ALGORITHMIC TRADING

2 : Programming simple options pricing models

NaiveOption Class

#1

```
public class NaiveOption{
        public OptionType Type;
        public double UnderlyingPrice;
        public double StrikePrice;
        public double TimeToExpiration;
        public double InterestRate;
        public double Volatility;
        public double Price; //Method
 }
 public enum OptionType {
        Call,
        Put
 }
```

Black-Scholes option pricing formula

$$C = S\,N(d_1) - X\,e^{-rT}\,N(d_2),$$

$$P = -S\,N(-d_1) + X\,e^{-rT}\,N(-d_2),$$

Where

$$d_1 = \frac{\ln\left(\frac{S}{X}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}, \quad d_2 = d_1 - \sigma\sqrt{T}$$

Black-Scholes options pricing

MODEL

$$C = S\,N(d_1) - K\,e^{-rT}\,N(d_2),$$

$$P = -S\,N(-d_1) + K\,e^{-rT}\,N(-d_2),$$

Where

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}},$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

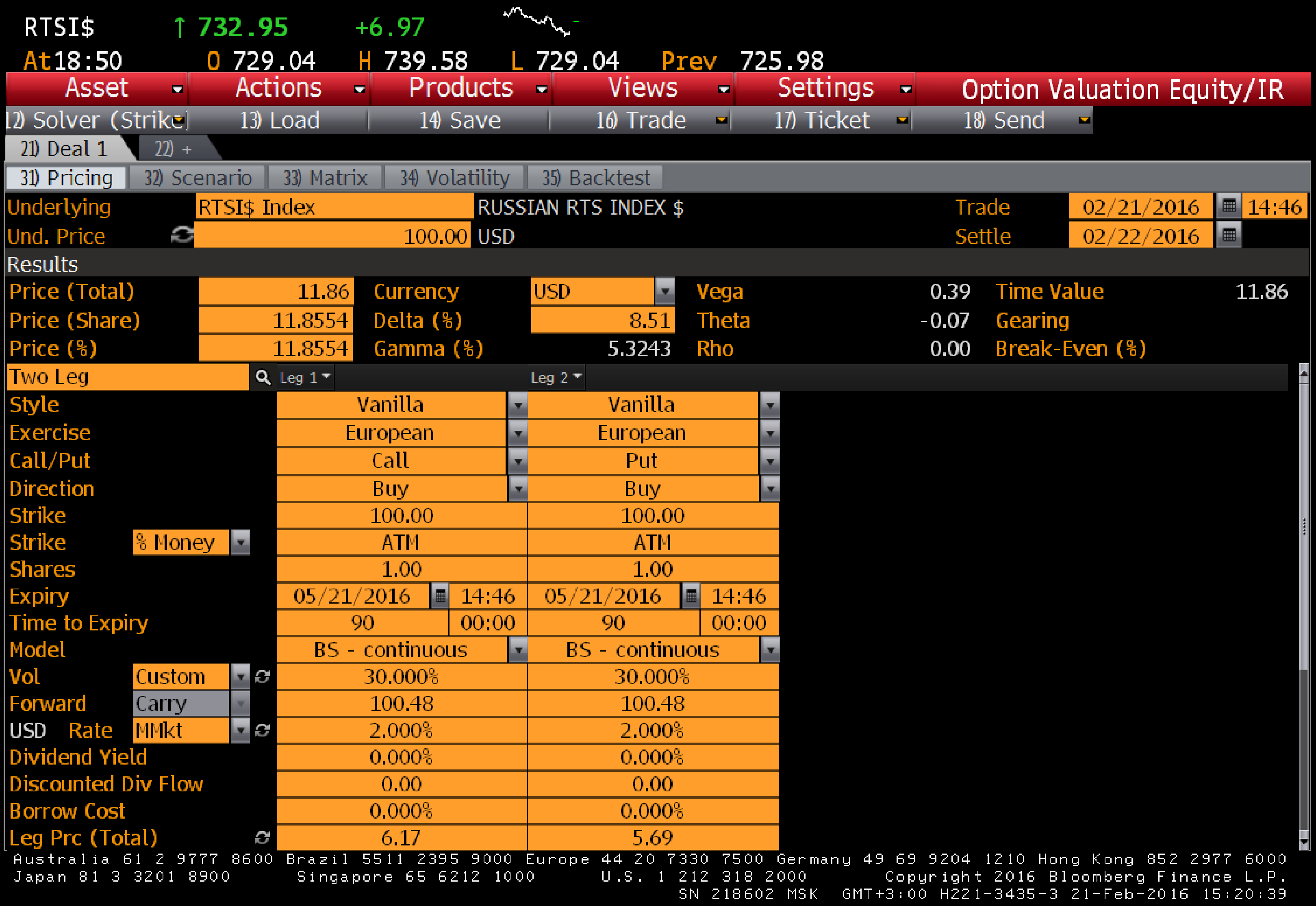
CODE

```
public class NaiveOption{

...

public double Price {

    get {

        var d1 = (Math.Log(UnderlyingPrice/StrikePrice) +
                  (InterestRate + Volatility*Volatility/2)*TimeToExpiration)/
                  (Volatility*Math.Sqrt(TimeToExpiration));

        var d2 = d1 - Volatility*Math.Sqrt(TimeToExpiration);

        double price;

        if (Type == OptionType.Call)
            price = UnderlyingPrice*NormalDistribution.Phi(d1) -
                    StrikePrice*Math.Pow(Math.E, -InterestRate*TimeToExpiration)*
                    NormalDistribution.Phi(d2);
        else
            price = -UnderlyingPrice * NormalDistribution.Phi(-d1) +
                    StrikePrice*Math.Pow(Math.E, -InterestRate*TimeToExpiration)*
                    NormalDistribution.Phi(-d2);

        return price;

    }

}
```

