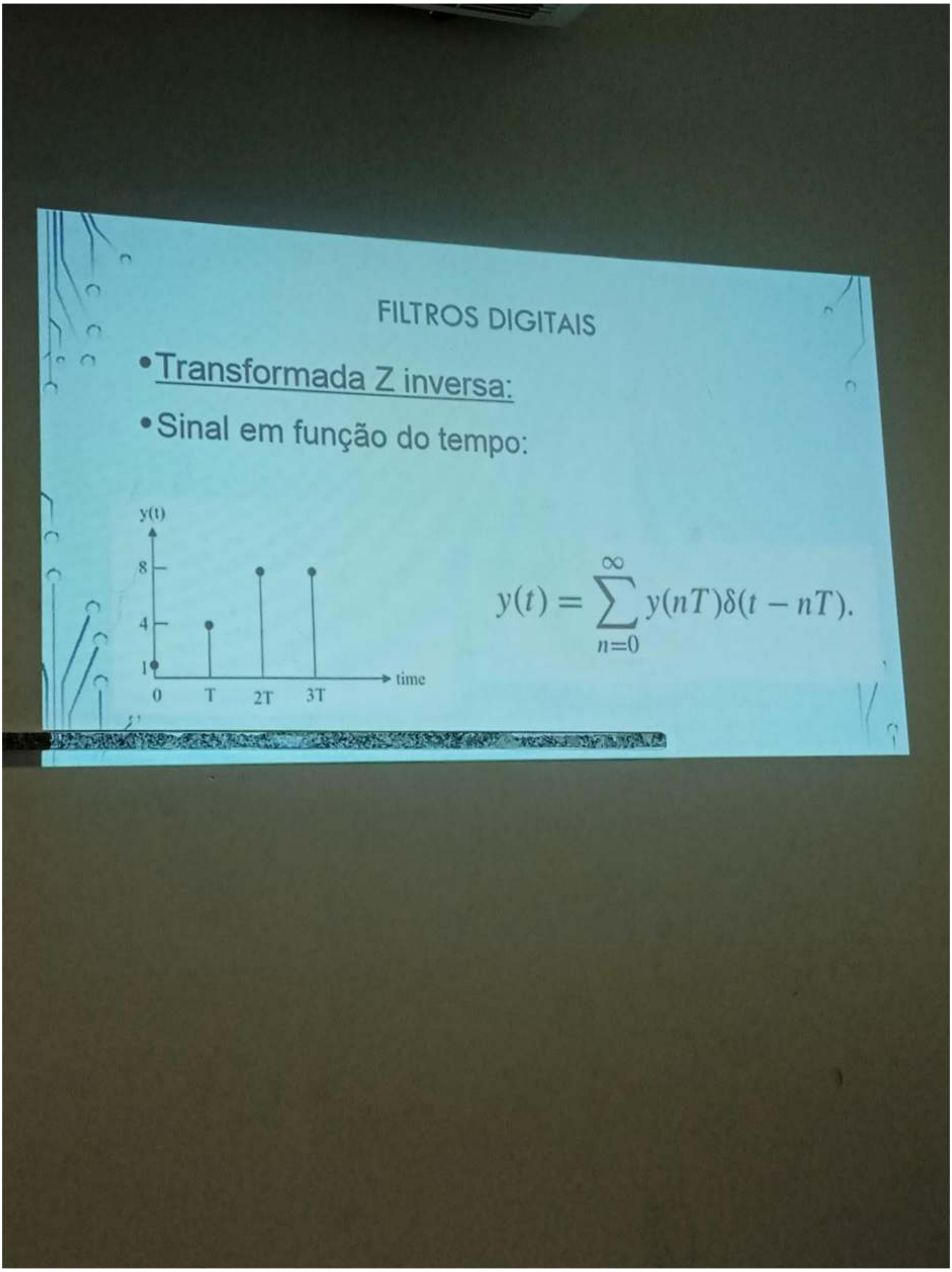
Aula 06-09 finalizando Transformada Z

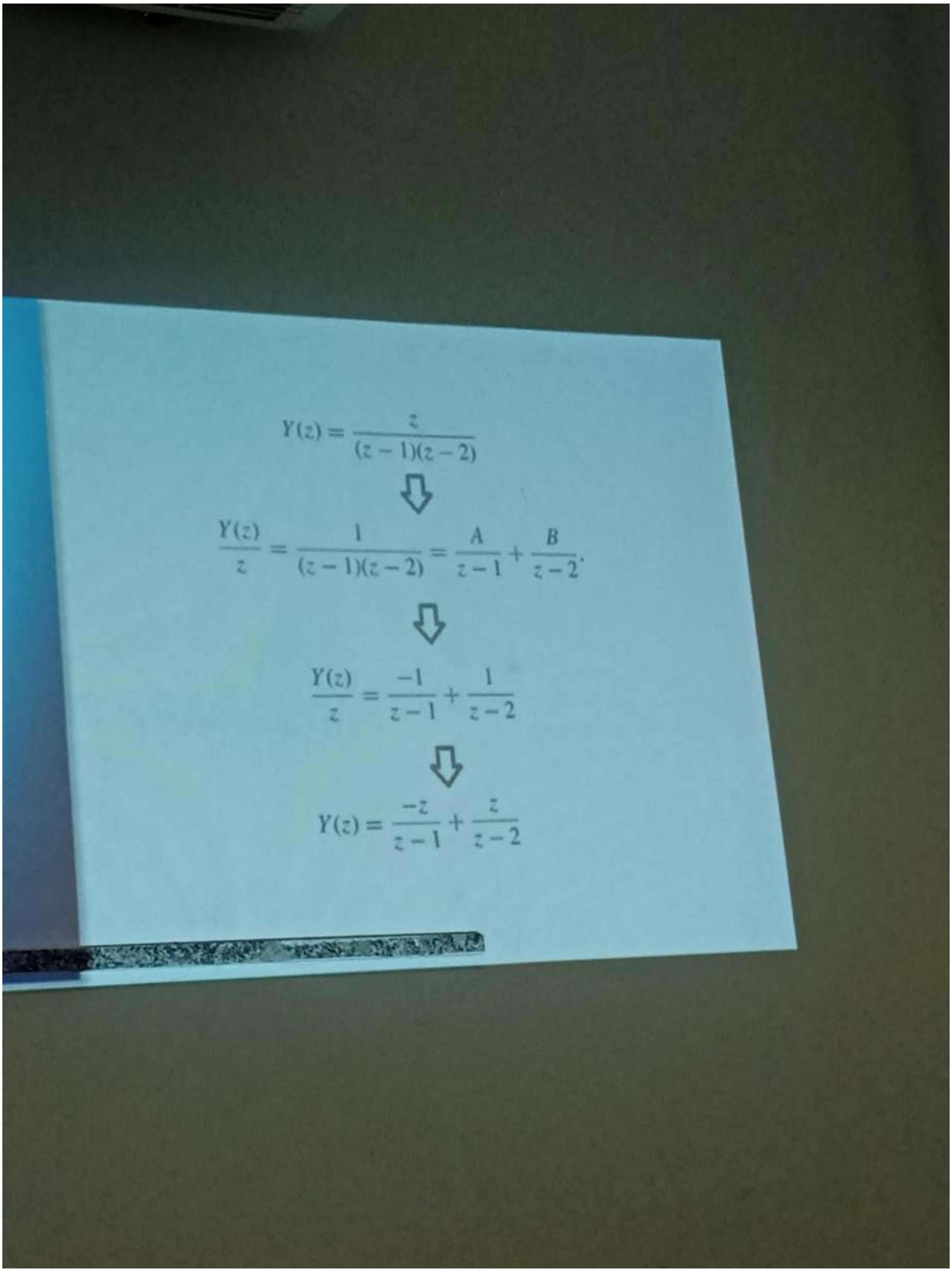
Transformada Z inversa

- Utilizada para converter uma função no domínio Z para o domínio do tempo
- A técnica mais simples é convertendo a função,no domínio Z, para a soma de frações parciais

