

Aula 20/09 Revisão para a prova 1 da N1

Prova 4 questões

Conteúdo:

- Revisão de filtros analógicos
- Filtros digitais
- Transformada de Laplace
- Transformada inversa de Laplace
- Transformada Z
- Transformada inversa Z

Lista de exercícios para a primeira avaliação para a N1

1. Aplique a transformada de Laplace para representar as FTs abaixo no domínio da frequência, no plano s.
 - c) $g(t) = 2 \cdot \exp(-2 \cdot t)$;
 - d) $k(t) = 2 \cdot \exp(-3 \cdot t) + 3 \cdot t$;
 - e) $k(t) = 2 \cdot \exp(-3 \cdot t) + 0.5 \cdot \exp(-2 \cdot t)$;

Solução

01ª QUESTÃO:

$$e^{-at} \xrightarrow{[L]} \frac{1}{s+a} \xrightarrow{[Z]} \frac{z}{z - e^{-at}} \xrightarrow{[Z^{-1}]} e^{-ant}$$

APLICAR A TRANSFORMADA DE LAPLACE:

c) $g(t) = 2 \cdot e^{-2t}$

$$G(s) = 2 \cdot \frac{1}{s+2} \Rightarrow \boxed{G(s) = \frac{2}{s+2}}$$

d) $g(t) = 2 \cdot e^{-3t} + 3 \cdot t$

$$G(s) = 2 \cdot \frac{1}{s+3} + 3 \cdot \frac{1}{s^2}$$

G(s)

G(s)

E)

K

V

ant

$$G(s) = \frac{2}{s+3} + \frac{3}{s^2}$$

$$G(s) = \frac{2s^2 + 3(s+3)}{s^2(s+3)} \Rightarrow G(s) = \frac{2s^2 + 3s + 9}{s^3 + 3s^2}$$

$$E) K(t) = 2 \cdot e^{-3t} + 0,5 \cdot e^{-2t}$$

$$K(s) = 2 \cdot \frac{1}{s+3} + 0,5 \cdot \frac{1}{s+2}$$

$$K(s) = \frac{2}{s+3} + \frac{0,5}{s+2}$$

$$K(s) = \frac{2(s+2) + 0,5(s+3)}{(s+2)(s+3)}$$

$$K(s) = \frac{2,5s + 4 + 1,5}{s^2 + 5s + 6}$$

$$K(s) = \frac{2,5s + 5,5}{s^2 + 5s + 6}$$

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Leista n/ prova 1

1) c)

$$e^{-at} \xrightarrow{[L]} \frac{1}{s+a} \xrightarrow{[Z]} \frac{z}{z-e^{-at}} \xrightarrow{[Z^{-1}]} e^{-ant}$$

$$G(s) = 2 \times \frac{1}{s+2} \Rightarrow \boxed{G(s) = \frac{2}{s+2}}$$

d) $G(s) = 2 \times \frac{1}{s+3} + 3 \times \frac{1}{s^2}$ \rightarrow Func. Propia

$$G(s) = \frac{2}{s+3} + \frac{3}{s^2}$$

$$G(s) = \frac{2 \cdot s^2 + 3 \cdot (s+3)}{s^2 \cdot (s+3)} \rightarrow G(s) = \frac{2s^2 + 3s + 9}{s^3 + 3s^2}$$

2) $K(s) = 2 \times \frac{1}{s+3} + 0,5 \cdot \frac{1}{s+2}$

$$K(s) = \frac{2}{s+3} + \frac{0,5}{s+2} \Rightarrow \boxed{K(s) = \frac{2,5s + 9,5}{s^2 + 5s + 6}}$$

$$K(s) = \frac{2(s+2) + 0,5(s+3)}{(s+2) \cdot (s+3)}$$

$$K(s) = \frac{2,5s + 4 + 1,5}{s^2 + 5s + 6}$$

2. Aplique a transformada de Laplace inversa para representar as Fts abaixo no domínio do tempo.

a) $F(s) = 2/(s+4)$;

c) $K(s) = 3/((s+2)*(s+3))$;

d) $H(s) = 1/(s+1) + 3/(s+2)$;

e) $F(s) = 2/((s+1)*(s+2)*(s+3))$;

$$e^{-at} \xrightarrow{[L]} \frac{1}{s+a} \xrightarrow{[Z]} \frac{z}{z-e^{-aT}} \xrightarrow{[Z^{-1}]} e^{-ant}$$

01ª QUESTÃO:

