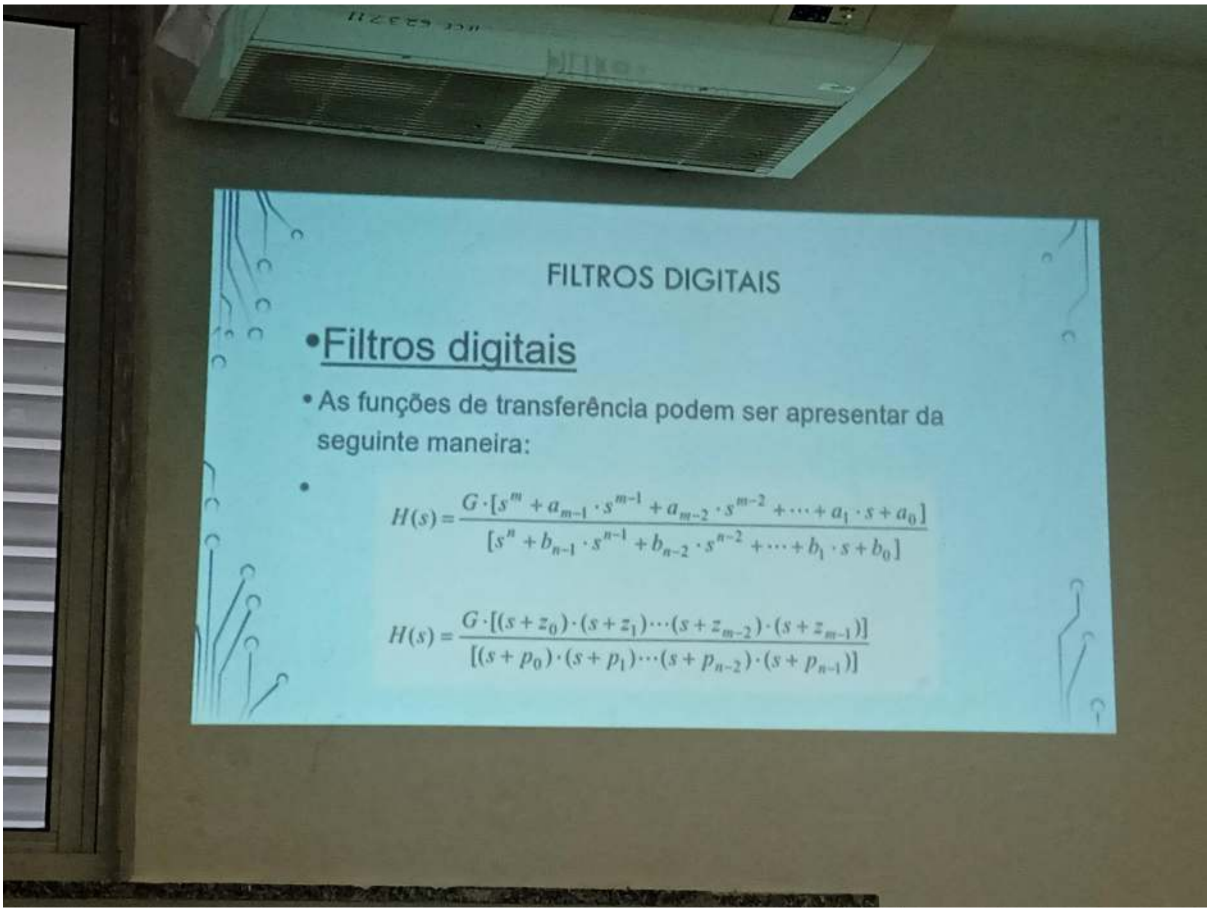
Aula 16/08/22

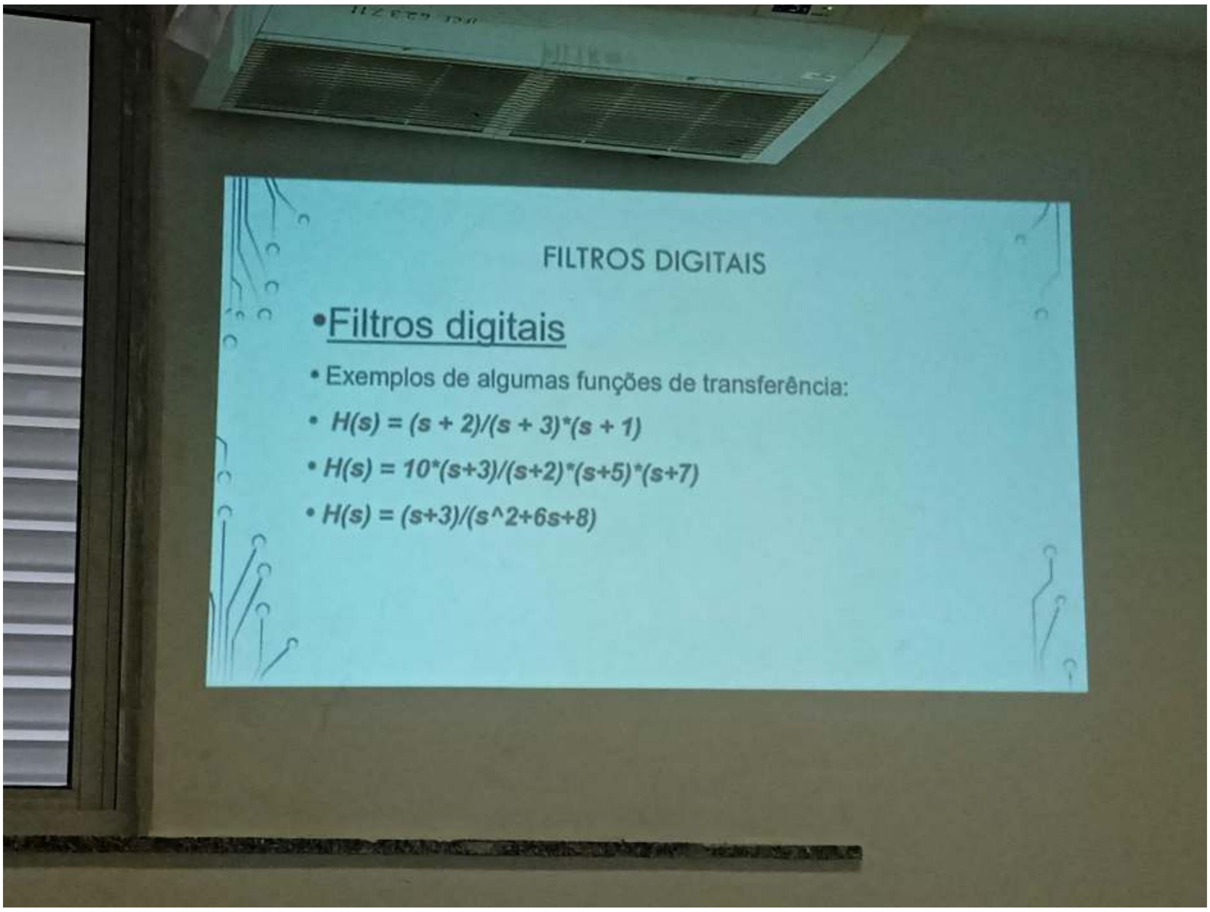
Email institucional:
pregis@ifce.edu.br

Revisão de filtros digitais
- Conceito de filtros
- Conceito de sinais analógicos e digitais
- H(s) função de transferência
- Transformada de Laplace

