Homework 4

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This document has been created as part of the fourth homework assignment at CMF.

Initially, the working directory, system locale are set and the required packages are loaded.

```
##### Initialisation #####
setwd("~/")
library(quantmod)
library(ggplot2)
library(ghyp)
library(FinTS)
library(copula)
library(fGarch)
library(tseries)
Sys.setlocale("LC_ALL","English")
```

The historical prices of S&P 500 and Dow Jones ETFs from 2014 up to present are loaded from Google Finance using *getSymbols* function from *quantmod* package.

```
# specify the tickers to load
tickers = c("SPY","DIA")

# environment where data will be stored
e <- new.env()

# suppress warning about future change in the function behaviour
options("getSymbols.warning4.0"=FALSE)

# load data from Yahoo Finance
getSymbols(tickers, src = "google", from="2014-01-01", env=e)

# merge data into a single xts taking only adjusted close prices
data = do.call(merge, eapply(e, Cl)[tickers])
names(data) = tickers
```

Daily returns are calculated and saved as *xts*.

```
##### Calculate returns #####
rets = apply(data, 2, function(x) diff(x)/x[-length(x)])
rets = xts(rets, order.by = index(data)[-1])
```

We now apply an LM-test with 12 lags for ARCH effects to both series.

```
##### LM-test #####
ArchTest(rets$SPY,lags=12)
```

```
##
```

```
##  ARCH LM-test; Null hypothesis: no ARCH effects
##
## data:  rets$SPY
## Chi-squared = 103.26, df = 12, p-value < 2.2e-16
```

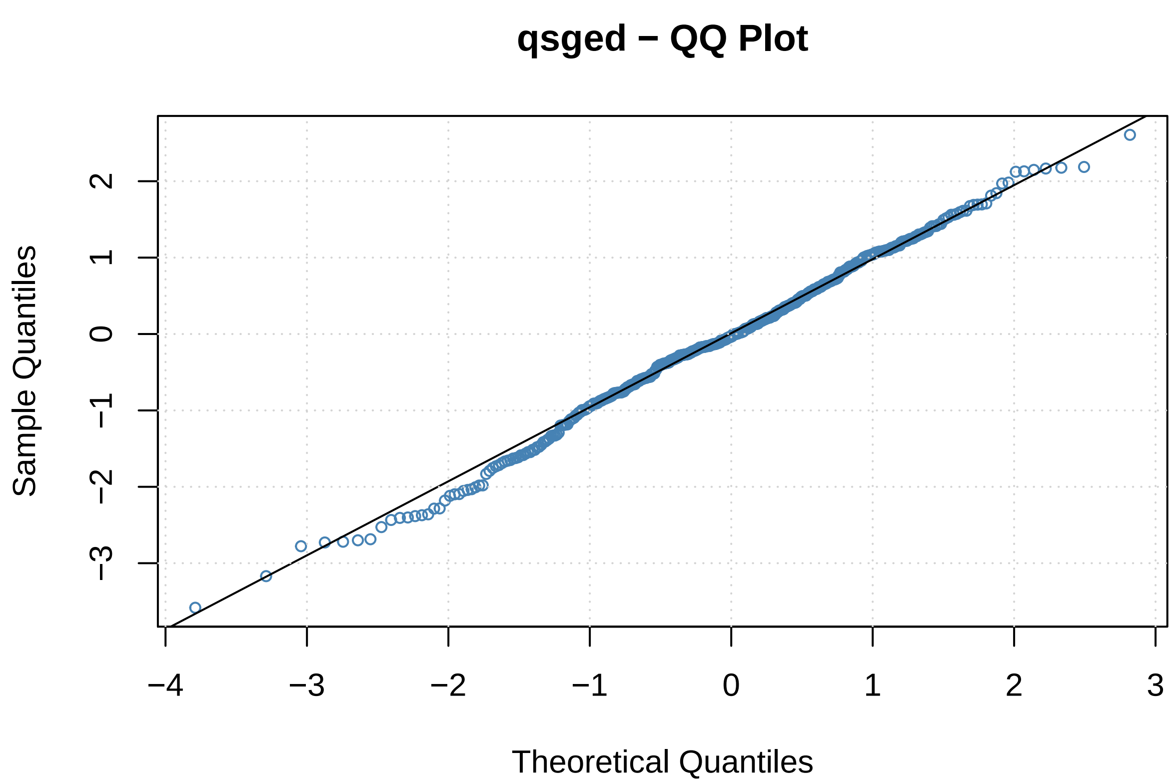
```r
ArchTest(rets$DIA,lags=12)
```

```
##
##  ARCH LM-test; Null hypothesis: no ARCH effects
##
## data:  rets$DIA
## Chi-squared = 114.07, df = 12, p-value < 2.2e-16
```

