

Travelling Salesman Problem

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1 Introduction

The travelling salesman problem asks the following question: "Given a list of cities and the distances between each pair of cities, what is the shortest possible route that visits each city exactly once and returns to the origin city?" The objective of this project is to find the shortest possible path among a set of given points (along with their coordinates) that traverses every single point exactly once, and then returns back to the starting point, thus attempting to solve the travelling salesman problem. This is done with the help of an algorithm, used to find the optimal path. The algorithm is explained hand-in-hand with a live example as that will be much easier to understand.

2 Working of the Algorithm

1. We assume that n distinct points are fed by the user to the algorithm, along with their coordinates.
2. Then, we create an $n \times n$ matrix which contains the distances from each point to every other point as the elements. (As in, the element $M[a, b]$, where M is the matrix, contains the distance of a from b .) Note that distance from one point to itself is considered as infinity, hence an entry of $M[i, i]$ is considered infinity for all i in n . A set of values in the example case has been taken and formed into a matrix as shown here

$$\begin{bmatrix} \infty & 20 & 30 & 10 & 11 \\ 15 & \infty & 16 & 4 & 2 \\ 3 & 5 & \infty & 2 & 4 \\ 19 & 6 & 18 & \infty & 3 \\ 16 & 4 & 7 & 16 & \infty \end{bmatrix}$$

3. Now, we "reduce" this matrix. This is done by finding the minimum element from across each row and subtracting them from each of the elements in the corresponding row. Then we repeat this for every column. The sum

of all the removed elements is called the reduced cost. The resultant matrix achieved is the first reduced matrix. It shall be referred to as C. The reduced cost as calculated is 25. The reduced matrix has been shown here

$$\begin{bmatrix} \infty & 10 & 17 & 0 & 1 \\ 12 & \infty & 11 & 2 & 0 \\ 0 & 3 & \infty & 0 & 2 \\ 15 & 3 & 12 & \infty & 0 \\ 11 & 0 & 0 & 12 & \infty \end{bmatrix}$$

4. Now, it is assumed that the path starts from one point. In this case, we consider point 1 to be the starting point (With regards to length of path, starting point won't matter).
5. First, consider the path $1 \rightarrow 2$. Now, to find the cost of node (or point) 2, we need to reduce the first reduced matrix in a particular way:
 - (a) First, make all the elements in the row of the starting point and the column of the ending point as infinity. So, all the elements in the row 1 and column 2 are made infinity.
 - (b) Then, as once we come from a point to another, it can't go back, the element of the order (from_point, to_point) must also be set to infinity. In this case, C (1,2) is set to infinity.
 - (c) The reduced matrix for path $1 \rightarrow 2$ is given here

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 11 & 2 & 0 \\ 0 & \infty & \infty & 0 & 2 \\ 15 & \infty & 12 & \infty & 0 \\ 11 & \infty & 0 & 12 & \infty \end{bmatrix}$$

- (d) Finally, the reduced cost for this node shall be calculated according to the formula:

$$M[from_node, to_node] + r$$

where r is the reduced cost of the current matrix. So, reduced cost for node 2 in this example is: $10 + 0 = 10$.

6. Similarly, the reduced matrix and the corresponding cost of reduction is found for all the points from the first node. In our example, the reduced costs for node 3,4 and 5 are 27, 0 and 6 respectively.
7. Then, the node with least cost of reduction is found, and that is assumed to be the next node in the optimal path. Hence, in our example, the point 4 will be next node.
8. Now, from the next node, the possible paths are further extended to possible points. Then, just like in the previous case, reduced matrixes are taken cost of reductions are found.

- (a) From node 4, the path can go to nodes 2 or 3. This matrix is then reduced, in similar manner as the previous case. The cost of reduction here is $C(\text{from_node}, \text{to_node}) + r$, where r is the cost of reduction of to_node .
 - (b) For the path $4 \rightarrow 2$, The elements of the 4th row and the 2nd column are turned to infinity. The elements $C(2,1)$ and $C(2,4)$ are also turned to infinity. Then, the cost of reduction is found. The cost of reduction of node 4 to node 2,3 and 5 happens to be 3, 25 and 11 respectively.
9. Again, the node with the least cost of reduction is chosen as the next point in the path. Hence, the path now becomes: $1 \rightarrow 4 \rightarrow 2$.
 10. Next, the similar method is followed to find the next node (from 3 and 5), by reducing the matrix and hence finding the cost of reduction for the remaining points.
 11. So, the cost of reduction for node 3 and 5 respectively.
 12. Hence, the next node would then be 5 and the node after that automatically becomes 3.
 13. The path eventually returns to where it started, so it returns to 1.

3 Conclusion

Hence, the most optimal path according to our algorithm is: $1 \rightarrow 4 \rightarrow 2 \rightarrow 5 \rightarrow 3 \rightarrow 1$. In this way, the algorithm is able to provide the optimal path for n number of inputted points