

# ALGORITHMIC TRADING

2 : Programming simple options pricing models

# NaiveOption Class

#1

```
public class NaiveOption{  
    public OptionType Type;  
    public double UnderlyingPrice;  
    public double StrikePrice;  
    public double TimeToExpiration;  
    public double InterestRate;  
    public double Volatility;  
    public double Price; //Method  
}  
  
public enum OptionType {  
    Call,  
    Put  
}  
  
}
```

# Black-Scholes option pricing formula

$$C = S N(d_1) - X e^{-rT} N(d_2),$$

$$P = -S N(-d_1) + X e^{-rT} N(-d_2),$$

Where

$$d_1 = \frac{\ln\left(\frac{S}{X}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}, \quad d_2 = d_1 - \sigma\sqrt{T}$$

# Black-Scholes options pricing

## MODEL

$$C = S N(d_1) - K e^{-rT} N(d_2),$$

$$P = -S N(-d_1) + K e^{-rT} N(-d_2),$$

Where

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}},$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

## CODE

```
public class NaiveOption{  
    ...  
    public double Price {  
        get {  
            var d1 = (Math.Log(UnderlyingPrice/StrikePrice) +  
                (InterestRate + Volatility*Volatility/2)*TimeToExpiration)/  
                (Volatility*Math.Sqrt(TimeToExpiration));  
            var d2 = d1 - Volatility*Math.Sqrt(TimeToExpiration);  
            double price;  
            if (Type == OptionType.Call)  
                price = UnderlyingPrice*NormalDistribution.Phi(d1) -  
                    StrikePrice*Math.Pow(Math.E, -InterestRate*TimeToExpiration)*  
                    NormalDistribution.Phi(d2);  
            else  
                price = -UnderlyingPrice * NormalDistribution.Phi(-d1) +  
                    StrikePrice*Math.Pow(Math.E, -InterestRate*TimeToExpiration)*  
                    NormalDistribution.Phi(-d2);  
            return price;  
        }  
    }  
}
```

# Console application with pricing

```
var option1 = new NaiveOption() {  
    UnderlyingPrice = 100,  
    Type = OptionType.Call,  
    StrikePrice = 100,  
    TimeToExpiration = 90.0/365,  
    Volatility = 0.3,  
    InterestRate = 0.02  
};  
Console.WriteLine(option1.Price);  
// 6.1721...
```

```
var option2 = new NaiveOption() {  
    UnderlyingPrice = 100,  
    Type = OptionType.Put,  
    StrikePrice = 100,  
    TimeToExpiration = 90.0 / 365,  
    Volatility = 0.3,  
    InterestRate = 0.02  
};  
Console.WriteLine(option2.Price);  
// 5.6802...
```

RTSI\$ ↑ 732.95 +6.97

At 18:50 O 729.04 H 739.58 L 729.04 Prev 725.98

Asset	Actions	Products	Views	Settings	Option Valuation Equity/IR
1) Solver (Strike)	13) Load	14) Save	16) Trade	17) Ticket	18) Send
2) Deal 1	22) +				
3) Pricing	32) Scenario	33) Matrix	34) Volatility	35) Backtest	

Underlying RTSI\$ Index RUSSIAN RTS INDEX \$ Trade 02/21/2016 14:46

Und. Price 100.00 USD Settle 02/22/2016

Results

Price (Total)	11.86	Currency	USD	Vega	0.39	Time Value	11.86
Price (Share)	11.8554	Delta (%)	8.51	Theta	-0.07	Gearing	
Price (%)	11.8554	Gamma (%)	5.3243	Rho	0.00	Break-Even (%)	

Two Leg Leg 1 Leg 2

Style	Vanilla	Vanilla
Exercise	European	European
Call/Put	Call	Put
Direction	Buy	Buy
Strike	100.00	100.00
Strike % Money	ATM	ATM
Shares	1.00	1.00
Expiry	05/21/2016 14:46	05/21/2016 14:46
Time to Expiry	90 00:00	90 00:00
Model	BS - continuous	BS - continuous
Vol Custom	30.000%	30.000%
Forward Carry	100.48	100.48
USD Rate MMkt	2.000%	2.000%
Dividend Yield	0.000%	0.000%
Discounted Div Flow	0.00	0.00
Borrow Cost	0.000%	0.000%
Leg Prc (Total)	6.17	5.69

Australia 61 2 9777 8600 Brazil 5511 2395 9000 Europe 44 20 7330 7500 Germany 49 69 9204 1210 Hong Kong 852 2977 6000  
 Japan 81 3 3201 8900 Singapore 65 6212 1000 U.S. 1 212 318 2000 Copyright 2016 Bloomberg Finance L.P.  
 SN 218602 MSK GMT+3:00 H221-3435-3 21-Feb-2016 15:20:39