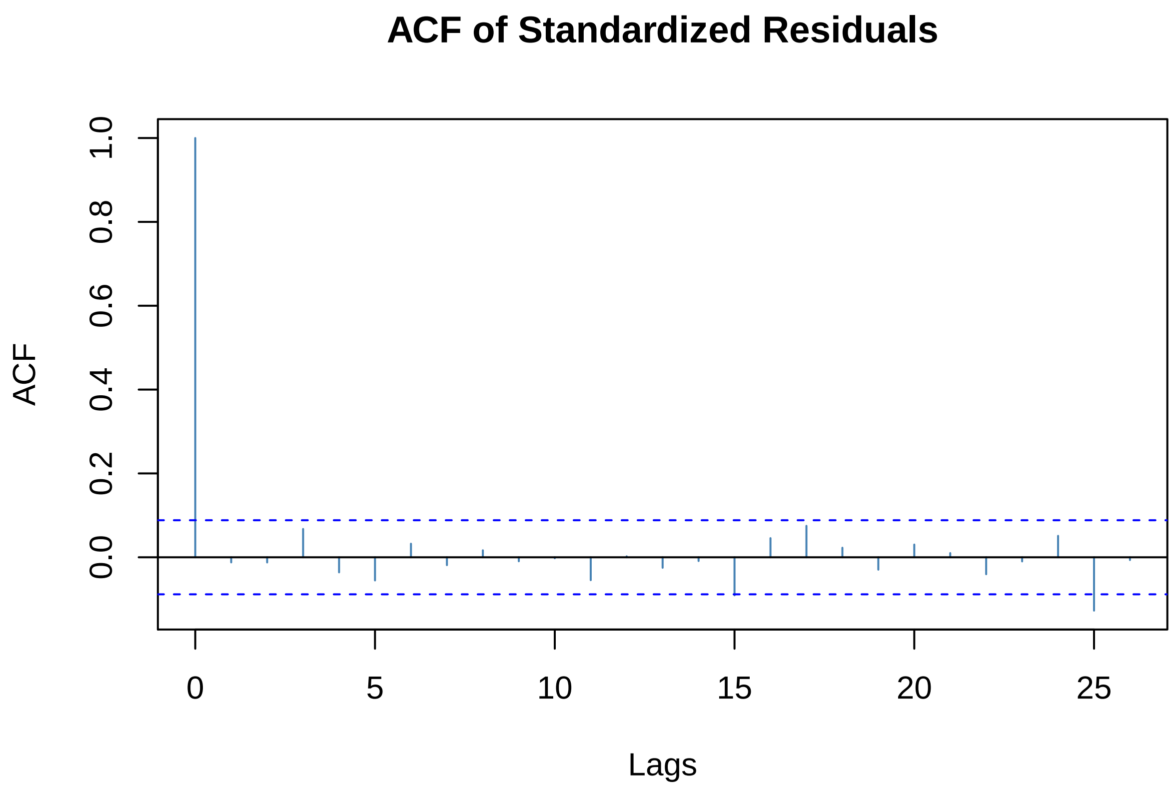
We reject the null hypothesis with a very low p-value for both series and thus conclude that there are ARCH effects in the returns series.

We now fit a GARCH model including leverage and shape (set to 1.2) parameters to both series. We use the Skew Generalized Error Distribution as a conditional distribution.

