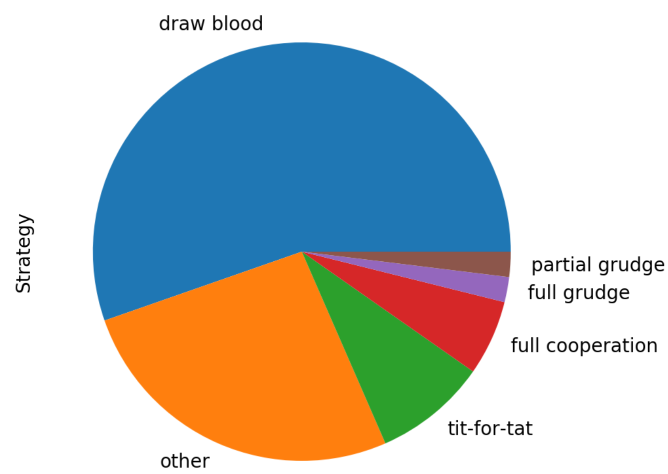
While our experiments with this model did not yield any conclusive results, we believe the random forest time series prediction approach has much potential. The accuracy of our model shows that it does have the potential to learn the relationships between a child's decision, past partner decisions, and aggressiveness. With more data, we could expose the model to many more data points at each level of aggressiveness. This should allow it to learn more general trends across aggression while being less sensitive to outliers and overfitting.

3.4. Classification Model

In an attempt to better understand types of strategies used by players, we classified each child's strategy. These strategies are meant to encapsulate each child's set of decisions in the game. We defined these strategies in two different ways. The first way we defined strategies is as follows:

- **draw blood**: defects in the first three rounds
- **Full grudge**: cooperates in first three rounds and defects for all of the last seven rounds
- **Partial grudge**: cooperates in the first seven rounds (allowing one retaliatory defection in round 4) and defects in the last three rounds
- **Tit-for-tat**: same as the simulated tit-for-tat, mimics partner's last move
- **Full cooperation**: cooperates in every rounds
- **Other**: any other sequence of moves

The prevalence of children with these different strategies is shown by the pie chart below. The table shows the percentage of each type of strategy in our data set.



Strategy	Percentage (represented as a decimal)
Draw blood	0.553398
Other	0.262136
tit-for-tat	0.087379
Full cooperation	0.058252
Full grudge	0.019417
Partial grudge	0.019417

Because there are so many children falling into the 'other' category - we hope to expand our classification to include more strategies. With the expansion of the data set (and thus the expansion of the number of children falling into each category), we expect that patterns between different strategies and other characteristics, will be more clearly defined.

Classification Model Future Potential

We hope to apply these classifications to the random forest model as well as other models. In the random forest time series predictor model, we could use these strategies to form input sequences to use for experiments and look at trends across aggression in the model's predictions. These trends in aggression could give us more insights about these classifications or help us define new ones. We hope the inclusion of these classifications will provide more insight about the data, as well as narrow errors in various models. Using these classifications, one could look at the average values of other features (for example average aggression scores) for each of the strategies defined.

4. Limitations

As we have touched on in previous sections, the main limitation we faced was the lack of enough data. Our dataset contained 215 children who played a total of 30 rounds each. The nature of this experiment is an intrinsic limitation as the cost and effort of obtaining data is significant compared to other types of data.

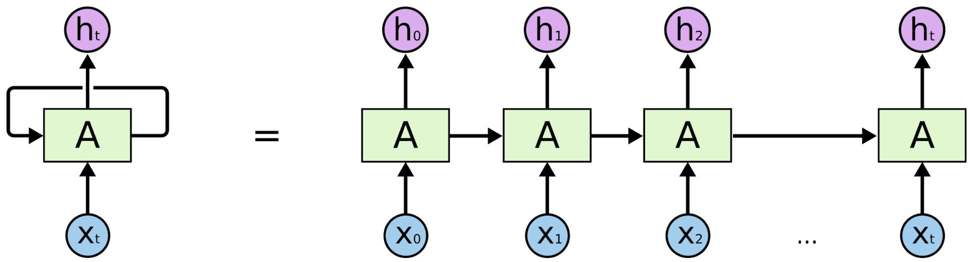
5. Future Direction

With a larger dataset, the random forest model could be substantially expanded upon. In addition, we recommend two additional models: the LSTM neural network and the Hidden Markov Model. With a larger data set, the risk of overfitting would decrease. Overfitting is a phenomenon when a model describes a specific dataset very well, but cannot be generalized and is in fact too specific to the data it was trained on. A larger dataset would also likely create more significant correlation coefficients when performing linear regressions.