# Bloomberg OV<Go> function

Call 6.17 & Put 5.69

# Greeks (for European no-dividend options)

From Black-Scholes equation it may be shown that:

Delta:  $\Delta_{\text{Call}} = N(d_1),$   $\Delta_{\text{Put}} = N(d_1) - 1$

Gamma:  $\Gamma = \frac{N'(d_1)}{S_0 \sigma \sqrt{T}}$

Theta:  $\Theta_{\text{Call}} = -\frac{S_0 N'(d_1) \sigma}{2\sqrt{T}} - rK e^{-rT} N(d_2),$   $\Theta_{\text{Put}} = -\frac{S_0 N'(d_1) \sigma}{2\sqrt{T}} + rK e^{-rT} N(-d_2)$

Vega:  $V = S_0 \sqrt{T} N'(d_1)$

Rho:  $\rho_{\text{Call}} = K T e^{-rT} N(d_2)$   $\rho_{\text{Put}} = -K T e^{-rT} N(-d_2)$

Where  $N'(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$

# Black-Scholes Delta calculation

## MODEL

$$\Delta_{\text{Call}} = N(d_1),$$

$$\Delta_{\text{Put}} = N(d_1) - 1$$

## CODE

```
public class NaiveOption{  
    ...  
    public double Delta {  
        get {  
            if (Type == OptionType.Call)  
                return NormalDistribution.Phi(_d1);  
            else  
                return NormalDistribution.Phi(_d1) - 1;  
        }  
    }  
}
```



# Black-Scholes Gamma calculation

## MODEL

$$\Gamma = \frac{N'(d_1)}{S_0 \sigma \sqrt{T}}$$

## CODE

```
public class NaiveOption{  
    ...  
  
    public double Gamma => NormalDistribution.Density(_d1)/  
        (UnderlyingPrice*Volatility*Math.Sqrt(TimeToExpiration));  
    ...  
}
```

# Black-Scholes Theta calculation

## MODEL

$$\Theta_{\text{Call}} = -\frac{S_0 N'(d_1) \sigma}{2\sqrt{T}} - r K e^{-rT} N(d_2),$$

$$\Theta_{\text{Put}} = -\frac{S_0 N'(d_1) \sigma}{2\sqrt{T}} + r K e^{-rT} N(-d_2)$$

## CODE

```
public class NaiveOption{
    ...

    public double Theta {
        get {
            if (Type == OptionType.Call)
                return -(UnderlyingPrice*NormalDistribution.Density( d1)*
                    Volatility)/(2*Math.Sqrt(TimeToExpiration)) -
                    InterestRate*StrikePrice*
                    Math.Pow(Math.E, -InterestRate*TimeToExpiration)*
                    NormalDistribution.Phi(_d2);
            else
                return -(UnderlyingPrice * NormalDistribution.Density( d1) *
                    Volatility) / (2 * Math.Sqrt(TimeToExpiration)) +
                    InterestRate * StrikePrice *
                    Math.Pow(Math.E, -InterestRate * TimeToExpiration) *
                    NormalDistribution.Phi(-_d2);
        }
    }
    ...
}
```

# Black-Scholes Vega calculation

## MODEL

$$V = S_0 \sqrt{T} N'(d_1)$$

## CODE

```
public class NaiveOption{  
    ...  
    public double Vega => UnderlyingPrice*Math.Sqrt(TimeToExpiration)*  
        NormalDistribution.Density(_d1);  
    ...  
}
```

# Black-Scholes Rho calculation

## MODEL

$$P_{\text{Call}} = KTe^{-rT}N(d_2)$$

$$P_{\text{Put}} = -KTe^{-rT}N(-d_2)$$

## CODE

```
public class NaiveOption{  
    ...  
    public double Rho {  
        get {  
            if (Type == OptionType.Call)  
                return StrikePrice*TimeToExpiration*  
                    Math.Pow(Math.E, -InterestRate*TimeToExpiration)*  
                    NormalDistribution.Phi(_d2);  
            else  
                return -StrikePrice * TimeToExpiration *  
                    Math.Pow(Math.E, -InterestRate * TimeToExpiration) *  
                    NormalDistribution.Phi(-_d2);  
        }  
    }  
    ...  
}
```

# Cash Greeks (back to definition)

Delta:  $\Delta = \frac{\partial P}{\partial S}$  (futures/forwards & equity usually called “delta 1” products)

Gamma:  $\Gamma = \frac{\partial^2 P}{\partial S^2} = \frac{\partial \Delta}{\partial S}$

Theta:  $\Theta = \frac{\partial P}{\partial T}$

Vega:  $V = \frac{\partial P}{\partial \sigma}$

Rho:  $\rho = \frac{\partial P}{\partial r}$

If we talk about portfolio of options we need to compare and to sum Greeks of different assets, so it's obvious that we need to transform it to cash values.