Passos para realizar a transformada Z inversa



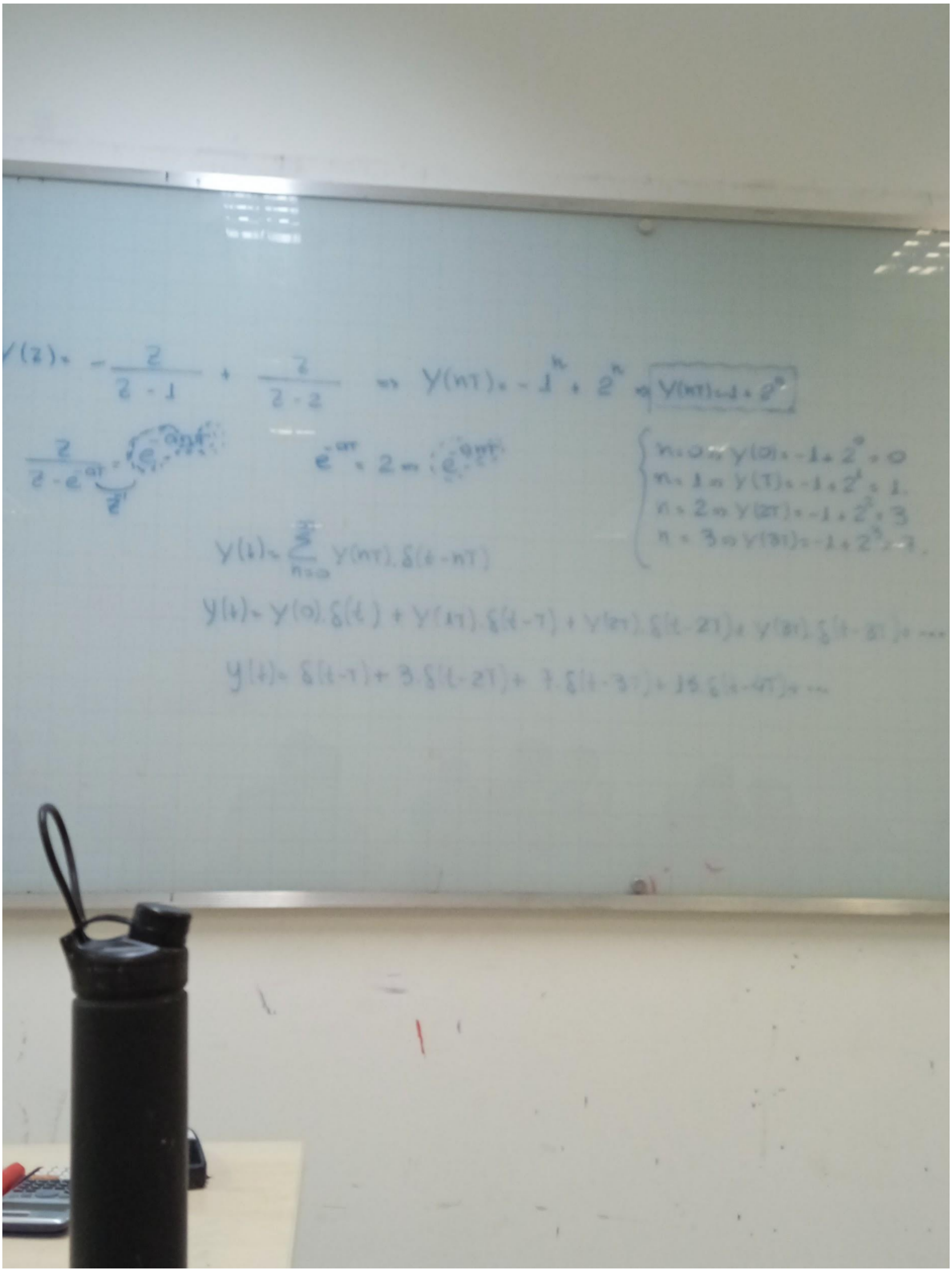
Pegando o resultado…



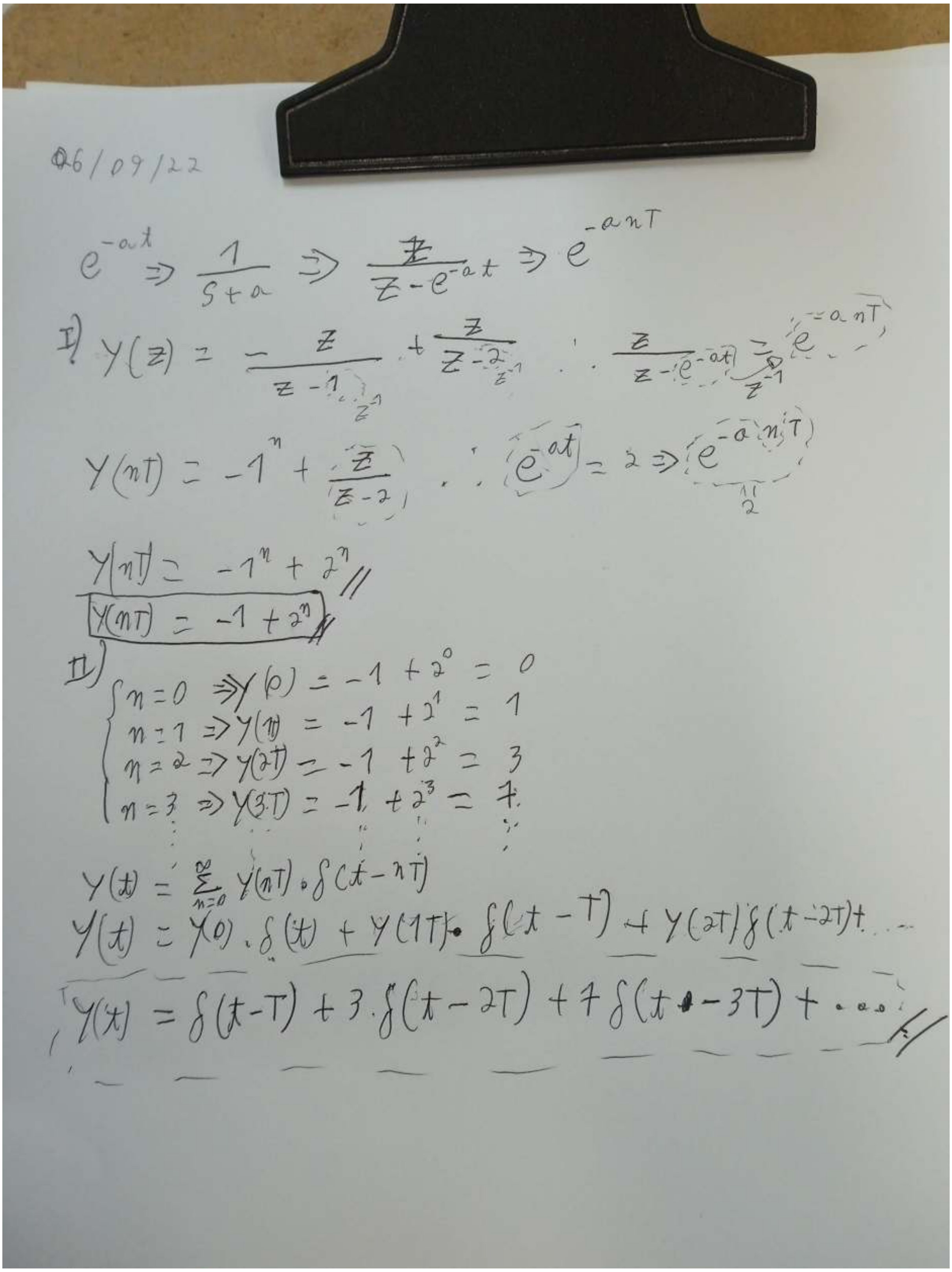

Pegando o resultado…

Anotação do professor

Anotação do Cristiano



Anotação do Cristiano



06/09/22

$$e^{-at} \Rightarrow \frac{1}{s+a} \Rightarrow \frac{z}{z-e^{-at}} \Rightarrow e^{-anT}$$

I) $Y(z) = -\dfrac{z}{z-1} + \dfrac{z}{z-2}$ ∴ $\dfrac{z}{z-e^{-at}} \Rightarrow e^{-anT}$

$Y(nT) = -1^n + \dfrac{z}{z-2}$ ∴ $e^{at} = 2 \Rightarrow e^{-anT}$

$Y(nT) = -1^n + 2^n$ //

$\boxed{Y(nT) = -1 + 2^n}$ //

II)

$$\begin{cases} n=0 \Rightarrow Y(0) = -1 + 2^0 = 0 \\ n=1 \Rightarrow Y(1T) = -1 + 2^1 = 1 \\ n=2 \Rightarrow Y(2T) = -1 + 2^2 = 3 \\ n=3 \Rightarrow Y(3T) = -1 + 2^3 = 7 \\ \vdots \end{cases}$$

$$Y(t) = \sum_{n=0}^{\infty} Y(nT)\cdot\delta(t-nT)$$

$$Y(t) = Y(0)\cdot\delta(t) + Y(1T)\cdot\delta(t-T) + Y(2T)\,\delta(t-2T) + \dots$$

$$Y(t) = \delta(t-T) + 3\cdot\delta(t-2T) + 7\,\delta(t-3T) + \dots$$ //