H(s)pode ser a relação entre a entrada e a saída



Numerador: Polos de entrada
Denominador: Polos de saída
Exemplo:


FILTROS DIGITAIS
•Filtros digitais
• Exemplos de algumas funções de transferência:
• H(s) = (s + 2)/(s + 3)*(s + 1)
• H(s) = 10*(s+3)/(s+2)*(s+5)*(s+7)
• H(s) = (s+3)/(s^2+6s+8)

⇒ Zeros:

$$s + 2 = 0$$

$$s = -2$$

⇒ Polos:

$$s_1 + 3 = 0$$

$$\boxed{s_1 = -3}$$

$$s_2 + 1 = 0$$

$$\boxed{s_2 = -1}$$

Representação polinomial dos polos

#representação Polinomial dos polos

$$H(S) = \frac{S+3}{S^2+6S+8} \rightarrow H(S) = \frac{S+3}{(S+2).(S+4)}$$

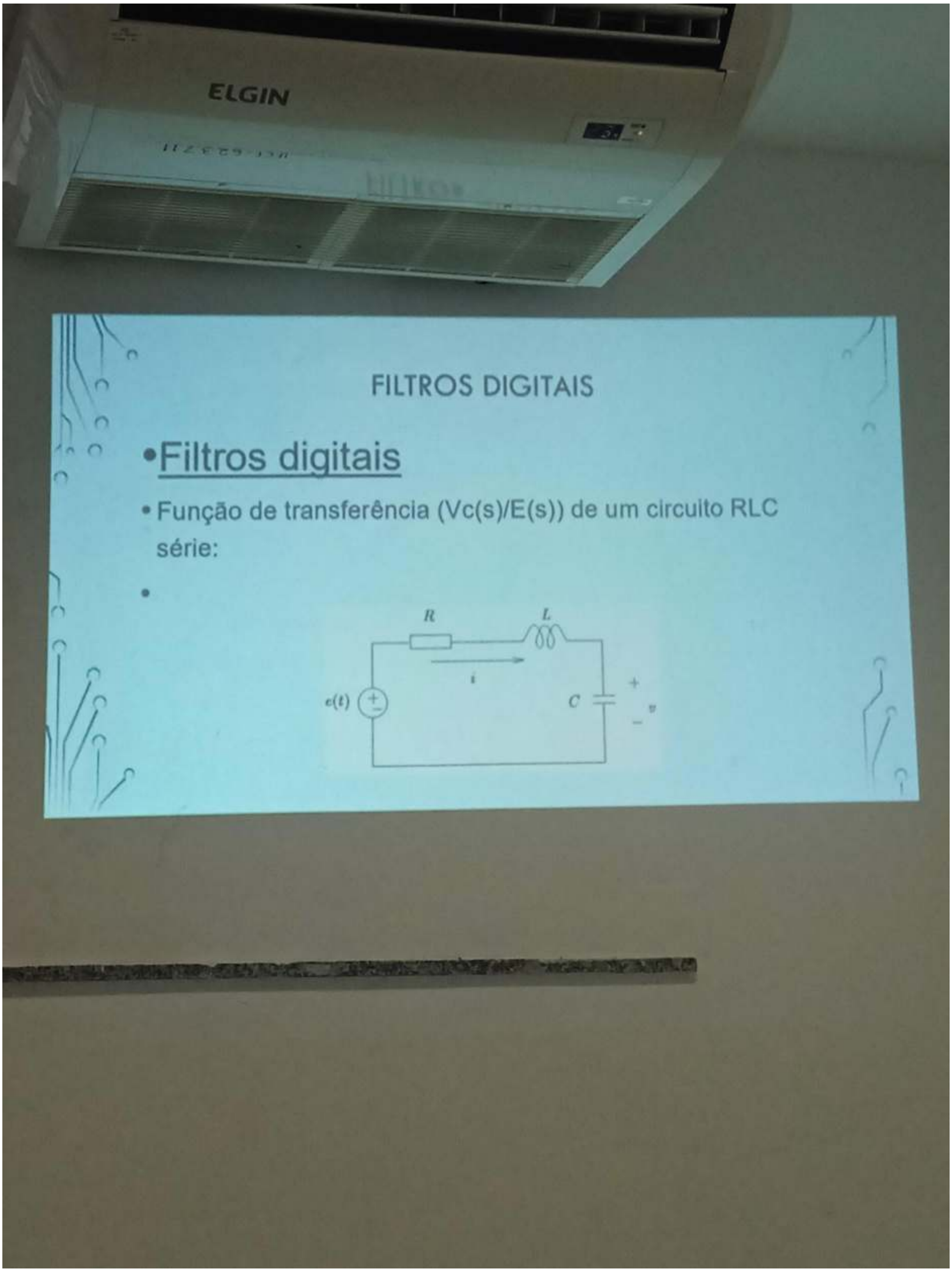
Zeros: $S+3=0$

$S=-3$

Polos: $\begin{cases} S_1+2=0 \\ S_1=-2 \\ S_2+4=0 \end{cases}$

$S_2=-4.$

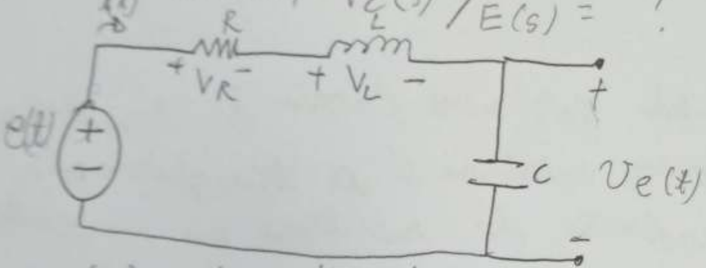
Aplicando



Método por equações diferenciais

Escrito por Cristiano Coutinho

⇒ Determinar $V_C(s)/E(s) = ?$



$e(t) = V_R + V_L + V_C(t)$ (LK das tensões) (L.K.V.)

$e(t) = R \cdot i(t) + L \cdot \dfrac{d\,i(t)}{dt} + V_C(t)$ ∴ $i(t) = \dfrac{d\,q(t)}{dt}$;

$e(t) = R \cdot \dfrac{d\,q(t)}{dt} + L \cdot \dfrac{d^2 q(t)}{dt^2} + v_C(t)$ $q = C \cdot u(t)$

$e(t) = R \cdot \dfrac{d(C \cdot \cancel{V(t)}^{\,v_C(t)})}{dt} + L \cdot \dfrac{d^2 \cdot (C \cdot \cancel{V(t)}^{\,v_C(t)})}{dt^2} + V_C(t)$

. Aplicando transf. de Laplace

$L[\,] = ?$

$E(s) = R \times C$

$L\left[\dfrac{d\,v_C(t)}{dt}\right] = s \cdot V_C(s)$

$$dt \qquad dt$$

$$e(t) = R \cdot \frac{d(C \cdot \overset{\to v_c(t)}{\cancel{V}(t)})}{dt} + L \cdot \frac{d^2 \cdot (C \cdot \overset{\to v_c(t)}{\cancel{V}(t)})}{dt^2} + v_c(t)$$