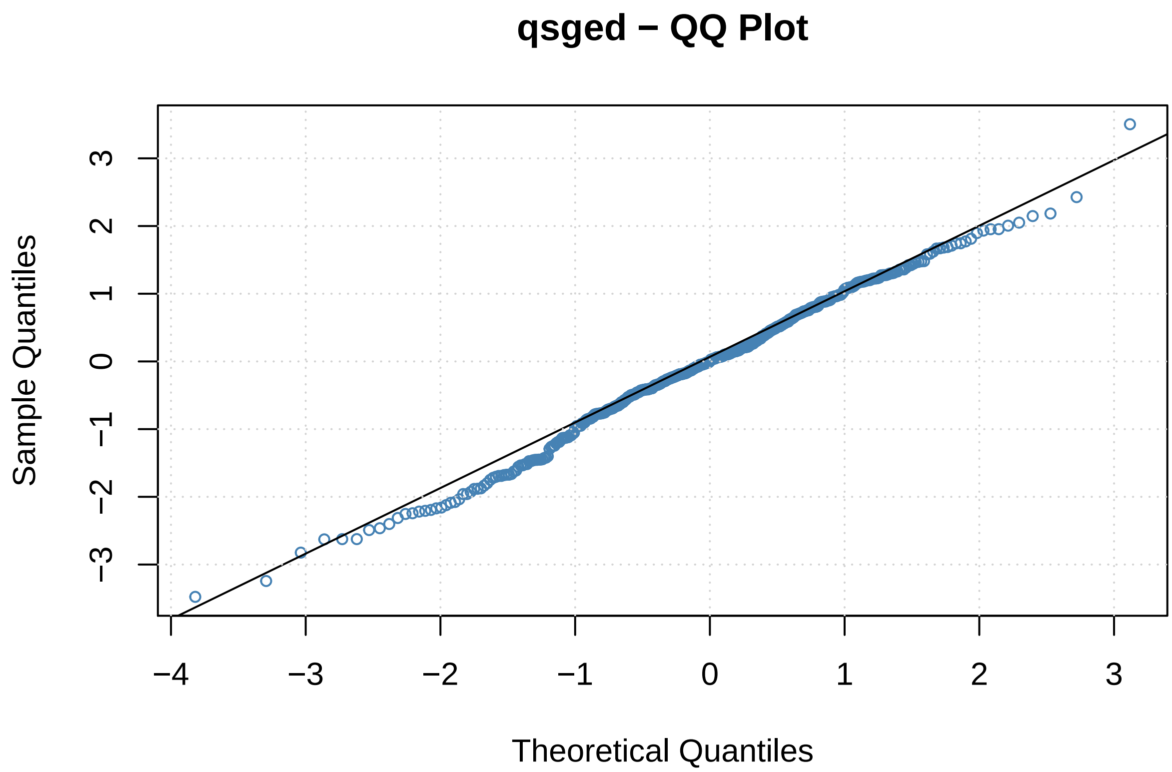
```r
##### GARCH modelling #####
#SPY
SPY.gfit <- garchFit(formula=~aparch(1,1),data=rets$SPY,delta=2,
                     include.delta=FALSE,leverage=TRUE,cond.dist="sged",
                     shape=1.2,include.shape=TRUE,trace=FALSE)

plot(SPY.gfit,which=13)
```