Pegando outro exemplo

Anotação do professor

$$\frac{Y(z)}{z} = \frac{1}{z.(z-1).(z-2)} = \frac{A}{z} + \frac{B}{z-1} + \frac{C}{z-2}$$

$$A=? \quad \frac{\cancel{z}}{\cancel{z}.(z-1).(z-2)} = \frac{A\,\cancel{z}}{\cancel{z}} + \frac{B\,\cancel{z}^{0}}{\cancel{z}-1} + \frac{C\,\cancel{z}^{0}}{\cancel{z}-2}\Bigg|^{z=0}$$

$$A = \frac{1}{(\cancel{z}-1).(\cancel{z}-2)}\Bigg|^{z=0} \Rightarrow \boxed{A=\frac{1}{2}}$$

$$B=? \quad \frac{\cancel{z-1}}{z.(\cancel{z-1}).(z-2)} = \frac{A.(\cancel{z}^{0}-1)}{\cancel{z}} + \frac{B(\cancel{z-1})}{\cancel{z-1}} + \frac{C.(\cancel{z}^{0}-1)}{\cancel{z}-2}\Bigg|^{z=1}$$

$$B = \frac{1}{z.(z-2)}\Bigg|^{z=1} \Rightarrow \boxed{B=-1}$$

$$C = ? \Rightarrow \frac{\cancel{z-2}}{z\cdot(z-1)\cdot(\cancel{z-2})} = \frac{A\cdot\cancel{(z-2)}^{0}}{\cancel{z}} + \frac{B\cdot\cancel{(z-2)}^{0}}{\cancel{z-1}} + \frac{C\cdot(\cancel{z-2})}{(\cancel{z-2})}\Bigg|^{z=2}$$

$$C = \frac{1}{z\cdot(z-1)}\Bigg|^{z=2} \Rightarrow \boxed{C = \frac{1}{2\cdot(1)} = \frac{1}{2}}$$

