

# Two Independent Sample Means: t Test

Cohen Chapter 7

EDUC/PSY 6600

“We cannot solve problems by using the same kind of thinking that we used when we created them.”

-- Albert Einstein

# Introduction

Same continuous (interval, ratio) **Dependent Variable (DV)** compared across 2 independent (random) samples

## Research Questions

- Is there a significant **difference** between the 2 group means?
- Do 2 samples come from **different** normal distributions with the same mean?

## Also called

- Independent-groups design
- Between-subjects design
- Between-groups design
- Randomized-groups design

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How to Compare Two Samples Using t Tests (11-1)



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## Hypothesis Test Steps

1. State the **Hypotheses**
  - Null: no mean difference *difference = 0*
  - Alternative: there *is* a mean difference
2. Select the **Statistical Test & Significance Level**
  - $\alpha$  level, *default = .05*
  - One vs. Two tails, *default = 2 tails*
3. Select random **samples** and collect data
  - 2 simple random samples, *independent*
  - 1 sample *divided* into independent groups
4. Find the **Region of Rejection**
  - Based on  $\alpha$  & # of tails
  - *only really do if working by hand*
5. Calculate the **Test Statistic**
  - Examples include: z, t, F,  $\chi^2$
  - *use a p-value more by computer*
6. Make the **Statistical Decision**
  - big p-value: "no evidence of a difference"
  - tiny p-value: "evidence of a difference"
  - **make sure it is in context**

## Test Statistic Format

$$\text{Test Statistic} = \frac{\text{Estimate} - \text{Hypothesis}}{SE}$$

### For a Single MEAN

$$\text{Test Statistic} = \frac{\text{Estimate}_{\text{Mean}} - \text{Hypothesis}_{\text{Mean}}}{SE_{\text{Mean}}}$$

- Assume the population's SD...

$$z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{N}}}$$

- Use the sample's SD...

$$t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{N}}}$$

$$df = N - 1$$

### For the DIFFERENCE in Two Means

$$\text{Test Statistic} = \frac{\text{Estimate}_{\text{Difference}} - \text{Hypothesis}_{\text{Difference}}}{SE_{\text{Difference}}}$$

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - 0}{SED}$$

|| Note: use z instead of t IF N's > 100

- Null Hypothesis is "No Difference"

$$H_0 : \bar{D} = 0$$

$$H_0 : \mu_1 - \mu_2 = 0$$

- The Standard Error for the Difference (SED) & degrees of freedom is more complex

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## Homogeneity of Variance (HOV)

|| HOV: both populations have the SAME spread

**Levene's Test** used to check for evidence that HOV is violated (Null = nothing going on...HOV is fine)

$$H_0 : \sigma_1 = \sigma_2$$

$$H_1 : \sigma_1 \neq \sigma_2$$

|| Note: Levene's doesn't work as well for small samples

**IF p-value is BIGGER than alpha**

- $F(1, 98) = 0.78, p = .377$

Does **NOT** provide evidence that HOV is violated

**IF p-value is SMALLER than alpha**

- $F(1, 77) = 4.58, p = .013$

**DOES** provide evidence that HOV is violated

## Pooled Variance Test

### Requirement

- groups have the **SAME SD's** (HOV)

### Default in R's `t.test()`

- `var.equal = TRUE`

### Degrees of Freedom

$$df_{pooled} = n_1 + n_2 - 2$$

### Standard Error

$$SE_{pooled} = \sqrt{s_p^2 \left( \frac{1}{n_1} + \frac{1}{n_2} \right)}$$

first, pool standard deviations

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

## Seperate Variance Test

### Requirement

- groups may have **different SD's**

### Optional in R's `t.test()`

- `var.equal = FALSE`

### Degrees of Freedom

$$\min(n_1, n_2) - 1 < df_{SV} < n_1 + n_2 - 2$$

### Standard Error

$$SE_{SV} = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

- Use is somewhat controversial
- Welch's *df* are more complex
- Easier to compute the SE by hand
- Use with unequal N's, especially small