So here P might be both options price and options portfolio value.

# Cash Greeks

When considering options position (portfolio) we're interested in Cash Greeks:

1. **Delta (Cash)**

What worth of underlying we need to buy/sell to offset market risk of option position or other words what position in underlying our options position is equivalent in the moment

2. **Gamma (1%)**

What delta portfolio going to have with Underlying Price move for 1% up / down

3. **Theta (1 day)**

How much do we pay (or receive) for holding option position for 1 day

4. **Vega (1% vol shift)**

How value of our portfolio going to change if we increase/decrease volatility in the model for 1%

5. **Rho (1% rate shift )**

How value of our portfolio going to change if we increase/decrease volatility in the model for 1%

# Cash Greeks

When considering options position (portfolio) we're interested in Cash Greeks:

1. Delta (Cash) :  $\Delta_{t,Cash} = \Delta_t S_t$
2. Gamma (1%) :  $\Gamma_{t,Cash} = \Gamma_t \frac{S_t^2}{100}$
3. Theta (1 day) :  $\Theta_{t,Cash} = \frac{\Theta_t}{365}$
4. Vega (1% vol shift) :  $V_{t,Cash} = 0.01 V_t$
5. Rho (1% rate shift) :  $P_{t,Cash} = 0.01 P_t$

# Cash Greeks Code

## MODEL

$$\Delta_{t,Cash} = \Delta_t S_t$$

$$\Gamma_{t,Cash} = \Gamma_t \frac{S_t^2}{100}$$

$$\Theta_{t,Cash} = \frac{\Theta_t}{365}$$

$$V_{t,Cash} = 0.01 V_t$$

$$P_{t,Cash} = 0.01 P_t$$

## CODE

```
public class NaiveOption{  
    ...  
  
    public double CashDelta => Delta*UnderlyingPrice;  
  
    public double CashGamma => Gamma*UnderlyingPrice*UnderlyingPrice/100;  
  
    public double CashTheta => Theta / 365;  
  
    public double CashVega => Vega /100;  
  
    public double CashRho=> Rho / 100;  
  
    ...  
}
```



# Greeks for position

Derivatives traders always looks at this type of position snapshot:

Underlying Name	CashDelta	CashGamma	CashVega	Theta	Rho
Asset A	\$4 500 000	\$2 700 000	\$185 000	-\$35 000	\$1 150
Asset B	-\$3 000 000	-\$1 200 000	-\$88 500	\$16 000	-\$530
Total	\$1 500 000	\$1 500 000	\$96 500	-\$18 000	\$620

What can we tell about portfolio?