$$\frac{Y(z)}{z} = \frac{1/2}{z} - \frac{1}{z-1} + \frac{1/2}{z-2}$$

$$Y(z) = \frac{1/2\cdot\cancel{z}}{\cancel{z}} - \frac{z}{z-1} + \frac{1}{2}\cdot\frac{z}{z-2}$$

$$Y(z) = \frac{1}{2} - \frac{z}{z-1} + \frac{1}{2}\cdot\frac{z}{z-2}$$

Y(n

Y(

$$\frac{C.(z-2)}{(z-2)}\Big|_{z=2} \qquad e^{-ab} \Rightarrow \frac{1}{s+a} \Rightarrow \frac{z}{z-e^{-aT}} \Rightarrow e^{-anT} = 1^n$$

$$\frac{z}{z-1} \Rightarrow 1 = e^{-aT}$$

$$\frac{z}{8} \qquad e^{aT}$$

$$= \frac{1}{2}$$

$$y(nT) = a - 1^n + \frac{2^n}{2}$$

$$y(nT) = a - 1^n + 2^{n-1} \Rightarrow y(nT) = a - 1 + 2^{n-1}.$$

$$n=0 \Rightarrow y(0) = \frac{1}{2} - 1 + 2^{-1} \Rightarrow y(0) = \frac{1}{2} - 1 + \frac{1}{2}$$

