Tarea 2

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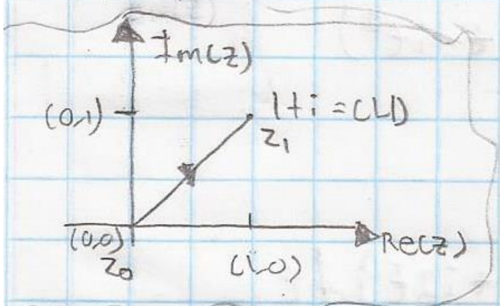
Evalúa $I = \int_C (1+i-2\bar{z})\,dz$

por las líneas que conectan los puntos $z_1 = 0$ y $z_2 = 1+i$

A. Por la recta

B. Por la parábola $y = x^2$

Sol. A.



Parametrizando

$x = x(t) = t \Rightarrow y = y(t) = t \qquad 0 \le t \le 1$

$z(t) = x(t) + i\,y(t) \qquad a \le t \le b$

$x = x(t) = t$ y $y = y(t) = t$ si $0 \le t \le 1$

$z = z(t) = t + it \Rightarrow dz = (1+i)\,dt$

Como $f(z) = (1+i-2\bar{z}) = 1+i-2(x-iy) = 1+i-2(t-it)$

$= 1+i-2t+2it = -2t+1+i(1+2t)$

tenemos así:

$$I = \int_0^{1+i} (1+i-2\bar{z})\,dz = \int_0^1 \left[-2t+1+i(1+2t)\right]\left[1+i\right]dt$$

$$= \int_0^1 -2t-2ti+1+i+i(1+2t)+i^2(1+2t)\,dt$$

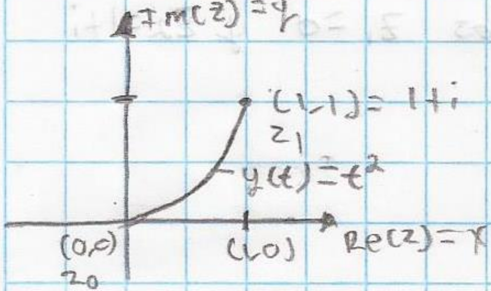
$$= \int_0^1 (-2t \cancel{-2ti} \cancel{+1} + i + i + i\cancel{2t} \cancel{-1} - 2t)\,dt = \int_0^1 (-4t+2i)\,dt$$

$$= -2t^2 + 2it \Big|_0^1 = (-2(1)+2i(1)) - (-2(0)+2i(0))$$

$= -2+2i \quad \therefore \quad \underline{I = -2+2i}$ //

B. Por la parábola $y = x^2$

Sol.



$x(t) = t, \quad y(t) = t^2, \quad 0 \leq t \leq 1$

$z = z(t) = x(t) + iy(t) = t + it^2 \quad 0 \leq t \leq 1$

$dz = (1 + i2t)\,dt$

$f(z) = 1 + i - 2\bar{z} = 1 + i - 2(x - iy) = 1 + i - 2(t - it^2)$

$= 1 + i - 2t + i2t^2 = 1 - 2t + i(1 + 2t^2)$

Así tenemos

$$I = \int_0^1 (1 + i - 2\bar{z})\,dz = (1 - 2t + i(1 + 2t^2))(1 + i2t)\,dt$$

$$= \int_0^1 (1 + i2t - 2t - i4t^2 + i(1 + 2t^2) + i^2 2t(1 + 2t^2))\,dt$$

$$= \int_0^1 (1 + i2t - 2t - i\cancel{4}t^2 + i + i2\cancel{t}^2 - 2t - 4t^3)\,dt$$

$$= \int_0^1 (1 + i2t - 4t - i2t^2 + i - 4t^3)\,dt = \left[t + it^2 - 2t^2 - \frac{i2t^3}{3} + it - t^4 \right]_0^1$$

$$= 1 + i - 2 - \frac{i2}{3} + i - 1 = -2 + 2i - \frac{i2}{3} = -2 + i\left(2 - \frac{2}{3}\right)$$

$$= -2 + \frac{i4}{3} \quad \therefore \; I = -2 + \frac{i4}{3}$$