CAREY, Gary Roland, 1940-
AN OPTIMAL DYNAMIC POLICY FOR THE DESIGN AND MAINTENANCE OF FLEXIBLE PAVEMENTS.

Texas A and M University, Ph.D., 1969
Engineering, industrial

University Microfilms, Inc., Ann Arbor, Michigan

AN OPTIMAL DYNAMIC POLICY FOR THE DESIGN AND MAINTENANCE OF FLEXIBLE PAVEMENTS

A Dissertation

by

GARY ROLAND CAREY

Submitted to the Graduate College of the
Texas A&M University in
partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

May 1969

Major Subject: Industrial Engineering

AN OPTIMAL DYNAMIC POLICY FOR THE DESIGN AND MAINTENANCE OF FLEXIBLE PAVEMENTS

A Dissertation

by

GARY ROLAND CAREY

Approved as to style and content by:

(Chairman of Committee)

(Head of Department)

(Member)

(Member)

(Member)

R R Hocking

(Member)

May 1969

ABSTRACT

An Optimal Dynamic Policy for the Design and
Maintenance of Flexible Pavements. (May 1969)

Gary R. Carey, B. S., Texas A&M University;
M. S., Texas A&M University;
Directed by: Dr. Glen D. Self

The primary objective of this research is to provide an analytical solution to the optimal flexible pavement design problem. The procedure is based upon empirical equations for deflection, performance, and traffic. These physical relationships are described in detail, both quantitatively and qualitatively. Along with other physically constraining factors, these equations provide a framework for the optimization techniques that are presented.

Optimization is with respect to providing, at minimum cost, a pavement structure that is acceptable for normal travel. Models are developed for predicting the relevant cost components of a pavement structure. These include initial construction, maintenance, and salvage value at the end of the analysis period. Maintenance is further divided into the categories of overlay construction, routine maintenance, seal coating, and the motorist's inconvenience costs during these operations.

The initial design problem is formulated as an integer programming problem and is solved by a specialized branch and

bound algorithm. Many initial designs are obtained, each the longest lived for a specific investment in initial construction. On the assumption that overlays are applied only when required, overlay policies are examined to determine the optimal maintenance policy for each of the initial designs. This will allow optimal design decisions to be made on the basis of cumulative cost rather than initial construction alone. An example problem is given in the appendix.

Finally, the flexible pavement problem is generalized in a specialized dynamic programming formulation. The first stage of the formulation is associated with initial construction, and the remaining stages are associated with future performance periods. Because of the complexity of the algorithm, iteration is required in order to evaluate the recursive relationship at each stage. For this reason, a special set of steps are given for stepping through the algorithm. A discussion is also contained concerning the applicability of the algorithm with respect to stage dependency under certain traffic conditions.

ACKNOWLEDGMENT

This research topic was originated by the staff of the Texas Transportation Institute at Texas A&M University. This staff was responsible for the development of the physical equations that form the basis of the proposed design procedure. I am extremely grateful to the entire staff for their suggestions, guidance, and comments during the time that this research was being conducted. I would like most to express my gratitude to Mr. Frank H. Scrivner, Research Engineer and Head of the Pavement Design Section of the Texas Transportation Institute. As well as serving on my graduate committee, Mr. Scrivner provided the invaluable technical assistance and background necessary for a firm understanding of the flexible pavement problem.

Special appreciation goes to Mr. Frank McFarland for his assistance in the economic considerations and the development of the models for detouring traffic. A special thanks also to Mr. Jim Brown of the Texas Highway Department for his consultations to insure that the end product of this research would be realistic and amenable to continued research.

I am extremely grateful to Dr. Glen D. Self of the Industrial Engineering Department for serving as chairman of my graduate committee. He originally suggested this research topic and without his encouragement and untiring support, this research would not have been completed at this time.

I would also like to express my appreciation to Dr. A. W. Wortham, Head of the Industrial Engineering Department; Dr. W. L. Meier, Industrial Engineering Department; Dr. R. R. Hocking, Institute of Statistics; and Dr. J. E. Pearson, Dean of the College of Business Administration, for their time and effort spent on my behalf while serving as members of my graduate committee.

Acknowledgments would not be complete without expressing my utmost appreciation to my loving wife, Judy, for her extreme patience and support during the long hours that this research was being conducted. In addition, she typed both the rough draft and the final manuscript of this dissertation.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER I

INTRODUCTION

The construction of roads and highways in the world today is a multi-billion dollar operation. Hundreds of different types of construction designs are required for the many needs that roads must satisfy. Highways, because of higher traffic, require a more durable design than that of less frequently used roads. In addition, geographical location has an important environmental role in the determination of the requirements for a specific design. These are only two of the possible considerations that go into the selection of a pavement design for a specific use.

There are two distinct classes of pavements: rigid and flexible. Concrete pavements are classified as rigid pavements, while asphalt pavements are commonly referred to as flexible pavements. The structural behavior of rigid pavements is well defined because the theory of elasticity appears to be directly applicable to these structures. Flexible pavements, however, generally have a multiple layer construction, each layer consisting of a different material. Each layer's exact contribution to the overall strength of the pavement is very difficult to determine because these structures do not, in the opinion of many investigators, deform under load in accordance with elastic

---

"The citations on the following pages follow the style of the Journal of the Operations Research Society of America."

theory. As a result, classical methods of stress analysis may not be meaningful.

Even now, very little actual theory exists for predicting flexible pavement behavior under load conditions. Because of this, pavement designers have long been handicapped when it came to predicting a useful life of the pavement. The effects of variations in climate, material strengths and other factors have not been fully understood. Even more important, there has been no definite basis for measuring pavement strengths. How then could pavements be constructed, without overdesigning, so that some degree of assurance could be placed upon its anticipated life?

The concept of using vertical deflection under load as the criterion for measuring flexible pavement strength was first implemented by the Western Association of State Highway Officials (WASHO) in a series of experiments which began in 1952. These experiments had quite an impact on the field of pavement design for they paved the way for another more significant series of experiments conducted by the American Association of State Highway Officials (AASHO) beginning in 1961. Now a milestone in flexible pavement research, the AASHO Road Test provided the missing link - a dependent measure of road performance called the "serviceability index". A relationship was also formed at that time to relate this performance factor to design and load factors. In addition, the numerous data that were collected still serves as the basis for much of the present day design research. A complete review of these historical activities along with later events is given in Chapter II.

Objectives

In a recent study by the Stanford Research Institute [19], it was reported that the total cost of highway construction during the 1960's may reach a total of $125 billion, with an additional $50 billion for maintenance of highways and streets. The Interstate system alone is expected to cost $40 billion by the time it is completed. Even a small savings in the cost of each mile of roadway could amount to a billion dollars savings each year. In order to accomplish this, both federal agencies and state highway departments should have all of the pertinent facts that are available concerning the economics of pavement design. Unfortunately, such facts are not generally available at this time. Very few studies of paving economics have been made, and the few that have been made are either applicable to rigid pavements or they are incomplete.

This research is being conducted under the auspices of the Texas Transportation Institute at Texas A&M University through contracts first granted in 1962 by the Texas Highway Department and the Bureau of Public Roads. Simply stated, the broad objective is to make available to the Texas Highway Department a recommended procedure for the design of flexible pavements.

In perspective, the problem has two logical phases. The first and possibly the most formidable was and still is the task of developing accurate empirical relationships to be used as predictors. The first of these is a mathematical relationship for describing transient deflection under traffic loads as a function of design parameters and variables. The second is that of relating this deflection to a performance criterion so that a usable life for the pavement can be predicted. A complete description of these equations as well as all variables and parameters is given in Chapter III. Chapter II contains a complete review of Phase I activities along with a literature review of similar research efforts.

The principal focus of this research is upon Phase II of the overall project. Broadly speaking, the objective is the detailed economic analysis of all possible pavement design alternatives based upon physical relationships that were developed during Phase I of the project. This analysis involves the development of an algorithm for the selection of the most optimal and several near optimal alternative designs from the large number of possible alternatives. The procedure takes into account both physical and cost variables, and provides a means for making design decisions based on probable overall costs, rather than on initial construction cost alone. Because of the number of variables involved, and the need to investigate all possible designs meeting selected criteria, initial attempts to prepare the usual curves or nomograms for the designer's use were quickly abandoned. Instead the algorithm calls for the integration of the previously described empirical relationships for deflection and performance, as well as time-traffic data supplied by the Texas Highway Department and several pertinent cost models which must be developed.

These models attempt to include all revelant cost considerations that might be important from a design comparison standpoint. The cost area, both present and future, are

I. Initial construction cost

II. Future costs

  A. Overlay construction costs

  B. Annual routine maintenance costs

  C. Seal coat costs

  D. User costs during each type of maintenance

III. Salvage value

Considerable attention is given to each of these models in Chapter IV.

In addition to the mathematical and verbal descriptions, each cost model is also discussed with respect to the assumptions made and the data requirements. In some cases the user will be expected to supply the data while in others the data development will be enclosed.

The user cost models deserve special mention. User cost is described to be the increase in motorist cost due to time delay and vehicle operation cost as a result of an overlaying operation. There are five different models presented, each depicting a different pattern for detouring traffic. It is felt that this section is particularly important because of its possible application to areas other than pavement design such as the cost of detouring traffic because of an accident. Another possibility that is presently under consideration by the Texas Transportation Institute is the application to traffic delays during the minor repair of bridge decks.

Comments on Applications

Research in the field of pavement design is still in the embryonic stage. For this reason a deliberate attempt has been made in formulating the algorithm to provide for ease of change, so that as new findings are made in flexible pavement research, they can be incorporated into the algorithm with a minimum of effort. Meanwhile, the algorithm is recommended as an aid not only to the design engineer but also to the research engineer in establishing where emphasis should be placed in pavement research.

There is a large number of design parameters used by the algorithm. In some cases, the parameters may also be treated as variables so that a sensitivity analysis can be performed. There is one principal warning at this point; even though a tremendous effort has been made to collect accurate data, there is no guarantee at this time that all relevant factors have been taken into account. In some cases, available data is representative of only a few regions in Texas. In other cases, "good" data is simply not available. As a result, some of these analyses cannot be expected to be as accurate as appearance might indicate.

There is one additional application that is noteworthy. The algorithm was originally intended as a guide to designing new pavement projects. Recently, a feature has been added so that optimal overlaying strategies can also be determined for existing pavements.

CHAPTER II

REVIEW OF THE LITERATURE

This study represents a comprehensive formulation of a flexible pavement design process that provides for the integration of both the technological and economic attributes of flexible pavements. The development of the design process has only begun to approach a degree of refinement in the last three years and in each case, independent research organizations have approached the problem differently. The logical continuation of the research toward the development of economic analyses has not been done in any detail. Individual cost components have been studied but not with the intent of relating them to the larger economic problem. It would seem that the technological development of the flexible pavement design process and the economic analysis of pavement design alternatives would be independent areas of study. This has not been the case. It has been found that the required optimization procedures involved in an analysis of this type are a function of the complexity of the problem formulation. This will be demonstrated later in this chapter.

This study relies heavily upon the particular design approach taken by the Texas Transportation Institute in its role as a research agency of the Texas Highway Department. Even though the technological development of the approach is outside the scope of this research, it is necessary to at least review the concepts involved. This is done in Chapter III. The literature review contained in the remainder of this chapter encompasses the primary areas of:

1. a brief historical review of the general flexible pavement design problem and the resulting flexible pavement design procedures relating to the approach assumed in this study
2. related pavement economic studies
3. branch and bound approaches as related to the type of problem considered in this research.

Review of the Flexible Pavement Design Problem

Before 1950, there was virtually no state-of-the-art associated with the field of flexible pavement design. As is often the case, even today, flexible pavements were designed by engineers who had enough experience to give an "educated" guess as to what type of construction might be required to withstand the physical demands that would be placed upon the pavement. As would be expected, the resulting pavements had a very wide variance between either falling very short of anticipated life or being greatly overdesigned, resulting in unnecessary costs. A partial solution to the problem was to be found in the analysis of pavement behavioral data collected from several road tests. Although the results of these tests are now considered to be relatively insignificant, the door was opened for a new and more significant test sponsored by the Western Association of State Highway Officials, beginning in 1951. This test began with the Highway Research Board supervising the construction and testing of a road to determine the effect of various axles loads on certain designs of flexible pavements. This project was named the WASHO Road Test. The WASHO Road Test - Part 1 [73] contains a complete description of the objectives as well as the construction, materials, vehicles, equipment, and techniques employed. The WASHO Road Test - Part 2 [74] presents the results of the test, including a summary of all data compiled. Two aspects are particularly significant about this test (1) it was the first experiment of its type to be performed under completely controlled conditions, and (2) the concept of using vertical deflection or displacement of the pavement under load as the principal criterion for measuring the strength of flexible pavements was formalized. There was still no theory available, however, to determine the useful life of a pavement based upon these deflection measurements. This was to be provided through still another series of road tests sponsored by the American Association of State Highway Officials (AASHO) beginning in 1961.

The initial objective of the AASHO Road Test was to adopt and quantitatively define a dependent variable between design parameters and pavement performance called the "present serviceability index", PSI. This index, first reported by Carey and Irick [7], is a subjective measure representing the public's opinion of the relative quality of the pavement surface and is primarily dependent upon roughness. It is a non-dimensional quantity that is measured on a linear scale ranging from 0.0 to 5.0. Average beginning serviceability indices of today's flexible pavements usually range from 4.0 to 4.5; while pavements with indices of about 2.0 are considered to be unacceptable for normal traffic.

The AASHO project, costing $27 million, was supervised by the Highway Research Board of the National Academy of Sciences and was to be the largest statistically designed experiment in the world. A total of 836 test sections were constructed to several different design specifications and then subjected to very high concentrations of load applications, 18 hours per day, for two years. Pavements consisting of three different layers were constructed at most of the test sections, each layer having three levels of depth. As a result, 3x3x3 factorial designs in complete blocks represented a single replication of an individual experiment. Experiments differed because of other factors, such as construction materials and types of load applications. Several independent factors, including serviceability index, were observed every two weeks at each test section. Finally an empirical relationship, called the performance equation, was developed from this data for predicting PSI as a function of design, number and type of load applications, and other analysis parameters. The performance equation is represented by

$$PSI = 4.2 - 2.7 \left(\frac{W}{\rho}\right)^{\beta} \tag{2.1}$$

where W is a specific number of load applications. The quantity β is a shape parameter for the serviceability loss function and ρ is equal to the number of load applications at which PSI = 1.5. These are given by

$$\beta = 0.4 + \frac{B_0(L_1+L_2)^{B_2}}{(D+1)^{B_1} L_2^{B_3}} \tag{2.2}$$

$$\rho = \frac{A_0(D+1)^{A_1} L_2^{B_3}}{(L_1+L_2)^{A_2}} \tag{2.3}$$

where D is a thickness index given by

$$D = b_1D_1 + b_2D_2 + b_3D_3 \; . \tag{2.4}$$

The remaining unknowns are parameters which were computed from the accumulated data. The quantities b_1, b_2, and b_3 are relative strength coefficients of the materials having thickness equal to D_1, D_2, and D_3 respectively. It is noteworthy that the quantity 4.2 in equation (2.1) was the average beginning serviceability index for all test sections. The significance is that this figure would probably be different for other experiments of this nature. Correspondingly, this equation would not necessarily be applicable to other pavements if the beginning PSI was not 4.2 because of differences between other factors that were constant at the test site.

This experiment and its results are contained in Report 5 of the AASHO Road Test [72]. Since 1962, numerous articles have appeared in the literature that generally discuss the serviceability index. Of particular interest is the work by Hutchinson [33,34,35,36] who has been the principal reporter on the subject.

Report 5 of the AASHO Road Test [72] has become a "Bible" to pavement design engineers in the sense that it represents the only complete set of data describing pavement performance throughout the useful lives of various pavement structures. Unfortunately, the data is valid only for the conditions at that site. Virtually all pavement design research since 1964 has been toward generalizing the use of this data in order that pavement performance could be related to design as a function of the in situ conditions of climate, subgrade strength, and material parameters. To be significant, any flexible pavement design procedure that is developed must accurately fit the AASHO Road Test data when translated to those conditions.

Another significant result of the AASHO Road Test was the development of a deflection equation (2.5) relating the design parameters of a pavement structure to the deflection that the pavement would sustain from the application of various loads. This empirical relationship is seen to be a function of the thickness index D previously defined.

$$\log_{10} d = \log_{10} B_0 + B_1 \log_{10} L_1 - B_2 \log_{10}(D+1) \qquad (2.5)$$

where d = deflection in thousandths of an inch

$D = C_1D_1 + C_2D_2 + C_3D_3$

L_1 = axle load in kips

B_i = constants determined by regression analysis and vary with climate.

Another report, called the AASHO Interim Design Guides [71], was developed in 1962 by the AASHO Design Committee and was intended to be a guide to the application of the newly developed design procedures. Also extensively discussed were suggested methods of obtaining data and the concept of equivalent load applications. In 1962, the Interim Guides were issued to the states to be used for a one-year trial period in parallel with their existing procedures. The purpose of this trial period was to allow the states to review the design procedures, and to check their validity in actual practice. After the trial period, and subsequent receipt of comments by the states, the AASHO Design Committee did not consider it necessary to revise the Guides or the instructions at that time, and they were retained in their interim status.

The concept of equivalent load applications deserves some comment. During the Road Test, traffic was accumulated on each test section by heavier than average vehicles with identical axle loads and axle configuration (as opposed to mixed traffic). This was done for two reasons:

1. to preserve consistency in the statistically designed experiment
2. to rapidly subject the test sections to a high volume of traffic so that the results could be analyzed after a reasonable time period of two years.

To use the AASHO Interim Guide performance equation, mixed traffic must be converted to equivalent wheel load applications to account for heavy vehicles being more detrimental to pavements than lighter vehicles. The 18-kip single axle load was used as a base for converting mixed traffic, since this was the legal load limit in most states at that time.

Most states collect loadometer data which accumulates the number of axles observed within each wheel load group. These groups are usually in 2000 pound intervals. State traffic agencies can then use this data to predict the number of axles of each load group expected during a future period of time. The equivalent wheel load for each group is then combined to give one number that is representative of mixed traffic as follows:

$$W_t = N_t \sum_{i=1}^{n} P_i e_i \tag{2.6}$$

where W_t = total equivalent 18,000 pound single axle loads due to mixed traffic during time t

N_t = total number of axles expected in time t

P_i = fraction of the axles in the i^{th} load group

e_i = load equivalency factor for a given axle group.

It is noted that one important assumption is that the percentage of axles in each load group remains constant over time t. The empirical formula that was developed for computing the 18-kip load equivalency factors for flexible pavements is

$$e_i = \frac{(L_i+M)^{4.79}}{(18+1)^{4.79}} \cdot \frac{10^{G/B_{18}}}{10^{G/B_i}\ M^{4.33}} \qquad (2.7)$$

where L_i = average axle load for a given axle load group and type

M = designation of axle type (M=1 for single axles and M=2 for tandem axles)

G = $\log \dfrac{P_0 - P_N}{P_0 - 1.5}$

P_N = terminal serviceability of the pavement

B_i = the destruction factor for loads in the i^{th} load group and is dependent upon the type of pavement structure.

The equivalent wheel loads derived in this procedure represent the totals for all lanes in both directions. Within this, most states use a 50-50 directional split, with a note that special conditions may warrant some other split.

The move toward generalizing the use of the AASHO Road Test data and the deflection equation (2.5) began through efforts by the National Cooperative Highway Research Program (NCHRP). The individual states were to set up satellite programs for extending the road test results to their respective regions. Irick and Hudson [41] reported on the guidelines that were to be followed to insure that common measurements were taken and to insure the maximum amount of continuity and coordination between states. A survey by Huff [32], on the use of satellite studies, revealed that 60 percent of the states replying to a questionnaire had not initiated such studies, and that only a few of these states indicated that such work would be considered in the future. Several states, however, have become forerunners in that their findings have already been extended to areas outside the scope of the original satellite objectives. Of those procedures that have been published, most have preserved the basic form of the original equations developed by the AASHO Committee. The procedure developed by the Texas Transportation Institute, though based in part on AASHO Road Test data, does not employ the AASHO Road Test performance equation (2.1); it will be discussed in Chapter III. Here the intent is to briefly describe the progress of the forerunning states as well as that of some independent researchers.

Painter [57] was one of the first to develop a more general mathematical expression for determining the present serviceability index.

$$P_t = P_0 e^{-bW_t} \tag{2.8}$$

where P_t = present serviceability index at time t

P_0 = the same index at start of traffic (t=0)

b = deterioration rate parameter

W_t = millions of accumulated load applications to time t.

The deterioration rate b was found to be constant throughout the year, except for the spring thaw periods. During these times, the deterioration rate increased to some higher rate that could be computed from

$$b_i = b_0[1+k(X_i-2X_i^2+X_i^2)] \tag{2.9}$$

where b_i = value of b at time i

b_0 = value of b prevailing before the thaw period

k = thaw parameter

X_i = fraction of the spring thaw already passed at time i.

Painter developed an equation relating the rate, b_0, to design and load that contained terms quite similar to the design index, D, previously mentioned,

$$\ln b_0 = a_0 + a_1D_1 + a_2D_2 + a_3D_3 + a_4L . \tag{2.10}$$

Painter further describes a solution to the problem of determining the minimum cost design. Equation (2.10) along with restraints on the terminal values of the performance measures, make up the constraining equations for a simple linear programming problem whose objective is to minimize initial construction cost. The starting point of this formulation, however, is that the pavement is designed for a specific accumulated number of load applications, thereby fixing the terminal values of the performance measures. In fact, a different "optimal" solution results from designing for more or less traffic. This is the drawback of optimizing with respect to initial construction cost rather than total cost over a long period of time.

Another approach has been taken by Vaswani [77] of the Virginia Highway Research Council. In his discussion, Vaswani states that pavements could be designed on the basis of elastic theory and load deflections. He also contends that Burmister's elastic theory [6] shows that contrary to the AASHO deflection equation (2.5), the deflection of a pavement is not merely a summation of the strength of the individual layers, but also a ratio of the strengths of adjoining layers. Thus, deflection would not only be a function of b_1D_1 but also of

$$\frac{b_1D_1}{b_2D_2 + b_3D_3 + \cdots} \quad . \tag{2.11}$$

If, as Vaswani states, the ratios of the strengths of adjoining layers remain relatively constant, the AASHO deflection equation would be applicable. On this basis, Vaswani developed a theory comparable to that of AASHO.

constraining equations for a simple linear programming problem whose objective is to minimize initial construction cost. The starting point of this formulation, however, is that the pavement is designed for a specific accumulated number of load applications, thereby fixing the terminal values of the performance measures. In fact, a different "optimal" solution results from designing for more or less traffic. This is the drawback of optimizing with respect to initial construction cost rather than total cost over a long period of time.

Another approach has been taken by Vaswani [77] of the Virginia Highway Research Council. In his discussion, Vaswani states that pavements could be designed on the basis of elastic theory and load deflections. He also contends that Burmister's elastic theory [6] shows that contrary to the AASHO deflection equation (2.5), the deflection of a pavement is not merely a summation of the strength of the individual layers, but also a ratio of the strengths of adjoining layers. Thus, deflection would not only be a function of b_1D_1 but also of

$$\frac{b_1D_1}{b_2D_2 + b_3D_3 + \cdots} \quad . \tag{2.11}$$

If, as Vaswani states, the ratios of the strengths of adjoining layers remain relatively constant, the AASHO deflection equation would be applicable. On this basis, Vaswani developed a theory comparable to that of AASHO.

Materials, Research & Development, Inc., located in California, has also conducted considerable research in the area of flexible pavement design. Their most recent contribution was in 1968 when they undertook to

1. collect, review, and summarize current state highway department pavement design procedures,
2. develop recommendations for revisions to the AASHO Interim Guides.

Their report [54] is based upon the results of a recent survey in the form of a comprehensive questionnaire directed toward highway agencies of the 50 states. The questionnaire consisted of 72 questions prepared to elicit specific information relative to the procedures currently being used for the design of flexible, rigid, and overlay pavements. Not only is a description of an idealistic design procedure, based upon many proposed minor changes in the AASHO Interim Guides, presented, but an outline of the more important current research projects being carried on by the states is presented as well. The pavement design procedure currently used by Texas is presented at the beginning of Chapter V. The idealistic design procedure differs widely from that of Texas and is, therefore, presented here for comparison. It is noteworthy, that this idealistic procedure is currently being used by a majority of the states surveyed.

The basic design equation (2.12) encompasses all of the variables associated with the design procedure and is given as

$$SN = \frac{1.051\ W_t^{.1068}\ R^{.1068}}{10^{.03973(SS-3)}\ 10^{.1068G_t/B}} - 1 \qquad (2.12)$$

$$G_t = \log \frac{P_0 - P_t}{P_0 - 1.5} \qquad (2.13)$$

$$B = 0.40 + \frac{0.081\ (L+N)^{3.23}}{(SN+1)^{5.19}} \qquad (2.14)$$

$$SN = b_1D_1 + b_2D_2 + b_3D_3 \qquad (2.15)$$

where SN = structural number (or index) of a pavement structure that is a combination of the various layers

W_t = total equivalent 18-kip axle loads expected during the design life of the facility

R = regional climate factor

SS = soil support term for the foundation material

P_0 = initial serviceability index of the pavement

P_t = terminal serviceability index of the pavement

b_1, b_2, b_3 = structural layer coefficients of the respective layers and are representative of material quality

D_1, D_2, D_3 = thickness of the respective layers of the pavement structure.

Equations (2.12), (2.13), and (2.14) are solved iteratively for SN as a function of the predicted traffic in terms of wheel load data, the value of soil support, and the regional factor. Finally, the thickness D_i of each layer may take on any realistic value such that equation (2.15) is satisfied. Nomographs have been constructed to facilitate the solution of these equations. Although no optimization procedure is given in the report, it is expected that a simple linear programming formulation could be used to solve for the respective D_i that result in the minimum cost of construction.

There are three very important points that deserve some discussion. These are with respect to the local soil support value, the local region factor, and the assumption of a four layer design. It is evident that these were constant factors, applicable only to the Road Test or the same conditions elsewhere. The principal objective of the satellite studies was to make it possible to establish the generalized effect of these factors. Since very few satellite studies were conducted, it was apparent that some other means had to be used to obtain the same result. The foregoing discussion is concerned with the methods that have been developed in order to generalize these effects.

The only known soil support value is that of the Road Test. Previously, soil support values for local conditions had to be assumed. The assumption has been made [54] that the application of layer elastic theory should provide a step toward a rational soil support scale. This must be true even though this is an unproven theory when applied to flexible pavements, it is obviously a more rational approach than guessing. On this basis, however, a procedure is presented whereby a soil support value may be developed on the basis of resilient modulus tests. An analysis has been made for several levels of both surface and subgrade modulus. It is hypothesized that subgrade modulus can be predicted and then correlated to a soil support value. Other soil support values may be found by interpolation. The fact remains that the correlations are made with respect to unsupported soil support values because there is virtually no data available for determining general soil support values.

The second factor is associated with environment and is called the regional factor. A limited sensitivity analysis of the parameters of the design equation has shown that an error in selecting the regional factor can have a pronounced effect on the solution. Of the parameters considered in the design equation, it is the least well defined. On the basis of replies to the questionnaire mentioned earlier, hypothetical contours of equal regional factors could be constructed for the United States. There would be, however, only seven different levels of this factor. Texas has conducted a similar but independent study and defined 16 different levels of a similar type of regional factor which are shown in Table (4.1). For this reason, there should be a natural tendency to be cautious about relying heavily upon the values of this regional factor. Indeed, if there is ever to be a rational design approach which will incorporate the effects of environment and region, it is almost imperative that a systematic program be layed out, with field test sections throughout the United States.

The final point of discussion is concerned with the assumption of a four layer design. This was the case in all test sections of the Road Test and even the present design equation is appropriate for only four layer design consisting of a surface, base, subbase, and a subgrade or foundation. There is no way, short of extrapolation of the design equation, to consider any other number of layers in a pavement structure. This is a very limiting factor where optimization of pavement designs is anticipated.

Other than the specific review that has been given, there is a multitude of articles in the literature of the type indicated by [8,9,22,38,66]. The majority of these articles simply relate the experience gained by the states' participation in either the satellite program or in other state supported research. In each case, however, investigators have invariably chosen to consider much small problems than those of interest in the remainder of this manuscript. As a result, they contribute very little to the current research effort.

Pavement Economic Studies

Virtually all previous highway cost studies have been directed toward individual aspects of the overall cost of pavement structures. There are basically two reasons for this. The first has to do with the size of these individual cost components. As will be seen, there are very few small problems in modeling the individual cost components. Very often, the required data is simply not available in a usable form. This makes it rather difficult to infer the desired cost relationships. In other cases, the amount of data that is available is so voluminous that it is difficult to inspect the data and identify particularly important areas.

The second reason that comprehensive economic studies, based upon overall cost, have not been made can be attributed to a lack of technology in the field of pavement design. As was described earlier, only recently has the pavement design process been quantified so that reasonably long analysis periods could be considered.

Comparable economic theories can be applied to the analysis of the rigid and flexible pavement design processes. Although these respective economics will eventually have to be compared to determine which type of design is most economical, this study is specifically concerned with only the flexible pavement analysis. Another assumption is that it is not necessary to specifically include all types of pavement costs. For example, if curbs are to be included in the design, their cost will be independent of the types and thicknesses of the materials used in the construction of the pavement. Costs of this type are not included in this analysis.

Of the types of costs considered in this study, only two have received notable attention in the literature - maintenance costs and user (motorists') costs as a result of the maintenance.

The formulation of a cost model to predict future maintenance costs is very difficult to generalize. One reason is that there is seldom a single procedure for maintenance operation that is fully applicable to a number of different locations. Limitations related to design and operational characteristics of the roadway, types of equipment available, and a variety of local factors influence these procedures. Within these constraints, most states have compiled sufficient historical data to be of use in an analysis of this type.

Fundamental to the development of cost models for maintenance costs is a definition of "maintenance". A generally accepted definition that originally adopted by the American Association of State Highway Officials [52] states that

> "...highway maintenance is the act of preserving and keeping, including all its elements, in as nearly as practical its original as-constructed condition or its subsequent improved condition; and the operation of a highway facility, and services incidental thereto, to provide safe, convenient, and economical highway transportation."

In a discussion of maintenance costs, it is usually difficult to distinguish maintenance expenditures from betterment expenditures. As is common, betterment expenditures are those that improve the condition of the pavement and are, as a result, considered construction rather than maintenance. In this manuscript, resurfacing with bituminous material is classified as betterment if the resurfacing depth is equal to or greater than one-half inch and as maintenance if the depth is less than one-half inch.