Console application with pricing

```
var option1 = new NaiveOption() {
    UnderlyingPrice = 100,
    Type = OptionType.Call,
    StrikePrice = 100,
    TimeToExpiration = 90.0/365,
    Volatility = 0.3,
    InterestRate = 0.02
};
Console.WriteLine(option1.Price);
// 6.1721...
```

```
var option2 = new NaiveOption() {
    UnderlyingPrice = 100,
    Type = OptionType.Put,
    StrikePrice = 100,
    TimeToExpiration = 90.0 / 365,
    Volatility = 0.3,
    InterestRate = 0.02
};
Console.WriteLine(option2.Price);
// 5.6802...
```



Bloomberg OV<Go> function

Call 6.17 & Put 5.69

Greeks (for European no-dividend options)

From Black-Scholes equation it may be shown that:

Delta: $\Delta_{\text{Call}} = \mathrm{N}(\mathrm{d}_1)$, $\Delta_{\text{Put}} = N(d_1) - 1$

Gamma: $\Gamma = \frac{\mathrm{N}'(\mathrm{d}_1)}{\mathrm{S}_0 \sigma \sqrt{T}}$

Theta: $\Theta_{\text{Call}} = -\frac{\mathrm{S}_0 \mathrm{N}'(\mathrm{d}_1)\sigma}{2\sqrt{T}} - rKe^{-rT}N(d_2)$, $\Theta_{\text{Put}} = -\frac{\mathrm{S}_0 \mathrm{N}'(\mathrm{d}_1)\sigma}{2\sqrt{T}} + rKe^{-rT}N(-d_2)$

Vega: $\mathrm{V} = \mathrm{S}_0 \sqrt{T} N'(d_1)$

Rho: $\mathrm{P}_{\text{Call}} = \mathrm{KTe}^{-\mathrm{rT}}\mathrm{N}(\mathrm{d}_2)$ $\mathrm{P}_{\text{Put}} = -\mathrm{KTe}^{-\mathrm{rT}}\mathrm{N}(-\mathrm{d}_2)$

Where $N'(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$

Black-Scholes Delta calculation

MODEL

$\Delta_{\text{Call}} = \mathrm{N}(\mathrm{d}_1),$

$\Delta_{\text{Put}} = N(d_1) - 1$

CODE

```
public class NaiveOption{
...
public double Delta {
        get {
                if (Type == OptionType.Call)
                    return NormalDistribution.Phi(_d1);
                else
                     return NormalDistribution.Phi(_d1) - 1;
        }
    }
}
```

Black-Scholes Gamma calculation

MODEL

$$\Gamma = \frac{N'(d_1)}{S_0 \sigma \sqrt{T}}$$

CODE

```
public class NaiveOption{
...
public double Gamma => NormalDistribution.Density(_d1)/
                         (UnderlyingPrice*Volatility*Math.Sqrt(TimeToExpiration));
...
}
```

Black-Scholes Theta calculation

MODEL

$$\Theta_{\text{Call}} = -\frac{\mathrm{S_0 N'(d_1)}\sigma}{2\sqrt{T}} - rKe^{-rT}N(d_2),$$

$$\Theta_{\text{Put}} = -\frac{\mathrm{S_0 N'(d_1)}\sigma}{2\sqrt{T}} + rKe^{-rT}N(-d_2)$$

CODE

```
public class NaiveOption{

...

public double Theta {

    get {

        if (Type == OptionType.Call)

            return -(UnderlyingPrice*NormalDistribution.Density( d1)*
                    Volatility)/(2*Math.Sqrt(TimeToExpiration)) –
                    InterestRate*StrikePrice*
                    Math.Pow(Math.E, -InterestRate*TimeToExpiration)*
                    NormalDistribution.Phi(_d2);
        else

            return -(UnderlyingPrice * NormalDistribution.Density( d1) *
                    Volatility) / (2 * Math.Sqrt(TimeToExpiration)) +
                    InterestRate * StrikePrice *
                    Math.Pow(Math.E, -InterestRate * TimeToExpiration) *
                    NormalDistribution.Phi(-_d2);

    }

}

...
```

Black-Scholes Vega calculation

MODEL

$$V = S_0\sqrt{T}N'(d_1)$$

CODE

```
public class NaiveOption{

...

public double Vega => UnderlyingPrice*Math.Sqrt(TimeToExpiration)*
                      NormalDistribution.Density(_d1);

...

}
```

Black-Scholes Rho calculation

MODEL

$$P_{Call} = KTe^{-rT}N(d_2)$$

$$P_{Put} = -KTe^{-rT}N(-d_2)$$

CODE

```
public class NaiveOption{
...
public double Rho {
    get {
        if (Type == OptionType.Call)
            return StrikePrice*TimeToExpiration*
                   Math.Pow(Math.E, -InterestRate*TimeToExpiration)*
                   NormalDistribution.Phi(_d2);
        else
            return -StrikePrice * TimeToExpiration *
                   Math.Pow(Math.E, -InterestRate * TimeToExpiration) *
                   NormalDistribution.Phi(-_d2);
    }
}
...
}
```

Cash Greeks (back to definition)

Delta: $\Delta = \frac{\partial P}{\partial S}$ (futures/forwards & equity usually called “delta 1” products)

Gamma: $\Gamma = \frac{\partial^2 P}{\partial S^2} = \frac{\partial \Delta}{\partial S}$

Theta: $\Theta = \frac{\partial P}{\partial T}$

Vega: $\mathrm{V} = \frac{\partial \mathrm{P}}{\partial \sigma}$

Rho: $\mathrm{P} = \frac{\partial \mathrm{P}}{\partial r}$

If we talk about portfolio of options we need to compare and to sum Greeks of different assets, so it’s obvious that we need to transform it to cash values.

