UNIVERSITY OF ALBERTA

# Dark-Hex:
# An Imperfect Information Game

A thesis submitted in partial fulfillment
of the requirements for the degree of

**Master's in Computing Science**

**University of Alberta**

March 12, 2021

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## Abstract

Dark-Hex is the imperfect information version of the game Hex. In this work, we have examined the game's interesting properties and provided the solution to some of the small board sizes. We were mainly interested in an approximate solution and a definitive player. We got Nash solution on small boards using pure strategy LP on different game modes. This allowed us to explore and analyze the properties of the different versions of the games we describe here. Next, we evaluated Sequence-LP on different board sizes, the results showed that for bigger board sizes we needed more approximate solutions. For the last part of our evaluation, we worked with the well-known algorithm for partial information environments; Counterfactual Regret Minimization. We did analyze vanilla CFR alongside evolved versions of it, RCFR, f-RCFR. We lastly present an increment to f-RCFR ¡appendName¿ ... ¡appendDifferences¿. The end of the thesis is our player. We developed a state-of-the-art player (**not sure if we can call it state of the art as there is no player exist for the domain at the moment**). In this chapter, we have trained some classical Reinforcement Learning players, vanilla CFR, RCFR, f-RCFR, ¡appendName¿ and some other popular algorithms (**Should include some combination etc. as well, I cannot name what exactly as I need to work on them before saying anything**). We got the benchmark by setting a league-like system and constantly competing with the players. Our player ¡appendName¿ got the top place **like this...**.

# 1 Notes

## 1.1 Dark Hex Versions

Dark Hex is a very poorly researched topic as far as I see. First thing to do seems to be to bring out the interesting points of the game, why is it worth researching on, what properties lays underneath? The rules of the game is hardly described anywhere, therefor it seems like we have a quite wide definition window. We came up with multiple versions of the game, we are not sure what will be the version we investigate at the moment. I will list the current versions discussed, and look into them in detail when deciding the interesting properties of the game. We might look into more than one versions in this thesis. It's yet to be decided.

### 1.1.1 Classic Dark Hex (Kriegspiel Hex)

Dark Hex is the extension for Kriegspiel Chess on Hex game. It turns the perfect information game of Hex to its imperfect version. The rules of winning and losing still stands as Classic Hex. What changes is that the players are not exposed to opponents move information. So for a player the current state is his/her own stones on the board, the number of moves made by opponent, cells where there is no known stone, and opponent stones where the player tried to make a move and got rejected.
**Rejection:** If an opponent stone is on the position the player is trying to play on it will result in rejection. Rejection is not a terminal for the current player, meaning the player still needs to make a move, only this time he/she has information on one more of the opponents stones.
**Example Game:** TODO

### 1.1.2 Abrupt Dark Hex

Abrupt or short Dark Hex is also an interesting version of Dark Hex. The difference is that instead of rejection Abrupt DH has collisions. Other than that the two games follow same properties.
**Collision:** If an opponent already has a stone on the location the player tries to make a move, collision happens, the player who tried to make a move loses his/her turn.

### 1.1.3 Noisy Dark Hex

Noise is an extension for the other versions of DH. The game follows exact version of Abrupt DH or Classical DH with the exception of noise.
**Noise:** Knowledge of the attempted move is publicly available for both players instead of only the player who tried to make the move. i.e. Player 1 makes a move on a2, there is a Player 2 stone on a2 therefor collision or rejection happens (depending on the type of the game), Player 1 knows the exact location of where one of Player 2's stones are is, Player 2 hears 'a noise' meaning that he/she will have the information that Player 1 tried to make a move on one of his/her stones, but won't know which location that is.

### 1.1.4 Flash Dark Hex

Same as Noisy Dark Hex except that instead of a noise, flash takes place.
**Flash:** Instead of noise this time opponent gets a 'flash' for the collision or rejections. Flash's are shown on exact location where collision or rejections has happened.

# 2 Abrupt Dark Hex Practices

Here is the full pay-off matrix. I think 0-1 without any context looks very much confusing. As far as I can see if the first move of the player B is not blocking the first players second move strategy it's a win otherwise it's a loss. So; Couple other observations:

1. Diagonal is always first player win, since the strategies are the same second player will get rejected both moves and lose.

2. Pattern is the same for each strategy, first players second move is the only important move. If the move is

| | 01 | 02 | 03 | 10 | 12 | 13 | 20 | 21 | 23 | 30 | 31 | 32 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 01 | 1, 0 | 1, 0 | 1, 0 | 0, 0 | 0, 0 | 0, 1 | 1, 0 | 1, 0 | 1, 0 | 1, 0 | 1, 0 | 1, 0 |
| 02 | 0, 0 | 0, 0 | 0, 0 | 0, 0 | 0, 0 | 0, 1 | 0, 0 | 0, 0 | 0, 0 | 0, 1 | 0, 0 | 0, 0 |
| 03 | 0, 0 | 0, 0 | 0, 0 | 0, 0 | 0, 1 | 0, 0 | 0, 0 | 0, 1 | 0, 0 | 0, 0 | 0, 0 | 0, 0 |
| 10 | 0, 0 | 0, 0 | 0, 0 | 1, 0 | 1, 0 | 1, 0 | 1, 0 | 1, 0 | 1, 0 | 1, 0 | 1, 0 | 1, 0 |
| 12 | 1, 0 | 1, 0 | 1, 0 | 1, 0 | 1, 0 | 1, 0 | 0, 0 | 0, 0 | 0, 0 | 1, 0 | 1, 0 | 1, 0 |
| 13 | 0, 0 | 0, 1 | 0, 0 | 0, 0 | 0, 0 | 0, 0 | 0, 1 | 0, 0 | 0, 0 | 0, 0 | 0, 0 | 0, 0 |
| 20 | 0, 0 | 0, 0 | 0, 0 | 0, 0 | 0, 0 | 0, 1 | 0, 0 | 0, 0 | 0, 0 | 0, 0 | 0, 1 | 0, 0 |
| 21 | 1, 0 | 1, 0 | 1, 0 | 0, 1 | 0, 0 | 0, 0 | 1, 0 | 1, 0 | 1, 0 | 1, 0 | 1, 0 | 1, 0 |
| 23 | 1, 0 | 1, 0 | 1, 0 | 1, 0 | 1, 0 | 1, 0 | 1, 0 | 1, 0 | 1, 0 | 0, 0 | 0, 1 | 0, 0 |
| 30 | 0, 0 | 0, 0 | 0, 0 | 0, 0 | 0, 1 | 0, 0 | 0, 0 | 0, 1 | 0, 0 | 0, 0 | 0, 0 | 0, 0 |
| 31 | 0, 0 | 0, 1 | 0, 0 | 0, 0 | 0, 0 | 0, 0 | 0, 1 | 0, 0 | 0, 0 | 0, 0 | 0, 0 | 0, 0 |
| 32 | 1, 0 | 1, 0 | 1, 0 | 1, 0 | 1, 0 | 1, 0 | 0, 0 | 0, 1 | 0, 0 | 1, 0 | 1, 0 | 1, 0 |