The Highway Research Board's Maintenance Cost Committee has made the earliest and most numerous attempts to analyze historical records and develop relationships between maintenance costs and the factors influencing these costs. In 1956 the Committee reported [58] on a study of 560 miles of roads of various surface types which showed relationships between surface maintenance costs and traffic, road thickness, and subgrade. The relationships were converted to curves, which could be used to predict surface maintenance costs for a given road. Procedures for using the relationships were illustrated, but no record was found indicating that the relationships have ever been tested in practice.

As evidenced by recent studies [20,50,51,68], the State of Louisiana has been particularly progressive in its attempts to develop quantitative measures of maintenance requirements. Of special interest has been the development of a formula to predict maintenance costs on a per-mile basis [44]. The development consisted of defining a "base mile" of surfaced road in terms of a fixed age, traffic volume, subgrade classification, and width. A section of road meeting the criteria was given a factor of 1 and a fixed annual per-mile maintenance cost. The maintenance costs varied, depending on sections of the state, so that local cost factors were reflected. A schedule was determined for each of the components, age, traffic, subgrade and width, to reflect increases in cost due to variations from the base mile criteria. These schedules were generated by examining the maintenance costs on roads which were adequately maintained and were different from the base mile. Using trial-and-error techniques, factors were determined to reflect cost variations. With this information it became possible to examine any road, assign it appropriate modifying factors, and predict its future maintenance costs. It is entirely possible that a similar approach could be standardized to predict the future maintenance costs of yet unconstructed roadways.

An Ohio maintenance study initiated in 1961 is still in progress [55]. The objective is to identify major factors that influence maintenance costs with the intent of taking steps to reduce the influence of these factors to a minimum.

Many other states have reported they are conducting maintenance studies similar to those that have been described. A rather comprehensive bibliography of some 300 references can be found in NCHRP Report 42 [39]. In addition, a number of these references contain figures obtained through recent maintenance cost studies along with procedures that were carried to obtain the data.

It is desirable to isolate as many contributors of the total cost of maintenance as possible. This is sometimes very difficult to do because state highway agencies do not always categorize these various costs in their bookkeeping systems. A survey by the National Cooperative Highway Research Program did indicate, however, that very few states have relatively sophisticated cost accounting procedures. As a result, the NCHRP Report 42 [39] contains a significant approach to the isolation of maintenance cost factors. Based on data collected from 28 test sections located within several states, a multiple linear regression analysis was performed to determine general maintenance work loads for seven different factors. In each case the factors were related to a single maintenance requirements value representing the units of labor, equipment and material expended to maintain adequately a one mile section of a four-lane divided interstate highway. Finally, factors were computed to convert the requirement units into appropriate hour and dollar values for each of the seven models. The activity groupings and the requirement models for each are as follows:

Pavement and Shoulders

$$Y_p = 19.72X_1^2 + 13.72X_2 - 183 \qquad (2.16)$$

where Y_p = pavement and shoulder maintenance requirement units

X_1 = surface age, in years

X_2 = number of days when the maximum daily temperature was below 32° F.

Drainage and Erosion

$$Y_D = 4.13X_1 + 2.68X_2 + 73 \tag{2.17}$$

where Y_D = drainage and erosion control requirement units

X_1 = a terrain factor showing the percentage of side slopes 2:1 or steeper

X_2 = annual average rainfall, in inches.

Vegetation Control

$$\begin{aligned} Y_{VR} &= 97.52X_1 + 35.12X_2 + 0.00975X_3 - 744 \\ Y_{VU} &= 97.25X_1 + 35.12X_2 + 0.01950X_3 - 744 \end{aligned} \tag{2.18}$$

where Y_{VR} = vegetation control requirement units in a rural location

Y_{VU} = vegetation control requirement units in an urban location

X_1 = length of the mowing season, in months

X_2 = precipitation during the mowing season, in inches

X_3 = average daily traffic volume.

Structures

$$Y_S = N_1(1.63X_1+28) + 1.80N_2A/f \qquad (2.19)$$

where Y_S = maintenance requirement units for structures

N_1 = average number of structures per mile

X_1 = number of days of snow cover

N_2 = average number of painted steel structures per mile

A = average deck area of an N_2-type structure, in square yards

f = number of years between repaintings.

Snow and Ice Control

$$Y_{IS} = 14.8X_1 - 37.5X_2 + 24.3X_3 + 51.0X_4 \qquad (2.20)$$

where Y_{IS} = snow and ice control maintenance requirement units

X_1 = average annual snowfall, in inches

X_2 = number of days of snowfall

X_3 = number of days with snow cover on the ground

X_4 = number of days when the maximum daily temperature was below 32° F.

Traffic Control Facilities

$$Y_{TC} = 0.0321X_1 + 165 \qquad (2.21)$$

where Y_{TC} = maintenance requirement units for all traffic control facilities, except rest or service areas, weighing or inspection facilities, and the cost of electric power

X_1 = average daily traffic volume.

Litter Removal and Sweeping

$$Y_{LSR} = 0.0051X_1 + 5.09X_2 + 113 \qquad (2.22)$$

$$Y_{LSU} = 0.0051X_1 + 5.09X_2 + 893$$

where Y_{LSR} = maintenance requirement units for litter removal and sweeping in a rural area

Y_{LSU} = maintenance requirement units for litter removal and sweeping in an urban area

X_1 = average daily traffic volume

X_2 = a terrain factor as percentage of side slopes 2:1 or steeper.

It should be mentioned that this study would not be applicable to most states because they do not have the required data immediately available. This is, however, the direction that maintenance cost predictions will eventually take.

As can be seen, there are many different types of maintenance charges associated with the upkeep of a roadway. Only by including as many factors as possible can the real cost of upkeep be accurately preidcted. It is also possible to be too extreme in identifying these various components, thereby increasing the sophistication without gaining any real accuracy. The overall objective of evaluating various pavement design alternatives should be kept in mind. Of the seven factors included in the NCHRP study, for example, only equation (2.16), the maintenance cost of the pavement and shoulders, is relevant to the objectives of this research. While the other models may be applicable to other types of studies, they can be considered constant for purposes of this study.

Under most circumstances, maintenance charges of all types are viewed with repsect to the physical cost of performing the operation along with the associated overhead and material costs. There is one area, however, that is often overlooked even though it has a cost that is as real as the physical costs; this is the users' or motorist's increase in cost due to being delayed while maintenance is performed. Total user cost can be divided into costs of two types:

1. the motorist's inconvenience cost of being delayed
2. the increase in vehicles operating costs.

Although it may seem that the operating costs of the vehicle would not be appreciably increased by these delays, comprehensive studies [10,11,81] have indicated that these costs can be significant.

There has not been a comprehensive study made for the purpose of analyzing user cost as applied in this study. It is possible, however, to draw some conclusion as to what values of user cost might be expected by combining the results of many similar, but less extensive studies. Studies of delay time, including delays arising from stops and slowdowns caused by specific road and traffic conditions, are particularly important to road user cost studies of the type considered here. Among studies which have developed useful data on motor vehicle delay times in urban areas are those conducted by Berry in and near San Francisco, California [4]. The data presented in Berry's report shows stopped time delays and slowdown delays at traffic signals as affected by type of signal, by signal timing and by traffic volume. These data, together with data developed by Webster [80] and Volk [78] are valuable for road user cost determination in urban areas.

Other data on delay times and fuel consumption for stops and slowdowns that are useful for user cost determination were reported in 1960 by Kent [42], Sawhill and Firey [60], and Claffey [12]. These papers were based on studies conducted primarily on rural roads. Measurements of motor vehicle fuel consumption in urban areas were also made by Bone [5], May [53], and Sawhill and Crandall [59] in connection with their investigation of travel times.

A very exhaustive set of experiments has been made by Claffey [10] on the cost of operating motor vehicles on primary routes in and through cities. The bulk of the data presented relates to fuel consumption rates and vehicle speeds for many road and traffic conditions. Extensive experimental data are also shown for the cost of tire wear, oil consumption, and maintenance. The experiments were conducted for a typical passenger car, a transit bus, a single unit truck, and a tractor semi-trailer combination truck. Traffic conditions vary from completely free-flowing freeways to congested downtown streets. In a further study, Claffey [11] also considers the effects of stop-and-go driving conditions upon vehicle performance.

Another exhaustive set of tables, showing the running cost of motor vehicles, is given by Winfrey [81]. The tables were prepared for five typical vehicles giving the dollars of running cost and gallons of fuel consumption for operation at uniform speed on minus grades, plus grades, and horizontal curves and for changes in speed. The types of vehicles considered were:

1. 4-kip passenger car
2. 5-kip commercial delivery truck
3. 12-kip single unit truck
4. 40-kip 2-S2 gasoline truck
5. 50-kip 3-S2 diesel truck.

Running costs are tabulated to include fuel, tires, motor oil, maintenance, and depreciation.

Data of the type that has been described can be useful in determining the addition vehicular cost due to being delayed. The only difficulty is to develop a model of the desired traffic condition around the available data. Assumptions must be made that simplify the entire problem, sometimes greatly. However, there has not been a theoretical approach developed that would be useful under the present circumstances. Traffic flow models have been developed by Drew [15,16,17,18] that completely describe the quality of flow on a facility in terms of the traffic parameters of volume, density, speed, and queueing of vehicles. His momentum-kinetic energy model [17] utilizes the generalized equations of motion to model a general traffic system. By differentiating this equation, the optimal vehicular speed can be obtained that maximizes the kinetic energy in the traffic stream. Drew further defines acceleration noise as the standard deviation of the distribution of the number of speed variations as a function of time and traffic. He then shows how the previously obtained optimal speed also minimizes the acceleration noise in the system. In itself, this concept may eventaully be applied to determine optimal speeds through areas congested by maintenance operations. A variation of this model is presently being investigated at the Texas Transportation Institute at Texas A&M University with the possibility of incorporating a traffic merging model. It is anticipated that this model can be used as a bulk measure for determining the total delay experienced by a group of vehicles traveling through an area that is restricted by a maintenance operation. No other theoretical models have been found in the literature that are applicable to the problem of predicting delay time as is considered here.

Branch and Bound Methods

The optimization of the initial construction design is the first of two such optimization problems considered in this research. This problem is formulated in Chapter V as an integer programming problem and is solved by a technique called "branch and bound". This approach has received considerable attention in recent years as being a rather clever technique for solving certain types of integer programming problems.

Many of the algorithms appearing in the literature rely on the partial enumeration of the set of all possible solutions. A fundamental property of the branch and bound procedure is that these techniques exclude large sets of possible solutions from consideration. Geoffrion [24] has called this scheme "implicit enumeration". The word implicit means that, while the technique does not totally enumerate all solutions, the exclusion of solutions does not cause a possibly optimal solution to be ignored.

The branch and bound procedure was first reported by Balas [2] in 1965. Since that time many have become cognizant of the tremendous potential offered by such solution methods. Perhaps Balinsky [3] indicated this potential when he stated that "... various clever methods of enumeration can be the most efficacious means existent by which to obtain solutions to practical problems".

Although several authors [43,45,75,79] have noted the applicability of the branch and bound technique to non-linear programming problems, most algorithms have been directed primarily toward the linear integer programming problem which can be generally stated as

$$\text{Maximize} \quad Z = C'X \tag{2.23}$$

$$\text{subject to} \quad A'X \leq b$$

$$x_j \geq \text{non-negative integer,}$$

$$j = 1,2,\ldots,n.$$

As an additional requirement, some algorithms [2,3,13,25] assume that all of the integers are restricted to the values 0 or 1. Without loss of generality, any x_j can be transformed to zero-one variables by the binary transformation

$$x_j = \sum_{i=0}^{k_j} 2^i \, y_{ij} \qquad , y_{ij} = 0 \text{ or } 1, \tag{2.24}$$

if the x_j have some physical bound or upper limit. The k_j must be sufficiently large to allow representation of all feasible values of x_j.

The general linear programming problem is solved by a very precise and well defined algorithm. The branch and bound technique, on the other hand, is a heuristic approach that requires a special algorithm for each problem. There is, however, a very general sequence of steps that describe the basic concepts of the branch and bound technique. A brief description of these steps will be presented. It is worth noting that the terms used in this discussion were originally coined by Geoffrion [24] but have become commonplace in the literature. Discussion and terminology are made with respect to the general problem (2.23) after a binary transformation has been made on the variables. The general discussion is independent of the linearity or non-linearity of the objective function.

A partial solution is defined as an assignment of binary values to a subset of n variables in X. Any variable not assigned a value is called free. A completion of a partial solution is the solution determined by an assignment of binary values to all free variables. For a given partial solution, it may be possible to find a feasible completion that maximizes the objective function (as in equation (2.23)) among all feasible completions. Secondly it may be possible to determine that the partial solution has no feasible completion better than the incumbent solution (the current best-known feasible solution). In either case, the partial solution is said to have been fathomed. Once a partial solution is fathomed, the implicit enumeration scheme backtracks to a partial solution which will allow further augmentation. A partial solution is said to be augmented when a free variable is assigned a fixed value. When this occurs, an attempt is made to fathom the new partial solution.

A block diagram of the general branch and bound scheme for a maximization problem is shown in Figure 2.1. It is important

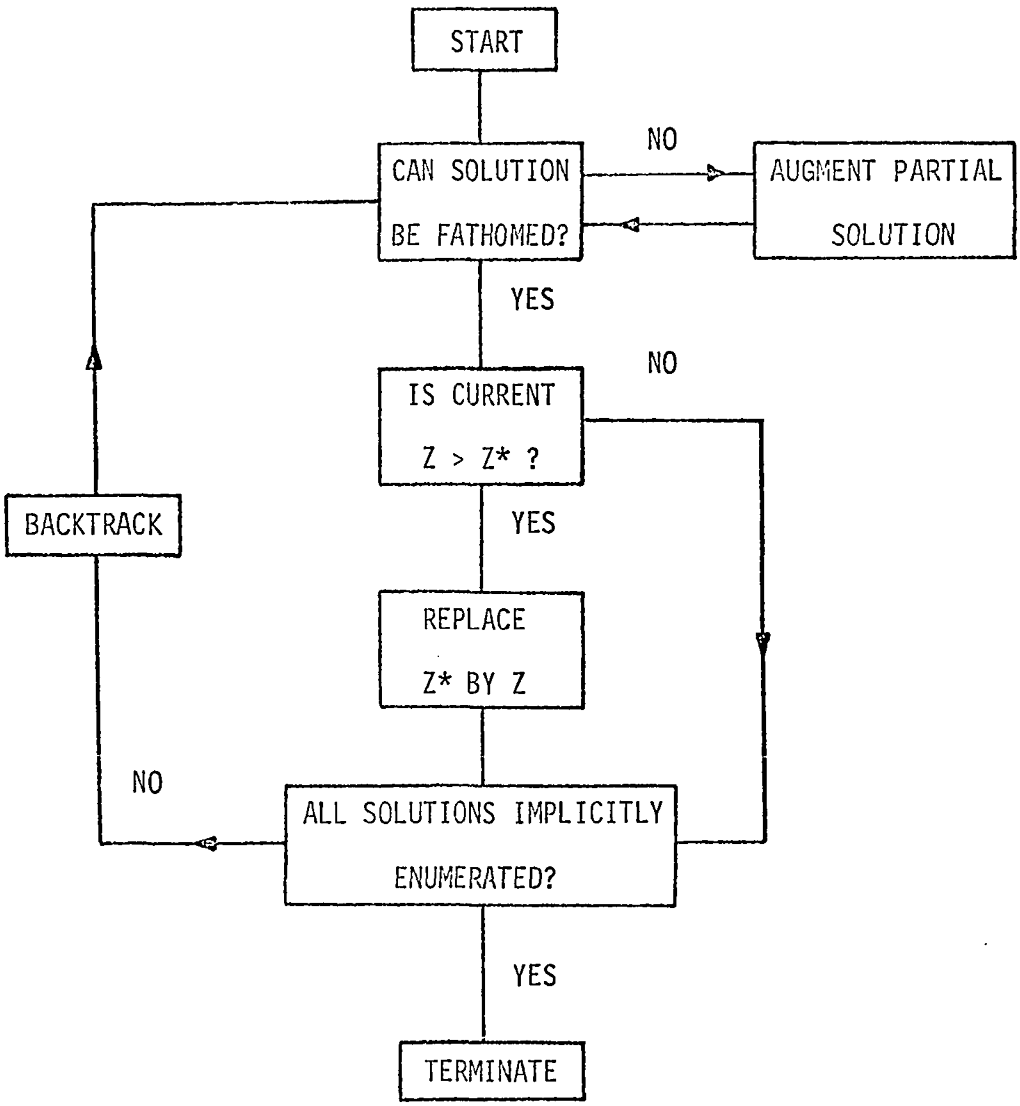

Figure 2.1 Block diagram for implicit enumeration (maximization)

to repeat that there are three ways that a partial solution may be fathomed:

1. by concluding that no completion of the partial solution can lead to feasibility
2. by concluding that no feasible completion will yield a value of the objective function better than the incumbent solution
3. by identifying a new incumbent solution.

If a partial solution cannot be fathomed, the current solution is augmented by setting one of the free variables equal to zero or one. The development of intelligent rules for the selection of this free variable along with clever methods of fathoming a partial solution, augmentation, and backtracking are the key factors that affect the efficiency of various algorithms. Most algorithms are distinguished by their method of performing one or more of these factors. It is also essential that these rules be developed so that a non-redundant sequence of partial solutions will be generated.

Many variations and applications of the branch and bound techniques have appeared in the literature. Greenberg [27] and Lomnicki [48,49], for example, have applied a mixed integer formulation to solve machine scheduling problems. Another example has been an all integer formulation of the plant location problem by Efroymson and Ray [21].

Wagner [79] has presented an interesting formulation to determine the optimal component sparing policy that maximizes system reliability for extended manned space missions. His algorithm initially utilizes a dynamic programming formulation to obtain a feasible lower bound (incumbent) on the optimal solution, thereby reducing the enumerable feasible region.

One other variation has been the application of surrogate constraints to the general branch and bound technique by Glover [26] and Geoffrion [23]. A surrogate constraint is an inequality implied by other constraints of an integer problem, and is designed to capture useful information that cannot be obtained from the parent constraints individually but is nevertheless a consequence of their conjunction.

The mathematical formulations of branch and bound algorithms can be, to say the least, very complicated and have been omitted through this section. Surveys of the more general algorithms can be found in [28,46]. Also described are some of the subtle differences that distinguish various generalized algorithms.

The research which is reported in this dissertation will combine the information contained in this chapter in order to provide an integrated presentation of the results. The physical aspects of pavement design, the economics of design, maintenance and user costs, and the principles of optimization will be combined to present an original and hopefully comprehensive approach to the design and maintenance of highway systems.

CHAPTER III

PHYSICAL VARIABLES AND MATHEMATICAL RELATIONSHIPS RELATING THESE VARIABLES

There is a large group of physical variables associated with materials, construction, traffic and environment that affect pavement performance. Of these, a few have been quantified and their effect on pavement performance has been established, at least approximately. The objective of this chapter is two-fold: (1) to define the physical variables considered in this design procedure and describe, qualitatively, their effect on pavement performance and (2) to present a set of empirical equations relating these physical variables.

The physical variables to be considered are categorized into the groups of design, deflection, traffic, and performance variables. These variables are related through empirical relationships for deflection, traffic, and performance.

Frequent reference will be made to these relationships in later chapters. For this reason it is felt that a thorough understanding of both variables and relationships is fundamental to these later discussions. Even though a large portion of this chapter is a review of previous work performed by the Texas Transportation Institute at Texas A&M University, the material was not treated in Chapter II because of its relevance to the objectives of this research. Moreover, it will be noted that the data upon which these relationships are based and the lengthy procedure followed in their derivation have been omitted.

Design Variables

The sketch in Figure 3.1 represents a pavement composed of n layers above the subgrade level. The material in each of these layers, and in the foundation layer, is characterized by a strength coefficient, a_i, $i = 1,2,...,n+1$, where a_{n+1} is the strength coefficient of the subgrade. The thickness of any

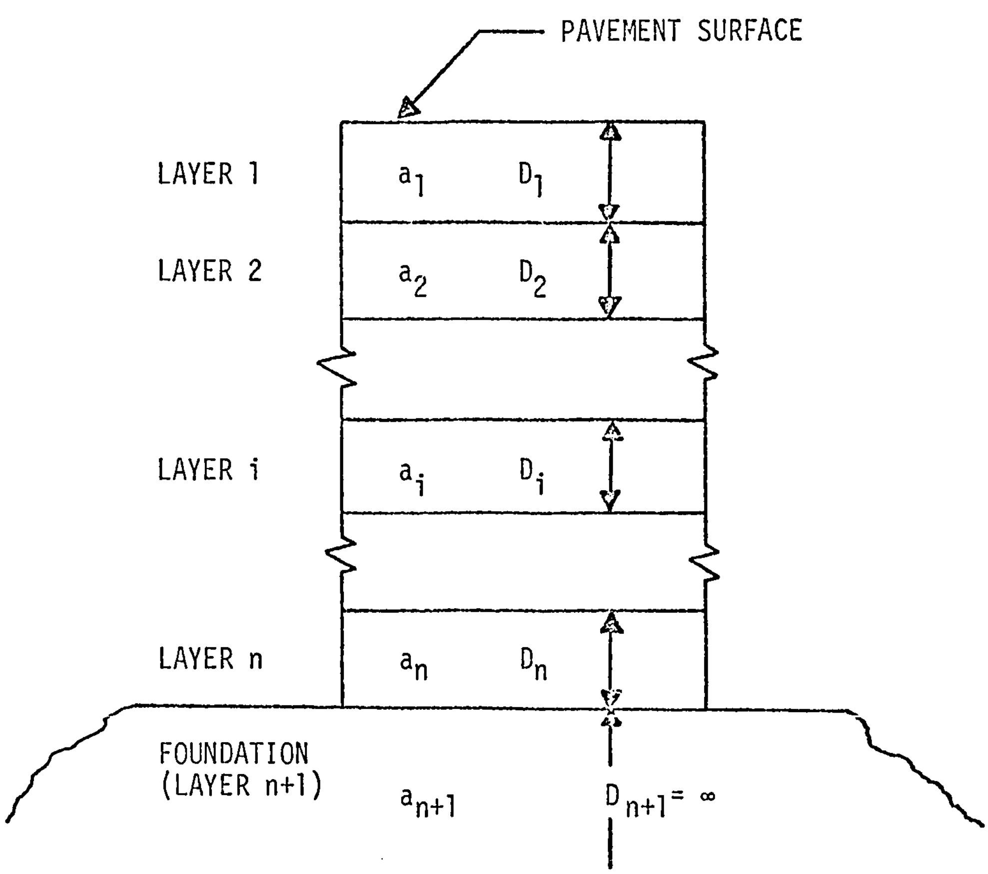

Figure 3.1 A pavement section of n layers above the subgrade level

layer above the foundation is denoted by D_i, $i = 1,2,\ldots,n$. The subgrade is considered to be of infinite thickness.

Measured in situ values of typical strength coefficients, range from about 0.17 for a wet clay to about 1.00 for a strongly stabilized base material. No way has been found for predicting these values with suitable accuracy from laboratory tests. For the present, the strength coefficient of a material proposed for use in a new pavement in a particular locality must be estimated from deflection measurements made on the same type of material in an existing pavement located in the same general area. A procedure for computing the strength coefficients for various types of materials is contained in Research Project 32-11 [63].

Deflection Variables

An important feature of this design procedure is the use of the Dynaflect† for measuring deflections on existing highways. Descriptions of the instrument and examples of its use in pavement research can be found in [56,64,65]. Basically, deflections measurements are obtained by the instrument resulting from a dynamic load of 1000 lbs., oscillating sinusoidally with time at 8 cps, being applied through two steel load wheels to the pavement, as indicated in Figure 3.2. Five sensors, resting on the pavement at the numbered points shown in the figure,

---

†Registered Trademark, Dresser Industries, Inc., Dallas, Texas

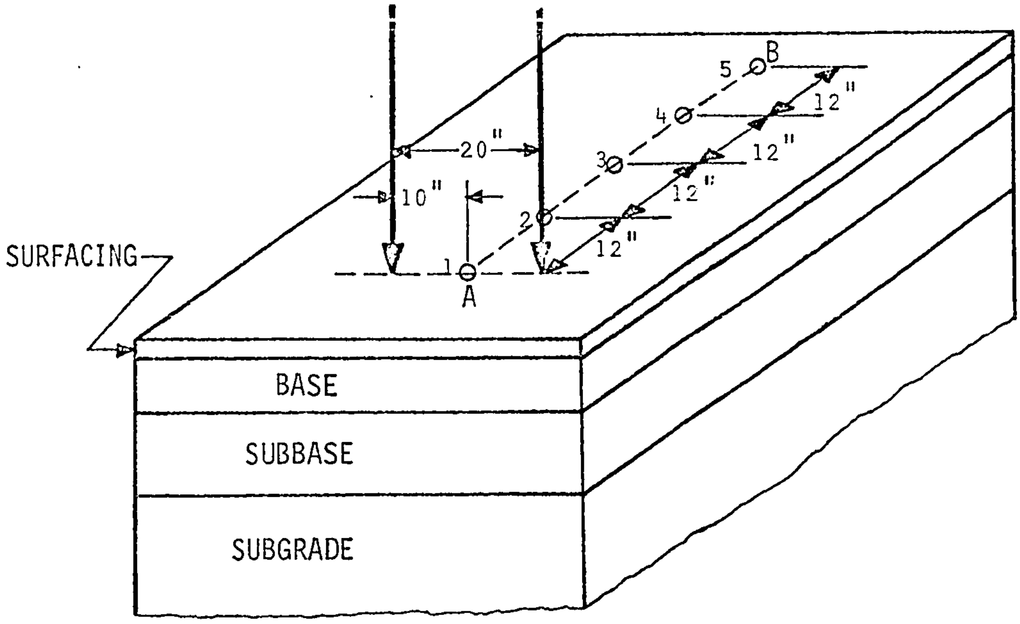

Figure 3.2 Position of Dynaflect sensors and load wheels during test

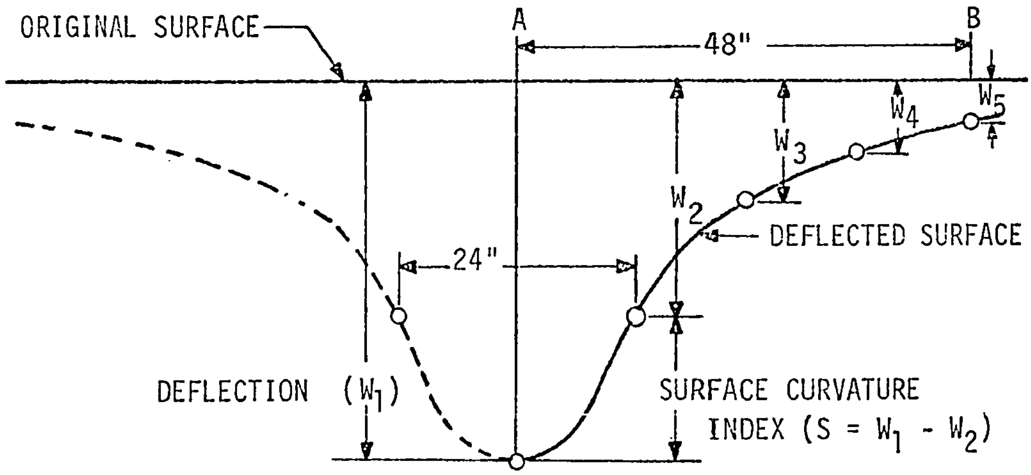

Figure 3.3 Typical deflection basin reconstructed from Dynaflect readings

register the vertical amplitude of the motion at those points in thousandths of an inch (mils).

A deflection basin of the type illustrated in Figure 3.3 results from the Dynaflect loading. The symbol W_i represents the amplitude (or deflection) occurring at the point i, i = 1,...,5, in the figure.

A deflection variable particularly important to this design procedure is the surface curvature index, S, shown in Figure 3.3 and defined by the equation, $S = W_1 - W_2$. The value of S for a proposed design can be predicted with reasonable accuracy from the deflection equation (3.3) to be given in a later section, provided that the values of the design variables are known. Two designs are assumed to be physically equivalent if they have the same value of S, and the same swelling clay parameters (also discussed in a later section). Of two designs with differing values of S, but with the same swelling clay parameters, the design with the lower value of S is the stronger and as a result, longer lived. It is important to note that other design procedures might make use of the data from all five sensors while this procedure makes use of only the first two.

Traffic Variables

In describing traffic, it is first necessary to select the total period of time following initial construction that is of interest to the design engineer. This period of time is usually on the order of 20 to 40 years, and is termed the analysis period, C. Definitions of the other traffic variables are

t = time in years, $t = 0$ at the time of initial construction

r_0 = the average daily traffic (ADT) in one direction at time $t = 0$

r_f = the ADT, in one direction, at time $t = f$

N_t = the total number of equivalent applications of an 18-kip single axle load (computed by the standard AASHO method) that will have passed over the pavement in one direction by any time, t

N_f = N at time $t = f$.

It is noted that an equivalency of applications is required to account for heavy vehicles being more detrimental to pavements than lighter vehicles. Data for r_0, r_f, and N_f, for f equal to both 20 and 30 years, can presently be obtained from the Planning Survey Division of the Texas Highway Department for any particular location.

Typical plots of N versus t for light, moderate, and heavy traffic are shown in Figure 3.4.