So here P might be both options price and options portfolio value.

Cash Greeks

When considering options position (portfolio) we're interested in Cash Greeks:

1. Delta (Cash)

   What worth of underlying we need to buy/sell to offset market risk of option position or other words what position in underlying our options position is equivalent in the moment

2. Gamma (1%)

   What delta portfolio going to have with Underlying Price move for 1% up / down

3. Theta (1 day)

   How much do we pay (or receive) for holding option position for 1 day

4. Vega (1% vol shift)

   How value of our portfolio going to change if we increase/decrease volatility in the model for 1%

5. Rho (1% rate shift)

   How value of our portfolio going to change if we increase/decrease volatility in the model for 1%

Cash Greeks

When considering options position (portfolio) we're interested in Cash Greeks:

1. Delta (Cash) : $\Delta_{t,Cash} = \Delta_{\mathrm{t}} S_t$
2. Gamma (1%) : $\Gamma_{t,Cash} = \Gamma_t \frac{S_t^2}{100}$
3. Theta (1 day) : $\Theta_{t,Cash} = \frac{\Theta_{\mathrm{t}}}{365}$
4. Vega (1% vol shift) : $\mathrm{V}_{t,Cash} = 0.01 V_t$
5. Rho (1% rate shift) : $\mathrm{P}_{t,Cash} = 0.01 \mathrm{P}_t$

Cash Greeks Code

MODEL

$$\Delta_{t,Cash} = \Delta_t S_t$$

$$\Gamma_{t,Cash} = \Gamma_t \frac{S_t^2}{100}$$

$$\Theta_{t,Cash} = \frac{\Theta_t}{365}$$

$$V_{t,Cash} = 0.01V_t$$

$$P_{t,Cash} = 0.01P_t$$

CODE

```
public class NaiveOption{
...
public double CashDelta => Delta*UnderlyingPrice;
public double CashGamma => Gamma*UnderlyingPrice*UnderlyingPrice/100;
public double CashTheta => Theta / 365;
public double CashVega => Vega /100;
public double CashRho=> Rho / 100;
...
}
```

Greeks for position

Derivatives traders always looks at this type of position snapshot:

Underlying Name	CashDelta	CashGamma	CashVega	Theta	Rho
Asset A	$4 500 000	$2 700 000	$185 000	-$35 000	$1 150
Asset B	-$3 000 000	-$1 200 000	-$88 500	$16 000	-$530
Total	$1 500 000	$1 500 000	$96 500	-$18 000	$620

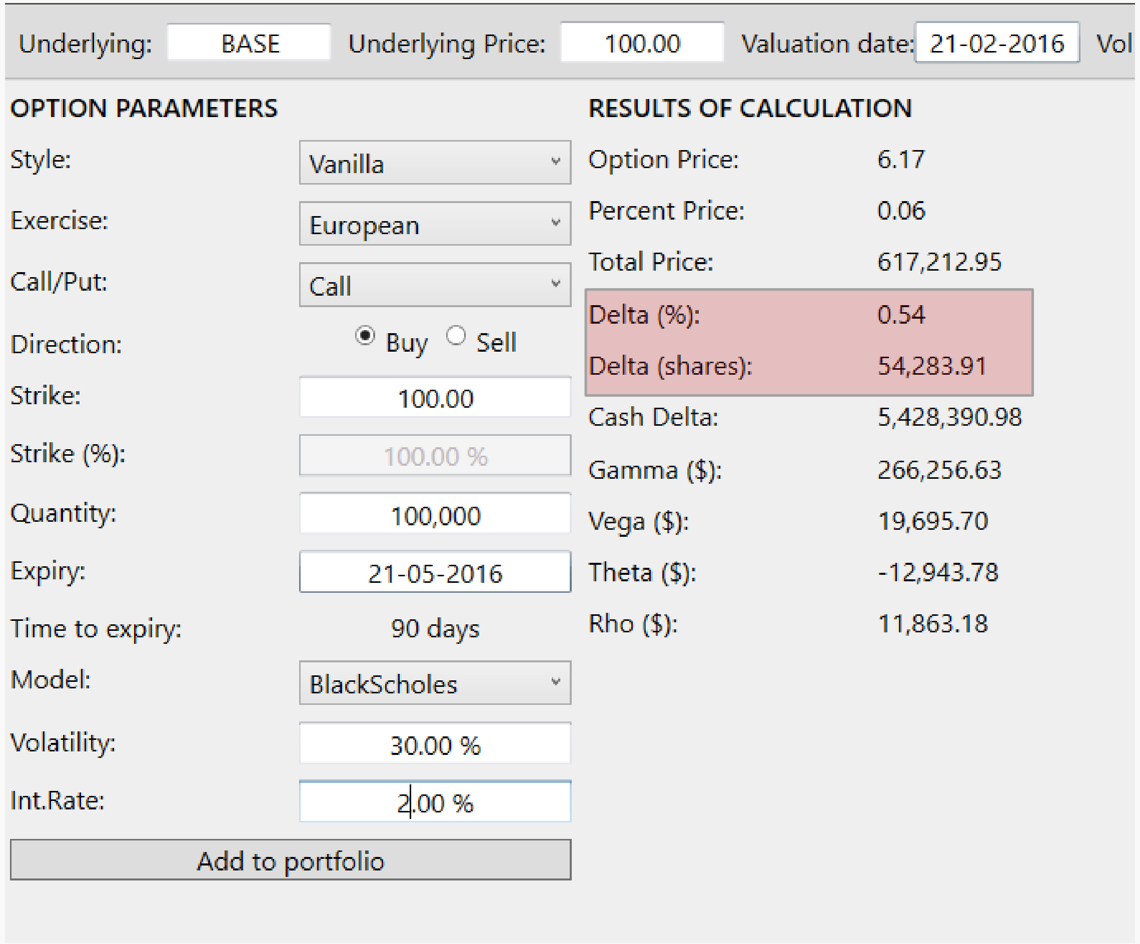
What can we tell about portfolio?
Long Delta / Long Gamma / Long Vega ? What else