Long Delta / Long Gamma / Long Vega ? What else

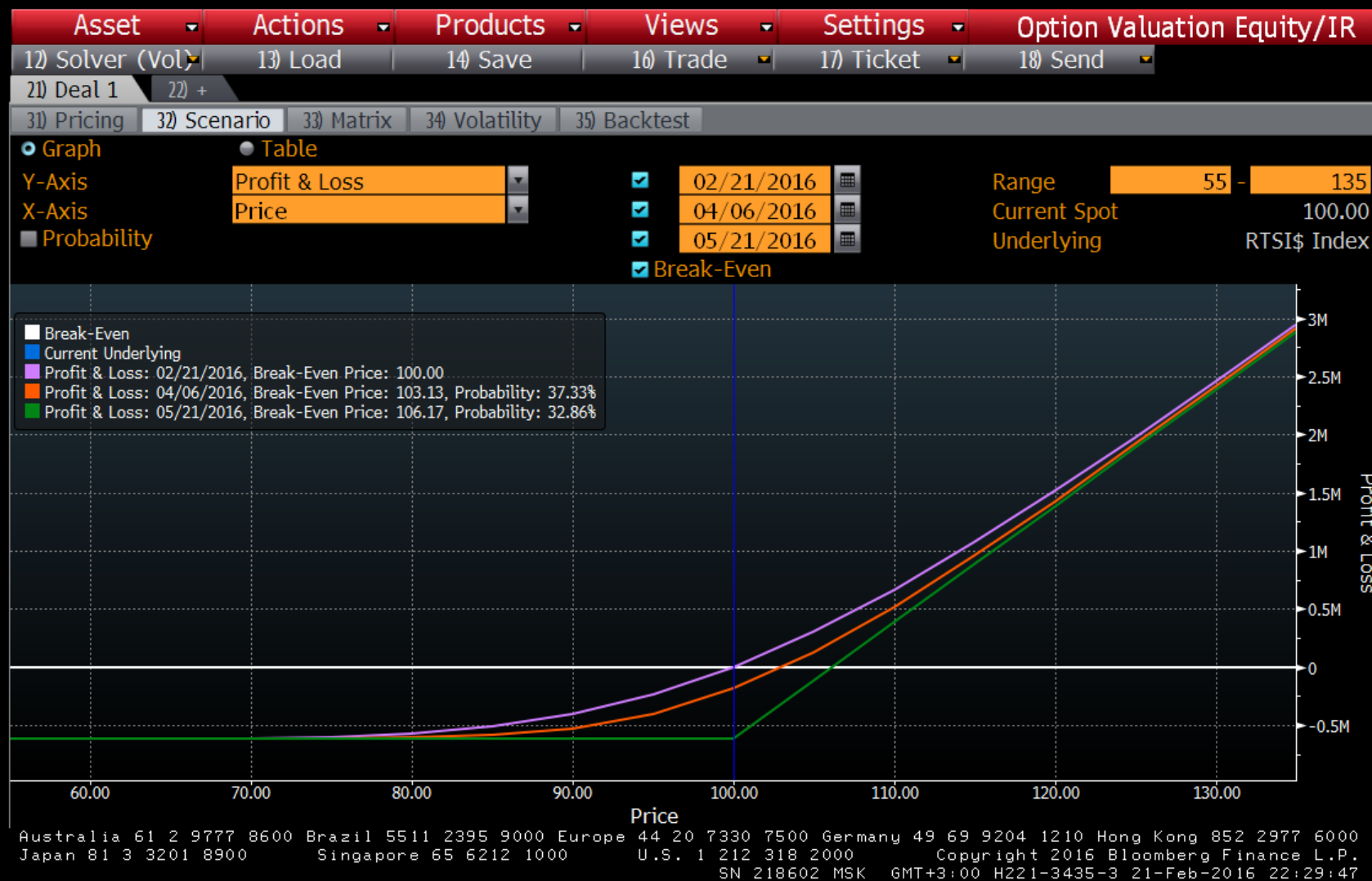
# Delta hedging

Delta hedging means “choosing” Delta such that the portfolio value doesn’t depend on the direction of the stock.

I.E. we hedge delta of the portfolio with offsetting delta positions using instruments with the same underlying (Spot, Futures, Forwards, Options etc)

# Delta hedging (option greeks)

Underlying:	BASE	Underlying Price:	100.00	Valuation date:	21-02-2016	Vol
<b>OPTION PARAMETERS</b>				<b>RESULTS OF CALCULATION</b>		
Style:	Vanilla			Option Price:	6.17	
Exercise:	European			Percent Price:	0.06	
Call/Put:	Call			Total Price:	617,212.95	
Direction:	<input checked="" type="radio"/> Buy <input type="radio"/> Sell			Delta (%):	0.54	
Strike:	100.00			Delta (shares):	54,283.91	
Strike (%):	100.00 %			Cash Delta:	5,428,390.98	
Quantity:	100,000			Gamma (\$):	266,256.63	
Expiry:	21-05-2016			Vega (\$):	19,695.70	
Time to expiry:	90 days			Theta (\$):	-12,943.78	
Model:	BlackScholes			Rho (\$):	11,863.18	
Volatility:	30.00 %					
Int.Rate:	2.00 %					
<button>Add to portfolio</button>						