Performance Variables

The serviceability index (P) versus time (t) for a pavement is referred to as the performance curve of the pavement. A performance variable is any variable describing, or in any way

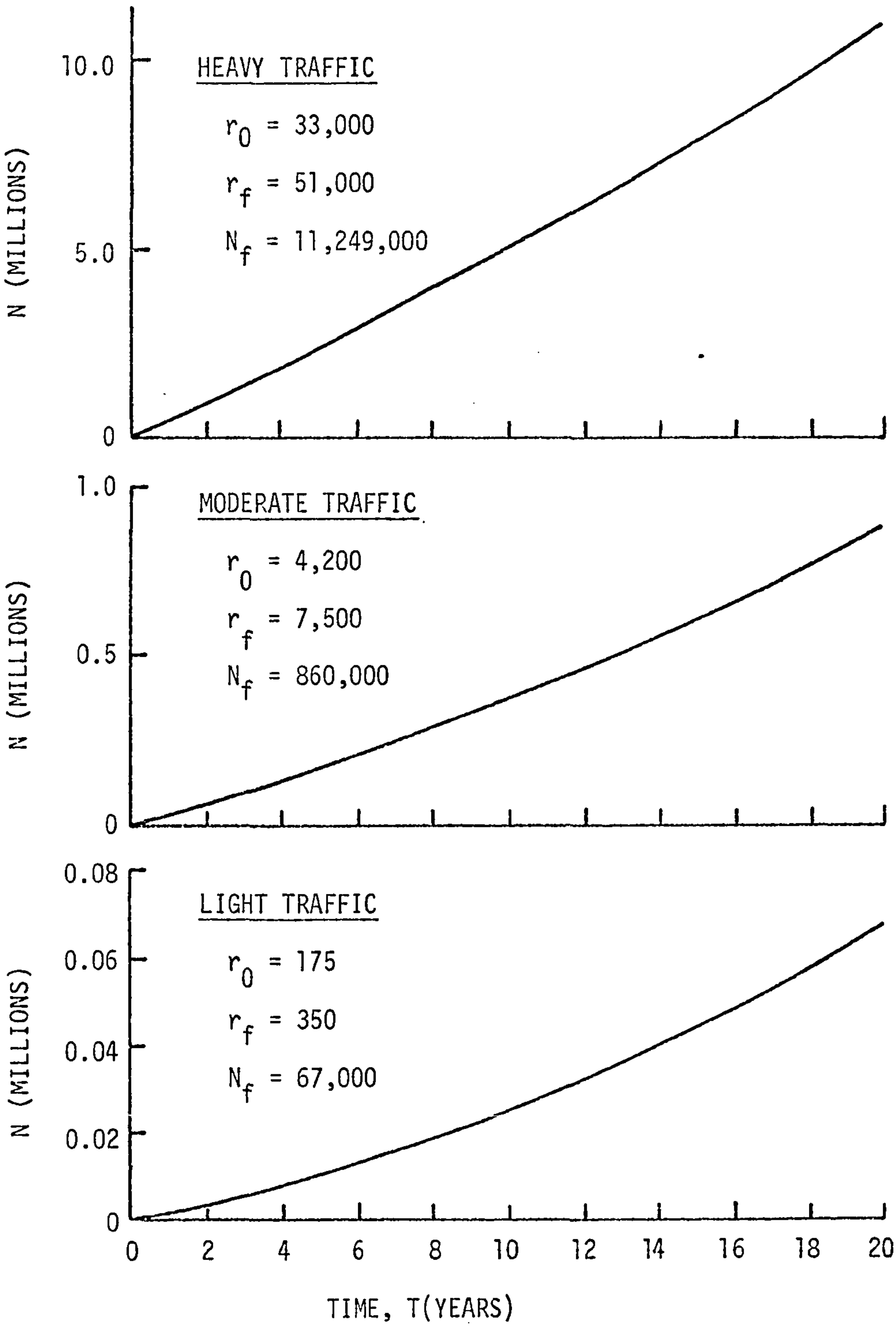

Figure 3.4 Typical traffic curves

affecting, the performance curve. In this respect, the design, deflection, and traffic variables previously described qualify as performance variables. They are not generally categorized as such, however, because the term performance variables is usually

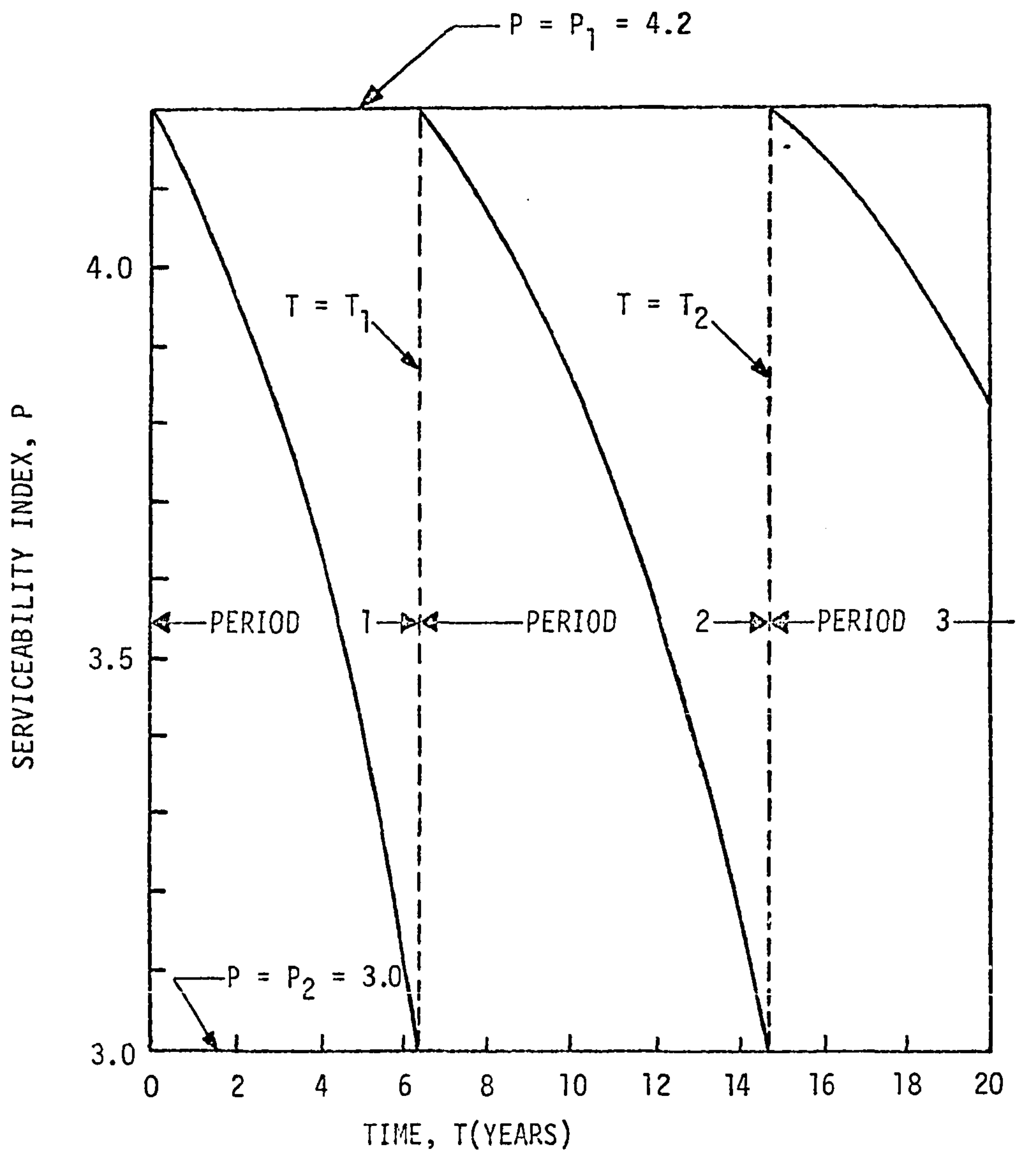

Figure 3.5 A performance curve for an analysis period of 20 years involving two overlays constructed at $T = T_1$ and $T = T_2$, and three performance periods

reserved for other variables that are directly related to the serviceability of the pavement.

Figure 3.5 shows a computed performance curve for a flexible pavement to which two overlays have been applied within an analysis period of 20 years. A performance period is defined as the time, in years, from the completion of initial or overlay construction, when $P = P_1$, to the time when the serviceability index next reaches its predetermined minimum, P_2. Performance periods are numbered in chronological order, Period 1 being the first.

Except in the computation of user costs due to traffic delays, the time required to construct an overlay is neglected. The value of t at the end of the k^{th} period, or at the beginning of the next period, is denoted by T_k.

An important phenomenon affecting the performance curve is that of uneven volumetric expansion and contraction of materials in the foundation [63]. This effect is illustrated in Figure 3.6, which portrays the computed serviceability history of four pavements constructed on foundation materials of four different types. These performance curves represent pavements <u>not</u> subjected to traffic - only the effect of foundation movements is shown.

In the case of the three lower curves in Figure 3.6 the variable, P, approached an ultimate value, P_2', taken here as 1.5. The variable, b_1, represents the relative rate at which P

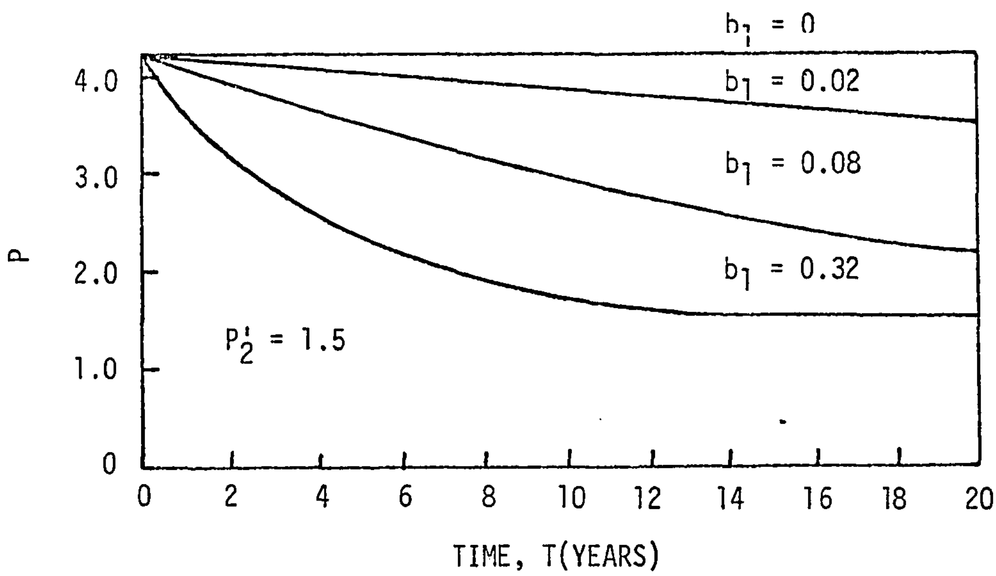

Figure 3.6 Four performance curves, each approaching an assumed lower limit of P = 1.5 and illustrating the effect of foundation movements in the absence of traffic

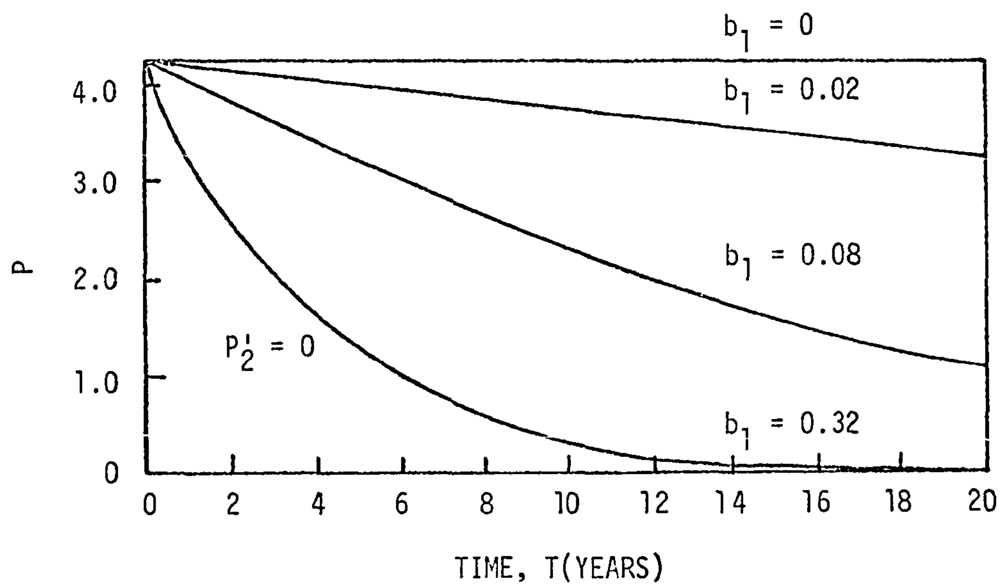

Figure 3.7 Four performance curves, each approaching an assumed lower limit of P = 0

approaches its ultimate value. Thus, the curve labeled "$b_1 = .02$" approaches the 1.5 serviceability level much more slowly than the curve labeled "$b_1 = .32$". The quantities P_2' and b_1 are termed "swelling clay parameters".

Not much is known about the quantity P_2', since the relevant serviceability index data are not available. The ultimate value of P in the absence of traffic may be greater or less than the value of 1.5 used in Figure 3.6. For comparison with Figure 3.6, performance curves in the absence of traffic for $P_2' = 0$ are shown in Figure 3.7.

If P_2' is fixed at an assumed value, b_1 can be estimated from serviceability index data on new pavements which have not been opened to traffic but nevertheless have roughened due to swelling clays. Very limited data of this type have indicated an upper limit of approximately 0.3 for b_1 when P_2' is assumed to be zero. The theoretical lower limit of b_1 is zero, for any value assigned to P_2'.

In the development of a relationship between performance variables based on AASHO Road Test data, it is desirable to define a serviceability loss function, Q as

$$Q = \sqrt{5 - P} - \sqrt{5 - P_1} \tag{3.1}$$

where P and P_1 are as previously defined, and the constant, 5, is the theoretical upper limit of P. With P_1 fixed (a commonly used value is 4.2), Q increases as the serviceability index, P, decreases, as shown in Figure 3.8. In this figure one of the performance curves given in Figure 3.7 has been replotted along with the corresponding curve for Q. The Q versus t curve is referred to as the serviceability loss curve, to distinguish it from the P versus t curve already defined as the performance curve. For this example, the P and Q curves approach ultimate values of zero and 1.34 respectively ($P_2' = 0$, $Q_2' = 1.34$).

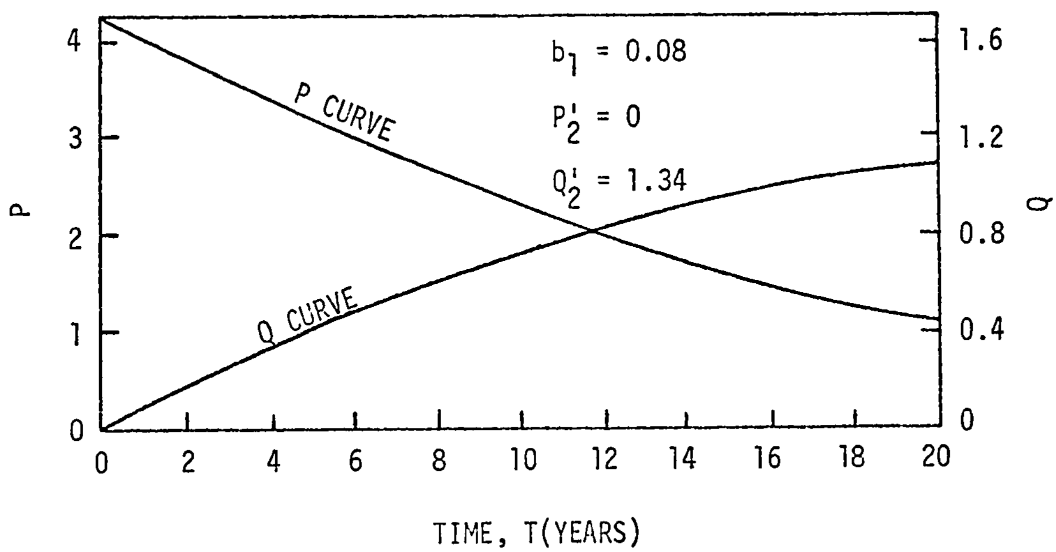

Figure 3.8 A performance curve and its corresponding serviceability loss curve, with no traffic operating

As previously was indicated, the serviceability loss or Q-curve of Figure 3.8 is taken to represent a pavement which has carried no traffic. It was next necessary to form an hypothesis as to how the placement of an overlay would affect such a Q-curve.

The hypothesis adopted was simply that (1) the slope of the curve at $t = T_k$ is the same before and after placement of an overlay, and (2) the ultimate value, P_2', of the serviceability index remains unchanged. The graph shown in Figure 3.9, wherein the slope of the Q-curve is the same at points A and B, illustrates this hypothesis.

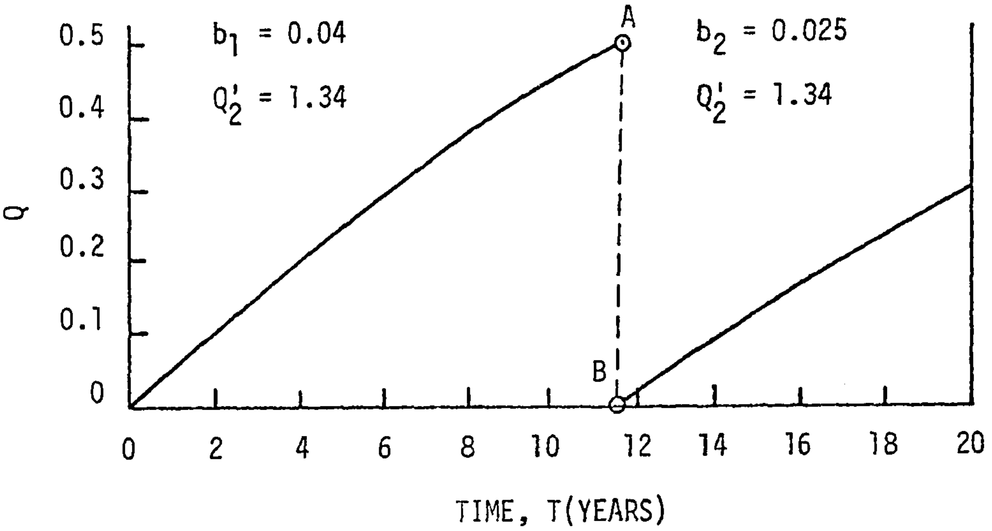

Figure 3.9 The assumed effect on the serviceability loss curve of the construction of an overlay at Point A. The slope of the curve at A and B is the same; both branches approach the same asymptote.

The adoption of the hypothesis requires the computation of a new value of b at the beginning of each performance period, based on the value applicable to the preceding period. In general, the value of b_{k+1} at the beginning of the $k+1^{st}$ performance is computed recursively by

$$b_{k+1} = b_k \, e^{-b_k(T_k - T_{k-1})} \quad . \tag{3.2}$$

Thus, for the Q-curve shown in Figure 3.9, the value of b_1 assigned to the first period is .04, while the computed value of b_2 applying to the second period is .025. Both branches of this Q-curve would, if extended, approach an ultimate value of 1.34 ($Q_2' = 1.34$) corresponding to $P_2' = 0$.

The effect of this hypothesis is conceptually appealing. With the application of an overlay, it would be expected that the serviceability loss during the second performance would be relatively smaller than in the first performance period becuase the pavement has been strengthed.

One other performance variable must be mentioned, a daily temperature constant, α, and its effective value over one year, $\bar{\alpha}$. This variable arose during an analysis of the AASHO Road Test data, and is believed to represent an increased susceptibility of asphaltic concrete to cracking under traffic in cold weather. The daily constant, α, is defined as the average of the maximum and minimum daily temperature, less 32° F. The change in the loss function, Q, occurring as the result of one day's traffic depends in part on the value of α for that day. By summing the daily changes in Q, a single equivalent value of α, designated $\bar{\alpha}$, can be used to estimate the change in Q for a full year. Table 3.1 contains estimates of $\bar{\alpha}$ for each district in Texas based on mean values of the high and low daily temperatures averaged over a ten year period. A map of Texas, depicting the 25 districts is shown in Figure 3.10.

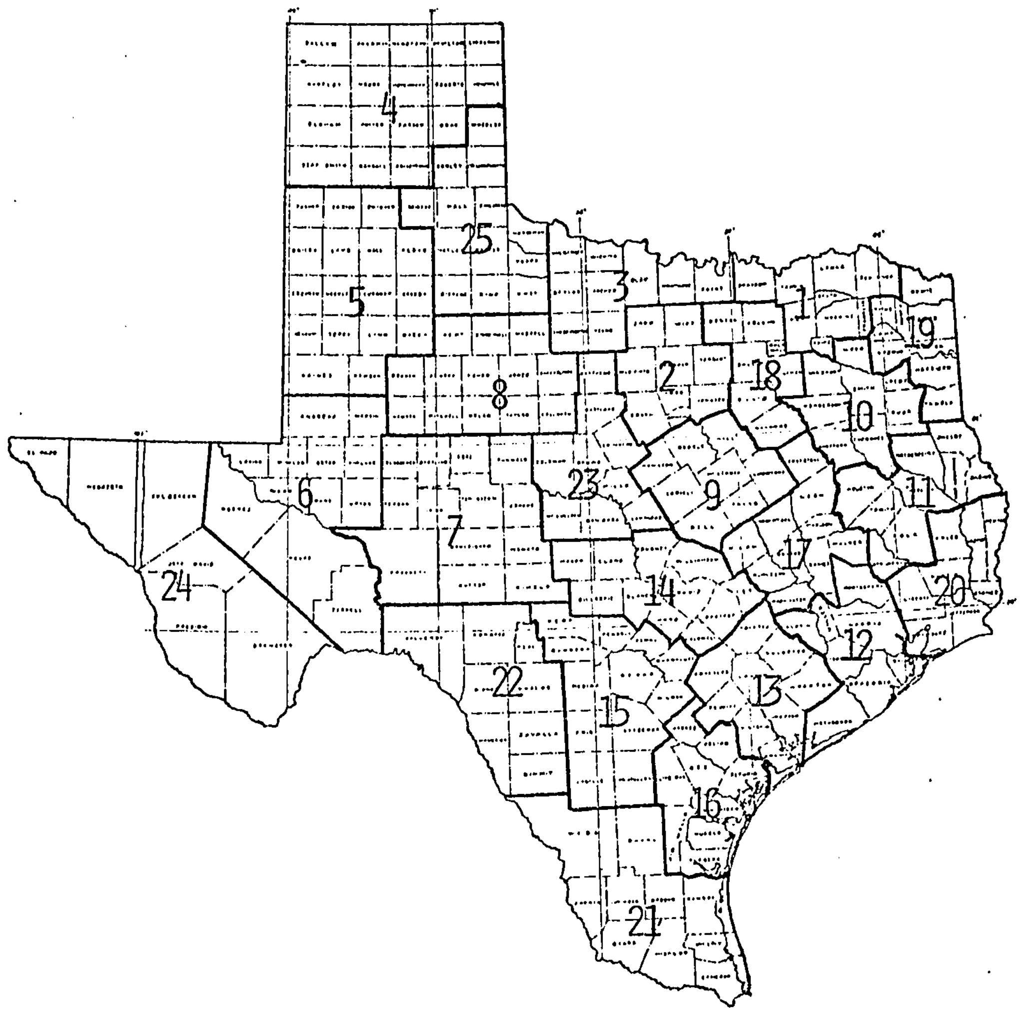

Figure 3.10 Map depicting the 25 districts in Texas

Dist.	Temp. Const. ($\overline{\alpha}$)	Dist.	Temp. Const. ($\overline{\alpha}$)	Dist.	Temp. Const. ($\overline{\alpha}$)	Dist.	Temp. Const. ($\overline{\alpha}$)	Dist.	Temp. Const. ($\overline{\alpha}$)
1	21	6	23	11	28	16	36	21	38
2	22	7	26	12	33	17	30	22	31
3	22	8	26	13	33	18	26	23	25
4	9	9	28	14	31	19	25	24	24
5	16	10	24	15	31	20	32	25	19

TABLE 3.1 TEMPERATURE CONSTANTS FOR EACH OF THE 25 DISTRICTS IN TEXAS

Deflection Equation

Presented in this section is a model for predicting the deflection of flexible pavement in the form of a "surface curvature index", S, from the design variables a_i and D_i. This is an empirical equation recently developed from deflection data gathered on the A&M Pavement Test Facility located at Texas A&M University's Research Annex near Bryan, Texas. A description of the facility is contained in Research Project 32-9 [62].

This is a new formulation that differs from previous deflection equations by featuring failure due to bending and fatigue as the most important consideration in predicting deflection rather than vertical displacement alone. The equation is

$$S = W_1 - W_2 \quad . \tag{3.3}$$

Each W_j can be expanded to

$$W_j = \sum_{m=1}^{n+1} \Delta_{jm} \tag{3.4}$$

$$\Delta_{jm} = \frac{k_0}{a_m^{k_1}} \left[\frac{1}{r_j^2 + k_2\left(\sum_{i=0}^{m-1} a_i D_i\right)^2} - \frac{1}{r_j^2 + k_2\left(\sum_{i=0}^{m} a_i D_i\right)^2} \right] \tag{3.5}$$

where W_j = vertical displacement detected by the j^{th} sensor

Δ_{jm} = the compression in the m^{th} layer as detected by the j^{th} sensor

k_0 = 0.891087

k_1 = 4.50292

k_2 = 6.25

a_0 = D_0 = 0

r_j = distance from either of the load wheels to the j^{th} sensor

Other variables have been previously defined. From Figure 3.2, it can be seen that for the first two sensors, $r_1^2 = 100$ and $r_2^2 = 244$. By manipulating equations (3.3), (3.4), and (3.5), S can be reduced to:

$$S = k_0 \left\{ a_1^{-k_1} \left[\frac{1}{100} - \frac{1}{244} \right] \right. \tag{3.6}$$

$$+ \left(a_2^{-k_1} - a_1^{-k_1}\right) \left[\frac{1}{100+6.25(a_1D_1)^2} - \frac{1}{244+6.25(a_1D_1)^2} \right]$$

$$+ \left(a_3^{-k_1} - a_2^{-k_1}\right) \left[\frac{1}{100+6.25(a_1D_1+a_2D_2)^2} - \frac{1}{244+6.26(a_1D_1+a_2D_2)^2} \right]$$

$$+\cdots+ \left(a_{n+1}^{-k_1} - a_n^{-k_1}\right) \left[\frac{1}{100+6.25(a_1D_1+\cdots+a_nD_n)^2} \right.$$

$$\left. \left. - \frac{1}{244+6.25(a_1D_1+\cdots+a_nD_n)^2} \right] \right\} .$$

A complete documentation of the development of this equation can be found in Research Report 32-12 [61].

Special mention should be made concerning the flexibility of this equation with respect to collapsibility and consistency. As would be expected, if one or more $D_j = 0$, the equation reduces to the correct form for predicting the SCI of the reduced system with the appropriate number of layers. In addition, if two or more adjacent layers are constructed from the same material, consistency can be maintained by equating the strength coefficients of the respective layers.

Traffic Equation

In general, the definition of the traffic parameters stated earlier can be extended to include traffic variables at any time t. Let

r_t = the ADT at any time t

r_0 = value of ADT at t = 0

r_f = value of ADT at t = f

R_t = accumulated number of vehicles at any time t

R_f = value of R at t = f

N_t = accumulated number of equivalent 18-kip axles at any time t

N_f = value of N_t at t = f.

There are two basic assumptions associated with future traffic. The first of these indicated by equation (3.7), is that the ratio of accumulated number of equivalent 18-kip axles to accumulated number of vehicles remains constant over time. This is equivalent to assuming that among the total number of vehicles, the proportions of the various types of vehicles remains constant.

$$k = \frac{N_t}{R_t} \tag{3.7}$$

The second assumption is that the average daily traffic increases linearly with time as:

$$r_t = r_0 + \left(\frac{r_f - r_0}{f}\right) t \ . \tag{3.8}$$

Integrating with respect to t yields

$$R_t = \int_0^t r_t dt = r_0 t + \left(\frac{r_f - r_0}{2f}\right) t^2 \ , \tag{3.9}$$

so that when $t = f$,

$$R_f = \left(\frac{r_0 + r_f}{2}\right) f \ . \tag{3.10}$$

Substituting equation (3.10) into equation (3.7) yields

$$k = \frac{2N_f}{(r_0 + r_f)f} \tag{3.11}$$

so that multiplying equations (3.9) and (3.11) yields a general relationship for $\dot{N}$ at any time t. Specifically of interest will be the times at the end of the performance period, T_k, $k = 1,2,\ldots,K$.

$$N_k = \left(\frac{N_f}{f(r_0 + r_f)}\right) 2r_0 T_k + \left(\frac{r_f - r_0}{f}\right) T_k^2 \tag{3.12}$$

Performance Equation

The performance equation (3.13) is an empirical relationship that relates the serviceability loss Q_2 to (1) the equivalent number of applications, N, required to reduce the serviceability index P from a maximum value of P_1 to a prescribed minimum of P_2 and (2) the time t required to accumulate this number of applications. The time t is defined as one performance period. This equation is given by:

$$Q_2 = \frac{53.6\ (N_k - N_{k-1})\ S^2}{\alpha} + Q_2' \left[1 - e^{-b_k (T_k - T_{k-1})} \right] \qquad (3.13)$$

where

$Q_2 = \sqrt{5 - P_2} - \sqrt{5 - P_1}$

$N_k - N_{k-1}$ = the equivalent number of 18-kip axles accumulated during the k^{th} performance period

$Q_2' = \sqrt{5 - P_2'} - \sqrt{5 - P_1}$

$T_k - T_{k-1}$ = time of the k^{th} performance period.

The first term of the equation has the same general form (apart from constants) as the original performance equation developed during the AASHO Road Test, while the second term has recently been added to account for swelling clays. It is noteworthy that there is no guarantee that the swelling clay concept is the only significant component to serviceability loss in the absence of traffic, but it is the only concept that has been quantified.

Pavement designers know that it is virtually impossible to design a pavement that will last for an extended period of time, say 20 years. Trial computations have indicated that the mathematical inclusion of the swelling clay component forces the performance equation to simulate actual pavement performance under varied conditions much more closely than previous empirical equations.

This chapter has presented an overview of the quantitative work that has been advanced in the area of pavement design in association with pavement performance predictions. This quantification provides the basis for the optimization techniques which are to be applied in later chapters and which represent the major contributions of this research. The problem becomes relatively complex due to the interrelationship of equations (3.6), (3.12), and (3.13). The information of this chapter was presented in this form to provide the background for the optimization algorithms presented in Chapters V and VI.

CHAPTER IV

DEVELOPMENT OF COST MODELS

There are a number of cost considerations associated with the investment in a road. Other than initial construction, this investment is accumulated throughout the life of the structure in the form of different types of maintenance charges. In addition, the pavement will usually have some value (although possibly a negative value) at the end of its life.

The purpose of this chapter is to describe cost models, both verbally and mathematically, to be used in evaluating the total cost of each alternative investment. Optimal design decisions are to be made on the basis of comparison of these accumulated investments. For this reason, proper account of each consideration must be taken if the analysis is to be accurate and complete.

The length of the analysis period deserves some discussion. In general, it is the period of time over which costs are computed. Ideally this period should end when the road is expected to be abandoned or when it is expected that major reconstruction work will be required. Unfortunately this length of time is not constant for all initial construction designs because some structures are stronger than others. There are two common methods [14] for making economic comparisons of structures with unequal lives. The first method is to adopt a study period that is a multiple of the lives of the alternatives being studied.

This method is undesirable for two reasons - (1) the large number of alternatives and (2) the study period would be very long, leaving a question as to the accuracy of future data forecasts.

The second and most meaningful method involves assigning salvage values to the alternatives after a shorter study period. The length of this period should obviously be selected so that data requirements can be estimated reasonably well. In practice, this period is between 15 and 30 years, with 20 years being the median. Analysis periods of this magnitude are common, long enough to reflect all potential differences between designs and short enough for most data forecasts. The Texas Highway Department, for example, makes both 20-year and 30-year forecasts of daily traffic levels in all districts of the state.

The majority of the various costs under consideration are to be accrued at irregular intervals in the future. For this reason the principles of engineering economy are used throughout the analysis. Only by discounting each future cost to the present can an accurate estimate of each alternative be evaluated for comparative purposes.