. Aplicando transf. de Laplace

$$L[\;] = ?$$

$$E(s) = R \times C$$

$$L\left[\frac{d\, v_c(t)}{dt}\right] = s \cdot V_c(s)$$

$$L\left[\frac{d^2\, v_c(t)}{dt^2}\right] = s^2 \cdot V_c(s)$$

$$E(s) = R.C.s \cdot V_c(s) + L\,C\,s^2\,V_c(s) + V_c(s)$$

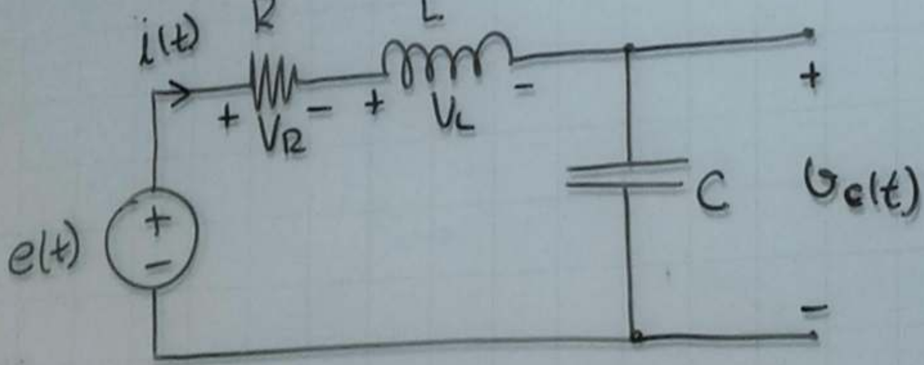
$$E(s) = V_c(s) \cdot (R.C.s + L\,C\,s^2 + 1)$$

$$\frac{V_c(s)}{E(s)} = \frac{1}{L.C.s^2 + R.C.s + 1}$$

$$\boxed{\frac{V_c(s)}{E(s)} = \frac{\frac{1}{LC}}{s^2 + \frac{R}{L} \cdot s + \frac{1}{LC}}}$$

Escrito pelo professor

⇒ DETERMINAR $V_C(s)/E(s) = ?$



$$e(t) = V_R + V_L + V_C(t)$$

$$e(t) = R.i(t) + L.\frac{di(t)}{dt} + v_c(t)$$

$$i(t) = \frac{dq(t)}{dt} \quad ; \quad q = C.U$$

$$q(t) = C.v_c(t)$$

$$e(t) = R \times \frac{dq(t)}{dt} + L.\frac{d^2 q(t)}{dt^2} + v_c(t)$$

$$e(t) = R \times \frac{d(C.v_c(t))}{dt} + L.\frac{d^2(C.v_c(t))}{dt^2} + v_c(t)$$

$$L[\;] = ?$$

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$$E(s) = R.C s . V_c(s) + L.C s^2 . V_c(s) + V_c(s).$$

$$\mathcal{L}\left[\frac{d\, v_c(t)}{dt}\right] = s . V_c(s)$$

$$\mathcal{L}\left[\frac{d^2\, v_c(t)}{dt^2}\right] = s^2 . V_c(s)$$

$$E(s) = V_c(s)\left(R.Cs + LCs^2 + 1\right)$$

$$\frac{V_c(s)}{E(s)} = \frac{1}{LC.s^2 + RC.s + 1} \Rightarrow \boxed{\frac{V_c(s)}{E(s)} = \frac{1/LC}{s^2 + \frac{R}{L}s + \frac{1}{LC}}}$$

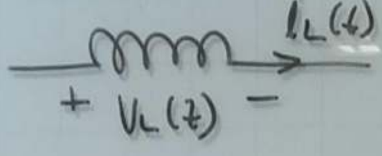
$$\frac{d\,(i(t))}{dt} = \frac{d\left(\frac{d\,q(t)}{dt}\right)}{dt}$$

$$= \frac{d^2\,q(t)}{dt^2}$$

Método representando a reatância no plano S

$$R = \frac{V}{I}$$

⇒ INDUTOR:



$$\Rightarrow V_L(t) = L \cdot \frac{di(t)}{dt}$$

$$\mathcal{L}\left[V_L(t) = L \cdot \frac{di(t)}{dt} \right] \Rightarrow V_L(s) = L . S . I(S)$$

$$\boxed{X_L(s) = \frac{V_L(s)}{I(s)} = L . S}$$

⇒ CAPACITOR:

$$\mathcal{L}\left[i_c(t) = C \cdot \frac{dv_c(t)}{dt} \right] \Rightarrow I_c(s) = C . S . V_c(S)$$

$$\boxed{X_c(s) = \frac{V_c(S)}{I_c(S)} = \frac{1}{C . S}}$$

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Representando as reatâncias no plano S (~~mesmo circuito~~)

• Domínio da frequência

⇒ indutor:

$i_L(t)$, $+\ V_L(t)\ -$ ⇒ $V_L(t) = L \cdot \frac{di(t)}{dt}$

X_L: reatância indutiva. Semelhante resistor (comportamento resistivo)

$L\left[v_L(t) = L \cdot \frac{di(t)}{dt}\right] = V_L(S) = L.S.I(S)$ ∴

$$\boxed{X_L(S) = \frac{V_L(S)}{I(S)} = L \cdot S} \quad \therefore R = \frac{V}{I}$$

$$\boxed{X_L(S) = L.S}$$

⇒ capacitor

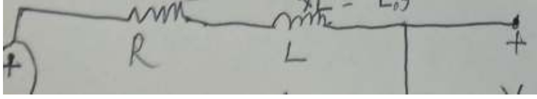
X_C: reatância capacitiva.

$$i_C(t) = C \frac{dv_C(t)}{dt}$$

$$L\left[i_C(t) = C \frac{dv_C(t)}{dt}\right]$$

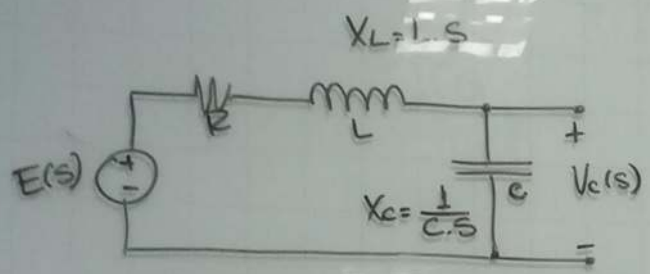
$$\Rightarrow I_C(S) = C.S.V_C(S)$$

$$X_C(S) = \frac{V_C(S)}{I_C(S)} = \frac{1}{C(S)}$$



$V_C(S) = \frac{\frac{1}{CS}}{\ }$ E(S)