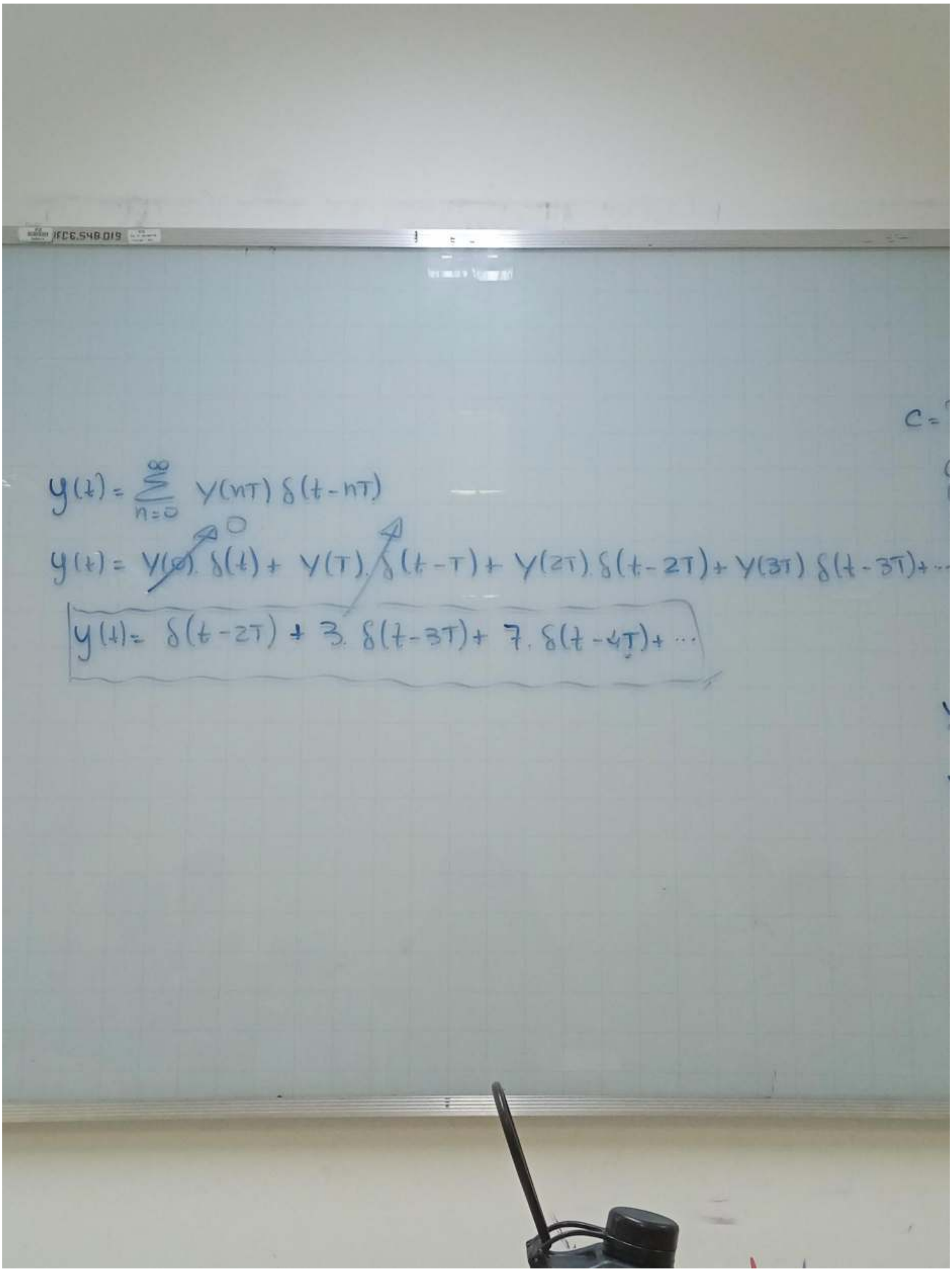
$$y(0) = 0;$$

$$y(T) = -1 + 2^0 \Rightarrow \boxed{y(T) = 0}$$

$$y(2T) = -1 + 2^1 \Rightarrow \boxed{y(2T) = 1}$$

$$y(3T) = -1 + 2^2 \Rightarrow \boxed{y(3T) = 3}$$

$$y(4T) = -1 + 2^3 \Rightarrow \boxed{y(4T) = 7}$$




Anotação do Cristiano

06/09/22

$$e^{-at} \Rightarrow \frac{1}{s+a} \Rightarrow \frac{z}{z-e^{-at}} \Rightarrow e^{-anT}$$

I) $Y(z) = -\frac{z}{z-1} + \frac{z}{z-2}$ $\therefore$ $\frac{z}{z-e^{-at}} \Rightarrow e^{-anT}$

$Y(nT) = -1^n + \frac{z}{z-2}$ $\therefore$ $e^{-at} = 2 \Rightarrow \underset{2}{e^{-anT}}$

$Y(nT) = -1^n + 2^n$ //

$\boxed{Y(nT) = -1 + 2^n}$ //

II)

$$\begin{cases} n=0 \Rightarrow Y(0) = -1 + 2^0 = 0 \\ n=1 \Rightarrow Y(T) = -1 + 2^1 = 1 \\ n=2 \Rightarrow Y(2T) = -1 + 2^2 = 3 \\ n=3 \Rightarrow Y(3T) = -1 + 2^3 = 7 \\ \vdots \end{cases}$$

$Y(t) = \sum_{n=0}^{\infty} Y(nT) \cdot \delta(t-nT)$

$Y(t) = Y(0) \cdot \delta(t) + Y(1T) \cdot \delta(t-T) + Y(2T)\delta(t-2T) + \ldots$

$Y(t) = \delta(t-T) + 3 \cdot \delta(t-2T) + 7\delta(t-3T) + \ldots$ //

Ex2)

$$\frac{Y(z)}{z} = \frac{1}{z \cdot (z-1) \cdot (z-2)} = \frac{A}{z} + \frac{B}{z-1} + \frac{C}{z-2}$$

I) $A = ?$ e $\times (z)$