Mathematically, the present value of the total cost of each alternative is

$$TC = IC + OC + MC + SC + UC - SV \qquad (4.1)$$

where TC = the present value of the total cost of the alternative

IC = the cost of initial construction

OC = the discounted sum of overlay construction charges

MC = the discounted sum of annual routine maintenance charges

SC = the discounted sum of all future seal coat costs

UC = the discounted sum of user costs during all types of maintenance

SV = the salvage value of the structure at the end of the analysis period, discounted to the present.

It is standard practice for each of these quantities to be computed on the basis of cost per square yard of pavement. A description of the evaluation of each quantity is given in the following sections.

Cost of Initial Construction

The cost of initial construction consists not only of the cost of raw materials, but includes the cost of laying the pavement. It is common practice, however, to include these ancillary considerations under the general term of in-place material costs. As a result the initial construction cost can be computed rather simply as:

$$IC = \sum_{j=1}^{n} D_j C_j \tag{4.2}$$

where n = number of layers above the subgrade

D_j = depth of the j^{th} layer (inches), $j = 1,2,\ldots,n$

C_j = in-place cost/sq.yd./inch of the material used in the j^{th} layer, $j = 1,2,\ldots,n$.

For research purposes, average material costs per compacted cubic yard in-place are available from several sources. Other than construction bid sheets, one comprehensive reference is by Teinert and Long [70].

Other costs, such as the cost of building curbs, medians or shoulders, are not considered since these costs are generally independent of the initial construction design. In other words, it will be decided in advance whether or not curbs will be constructed and this cost will be constant for all alternative construction designs.

No present worth factor is required since the analysis period begins with initial construction.

Overlay Construction Costs

After a period of time, because of constant use of the road, the serviceability index of the pavement will be reduced to its minimum acceptable value P_2. When this occurs, there are several alternatives: (1) the pavement may be resurfaced or overlayed, (2) a major redesign of the pavement may be undertaken, or (3) the road may be abandoned. Of these alternatives, unless the entire analysis period has elasped, it is assumed throughout this research that the pavement will be resurfaced.

Before the pavement is resurfaced, it is common practice to level the pavement by filling in the low places with asphalt. The assumption is that this action increases the cost of the overlay but does not increase the strength of the pavement. Otherwise, resurfacing of the pavement is assumed to be accomplished by means of an asphalt overlay placed directly upon the old pavement. While resurfacing is generally required because of surface defects, the pavement is also being improved to a standard of serviceability at least equal to the condition of the initial design. This is particularly important from a theoretical point of view because the pavement can then be considered as a "new but stronger" pavement during the succeeding performance period.

The model for computing the present worth of the total cost attributed to the construction of overlays during the analysis period is represented by:

$$OC = \sum_{k=1}^{K-1} \frac{C_1 O_k + C_1}{(1+i)^{T_k}} \tag{4.3}$$

where $K-1$ = number of overlays constructed during the analysis period (there are K performance periods)

C_1 = in-place cost/sq.yd./inch of asphaltic concrete

O_k = overlay thickness, not including the level-up inch, in the k^{th} performance period

i = interest rate

T_k = time at the end of the k^{th} performance period.

The term C_1 is added to the cost of each overlay as an additional charge for level-up. Also, note the demoninator as being a factor for obtaining the present worth of each future expenditure.

Annual Routine Maintenance Costs

Annual routine maintenance, as viewed in this research, is a general term that encompasses all types of repair that are not specifically categorized into other cost models in this chapter. It is concerned solely with the pavement surface, the cost of which includes chiefly the cost of patching the roadway. The assumption is that routine maintenance is incurred in order to maintain the pavement in a serviceable condition but not necessarily in a condition equal to the original pavement. As a result, the serviceability index of the pavement is assumed to be unaffected by this service.

There are two basic approaches to assessing the future cost of routine maintenance. One procedure would be to treat this annual cost as a discrete cost that occurs instantaneously on a year-end basis. A more realistic approach would be to recognize that these charges occur throughout the year on a continuous basis. The latter concept can lead to a more generalized analysis.

In general, Figure 4.1 represents the increase in the cumulative cost of routine maintenance with respect to time.

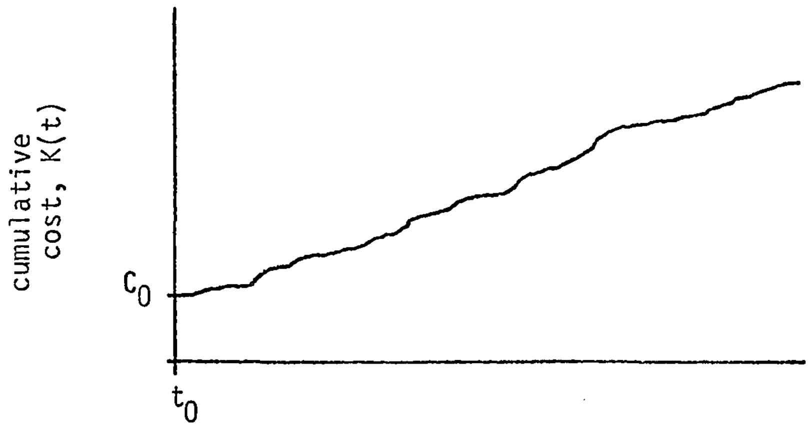

Figure 4.1 General course of increasing cumulative cost

Consider the instantaneous rate of change of this cost relative to a particular value of K(t) as:

$$\frac{\frac{dK(t)}{dt}}{K(t)} \; dt \; .$$

Integrating this function over time and averaging the result will yield:

$$r = \frac{1}{t-t_0} \int_{t_0}^{t} \frac{\frac{dK(t)}{d_t}}{K(t)} \; dt \; .$$

$$r = \frac{1}{t-t_0} \; \ln \left[\frac{K(t)}{K(t_0)}\right]$$

where r is the average rate of change in cumulative cost in $[t_0,t]$ and $K(t_0) = C_0$ as shown in Figure 4.1.

Finally, a general relation for K(t) in terms of r is obtained from the previous relationship by expontentiating both sides and simplifying to give:

$$K(t) = C_0 e^{r(t-t_0)} . \tag{4.4}$$

In general, r would be a function of age and traffic, but it may also be a function of other parameters. In some cases, it may be possible to formulate a single function for r that would be applicable to an entire analysis period. Otherwise, the rate may vary; thus requiring several different functions, each applicable to a specific period of time.

The cost model for predicting future maintenance costs assumes that the annual cost of repair increases linearly with time in the form of equation (4.5).

$$\text{annual cost} = R_1 + (m-1)R_2 \tag{4.5}$$

where R_1 = routine maintenance cost/sq.yd. during the first year after initial or overlay construction

m = number of years after initial or overlay construction

R_2 = incremental increase in routine maintenance cost/sq.yd. per year.

The application of an overlay at a future date has the effect of smoothing the pavement surface so that the routine maintenance charges during the following year will again be R_1.

More realistically, these routine maintenance charges generally have the effect of increasing linearly during the first few years and then leveling out to a relatively constant rate. This fact is substantiated by several sources of data [19,39,40]. In each case, however, the data represents routine maintenance costs for pavements whose serviceability index has dropped to a very low value. Since the design procedure offered in this research represents general pavement maintenance under controlled conditions, overlay construction would have been advised before the serviceability index reached the low value. By truncating the maintenance cost data at this reasonable level, the assumption of linearity becomes a close approximation.

Unlike instantaneous costs, the present worth of costs that occur on a continuous basis should be determined by continuous compound interest factors. The only difference between this and discrete compounding is the replacement of e^{in} for $(1+i)^n$. Thus, the present worth of all routine maintenance charges during the analysis period can be represented as:

$$RM = \sum_{k=1}^{K} \sum_{m=1}^{M_k} \frac{R_1 + (m-1)R_2}{e^{i(L_k+m)}} \tag{4.6}$$

where RM = present value of the total cost/sq.yd. of all routine maintenance during the analysis period

i = interest rate

M_k = number of years in the k^{th} performance period and is defined as the difference in the rounded values of

T_k and T_{k-1}

K = number of performance periods in the analysis period

$$L_k = \begin{cases} \sum_{j=1}^{k-1} M_j & \text{for } k = 2,3,\ldots,K \\ 0 & \text{for } k = 1. \end{cases}$$

In words, L_k is simply the number of years before the beginning of the k^{th} performance period. For consistency, the k^{th} performance period is truncated at the end of the analysis period so that maintenance only during the analysis period is considered.

This research has already begun to point out the need for more sophisticated techniques of accumulating routine maintenance cost data. As more is learned about predicting these future costs, cumulative cost functions of the type in equation (4.6) will be investigated. The difficulty at the present time is in developing accurate expressions for r. One approach would be to use the linear relationship for annual cost given in equation (4.5).

Consider C_m to be the cumulative cost through the m^{th} year. Then

$$C_m = mR_1 + \sum_{i=1}^{m} (i-1)\, R_2$$

or

$$C_m = mR_1 + \frac{m(m-1)}{2} R_2 \,.$$

Letting r_m be the rate of change in cumulative cost during the m^{th} year, the ratio of cumulative cost in successive years can be used to obtain:

$$r_m = \ln\left[\frac{2mR_1 + m(m-1)\ R_2}{2(m-1)\ R_1 + (m-1)(m-2)\ R_2}\right],\ m = 2,3,\ldots$$

where $t - t_0 = 1$. This model allows RM to be treated as a continuous function. It is noted that this value of r_m will produce the same estimates of total routine maintenance cost as equation (4.6).

Equation (4.6) will be particularly important in evaluating routine maintenance costs during the optimization of overlay policies in Chapter V.

Seal Coat Costs

The basic reason for applying seal coats to a pavement is to fill surface cracks. Like routine maintenance, the application of a seal coat does not affect the serviceability index of the pavement. Since the construction of an overlay accomplishes the same result, there is rarely any need to seal coat a pavement in the same vicinity of time that the overlay is applied. The effects of seal coating and routine maintenance, however, are assumed to be independent.

Seal coats are frequently scheduled periodically around overlays with the time to the first seal coat generally being longer than the time between subsequent seal coats. This is because maintenance engineers prefer to preserve, as long as possible, the smooth finish produced by initial or overlay construction.

The model for generalizing the cost of seal coat applications during the analysis period requires a schedule of

1. τ_1, the time before the first seal coat after initial or overlay construction
2. τ_2, the time between subsequent seal coats
3. τ_3, the minimum time prior to a scheduled overlay that a seal coat will not be applied.

The times τ_1 and τ_2 may be identical without loss of generality.

The determination of the present value of all seal coats is actually very simple to describe but rather complicated to formulate mathematically. The basic idea is that of taking the following sum

$$SC = \sum_{j=1}^{NUM} \frac{SCC}{(1+i)^{y_j}}$$

where SC = present value of the cost/sq.yd. of all future seal coats

SCC = cost/sq.yd. of a seal coat

NUM = total number of seal coats applied during the analysis period

y_j = year in which the j^{th} seal coat is applied.

To be more specific,

$$SC = \sum_{k=1}^{K} \sum_{j=1}^{J_k} \frac{\xi}{(1+i)^{L_k}} \tag{4.7}$$

where SC = the present worth of all seal coats performed during the analysis period

$$J_k = \begin{cases} \dfrac{T_k - T_{k-1} - \tau_1 - \tau_3}{\tau_2} + 1, & \text{for } T_k \geq \tau_1 + \tau_3 \\ 0 & \text{, otherwise} \end{cases}$$

$$\xi = \begin{cases} 0 & \text{for } J_k = 0 \\ SCC & \text{, otherwise} \end{cases}$$

$$L_k = T_{k-1} + \tau_1 + (j-1)\tau_2 \ .$$

Actually, J_k is simply the number of seal coats performed in the k^{th} performance period.

User Costs

The purpose of this section is to describe the procedure for calculating users' or motorist's increase in cost due to being delayed while the various types of maintenance are being carried out. These costs include those associated with delay (time) and vehicle operation. Models will be developed for calculating these traffic costs. Five different models of handling traffic are discussed, and it is expected that other desired variations would require procedures very closely related to those given. It should be noted that the term "maintenance" as referred to in this section includes not only routine maintenance but seal coating and overlaying.

This section is divided into four sub-sections, the first of which describes the assumed speed profiles of vehicles in the vicinity of the zone where the maintenance is being performed. The second sub-section gives the time and operating costs associated with the different movements described by the speed profiles. In the third sub-section the five models for diverting traffic are described. Equations are given for computing the average proportion of vehicles stopped and the average time stopped in the fourth sub-section. These are for conditions in which traffic congestion arises because of traffic delay. The final sub-section gives relationships for computing the total user cost per square yard of pavement.

Speed profiles

It is assumed that all vehicles approach the maintenance area at the same speed, called the "approach speed". It is further assumed that there is a "restricted area" through which vehicles travel at a reduced speed. For some methods of handling traffic, this reduced speed may be the same for vehicles traveling in both directions; while in others, speeds may be different for each direction. Speeds are assumed to always be equal for vehicles going in the same direction. Generally, the vehicles traveling in the same direction as that in which the maintenance is being performed (hereafter referred to as direction A) will have their speed reduced as much or more than will those traveling in the opposite direction (direction B).

In general, the effective length of the restricted area is different for vehicles traveling in each direction. These lengths are denoted by L_A and L_B, respectively. The amount by which L_A is longer than L_B will be determined by the road geometrics and the method of handling traffic. This is discussed more fully in the section where the models for handling traffic are discussed.

While maintenance is being performed, vehicles travel through their respective restricted zones at "through speeds" of S_A and S_B, respectively. It is assumed that vehicles maintain these "through speeds" all the way through the restricted area.

In general, a proportion (P_{A1} in direction A and P_{B1} in direction B) of all vehicles will be stopped because of traffic congestion as they approach the restricted area. It is assumed that these vehicles stop and then accelerate back to their through speeds, reaching these speeds at the moment they enter their respective restricted zones. The vehicles travel at the reduced speed (S_A or S_B) through the restricted area and upon leaving the restricted area, they return to a speed equal to their approach speed.

Figure 4.1 shows the speed profile in direction A for vehicles that are stopped. The letters along the horizontal axis denote points where speeds are changed. The vehicle approaches the restricted area at a speed of AS, begins decelerating at point V and is stopped by the time it reaches point W. It remains stopped for a time (D_{A1} in direction A and D_{B1} in direction B) at point W, then accelerates back to the through speed S_A or S_B. This speed is reached at point X, the beginning of the restricted area. The vehicle then travels from point X to point Y at the through speed, and at point Y begins accelerating back to the approach speed which is reached at point Z. A comparable figure could be drawn for vehicles traveling in direction B.

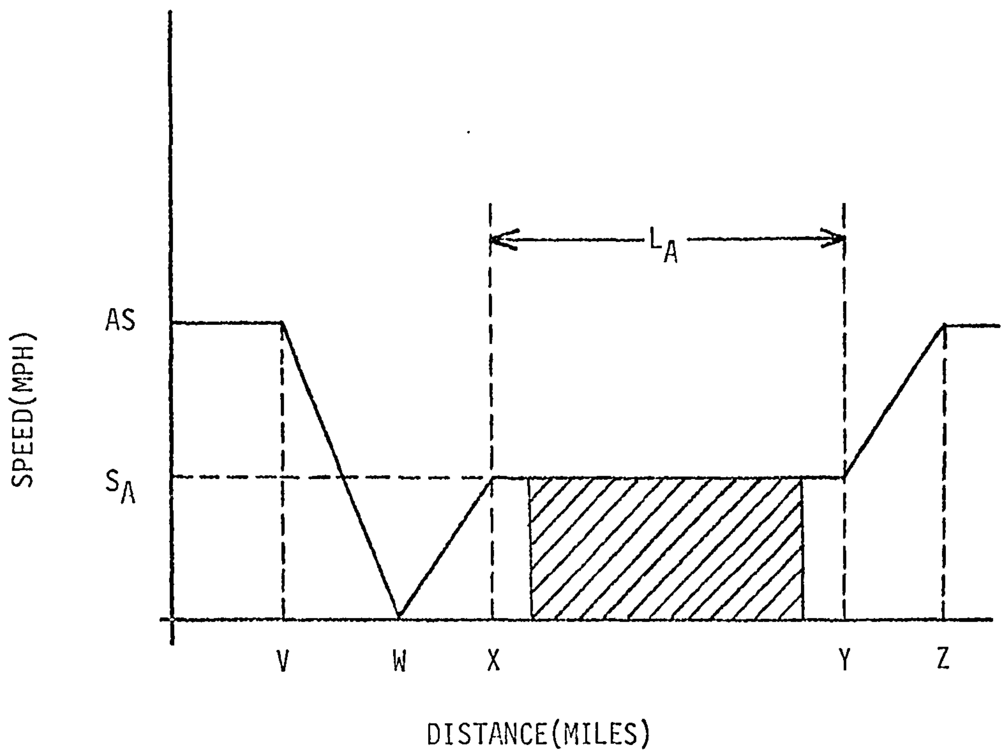

Figure 4.2 Speed profile in direction A for vehicles which are stopped because of traffic delay during maintenance operations

Vehicles which do not stop are slowed when they pass through the restricted area and it is assumed that their deceleration is such that they reach the through speed (S_A or S_B) at the moment they enter the restricted area. The proportion of vehicles which do not stop equals $(1-P_{A1})$ for direction A and $(1-P_{B1})$ for direction B.

Figure 4.3 shows the speed profile for a vehicle traveling in direction A but is slowed rather than being stopped by the maintenance operation. The letters along the horizontal axis again denote points where speeds are changed. These changes in speed have interpretations similar to those given in Figure 4.2.

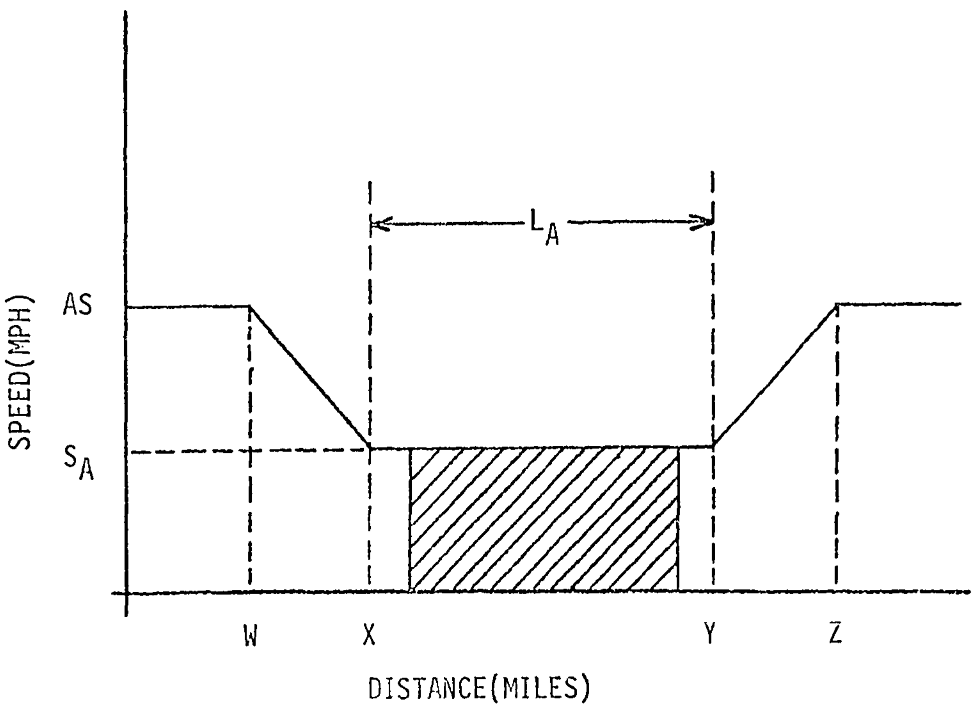

Figure 4.3 Speed profile for vehicles which are *not* stopped but are slowed by maintenance operations

In addition, a proportion (P_{A2} in direction A and P_{B2} in direction B) of all vehicles will be stopped inside of the restricted area because of movement of maintenance personnel and equipment. These vehicles are stopped for average times of D_{A2} and D_{B2} in the respective directions. It is to be noted that delays of types 1 and 2 are assumed to be independent of each other.

Because of maintenance operations, vehicles will travel through the restricted area with speed profiles as shown in Figure 4.2 and 4.3. In the absence of maintenance operations, they would have traveled at the approach speed through the restricted area. The excess traffic costs due to this inconvenience includes,

1. the excess time and operating costs as a result of stopping from the approach speed (from V to W in Figure 4.2),
2. the excess time and operating (idling) costs due to being stopped (at point W in Figure 4.2),
3. the excess time and operating costs due to slowing from the approach speed to the through speed (from W to X in Figure 4.3),
4. the excess time and operating costs due to traveling a distance L_A or L_B (from X to Y in Figure 4.2) at a reduced speed (S_A or S_B) instead of traveling at the approach speed (AS), and
5. the excess time and operating costs due to being stopped and delayed (idling) while inside the restricted area (not shown in either figure).

The excess costs of the aforementioned types are discussed in the next section.

Time and operating costs

The excess user costs because of maintenance includes the excess cost of stopping and slowing down, the cost of delay while stopped, and the excess cost of traveling at a reduced speed through the restricted area. The information required to compute these costs has been drawn from several sources. The primary consideration in determining these costs is the vehicle distribution of the traffic that is subjected to the delay. Based on exhaustive studies, Winfrey [81] has published tables containing various types of operating costs along with excess time requirements for making various speed changes under average conditions. These data were collected for the following five types of vehicles:

1. 4-kip passenger car
2. 5-kip commercial delivery truck
3. 12-kip single unit truck
4. 40-kip 2-S2 gasoline truck
5. 50-kip 3-S2 diesel truck.

Data from these tables are not reproduced here; rather, results of their use will be given.

These five types of vehicles are assumed to be indicative of normal traffic flow and are used as such throughout this section.

Furthermore, all other types of vehicles will be grouped into one of these categories. Table 4.1 contains proportions of vehicles in each of these groups for average urban and rural areas in Texas. These data have been reduced from 1966 studies by the Planning Survey Division of the Texas Highway Department [76].

	Proportions	
Group	Rural	Urban
1	.7209	.7961
2	.1193	.1432
3	.0419	.0436
4	.0393	.0067
5	.0786	.0104
	1.0000	1.0000

TABLE 4.1 AVERAGE PROPORTIONS OF VEHICLES IN RURAL AND URBAN AREAS IN TEXAS

	Value of an Hour	
Group	Rural	Urban
1	$ 3.00	$ 3.00
2	3.00	3.00
3	3.89	3.88
4	5.01	4.96
5	6.45	11.95

TABLE 4.2 WEIGHTED AVERAGE VALUES OF TIME FOR MOTORISTS IN TEXAS

Table 4.2 contains average values of time for average motorists in each of the five groups of vehicles. These values are based on information in studies by the Stanford Research Institute [29], Lisco [47], and Adkins [1].

With this information, weighted average values of the combined cost of operation and delay have been computed from

$$W = \sum_{i=1}^{5} P_i \left(C_i + H_i V_i\right) \qquad (4.8)$$

where W = weighted average of costs

P_i = proportion of traffic in the i^{th} vehicle group

C_i = excess operating cost for vehicles in the i^{th} group

H_i = excess time for vehicles in the i^{th} group

V_i = average value of time for motorists of vehicles in the i^{th} group.

These total cost values are given in Tables 4.3 through 4.6. Table 4.3 gives the average excess time and operating costs for speed changes on rural roads in Texas. Table 4.4 gives comparable information for urban roads in Texas. Time and operating costs of traveling at uniform speeds are given in Table 4.5. Table 4.6 gives the average time and operating costs of idling.

Methods of diverting traffic

There are several methods of diverting traffic during maintenance operations. The method used in any instance will depend primarily upon highway geometrics, especially the number

Initial Speed MPH	DOLLARS PER 1000 CYCLES, BY SPEED REDUCED TO AND RETURNED FROM (MPH)					
	0	10	20	30	40	50
10	8.473					
20	18.200	9.413				
30	31.550	21.491	11.354			
40	50.360	39.609	28.422	15.795		
50	77.932	66.233	53.917	39.941	22.612	
60	120.546	106.979	92.482	76.022	56.405	32.485

TABLE 4.3 DOLLARS OF EXCESS OPERATING AND TIME COST OF SPEED CHANGE CYCLES - EXCESS COST ABOVE CONTINUING AT INITIAL SPEED, FOR RURAL ROADS IN TEXAS

Initial Speed MPH	DOLLARS PER 1000 CYCLES, BY SPEED REDUCED TO AND RETURNED FROM (MPH)					
	0	10	20	30	40	50
10	5.869					
20	11.769	5.602				
30	19.500	12.857	6.501			
40	30.030	22.933	15.976	8.607		
50	45.002	37.338	29.610	21.448	11.856	
60	67.868	58.992	49.114	40.242	29.360	16.432

TABLE 4.4 DOLLARS OF EXCESS OPERATING AND TIME COST OF SPEED CHANGE CYCLES - EXCESS COST ABOVE CONTINUING AT INITIAL SPEED, FOR URBAN ROADS IN TEXAS

Uniform Speed MPH	DOLLARS PER 1000 MILES RURAL ROADS	URBAN ROADS
5	$ 750.20	$ 692.06
10	393.47	362.43
15	274.15	252.21
20	214.53	197.06
25	179.13	164.16
30	156.05	142.57
35	140.18	127.58
40	129.03	116.84
45	121.06	108.98
50	115.51	103.24
55	111.94	99.18
60	110.16	96.73

TABLE 4.5 DOLLARS OF OPERATING AND TIME COST PER 1000 VEHICLE-MILES AT UNIFORM SPEEDS IN TEXAS

TYPE OF DELAY COST	DOLLARS PER 1000 HOURS RURAL ROADS	URBAN ROADS
Operating	$ 132.22	$ 122.89
Time	$ 3,367.54	$ 3,140.22
Total	$ 3,499.76	$ 3,263.11

TABLE 4.6 DOLLARS OF OPERATING AND TIME COST OF DELAY (OR IDLING) PER 1000 VEHICLE-HOURS, IN TEXAS

of lanes, the type of median (if any), and the presence or absence of pavement shoulders, frontage roads, or other alternate routes. The five models presented here are considered to be representative of the most commonly used methods, and it is expected that variations in these models will not be difficult.

The first two models for diverting traffic are to be used for two-lane roads (with or without shoulders), and the other three models are used for roads with four or more lanes. These five methods are depicted by Figures 4.4 through 4.8. In each case, consideration is given only for maintenance being performed on lanes in one direction (direction A, as previously described). It is assumed that comparable delays will be experienced when the maintenance is eventually performed on the lane(s) in direction B. The length L_A is the average distance over which traffic traveling in direction A is slowed by the maintenance operation at any one time. Traffic flowing in direction B will similarly be affected, but the restricted length L_B will generally be smaller than L_A. In some cases it might be desired to set L_B equal to zero. It is to be noted that these distances are not necessarily constant. This is particularly true in Models IV and V. As maintenance continues, the restriction area will also continuously flow resulting in varied lengths of L_A. In this case an average value of L_A can be used. Another alternative would be to divide the total distance to be maintained into several sections and separately consider the traffic costs for each section.

