

Лабораторная работа 7

Департамент
Бизнеса
Технологий

ВКГ-41

$$y^2 = x^3 + x + 2 \pmod{29}$$

$$P_0 = (7, 2) \quad P = (7, 2) \quad C = 3 \quad n = 4$$

$$X_0 = (7, 2)$$

$$i = 1$$

$$x_0^{-1} = ?$$

$$x = 7 \Rightarrow x^{-1} \pmod{29} = 25 \quad y = 2$$

$$3 \cdot (25, 2) \quad \lambda = \frac{3 \cdot 25^2 + 1}{2 \cdot 2} = \frac{1876}{4} \pmod{29}$$

$$1876 \pmod{29} = 20$$

$$\lambda = 20 \cdot 22 = 440 \pmod{29} = 5 \quad y^{-1} \pmod{29} = 22$$

$$x_3 = 5^2 - 2 \cdot 25 = 25 - 50 = -25 \pmod{29} = 4$$

$$y_3 = 5 \cdot (25 - 7) - 2 = 105 - 2 = 103 \pmod{29} = 16$$

$$2P = (7, 16)$$

$$3 \cdot (25, 2) = (5(16) + (25, 2)) \quad \lambda = \frac{2 - 16}{25 - 5} = \frac{-14}{21} = 15 \cdot 21^{-1}$$

$$21^{-1} \pmod{29} = 29$$

$$\lambda = 15 \cdot 29 = 360 \pmod{29} = 12$$

$$x_3 = 12^2 - 4 - 25 = 144 - 29 = 115 \pmod{29} = 28$$

$$y_3 = 12(4 - 22) - 16 = 12 \cdot (-18) - 16 = -202 - 16 = -218 \pmod{29} = 27$$

$$X_1 = (22, 27) + (7, 2)$$

$$\lambda = \frac{2 - 27}{2 - 22} = \frac{-25}{-20} = 4 \cdot 2 = 32 \pmod{29} = 3$$

$$\lambda = \frac{4}{5} \Rightarrow 8 \cdot 11 = 88 \pmod{29} = 15$$

$$x_3 = 15^2 - 28 - 7 = 190 \pmod{29} = 10$$

$$y_3 = 15(22 - 16) - 27 = 15 \cdot 6 - 27 = 90 - 27 = 63 \pmod{29} = 5$$

$$X_1 = (10, 5)$$

$$i=2$$

$$X_1 = (11, 2) \Rightarrow X^{-1} = 11$$

$$X_1^{-1} (11, 2)$$

$$3 \cdot (11, 2)$$

$$\lambda = \frac{3 \cdot 11^2 + 1}{2 \cdot 2} = \frac{365}{16} \quad 365 \bmod 29 = 12 \quad 16^{-1} \bmod 29 = 20$$

$$\lambda = 12 \cdot 20 \bmod 29 = \underline{21}$$

$$3P = (13, 2) + (11, 2)$$

$$\lambda = \frac{2-2}{13-11} = 0 \quad X_3 = -24 = 5$$

$$Y_3 = -2 = 21 \Rightarrow 3 \cdot (11, 2) = (5, 21)$$

$$X_2 = (5, 21) + (7, 2)$$

$$\lambda = \frac{2-21}{7-5} = \frac{-19}{2} = -10 \cdot 2^{-1} \bmod 29 = 5$$

$$X_3 = 5^2 - 5 - 7 = 25 - 12 = 13$$

$$Y_3 = 5(5 - 13) - 21 = -61 \bmod 29 = 26$$

$$X_2 = (13, 26)$$

$$i=3$$

$$X = 13 \quad X^{-1} = 9 \quad X_2^{-1} = (9, 26)$$

$$3 \cdot (9, 26)$$

$$\lambda = \frac{3 \cdot 21 + 1}{2 \cdot 26} = \frac{244}{52} = 12 \cdot 52^{-1} \bmod 29 = 12$$

$$X_3 = 2^2 - 2 \cdot 9 = 4 - 18 = -14 \bmod 29 = 15$$

$$Y_3 = 2(9 - 15) - 26 = -12 - 26 = -38 = 20$$

$$2P = (15, 20)$$

$$3P = (15, 20) + (9, 26)$$

$$\lambda = \frac{26-20}{9-15} = \frac{6}{-6} = -1 = 28$$

$$X_3 = 22^2 - 15 - 9 = 760 \bmod 29 = 7$$

$$Y_3 = 22(15 - 7) - 20 = 204 \bmod 29 = 1$$

$$3 \cdot (9, 26) = (7, 1)$$

$$(7, 1) + (7, 2) = \underline{0}$$

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Apmj-