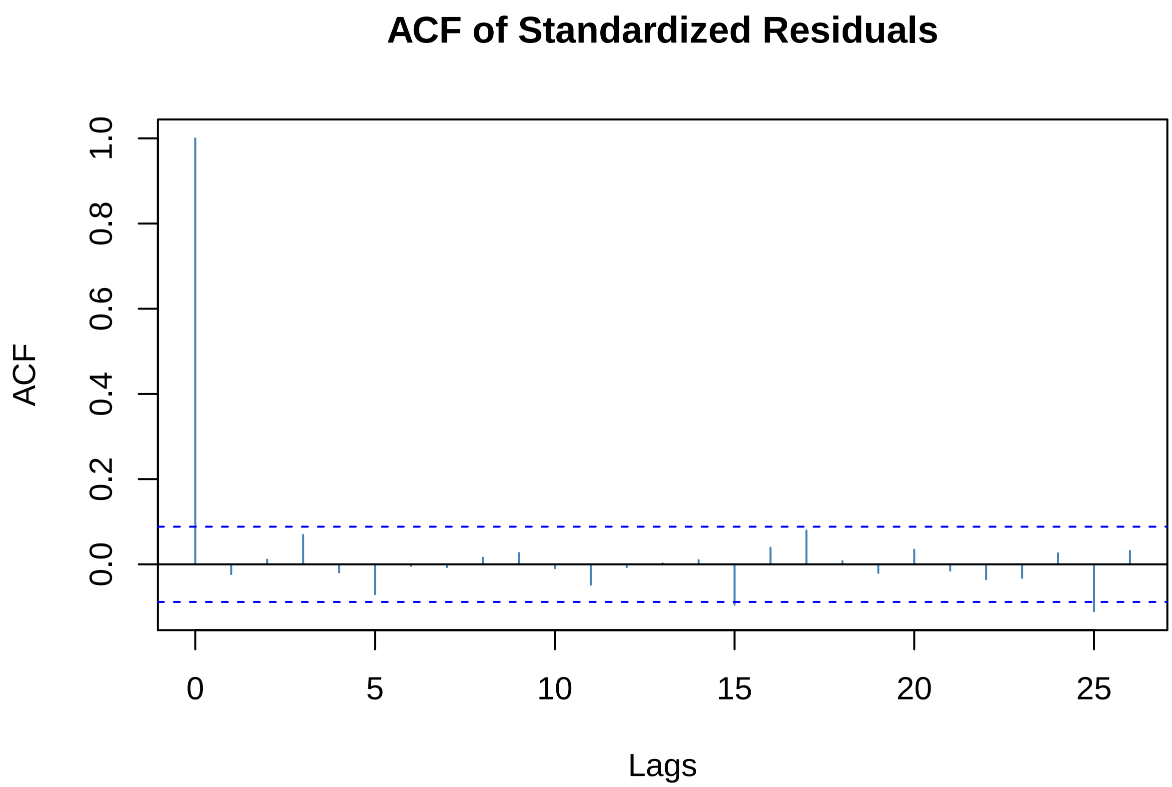
```r
plot(SPY.gfit,which=10)
```




```r
#DIA
DIA.gfit <- garchFit(formula=~aparch(1,1),data=rets$DIA,delta=2,
                     include.delta=FALSE,leverage=TRUE,cond.dist="sged",
                     shape=1.2,include.shape=TRUE,trace=FALSE)

plot(DIA.gfit,which=13)
```




```
plot(DIA.gfit,which=10)
```



We now conduct ADF-, PP- and KPSS-tests for stationarity.

```
##### Stationarity tests #####
adf.test(rets$SPY)
adf.test(rets$DIA)

pp.test(rets$SPY)
pp.test(rets$DIA)

kpss.test(rets$SPY, null="Level")
kpss.test(rets$DIA, null="Level")
```

We can conclude about the stationarity of the series.

We now standardise the residuals and compute their partial distributions using the CDF of the Skew Generalized Error Distribution.

```
##### Standardized residuals #####
z <- matrix(nrow=length(rets$SPY),ncol=2)
z[,1] <- SPY.gfit@residuals / SPY.gfit@sigma.t
z[,2] <- DIA.gfit@residuals / DIA.gfit@sigma.t

##### Residuals partial distributions #####
mean <- c(0,0); sd <- c(1,1); nu <- c(1.2,1.2)

xi <- c(SPY.gfit@fit$par["skew"], DIA.gfit@fit$par["skew"])
```

```
cdf <- matrix(nrow=length(rets$SPY),ncol=2)
for (i in 1:2) cdf[,i] = psged(z[,i],mean=mean[i],sd=sd[i],nu=nu[i],xi=xi[i])
```