Exemplo anterior

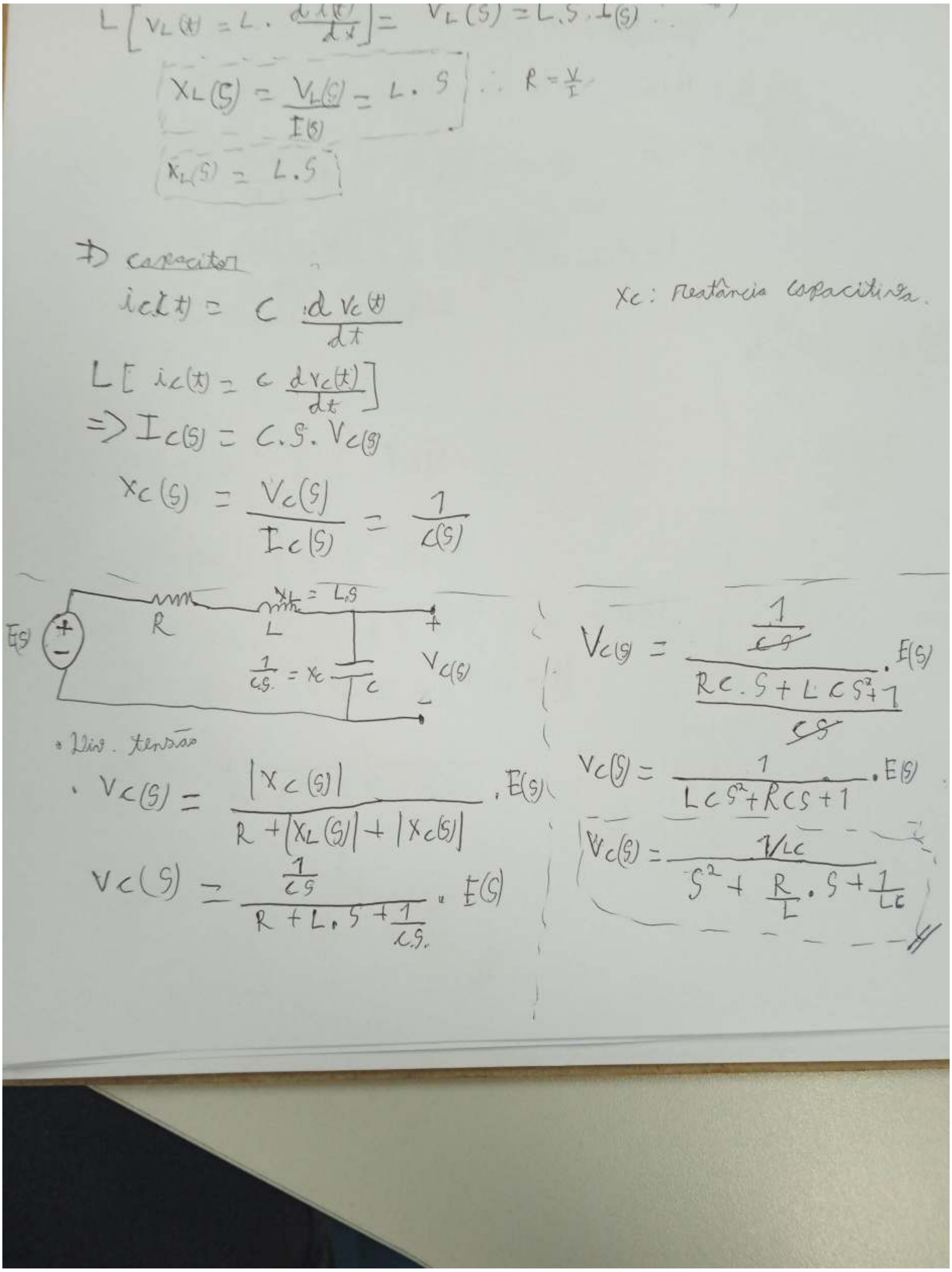


=> **DIVISOR DE TENSÃO:**

$$V_c(S) = \frac{|X_c(S)|}{R + |X_L(S)| + |X_c(S)|} \cdot E(S)$$

$$V_c(S) = \frac{\frac{1}{C.S}}{R + L.S + \frac{1}{C.S}} \cdot E(S) \Rightarrow V_c(S) = \frac{1/\cancel{CS}}{\frac{RC.S + LCS^2 + 1}{\cancel{CS}}} \cdot E(S)$$

$$V_c(S) = \frac{1}{LCS^2 + R.CS + 1} \cdot E(S) \Rightarrow \frac{V_c(S)}{E(S)} = \frac{1/LC}{S^2 + \frac{R}{L}S + \frac{1}{LC}}$$



$$\mathcal{L}\left[v_L(t) = L \cdot \frac{di(t)}{dt}\right] = V_L(s) = L \cdot s \cdot I(s)$$

$$\boxed{X_L(s) = \frac{V_L(s)}{I(s)} = L \cdot s} \quad \therefore \quad R = \frac{V}{I}$$

$$\boxed{X_L(s) = L \cdot s}$$

→ capacitor

$$i_c(t) = C\,\frac{dv_c(t)}{dt}$$

X_c: Reatância capacitiva.

$$\mathcal{L}\left[i_c(t) = C\,\frac{dv_c(t)}{dt}\right]$$

$$\Rightarrow I_c(s) = C \cdot s \cdot V_c(s)$$

$$X_c(s) = \frac{V_c(s)}{I_c(s)} = \frac{1}{C(s)}$$

• Div. tensão

$$V_c(s) = \frac{|X_c(s)|}{R + |X_L(s)| + |X_c(s)|} \cdot E(s)$$

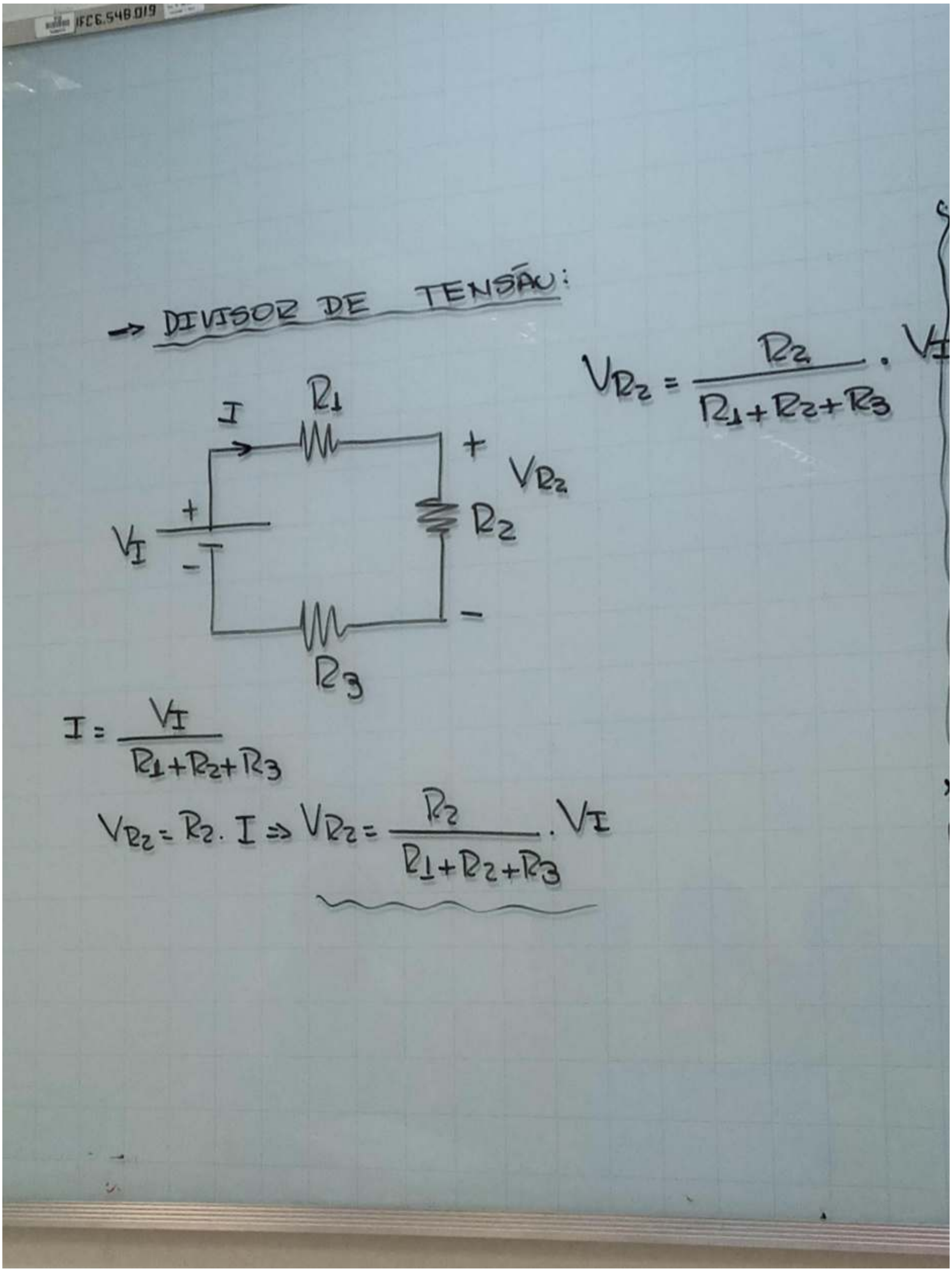
$$V_c(s) = \frac{\frac{1}{Cs}}{R + L \cdot s + \frac{1}{C.s}} \cdot E(s)$$