$$\frac{\cancel{z}}{\cancel{z} \cdot (z-1) \cdot (z-2)} = \frac{A \cdot \cancel{z}}{\cancel{z}} + \frac{B \cdot \cancel{z}^{\,0}}{z-1} + \frac{C \cdot \cancel{z}^{\,0}}{z-2} \Bigg|_{z=0}$$

$$A = \frac{1}{(z-1) \cdot (z-2)} \Rightarrow \boxed{A = \frac{1}{2}}$$

II) $B = ?$ e $\times(Z-1)$

$$\frac{\cancel{Z-1}}{Z\cdot(\cancel{Z-1})\cdot(Z-2)} = \frac{A\cdot\cancel{(Z-1)}^{\,0}}{Z} + \frac{B\cdot\cancel{(Z-1)}}{\cancel{Z-1}} + \frac{C\,\cancel{(Z-1)}^{\,0}}{Z-2}\Bigg|_{Z=1}$$

$$B = \frac{1}{Z\cdot(Z-2)}\Bigg|_{Z=1} \Rightarrow \boxed{B=-1}$$

III) $C = ?$ e $\times(Z-2)$

$$\frac{\cancel{Z-2}}{Z\cdot(Z-1)\cdot(\cancel{Z-2})} = \frac{A\cdot\cancel{(Z-2)}^{\,0}}{Z} + \frac{B\cdot\cancel{(Z-2)}^{\,0}}{(Z-1)} + \frac{C\cdot\cancel{(Z-2)}}{\cancel{(Z-2)}}\Bigg|_{Z=2}$$

$$C = \frac{1}{Z\cdot(Z-1)}\Bigg|_{Z=2} \Rightarrow \boxed{C=\frac{1}{2}}$$

IV) Representando as funções de transferência por frações parciais

* Termo Independente

$$\frac{Y(Z)}{Z} = \frac{1/2}{Z} - \frac{1}{Z-1} + \frac{1/2}{Z-2}$$

$$Y(Z) = \frac{1/2\cdot\cancel{Z}}{\cancel{Z}} - \frac{Z}{Z-1} + \frac{1}{2}\cdot\frac{Z}{Z-2}$$

$$Y(Z) = \overset{*}{\frac{1}{2}} - \frac{Z}{Z-1} + \frac{1}{2}\cdot\frac{Z}{Z-2}$$