We initialise four different copulas and fit them to our data.

```
##### Initialise copulas #####
norm.cop <- normalCopula(dim=2,param=0.5,dispstr="un")
stud.cop <- tCopula(dim=2,param=0.5,df=5,df.fixed=TRUE,dispstr="un")
gumb.cop <- gumbelCopula(dim=2,param=2)
clay.cop <- claytonCopula(dim=2,param=2)

##### Fit copulas #####
norm.fit <- fitCopula(cdf,copula=norm.cop)
stud.fit <- fitCopula(cdf,copula=stud.cop)
gumb.fit <- fitCopula(cdf,copula=gumb.cop)
clay.fit <- fitCopula(cdf,copula=clay.cop)
```

We now conduct a Monte-Carlo simulation of the returns based on the best-fit Student copula and calculate the portfolio returns assuming equal weights of the assets in the portfolio.

```
##### Monte-Carlo #####
N = 100000
cdf.sim <- rcopula(n=N,copula=stud.fit@copula)
z.sim <- matrix(nrow=N,ncol=2)
for (i in 1:2) z.sim[,i] <- qsged(cdf.sim[,i],
                                  mean=mean[i],sd=sd[i],nu=nu[i],xi=xi[i])
frc1 <- predict(SPY.gfit,n.ahead=1)
frc2 <- predict(DIA.gfit,n.ahead=1)
mu <- c(frc1[,1],frc2[,1])
sigma <- c(frc1[,3],frc2[,3])

##### Modelled portfolio returns #####
w = c(0.5, 0,5)
prt.sim <- w[1]*(mu[1]+sigma[1]*z.sim[,1]) +
  w[2]*(mu[2]+sigma[2]*z.sim[,2])
```

We now calculate Value-at-Risk and Expected Shortfall for the portfolio..

```
alpha <- 0.10
prt.sim <- sort(prt.sim)
VaR <- prt.sim[alpha*N]
ES <- mean(prt.sim[1:(alpha*N-1)])

print(VaR)
```

```
## [1] -0.007524986
```

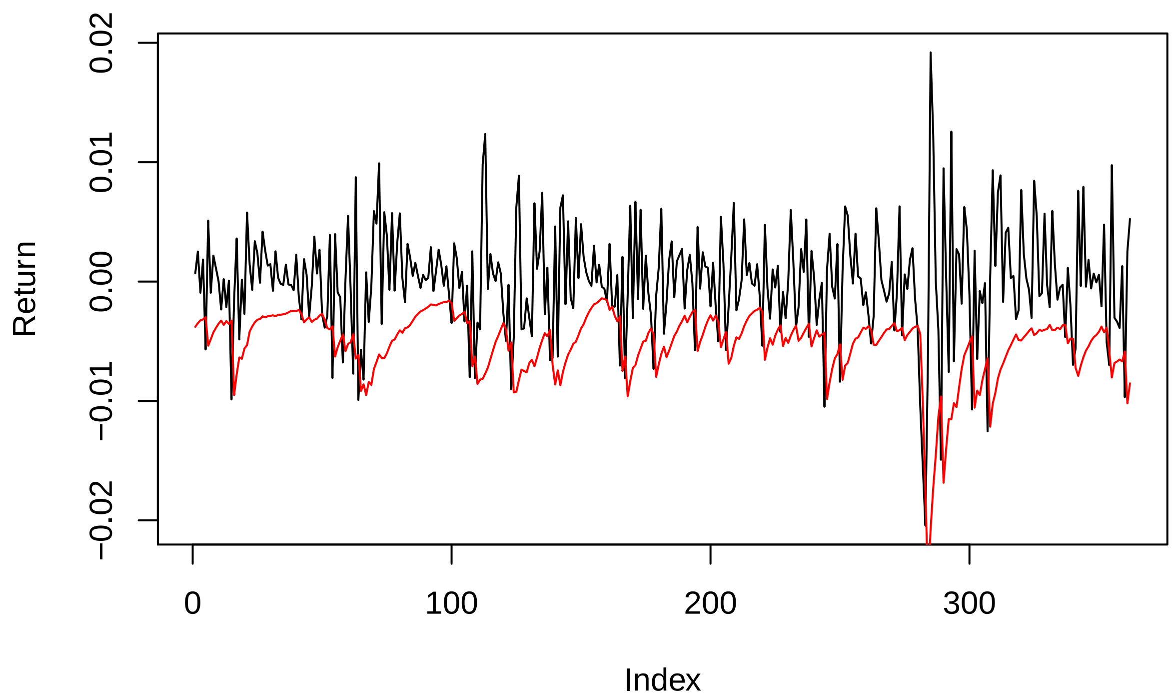
```
print(ES)
```

```
## [1] -0.0120965
```

