Tarea 3

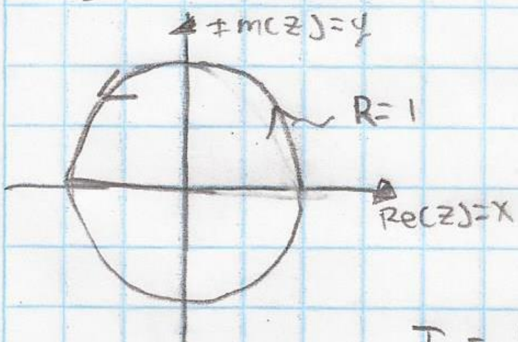
Meza Vargas Brandon David - 2CM11

Calcula la integral

$$I = \int_C (z^2 + z\bar{z})\,dz$$

Donde C es el arco de circunferencia $|z| = 1$ con $(0 \leq \theta \leq \pi)$

Sol.



$$z(t) = R\,e^{it} = e^{it}$$
$$dz = i e^{it}\,dt \qquad 0 \leq t \leq \pi$$

Así tenemos:

$$I = \int_C (z^2 + z\bar{z})\,dz = \int_0^{\pi} \left((e^{it})^2 + e^{it}(e^{-it}) \right) i e^{it}\,dt$$

$$I = \int_0^{\pi} \left((e^{2it}) + 1 \right) i e^{it}\,dt = \int_0^{\pi} \left(i e^{3it} + i e^{it} \right) dt$$

$$= \left[\frac{\cancel{i} e^{3it}}{3\cancel{i}} + \frac{\cancel{i} e^{it}}{\cancel{i}} \right]_0^{\pi} = \frac{e^{3it}}{3} + e^{it} = \left[\frac{e^{3i\pi}}{3} + e^{i\pi} \right] - \left[\frac{1}{3} + 1 \right]$$

$$= \left[\frac{\cos(3\pi) + \cancelto{0}{i\,\text{sen}\,3\pi}}{3} + \cos\pi + i\,\text{sen}\,\pi \right] - \frac{4}{3}$$

$$= -\frac{1}{3} - 1 - \frac{4}{3} = -\frac{8}{3} \quad \therefore \quad \boxed{I = -\frac{8}{3}}$$