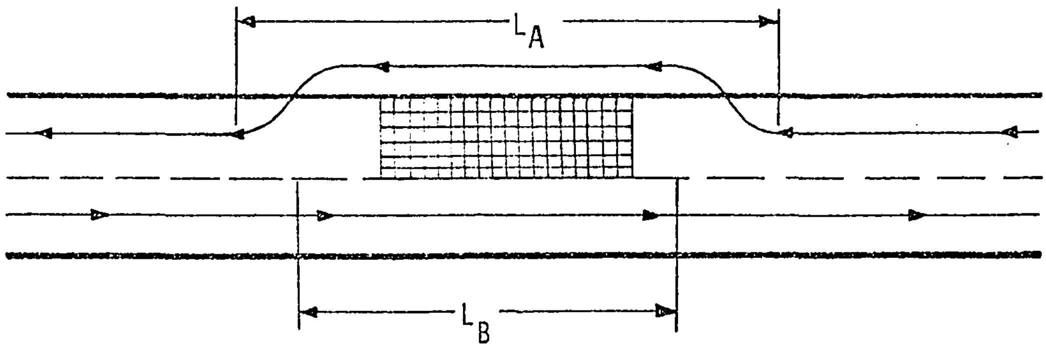

Figure 4.4 Model I. Traffic routed to shoulders

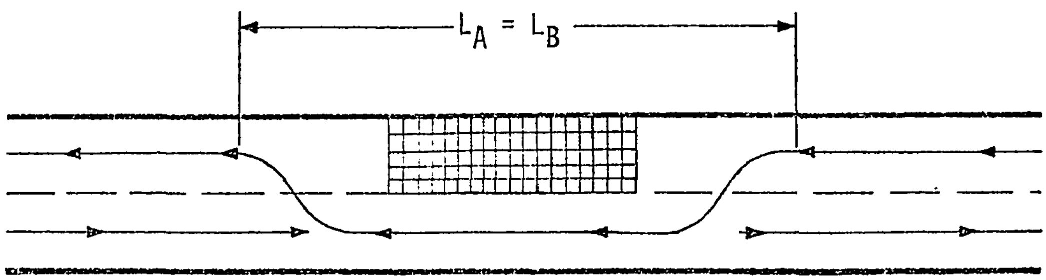

Figure 4.5 Model II. Alternating traffic in one lane

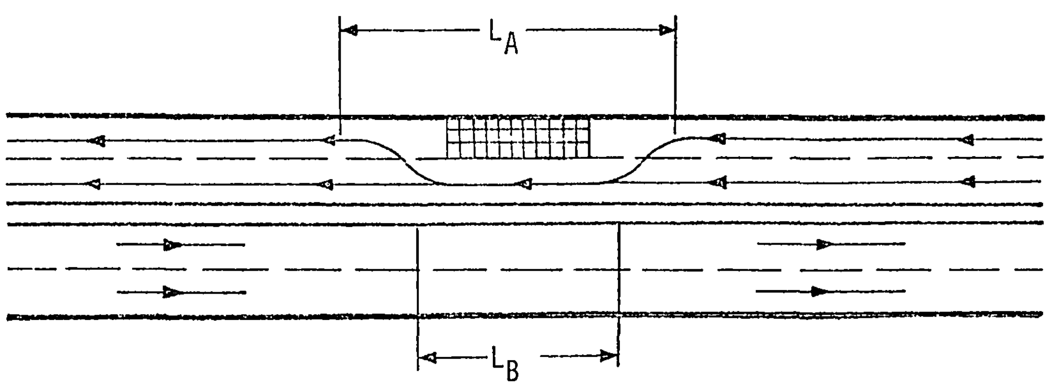

Figure 4.6 Model III. Two lanes merge

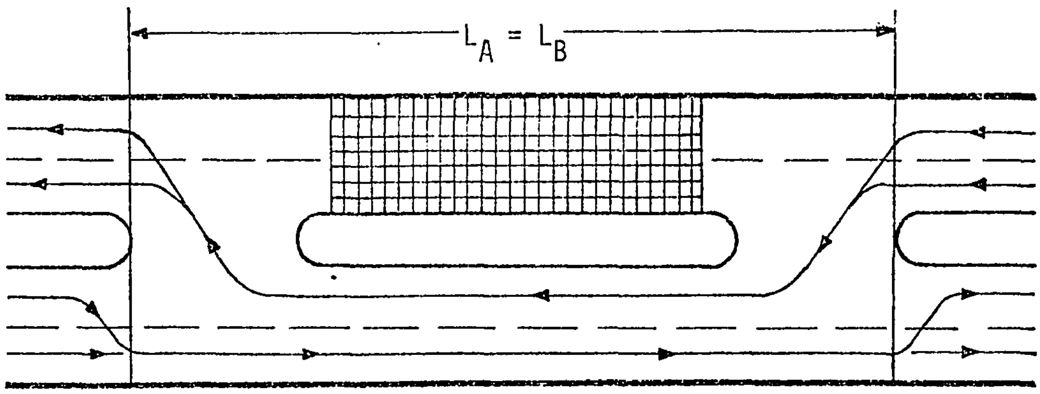

Figure 4.7 Model IV. Traffic in direction A routed to oncoming lanes

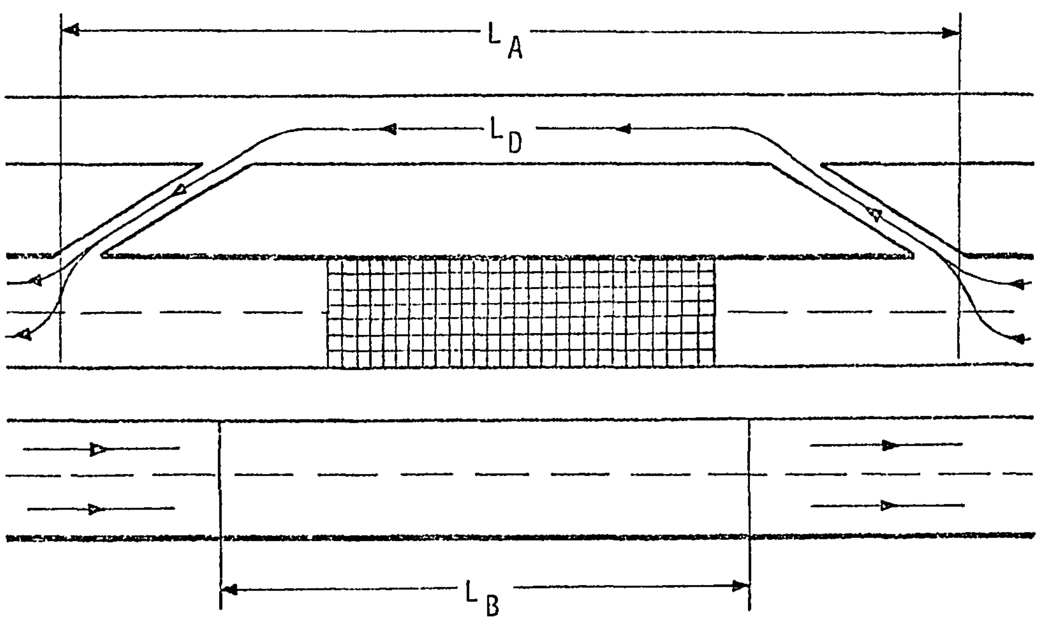

Figure 4.8 Model V. Traffic routed to frontage road or other parallel route

Model I, as depicted in Figure 4.4, is for two-lane roads with shoulders. Direction A traffic will normally be diverted onto the shoulder and such traffic must generally slow down. Traffic in direction B proceeds as usual but must also slow down though probably not as much as the traffic traveling in direction A. Another version of this model is to divert the direction B traffic onto its shoulder and divert the direction A traffic into the on-coming lane. In addition to the delay due to traveling at a reduced speed, a proportion of the traffic may be additionally delayed by having to stop due to movement of maintenance personnel and equipment in the restricted area and by the inability to overtake other vehicles in the restricted area. Since these proportions very often vary between maintenance operations and locations, they are assumed to be predictable by the design engineer.

Model II is shown in Figure 4.5. For two-lane roads without shoulders, it is sometimes necessary to post flagmen at each end of the restricted area to stop traffic in one direction while traffic from the other direction proceeds through the restricted area. The flagmen determine from which direction traffic is to flow through the restricted area at any one time. Vehicles arriving first usually have priority. If an additional vehicle arrives while other vehicles going in the same direction are proceeding through the restricted area, then this additional vehicle usually may also proceed through the area, except that it will be stopped when the vehicles in the queue from the other direction are of a number or have been waiting a time which justifies priority for them. Traffic from each direction travels through the restricted area at a reduced speed. Vehicles may also be stopped to allow for the maintenance personnel and equipment in the restricted area. In general, the proportion of traffic stopped to give way to vehicles from the opposite direction will be higher for longer lengths of $L_A (=L_B)$ and for higher volumes of traffic.

There are several possible ways of diverting traffic when there are two or more lanes in each direction. In Figure 4.6, Model III is useful when there is a non-transversable median. It is assumed that at least one lane in direction A remains open for traffic. For low traffic volumes the effects on traffic in direction A will be the delays due to traveling at a reduced speed through the area and occasional stops due to movement of maintenance personnel and equipment. For volumes in which the flow of traffic is above the capacity of the restricted roadway, a queue will result upstream of the maintenance operation which will lead to vehicles being stopped due to congestion.

For other roads with two or more lanes in each direction, Model IV may be applicable. Figure 4.7 represents this model as having either

1. no medians,

2. low medians that can be crossed at any point, or

3. median openings at regularly spaced intervals.

In these cases it is sometimes desirable to block all lanes in direction A and divert the traffic onto the on-coming lanes. Figure 4.7 depicts the case for a road with median openings. The restricted length L_A will generally be much longer than the actual distance required by the maintenance operation whereas in the other two cases L_A would be approximately equal to the L_A of Model III. Under these conditions both directions of traffic will be affected, and the method of calculating the average delay and proportion of vehicles stopped in each direction is the same as the computation for direction A in Model III.

Model V is a variation of Model III in that traffic in direction A is generally squeezed to an off-lane ramp whereas direction B traffic is unaffected except for slowing down and occasional stops due to equipment and personnel. In this case, traffic is diverted to an alternate route such as a frontage road or a parallel street. Unlike other models, vehicles may be required to travel a significantly farther distance than L_A before returning to the original roadway. Accurate account must be taken of the difference between the actual distance traveled, L_D, and the distance that would have been traveled had there been no detour. It is noted that the restricted length, L_A, requires a different interpretation for this model.

Average delay and proportion of vehicles stopped

The delay to traffic due to maintenance is of four basic types. These are with respect to a vehicle

1. traveling at a reduced "uniform" speed in the restricted area,
2. not having the ability to overtake and pass other vehicles traveling in the same direction,
3. having to stop because of the movement of personnel and equipment in the restricted area, and
4. having to stop because of congestion when the traffic demand exceeds the capacity of the restricted area.

The delay per vehicle due to traveling at a reduced speed is the difference between the travel time at the reduced speed through the restricted area and the travel time vehicles would have had through the same area had it not been restricted.

The delay due to vehicles not having the ability to overtake and pass other vehicles traveling in the same direction because of the maintenance operation may result both inside and outside of the restricted area. The delay within the restricted area is included in the delay discussed in the preceding paragraph if accurate estimates of reduced speeds are used. The delay due to the inability to pass outside the restricted area because of maintenance is more complicated and is probably largest in cases where the capacity of the restricted area is smaller than the demand (or input) to the area. In such cases, wherein demand exceeds capacity, there will be congestion and queueing of vehicles. These queues must disperse after leaving the restricted area and before such dispersion occurs there probably will be some inability to pass other vehicles. Such effect probably will be small in all cases except Model II with considerable queueing. If the length of the restricted area is fairly long and/or traffic volumes are fairly large, then in Model II, fairly long queues of vehicles will arise. Such queues develop in one direction while vehicles are traveling through the restricted area from the other direction. When the queue is finally allowed to proceed through the restricted area, it will emerge as a moving queue, sometimes of considerable length, and extra delay will result until the queue disperses. The rate of dispersion of this moving queue will depend chiefly upon the amount of traffic from the opposite direction, the road geometrics, and the type of vehicles in the queue. This moving queue will also impede passing by traffic from the opposite direction; however, there will be a considerable length of road in front of this queue in which there will be no (or very few) vehicles. This will somewhat offset the effects of the moving queue on traffic in the opposite direction. Since the type of delay discussed in this paragraph probably is small in most cases and since it is difficult to predict, it is ignored (i.e., assumed to be zero).

The third type of delay, results from vehicles having to stop because of the movement of maintenance personnel and equipment in the restricted area. It is computed as the product of the average number of vehicles stopped and the excess cost per vehicles of stopping and remaining stopped for an average period of time. Delays of this type often vary considerably between maintenance operations. For this reason, the design engineer is expected to predict

1. P_{A2} and P_{B2}, the proportions of vehicles stopped due to movement of maintenance personnel and equipment, and
2. D_{A2} and D_{B2}, the average times that vehicles remain stopped.

It is expected that the maintenance operation will be performed in such a manner that the number of vehicles stopped for these reasons will be small. Therefore, it is not infrequent that estimates of the proportions of such stops are near or equal to zero.

The fourth type of delay results from vehicles having to stop because of congestion that results from the traffic demand exceeding the capacity of the roadway in the restricted area. In Model I this type of delay should be almost nonexistent and is assumed to be zero. For the other four models, the method of computing the proportion of vehicles stopped (P_{A1} in direction A and P_{B1} in direction B) and the average time stopped for each stopped vehicle (D_{A1} and D_{B1}) will next be described.

Model II assumes that the hourly traffic is evenly divided between directions. It also assumes that the number of vehicles arriving during an interval of time from each direction follows a Poisson distribution. The proportion of vehicles stopped due to congestion (P_{A1} or P_{B1}) can be estimated from a queueing model developed by Tanner [69]. The resulting equation (4.9) has not been operationally verified but simulations have indicated that realistic proportions can be obtained. The equation is

$$P_{A1} = P_{B1} = 0.5(1-e^{aI})^2 \tag{4.9}$$

where a = the average time required to travel through the restricted area ($a = L_A/S_A$)

I = the average number of vehicles arriving at the restricted area per hour.

Based on the same study, Tanner also developed the following relationship for estimating the average time stopped for stopped vehicles (D_{A1} or D_{B1}):

$$D_{A1} = D_{B1} = \frac{(1+e^{2aI})\ (e^{aI}-aI-1)}{2I(e^{2aI}-e^{aI}+1)\ P_{A1}} \ . \tag{4.10}$$

The California Division of Highways [67] developed a theoretical traffic model to determine the average total delay experienced by vehicles during a period of lane restriction. Their approach can be extended to also obtain relationships for estimating P_{A1}, P_{B1}, D_{A1}, and D_{B1} for roadways with four or more lanes. These relationships will be useful for the analysis of Models III, IV, and V.

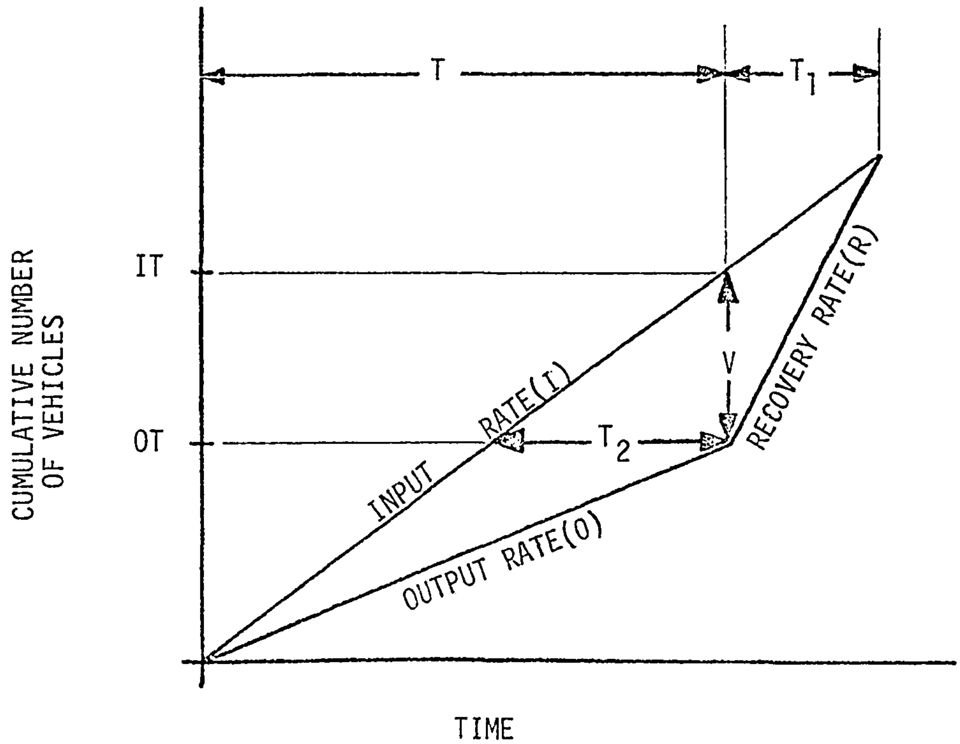

Figure 4.9 Delay in terms of capacitated flow rates

Figure 4.9 shows the relation between capacity and delay when the demand rate-of-flow exceeds capacity. It is assumed that the input (demand) rate I is the same for both directions of travel. Normally, lane restrictions for maintenance purposes will take place during daylight hours. The average hourly traffic for these times will necessarily be greater than 1/24th of the average daily traffic. Hourly estimates of 6 percent of average daily traffic in rural areas and 5 percent of average daily traffic in urban areas have been found to be quite accurate [31].

The symbols in Figure 4.9 may be interpreted as:

I = input rate in vehicles/hour

O = restricted output rate in vehicles/hour

R = recovery rate in vehicles/hour

T = hours/day of lane restriction

T_1 = hours required for traffic to return to normal after the lane restriction is removed

V = maximum number of vehicles delayed at one time (just before the lane restriction is removed)

T_2 = maximum delay to an individual vehicle in hours

D = total delay in vehicle-hours.

Based on Figure 4.9, it can be seen that V can be expressed by

$$V = T(I-O) \text{ vehicles.} \tag{4.11}$$

The time, T_1, required for traffic to return to normal conditions can be determined from

$$RT_1 = V+IT_1. \tag{4.12}$$

Substitution of 4.11 into 4.12 provides for the determination of T_1, as

$$T_1 = \frac{T(I-O)}{R-I} \text{ hours.} \tag{4.13}$$

The total delay for the period of lane restriction, T, and the recovery period, T_1, is represented by the area between the two curves. Mathematically, this is

$$D = \frac{TV}{2} + \frac{T_1 V}{2} \tag{4.14}$$

which can be reduced, by substitution of equation (4.11), to:

$$D = \frac{T}{2}\,(I-O)(T+T_1)\ . \tag{4.15}$$

This total delay time can be split into the two components

1. total traveling time, and
2. total stopped time.

Assuming that vehicles travel at the through speed when they are traveling, these components can be related as,

$$\frac{D}{I(T+T_1)} = \frac{L_A'}{S_A} + T_S \tag{4.16}$$

where $I(T+T_1)$ = total number of input vehicles before recovery is complete

L_A' = effective restricted length (including the accumulated queue)

T_S = total stopped time.

On the average, the queue length is

$$L_A' - L_A = \frac{VM'L}{2M} \tag{4.17}$$

where V = the number of vehicles delayed at the time recovery begins

M' = number of lanes open during restricted operation

M = number of lanes open during normal operation

L = vehicles spacing (including the average length of vehicles), in miles/vehicle.

Only those vehicles that do get stopped contribute to the total stopped time, and this is accounted for by

$$T_S = IT\, P_{A1} D_{A1} \tag{4.18}$$

where P_{A1} = proportion of vehicles stopped

D_{A1} = average stopped time per stopped vehicle.

This assumes that no vehicles will be stopped after recovery begins.

Equations (4.17) and (4.18) can be substituted into equation (4.16) and simplified to find that

$$D_{A1} = \frac{(T/I)(I-O)MS_A - 2ML_A - T(I-O)M'L}{2MIT\, S_A P_{A1}} . \tag{4.19}$$

Equation (4.11) estimates the maximum number of vehicles that will be <u>delayed</u> at any one time. This can also be interpreted as the minimum number of vehicles that will be <u>stopped</u>. From this, the total number of stopped vehicles can be approximated by

$$P_{A1} = \frac{V}{IT} \left(\frac{I}{O}\right)^2 \tag{4.20}$$

where the squared term is an input-output scaling factor which forces P_{A1} to increase parabolically as the ratio I/O increases. In this form it is possible for P_{A1} to be greater than 1, but this occurance can be properly interpreted as some vehicles have to stop more than once.

Finally, equation (4.20) can be used to solve for D_{A1} in equation (4.19). For use in determining user costs applicable to Models III, IV, and V, these relationships must be used with Table 4.7.

Percent Trucks	No. of Lanes One Direction (Normal Operation)		2	3	4
	No. of Lanes One Direction (Restricted Operation)		1	2	3
0 - 10	Output Rate	0	1400	2800	4500
(Urban)	Recovery Rate	R	3000	4700	6400
over 10	Output Rate	0	1350	2700	4350
(Rural)	Recovery Rate	R	3000	4500	6200

TABLE 4.7 CAPACITY TABLE OF FLOW RATES IN VEHICLES PER HOUR

This table, prepared during the California study, shows the flow capacities on various types of roads when operated under restricted and unrestricted conditions. Output rate is the capacity of the restricted system while recovery rate is the capacity of the unrestricted system. These capacities pertain to rates at which excessive congestion occurs rather than being absolute capacities. Input rates greater than the listed output rates will normally result in the formation of a queue.

User cost equations

The purpose of this section is to develop relationships, based on the delays discussed in the previous section, for computing the total user cost associated with a single maintenance operation. The basic approach is that of determining the hourly cost as a function of the traffic volume and the fixed proportions of this traffic that will be subjected to the various types of delay. Mathematically, this hourly rate is

$$\begin{aligned} UCR = I\,[&P_{A1}(C_{A1}+C_{A2}+C_{A3})+(1-P_{A1})(C_{A3}+C_{A4})+P_{A2}(C_{A5}) \\ &+P_{B1}(C_{B1}+C_{B2}+C_{B3})+(1-P_{B1})(C_{B3}+C_{B4})+P_{B2}(C_{B5})] \end{aligned} \tag{4.21}$$

where the subscript "A" refers to vehicles traveling in direction A and the subscript "B" refers to vehicles traveling in direction B. In addition, the variables are defined as:

UCR	=	the user cost rate in dollars per hour
I	=	the hourly traffic volume in one direction
P_{A1} and P_{B1}	=	the proportions of traffic which are stopped due to the capacity of the restricted section being less than the demand (the proportions stopped because of congestion)

P_{A2} and P_{B2} = the proportions of traffic which are stopped due to the movement of maintenance personnel and equipment in the restricted area

C_{A1} and C_{B1} = the excess costs per vehicle of stopping from the approach speed (values given in the first columns of Tables 4.3 and 4.4, divided by 1000)

C_{A2} and C_{B2} = the costs per vehicle of idle time for vehicles stopped (derived by multiplying D_{A1} and D_{B1} by values for idling from Table 4.6, divided by 1000)

C_{A3} and C_{B3} = the excess costs per vehicle of traveling through the restricted area of length L_A and L_B at a reduced speed (equal to L_A or L_B multiplied by the cost of operating at a reduced speed S_A or S_B, minus the cost of operating at the approach speed AS, all taken from Table 4.5 and divided by 1000)

C_{A4} and C_{B4} = the excess costs per vehicle of slowing from the approach speed AS to through speeds S_A and S_B and returning to the approach speed (taken from Tables 4.3 and 4.4, divided by 1000)

C_{A5} and C_{B5} - the excess costs per vehicle of stopping from the through speed S_A or S_B (taken from Tables 4.3 and 4.4 and divided by 1000), plus the cost of idling while stopped due to the movement of maintenance personnel and equipment (obtained by multiplying D_{A2} and D_{B2} by appropriate values from Table 4.6).

As noted, UCR is the hourly rate of maintaining direction A, only. Fortunately, it is also generally true for direction B. The possible exception is in Model V where the detoured distances may be different for each direction of traffic.

The next step is to convert this cost per hour to a cost per square yard of pavement. Here it is necessary, for the first time, to be specific about the type of maintenance under consideration. In the case of user cost for an overlay operation, the following relationship is defined:

$$U_0 = (ACCD)(OT + \frac{1}{36})(UCR)/(ACPR) \qquad (4.22)$$

where U_0 = user cost in dollars per square yard of pavement surface

ACCD = asphaltic concrete compacted density in tons per compacted cubic yard

OT = overlay thickness in yards

UCR = user cost rate in dollars per hour

ACPR = asphaltic concrete production rate in tons per hour.

The extra inch added to the overlay thickness is referred to as a level-up inch and has the purpose of smoothing the pavement before actually overlaying it.

The user cost associated with a seal coat application is similar except that there is no thickness to consider. The relationship is simply

$$U_S = (UCR)/(SCPR) \tag{4.23}$$

where U_S = user cost in dollars per square yard of pavement surface

SCPR = seal coat production rate in yards per hour.

Similarly, a relationship for routine maintenance user cost is

$$U_R = (UCR)/(RMPR) \tag{4.24}$$

where the quantities U_R and RMPR are similar to their counterparts in the seal coat model. In actual practice it may not be desired to include the user cost associated with routine maintenance. This cost is relatively insignificant when compared to other costs, principally because of its faster production rate. In addition, traffic diversion techniques are not exactly compatible with the concepts of traffic diversion with respect to seal coating and overlaying. A third reason is the amount of variance associated with the actual performance of routine maintenance. As considered in this research, routine maintenance is a function that should continuously take place in order to insure maximum serviceability. Unfortunately, this policy is not always followed and routine maintenance is very often performed whenever possible, rather than whenever required.

As in the case of the physical cost models, present worth factors must be applied to the user costs to discount these future costs back to the present.

Salvage Value

In previous discussions it has been assumed that alternative designs are compared on the basis of total cost within a specific analysis period. Seldom will the pavement be abandoned at the end of this analysis period. It would be more general to assume that a major reconstruction would be performed at that time. In either case some return would normally be derived from the structure. It is assumed in this research that this net return will be the value of the usable materials in the pavement less the cost of again making them usable or disposing of them.

The salvage value of a pavement is variable and depends upon several factors. For example, if a major change in vertical or horizontal alignment is made at the time of reconstruction, the salvage value may be negligible. In some cases materials must be removed. In these instances the salvage value would be the value of the materials (if they were to be reused) less the cost of removing the materials. This value, of course, may be negative.

The salvage value is dependent upon the future plans for the roadway. This dependence is incorporated into a factor for the expected percent of material worth. Based upon these considerations, the salvage value per square yard, SV, is computed from the following relationship:

$$SV = P_{sv} \left[\frac{\sum_{j=1}^{N} D_j C_j + \sum_{k=1}^{K-1} C_1 O_k}{(1+i)^C} \right] \tag{4.25}$$

where P_{sv} = the percent of the value of the materials in the pavement at the end of the analysis period

D_j = the depth of the j^{th} layer in the initial construction

C_j = the cost per square yard/inch of the material in the j^{th} layer

C_1 = the cost per square yard/inch of overlay construction

O_k = the depth in inches of the k^{th} overlay

N = the number of layers of pavement in the initial construction

K = the number of performance periods (K-1 is the number of overlays)

C = the length of the analysis period.

The quantity P_{sv} might equally well be interpreted as a probability so that equation (4.25) would simply be an expected value computation. As in previous equations, the fact that P_{sv} is subjectively determined is of no great consequence as long as it is constant for alternative designs.

In most cases, there is not a great deal of analysis associated with the cost models presented in this chapter. Additional research in some areas may provide valuable insight into the possible improvement of these models. For the present, they serve the purpose of being realistic estimates of the future costs of a roadway. In this capacity, they will form the basis of the optimization algorithms to be presented in Chapters V and VI.

CHAPTER V

FORMULATION OF THE OPTIMAL FLEXIBLE PAVEMENT DEISGN PROBLEM

The flexible pavement design problem was described in basic terms in Chapter I. The purpose of this chapter is to state the problem mathematically and to describe the underlining assumptions to the problem.

The flexible pavement design process can be viewed as a sequential decision process over an analysis period of C years in length. On a time scale, the design process can be represented as in Figure 5.1, the decisions being made at times T_k, $k = 0,1,\ldots,K$. Initial construction is assumed to take place at time $T_0 = 0$ with pavement overlays being constructed at times T_k, $k = 1,2,\ldots,K$.

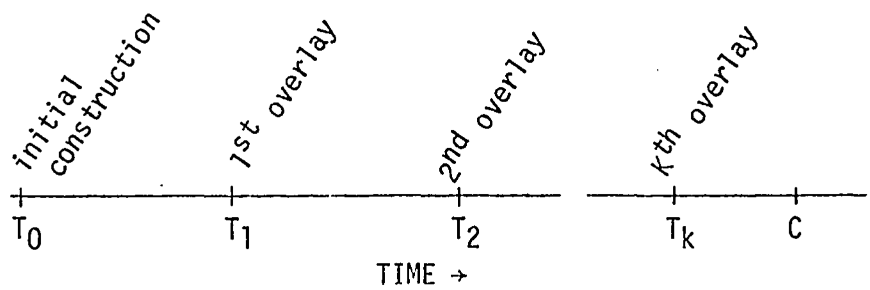

Figure 5.1 Sequential decision process represented over time

The decision variables in the process are those associated with;

1. the initial construction depths D_j for each layer $j = 1,2,\ldots,N$
2. the times T_k, $k = 1,2,\ldots,K$, at which overlay construction takes place
3. the thickness O_k of the k^{th} overlay.

Values of these decision variables are to be determined such that the total cost of the structure during the analysis period is a minimum.

It is assumed that there are N different materials available for initial construction. Because of the collapsibility and consistency of the deflection equation (3.6), an initial design can, in general, be viewed as consisting of N layers with any D_j, $j = 2,3,\ldots,N$, being zero. It is assumed that the first layer will always be asphaltic concrete. Other than this, all combinations of materials are allowed, within the constraint that they be ordered in decreasing strength. This restriction disallows physically unrealistic designs consisting of stronger materials being located above weaker materials. Occasional reference will be made to designs consisting of n layers. This will imply that there are n layers of non-zero thickness.

The total cost of the structure is composed of two basic components. These are:

1. the cost of initial construction
2. the future cost of all types of maintenance.

As described in Chapter III, there is only one mathematical link between these two components. This link is the set of relationships for deflection (3.6), traffic (3.12), and performance (3.13). These relationships are of such a complex nature that it is not possible to formulate a single objective function containing all of the decision variables. As a result, it is necessary to consider the optimization process as being composed of two smaller problems, one for each basic cost component. Of course, an optimal design policy would be based upon the cumulative cost.

Solving two optimization problems, instead of one, necessarily means that the size of the overall problem is increased many-fold because it is necessary to determine the optimal overlay policy for each feasible initial construction design. The procedure for determining initial construction designs will be that of determining the "best" design for each possible incremental investment in initial construction cost. This problem is formulated as an integer programming problem in the next section. The algorithm for overlay optimization is considered in the section following that of initial construction.

The Initial Design Problem

The objective of the initial design problem is to determine the layer depths D_j, $j = 1,2,\ldots,N$, that result in the strongest pavement (minimum deflection) with respect to the initial investment and other restrictions that are to be specified.

This problem can be stated as

$$\min S = k_0 \left\{ a_1^{-k_1} \left[\frac{1}{100} - \frac{1}{244} \right] \right. \tag{5.1}$$

$$+ \left(a_2^{-k_1} - a_1^{-k_1}\right) \left[\frac{1}{100+6.25(a_1 D_1)^2} - \frac{1}{244+6.25(a_1 D_1)^2} \right]$$

$$+ \left(a_3^{-k_1} - a_2^{-k_1}\right) \left[\frac{1}{100+6.25(a_1 D_1 + a_2 D_2)^2} - \frac{1}{244+6.25(a_1 D_1 + a_2 D_2)^2} \right]$$

$$+\cdots+ \left(a_{n+1}^{-k_1} - a_n^{-k_1}\right) \left[\frac{1}{100+6.25(a_1 D_1 + \cdots + a_n D_n)^2} \right.$$

$$\left. \left. - \frac{1}{244+6.25(a_1 D_1 + \cdots + a_n D_n)^2} \right] \right\}$$

subject to:

$$\sum_{j=1}^{N} C_j D_j \leq \gamma$$

$$D_j = 0,\ \theta_j,\ \theta_j + \Delta,\ \theta_j + 2\Delta, \ldots, \phi_j \quad , \ j = 1,2,\ldots,N$$

$$\sum_{j=1}^{N} D_j \leq \psi$$

where C_j = cost per inch thickness of a compacted square yard of the j^{th} material

D_j = depth of the j^{th} layer

θ_j = minimum allowable depth of the j^{th} layer, if $D_j > 0$

ϕ_j = maximum allowable depth of the j^{th} layer

Δ = increment in the values of depth that each layer may assume

ψ = maximum total thickness allowed in initial construction.

The quantity γ is to take on discrete values between 0 and CM, where CM can be interpreted as the maximum funds per square yard available for investment in initial construction. The quantity CM must arbitrarily be as large as is necessary to insure that all feasible initial designs are considered. Even more important than being required by the algorithm, this is very often a budgeted quantity that cannot be exceeded. As a result, even more emphasis is placed upon the need for a design optimization procedure. When budget is not a limiting factor, amounts in the neighborhood of \$4 for CM have been found to be satisfactory. Experience has also indicated that γ should be incremented by an amount ranging from \$0.01 to \$0.05.

The quantities θ_j and ϕ_j serve as physical minimum and maximum restrictions, respectively. The material, crushed stone, exemplifies the need for a minimum allowable thickness restriction. Since the average diameter of these stones may be one inch, a three or four inch minimum thickness restriction would normally be appropriate. For other types of materials, it may be physically unrealistic to exceed a prescribed maximum.

Solution Approach to the Algorithm

To qualify as in integer programming problem, it is not required that the decision variables take on strictly integral values. It is assumed in this design procedure that all of the D_j, $j = 1,2,\ldots,N$, be in one-half inch increments ($\Delta = 1/2$) so that a simple transformation will result in integral values of the D_j.