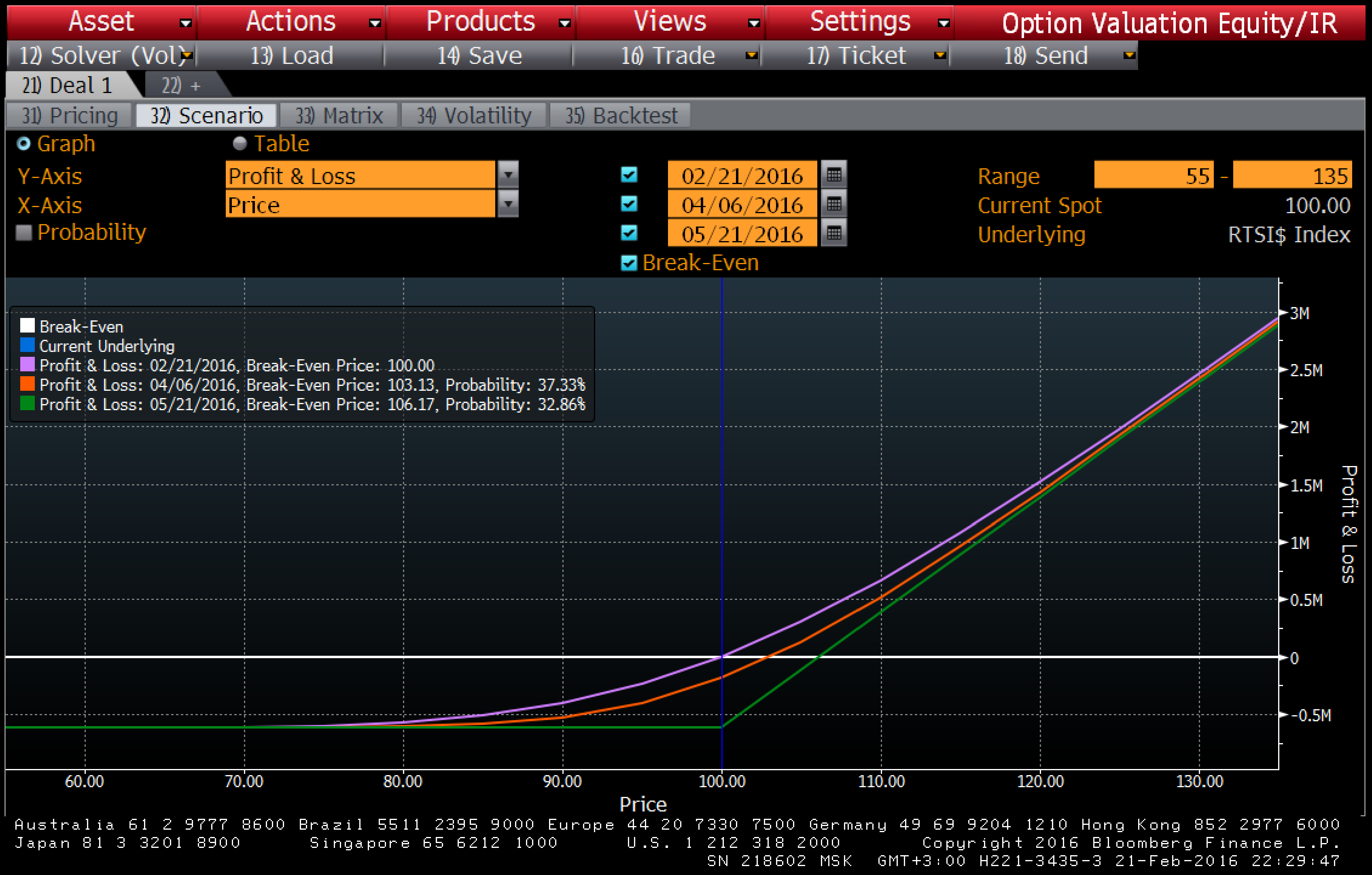
Delta hedging

Delta hedging means “choosing” Delta such that the portfolio value doesn’t depend on the direction of the stock.

I.E. we hedge delta of the portfolio with offsetting delta positions using instruments with the same underlying (Spot, Futures, Forwards, Options etc)

Delta hedging (option greeks)