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## Confidence Intervals

We construct the confidence interval for the DIFFERENCE between the 2 means

### General Form for All Confidence Intervals

$$\text{Point Estimate} \pm CV_{\text{Estimate}} \times SE_{\text{Estimate}}$$

### General Form for CIs for the Difference in Means

$$(\bar{x}_1 - \bar{x}_2) \pm t_{CV} \times SE_{Diff}$$

## Pooled Variance CI

- groups have the **SAME SD's** (HOV)

$$(\bar{x}_1 - \bar{x}_2) \pm t_{CV} \times \sqrt{s_p^2 \left( \frac{1}{n_1} + \frac{1}{n_2} \right)}$$

## Seperate Variance CI

- groups may have **different SD's**

$$(\bar{x}_1 - \bar{x}_2) \pm t_{CV} \times \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

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## Comparing Means

Situation		Confidence Intervals			Hypothesis Testing		Notes
		Estimate	C.V.	SE <sub>estimate</sub>	Hypothesis	Test-statistic (df)	
X = measurement in sample <b>μ = POPULATION MEAN</b>	1 SAMPLE	$\bar{x}$ Sample mean	$\pm z^*$	$SE_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$	$H_0: \mu_x = \mu_0$ $H_a: \mu_x [\neq > <] \mu_0$	$z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$	Use if you know the population SD or when sample is very large
			$\pm t^*$	$SE_{\bar{x}} = \frac{s}{\sqrt{n}}$		$t = \frac{\bar{x} - \mu_0}{s / \sqrt{n}}$ $df = n - 1$	Use with a small sample or using the sample's SD instead of the population's $d = \bar{x} / SE_{\bar{x}}$
	MATCHED PAIRS (n = # of pairs)	$D = x_1 - x_2$  $\bar{D} = \bar{x}_1 - \bar{x}_2$	$\pm t^*$	$SE_{\bar{D}} = \frac{s_D}{\sqrt{n}}$	$H_0: \mu_D = \mu_0$ $H_a: \mu_D [\neq > <] \mu_0$  ---or--- $H_0: \mu_1 - \mu_2 = 0$ $H_a: \mu_1 - \mu_2 [\neq > <] 0$	$t = \frac{\bar{D} - \mu_0}{s_D / \sqrt{n}}$ $df = n - 1$	"Direct-Differences Method" First must subtract all pairs to create a new variable D and then find $s_D$ which is the SD of D's
			$\pm t^*$	$SE_{\bar{D}} = \sqrt{\frac{s_1^2 + s_2^2}{n} - \frac{2rs_1s_2}{n}}$		$t = \frac{\bar{D} - \mu_0}{\sqrt{\frac{s_1^2 + s_2^2}{n} - \frac{2rs_1s_2}{n}}}$ $df = n - 1$	"Correlation Method" Instead of subtracting all pairs, find each variables' M & SD, as well as the r between the variables in the sample
	2 INDEPENDENT SAMPLES (not paired)	$\bar{D} = \bar{x}_1 - \bar{x}_2$  Difference in 2 sample means	$\pm z^*$	$SE_{\bar{D}} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$	$H_0: \mu_1 - \mu_2 = 0$ $H_a: \mu_1 - \mu_2 [\neq > <] 0$	$z = \frac{\bar{D}}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$	Know both populations SD or when samples are large
			$\pm t^*$	$SE_{\bar{D}} = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$  $SE_{\bar{D}} = \sqrt{s_p^2 \left( \frac{1}{n_1} + \frac{1}{n_2} \right)}$  $s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$ $s_p^2 \xrightarrow{n_1=n_2} \frac{s_1^2 + s_2^2}{2}$		$t = \frac{\bar{D}}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$ $\min(n's) - 1 < df_{SV} < n_1 + n_2 - 2$  $t = \frac{\bar{D}}{\sqrt{s_p^2 \left( \frac{1}{n_1} + \frac{1}{n_2} \right)}}$ $df_{pooled} = n_1 + n_2 - 2$	"Separate Variances t-test" Use with equal n's - or - if violated HOW (var.equal = FALSE). Also use with equal n's if the larger sample as the smaller SD $d = \bar{D} / SE_{\bar{D}}$  "Pooled Variance t-test" Assumes the populations have equal SD's (test HOW w/Levene's Test, var.equal = TRUE). Also use then the n's are not equal and the larger sample has the larger SD $d = \bar{D} / s_p$