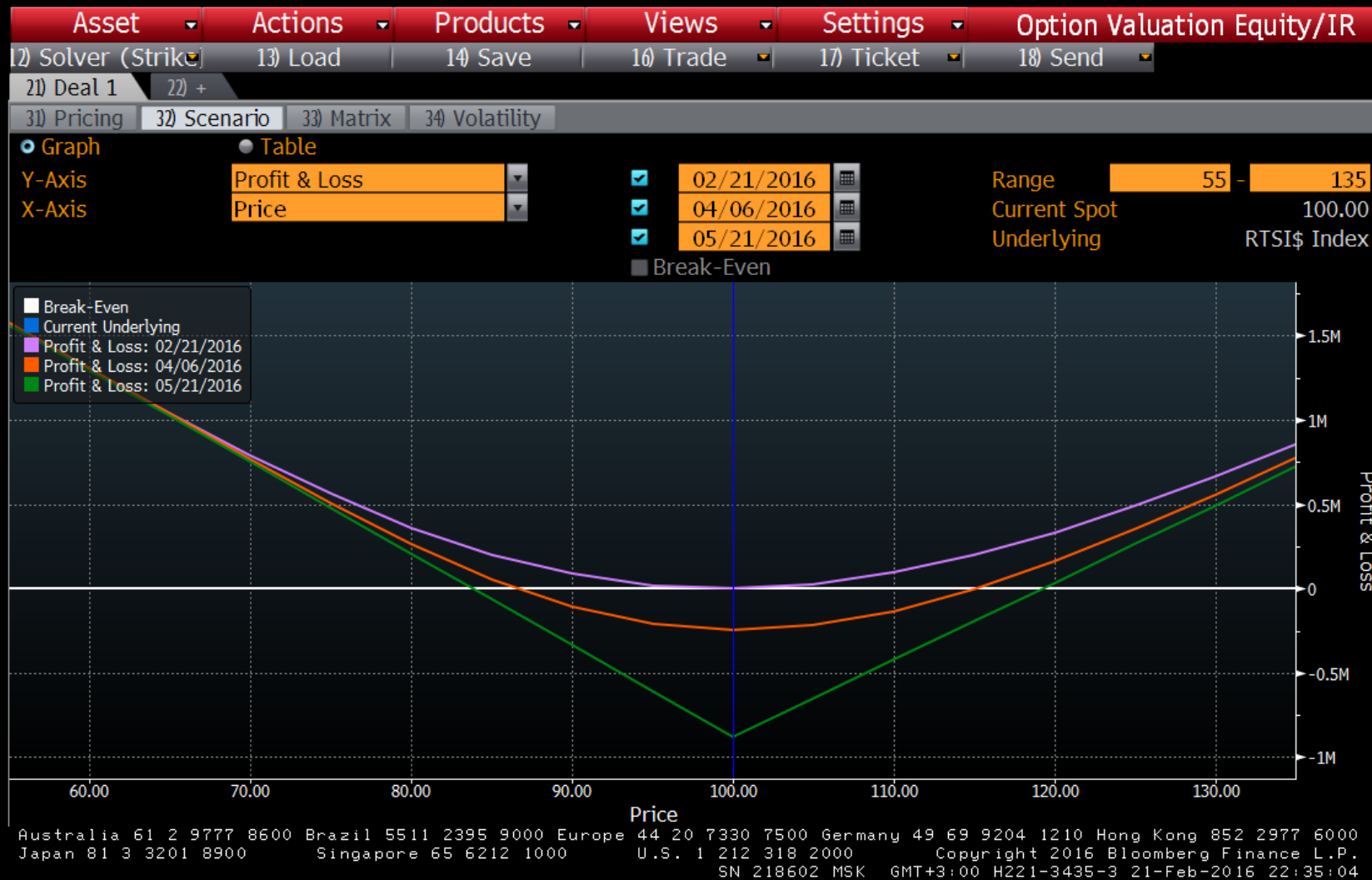


Naked call option price sensitivity

# Delta hedging (explanation)

From pricer window we see that BUYING 100'000 call options in terms of 'DELTA RISK' is equivalent **AT THIS PARTICULAR MOMENT** to BUY 54'284 shares of underlying asset.

So to offset MARKET RISK or BETA RISK or DELTA RISK (which is equivalent) we need to sell 54'284 shares. Let's see on PL profile of delta hedged position.



Hedged call option price sensitivity

# Delta hedging (continuation)

We see that delta hedged position is almost symmetrical on equity upside and downside.

**BUT** why when we buy CALL and hedge it we make more money on the downside than on the upside?

Let's do same exercise with PUT (compare NAKED and HEDGED positions)

