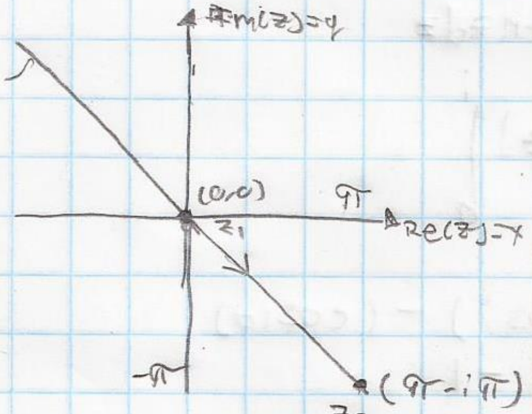
Meza Vargas Brandon David - 2CM11
Tarea 4

Evalúe

$$I = \int_C e^{\bar{z}}\, dz$$

Donde C es el segmento de la recta $y = -x$ que conecta a los puntos $z_1 = 0$ y $z_2 = \pi - i\pi$



$x = x(t) = t \Rightarrow y = y(t) = -t \quad 0 \le t \le 1$

Sabemos

$z(t) = z_0 + t(z_1 - z_0), \quad 0 \le t \le 1$

$z(t) = t(\pi - i\pi) = t\pi - i\pi t$

$\dot{z}(t) = \pi - i\pi$

$\bar{z}(t) = t\pi + i\pi t$

Así tenemos

$$I = \int_0^1 e^{\bar{z}} dz = \int_0^1 e^{(t\pi + i\pi t)} (\pi - i\pi)\, dt = (\pi - i\pi) \int_0^1 e^{t\pi} e^{i\pi t}\, dt$$

$$= (\pi - i\pi) \int_0^1 e^{t\pi + i\pi t}\, dt = \frac{(\pi - i\pi)}{(\pi + i\pi)} \int_0^1 e^{u}\, du = \frac{(\pi - i\pi)}{(\pi + i\pi)} e^{t\pi + i\pi t} \Big|_0^1$$

$u = t\pi + i\pi t$

$du = \pi + i\pi\, dt$

$$= \frac{(\pi - i\pi)}{(\pi + i\pi)} (e^{\pi + i\pi} - 1) = \frac{\pi(1-i)}{\pi(1+i)} (e^{\pi + i\pi} - 1) = \frac{(1-i)}{(1+i)} (e^{\pi + i\pi} - 1)$$

$$\therefore I = \frac{(1-i)}{(1+i)} (e^{\pi + i\pi} - 1)$$