$$V_c(s) = \frac{\frac{1}{\cancel{Cs}}}{\frac{RC.s + LCs^2 + 1}{\cancel{Cs}}} \cdot E(s)$$

$$V_c(s) = \frac{1}{LCs^2 + RCs + 1} \cdot E(s)$$

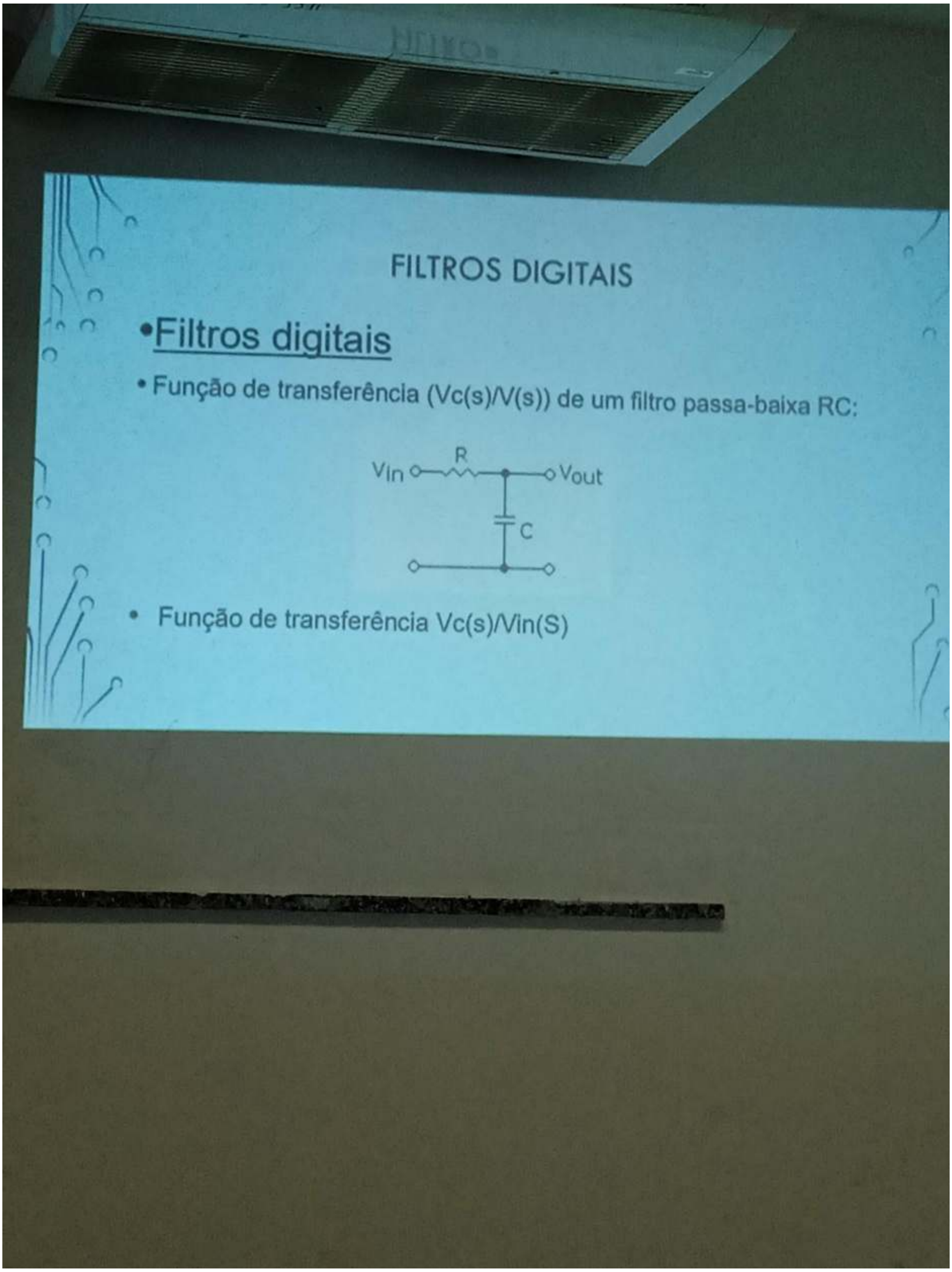
$$\boxed{V_c(s) = \frac{1/LC}{s^2 + \frac{R}{L} \cdot s + \frac{1}{LC}}}$$

Revisão de Divisor de tensão



Aplicando esse método em Filtros

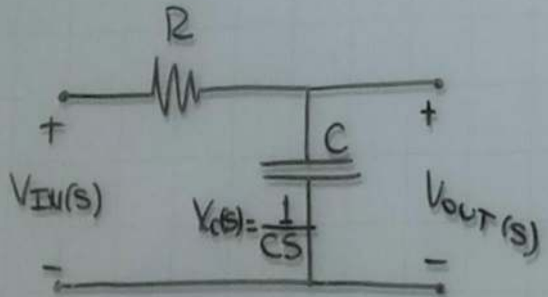
Filtro passa-baixa


FILTROS DIGITAIS
•Filtros digitais
• Função de transferência (Vc(s)/V(s)) de um filtro passa-baixa RC:
R
Vin
Vout
C
• Função de transferência Vc(s)/Vin(S)

Vc(s) = Vout(s)

⇒ FILTRO PASSA-BAIXA PASSIVO:

DETERMINAR $\dfrac{V_{OUT}(S)}{V_{IN}(S)} = ?$



=> APLICANDO O DIVISOR DE TENSÃO:

$$V_{OUT}(s) = \frac{|X_C(s)|}{R + |X_C(s)|} \cdot V_{IN}(s)$$

$$V_{OUT}(s) = \frac{1/CS}{R + \frac{1}{C.S}} \cdot V_{IN}(s)$$

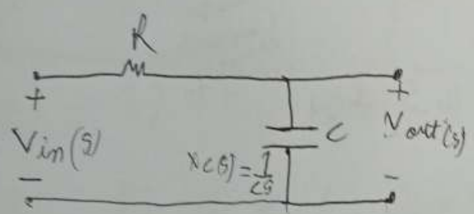
$$V_{OUT}(s) = \frac{\frac{1}{\cancel{CS}}}{\frac{RC.S+1}{\cancel{CS}}} \cdot V_{IN}(s)$$

$$V_{OUT}(s) = \frac{1}{RCS+1} \cdot V_{IN}(s) \Rightarrow \boxed{\frac{V_{OUT}(s)}{V_{IN}(s)} = \frac{\frac{1}{RC}}{S+\frac{1}{RC}}}$$

Aplicando em Filtros Digitais.