→ TRANSFORMADA DE LAPLACE INVERSA:

$$A) F(s) = \frac{2}{s+4} \Rightarrow F(s) = 2 \cdot \frac{1}{s+4}$$

$$f(t) = 2 \cdot e^{-4t}$$

$$C) K(s) = \frac{3}{(s+2)(s+3)} = \frac{A}{s+2} + \frac{B}{s+3}$$

$$A = ? \quad \frac{3 \cdot (s+3)}{(s+2)(s+3)} = \frac{A \cdot (s+3)}{s+2} + \frac{B \cdot (s+3)}{s+3} \quad | s = -2$$

$$A = \frac{3}{s+3} \quad | s = -2 \Rightarrow A = 3$$

$$\frac{3 \cdot (s+3)}{(s+2)(s+3)}$$

$$B = -$$

$$K(s)$$

$$K(s)$$

$$\rightarrow e^{-at} \\ [2-4]$$

$$\frac{3 \cdot (s+3)}{(s+2)(s+3)} = \frac{A \cdot \cancel{(s+3)}}{s+2} + \frac{B \cdot (s+3)}{s+3} \quad | s = -3$$

$$B = \frac{3}{s+2} \quad | s = -3 \Rightarrow \boxed{B = -3}$$

$$K(s) = \frac{3}{s+2} - \frac{3}{s+3} \Rightarrow K(s) = 3 \cdot \frac{1}{s+2} - 3 \cdot \frac{1}{s+3}$$

$$k(t) = 3 \cdot e^{-2t} - 3 \cdot e^{-3t}$$

$$\begin{array}{c}
 \xrightarrow{[L^{-1}]} \\
 \xleftarrow{ab} e \xrightarrow{[L]} \frac{1}{s+a} \xrightarrow{[Z]} \frac{z}{z - e^{-at}} \xrightarrow{[Z^{-1}]}
 \end{array}$$

01ª QUESTÃO:

→ TRANSFORMADA DE LAPLACE INVERSA:

$$\begin{aligned}
 D) \quad H(s) &= \frac{1}{s+1} + \frac{3}{s+2} \\
 \boxed{h(t) &= e^{-t} + 3 \cdot e^{-2t}}
 \end{aligned}$$

$$\begin{aligned}
 E) \quad H(s) &= \frac{2}{(s+1)(s+2)(s+3)} = \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{s+3} \\
 A=? \quad \frac{2 \cdot \cancel{(s+1)}}{(s+1)(s+2)(s+3)} &= \frac{A \cdot \cancel{(s+1)}}{\cancel{s+1}} + \frac{B \cdot \cancel{(s+1)}}{s+2} + \frac{C \cdot \cancel{(s+1)}}{s+3} \quad \text{Ponto} \\
 A &= \frac{2}{(s+2)(s+3)} \Big|_{s=-1} \Rightarrow \boxed{A=1}
 \end{aligned}$$

e^{-at}

$$\frac{2 \cdot \cancel{(s+2)}}{(s+1) \cdot \cancel{(s+2)} \cdot (s+3)} = \frac{A \cdot \cancel{(s+2)}}{\cancel{s+1}} + \frac{B \cdot \cancel{(s+2)}}{\cancel{s+2}} + \frac{C \cdot \cancel{(s+2)}}{s+3} \Big|_{s=-2}$$

$$B = \frac{2}{(s+1) \cdot (s+3)} \Big|_{s=-2} \Rightarrow B = \frac{2}{-1 \cdot -1} \Rightarrow B = 2$$

$C = ?$

$$\frac{2 \cdot \cancel{(s+3)}}{(s+1) \cdot (s+2) \cdot \cancel{(s+3)}} = \frac{A \cdot \cancel{(s+3)}}{s+1} + \frac{B \cdot \cancel{(s+3)}}{s+2} + \frac{C \cdot \cancel{(s+3)}}{\cancel{s+3}} \Big|_{s=-3}$$

$$C = \frac{2}{(s+1) \cdot (s+2)} \Big|_{s=-3} \Rightarrow C = \frac{2}{-2 \cdot -1} \Rightarrow C = 1$$

$$H(s) = \frac{1}{s+1} - 2 \cdot \frac{1}{s+2} + \frac{1}{s+3}$$

$$h(t) = e^{-t} - 2 \cdot e^{-2t} + e^{-3t}$$

Extra: questão de uma função do segundo grau com transformada de Laplace

$$e^{-at} \xrightarrow{[L]} \frac{1}{s+a} \xrightarrow{[Z]} \frac{z}{z-e^{-a}}$$

01ª QUESTÃO:

→ TRANSFORMADA DE LAPLACE:

$$h(t) = 2t^2 + 3t + 4$$

$$H(s) = 2 \cdot \frac{2}{s^3} + 3 \cdot \frac{1}{s^2} + 4 \cdot \frac{1}{s}$$

$$H(s) = \frac{4}{s^3} + \frac{3}{s^2} + \frac{4}{s}$$

$$H(s) = \frac{4 + 3s + 4s^2}{s^3} \Rightarrow H(s) = \frac{4s^2 + 3s + 4}{s^3}$$