V) Representando em tempo discreto, para n amostras

$$y(nT) = a - 1^n + \frac{2^n}{2}$$

$$y(nT) = a - 1^n + 2^{n-1} \Rightarrow \boxed{y(nT) = a - 1 + 2^{n-1}}$$

III) Representando em séries infinitas (Atenção a OBS)

$n=0 \Rightarrow Y(0) = \frac{1}{2} - 1 + 2^{-1} \Rightarrow Y(0) = \frac{1}{2} - 1 + \frac{1}{2}$

$Y(0) = 0.$

$Y(T) = -1 + 2^{0} \Rightarrow Y(T) = 0$

$Y(2T) = -1 + 2^{1} \Rightarrow Y(2T) = 1$

$Y(3T) = -1 + 2^{2} \Rightarrow Y(3T) = 3$

$Y(4T) = -1 + 2^{3} \Rightarrow Y(4T) = 7$

$$y(t) = \sum_{n=0}^{\infty} Y(nT)\, \delta(t - nT)$$

$$Y(t) = \cancel{Y(0)\, \delta(t)}^{0} + \cancel{Y(T) \cdot \delta(t-T)}^{0} + Y(2T) \cdot \delta(t-2T) + \ldots$$

$$y(t) = \delta(t-2T) + 3\,\delta(t-3T) + 7 \cdot \delta(t-4T) + \ldots //$$

Obs:
Termo independente: só influenciará para quando Y(0) ou seja Z= 0, mas para os outros termos não precisará ser considerado por ser considerado um "Ponto de Partida". Para Y(T), Y(2T) em diante, ele será desconsiderado para os cálculos.

Cobrar o professor do exemplo prático desse caso no Matlab quinta

Ex2:

Anotação do professor

$$\frac{1}{s+a} \xrightarrow{|Z|} \frac{z}{z-e^{-aT}}$$

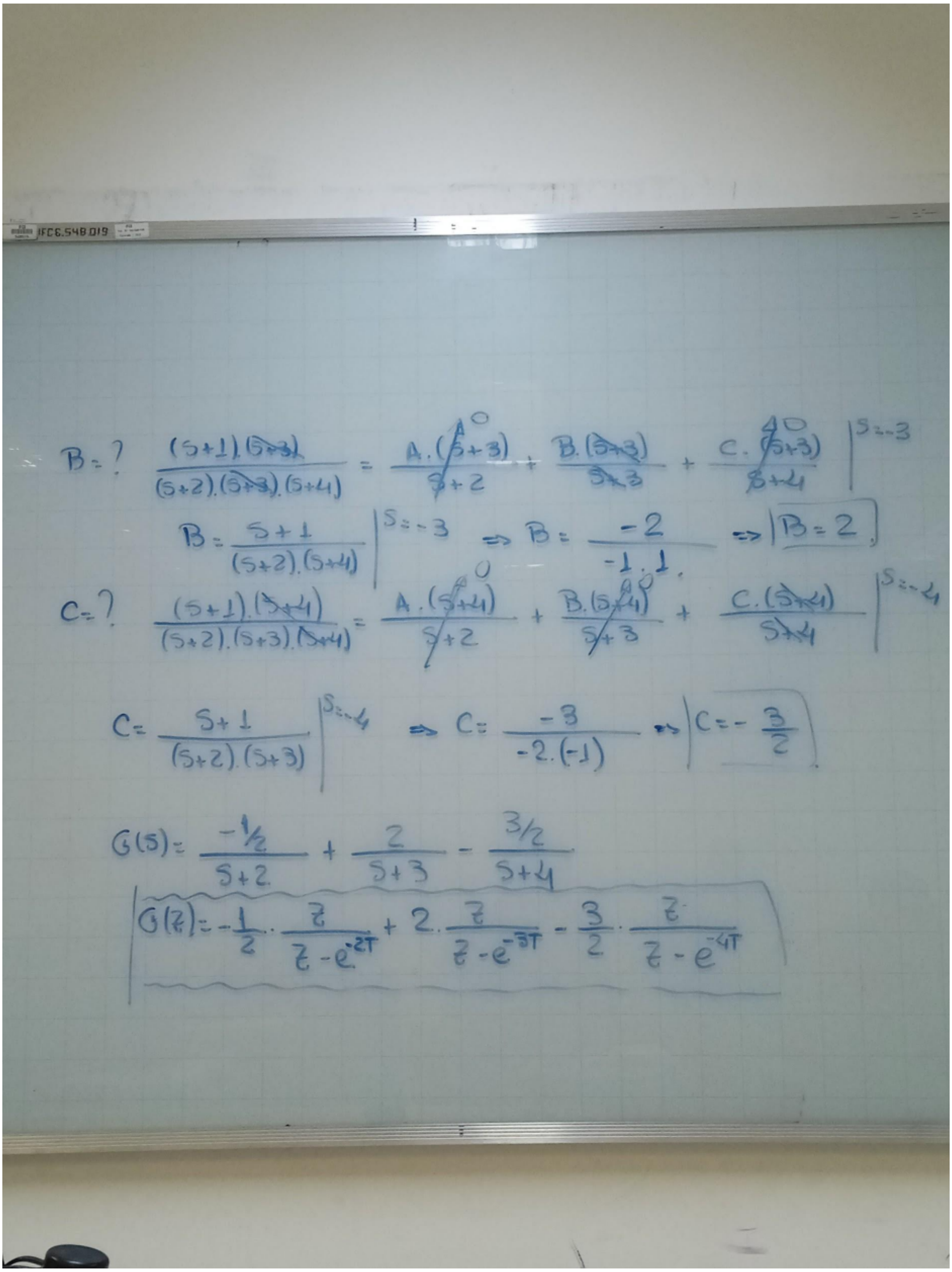
**EXERCÍCIO:** UTILIZE A TRANSFORMADA Z PARA REPRESENTAR A SEGUINTE FT NO DOMÍNIO Z.