Thus, we see the collection of more data as an important next step. As it is difficult for one research lab to conduct enough experiments to produce a large quantity of data, we think the best solution is for researchers across the country who are doing similar studies to collaborate and pool their data. We recognize that this would be no easy task, but with clearly defined and agreed upon experiment design and data collection guidelines, we think this is a possibility.

5.1 LSTM

A more robust time series model could be created with an LSTM (long short-term memory) neural network. An LSTM is a type of Recurrent Neural Network (RNN), which is a class of neural networks that are good at learning time series and sequence data by allowing the output of each step to be used as the input for the next step using feedback loops. The only problem with a basic RNN is that the children aren't only thinking about what their partner did in the single previous round, they are also thinking about what they did many rounds ago. An LSTM is a type of RNN that can solve this. LSTMs have a 'memory' that can maintain information over long periods of time, allowing us to use information from many previous steps to predict the next child decision.

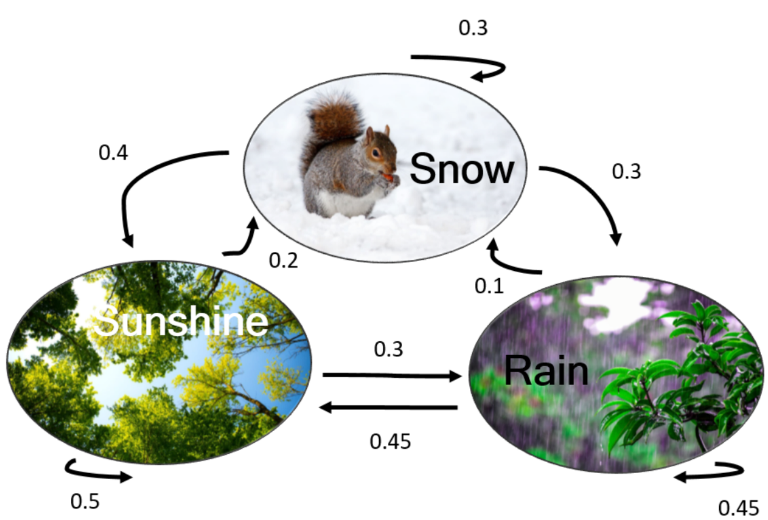


The shortcoming of the LSTM is that it is a complex model that is susceptible to overfitting. Neural networks in general usually require a lot of samples because they need a lot of data to converge on the hidden relationships between variables. The dataset we have is too small to train such a model without it simply memorizing the data rather than learning hidden trends in it. With more data, we think that an LSTM could provide a very exciting predictive model.

5.2 Hidden Markov Model

5.2.1. Standard Markov Model

A standard Markov Model is a way of observing transitions between states, in particular the probability of moving from one state to the other, and in turn the long run probability of being in each state. When drawn, the probability of going from one state to the next is represented by a decimal above the arrow corresponding to that transition. A simple example of a markov chain is shown below. In this case, the states are different weather forecasts.

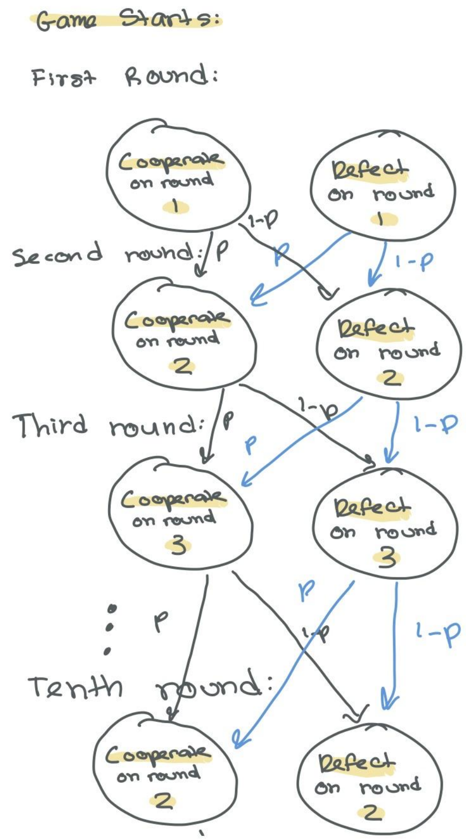


[1]