Finally, we now want to calculate the VaR curve through a 130 day window. We fit GARCH and copula models every iteration.

```
##### Calculate VaR curve #####
VaR_curve <- numeric()
h <- 0.5 * 260 # window length

for (i in (h+1):length(rets[,1]))
{
  h.rets <- rets[(i-h):(i-1),]

  SPY.gfit <- garchFit(formula=~aparch(1,1),data=h.rets$SPY,delta=2,
                       include.delta=FALSE,leverage=TRUE,cond.dist="sged",
                       shape=1.2,include.shape=TRUE,trace=FALSE)

  DIA.gfit <- garchFit(formula=~aparch(1,1),data=h.rets$DIA,delta=2,
                       include.delta=FALSE,leverage=TRUE,cond.dist="sged",
                       shape=1.2,include.shape=TRUE,trace=FALSE)

  z <- matrix(nrow=length(h.rets$SPY),ncol=2)
  z[,1] <- SPY.gfit@residuals / SPY.gfit@sigma.t
  z[,2] <- DIA.gfit@residuals / DIA.gfit@sigma.t

  mean <- c(0,0); sd <- c(1,1); nu <- c(1.2,1.2)
  xi <- c(SPY.gfit@fit$par["skew"], DIA.gfit@fit$par["skew"])
  cdf <- matrix(nrow=length(h.rets$SPY),ncol=2)
  for (j in 1:2) cdf[,j] = psged(z[,j],mean=mean[j],sd=sd[j],nu=nu[j],xi=xi[j])

  stud.fit <- fitCopula(cdf,copula=stud.cop)

  N = 100000
  cdf.sim <- rcopula(n=N,copula=stud.fit@copula)
  z.sim <- matrix(nrow=N,ncol=2)
  for (j in 1:2) z.sim[,j] <- qsged(cdf.sim[,j],
                                    mean=mean[j],sd=sd[j],nu=nu[j],xi=xi[j])
  frc1 <- predict(SPY.gfit,n.ahead=1)
  frc2 <- predict(DIA.gfit,n.ahead=1)
  h.mu <- c(frc1[,1],frc2[,1])
  h.sigma <- c(frc1[,3],frc2[,3])

  h.prt.sim <- w[1]*(h.mu[1]+h.sigma[1]*z.sim[,1]) +
    w[2]*(h.mu[2]+h.sigma[2]*z.sim[,2])

  h.prt.sim <- sort(h.prt.sim)

  VaR_curve[i-h] <- h.prt.sim[alpha*N]
}
```

```
fact <- rets[(h+1):length(rets[,1]),1]*w[1] + rets[(h+1):length(rets[,2]),2]*w[2]
plot(as.numeric(fact),type="l", ylab = "Return")
lines(VaR_curve,col="red")
```



In order to check the adequacy of the VaR curve we conduct the Kupiec test.

```
##### Kupiec test #####
K <- sum(fact<VaR_curve); alpha0 <- K/length(rets[,1])
S <- -2*log((1-alpha)^(length(rets[,1])-K)*alpha^K)+
  2*log((1-alpha0)^(length(rets[,1])-K)*alpha0^K)
p.value <- 1-pchisq(S,df=1)
```

Kupiec test shows that the actual number of drawdowns (alpha0 = 0.089) is not statistically different from the modelled one (alpha = 0.1) with a high p-value (0.43).