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## Comparing Means

Situation		Confidence Intervals			Hypothesis Testing		Notes
		Estimate	C.V.	SE <sub>estimate</sub>	Hypothesis	Test-statistic (df)	
X = measurement in sample <b>μ = POPULATION MEAN</b>	1 SAMPLE	$\bar{x}$ Sample mean	$\pm z^*$	$SE_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$	$H_0: \mu_x = \mu_0$ $H_a: \mu_x [\neq > <] \mu_0$	$z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$	Use if you know the population SD or when sample is very large
			$\pm t^*$	$SE_{\bar{x}} = \frac{s}{\sqrt{n}}$		$t = \frac{\bar{x} - \mu_0}{s / \sqrt{n}}$ $df = n - 1$	Use with a small sample or using the sample's SD instead of the population's $d = \bar{x} / SE_{\bar{x}}$
	MATCHED PAIRS (n = # of pairs)	$D = x_1 - x_2$  $\bar{D} = \bar{x}_1 - \bar{x}_2$	$\pm t^*$	$SE_{\bar{D}} = \frac{s_D}{\sqrt{n}}$	$H_0: \mu_D = \mu_0$ $H_a: \mu_D [\neq > <] \mu_0$  ---or--- $H_0: \mu_1 - \mu_2 = 0$ $H_a: \mu_1 - \mu_2 [\neq > <] 0$	$t = \frac{\bar{D} - \mu_0}{s_D / \sqrt{n}}$ $df = n - 1$	"Direct-Differences Method" First must subtract all pairs to create a new variable D and then find $s_D$ which is the SD of D's
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			$\pm t^*$	$SE_{\bar{D}} = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$  $SE_{\bar{D}} = \sqrt{s_p^2 \left( \frac{1}{n_1} + \frac{1}{n_2} \right)}$  $s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$ $s_p^2 \xrightarrow{n_1=n_2} \frac{s_1^2 + s_2^2}{2}$		$t = \frac{\bar{D}}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$ $\min(n's) - 1 < df_{SV} < n_1 + n_2 - 2$  $t = \frac{\bar{D}}{\sqrt{s_p^2 \left( \frac{1}{n_1} + \frac{1}{n_2} \right)}}$ $df_{pooled} = n_1 + n_2 - 2$	"Separate Variances t-test" Use with equal n's - or - if violated HOW (var.equal = FALSE). Also use with equal n's if the larger sample as the smaller SD $d = \bar{D} / SE_{\bar{D}}$  "Pooled Variance t-test" Assumes the populations have equal SD's (test HOW w/Levene's Test, var.equal = TRUE). Also use then the n's are not equal and the larger sample has the larger SD $d = \bar{D} / s_p$

