BRSU

Neural Networks
Assignment 2

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1 Read chapter 2 from Haykin's book until 2.13 (leaving out Statistical learning theory to end of chapter) and summarize or sketch your insights in mind-map or an outline or a summary.

See figure 1.1.

2 Design a perceptron that computes the following Boolean function:

2.1 $f(x_1, x_2) = NOT(x_1 AND x_2)$

A single layer perceptron displays a simple linar function. For this problem it looks like:
$STEP(w_1 x_1 + w_2 x_2 + w_0) = y$
For the given functions the following equations have to hold true:
$w_1 + w_2 < -w_0$
$w_1 > -w_0$
$w_2 > -w_0$
$0 > -w_0$

This is true for $w_0 = 0.2, w_1 = w_2 = -0.15$
(See figure 2.1)

2.2 $f(x_1, x_2, x_3) = (x_1 AND x_2) OR x_3$

The SLP function does look like:
$STEP(w_1 x_1 + w_2 x_2 + w_3 x_3 + w_0) = y$

The following equations have to hold true:
$w_1 + w_2 + w_3 > -w_0$
$w_2 + w_3 > -w_0$
$w_1 + w_3 > -w_0$
$w_3 > -w_0$
$w_1 + w_2 > -w_0$
$w_1 < -w_0$
$w_2 < -w_0$

This is true for $w_0 = -0.2, w_1 = w_2 = 0.15, w_3 = 0.3$
(See figure 2.2)

3

3.1 A basic limitation of the perceptron is that it cannot implement the EXCLUSIVE OR function. Explain the reason for this limitation.

For XOR, the following equations have to be true:
$w_1 + w_2 < -w_0$
$0 < -w_0$
$w_1 > -w_0$
$w_2 > -w_0$

From these equations:
$w_0 > 0 \rightarrow w_1 > 0 \wedge w_2 > 0$
$w_1 + w_2 < -w_0 < w_1 \wedge w_1 + w_2 < -w_0 < w_2$
$\rightarrow w_1 < 0 \wedge w_2 < 0$

This contradiction shows that XOR is not solvable using a Single Layer Perceptron.

3.2 Show that neural networks with one hidden layer can describe all Boolean functions.

Every boolean function can be written in KNF, i.e. a conjunction of disjunctions.
The connection from the input layer to the hidden layer can be seen as n SLPs, where n is the number of neurons in the hidden layer. So it would be possible to let each of those SLPs

represent one disjunction.
The connection from the hidden layer to the single output neuron can again be seen as a single SLP.
This SLP is able to represent the conjunction of all former disjunctions.
Thus a network with one hidden layer is able to describe any boolean function because any boolean function can be written as a conjunction of disjunctions.