• Filtro Passa-baixo passivo:

• Determine $\frac{V_{out}(S)}{V_{in}(S)} = ?$



• Div. de tensão

$$V_{out}(S) = \frac{|X_C(S)|}{R + X_C(S)} \cdot V_{in}(S)$$

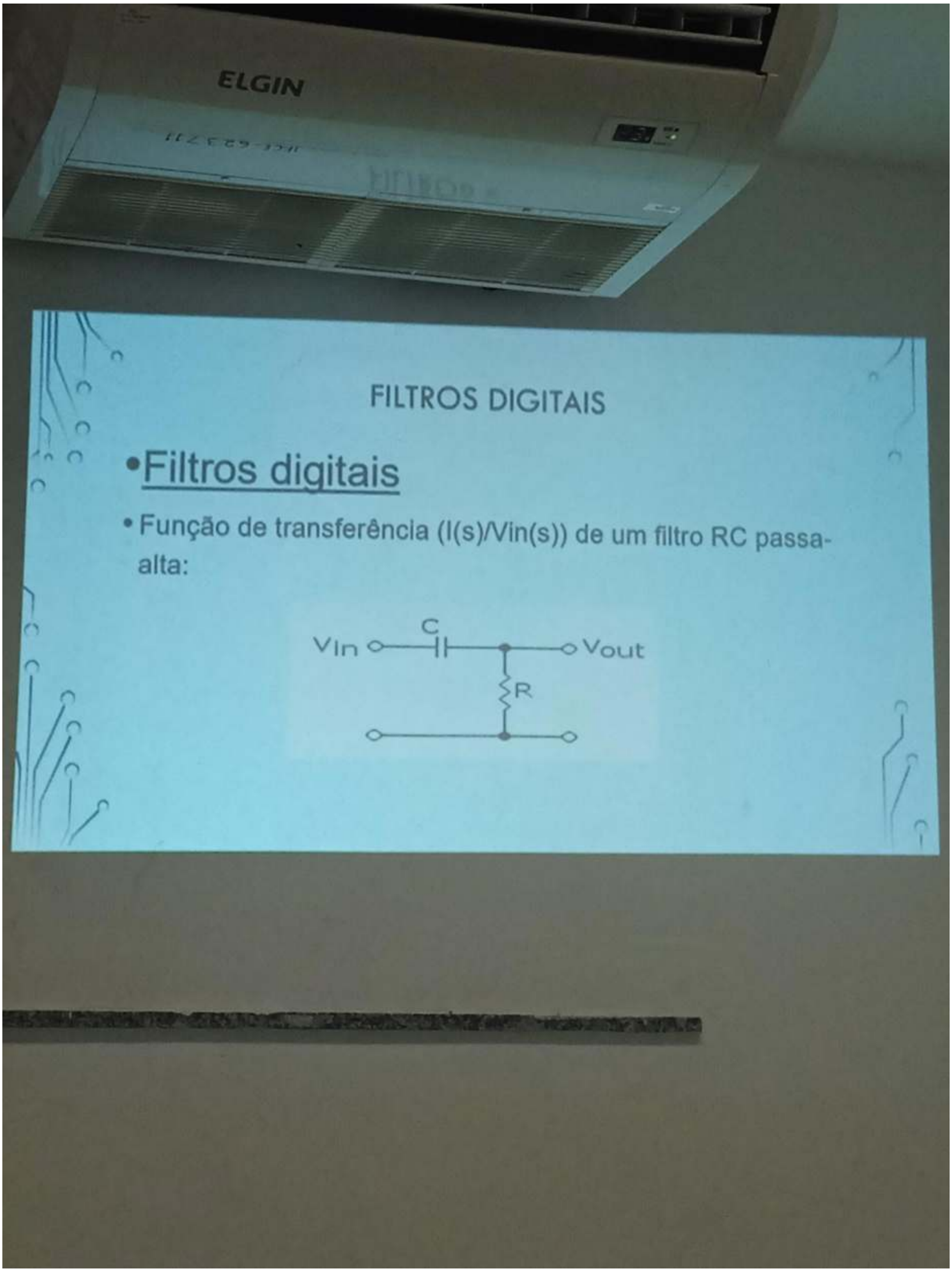
$$V_{out}(S) = \frac{\frac{1}{CS}}{R + \frac{1}{CS}} \cdot V_{In}(S)$$

$$V_{out}(S) = \frac{\frac{1}{\cancel{CS}}}{\frac{RCS+1}{\cancel{CS}}} \cdot V_{in}(S)$$

$$V_{out}(S) = \frac{1}{RCS+1} \cdot V_{in}(S)$$

$$\boxed{\frac{V_{out}(S)}{V_{in}(S)} = \frac{\frac{1}{RC}}{S + \frac{1}{RC}}}$$

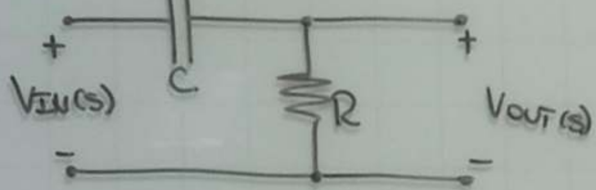
Filtro RC Passa-alta


ELGIN
FILTROS DIGITAIS
•Filtros digitais
• Função de transferência (I(s)/Vin(s)) de um filtro RC passa-alta:
C
Vin
Vout
R

=> FILTRO PASSIVO PASSA-ALTA:

DETERMINAR $\dfrac{V_{OUT}(s)}{V_{IN}(s)} = ?$

$$X_C(s) = \frac{1}{C.S.}$$



=> APLICANDO O DIVISOR DE TENSÃO:

$$V_{OUT}(s) = \frac{R}{R + |X_C(s)|} . V_{IN}(s)$$

$$V_{OUT}(s) = \frac{R}{R + \frac{1}{C.S}} . V_{IN}(s) \Rightarrow V_{OUT}(s) = \frac{R}{\frac{R.C.S + 1}{C.S}}$$