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## Comparing Means

Situation	Confidence Intervals			Hypothesis Testing		Notes
	Estimate	C.V.	SE <sub>estimate</sub>	Hypothesis	Test-statistic (df)	
1 SAMPLE Sample mean	$\bar{x}$	$\pm z^*$	$SE_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$	$H_0: \mu_x = \mu_0$ $H_a: \mu_x [\neq > <] \mu_0$	$z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$ $t = \frac{\bar{x} - \mu_0}{s / \sqrt{n}}$ $df = n - 1$	Use if you know the population SD or when sample is very large Use with a small sample or using the sample's SD instead of the population's $d = \bar{x} / SE_{\bar{x}}$
		$\pm t^*$	$SE_{\bar{x}} = \frac{s}{\sqrt{n}}$			
MATCHED PAIRS (n = # of pairs)	$D = x_1 - x_2$	$\pm t^*$	$SE_D = \frac{s_D}{\sqrt{n}}$	$H_0: \mu_D = \mu_0$ $H_a: \mu_D [\neq > <] \mu_0$	$t = \frac{\bar{D} - \mu_0}{s_D / \sqrt{n}}$ $df = n - 1$	"Direct-Differences Method" First must subtract all pairs to create new variable D and then find $s_D$ which is the SD of D's
	$\bar{D} = \bar{x}_1 - \bar{x}_2$	$\pm t^*$	$SE_{\bar{D}} = \sqrt{\frac{s_1^2 + s_2^2}{n} - \frac{2rs_1s_2}{n}}$	$H_0: \mu_1 - \mu_2 = 0$ $H_a: \mu_1 - \mu_2 [\neq > <] 0$	$t = \frac{\bar{D} - \mu_0}{\sqrt{\frac{s_1^2 + s_2^2}{n} - \frac{2rs_1s_2}{n}}}$ $df = n - 1$	"Correlation Method" Selecting all pairs, find each variables' M & SD, as well as the r between the variables in the sample
2 INDEPENDENT SAMPLES (not paired)		$\pm z^*$	$SE_{\bar{D}} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$		$z = \frac{\bar{D}}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$	Know both populations SD or when samples are large
			$SE_{\bar{D}} = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$	$H_0: \mu_1 - \mu_2 = 0$ $H_a: \mu_1 - \mu_2 [\neq > <] 0$	$t = \frac{\bar{D}}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$ $\min(n's) - 1 < df_{SV} < n_1 + n_2 - 2$	"Separate Variances t-test" Use with equal n's - or - if violated HOW (var.equal = FALSE). Also use with equal n's if the larger sample as the smaller SD $d = \bar{D} / SE_{\bar{D}}$
		$\pm t^*$	$SE_{\bar{D}} = \sqrt{s_p^2 \left( \frac{1}{n_1} + \frac{1}{n_2} \right)}$	$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$ $s_p^2 \xrightarrow{n_1=n_2} \frac{s_1^2 + s_2^2}{2}$	$t = \frac{\bar{D}}{\sqrt{s_p^2 \left( \frac{1}{n_1} + \frac{1}{n_2} \right)}}$ $df_{pooled} = n_1 + n_2 - 2$	"Pooled Variance t-test" Assumes the populations have equal SD's (test HOW w/Levene's Test, var.equal = TRUE). Also use then the n's are not equal and the larger sample has the larger SD $d = \bar{D} / s_p$

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## Example 1) A Fully Randomized Experiment

In order to assess the efficacy of a new antidepressant drug, 10 clinically depressed participants are **randomly assigned** to one of **TWO** groups.

- Assume that prior to introducing the treatments, the experimenter confirmed that the level of depression in the 2 groups was equal.

After 6 months, all participants are rated by a psychiatrist (**blind to participant assignment**) on their level of depression.

Five participants are assigned to Group 1, which is administered the **antidepressant drug** for 6 months.

11, 1, 0, 2, 0

The other five participants are assigned to Group 2, which is administered a **placebo** during the same 6 month period.

11, 11, 5, 8, 4

**Research Question:** Is there evidence that the antidepressant drug reduces depression more than a placebo?

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## Example 1) Entering Small Dataset by Hand

### Load Packages

```
library(tidyverse)
library(furniture)
library(car)
```

### Enter the data

Two people per line, but don't really pair up

```
df1_wide <- data.frame(drug = c(11, 1, 3, 4, 0),
                      placebo = c(11, 11, 5, 8, 4))
```

### View the data frame

```
df1_wide
  drug placebo
1   11      11
2    1      11
3    3       5
4    4       8
5    0       4
```

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## Example 1) Pivoting Wide-to-Long