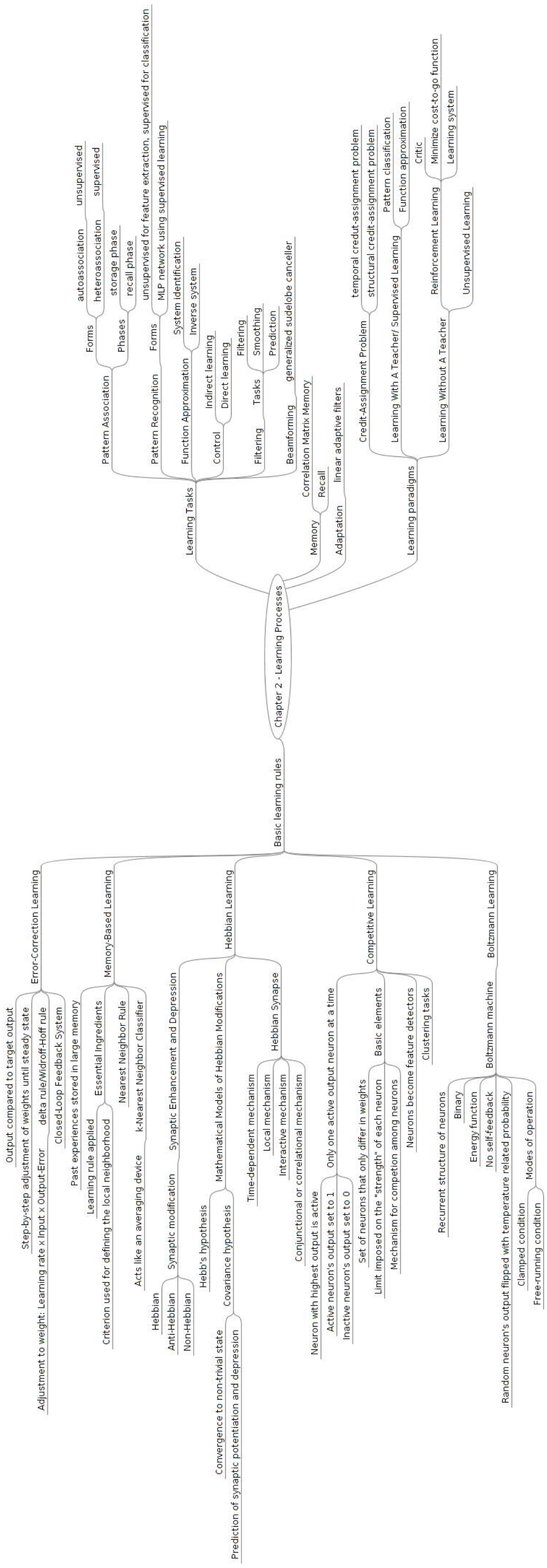

Figure 1.1: Mindmap

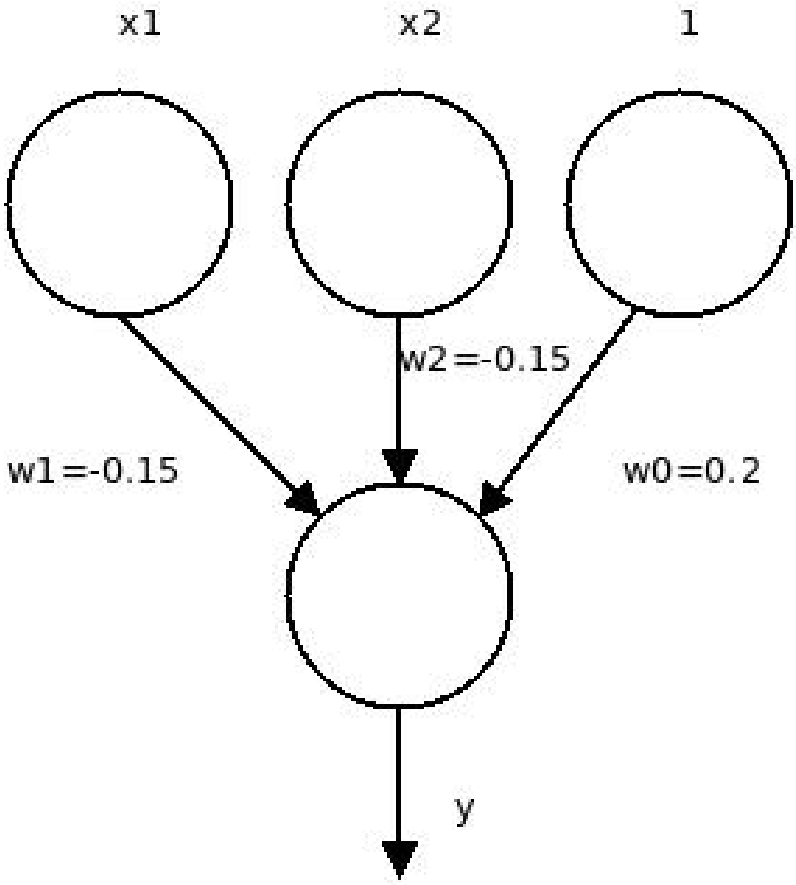

Figure 2.1: NotAnd

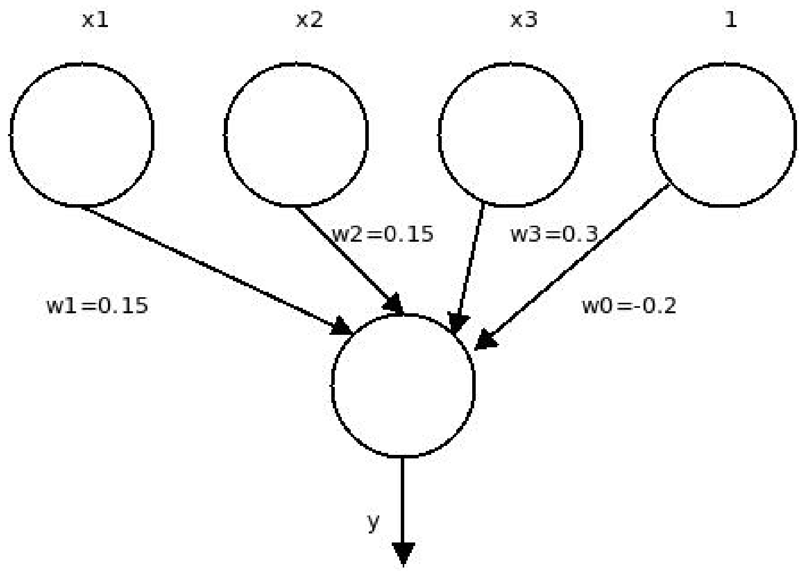

Figure 2.2: AndOr