Since the majority of the discussion in the remainder of this chapter will be in terms of binary variables, it is convenient at this point to restate problem (5.1) in terms of only binary variables. This is done through the transformation (2.24). In condensed form the new problem is

$$\min S = k_0 a_1^{-k_1}\left[\frac{1}{100} - \frac{1}{244}\right] \tag{5.2}$$

$$+ \sum_{n=1}^{N} k_0 \left[a_{n+1}^{-k_1} - a_n^{-k_1}\right]\left\{\frac{1}{\left[100+6.25\left[\sum_{i=1}^{n} \frac{a_i}{2} \sum_{j=1}^{J_i} 2^{j-1}\, d_{ij}\right]^2\right.}\right.$$

$$\left. - \frac{1}{244+6.25\left[\sum_{i=1}^{n} \frac{a_i}{2} \sum_{j=1}^{J_i} 2^{j-1}\, d_{ij}\right]^2}\right\}$$

subject to:

$$\sum_{i=1}^{N} 0.5C_i \sum_{j=1}^{J_i} 2^{j-1} d_{ij} \leq \gamma \tag{5.3}$$

$$0.5 \sum_{j=1}^{N} 2^{j-1} d_{ij} = 0, \theta_i, \theta_i + \Delta, \theta_i + 2\Delta, \ldots, \phi_i, \; i=1,2,\ldots,N$$

$$\sum_{i=1}^{N} 0.5 \sum_{j=1}^{J_i} 2^{j-1} d_{ij} \leq \psi$$

$$d_{ij} = 0 \text{ or } 1 \qquad \text{for all } i \text{ and } j$$

where J_i is the number of binary variables required to represent all of the possible values of D_j. For a constant γ, each J_i is determined on the basis of two criteria. The two restrictions are:

1. $D_j \leq \phi_j$, $j = 1,?,\ldots,N$
2. $D_j \leq \gamma/C_j$, $j = 1,2,\ldots,N$.

Therefore, $D_j \leq \min(\phi_j, \gamma/C_j)$. The second criteria simply states that if all available funds were invested in layer j, its depth would be γ/C_j.

Examination of problem (5.2) will indicate that it cannot be solved by most optimization techniques because it is both non-linear and non-separable. For this reason, it appears to be amenable to practical solution only by a branch and bound approach. The branch and bound algorithm that is presented in this research is very similar to Geoffrion's modification to Balas' method [24]. The specific algorithm will be given later in this section. The intention is to first indicate how the branch and bound procedure is adaptable to problem (5.2).

It is felt that a small example will greatly enhance the discussion that follows. Consider the initial design problem with only two materials available, possible layer depths being any combination of the specific values

$$D_1 = 1/2,\ 1,\ 1\text{-}1/2 \qquad (5.4)$$
$$D_2 = 0,\ 1,\ 1\text{-}1/2,\ 2,\ 2\text{-}1/2.$$

A transformation to binary variables is not required by the general branch and bound method but is used by this procedure principally because the decision rules become easier to formulate. The symbol D_j, as before, is the thickness in inches of the j^{th} layer. The symbol d_{ij} will denote binary variables according to the transformation

$$D_j = 0.5 \sum_{i=1}^{k_j} 2^{i-1} d_{ij} \quad , \quad d_{ij} = 0 \text{ or } 1. \qquad (5.5)$$

For a fixed j, k_j will be the number of binary variables required to allow D_j to assume any of its possible values. In the example

$$k_1 = 2$$
$$k_2 = 3$$

results in the possible values

$$D_1 = 0.5(2^0 d_{11} + 2^1 d_{21})$$

$$D_2 = 0.5(2^0 d_{12} + 2^1 d_{22} + 2^2 d_{23}).$$

By allowing all of the d_{ij} to assume the values zero and one, all possible solutions can be generated for the D_j. Each solution can be represented as a terminal node of a solution tree. The solution tree for this example is illustrated in Figure 5.2. The left branch following each node represents the value of the variable equal to 0 and the right branch, the value 1. In general, a feasible solution is any solution that satisfies the restrictions to problem (5.1). In this example only the values in (5.4) are feasible. Feasible paths are indicated by solid lines in Figure 5.2 while infeasible paths are indicated by dotted lines. Since there are five binary variables, there are 2^5 solution sets for the variables, of which 15 satisfy the imposed restrictions. Even more paths may be excluded by exercising restrictions on both total cost and total thickness. It is the task of the branch and bound algorithm to exclude some of the infeasible paths by a bounding technique while determining the optimal path. In general, the efficiency of any exclusion process will be completely determined by the nature of the decision rules which constitute the algorithm.

The basic solution approach for a problem with a variable

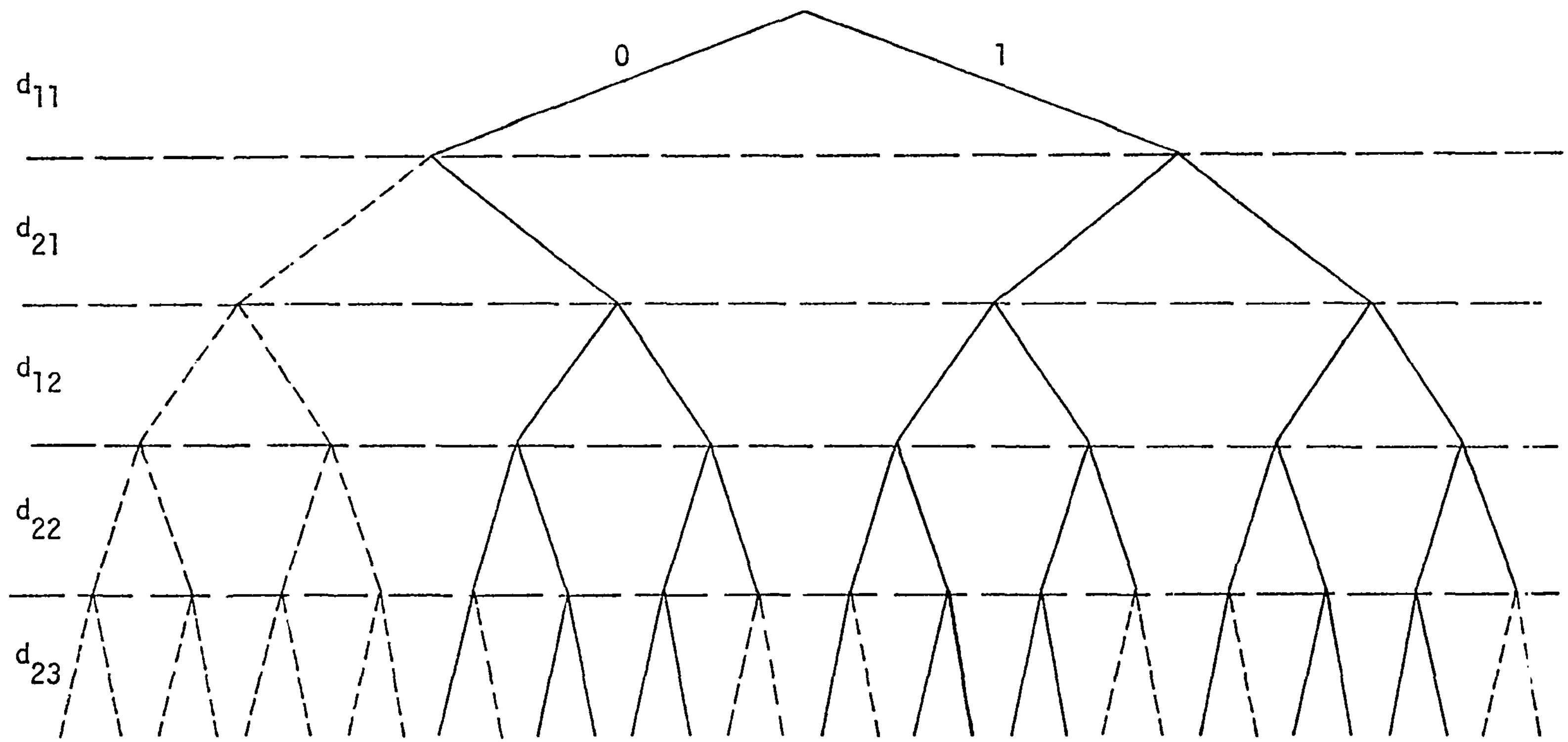

Figure 5.2 Solution tree for example (5.4)

constraint (such as the cost constraint in this problem) varies somewhat from the solution approach for a similar problem with a fixed constraint. The approach for a fixed value of γ will first be discussed, followed by the modifications required to allow γ to vary between 0 and CM.

It must first be established which value (0 or 1) is the "best" value for all binary variables. (Best in this sense means those values which make S as small as possible.) Examination of problem (5.2) will verify that S takes on its smallest possible value when all $d_{ij} = 1$. Since this trial solution is obviously infeasible, one variable (d_{ij}) is selected to be assigned a value of 0. The criterion for the selection of this variable will be on the basis of reducing the maximum amount of infeasibility in the total cost restriction (5.3). As a result, the variable with the largest coefficient in (5.3) is assigned a value of 0. In general, S is again evaluated and the procedure is repeated until a solution is found which satisfies all of the restrictions. The current branch of the tree is said to have been fathomed and S becomes the incumbent solution, S*. This establishes S* as the current upper bound on the optimal solution. Other branches must be systematically evaluated, each time updating S* when a feasible solution is found with a value of S that is smaller than S*.

As discussed in the literature review, one means of reducing the portion of the solution tree that must be explicitly considered is to bound the solutions in some of the branches and utilize these bounds as criteria for determining which solution sets must be evaluated. For example, upon evaluating a new branch of the solution tree, it might be found that the best completion (all free or unassigned variables having trial values of 1) of the partial solution (consisting of those variables which had previously assigned values of 0 and 1) does not have a value of S that is smaller than the incumbent solution. Consequently, this branch, regardless of its feasibility, can be excluded from further consideration. It is noted that the best completion must have a higher value of S than a completion with some of its free variables having a value of 0. The branch is thus fathomed because its best solution is bounded by the current incumbent solution.

Another type of exclusion of a branch can be identified when there is no possible completion of a partial solution that can result in a feasible solution. Each of these types of bounding can be identified and more easily understood by studying the example problem that will be given after the formal presentation of the algorithm.

For the problem under consideration in this research, the procedure is different in that many problems must be considered instead of only one. For illustration purposes, the approach is somewhat related to evaluating the objective functions associated with all possible branches of the solution tree and determining the cost of each alternative. Logically, this collection of solutions could be sorted on their respective costs to determine an ordering of optimal solutions depending on costs.

Although considerably less efficient than for a single problem with a constant value of γ, the branch and bound procedure can still offer a sizable reduction in the number of alternatives that must be evaluated. The scheme that is employed in this research is to use the constraint as a cost indicator rather than simply as a constraint. To be more specific, each time a solution is identified (feasible or infeasible) the procedure will be to compute the associated cost of the solution. The value of S associated with the solution can be compared to the incumbent solution for the same value of cost and a decision can be made as to whether to exclude the solution or to retain it as the new incumbent solution.

Bounding cannot occur as frequently for this variably constrainted problem because a solution that is infeasible for one value of cost may be feasible for another value of cost. The thickness restrictions (both layer depths and total thickness), on the other hand, can be used for excluding some branches. For this reason, it is important that these restrictions be as realistic as possible. This is emphasized because unless there are actual physical restrictions on these depths, there may be a natural tendency to formulate these constraints to allow for a very wide range of depths. For example, a design engineer may not desire to design a pavement with a total thickness that is greater than 30 inches; but because he would rather that the optimization technique tell him which thickness is most economical, he may stipulate a maximum thickness of an absurd amount, such as 100 inches. These types of restrictions would cause the branch and bound procedure to be very inefficient because the solution procedure would approach that of total enumeration.

An example problem utilizing the procedures discussed here will be given following the algorithm that is presented in the next section.

The Initial Design Algorithm

The following algorithm is presented to quantify the approach that was discussed in the previous section. The decision variables are of the type that assume only the values zero or one. Unless otherwise noted, the steps are to be followed consecutively.

1. Begin by making the following transformation on the subscripts to obtain a single-subscripted variable:

$$\delta_k\ ,\quad k = 1,2,\ldots,J_1,J_1+1,\ldots,J_1+J_2,J_1+J_2+1,\ldots,K$$

where $K = J_1+J_2+\cdots+J_N$

such that:

$$d_{1j} = \delta_j \qquad , j = 1,2,\ldots,J_1$$

$$d_{2j} = \delta_{J_1+j} \qquad , j = 1,2,\ldots,J_2$$

$$\vdots$$

$$d_{Nj} = \delta_{J_1+J_2+\cdots+J_{N-1}+j} \qquad , j = 1,2,\ldots,J_N.$$

2. Order the indices k, $k = 1,2,\ldots,K$, in $\underset{\sim}{U}$ with respect to the k^{th} variable's contribution in the total cost constraint such that u_i contains the i^{th} "most expensive" assignment, $i = 1,2,\ldots,K$. The k^{th} variable, δ_k, is to have the i^{th} largest coefficient in the cost constraint.
3. Set $i = 0$, $\delta_k = 1$, $k = 1,2,\ldots,K$ and $S^*_m = 10^{20}$, $m = 1,2,\ldots,M$. There may be M different optimal solutions as a result of spliting CM into M increments.
4. Compute $S(\underset{\sim}{\delta})$ and γ, the cost associated with this current solution. If $S(\underset{\sim}{\delta}) \geq S^*_\gamma$, skip to step 8.
5. Compute $D_n = \sum_{j=1}^{J_n} 2^{j-1} d_{nj}$, $n = 1,2,\ldots,N$, and skip to step 8 if

$$D_n > \phi_n ,$$

$$D_n \neq 0 \text{ and } D_n < \phi_n ,$$

$$\sum_{n=1}^{N} D_n > \psi ,$$

$$\text{or} \quad \sum_{n=1}^{N} C_n D_n > CM.$$

6. Replace S^*_γ by $S(\underset{\sim}{\delta})$ and $\delta^*_{\gamma k}$, $k = 1,2,\ldots,K$.
7. Repeat step 6, each time increasing γ by one increment until either $S(\underset{\sim}{\delta}) \geq S^*_\gamma$ or $\gamma \geq CM$.
8. If $i \neq K$, proceed to step 11. Otherwise assign $\delta_{u_i} = 1$.
9. If $i = 1$, terminate. The optimal solutions are contained in S^*_m and δ^*_{mk}, $m = 1,2,\ldots,M$, $k = 1,2,\ldots,K$.
10. Decrease i by one and return to step 9, if $\delta_{u_i} = 1$. Otherwise assign $\delta_{u_i} = 1$.
11. Increase i by one and assign $\delta_{u_i} = 0$. Evaluate

$$D_n = \sum_{j=1}^{J_n} 2^{j-1} X_{nj} \ , \ n = 1,2,\ldots,N$$

where

$$X_{nj} = \begin{cases} \delta_k, \ n,j \varepsilon u_k, k = 1,2,\ldots,i \\ \text{anything to allow } D_1 \text{ to be feasible} \\ \qquad i,j\varepsilon u_k, k = i+1,\ldots,K, \\ 0 \quad , \ \text{otherwise.} \end{cases}$$

If

$$\sum_{n=1}^{N} C_n D_n > CM \quad ,$$

$$\sum_{n=1}^{N} D_n > \psi \; ,$$

or $\quad D_n > \phi_n, \; n = 1,2,\ldots,N$,

return to step 10.

Skip to step 12 if $D_1 < \theta$.

Otherwise, return to step 4.

12. Assign $\delta_{u_i} = 1$ and repeat step 11.

Steps 1, 2, and 3 of the algorithm are provided as a starting point for the procedure. Tables are established that will eventually contain the optimal values of the objective function, along with the associated values of the decision variables, for each problem under consideration. The task at step 4 is to complete the value of S, based upon the values of the decision variables (δ_k, $k = 1,2,\ldots,K$), and to compare this value with the incumbent solution for the same cost.

Special mention should be made concerning the differences in steps 5 and 11. Upon entering step 5, except for the first iteration, assigned values of 0 or 1 will have been made to some of the variables. The objective at step 5 is to obtain the best completion of this partial solution (the completion with all remaining free variables set to 1) and to determine whether or not this completion is feasible. Contrary to this, the question that is posed at step 11 is whether or not any completion of the partial solution will be feasible. If the answer is no, the branch is excluded from further consideration.

The purpose of steps 6 and 7 is to replace the old incumbent solution S^*_γ with the new value of S for each value of cost greater than or equal to γ, as long as $S(\underset{\sim}{\delta})$ remains less than S^*_γ. The rules in steps 8, 10, and 12 govern the procedure for indexing through the solution tree.

Upon termination at step 9, the tables for S and δ contain the respective optimal solutions for each increment of γ between 0 and CM. An example of the solution procedure and the final tables for S and δ are contained in the following section.

An Example Problem

The simple example that was discussed in connection with the restrictions (5.4) and illustrated in Figure 5.2 will be used to demonstrate the solution procedure. Table 5.1 contains the set of

	PARAMETERS			
Layer j	Strength Coefficients a_j	Material Costs C_j	Minimum Depths θ_j	Maximum Depths ϕ_j
1	.80	.30	0.5	1.5
2	.50	.20	1.0	2.5
subgrade	.20	-	-	-

TABLE 5.1 VALUES OF PARAMETERS FOR EXAMPLE PROBLEM

values for the parameters used in the example problem. In addition, let CM = \$0.75, ψ = 3.0, and the increments in γ be \$0.05.

The objective function can be obtained from either (5.1) or (5.4) and the constraints can be reduced to:

$$15\delta_1 + 30\delta_2 + 10\delta_3 + 20\delta_4 + 40\delta_5 \leq \gamma$$

$$D_1 \leq 1.5$$

$$D_2 \leq 2.5$$

$$D_1 + D_2 \leq 3.0$$

where $0 \leq \gamma \leq 0.75$

$\delta_k = 0 \text{ or } 1 \ , \ k = 1,2,\ldots,5.$

The depths D_1 and D_2 are represented by

$$D_1 = 0.5\delta_1 + \delta_2$$

$$D_2 = 0.5\delta_3 + \delta_4 + 2\delta_5.$$

The u_i in the order of decreasing cost are: 5, 2, 4, 1, 3.

Figure 5.3 illustrates the solution tree for the δ_k variables. The light solid lines represent the solutions which are enumerated but infeasible, while the heavy solid lines represent the solutions which are evaluated by the objective function. The branches represented by dotted lines are excluded by the algorithm.

As an alternative to listing the individual steps of the algorithm, Table 5.2 contains the iterative solutions in the order

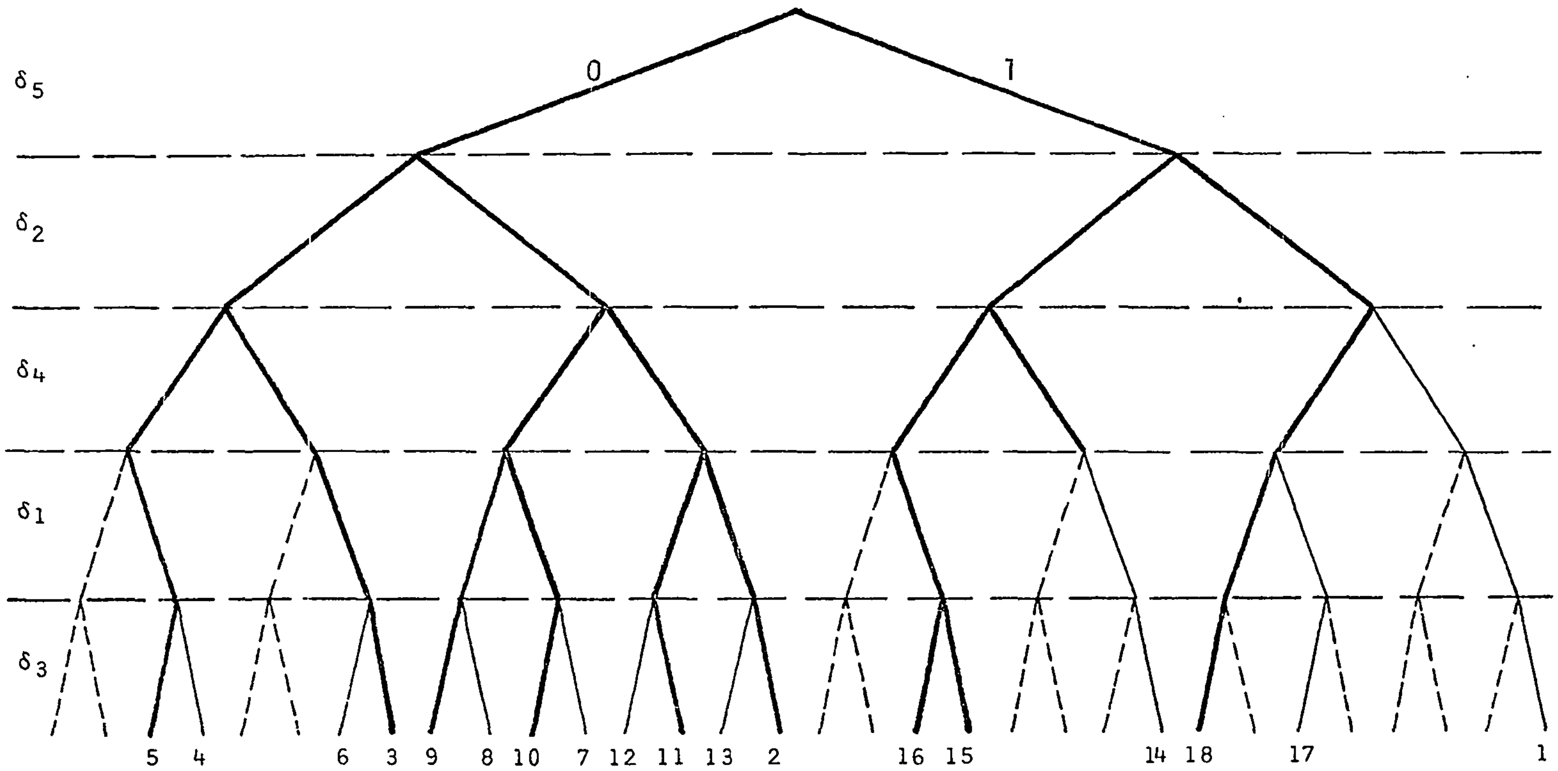

Figure 5.3 Solution tree for example problem contained in Table 5.1

ORDER	δ_k , k = 1,2,3,4,5	FEASIBLE OR INFEASIBLE	COST (IF COMPUTED)	S
1	1, 1, 1, 1, 1	I		3.955
2	1, 1, 1, 1, 0	F	.75	5.456
3	1, 0, 1, 1, 0	F	.45	6.608
4	1, 0, 1, 0, 0	I		7.120
5	1, 0, 0, 0, 0	F	.15	7.281
6	1, 0, 0, 1, 0	F	.35	6.892
7	1, 1, 1, 0, 0	I		6.200
8	0, 1, 1, 0, 0	I		6.723
9	0, 1, 0, 0, 0	F	.30	6.986
10	1, 1, 0, 0, 0	F	.45	6.535
11	0, 1, 1, 1, 0	F	.60	6.064
12	0, 1, 0, 1, 0	F	.50	6.412
13	1, 1, 0, 1, 0	F	.65	5.836
14	1, 0, 1, 1, 1	I		5.157
15	1, 0, 1, 0, 1	F	.65	5.922
16	1, 0, 0, 0, 1	F	.55	6.280
17	1, 1, 0, 0, 1	I		5.070
18	0, 1, 0, 0, 1	F	.70	5.692

TABLE 5.2 ITERATIVE SOLUTIONS OBTAINED BY THE INITIAL DESIGN ALGORITHM

COST, γ	S^*_γ	D_1 , D_2
\$ 0.0	-	-
.05	-	-
.10	-	-
.15	7.281	0.5 , 0.0
.20	7.281	0.5 , 0.0
.25	7.281	0.5 , 0.0
.30	6.986	1.0 , 0.0
.35	6.892	0.5 , 1.0
.40	6.892	0.5 , 1.0
.45	6.535	1.5 , 0.0
.50	6.412	1.0 , 1.0
.55	6.280	0.5 , 2.0
.60	6.064	1.0 , 1.5
.65	5.836	1.5 , 1.0
.70	5.692	1.0 , 2.0
.75	5.456	1.5 , 1.5

TABLE 5.3 SUMMARY OF OPTIMAL SOLUTIONS FOR EACH VALUE OF COST

in which they were considered by the algorithm. This order is also indicated at the terminal nodes of the solution tree. Table 5.3 is a summary of the optimal solutions for each value of cost. It is noted that nearly half of the possible solutions were excluded and thus not considered explicitly. The remaining 18 solutions that were considered would be a rather large number of solutions to consider in a general branch and bound procedure. It is noted, however, a general branch and bound algorithm searches for the optimal solution to only one problem, instead of many. It can be seen, by making the appropriate changes in the algorithm, that the solution to the single problem with γ = CM would require only five solutions to be evaluated.

This difference in efficiency is the result of not being able to formulate the algorithm for total cost into one large problem, instead of two problems. In the former case it would not have been necessary to initially determine all possible initial designs.

Overlay Optimization

The second optimization problem is associated with the future costs of the pavement. Relationships for each of the contributing cost components were developed and discussed in Chapter IV. The purpose of this section is to develop an algorithm for utilizing these relationships to determine the optimal policies for all types of maintenance. Actually, the only decisions to be made, regarding future maintenance costs, are those associated with the overlay policy. As indicated in Chapter IV, the annual cost of routine maintenance is assumed to increase at a linear rate, beginning with a constant annual cost of R_1 after initial or overlay construction. An additional schedule, also discussed in Chapter IV, is associated with the frequency of seal coat applications. The schedule is based upon pavement requirements and is also a function of the overlay policy that is adopted.

There are three types of variables associated with an overlay policy,

1. the number of performance periods, K,
2. the times T_k , $k = 1,2,\ldots,K-1$, at which overlays are applied, and
3. the depths O_k of each overlay.

An optimal overlay policy is a function of the initial construction design and the traffic level that the pavement is expected to maintain. Depending on the traffic level, a very strong initial design might not require an overlay during the analysis period, while a weak initial design would require several overlays in order to hold the serviceability index above its minimum acceptable level of P_2. Of course, high traffic volumes would require overlays more often than lower traffic volumes.

The dependence of the initial construction optimization, considered in the previous section, now becomes clear. All that is required in order to determine an optimal overlay policy is

Table 5.3 which gives the optimal initial construction design and the associated value of deflection of each possible amount of investment in initial construction. The remaining problem is that of determining the optimal overlay policy for each initial construction design and then combine these costs to determine the overall cost of each initial construction design.

The only physical restriction on the overlay optimization problem is that the serviceability index P can never be allowed to drop below its minimum acceptable level of P_2. This restriction serves as a limiting factor on the time that can elapse before an overlay must be applied and it generally forces at least one overlay to be performed during the analysis period. The rate at which the serviceability index degenerates after initial or overlay construction is a function of two factors - traffic and the swelling clay parameters.

For fixed levels of these two factors, the deflection values from Table 5.3 can be used to compute the times at which overlays would be required for each initial design. These times, as well as the associated equivalent number of 18-kip load applications, are determined from the simultaneous solution of the performance equation (3.13) and the traffic equation (3.12).

When an overlay is applied, both the deflection and the swelling clay parameter b are reduced. It is assumed that the deflection of the new system is determined by simply increasing the depth of layer 1 by the amount of the overlay, so that the depths of the new design are:

$$D_1 + O_1, D_2, \ldots, D_N.$$

Overlay depths, similar to initial construction depths, are required to be in increments of one-half inch. The relationship for determining the new value of b was given by equation (3.2). It is noted that this evaluation is a function of the length of the previous performance period.

The total cost during any performance period is simply the sum of the costs for:

1. the routine maintenance performed during that period,
2. the seal coats applied during that period,
3. the overlay cost during that period, and
4. the user costs associated with each of these types of maintenance.

Relationships for these costs were developed in Chapter IV.

Under some conditions, the optimal overlay policy will be one in which overlays are applied only when required (when $P = P_2$). This policy is intuitively appealing from the viewpoint of utilizing the facility to the utmost. When this occurs, the times at which overlays are applied will be functions of the respective initial construction design and the depth of each previous overlay. This means that the only decisions to be made in determining the optimal policy are the respective overlay depths, O_k. Each of these possible alternatives can be conveniently represented by a tree of the type shown in Figure 5.4. Each branch of the tree represents an individual overlay policy.

Experience has indicated that the actual number of branches of such a tree for most problems are small. For example, the tree shown in Figure 5.4 is typically rather large as compared to trees

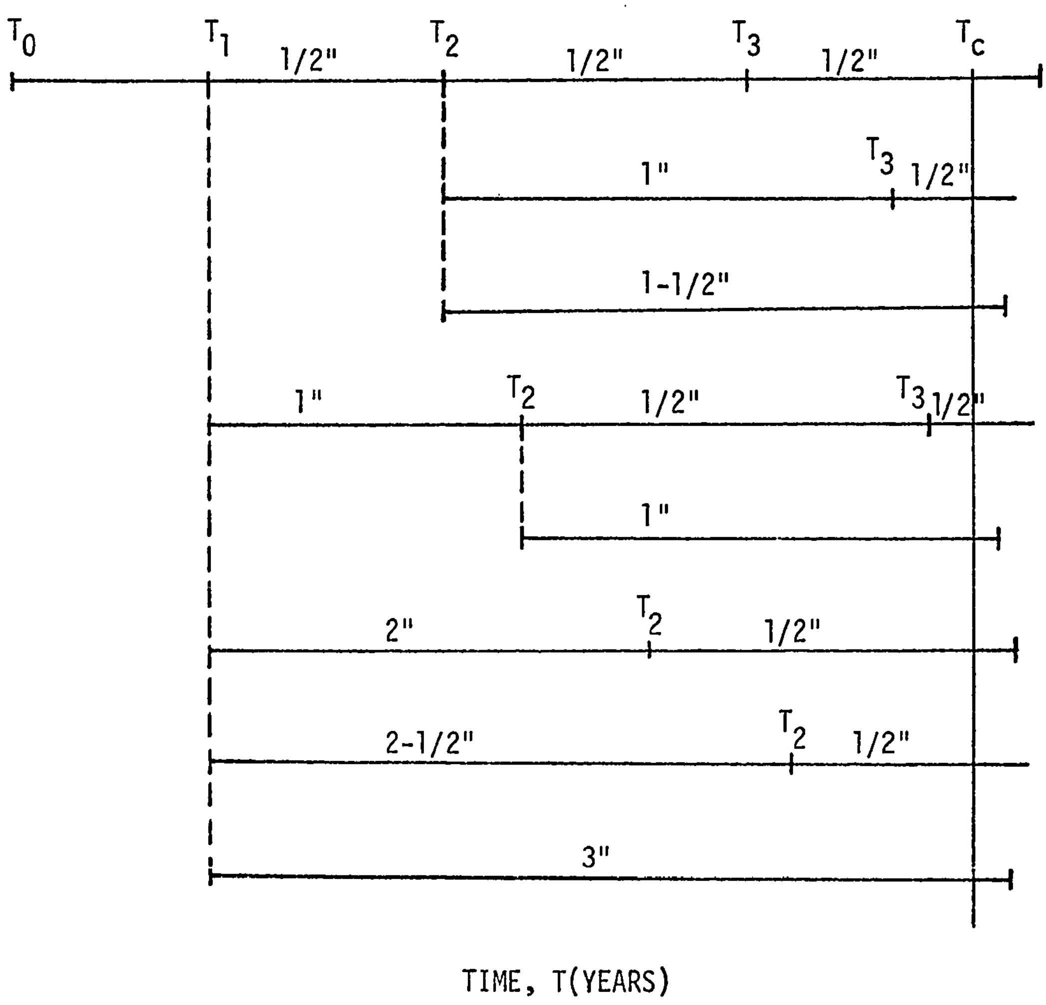

Figure 5.4 Typical tree of possible overlay policies

for most problems with 20 year analysis periods. Under these circumstances, the optimal overlay policy for a given initial design can be obtained rather easily by enumerating all of the overlay policies. The cost of each policy is evaluated by summing the previously described costs over all performance periods. These costs are accumulated on the basis of their present worth and added to the cost of initial construction to yield the overall cost of the structure.