$$G(s) = \frac{s+1}{(s+2).(s+3).(s+4)} = \frac{A}{s+2} + \frac{B}{s+3} + \frac{C}{s+4}$$

$$\frac{(s+1).\cancel{(s+2)}}{\cancel{(s+2)}.(s+3).(s+4)} = \frac{A.\cancel{(s+2)}}{\cancel{s+2}} + \cancel{\frac{B(s+2)}{s+3}}^{0} + \cancel{\frac{C.(s+2)}{s+4}}^{0} \Bigg|^{s=-2}$$

$$A = \frac{s+1}{(s+3).(s+4)} \Bigg|^{s=-2} \Rightarrow A = \frac{-1}{1.2} \Rightarrow \boxed{A = -\frac{1}{2}}$$

Anotação do Cristiano




Anotação do Cristiano

Ex2

$$G(s) = \frac{s+1}{(s+2)\cdot(s+3)\cdot(s+4)} = \frac{A}{(s+2)} + \frac{B}{(s+3)} + \frac{C}{(s+4)}$$

I) A = ? e x(s+2)

$$\frac{(s+1)\cdot\cancel{(s+2)}}{\cancel{(s+2)}\cdot(s+3)\cdot(s+4)} = \frac{A\cancel{(s+2)}}{\cancel{(s+2)}} + \frac{B\cancel{(s+2)}^{0}}{\cancel{(s+3)}} + \frac{C\cancel{(s+2)}^{0}}{\cancel{(s+4)}}\Bigg|_{s=-2}$$

$$A = \frac{(s+1)}{(s+3)\cdot(s+4)}\Bigg|_{s=-2} \Rightarrow A = \frac{-1}{1\cdot 2} \Rightarrow \boxed{A = -\frac{1}{2}}$$

$$\cancel{A = -\frac{1}{2}}$$

II) B = ? e x(s+3)

$$\frac{(s+1)\cdot\cancel{(s+3)}}{(s+2)\cdot\cancel{(s+3)}\cdot(s+4)} = \frac{A\cdot\cancel{(s+3)}^{0}}{\cancel{(s+2)}} + \frac{B\cdot\cancel{(s+3)}}{\cancel{(s+3)}} + \frac{C\cancel{(s+3)}^{0}}{\cancel{(s+4)}}\Bigg|_{s=-3}$$

$$B = \frac{(s+1)}{(s+2)\cdot(s+4)}\Bigg|_{s=-3} \qquad B = \frac{-2}{-1\cdot 1} \Rightarrow \boxed{B = 2}$$

III) C = ? e x(s+4)

$$\frac{(s+1)\cdot\cancel{(s+4)}}{(s+2)\cdot(s+3)\cdot\cancel{(s+4)}} = \frac{A\cdot\cancel{(s+4)}^{0}}{\cancel{s+2}} + \frac{B\cdot\cancel{(s+4)}^{0}}{\cancel{(s+3)}} + \frac{C\cdot\cancel{(s+4)}}{\cancel{(s+4)}}\Bigg|_{s=-4}$$

$$C = \frac{(s+1)}{(s+2)\cdot(s+3)}\Bigg|_{s=-4} \qquad C = \frac{-3}{(-2)\cdot(-1)} \Rightarrow \boxed{C = \frac{3}{2}}$$

IV)

IV)

$$G(s) = \frac{-\frac{1}{2}}{(s+2)} + \frac{2}{(s+3)} + \frac{-\frac{3}{2}}{(s+4)}$$

$$G(s) = -\frac{1}{2} \cdot \frac{1}{(s+2)} + \frac{2}{(s+3)} + -\frac{3}{2} \cdot \frac{1}{(s+4)}$$

$$\boxed{G(t) = -\frac{1}{2} \cdot \frac{z}{z-e^{-2T}} + 2 \cdot \frac{z}{z-e^{-3T}} + -\frac{3}{2} \cdot \frac{z}{z-e^{-4T}}}$$