Naked call option price sensitivity

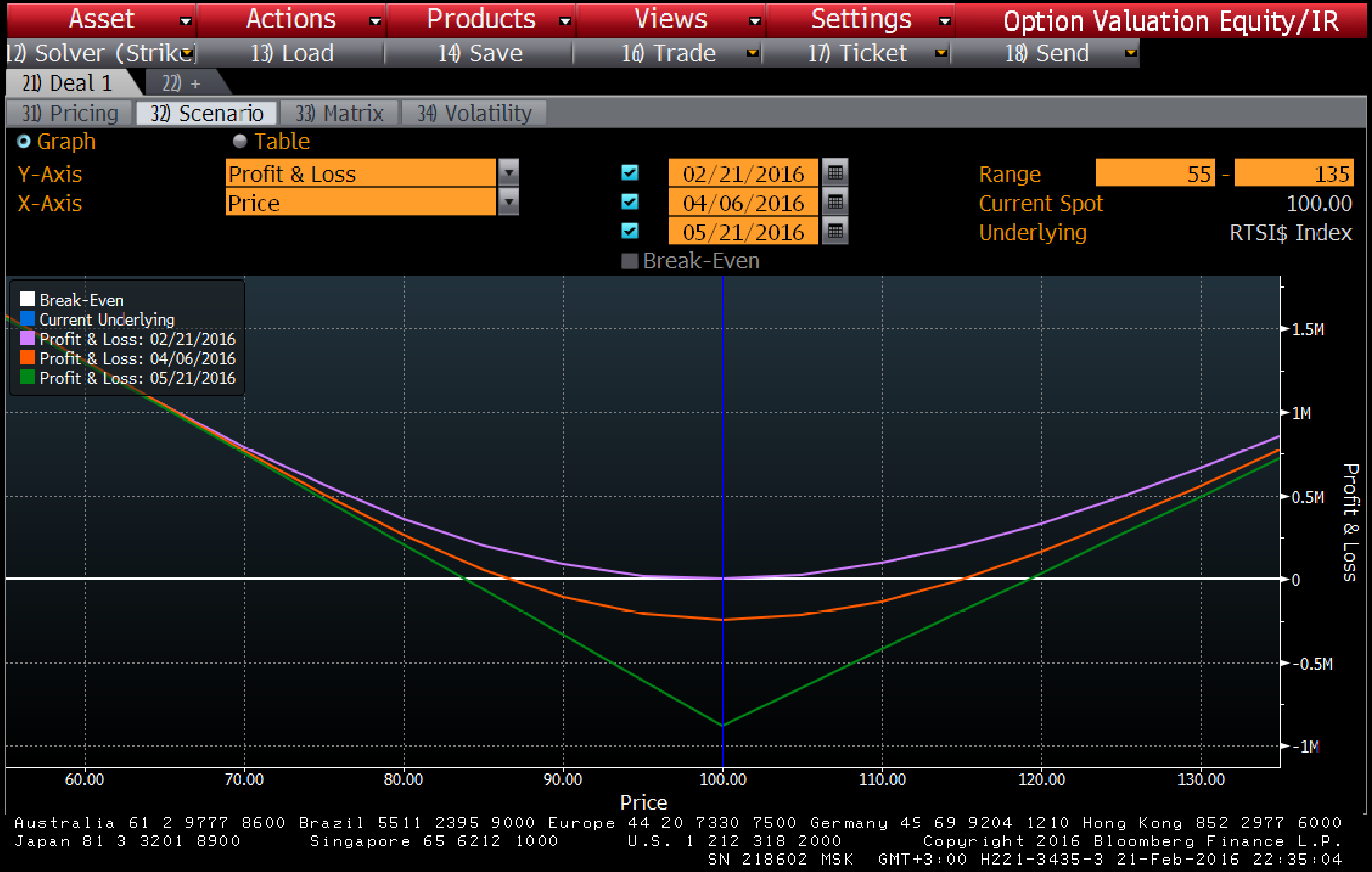
Delta hedging (explanation)

From pricer window we see that BUYING 100’000 call options

in terms of ‘DELTA RISK’ is equivalent AT THIS PARTICULAR MOMENT to

BUY 54’284 shares of underlying asset.

So to offset MARKET RISK or BETA RISK or DELTA RISK (which is equivalent) we need to sell 54’284 shares. Let’s see on PL profile of delta hedged position.



Hedged call option price sensitivity

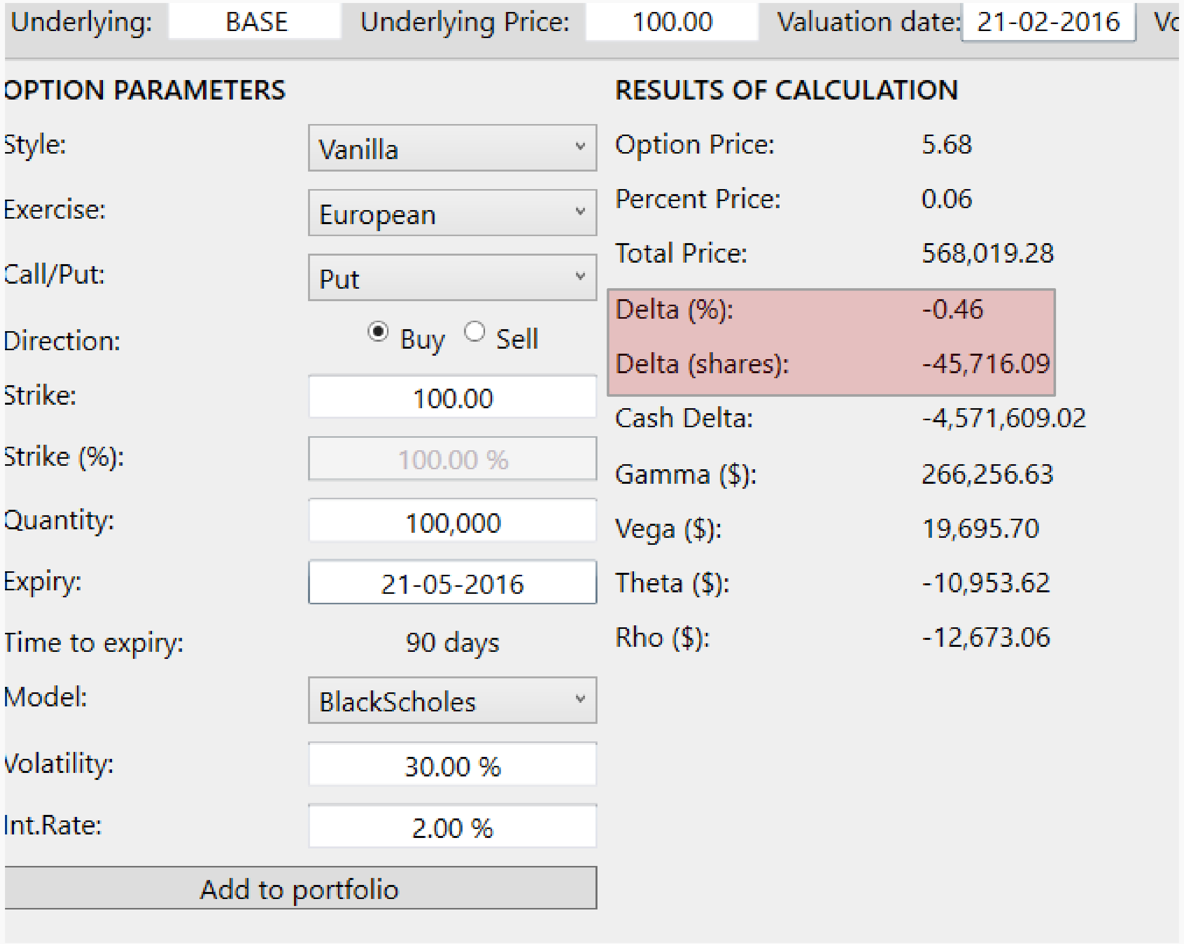
Delta hedging (continuation)