### Restructure the data frame

We want one line per person

```
df1_long <- df1_wide %>%
  tidyr::pivot_longer(cols = c(drug, placebo),
                     names_to = "pill",
                     names_ptypes = list(group = factor()),
                     values_to = "depression")
```

### View the data frame

```
df1_long
# A tibble: 10 x 2
  pill      depression
  <chr>         <dbl>
1 drug             11
2 placebo          11
3 drug              1
4 placebo          11
5 drug              3
6 placebo           5
7 drug              4
8 placebo           8
9 drug              0
10 placebo          4
```

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## Example 1) Exploratory Data Analysis

Summarize:  $N$ ,  $M$ ,  $SD$

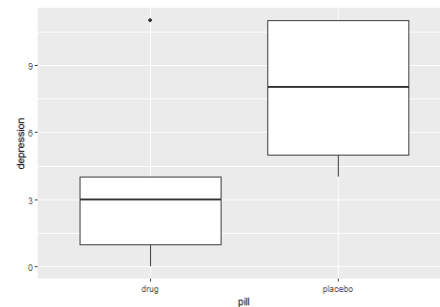
```
df1_long %>%
  dplyr::group_by(pill) %>%
  furniture::table1(depression,
    digits = 2,
    na.rm = FALSE,
    output = "markdown")
```

	drug	placebo
n	5	5
depression	3.80 (4.32)	7.80 (3.27)

Note: Normality is nearly impossible to assess in very small samples

Visualize: *boxplots*

```
df1_long %>%
  ggplot(aes(x = pill,
    y = depression)) +
  geom_boxplot()
```



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## Example 1) Levene's Test for HOV

BEFORE the t-test, check for violations of HOV

```
df1_long %>%
  car::leveneTest(depression ~ pill, # name of the data
    data = ., # continuous DV ~ 2-group IV
    center = mean) # pipe the data from above
# default is median
```

```
Levene's Test for Homogeneity of Variance (center = mean)
Df F value Pr(>F)
group 1 0.0526 0.8244
      8
```

### Conclusion

No evidence of violations of HOV were found,  $F(1, 8) = 0.05$ ,  $p = .824$ .

Choose to do the pooled variance t-test (`var.equal = TRUE`)

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## Example 1) Run a Pooled Variance t-Test

### Run the POOLED variance t-test

```
df1_long %>%  
  t.test(depression ~ pill,  
        data = .,  
        var.equal = TRUE) # name of the data  
                          # continuous DV ~ 2-group IV  
                          # pipe the data from above  
                          # do the pooled-variance version
```

```
Two Sample t-test  
  
data: depression by pill  
t = -1.6496, df = 8, p-value = 0.1376  
alternative hypothesis: true difference in means is not equal to 0  
95 percent confidence interval:  
-9.591763 1.591763  
sample estimates:  
mean in group drug mean in group placebo  
3.8 7.8
```

### Conclusion

No evidence of a difference in depression was found,  $t(8) = 1.80, p = .110$ .

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## Example 1) Writing Methods & Results

### Methods Section - analysis

To test the effectiveness of the drug at reducing depression, an **independent samples t-test** was performed, with pill type (*antidepressant drug vs. placebo*) functioning as the **independent variable** and depression score as the **dependent variable**.

**Levene's test** assessed for the assumption of homogeneity of variance to determine if a **pooled** or **separate variance** test should be performed.

### Results Section

After six months, the **five** participants taking the drug scored numerically lower on the depression scale ( $M = 2.80, SD = 4.66$ ), compared their **five** counter parts taking placebo ( $M = 7.80, SD = 3.27$ ).

The assumption of homogeneity of variances was tested and satisfied,  $F(1, 8) = 0.05, p = .824$ , thus the pooled test was conducted.

No evidence was found to support the claim that this antidepressant change depression,  $t(8) = 1.80, p = .138$ .