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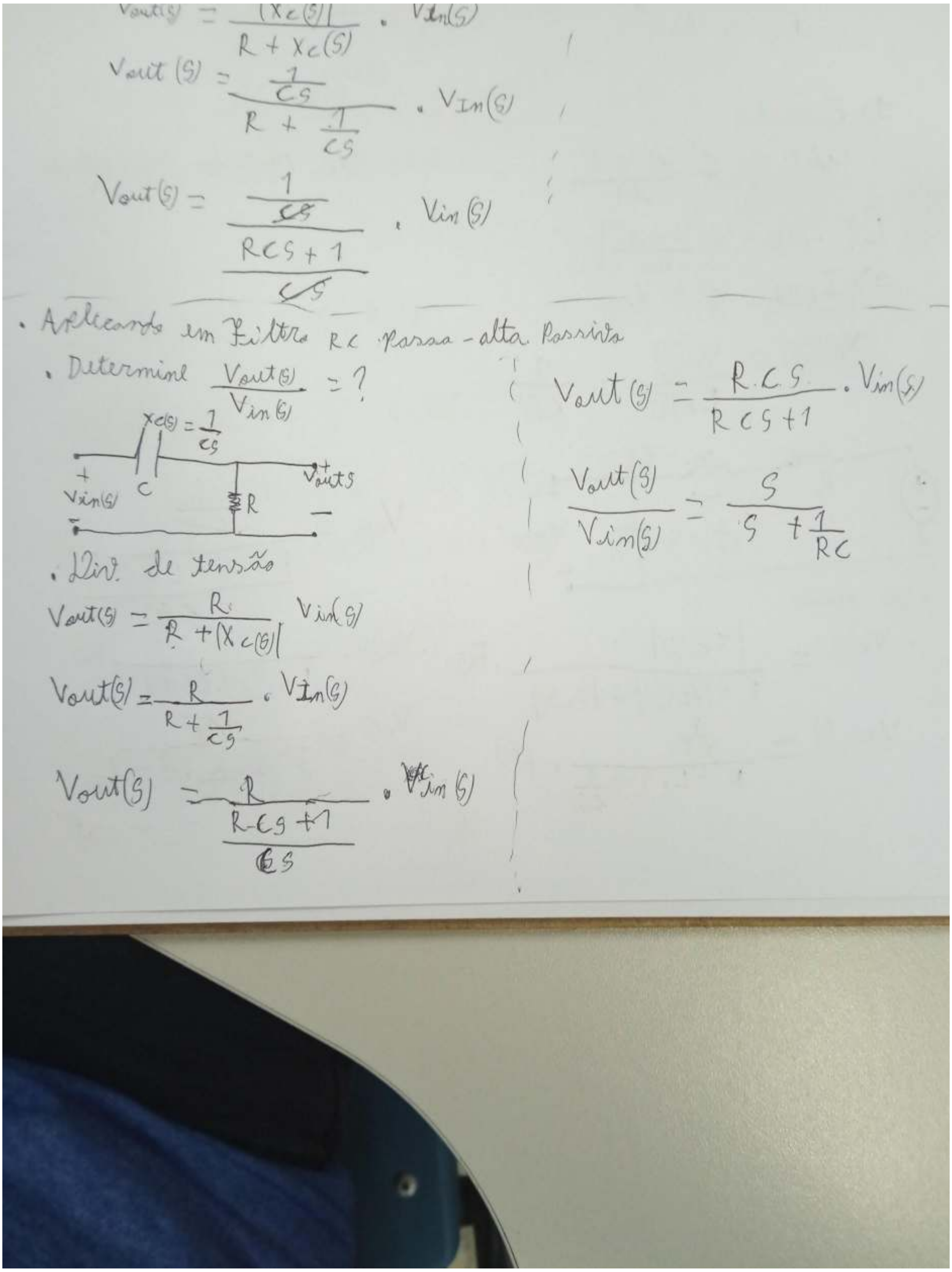
$$V_{OUT}(s) = \frac{R.C.S}{R.CS + 1} \cdot V_{IN}(s)$$

$$\boxed{\frac{V_{OUT}(s)}{V_{IN}(s)} = \frac{s}{s + \frac{1}{RC}}}$$



$$V_{out}(s) = \frac{|X_c(s)|}{R + X_c(s)} \cdot V_{in}(s)$$

$$V_{out}(s) = \frac{\frac{1}{Cs}}{R + \frac{1}{Cs}} \cdot V_{in}(s)$$

$$V_{out}(s) = \frac{\frac{1}{\cancel{Cs}}}{\frac{RCs+1}{\cancel{Cs}}} \cdot V_{in}(s)$$

- Aplicando em Filtro RC passa-alta passivo
- Determine $\frac{V_{out}(s)}{V_{in}(s)} = ?$

- Div. de tensão

$$V_{out}(s) = \frac{R}{R + |X_c(s)|} V_{in}(s)$$

$$V_{out}(s) = \frac{R}{R + \frac{1}{Cs}} \cdot V_{in}(s)$$

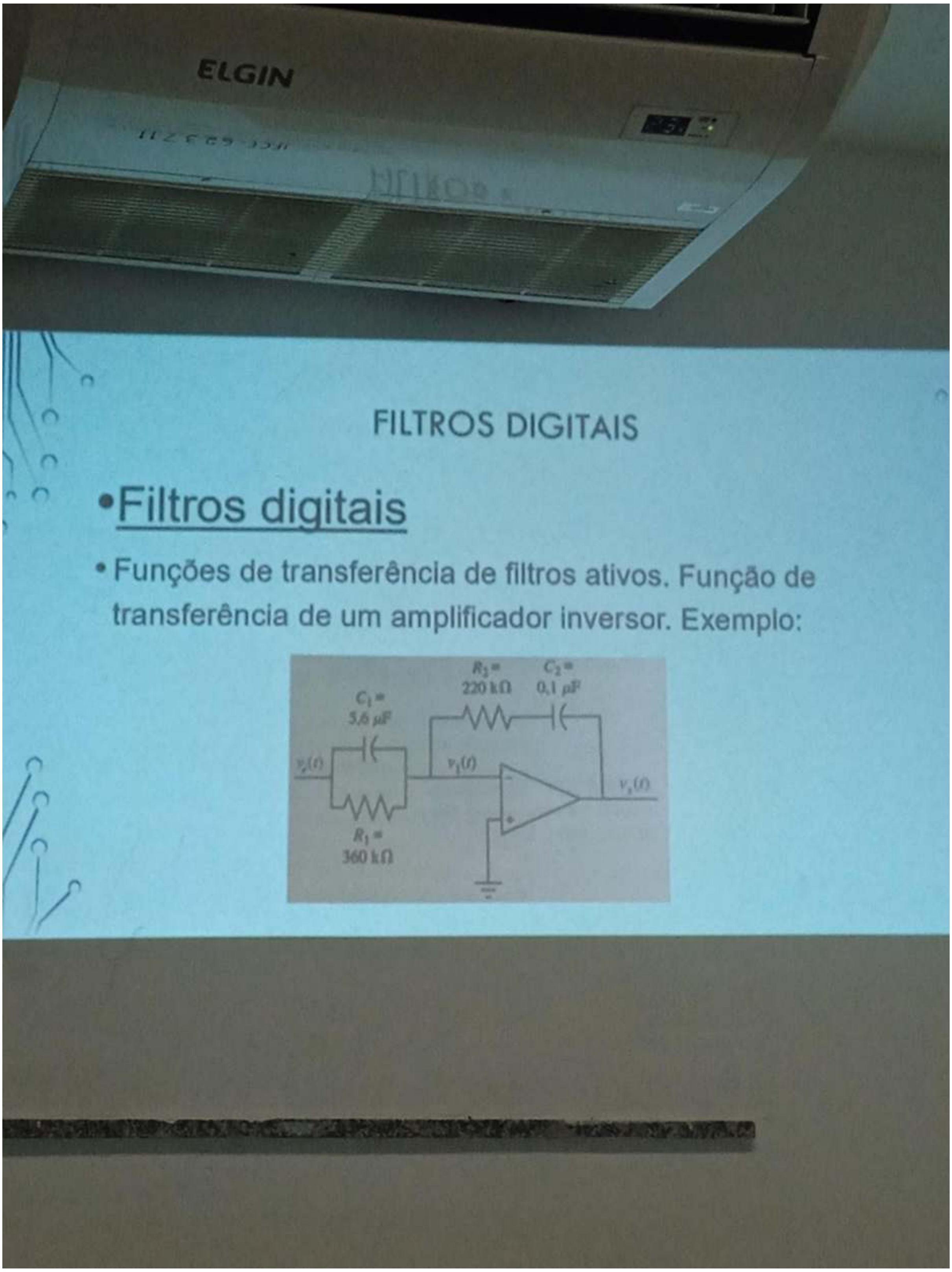
$$V_{out}(s) = \frac{R}{\frac{RCs+1}{Cs}} \cdot V_{in}(s)$$