# 3 Strategy space for 2x2

How many possible games are there for Dark Hex games? What will be our search space? How can we make sure that we capture the bounds correctly? While discussing this all, we have come to realize that even for the smallest boards it is not visible really quick how big the search space, or game strategies will be. Easiest way to grasp a concept is to go with an example, and tackle a problem along the way. So here we will discover the number of possible strategies on a 2 by 2 board.

## 3.1 Without end game rules

What we are considering is not an optimal play, so that we can evaluate every possible pure strategy rather than dominant strategies. For the sake of seeing the bigger picture, we will first make an assumption; a game is not over unless the board is filled. On a 2 by 2 board this means there will be exactly 4 moves executed.

**First move:** $b_1$

First move has no rejection possibility therefore the move will for sure go through. Since we do not care about the win at the moment (we only care about filling the board) we won't be checking the cells separately as advantageous or not. We also will ignore isomorphism. These conditions give us equally important 4 cells on the board. So Black first move can be on any of these cells, and since all of them has the same value for us, we will examine only one, and results will multiply accordingly. We will call first Black move $b_1$. Let's call Black's information state $B$ and White's information state $W$.

$$B = \{b_1\}$$
$$W = \{\}$$

**Second move:** $w_1$

Second move divides into two; if white hits the $b_1$ or not. They both will have three possible moves (hits $b_1$, and moves; or moves one of the other three cells). The information state differs depending on the rejection happening or not, therefore the two moves will have different continuations, let's call the strategy with rejection $S_r$ and without rejection $S_n$.

$$B_{S_r} = \{b_1\}$$
$$W_{S_r} = \{b_1, w_1\} \qquad \text{or}$$
$$W_{S_n} = \{w_1\}$$

**Third move:** $b_2$

Third move is exactly the same as the second move, this time for black. Black will make it's move either finding $w_1$ or making a move directly. Gives us 2 move possibilities for each.

$$
\begin{aligned}
B_{S_r(1)} &= \{b_1, b_2\} && \text{or} \\
B_{S_r(2)} &= \{b_1, w_1, b_2\} \\
W_{S_r} &= \{b_1, w_1\}
\end{aligned}
$$

**Last move:** $w_2$

Last move is where it gets a little more complicated. Depending on what white's prior knowledge is, we have to have separate the results. There are 5 possible $W$'s for the strategy $S_n$ and 2 for $S_r$.

$$
\begin{aligned}
W_{S_n} &= \{w_1, w_2\} && \text{or} \\
W_{S_n} &= \{w_1, b_1, w_2\} && \text{or} \\
W_{S_n} &= \{w_1, b_2, w_2\} && \text{or} \\
W_{S_n} &= \{w_1, b_1, b_2, w_2\} && \text{or} \\
W_{S_n} &= \{w_1, b_2, b_1, w_2\}
\end{aligned}
$$

$$
\begin{aligned}
W_{S_r} &= \{b_1, w_1, w_2\} && \text{or} \\
W_{S_r} &= \{b_1, w_1, b_2, w_2\}
\end{aligned}
$$

So we have;

$$
\begin{aligned}
&4 \times 3 \times (2 \times 2) \times 5 + \\
&4 \times 3 \times (2 \times 2) \times 2 = 336
\end{aligned}
$$

Here we have two products; one for $S_n$ and one for $S_r$, which are the same for the prior part since the only difference is on the second move where white either hits the black stone or not, and there are 3 different moves for both cases. We have resulted that 336 is the maximum number of games if we assume that the game doesn't end until the board is full. Now let's prune this tree by adding the end game rules.

## 3.2 With end game rules

It is a bit more manual to calculate the number of games when including the rules. We will examine the moves considering isomorphic properties, which will allow us to calculate the games as a whole when we can benefit. For the initial move for example we have 4 moves, but 2 isomorphic positions (0 is the same as 3, and 1 is the same as 2). So for the first move we will only consider two positions -close corner(2) and far corner(0)-.

### 3.2.1 First move to far corner

We have two far corners, and they are isomorphic. That's why investigating one and multiplying the result by 2 is going to suffice.

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### 3.2.2 First move to close corner

Same goes here, we will examine only the cell 2.

For the second move, white has two different options; either play to the virtual connection black has, or play next to the black move (and lose). So if the branch continues on VC the game might go on and white still has a probability to win, otherwise the game ends on 3 moves for sure since both moves gives black the connection.

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