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# Assumptions of a 2-sample t-Test

## 1. BOTH Samples were drawn at **RANDOM** -AND- are **INDEPENDENT** of each other (at least as representative as possible)

- Nothing can be done to fix NON-representative samples & you can **NOT** statistically test
- If not independent samples, use a *Paired-Samples t-Test*

## 2. The Continuous DV follows a **NORMAL** distribution in BOTH populations

- **NOT** as important if the sample is large, due to the **Central Limit Theorem**
- **CAN** statistically test:
  - Visual inspection of a **histogram**, **boxplot**, and/or **QQ plot** (*straight 45 degree line*)
  - Calculate the Skewness & Kurtosis... less clear guidelines
  - Conduct **Shapiro-Wilks** test ( $p < .05$  ??? *not normal*)

## 3. **Homogeneity of Variance** (HOV)

- BOTH populations have the **SAME** spreads (*SDs*)
- Use **Levene's Test** to assess for HOV (Null Hypothesis is HOV is valid)

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# Violating the Assumptions

## **Equal Size Groups** violations "hurt" less

Only violate **HOV**

- Small effect:  $p$  off by  $\pm .02$

Only violate **Normality**

- Small effect:  $p$  off by  $\pm .02$
- **HOWEVER**: If samples are highly skewed or are skewed in opposite directions  $p$ -values can be **very** inaccurate!

Violate **Both**

- Moderate effects if  $N$  is large,  $p$ -value can be inaccurate
- Large effects if  $N$  is small,  $p$ -value can be **very** inaccurate

## **UN-equal Size Groups** violations have "BIG" impacts

Only violate **HOV**

- Large effect

Only violate **Normality**

- Large effect

Violate **Both**

- Huge effect

$p$ -values can be **very** inaccurate with unequal  $n$ s AND violations of assumptions, especially when  $N$  is small

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# Alternatives for when you Violate Assumptions

Violation of **Normality** or your DV is **ordinal**...

None of these are covered in this class

- Two Sample Wilcoxon test (*aka, Mann-Whitney U Test*)
- Sample re-use methods
- Rely on empirical, rather than theoretical, probability distributions
  - Exact statistical methods
  - Permutation and randomization tests
  - Bootstrapping

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# Random Assignment & Limits on Conclusions

Random assignment to groups DECREASES experimenter biases & confounding

- Cases are enumerated and numbers drawn to assign groups

Randomization does NOT ensure **equality** of group characteristics, but does allow for group differences to be **attributed** to the randomized factor.

## True Experiment

- Random assignment & manipulation of IV
- Even better if it is **double blind**

## Quasi-experiment

- Either randomization or manipulation
- Could randomize groups or clusters

## Non-experiment or Observational Study

- Neither randomization or manipulation
- Participants self-select or form naturally occurring groups

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## Example 2) A Fully Randomized Experiment

An industrial psychologist is investigating the effects of different types of **motivation** on the performance of simulated clerical **tasks**.

The **ten** participants in the "**individual motivation**" sample are told that they will be rewarded according to how many tasks **they** successfully complete.

The **12** participants in the "**group motivation**" sample are told that they will be rewarded according to the average number of tasks completed **by all the participants** in their sample.

The number of tasks completed by each participant are as follows:

- Individual Motivation (Self): 11, 17, 14, 12, 11, 15, 13, 12, 15, 16
- Group Motivation (Collective): 10, 15, 4, 8, 9, 14, 6, 15, 7, 11, 13, 5

**Research Question:** Is there evidence that performance on clerical tasks is effected by the type of motivation?

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## Example 2) Entering Small Dataset by Hand

Do NOT worry about this...I will do this part in the assignments when needed

```
df_self <- data.frame(motivation = "Self",  
                     tasks_done = c(11, 17, 14, 12, 11, 15, 13, 12, 15, 16))
```

```
df_coll <- data.frame(motivation = "Collective",  
                     tasks_done = c(10, 15, 4, 8, 9, 14, 6, 15, 7, 11, 13, 5))
```

```
df2_long <- dplyr::full_join(df_self, df_coll) %>%  
  dplyr::mutate(motivation = factor(motivation))
```

```
tibble::glimpse(df2_long)
```

```
Rows: 22  
Columns: 2  
$ motivation <fct> Self, Self, Self, Self, Self, Self, Self, Self, Self, Se...  
$ tasks_done <dbl> 11, 17, 14, 12, 11, 15, 13, 12, 15, 16, 10, 15, 4, 8, 9,...
```