$$V_{out}(s) = \frac{RCs}{RCs+1} \cdot V_{in}(s)$$

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{s}{s + \frac{1}{RC}}$$

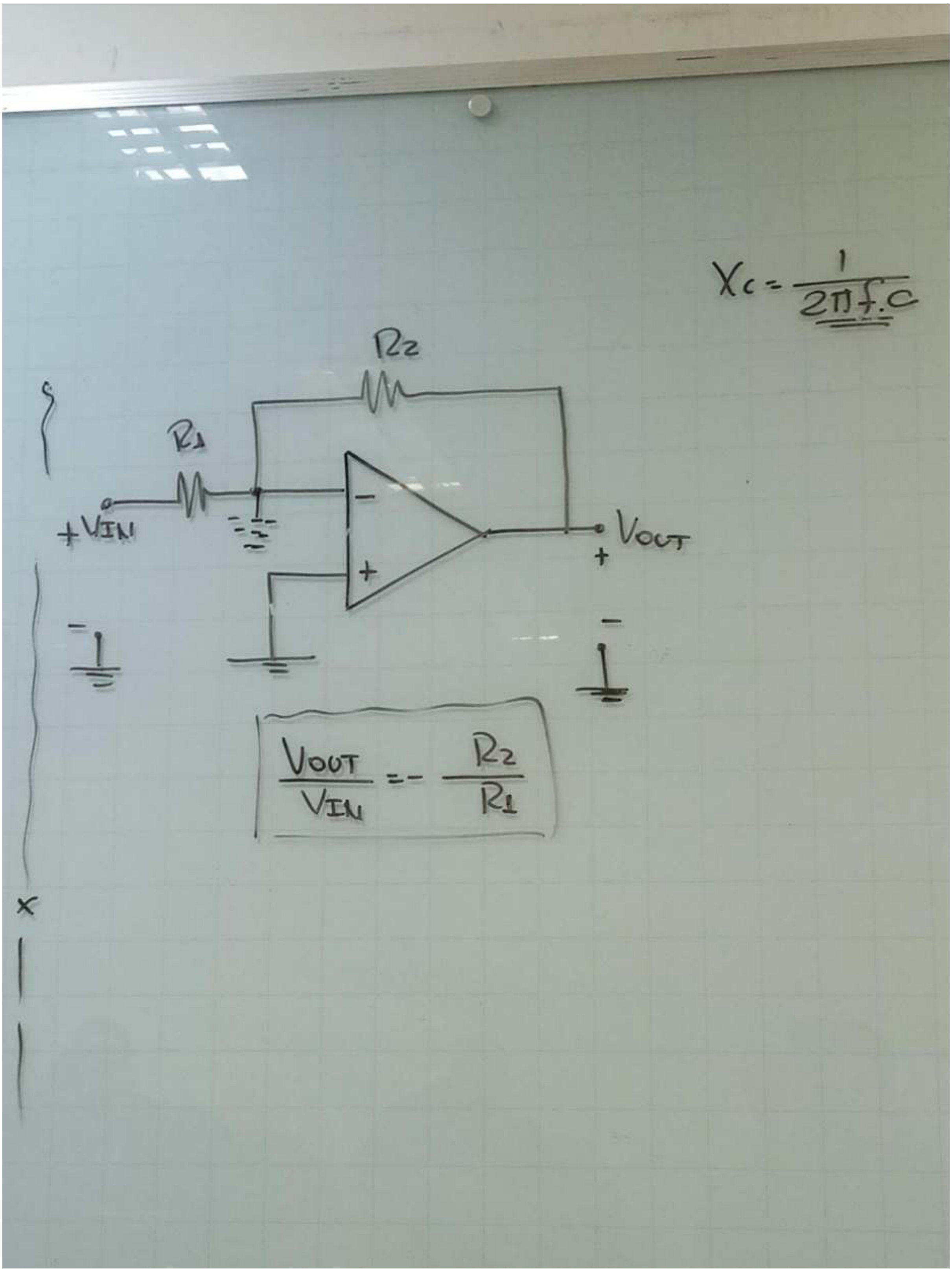
Funções de transferência de filtros ativos.

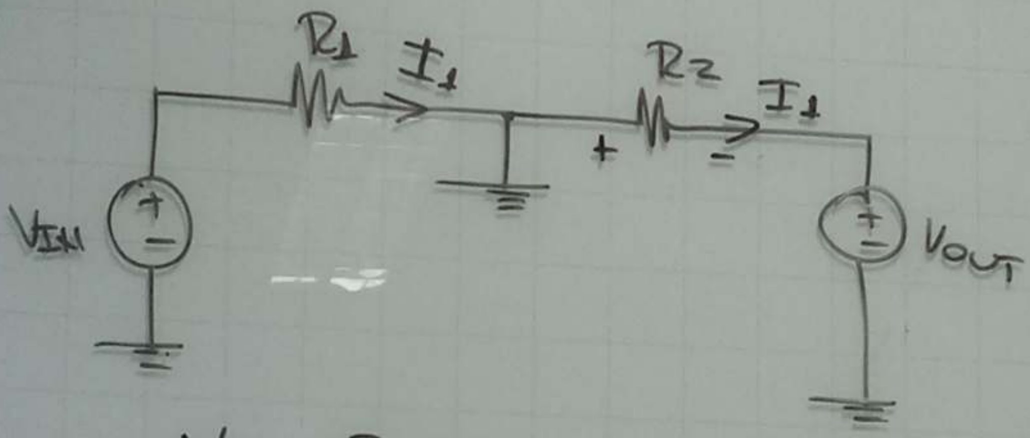
Função de transferência de um amplificador inversor



Obs: Filtro Passa-alta

Revisão Amplificador inversor


$X_C = \frac{1}{2\pi f . C}$
R2
R1
+VIN
VOUT
+
−
$\frac{V_{OUT}}{V_{IN}} = -\frac{R_2}{R_1}$



$$V_{IN} = R_1 \cdot I_1 \quad \Rightarrow \quad I_1 = \frac{V_{IN}}{R_1}$$

$$V_{OUT} = -R_2 \cdot I_1$$

$$V_{OUT} = -R_2 \cdot \frac{V_{IN}}{R_1} \quad \Rightarrow \quad \frac{V_{OUT}}{V_{IN}} = -\frac{R_2}{R_1}$$

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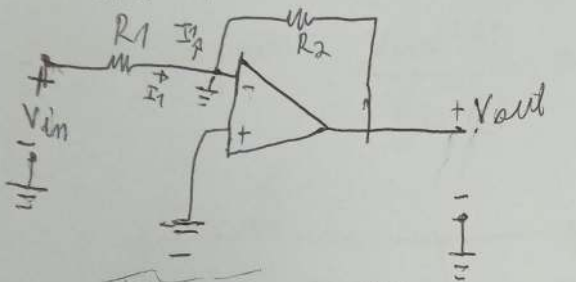
- Amplificador Inversor



$$\boxed{\frac{V_{out}}{V_{in}} = -\frac{R}{R_1}}$$

$\xrightarrow{\text{analise}}$ $\boxed{X_C = \frac{1}{2\pi f C}}$

- como chegar a relação

$V_{in} = R_1 \cdot I_1 \Rightarrow I_1 = \frac{V_{in}}{R_1}$

$V_{out} = -R_2 \cdot I_1$

$$V_{out} = -R_2 \cdot \frac{V_{in}}{R_1}$$

$$\boxed{\frac{V_{out}}{V_{in}} = -\frac{R_2}{R_1}}$$