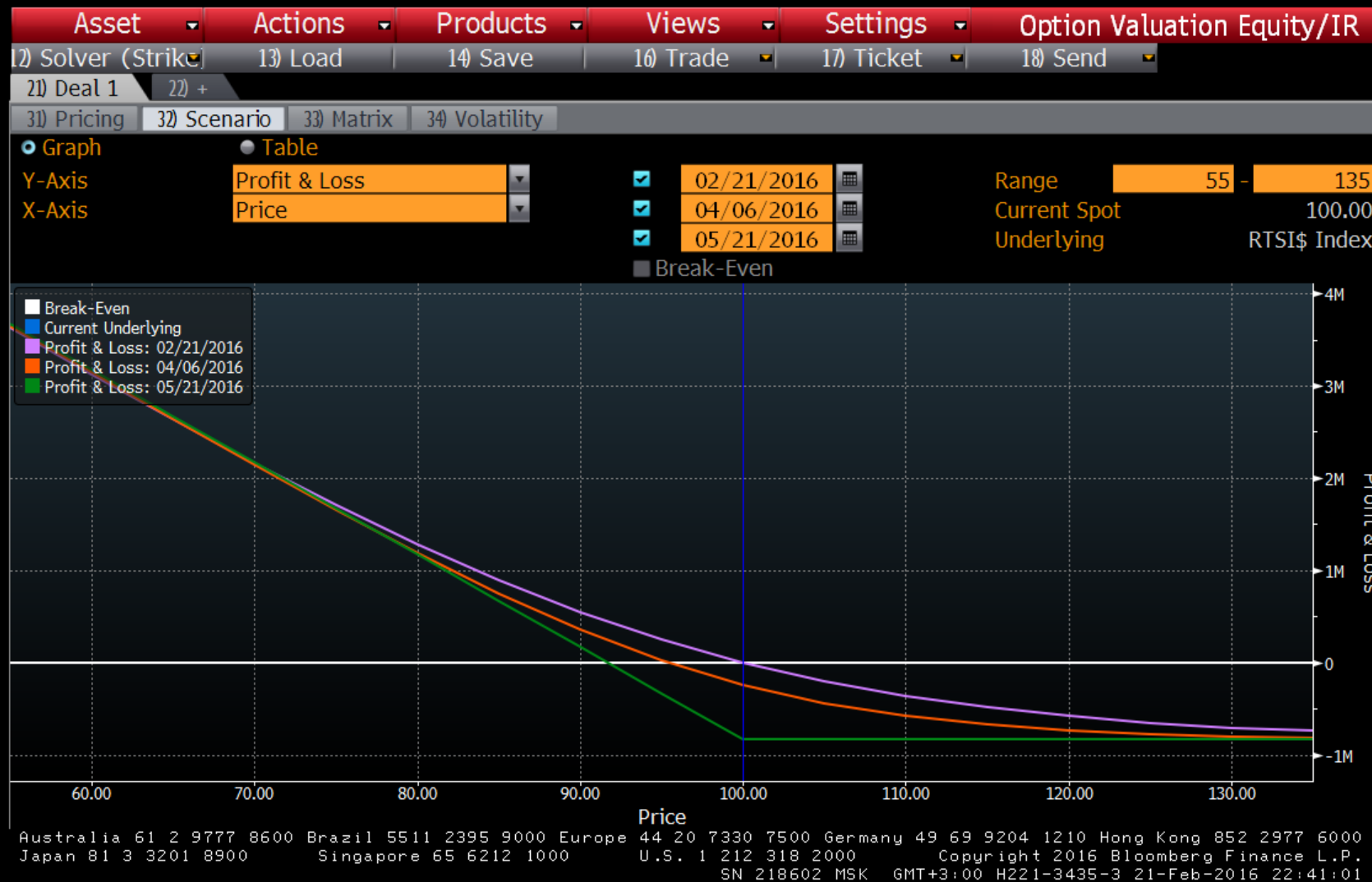
# Delta hedging (put)

Underlying:	BASE	Underlying Price:	100.00	Valuation date:	21-02-2016	Vc
OPTION PARAMETERS			RESULTS OF CALCULATION			
Style:	Vanilla		Option Price:	5.68		
Exercise:	European		Percent Price:	0.06		
Call/Put:	Put		Total Price:	568,019.28		
Direction:	<input checked="" type="radio"/> Buy <input type="radio"/> Sell		Delta (%):	-0.46		
Strike:	100.00		Delta (shares):	-45,716.09		
Strike (%):	100.00 %		Cash Delta:	-4,571,609.02		
Quantity:	100,000		Gamma (\$):	266,256.63		
Expiry:	21-05-2016		Vega (\$):	19,695.70		
Time to expiry:	90 days		Theta (\$):	-10,953.62		
Model:	BlackScholes		Rho (\$):	-12,673.06		
Volatility:	30.00 %					
Int.Rate:	2.00 %					
Add to portfolio						