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## Example 2) Exploratory Data Analysis

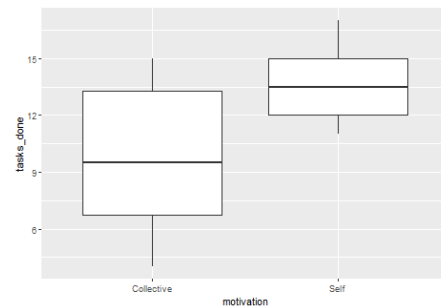
Summarize: *N, M, SD*

```
df2_long %>%  
  dplyr::group_by(motivation) %>%  
  furniture::table1(tasks_done,  
                    digits = 2,  
                    output = "markdown")
```

	Collective	Self
n	12	10
tasks_done	9.75 (3.89)	13.60 (2.12)

Visualize: *boxplots*

```
df2_long %>%  
  ggplot(aes(x = motivation,  
             y = tasks_done)) +  
  geom_boxplot()
```



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## Example 2) Levene's Test for HOV

BEFORE the t-test, check for violations of HOV

```
df2_long %>%  
  car::leveneTest(tasks_done ~ motivation, # name of the data  
                  data = .,                # continuous DV ~ 2-group IV  
                  center = mean)           # pipe the data from above  
                                           # default is median
```

```
Levene's Test for Homogeneity of Variance (center = mean)  
    Df F value Pr(>F)  
group 1  4.8287 0.03994 *  
    20  
---  
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

### Conclusion

Evidence found of violations of HOV,  $F(1, 20) = 4.83, p = .040$ . Also, samples are both small and unequal in size.

Choose to do the separate variance t-test (`var.equal = FALSE`)

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## Example 2) Run a Seperate Variance t-Test

### Run the SEPERATE variance t-test

```
df2_long %>%  
  t.test(tasks_done ~ motivation, # name of the data  
         data = .,                # continuous DV ~ 2-group IV  
         var.equal = FALSE)       # pipe the data from above  
                                # do the seperate-variance version
```

Welch Two Sample t-test

```
data: tasks_done by motivation  
t = -2.9456, df = 17.518, p-value = 0.008833  
alternative hypothesis: true difference in means is not equal to 0  
95 percent confidence interval:  
-6.601413 -1.098587  
sample estimates:  
mean in group Collective      mean in group Self  
          9.75                13.60
```

### Conclusion

Evidence of a difference in performace was found,  $t(17.52) = 2.95, p = .009$ .

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## Example 2) Writing Methods & Results

### Methods Section - analysis

To test if performance on clerical tasks is effected by the type of motivation, an **independent samples t-test** was performed, with type of motivation (*self vs. collective*) functioning as the **independent variable** and number of tasks completed as the **dependent variable**.

**Levene's test** assessed for the assumption of homogeneity of variance to determine if a **pooled** or **seperate variance** test should be performed.

### Results Section

The individually motivated participants completed more clerical tasks ( $n = 10, M = 13.60, SD = 2.12$ ), compared to the participants being motivated by their group's collective results ( $n = 12, M = 9.75, SD = 3.89$ ).

The assumption of homogeneity of variances was tested and evidence of violations were found,  $F(1, 20) = 4.83, p = .040$ , thus the seperated variance test was conducted.

Statistically significant evidence was found to support the claim that type of motivation effects performance. Thus, individual motivation does result in a mean **3.85** additional tasks completed compared to group motivation,  $t(17.52) = 2.95, p = .009, 95\% CI [1.10, 6.60]$ .

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