We see that delta hedged position is almost symmetrical on equity upside and downside.

BUT why when we buy CALL and hedge it we make more money on the downside than on the upside?

Let's do same exercise with PUT (compare NAKED and HEDGED positions)

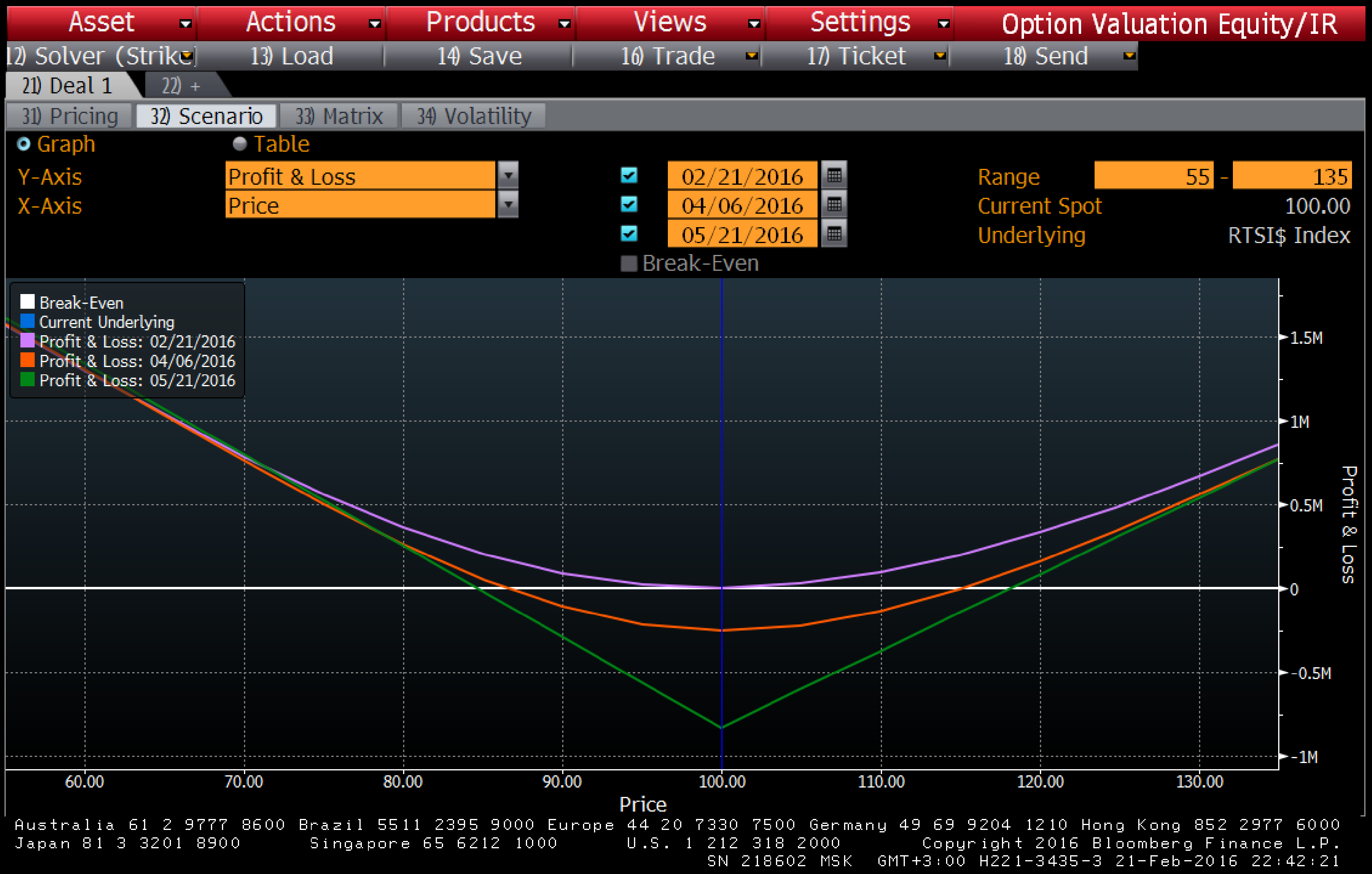
Delta hedging (put)



Buying 100 000 ATM Puts
Is equivalent to selling ~45k
underlying shares.