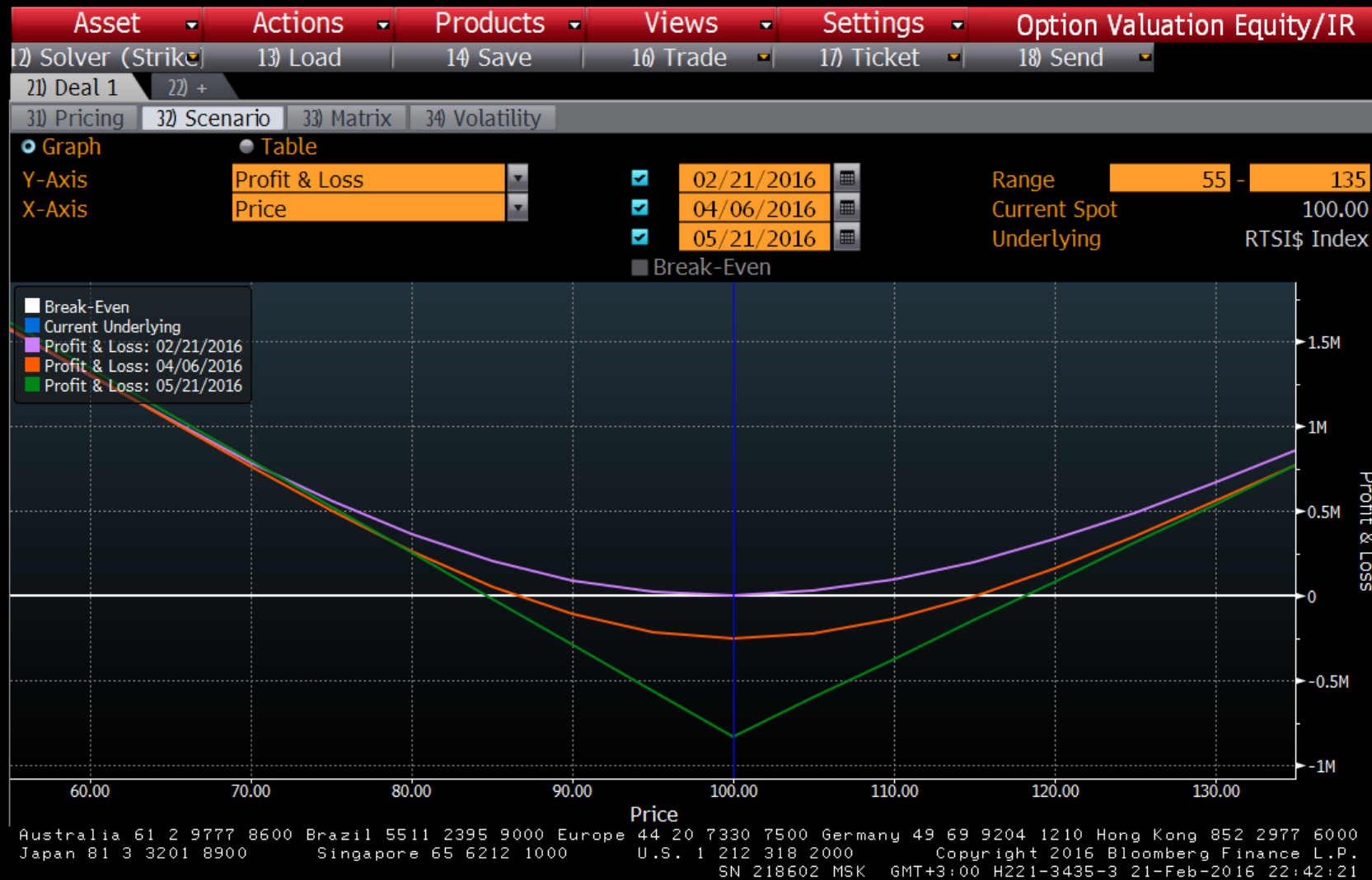
Buying 100 000 ATM Puts  
Is equivalent to selling ~45k  
underlying shares.

So to hedge it we need  
To BUY it on the market.





Naked putoption price sensitivity



Hedged put option price sensitivity

# Delta hedging (continuation)

**Home-task:** Why we make more money on the downside when buying options, than on the upside?

# We're ready to make some code (было)

Option Pricing Model

OPTION PARAMETERS

Underlying: BASE

Underlying Price: 100

Style: Knockin

Exercise: European

Call/Put: Call

Direction: ☒ Buy ☐ Sell

Strike: 100

Strike (%): 1

Quantity: 1

Valuation date: 17-Feb-16

Expiry: 17-May-16

Time to expiry: 90.00:00:00

Model: BlackScholes

Volatility: 0.3

Int.Rate: 0.05

Compute

Add to portfolio

COMPUTED PARAMETERS

Name	Value
Option Price	0
Delta	0
Cash Delta	0
Gamma	0
Cash Gamma	0
Vega	0
Cash Vega	0
Theta	0
Cash Theta	0
Rho	0
Cash Rho	0