Extra) $[L] = ?$

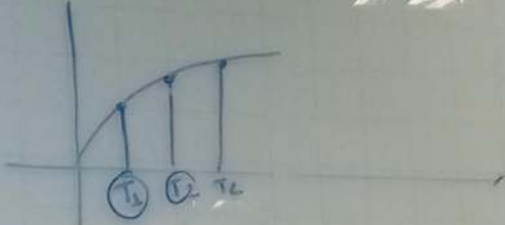
$$h(t) = 2 \cdot t^2 + 3t + 4$$

$$H(s) = 2 \cdot \frac{2}{s^3} + 3 \cdot \frac{1}{s^2} + 4 \cdot \frac{1}{s}$$

$$H(s) = \frac{4}{s^3} + \frac{3}{s^2} + \frac{4}{s} //$$

3. Aplique a transformada Z para representar as FTs abaixo no plano Z.
- a) $F(s) = 3/(s+4)$;
 - c) $K(s) = 3/((s+2)(s+3))$;

e^{-ant}



3ª QUESTÃO:

APLICAR A TRANSFORMADA Z.

A) $F(s) = \frac{3}{s+4} \rightarrow F(s) = 3 \cdot \frac{1}{s+4}$

$F(z) = \frac{3 \cdot z}{z - e^{-4T}}$

C) $K(s) = \frac{3}{(s+2)(s+3)} = \frac{A}{s+2} + \frac{B}{s+3}$

$A=? \Rightarrow A = \frac{3}{s+3} \Big|_{s=-2} \Rightarrow \boxed{A=3}$

$B=? \Rightarrow B = \frac{3}{s+2} \Big|_{s=-3} \Rightarrow \boxed{B=-3}$

$K(s) = \frac{3}{s+2} - \frac{3}{s+3} \Rightarrow$

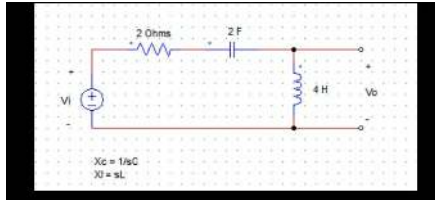
$K(z) = \frac{3z}{z - e^{-2T}} - \frac{3z}{z - e^{-3T}} \Rightarrow$

4. Aplique a transformada Z inversa para representar as FTs abaixo em tempo discreto (amostras).
- a) $F(z) = z/(z-4)$;
- b) $F(z) = z/(z-1) + z/(z-0.2)$;

c) $K(z) = z/((z-1)*(z-3))$;

5. Determine, no domínio da frequência (plano S), a relação $V_o(s)/V_i()$.

Dica para resolver: basta resolver as reatância (capacitiva e indutiva) no plano S



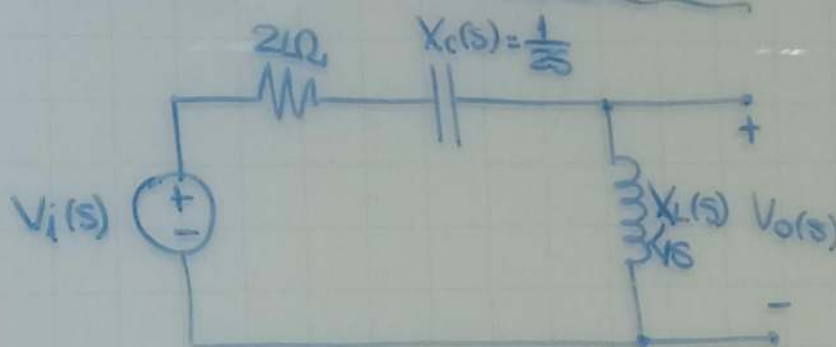
Resposta: $8s^2/(4s + 1 + 8s^2)$

Resolução

$$\frac{d}{dt} \rightarrow e^{-sT} [e^{-1}]$$

$$X_C(s) = \frac{1}{sC} \Rightarrow X_C(s) = \frac{1}{2s}$$

$$X_L(s) = Ls \Rightarrow X_L(s) = 4s$$



\Rightarrow APPLIED
DIVIDE

$$V_o(s)$$

$$\frac{V_o(s)}{V_i(s)} =$$

$$\frac{V_o(s)}{V_i(s)} =$$

$$\frac{V_o(s)}{V_L(s)}$$

⇒ DIA 20 DE SE
1ª AVALIAÇÃO DA
ETAPA.