So to hedge it we need
To BUY it on the market.

Naked putoption price sensitivity

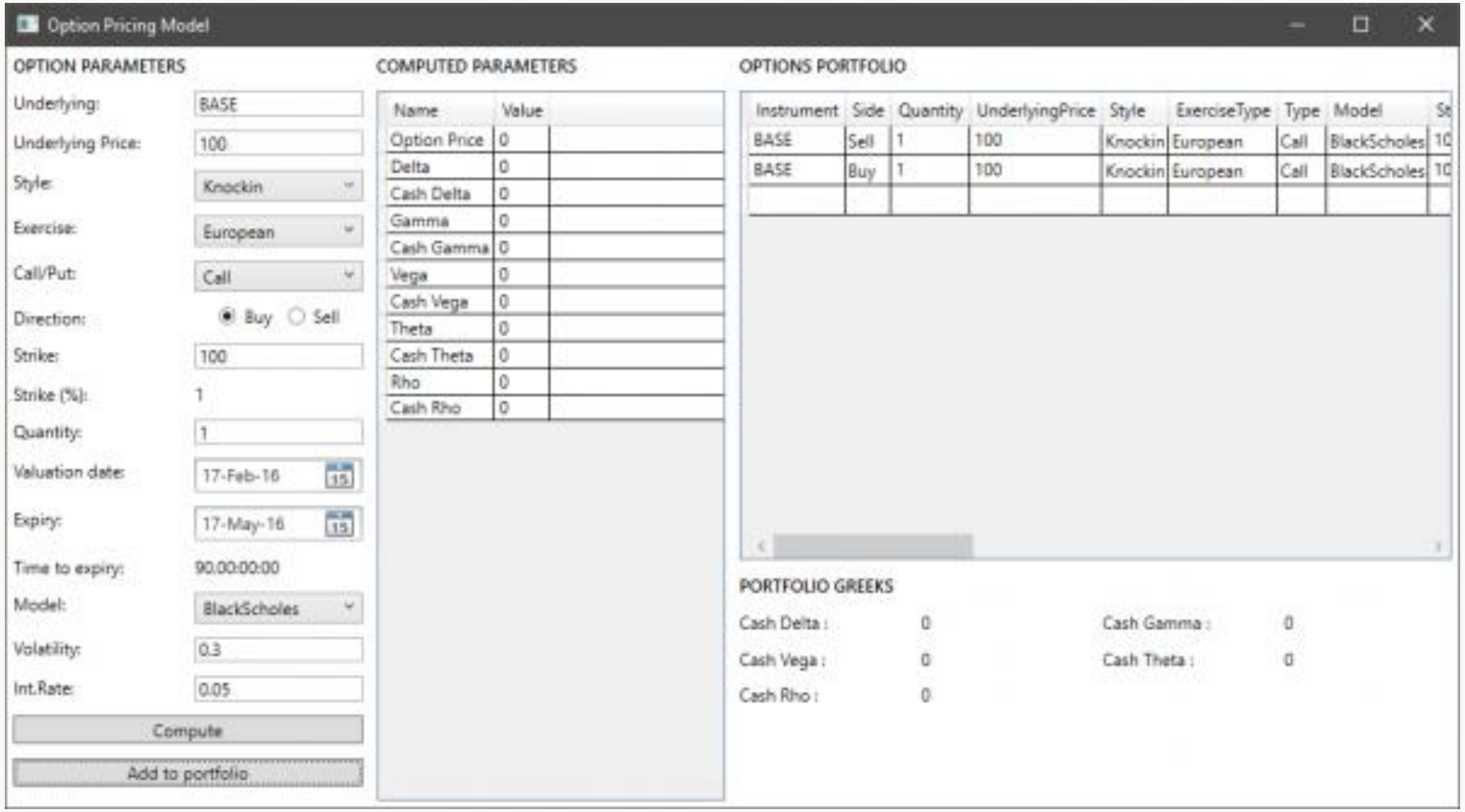


Hedged put option price sensitivity

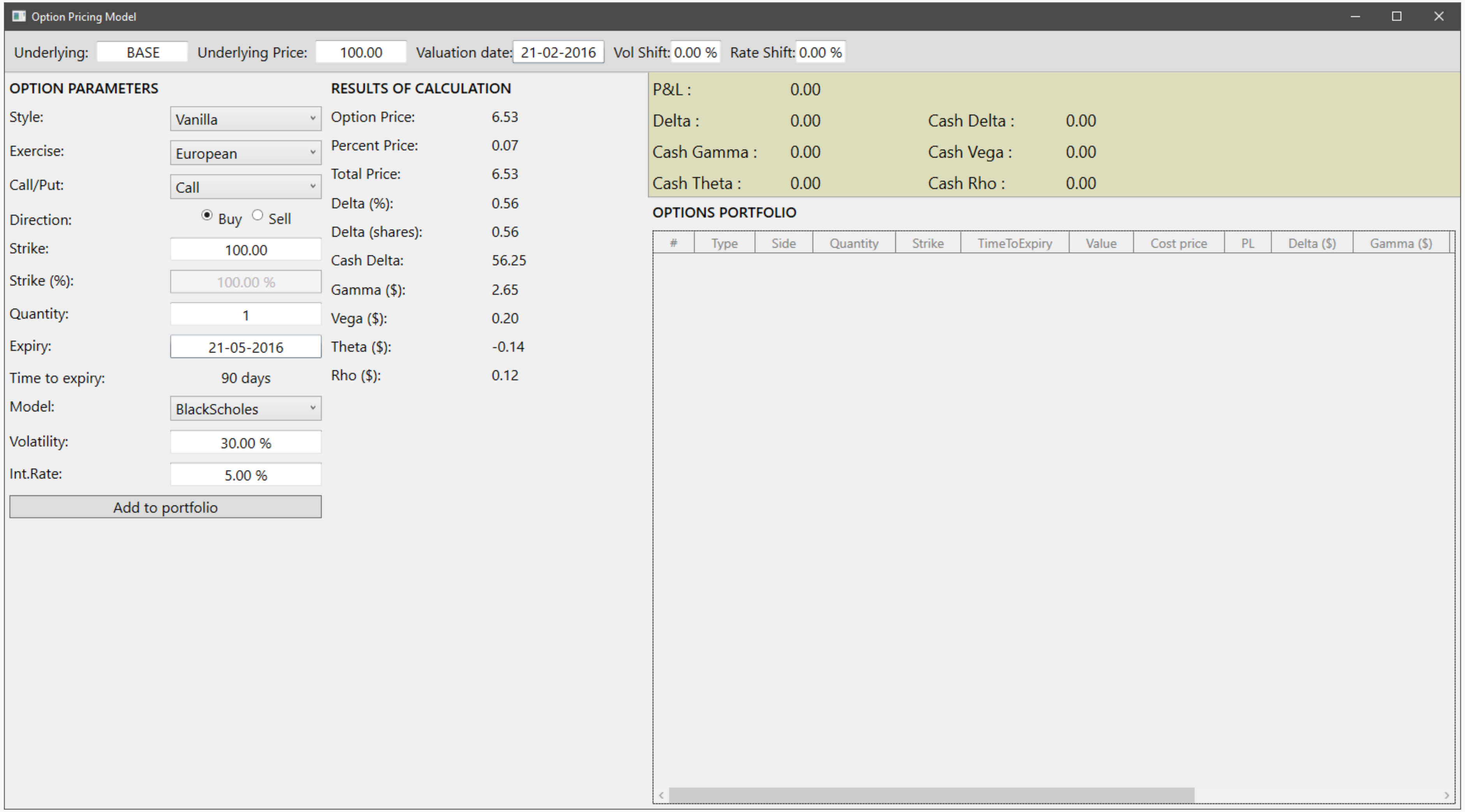
Delta hedging (continuation)

Home-task: Why we make more money on the downside when buying options, than on the upside?

We’re ready to make some code (было)



We’re ready to make some code (стало)



Reading List

1. J.C. Hull – Options, Futures, and Other Derivatives
2. N.Taleb – Dynamic Hedging
3. Espen Haug – Option Pricing Formulas

Homework

1. Practice
   1. *Modify Black-Scholes for dividends paying stocks*
2. Small essays (with graphical illustration)

Homework

1. Practice
    1. *Modify Black-Scholes for dividends paying stocks*
    2. *Implement Binomial pricing model*
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1. Practice:
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Homework

1. Practice:
   1. *Modify Black-Scholes for dividends paying stocks*
   2. *Implement Binomial pricing model*
   3. *Implement Trinomial pricing model*
   4. *Implement Explicit Finite Difference pricing model*
2. Small essays (with graphical illustration)

Homework

1. Practice:
    1. *Modify Black-Scholes for dividends paying stocks*
    2. *Implement Binomial pricing model*
    3. *Implement Trinomial pricing model*
    4. *Implement Explicit Finite Difference pricing model*
2. Small essays (with graphical illustration)
    1. *Volatility shifts impact on options portfolio*
    2. *Interest rate impact on options portfolio*

Homework

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   1. *Modify Black-Scholes for dividends paying stocks*
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   1. *Volatility shifts impact on options portfolio*
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   3. *Example of options position where Vega and Gamma have different signs*

Homework

1. Practice:
    1. *Modify Black-Scholes for dividends paying stocks*
    2. *Implement Binomial pricing model*