The previous discussion was made on the basis of overlays being applied only when they were required. The critical consideration that arises at this point is the question of whether or not there is anything to be gained by overlaying <u>before</u> P reached P_2. It is noted that the effect of overlaying one year earlier than was scheduled can be forced by considering a slightly inferior initial design. These circumstances are distinctly different from the proposed question.

In order to evaluate the alternative of overlaying one year earlier than is required, the cumulative effect of the following considerations must be evaluated:

1. The cost of routine maintenance during the overlay year would be R_1 instead of $R_1 + (M-1)R_2$, if M is the number of years since the previous overlay.
2. An additional seal coat may not be required in the previous performance period.

3. The user cost during the overlay operation would be smaller, one year earlier, because the traffic volume will be smaller.
4. The swelling clay parameter b for the next performance period would have a larger value than it would otherwise have. This is due to the recursive relationship used to compute the value of b for the next performance period, being a function of the time since the previous overlay.
5. Other factors being constant, the next performance period will be shorter than it would have otherwise been because of (4) above. As a result, an extra overlay may be required during the analysis period.
6. The present worth of the future expenditures under consideration will have a higher value because of the change in the discounting factor.

Effect 1, 2, and 3 listed above would result in a positive savings; while effect 4, 5, and 6 would result in a negative savings in the net change in the total cost. These effects coupled with similar effects that may occur at the end of the analysis period make the evaluation of this alternative unpredictable in the general case.

Because of the sequential nature of the decision process, the one optimization technique that would appear to be adaptable is dynamic programming. There are many types of variables that are contained in the overlay optimization problem. Most of these would be candidates for the standard problem variables associated with a dynamic programming formulation. The difficulty in applying this technique is in accounting for all of the interrelationships between the variables. An accurate recursive relationship for this process must include variables for the following considerations each time an overlay decision is to be made:

1. the initial construction depths,
2. which overlay is under consideration,
3. the time since the previous overlay,
4. the time at which the decision is being made, and
5. the accumulated depths of all previous overlays.

It is possible to formulate a multi-stage variable dynamic programming formulation to incorporate these concepts. However, multi-dimensional formulations are very often computationally intractable and single dimensional formulations, such as the one presented in Chapter VI, are always desired.

CHAPTER VI

A DYNAMIC PROGRAMMING FORMULATION

A dynamic programming formulation is presented in this chapter for the optimal flexible pavement design problem. This is a specialized formulation that eliminates the previously described difficulties associated with the dependent relationships for deflection (3.6), traffic (3.12), and performance (3.13). Both the initial construction problem and the overlay optimization problem are treated in the formulation, stage 1 being associated with initial construction and the remaining stages associated with future overlays. Therefore, stages in the formulation are represented by performance periods. The objective at each stage will be to maximize the length of the respective performance periods through a restatement of the performance equation. This maximization will be constrained by a cost restriction that includes all of the costs occurring within each stage. Cumulative cost is formulated as the state variable, so that the cost restriction acts as a transition function between stages. The decision variables at each stage are associated with depths of pavement. There are N decision variables in stage 1 associated with the depths of the N layers of initial construction. Cumulative overlay depth is the single decision at the remaining stages.

The objective of maximizing the time in each performance period, subject to a cost constraint, is appealing from the standpoint of "buying" time. The question that is asked is, how much time can be bought with the available resources? The dual to this problem would be to minimize overall cost, subject to a time constraint. Without going into detail, it is simply stated here that this dual problem cannot be structured to exhibit the required stagewise independence. Therefore, the problem as is formulated is not only a convenient formulation, but it also allows for both the initial and overlay construction problems to have the same state variable and objective function.

In equation (3.13) the performance equation was given as:

$$Q_2 = \frac{53.6(N_k - N_{k-1})S^2}{\alpha} + Q_2' \left[1 - e^{-b_k(T_k - T_{k-1})}\right] . \tag{6.1}$$

Each of the variables contained in this equation were described in the same section. Equation (6.1) can be solved for T_k to yield:

$$T_k = T_{k-1} + (1/b_k)\ln\left[\frac{\alpha Q_2'}{\alpha Q_2' - \alpha Q_2 + 53.6(N_k - N_{k-1})S^2}\right]. \tag{6.2}$$

For k = 1, T_0 and N_0 are equal to zero. This relationship will serve as the objective function in the formulation that follows.

The dynamic programming formulation is represented by the following set of recursive relationships. These relationships are written for the first two stages before the generalized relationship is given.

Stage 1:

$$f_1(\gamma_1) = \max_{D_1, D_2, \ldots, D_N} \left\{ T_1 \right\}$$

$$f_1(\gamma_1) = \max_{D_1, D_2, \ldots, D_N} \left\{ (1/b_1)\ln \frac{\alpha Q_2'}{\alpha Q_2' - \alpha Q_2 + 53.6 N_1 S_1^2} \right\}$$

subject to the cost restriction,

$$\sum_{j=1}^{N} C_j D_j + RM(T_1) + SC(T_1) + U_S(T_1) \leq \gamma \quad , \; 0 \leq \gamma \leq CM$$

$$\theta_j \leq D_j \leq \phi_j \quad \text{if } D \neq 0$$

where N_1 is a function of T_1 through the traffic equation represented by:

$$N_1 = \left[\frac{N_f}{f(r_0 + r_f)} \right] 2 r_0 T_1 + \left[\frac{r_f - r_0}{f} \right] T_1^2 \quad .$$

The quantity S_1 represents deflection and is related to the D_j by equation (5.1). It is written here simply as:

$$S_1 = f(D_1, D_2, \ldots, D_N) \; .$$

Furthermore, for a specific value of T_1, the swelling clay parameter at the beginning of the second performance period is computed as:

$$b_2 = b_1 e^{-b_1 T_1} \; .$$

Stage 2:

$$f_2(\gamma_2) = \max_{0.5 \leq O_1} \left\{ T_2 \right\}$$

$$f_2(\gamma_2) = \max_{0.5 \leq O_1} \left\{ (1/b_2)\ln \left[\frac{\alpha Q_2'}{\alpha Q_2' - \alpha Q_2 + 53.6(N_2 - N_1)S_2^2} \right] + f_1(\gamma - \gamma_2) \right\}$$

subject to the cost restriction,

$$OC(O_1) + U_0(T_1) + RM(T_2,T_1) + SC(T_2,T_1) + U_S(T_2) \leq \gamma \quad ,$$
$$0 \leq \gamma \leq CM$$

where N_2 is a function of T_2 through the traffic equation represented by:

$$N_2 = \left[\frac{N_f}{f(r_0+r_f)} \right] 2r_0T_2 + \left[\frac{r_f-r_0}{f} \right] T_2^2 \quad .$$

The quantity S_2 again represents deflection, and S_2 differs from S_1 only because D_1 has now become $D_1 + O_1$. Thus S_2 can be written as:

$$S_2 = f(D_1 + O_1, S_1) \; .$$

The new value for the swelling clay parameter at the beginning of the third period is computed as:

$$b_3 = b_2 e^{-b_2(T_2-T_1)} .$$

It is possible to write the general recursive relationship for stage n.

Stage n:

$$f_n(\gamma_n) = \max_{\frac{n}{2} \leq O_{n-1}} \left\{ T_n \right\}$$

$$f_n(\gamma_n) = \max_{\frac{n}{2} \leq O_{n-1}} \left\{ (1/b_n)\ln\left[\frac{\alpha Q_2'}{\alpha Q_2 - \alpha Q_2 + 53.6(N_n - N_{n-1})S_n^2}\right] + f_{n-1}(\gamma - \gamma_n) \right\}$$

subject to the cost restriction,

$$OC(O_{n-1}) + U_0(T_{n-1}) + RM(T_n, T_{n-1}) + SC(T_n, T_{n-1}) + U_S(T_n) \leq \gamma \quad , \; 0 \leq \gamma \leq CM.$$

The accumulated traffic at time T_n will be

$$N_n = \left[\frac{N_f}{f(r_0 + r_f)}\right] 2r_0T_n + \left[\frac{r_f - r_0}{f}\right] T_n^2 \quad . \tag{6.3}$$

The new value of deflection during the n^{th} performance period is a function of the initial design and the accumulated overlay depth, written as:

$$S_n = f(O_n, S_1) \; .$$

It should be noted that because of the nature of the problem, the dependence of the decision at stage n upon the decisions at previous stages does <u>not</u> violate the dependency principles of dynamic programming. This will be discussed later in this chapter.

The swelling clay value at the beginning of the next performance period is computed as:

$$b_{n+1} = b_n e^{-b_n(T_n - T_{n-1})} .$$

The optimization problem at stage 1 requires N decisions and can be solved by a modification of the branch and bound algorithm presented in Chapter V. The only required change in the algorithm is the adjustment of the cost function to include maintenance costs during the first performance period. It is noted from equation (6.2) that maximizing time within this period is equivalent to minimizing the deflection of the initial structure.

As indicated by the recursive relationship, the computations for deflection at future stages are dependent upon the initial designs. For this reason, the initial design decisions along with the associated values of the swelling clay parameter, b_n, must be retained for all intermediate solutions in future stages.

The optimization procedure for future stages is more complicated than at the first stage; in fact, the evaluation procedure for each decision variable is iterative in nature. The reason is that the variable to be maximized, T_n, also appears in the objective function through the traffic equation (6.3). Additionally, the objective function is directly constrained by the cost restriction since most of the cost components appearing

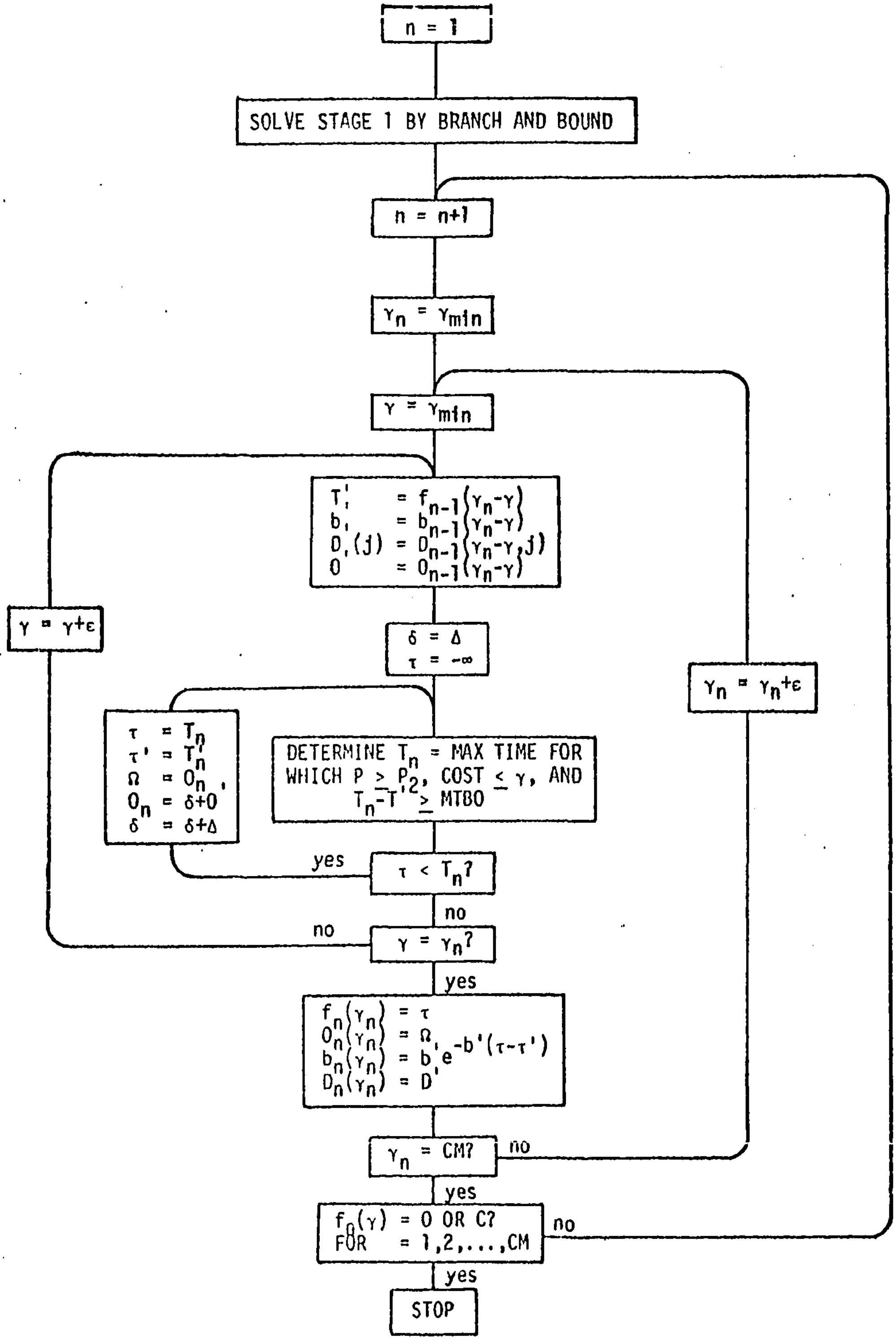

Figure 6.1 Summary flow diagram for the dynamic programming formulation

in the restriction are a function of T_n. Because of these curious factors, the recursive relations are rather difficult to implement. Figure 6.1 shows a summary flow diagram of the algorithm. The following steps describe in detail one method of stepping through the recursive relationships.

1. At stage n = 1, solve the initial construction problem by the branch and bound procedure presented in Chapter V. For each γ_1, $0 \leq \gamma_1 < CM$, determined the maximum time in the first performance period as:

$$f_1(\gamma_1) = \begin{matrix} T_1 & , T_1 \geq MTTO \\ 0 & , T_1 < MTTO \ . \end{matrix}$$

   For each $f_1(\gamma_1)$, determine $D_1(\gamma_1,j) = D_j$, $j=1,2,\ldots,N$. Also compute $b_1(\gamma_1) = b_1 e^{-b_1 T_1}$ and let $O_1(\gamma_1) = 0$ for $0 \leq \gamma_1 \leq CM$.

2. Continue to the next stage (n = n+1) and initialize $\gamma_n = \gamma_{min} + 1.5C_1 + RM(MTTO)$, where γ_{min} is the minimum cost that must be available in the previous stage so that the time in that stage $T_n - T_{n-1} \geq MTBO$.

3. Increase γ_n by ε, where ε is the minimum incremental change in cost that is considered. Initialize $\gamma = 1.5C_1 + RM(MTBO)$; where γ is to be the amount of γ_n to be allocated to stage n, leaving $\gamma_n - \gamma$ to be allocated to stage n - 1. Initialize $\tau = 0$, $\Omega = 0$, and $\tau' = 0$.

4. Let $T' = f_{n-1}(\gamma_n - \gamma)$. If T' equals either zero or C, skip to step 16.

5. Let $b' = b_{n-1}(\gamma_n - \gamma)$,

$$D'(j) = D_{n-1}(\gamma_n - \gamma, j) \ , \ j = 1,2,\ldots,N \ ,$$

$$0' = 0_{n-1}(\gamma_n - \gamma).$$

Initialize

$$\tau = -\infty$$

$$\delta = \Delta$$

$$0_n = \delta + 0'$$

6. Compute $S_n = h(D'(j), 0_n)$,

$$N' = g(T') \ ,$$

$$T_n = C.$$

7. Compute $N = g(T_n)$

$$P = p(N, N', T_n, T', b', S_n)$$

Repeat this step with $T_n = T_{n-1}$ if $P < P_2$.

8. Compute all cost during this performance period as:

$$RM = r(T_n, T')$$

$$SC = s(T_n, T')$$

$$U_S = U(T_n, T', N')$$

$$OC = C(\Delta)$$

$$U_0 = U(T_n, T', N', \Delta).$$

Then

$$COST = RM + SC + U_S + OC + U_0.$$

If $T_n \neq C$, skip to step 10.

9. Include salvage value in the cost during this period as:

$$COST = COST + SV(D'(j), O_n)$$

10. If $T_n - T' < MTBO$, set $T_n = 0$ and skip to step 12.

11. If COST is less than the amount γ being allocated to this stage, return to step 7 with $T_n = T_n - 1$.

12. If $\tau \geq T_n$, skip to step 15.

13. Store
$$\tau = T_n$$
$$\tau' = T'$$
$$\Omega = O_n$$

14. If $\tau = 0$, return to step 6 with $\delta = \delta + \Delta$ and $O_n = \delta + O'$.

15. If $\gamma < \gamma_n$, return to step 4 with $\gamma = \gamma + \varepsilon$.

16. Store
$$f_n(\gamma_n) = \tau$$
$$O_n(\gamma_n) = \Omega$$
$$b_n(\gamma_n) = b' e^{-b'(\tau - \tau')}$$

17. If $\gamma_n < CM$, return to step 3. Otherwise, for all γ_n, $0 \leq \gamma_n \leq CM$, write γ_n, $f_n(\gamma_n)$, $O_n(\gamma_n)$, and $b_n(\gamma_n)$.

18. Store for all γ_n, $0 \leq \gamma_n \leq CM$,
$$f_{n-1}(\gamma_n) = f_n(\gamma_n) \ ,$$
$$O_{n-1}(\gamma_n) = O_n(\gamma_n) \ ,$$
$$b_{n-1}(\gamma_n) = b_n(\gamma_n) \ ,$$
and return to step 2 if any $f_n(\gamma_n)$ is <u>not</u> equal to either zero or C. Otherwise, stop.

These rules are paraphrased in the discussion that follows.

For each amount γ_n available at stage n, γ, min $< \gamma \leq \gamma_n$, may be allocated to stage n, with $\gamma_n - \gamma$ being allocated to stage n - 1. Through the recursive relationship, the optimal return of T_{n-1} is determined as a function of $\gamma_n - \gamma$. Also determined at that time are the D_j corresponding to the initial design for the structure under consideration, the beginning value of b_n which is a function of the return at stage n - 1, and O_{n-1} which is the accumulation of all previous overlay depths.

The decision at stage n is to be O_n, the cumulative overlay depth at time T_{n-1} that maximizes T_n. For an overlay depth $O_n - O_{n-1}$ at stage n (the possible values are Δ, $\Delta + \delta$, $\Delta + 2\delta, \ldots$), a new value of deflection S_n is computed based upon the values of D_j that were obtained.

The optimal return T_{n-1} is the time at the beginning of the current stage. The corresponding traffic at time T_{n-1} must be determined from the traffic equation (6.3). Beginning with a large value of T_n, the traffic equation and the performance equation are repeatedly solved, each time reducing T_n by one year, until the final value of the serviceability index P is greater than or equal to P_2. Fianlly the cost equation is evaluated for this value of T_n to determine if the solution is feasible. If not, T_n must be further reduced until the cost restriction is satisfied.

If the cost restriction is satisfied for the value of T_n that satisfies the performance equation, the procedure is repeated with an increased overlay depth.

This discussion has been for a fixed value of γ, the amount of γ_n allocated to stage n. Next, γ is increased, until $\gamma = \gamma_n$, and the solution procedure is repeated, each time determining the associated maximum values of T_n. The largest of these maximum values is the optimal return for the amount γ_n available at stage n. Finally, γ_n must be allowed to increase to CM in order to consider all possible overlay policies within the n^{th} stage.

This procedure is repeated for all remaining stages, meaning that overlay policies are investigated for each value of γ_n. It is important to note that the initial design decisions at stage 1 are based upon minimizing the cost during the first performance period and that they are not allowed to vary throughout the remaining stages. Optimization at the succeeding stages is associated only with overlay policies. This does not violate the independence requirement of dynamic programming because these initial designs are considered as parameters at later stages. Only the depth of layer 1 is a variable and this occurs through its alteration as a result of overlays.

The only remaining consideration is a discussion of the apparent dependence of the decisions at one stage upon the overlay decisions made at previous stages. This dependence can best be described as a pseudo-dependence that does not violate dynamic programming principles. This is a result of two important dominating factors that are inherent in the flexible pavement problem.

It is well to consider how stage dependency is manifest in a dynamic programming formulation. Stage dependency precludes that as a result of fixing the decision at a certain stage i, it might be possible to obtain a better value of the objective function at a later stage n if the decision at the earlier stage could be changed. If it can be shown that this situation is not possible, then there can be no stage dependency (in the dynamic programming sense) and there can be no danger in considering this previous decision independent of future decisions. Thus, although the decision at stage n will be affected by the decision at stage i, there must be no way to improve the functional at stage n by changing the decision at stage i. Consequently, the decision at stage i must be best for any future decision.

This concept is inherent in the formulation that is presented in this chapter and is a result of two dominating factors that are present in the flexible pavement problem. A clear understanding of these two factors and their effects will serve to demonstrate that the pseudo-dependence exists. These factors are associated with (1) the monotonically decreasing function for the swelling clay parameter, b_k and (2) the increase in traffic with time.

It is recalled that b_k is a degradation factor associated with the effect of swelling clay upon the serviceability of a pavement structure. A large value of b_k indicates a faster degradation than a small value. The recursive function for predicting the swelling clay parameter for the k+1st period was given in Chapter III as:

$$b_{k+1} = b_k e^{-b_k(T_k - T_{k-1})}.$$

This indicates that b_{k+1} decreases at an increasing rate as the time in the kth performance period increases. It follows that a longer subsequent performance period can always be gained by waiting as long as possible to overlay (with respect to the available resourses in the cost constraint).

The second dominating factor is associated with the user costs during overlay construction. Because traffic increases with time, the user cost rate during overlay is less in early performance periods than in later periods. Consequently, for a given amount of resource available, a thicker overlay can be laid and more time can be gained in an early period than in a later period. This will be a dominating factor when the user cost rate increases faster than the discounting rate decreases. The circumstances for which this is true can be isolated by determining when the following is true:

$$U_1 < (1+i)^{-n} U_2 \qquad (6.4)$$

where U_1 = user cost rate at time t_1

U_2 = user cost rate at a time t_2, n years after t_1

$(1+i)^{-n}$ = the present worth discounting factor for n years.

Equations (4.21) and (4.22) gave the user cost rate as a function of the traffic input and the delay cost per vehicle. For high traffic in which congestion occurs during overlay operations, the delay cost per vehicle is a function of the traffic input. For traffic levels low enough so that significant congestion does not occur, the delay cost per vehicle is a constant and the expression (6.4) can be reduced to:

$$\frac{r_1}{r_2} < (1+i)^{-n} \tag{6.5}$$

where the r_j are traffic input rates. Equation (3.8) related this input rates to time as:

$$r_t = r_0 + \left(\frac{r_f - r_0}{f}\right) t$$

where r_0 = average daily traffic at time t = 0

r_f = average daily traffic at time t = f.

Along with the substitution $t_2 = t_1 + n$, the inequality (6.5) can be easily reduced to:

$$(1+i)^n < 1 + K_n/(r_0+Kt_1) \tag{6.6}$$

where $K = (r_f - r_0)/f$.

The conjecture is that when the inequality (6.6) holds, it is cheaper to overlay at time t_1 rather than n years later.

The general shape of the two sides of the inequality are shown in Figure 6.2.

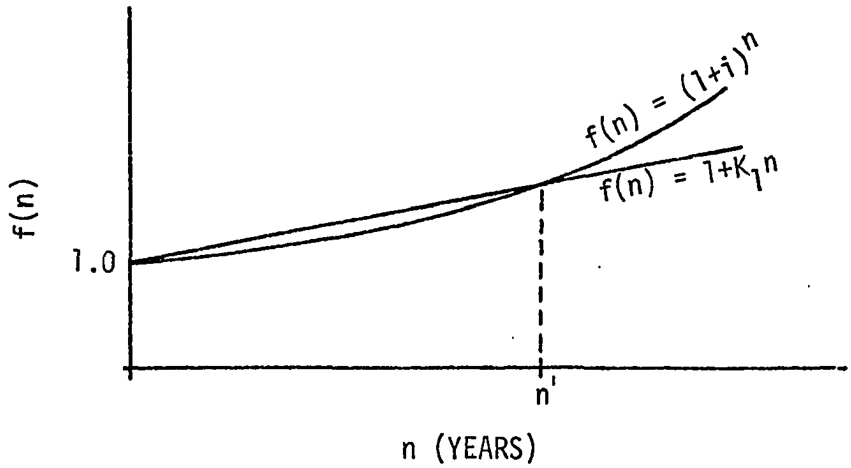

Figure 6.2 Plot of increasing interest and increasing user cost effects against time

The slope of the linear relationship, $K_1 = \frac{K}{r_0+Kt_1}$. As shown in the figure, the inequality (6.6) holds for n less than n'. It is noted, without proof, that for interest rates less than 6% and for low to medium traffic, the point n' is longer than performance periods would normally be. Because of the different traffic models, it is difficult to be more specific since it is difficult to generalize at what point traffic congestion will occur during overlay. With high traffic, the user cost rate increases even faster, indicating that a very high interest rate would be required to offset the user cost. It must be remembered that the relationship (6.6) is not applicable to high traffic because it does not include the additional user cost due to congestion delays.

Thus, under some conditions the dominance conjecture holds and the decisions in the dynamic programming formulation are essentially independent if and only if all except the last performance period is less than n' years in length. It is emphasized that, although the number of computations sometimes becomes prohibitive, multi-dimensional formulations are possible. One example would be to let cumulative inches of overlay be a second state variable with the decision at each stage being the overlay depth during that performance period.

As the subject area matures and additional research provides new or modified empirical results, it may be possible to define additional dominating factors that can lead to a more refined optimization procedure. The techniques developed during the course of this research have solved the design problem well beyond the precision of the design process. Therefore, the results offered are considered to be both original and an advancement in the technology to support future design of flexible pavements.

CHAPTER VII

SUMMARY AND CONCLUSIONS

The development of procedures for the analysis and optimization of the flexible pavement design problem has been described in previous chapters of this manuscript. The research was conducted as a joint effort with the Texas Transportation Institute, a research agency for the Texas Highway Department. The Texas Transportation Institute was responsible for the development of the design procedures that are reviewed in Chapter III of this text. This manuscript is not intended as a guide to the use of these procedures, rather to briefly acquaint the reader with the concepts of the procedures. It is felt that these concepts are of paramount importance to understanding and appreciating the complexities of both the flexible pavement design problem and the optimization techniques developed in this research.

As indicated in Chapter II, the AASHO Interim Guides [54] offer a design procedure that is felt by some to be inadequate for determining pavement designs under general conditions. Previous optimization techniques have adopted the associated behavioral relationships for the optimization of the initial construction design problem. These approaches have utilized linear programming algorithms. The investigation of optimal design procedures applicable to longer periods of time has not been treated in the literature.

The empirical relationships for deflection, performance, and traffic are an integral part of the algorithms, presented in Chapter V, for the determination of the optimal design policy. The initial construction problem was formulated as an integer programming problem and solved by a specialized branch and bound algorithm that permits the cost relationship to act as a variable constraint. This problem, coupled with the problem of determining optimal overlay policies for each initial design, represents a major contribution to the field of flexible pavement design. An additional application of the overlay optimization algorithm will be the implementation of the algorithm to determine optimal overlay strategies for existing pavements. It is expected that this application will have a greater demand than will the initial construction algorithm.

An important result of the optimization technique thus far has been in pointing out deficiencies in the design procedure. For example, the swelling clay parameters, included only recently, were inserted in the performance equation in order to limit the potential life of a pavement, thereby forcing designs to physically be more realistic. Before their inclusion, optimal initial designs were obtained that had lives of over 30 years without an overlay. This is virtually a physical impossibility.

Models for predicting future costs of a roadway were developed in Chapter IV. The precision of these models is handicapped by the fact that accurate and complete data are not available for a thorough investigation of cost relationships. This research has emphasized the need for new and more effective cost accounting procedures. As methods of collecting data are improved, cost predicting capabilities will also be enhanced.

A study has not been made to determine the effects of maintenance operations upon traffic congestion. The unpredictable interaction of motorists reacting to traffic delays can result in considerable error in the subjective estimates of delay times. In most cases user costs have been small, indicating that these factors may be negligible when compared to physical costs. Other example problems with very high traffic have yielded user costs of about the same magnitude as the cost of initial construction.

A computer program has been developed that utilizes the techniques described in this research. The output of an example problem is contained in the appendix of this text. One feature of this program is the determination of not one but the best 12 designs. These are included because of the inaccuracies in estimating costs. Realizing this, a designer would have little faith in the best single design, whereas noting that a particular combination of materials consistently appeared in the best 12 designs would give the designer considerably more confidence in his selection of an initial design.

This computer program is currently being used by the Center for Highway Research at the University of Texas to perform limited sensitivity analyses on various design parameters. The ultimate objective of the research program is the full scale use of this computer program not only in designing new pavement projects but in determining optimal strategies for existing pavements. Meanwhile, the computer program is recommended as an aid not only to the design engineer but also to the research engineer in establishing points of emphasis for future flexible pavement research.

CHAPTER VIII

RECOMMENDATIONS FOR FUTURE RESEARCH

The flexible pavement design project, originally instigated by the Texas Highway Department, has reached its first plateau. Needless to say, there are several areas that require further refinement. The most obvious of these are associated with the refinement of the empirical relationships for deflection and performance, as well as further study into the development of analytical cost models for predicting the future costs of a roadway. It is felt that the costs associated with delay times during maintenance operations have been adequately developed. The approach that has been taken to determine relationships for these delay times is admittedly weak. There is a real need for a detailed study to furnish analytical relationships for predicting delay times in terms of traffic and maintenance parameters. Such a study should also include an investigation of the interaction between delayed vehicles, as well as the associated affects upon motorists. Additional information concerning accidents, as a result of maintenance operations, may also be valuable in determining the total user cost.