⇒ APLICANDO O
DIVISOR DE TENSÃO: ANALÓGICOS

⇒ REVISÃO DE FI

⇒ FILTROS DIGITAIS

$$V_o(s) = \frac{X_L(s)}{R + X_L(s) + X_C(s)} \cdot V_i(s) \Rightarrow \text{TRANSFORMADA DE LAPLACE}$$

$$\frac{V_o(s)}{V_i(s)} = \frac{4s}{2 + 4s + \frac{1}{25}}$$

$$\frac{V_o(s)}{V_i(s)} = \frac{4s}{4s^2 + 8s + 1}$$

$$\frac{V_o(s)}{V_i(s)} = \frac{8s^2}{8s^2 + 4s + 1} \Rightarrow \frac{V_o(s)}{V_i(s)} = \frac{s^2}{s^2 + \frac{1}{2}s + \frac{1}{8}}$$

$$e^{\frac{L^2}{ab}} \xrightarrow{[L]} \frac{1}{s+a} \xrightarrow{[Z]} \frac{z}{z - e^{-at}} \xrightarrow{[Z]}$$

09ª QUESTÃO:

QUAL SERIA A SAÍDA DE UM FILTRO DIGITAL MEDIANO SE AS AMOSTRAS COLETADAS FOSSEM AS SEGUINTE:

ENT = (2, 4, 6, 100, 4, 2, 3, 50, 2, 3)

2, 2, 4 \Rightarrow 2, 2, 4 = 2
 2, 4, 6 \Rightarrow 2, 4, 6 = 4
 4, 6, 100 \Rightarrow 4, 6, 100 = 6
 6, 100, 4 \Rightarrow 4, 6, 100 = 6
 100, 4, 2 \Rightarrow 2, 4, 100 = 4

4, 2, 3 \Rightarrow 2, 3, 4 = 3
 2, 3, 50 \Rightarrow 2, 3, 50 = 3
 3, 50, 2 \Rightarrow 2, 3, 50 = 3
 50, 2, 3 \Rightarrow 2, 3, 50 = 3
 2, 3, 3 \Rightarrow 2, 3, 3 = 3

3

⇒ 3

⇒ 3

⇒ 3

⇒ 3

SAÍDA = (2, 4, 6, 6, 4, 3, 3, 3, 3).

Exerc 1) Filtros mediana. * Maria

Qual seria a saída de um filtro digital mediana se as amostras coletadas fossem as seguintes:

$$\text{Ent} = (2, 4, 6, 100, 4, 2, 3, 50, 2, 3)$$

$$\text{I)} \quad 2, 4, 4 \Rightarrow 2, \textcircled{4}, 4 = 4$$

$$2, 4, 6 \Rightarrow 2, \textcircled{4}, 6 = 4$$

$$4, 6, 100 \Rightarrow 4, \textcircled{6}, 100 = 6$$

$$6, 100, 4 \Rightarrow 4, \textcircled{6}, 100 = 6$$

$$100, 4, 2 \Rightarrow 2, \textcircled{4}, 100 = 4$$

$$4, 2, 3 \Rightarrow 2, \textcircled{3}, 4 = 3$$

$$\text{II)} \quad 2, 3, 50 \Rightarrow 2, \textcircled{3}, 50 = 3$$

$$4, 50, 2 \Rightarrow 2, \textcircled{3}, 50 = 3$$

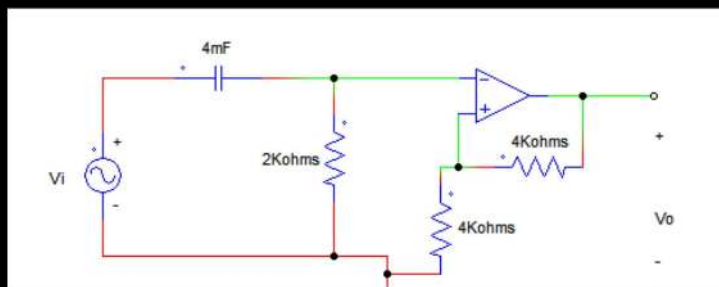
$$50, 2, 3 \Rightarrow 2, \textcircled{3}, 50 = 3$$

$$2, 3, 3 \Rightarrow 2, \textcircled{3}, 3 = 3$$

$$\text{Saída} = (2, 4, 6, 6, 4, 3, 3, 3, 3, 3)$$

6. Nos filtros abaixo, determine o tipo de filtro e a relação $A_v = V_o/V_i$.

C.



$$e^{-at} \xrightarrow{[L]} \frac{1}{s+a} \xrightarrow{[Z]} \frac{z}{z-e^{-aT}}$$

06ª QUESTÃO:

$$\frac{V_o(s)}{V_i(s)} = ?$$

$$\Delta V = \frac{V_o}{A_v} = V_{in1} - V_{in2}$$

c)