OPTIONS PORTFOLIO

Instrument	Side	Quantity	UnderlyingPrice	Style	ExerciseType	Type	Model	Se
BASE	Sell	1	100	Knockin	European	Call	BlackScholes	10
BASE	Buy	1	100	Knockin	European	Call	BlackScholes	10

PORTFOLIO GREEKS

Cash Delta :	0	Cash Gamma :	0
Cash Vega :	0	Cash Theta :	0
Cash Rho :	0		

# We're ready to make some code (стало)

Option Pricing Model

Underlying: BASE Underlying Price: 100.00 Valuation date: 21-02-2016 Vol Shift: 0.00 % Rate Shift: 0.00 %

OPTION PARAMETERS

Style: Vanilla Exercise: European Call/Put: Call Direction: ☒ Buy ☐ Sell Strike: 100.00 Strike (%): 100.00 % Quantity: 1 Expiry: 21-05-2016 Time to expiry: 90 days Model: BlackScholes Volatility: 30.00 % Int.Rate: 5.00 %

Add to portfolio

RESULTS OF CALCULATION

Option Price: 6.53 Percent Price: 0.07 Total Price: 6.53 Delta (%): 0.56 Delta (shares): 0.56 Cash Delta: 56.25 Gamma (\$): 2.65 Vega (\$): 0.20 Theta (\$): -0.14 Rho (\$): 0.12

P&L : 0.00 Delta : 0.00 Cash Delta : 0.00 Cash Gamma : 0.00 Cash Vega : 0.00 Cash Theta : 0.00 Cash Rho : 0.00

OPTIONS PORTFOLIO

#	Type	Side	Quantity	Strike	TimeToExpiry	Value	Cost price	PL	Delta (\$)	Gamma (\$)
---	------	------	----------	--------	--------------	-------	------------	----	------------	------------

# Reading List

1. J.C. Hull – Options, Futures, and Other Derivatives
2. N.Taleb – Dynamic Hedging
3. Espen Haug – Option Pricing Formulas

# Homework

1. Practice
  1. *Modify Black-Scholes for dividends paying stocks*
2. Small essays (with graphical illustration)

# Homework

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  1. *Modify Black-Scholes for dividends paying stocks*
  2. *Implement Binomial pricing model*
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# Homework

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  3. *Implement Trinomial pricing model*
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  4. *Implement Explicit Finite Difference pricing model*
2. Small essays (with graphical illustration)

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2. Small essays (with graphical illustration)
  1. *Volatility shifts impact on options portfolio*
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  3. *Example of options position where Vega and Gamma have different signs*

# Homework

## 1. Practice:

1. *Modify Black-Scholes for dividends paying stocks*
2. *Implement Binomial pricing model*
3. *Implement Trinomial pricing model*
4. *Implement Explicit Finite Difference pricing model*

## 2. Small essays (with graphical illustration)

1. *Volatility shifts impact on options portfolio*
2. *Interest rate impact on options portfolio*
3. *Example of options position where Vega and Gamma have different signs*
4. *When delta hedging portfolio consisting of one bought option and underlying hedges is it possible to loose more than premium paid for that option?*

# Homework

## 1. Practice (70%)

1. (5%) *Modify Black-Scholes for dividends paying stocks*
2. (15%) *Implement Binomial pricing model*
3. (20%) *Implement Trinomial pricing model*
4. (30%) *Implement Explicit Finite Difference pricing model*

## 2. Small essays (with graphical illustration) (30%)

1. (5%) *Volatility shifts impact on options portfolio*
2. (5%) *Interest rate impact on options portfolio*
3. (10%) *Example of options portfolios where Vega and Gamma have different signs*
4. (10%) *When delta hedging portfolio consisting of one bought option and underlying hedges is it possible to loose more than premium paid for that option?*

Deadline: 5-mar (upload @ edu.kb9.co.uk) Minimum : 65% (2/3)

# Next lecture

Date: 7-mar-2016

Topics:

1. Pricing & models:
  1. *path dependent options and other “vanilla” exotics*

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  1. *Stress testing portfolio*
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3. Practice:
  1. *Preparing framework for market making in options*

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## Topics:

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  2. *Notes on risks and risk-limits of options trading*
  3. ***Volatility surface of exchange traded options***
3. Practice:
  1. *Preparing framework for market making in options*
  2. *Automation of delta-hedging*



**KEEP  
CALM  
AND  
QUANT  
ON**