The branch and bound technique that is employed in this research for determining optimal initial construction designs is sound. However, it may be possible to further develop the bounding criteria used by this algorithm. One suggestion would be the implementation of an approximation technique to determine initial lower bounds on the optimal solutions for each value of cost. Having these initial incumbent solutions would allow more bounding to occur, thereby further reducing the number of branches that must be explicitly considered. Another way of reducing the number of computations required to solve the initial design problem would be by increasing the increment sizes for the possible values of γ in the cost constraint. Care must be taken to insure that the possible loss in accuracy would not be prohibitive.

In order to improve upon the overlay optimization algorithm, cost analyses may be required to determine possible dominating levels of the various maintenance costs. For example, if routine maintenance and seal coat costs were higher than the cost of overlay, an optimal policy would probably be to overlay as often as possible. On the other hand, depending upon the traffic volume, it may be possible to establish that overlays should only be applied when they are required. These types of dominating relationships would greatly improve the efficiency of the overlay optimization technique.

It is felt that the need for additional research in other areas will be emphasized as experience is gained from using the computer program for solving realistic problems. In spite of the limitations that are described in this research, there is a need for further quantification of the ideas and concepts presented in this research.

APPENDIX

The computer program that has been written utilizes the procedures described in Chapter V. The dynamic programming formulation has not been programmed but this is expected during future phases of the project. Contained in this section is the output from an example problem. First is a listing of the many input parameters describing materials, performance, traffic, and costs. Following this is the optimal design for each combination of materials.

Finally, the best 12 designs are tabulated on a summary sheet. These designs are listed in the order of increasing total cost. In each case, the complete design including the maintenance policy is shown. It is felt that this output is sufficiently documented to require no further discussion.

THE CONSTRUCTION MATERIALS UNDER CONSIDERATION ARE

MATERIALS	COST/CY	COST/SY/IN	ST. COEF.	MIN. DEPTH	MAX DEPTH
ASPHALTIC CONCRETE	12.00	0.3333	0.80	0.50	10.00
LIME TREATED GRAVEL	5.40	0.1500	0.65	4.00	32.00
LIME TREATED SUBGRADE	3.00	0.0833	0.35	4.00	8.00
SUBGRADE	0.0	0.0	0.22	0.0	0.0

THE TOTAL NUMBER OF MATERIALS UNDER CONSIDERATION IS 3
THE MAXIMUM FUNDS AVAILABLE TO BE SPENT PER SQ. YD. ON THE INITIAL DESIGN IS 4.00 DOLLARS
THE LENGTH OF THE ANALYSIS PERIOD IS 20.0 YEARS
THE INTEREST RATE OR TIME VALUE OF MONEY IS 5.0 PERCENT.
THE ASPHALTIC CONCRETE PRODUCTION RATE IS 75.0 TONS PER HOUR.
THE ASPHALTIC CONCRETE COMPACTED DENSITY IS 1.8000 TONS PER COMPACTED CUBIC YARD.
SALVAGE IS TO BE 75.0 PERCENT OF THE STRUCTURAL VALUE AT THE END OF THE ANALYSIS PERIOD.
THE MAXIMUM ALLOWABLE THICKNESS OF INITIAL CONSTRUCTION IS 40.0 INCHES.

THE DISTRICT TEMPERATURE CONSTANT IS = 28.0
THE MAXIMUM SERVICEABILITY INDEX P1 IS 4.2
THE MINIMUM SERVICEABILITY INDEX P2 IS 3.0
THE SWELLING CLAY PARAMETERS--
P2 PRIME IS 1.50
B1 IS 0.0250

THE ONE DIRECTION ADT AT THE BEGINNING OF THE ANALYSIS PERIOD IS 2100.
THE ONE DIRECTION ADT AT THE END OF THE ANALYSIS PERIOD IS 3750.
THE ONE DIRECTION 20 YEAR ACCUMULATED NUMBER OF EQUIVALENT 18-KIP AXLES IS 860000

```
THE MINIMUM TIME TO THE FIRST OVERLAY IS   5 YEARS.
THE MINIMUM TIME BETWEEN OVERLAYS IS   5 YEARS.
THE MINIMUM TIME TO THE FIRST SEAL COAT AFTER INITIAL OR OVERLAY CONSTRUCTION IS  5.0 YEARS.
THE MINIMUM TIME BETWEEN SEAL COATS IS  3.0 YEARS.
A SEAL COAT WILL NOT BE APPLIED WITHIN ONE YEAR PRIOR TO AN OVERLAY.
THE NUMBER OF OPEN LANES IN THE RESTRICTED ZONE IN THE OVERLAY DIRECTION IS    1
THE NUMBER OF OPEN LANES IN THE RESTRICTED ZONE IN THE NON-OVERLAY DIRECTION IS    1

THE NUMBER OF LANES IN EACH DIRECTION IS  1
THE DISTANCE,MEASURED ALONG THE C.L.,OVER WHICH TRAFFIC IS SLOWED IN THE OVERLAY DIRECTION IS 0.50 MILES.
THE DISTANCE,MEASURED ALONG THE C.L.,OVER WHICH TRAFFIC IS SLOWED IN THE NON-OVERLAY DIRECTION IS 0.25 MILES.
THE PROPORTION OF THE ADT WHICH ARRIVES DURING EACH HOUR OF CONSTRUCTION IS  6.0 PERCENT.
OVERLAY CONSTRUCTION TAKES PLACE 12.0 HOURS PER DAY.
THE ROAD IS IN A RURAL AREA.

THE PROPORTION OF VEHICLES STOPPED BY CONTRACTORS EQUIPMENT(OVERLAY DIRECTION) IS  2.0 PERCENT.
THE PROPORTION OF VEHICLES STOPPED BY CONTRACTORS EQUIPMENT(NON-OVERLAY DIRECTION) IS  2.0 PERCENT.
THE AVERAGE TIME STOPPED BY CONTRACTORS EQUIPMENT(OVERLAY DIRECTION) IS 0.0040 HOURS.
THE AVERAGE TIME STOPPED BY CONTRACTORS EQUIPMENT(NON-OVERLAY DIRECTION) IS 0.0040 HOURS.
THE AVERAGE APPROACH SPEED TO THE OVERLAY ZONE IS 60.0 MILES PER HOUR.
THE AVERAGE SPEED THROUGH THE OVERLAY ZONE IN THE OVERLAY DIRECTION IS 40.0 MILES PER HOUR.
THE AVERAGE SPEED THROUGH THE OVERLAY ZONE IN THE NON-OVERLAY DIRECTION IS 50.0 MILES PER HOUR.
TRAFFIC MODEL  1 IS USED IN THIS ANALYSIS.

THE FIRST YEAR COST OF ROUTINE MAINTENANCE IS  50.00 DOLLARS PER LANE MILE.
THE INCREMENTAL INCREASE IN COST PER YEAR IS  20.00 DOLLARS PER LANE MILE.
THE COST OF A SEAL COAT IS 1500.00 DOLLARS PER LANE MILE.
THE WIDTH OF EACH LANE IS 12.00 FEET.
```

```
FOR THE 1 LAYER DESIGN WITH THE FOLLOWING MATERIALS-

            MATERIAL                COST/CY  COST/SY/IN  ST. COEF.  MIN. DEPTH  MAX DEPTH
ASPHALTIC CONCRETE                   12.00     0.3333       0.80       0.50       10.00
SUBGRADE                              0.0      0.0          0.22       0.0         0.0

  1   THE OPTIMAL DESIGN FOR THE MATERIALS UNDER CONSIDERATION--
       FOR INITIAL CONSTRUCTION THE DEPTHS SHOULD BE
            ASPHALTIC CONCRETE               6.50 INCHES

       THE LIFE OF THE INITIAL CONSTRUCTION =  5.35 YEARS
       THE OVERLAY SCHEDULE IS
            1.00 INCHES AFTER  5.35 YEARS
            0.50 INCHES AFTER 12.93 YEARS
            TOTAL LIFE =      21.41 YEARS

       SEAL COATS SHOULD OCCUR AFTER
            (1)  10.35 YEARS
            (2)  17.93 YEARS

       THE TOTAL COSTS PER SQ. YD. FOR THESE CONSIDERATIONS ARE
            INITIAL CONSTRUCTION COST             2.157
            TOTAL ROUTINE MAINTENANCE COST        0.192
            TOTAL OVERLAY CONSTRUCTION COST       0.788
            TOTAL USER COST DURING
                      OVERLAY CONSTRUCTION        0.027
            TOTAL SEAL COAT COST                  0.219
            SALVAGE VALUE                        -0.754
            TOTAL OVERALL COST                    2.629

        THE NUMBER OF FEASIBLE DESIGNS EXAMINED FOR THIS SET OF MATERIALS WAS        8
```

FOR THE 2 LAYER DESIGN WITH THE FOLLOWING MATERIALS-

MATERIAL	COST/CY	COST/SY/IN	ST. COEF.	MIN. DEPTH	MAX DEPTH
ASPHALTIC CONCRETE	12.00	0.3333	0.80	0.50	10.00
LIME TREATED GRAVEL	5.40	0.1500	0.65	4.00	32.00
SUBGRADE	0.0	0.0	0.22	0.0	0.0

```
  2  THE OPTIMAL DESIGN FOR THE MATERIALS UNDER CONSIDERATION--
      FOR INITIAL CONSTRUCTION THE DEPTHS SHOULD BE
             ASPHALTIC CONCRETE              0.50 INCHES
             LIME TREATED GRAVEL             7.50 INCHES

      THE LIFE OF THE INITIAL CONSTRUCTION =  5.43 YEARS
      THE OVERLAY SCHEDULE IS
             0.50 INCHES AFTER  5.43 YEARS
             1.00 INCHES AFTER 11.56 YEARS
             TOTAL LIFE =      20.27 YEARS

      SEAL COATS SHOULD OCCUR AFTER
             (1)  10.43 YEARS
             (2)  16.56 YEARS

      THE TOTAL COSTS PER SQ. YD. FOR THESE CONSIDERATIONS ARE
             INITIAL CONSTRUCTION COST            1.237
             TOTAL ROUTINE MAINTENANCE COST       0.190
             TOTAL OVERLAY CONSTRUCTION COST      0.763
             TOTAL USER COST DURING
                      OVERLAY CONSTRUCTION        0.026
             TOTAL SEAL COAT COST                 0.224
             SALVAGE VALUE                       -0.506
             TOTAL OVERALL COST                   1.984

        THE NUMBER OF FEASIBLE DESIGNS EXAMINED FOR THIS SET OF MATERIALS WAS      36
```

```
FOR THE 2 LAYER DESIGN WITH THE FOLLOWING MATERIALS-

          MATERIAL              COST/CY  COST/SY/IN  ST. COEF.  MIN. DEPTH  MAX DEPTH
ASPHALTIC CONCRETE               12.00     0.3333      0.80       0.50       10.00
LIME TREATED SUBGRADE             3.00     0.0833      0.35       4.00        8.00
SUBGRADE                          0.0      0.0         0.22       0.0         0.0

 3   THE OPTIMAL DESIGN FOR THE MATERIALS UNDER CONSIDERATION--
      FOR INITIAL CONSTRUCTION THE DEPTHS SHOULD BE
             ASPHALTIC CONCRETE              4.00 INCHES
             LIME TREATED SUBGRADE           7.00 INCHES

      THE LIFE OF THE INITIAL CONSTRUCTION =  5.35 YEARS
      THE OVERLAY SCHEDULE IS
             1.00 INCHES AFTER  5.35 YEARS
             0.50 INCHES AFTER 12.89 YEARS
             TOTAL LIFE =      21.29 YEARS

      SEAL COATS SHOULD OCCUR AFTER
             (1)  10.35 YEARS
             (2)  17.89 YEARS

      THE TOTAL COSTS PER SQ. YD. FOR THESE CONSIDERATIONS ARE
             INITIAL CONSTRUCTION COST          1.910
             TOTAL ROUTINE MAINTENANCE COST     0.192
             TOTAL OVERLAY CONSTRUCTION COST    0.788
             TOTAL USER COST DURING
                    OVERLAY CONSTRUCTION        0.027
             TOTAL SEAL COAT COST               0.219
             SALVAGE VALUE                     -0.683
             TOTAL OVERALL COST                 2.453

       THE NUMBER OF FEASIBLE DESIGNS EXAMINED FOR THIS SET OF MATERIALS WAS      44
```

FOR THE 3 LAYER DESIGN WITH THE FOLLOWING MATERIALS-

MATERIAL	COST/CY	COST/SY/IN	ST. COEF.	MIN. DEPTH	MAX DEPTH
ASPHALTIC CONCRETE	12.00	0.3333	0.80	0.50	10.00
LIME TREATED GRAVEL	5.40	0.1500	0.65	4.00	32.00
LIME TREATED SUBGRADE	3.00	0.0833	0.35	4.00	8.00
SUBGRADE	0.0	0.0	0.22	0.0	0.0

```
 4   THE OPTIMAL DESIGN FOR THE MATERIALS UNDER CONSIDERATION--
      FOR INITIAL CONSTRUCTION THE DEPTHS SHOULD BE
             ASPHALTIC CONCRETE              0.50 INCHES
             LIME TREATED GRAVEL             5.50 INCHES
             LIME TREATED SUBGRADE           4.00 INCHES

      THE LIFE OF THE INITIAL CONSTRUCTION =  5.12 YEARS
      THE OVERLAY SCHEDULE IS
             0.50 INCHES AFTER  5.12 YEARS
             0.50 INCHES AFTER 10.94 YEARS
             0.50 INCHES AFTER 17.62 YEARS
             TOTAL LIFE =      25.20 YEARS

      SEAL COATS SHOULD OCCUR AFTER
             (1)  15.94 YEARS

      THE TOTAL COSTS PER SQ. YD. FOR THESE CONSIDERATIONS ARE
             INITIAL CONSTRUCTION COST           1.320
             TOTAL ROUTINE MAINTENANCE_COST      0.175
             TOTAL OVERLAY CONSTRUCTION COST     0.892
             TOTAL USER COST DURING
                       OVERLAY CONSTRUCTION      0.032
             TOTAL SEAL COAT COST                0.098
             SALVAGE VALUE                      -0.516
             TOTAL OVERALL COST                  2.001

        THE NUMBER OF FEASIBLE DESIGNS EXAMINED FOR THIS SET OF MATERIALS WAS       72
```

	1	2	3	4	5	6	7	8	9	10	11	12
DESIGN NUMBER	2	4	2	4	2	4	4	2	4	4	2	4
INIT. CONST. COST	1.287	1.320	1.437	1.360	1.517	1.470	1.510	1.367	1.590	1.440	1.667	1.400
OVERLAY CONST. COST	0.763	0.892	0.561	0.744	0.430	0.596	0.567	0.738	0.430	0.703	0.292	0.738
USER COST	0.026	0.032	0.021	0.026	0.015	0.022	0.021	0.026	0.015	0.025	0.011	0.026
SEAL COAT COST	0.224	0.098	0.280	0.218	0.368	0.286	0.280	0.292	0.368	0.286	0.409	0.292
ROUTINE MAINT. COST	0.190	0.175	0.205	0.190	0.243	0.198	0.205	0.199	0.243	0.204	0.250	0.199
SALVAGE VALUE	-0.506	-0.516	-0.502	-0.528	-0.523	-0.511	-0.523	-0.528	-0.544	-0.549	-0.518	-0.537
TOTAL COST	1.984	2.001	2.008	2.010	2.049	2.060	2.060	2.093	2.101	2.108	2.111	2.117
NUMBER OF LAYERS	2	3	2	3	2	3	3	2	3	3	2	3
D(1)	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
D(2)	7.5	5.5	8.5	5.5	9.0	6.5	6.5	8.0	7.0	6.0	10.0	6.0
D(3)	0.0	4.0	0.0	4.5	0.0	4.0	4.5	0.0	4.5	4.5	0.0	4.0
NUMBER OF PERF. PERIODS	3	4	3	3	2	3	3	3	2	3	2	3
T(1)	5.430	5.117	7.617	5.547	8.828	7.227	7.695	6.484	8.945	6.602	11.484	6.133
T(2)	11.562	10.937	15.977	11.797	20.352	15.234	16.211	15.312	20.586	15.547	23.984	14.531
T(3)	20.273	17.617	25.313	20.547	0.0	24.141	25.625	25.078	0.0	25.430	0.0	23.828
T(4)	0.0	25.195	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
OVERLAY POLICY(INCH)												
O(1)	0.5	0.5	0.5	0.5	1.0	0.5	0.5	1.0	1.0	1.0	0.5	1.0
O(2)	1.0	0.5	0.5	1.0	0.0	0.5	0.5	0.5	0.0	0.5	0.0	0.5
O(3)	0.0	0.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
NUMBER OF SEAL COATS	2	1	2	2	3	2	2	2	3	2	3	2
SEAL COAT SCHEDULE (YEARS)												
SC(1)	10.430	15.937	5.000	10.547	5.000	5.000	5.000	5.000	5.000	5.000	5.000	5.000
SC(2)	16.562	0.0	12.617	16.797	13.828	12.227	12.695	11.484	13.945	11.602	8.000	11.133
SC(3)	0.0	0.0	0.0	0.0	16.828	0.0	0.0	0.0	16.945	0.0	16.484	0.0
SC(4)	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

THE TOTAL NUMBER OF FEASIBLE DESIGNS CONSIDERED WAS 160

REFERENCES

1. W. G. Adkins, A. W. Ward, and W. F. McFarland, "Values of Time Savings of Commercial Vehicles," NCHRP Report 33, Highway Research Board, Washington, D. C., 1967.

2. Egon Balas, "An Additive Algorithm for Solving Linear Programs with Zero-One Variables," Operations Research, Vol. 13, 517-545 (1965).

3. M. L. Balinski, "Integer Programming: Methods, Uses, Computations," Management Science, Vol. 12, 253-313 (1965).

4. D. S. Berry, "Field Measurement of Delay at Signalized Intersection," Proceedings Thirty-Fifth Annual Meeting, Highway Research Board, Washington, D. C., 1956.

5. A. J. Bone, "Travel Time and Gasoline Comsumption Studies in Boston," Proceedings Thirty-First Annual Meeting, Highway Research Board, Washington, D. C., 1952.

6. D. M. Burmister, "Theory of Stresses and Displacements in Layered Systems and Applications to the Design of Airport Runways," Highway Research Board Proceedings, Vol. 23, 126-154, 1943.

7. W. N. Carey and P. E. Irick, "The Pavement Serviceability - Performance Concept," Highway Research Board Bulletin No. 250, 40-58 (1960).

8. W. E. Chostain, Sr., "Application of Road Test Formulas in Structural Design of Pavement," Highway Research Board Special Report 73, 1962.

9. _____ and D. R. Schwartz, "AASHO Road Test Equations Applied to the Design of Bituminous Pavements in Illinois," Highway Research Record No. 90 (1965).

10. P. J. Claffey, Road User Cost in Urban Areas, NCHRP Project 2-7, May, 1966.

11. _____, "Running Costs of Motor Vehicles as Affected by Highway Design and Traffic," NCHRP Report 13, Highway Research Board, Washington, D. C., December, 1966.

12. _____,"Time and Fuel Consumption for Highway User Benefit Studies," Highway Research Board Bulletin 276, Washington, D. C. (1960).

13. A. V. Cobot and A. P. Hunter, "An Approach to Zero-One Integer Programming," Operations Research, Vol. 16, 1206-1211 (1968).

14. E. Paul DeGarmo, Engineering Economy, 4th Ed., The Macmillian Company, pp. 265-269, 1967.

15. D. R. Drew, "The Energy-Momentum Concept of Traffic Flow," Traffic Engineering (1965).

16. _____ and C. L. Dudek, "Investigation of an Internal Energy Model for Evaluating Freeway of Service," Freeway Surveillance Project Research Report 24-11, Texas Transportation Institute, Texas A&M University, 1965.

17. _____, _____, and C. J. Keese, "Freeway Level of Service as Described by an Energy-Acceleration Noise Model," Highway Research Record No. 162 (1967).

18. _____ and C. J. Keese, "Freeway Level of Service as Influenced by Volume and Capacity Characteristics," Highway Research Record No. 99 (1965).

19. "Economics of Asphalt and Concrete for Highway Construction," Stanford Research Institute, Project IE-3153, 157-184, April, 1961.

20. C. C. Edwards, "Development of Maintenance Cost Formula and Its Applications," Highway Research News No. 16, 61-69, December, 1964.

21. M. A. Efroymson and T. L. Ray, "A Branch-Bound Algorithm for Plant Location," Operations Research, Vol. 14, 361-368 (1966).

22. L. F. Erickson, "An Evaluation of Flexible Pavement Design Methods," Research Division, Idaho Department of Highways, 1964.

23. A. M. Geoffrion, "Implicit Enumeration Using an Imbedded Linear Program," Working Paper No. 120, Western Management Science Institute, May, 1967.

24. _____, "Integer Programming by Implicit Enumeration and Balas' Method," Memo RM-4783-PR, Rand Corporation, Santa Monica, California, February, 1966.

25. Fred Glover, "A Multiphase-Dual Algorithm for the Zero-One Integer Programming Problem," Operations Research, Vol. 13, 879-919 (1965).

26. _____, "Surrogate Constraints," Operations Research, Vol. 16, 741-749 (1968).

27. H. H. Greenberg, "A Branch-Bound Solution to the General Scheduling Problem," Operations Research, Vol. 14 353-361 (1968).

28. R. L. Gue, J. C. Liggett, and K. C. Cain, "Analysis of Algorithms for the Zero-One Programming Problem," Comm. ACM, Vol. 11, 837-844 (1968).

29. D. G. Haney and T. C. Thomas, "The Value of Time for Passenger Cars," Stanford Research Institute, Menlo Park, California, May, 1967.

30. S. S. Hejal, T. R. Buick, and J. C. Oppenlander, "Optimal Selection of Flexible Pavement Components," Joint Highway Research Project Report No. 18, September, 1968.

31. "Highway Capacity Manual, 1965," Special Report 87, Highway Research Board, Washington, D. C., p. 32, 1965.

32. T. S. Huff, "Report on Satellite Studies on Pavement Performance," Presented at 1965 Annual Meeting of AASHO, New York, New York, 1965.

33. B. G. Hutchinson, "Principles of Subjective Rating Scale Construction," Highway Research Record No. 46, Highway Research Board, Washington, D. C., 60-70 (1964).

34. _____, "The Measurement of Highway Pavement Performance," Proceedings Second International Conference on the Structural Design of Asphalt Pavements, Ann Arbor, Michigan, 1-15, August, 1967.

35. _____, "The Rationalization of Pavement Evaluation Decisions," Proceedings Australian Road Research Board, 1966.

36. _____, "The Use of Canadian Pavement Evaluation Data for Design," Proceedings Canadian Good Roads Association, 1966.

37. _____ and R. C. G. Haas, "A Systems Analysis of the Highway Pavement Design Process," Highway Research Record No. 239, Highway Research Board, Washington, D. C. (1968).

38. F. N. Hveem and G. B. Sherman, "Thickness of Flexible Pavements by the California Formula Compared to AASHO Road Test Data," Highway Research Record No. 13, Highway Research Board, Washington, D. C. (1963).

39. "Interstate Highway Maintenance Requirements and Unit Maintenance Expenditure Index," NCHRP Report 42, Washington, D. C., 1967.

40. "Iowa State Highway Maintenance Study," HRB Special Report 65, Supplement I, National Academy of Sciences - National Research Council, Washington, D. C., 1961.

41. R. E. Irick and W. R. Hudson, "Guidelines for Satellite Studies of Pavement Performance," National Cooperative Highway Research Program Report 2A, Highway Research Board of the National Academy of Sciences - National Research Council, Washington, D. C., 1964.

42. M. F. Kent, "Fuel and Time Consumption Rates for Trucks in Freight Service," Highway Research Board Bulletin 276, Washington, D. C. (1960).

43. A. H. Land and A. Doig, "An Automatic Method of Solving Discrete Programming Problems," Econometric 28, 497-520 (1960).

44. E. A. Landry, "Methods of Allocating Maintenance Funds," Proceedings AASHO, 276-286, 1956.

45. E. L. Lawler and M. D. Bell, "A Method for Solving Discrete Optimization Problems," Operations Research, Vol. 14, 1098-1112 (1966).

46. _____ and D. E. Wood, "Branch-and-Bound Methods: A Survey," Operations Research, Vol. 14, 669-719 (1966).

47. T. E. Lisco, "The Value of Commuters' Travel Time: A Study in Urban Transportation," Proceedings Forty-Eighth Annual Meeting, Highway Research Board, Washington, D. C., January, 1968.

48. Z. A. Lomnicki, "A 'Branch-and-Bound' Algorithm for the Exact Solution of the Three-Machine Scheduling Problem," Operational Research Quarterly, Vol. 16, 89-100 (1965).

49. _____ and A. P. G. Brown, "Some Applications of the 'Branch-and-Bound' Algorithm to the Machine Scheduling Problem," Operational Research Quarterly, Vol. 17, 173-186 (1966).

50. L. Mann, Jr., "A Procedure to Predict Maintenance Costs for Highway Systems," Engineering Experiment Station Bulletin No. 85, Louisiana State University, 1965.

51. _____, "Predicting Highway Maintenance Cost," Highway Research News No. 19, 28-36 (1965).

52. "Manual of Uniform Highway Accounting Procedures," American Association of State Highway Officials, Washington, D. C., 1958.

53. A. D. May, "A Friction Concept of Traffic Flow," Proceedings Thirty-Eighth Annual Meeting, Highway Research Board, Washington, D. C., 1959.

54. B. F. McCullough, C. J. VanVil, B. A. Vallerga, and R. G. Hicks, "Evaluation of AASHO Interim Guides for Design of Pavement Structures," NCHRP Project 1-11, December, 1968.

55. Ohio Department of Highways, Maintenance Cost Study, Manual.

56. George M. Pace, "Evaluation of the Dynaflect for the Non-Destructive Testing of Portland Cement Concrete Pavements," Technical Report No. 4-61, Department of the Army, Ohio Division Laboratories, Corps of Engineers, Cincinnati, Ohio, 1967.

57. L. J. Painter, "Analysis of AASHO Road Test Asphalt Pavement Data by The Asphalt Institute," Highway Research Record No. 71, Highway Research Board, Washington, D. C., 15-38 (1965).

58. H. A. Radzikowski, "Progress Report of Committee on Maintenance Costs," Highway Research Board Bulletin No. 155, 1-9 (1956).

59. R. B. Sawhill and K. C. Crandall, "Some Measurable Quantities of Traffic Service Influenced by Freeways," Highway Research Record No. 49 (1964).

60. _____ and J. C. Firey, "Motor Transport Fuel Consumption Rates and Travel Time," Highway Research Board Bulletin No. 276, Washington, D. C. (1960).

61. F. H. Scrivner and W. M. Moore, "An Empirical Equation for Predicting Pavement Deflections," Research Report 32-12, Texas Transportation Institute, Texas A&M University, College Station, Texas, October, 1968.

62. _____ AND _____, "Some Recent Findings in Flexible Research," Research Project 32-9, Texas Transportation Institute, Texas A&M University, College Station, Texas, 1967.

63. _____, _____, W. F. McFarland, and G. R. Carey, "A Systems Approach to the Flexible Pavement Design Problem," Research Report 32-11, Pavement Design Department, Texas Transportation Institute, Texas A&M University, College Station, Texas, in press.

64. _____, Poehl, Rudell, and W. M. Moore, "Detecting Seasonal Changes in Load-Carrying Capabilities of Flexible Pavement," NCHRP Report in press, Highway Research Board, Washington, D. C.

65. _____, Swift, Gilbert, and W. M. Moore, "A New Research Tool for Measuring Pavement Deflection," Highway Research Record No. 129, Highway Research Board, Washington, D. C., 1-11 (1966).

66. E. L. Skok, Jr. and P. J. Diethelin, "Applications of AASHO Road Test Results to Design of Flexible Pavements in Minnesota," Proceedings AAPT, 1966.

67. W. P. Smith, "Delay to Traffic Due to Future Resurfacing Operations," Traffic Bulletin No. 7, Department of Public Works, Division of Highways, Sacremento, California, November, 1963.

68. Z. K. Sutarwala and L. Mann, Jr., "A Formula for the Allocation of Maintenance Funds for Highways Using a Mathematical Model to Predict Maintenance Cost," Engineering Experiment Station Bulletin No. 72, Louisiana State University, p.20, 1963.

69. J. C. Tanner, "A Problem of Interference Between Two Queues," Biometrica, Vol. 40, Parts 1 and 2, 58-69 (1953).

70. W. C. Teinert and E. G. Long, "A Highway Construction Bid Price Index," Report No. 43-1, Texas Highway Department, Austin, Texas, March, 1965.

71. "The AASHO Road Test," Highway Research Board Special Report 73, Highway Research Board, Washington, D. C., 1962.

72. "The AASHO Road Test; Pavement Research," Special Report 61E, Report 5, Publication 954, National Academy of Sciences - National Research Council, Washington, D. C., 1962.

73. "The WASHO Road Test - Part 1," Report 18, Highway Research Board, Washington, D. C., 1954.

74. "The WASHO Road Test - Part 2," Report 22, Highway Research Board, Washington, D. C., 1955.

75. F. A. Tillman and J. M. Liittschwager, "Integer Programming Formulation of Constrained Reliability Problems," Management Science, Vol. 13, 887-899 (1967).

76. "Truck Weight and Vehicle Classification State of Texas, 1966," Texas Highway Department, Planning Survey Division, Austin, Texas.

77. N. K. Vaswani, "Design of Pavements Using Deflection Equations from AASHO Road Test Results," Highway Research Record No. 239, Highway Research Board, Washington, D. C., 76-94 (1968).

78. W. N. Volk, "Effect of Type of Control on Intersection Delay," Proceedings Thirty-Fifth Annual Meeting, Highway Research Board, Washington, D. C., 1956.

79. D. E. Wagner, "Optimum Packaging Design and Component Sparing Policies for Extended Manned Space Missions, " Unpublished Ph.D. Thesis, Texas A&M University, May, 1968.

80. F. V. Webster, "Delays at Traffic Signals: Fixed Time Signals RN/2374," Great Britain Road Research Laboratory, 1955.

81. R. Winfrey, Motor Vehicles Running Costs for Highway Economy Studies, Arlington, Virginia, 1963.

VITA

Gary R. Carey was born in Houston, Texas, December 5, 1940, son of Mr. and Mrs. William C. Carey. He attended elementary, junior high and high school in Houston, graduating from Sam Houston High School in 1959.

In 1965, he received a Bachelor of Science Degree in Industrial Engineering from Texas A&M University. While employed as a graduate assistant in the Industrial Engineering Department at Texas A&M, he received a Master of Science Degree in Industrial Engineering in January, 1967. He remained at Texas A&M University and began doctoral studies in Industrial Engineering with an Operations Research option.

In June of 1967, Mr. Carey joined the staff of the Texas Transportation Institute as a Research Associate Engineer and in September of 1967, also became an Instructor in the Industrial Engineering Department. He currently holds a joint appointment, each part time, with these two departments.

Address: 1700 Jersey, Apt. 101, College Station, Texas

The typist for this dissertation was Mrs. Judy Carey.