    3. *Implement Trinomial pricing model*
    4. *Implement Explicit Finite Difference pricing model*
2. Small essays (with graphical illustration)
    1. *Volatility shifts impact on options portfolio*
    2. *Interest rate impact on options portfolio*
    3. *Example of options position where Vega and Gamma have different signs*
    4. *When delta hedging portfolio consisting of one <u>bought</u> option and underlying hedges is it possible to loose more than premium paid for that option?*

Homework

1. Practice (70%)
    1. *(5%) Modify Black-Scholes for dividends paying stocks*
    2. *(15%) Implement Binomial pricing model*
    3. *(20%) Implement Trinomial pricing model*
    4. *(30%) Implement Explicit Finite Difference pricing model*
2. Small essays (with graphical illustration) (30%)
    1. *(5%) Volatility shifts impact on options portfolio*
    2. *(5%) Interest rate impact on options portfolio*
    3. *(10%) Example of options portfolios where Vega and Gamma have different signs*
    4. *(10%) When delta hedging portfolio consisting of one <u>bought</u> option and underlying hedges is it possible to loose more than premium paid for that option?*

Deadline: 5-mar (upload @ edu.kb9.co.uk) <u>Minimum : 65% (2/3)</u>

Next lecture

Date: 7-mar-2016

Topics:

1. Pricing & models:
    1. *path dependent options and other "vanilla" exotics*

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2. Derivatives trading:
    1. *Stress testing portfolio*
    2. *Notes on risks and risk-limits of options trading*

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    3. ***Volatility surface of exchange traded options***

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   3. ***Volatility surface of exchange traded options***
3. Practice:
   1. *Preparing framework for market making in options*

Next lecture

Date: 7-mar-2016

Topics:

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3. Practice:
    1. *Preparing framework for market making in options*
    2. *Automation of delta-hedging*

KEEP
CALM
AND
QUANT
ON