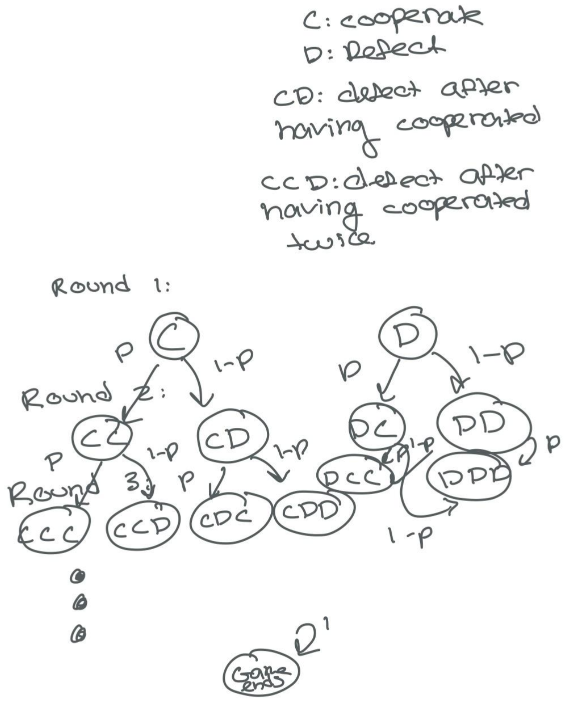
[1] https://towardsdatascience.com/introduction-to-hidden-markov-models-cd2c93e6b781

With this dataset, our proposed Markov chain would have states corresponding to decisions to cooperate and defect in each round. Because deciding to cooperate on the first round is different than deciding to cooperate on the second round, we would create twenty different states for each game. Because Markov chains must either continue looping through the states, or end in an *absorbing state*, we would add an absorbing state at the end, corresponding to the game being finished.

A simple layout of this model is shown below. Keep in mind theoretically there would be a different Markov chain for each child, because each child does not have the same probability of cooperating or defecting in each round. Also note that in the figure below the probability of going from one state to the other is denoted as “P”, however this is just a shorthand meaning the probability of cooperating or defecting, in the next round - and would theoretically be different for every round - treat this P as a placeholder rather than a variable.



To increase the specificity of this Markov chain, we could create states corresponding to the sequence of defecting or cooperating, so that a state for cooperating, then cooperating, is different from the state for defecting, than cooperating. Because there are two options in each round, and ten rounds, there would be a total of 2^{10} different states for each type of game. A general outline of this more complex model is shown below.



5.2.2. Hidden Markov Model

The hidden Markov Model is a special way of constructing a Markov Chain given incomplete information. Unlike in the standard Markov Model, we cannot observe all the states in the Hidden Markov Model - here we cannot observe all states, however we can observe certain variables that are dependent on these hidden states. An example of a hidden Markov model is one where you do not know the state of the weather, but you can observe what people are wearing (for example if someone is wearing a raincoat), and these observations are used to calculate the likelihood of being in each ‘hidden’ state.

With our data set, the hidden/unknown state would be each child’s true aggression or more simply, their propensity to defect. The observed variable dependent on this hidden state would be their decisions to defect or cooperate. There are several python packages that implement the hidden markov model. We are still looking through them to decide which would be the easiest to implement. Some packages with hidden markov model functions are Pyro, SKlearn, and NetworkX.

Works Cited:

(1997) The problem of multicollinearity. In: Understanding Regression Analysis. Springer, Boston, MA. https://doi.org/10.1007/978-0-585-25657-3_37