Confidence Intervals and the t Distribution

Cohen Chapter 6

EDUC/PSY 6600

"It is common sense to take a method and try it.
If it fails, admit it frankly and try another.
But above all, try something."

-- Franklin D. Roosevelt

Problems with z-tests

Usually we do **NOT** know σ, so we can **NOT** compute *Standard Error for the Mean (SEM)* $(SE_{\bar{x}})$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

Can you use the sample's *SD* (s) in place of populations's *SD* (σ) to calculate the SEM $(SE_{\bar{x}})$ as part of the z-test?

- Large samples – Yes (N > 300 participants)
- Small samples – No, **inaccurate** results

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} \xrightarrow{\text{If N is large}} z = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

Sample Size Matters

Small samples

As samples get smaller: $N \downarrow$

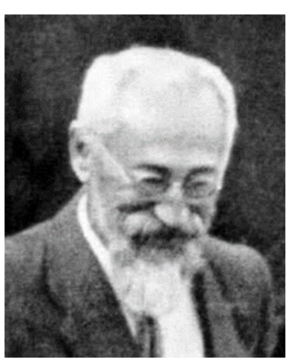
- the skewness of the sampling distribution of $s \uparrow$
- s **under**estimates σ
- z will $\uparrow$
- an **over**estimate $\uparrow$ risk of **Type I error**

LARGE samples

Compared to smaller samples:

- s **un**biased estimate of σ
- σ is a constant, *unknown truth*
- s is NOT a constant, since it varies from sample to sample
- As N increases, $s \to \sigma$

The t Distribution, "student's t"

1908, William Gosset

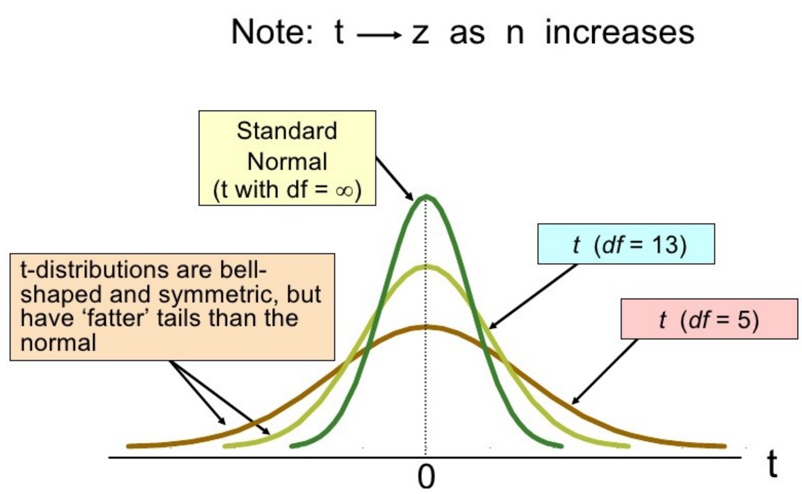
- Guinness Brewing Company, England
- Invented `t-test` for **small** samples for brewing quality control

Wrote paper using moniker "a student" discussing nature of SE_M when using s^2 instead of σ^2

- Worked with Fisher, Neyman, Pearson, and Galton

Priceonomics: The Guinness Brewer Who Revolutionized Statistics

Student's t Distributions



Student's t & Normal Distributions

Similarities between t & z

- Follows mathematical function
- Symmetrical, continuous, bell-shaped
- Continues to $\pm$ infinity
- Mean: $M = 0$
- Area under curve = $p(event[s])$
- When N is **large** (≈ 300), $t = z$

How t is Different from z

- Family of distributions
- Different distribution for each N (or df)
- Larger area in **tails** (%) for any value of t corresponding to z
  - $t_{cv} > z_{cv}$, for a given α
- More difficult to reject H_0 w/ t-distribution
  - $df = N - 1$
- As $df \uparrow$, the critical value of $t \rightarrow z$

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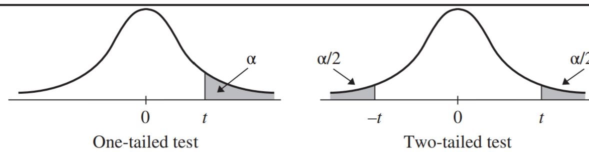
Critical Values of the t Distribution



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15	1.341	1.753	2.131	2.602	2.947	4.073
16	1.337	1.746	2.120	2.583	2.921	4.015

Calculating the t-Statistic

x is interval/ratio data (ordinal okay: $\geq 10 - 16$ levels or values)

Like z, the t-statistic represents a **SD** score (the # of SE's that $\bar{x}$ deviates from μ)

$$t = \frac{\bar{x} - \mu_x}{\frac{s_x}{\sqrt{N}}}$$

$$df = N - 1$$

When σ is known, t-statistic is sometimes computed (rather than `(z)`-statistic) if N is **small**

Estimate the population SEM with sample data:

Estimated SEM is the amount a sample's observed **mean**
may have deviated from
the true or population value
just due to random chance variation due to sampling.

Assumptions of a 1-sample t-test

1. Sample was drawn at RANDOM *(at least as representative as possible)*

- Nothing can be done to fix a NON-representative samples!
- Can **NOT** statistically test

2. SD of the sampled population = SD of the comparison population

- Nearly impossible to check, can **NOT** statistically test

3. Variable has a NORMAL distribution in the population

- **NOT** as important if the sample is large, due to the **Central Limit Theorem**
- **CAN** statistically test:
  - Visual inspection of a histogram, boxplot, and/or QQ plot *(straight 45 degree line)*
  - Calculate the Skewness & Kurtosis… less clear guidelines
  - Conduct Shapiro-Wilks test *(p < .05 ??? not normal)*

Formula Sheet

One-Sample Tests

Confidence Intervals			Hypothesis Testing		
Estimate	C.V.	SEestimate	Hypothesis	Test-statistic (df)	Notes
$\bar{x}$ Sample mean	± z*	$SE_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$	$H_0: \mu_x = \mu_0$ $H_a: \mu_x [\neq > <] \mu_0$	$z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$	Use if you know the population SD or when sample is very large
	± t*	$SE_{\bar{x}} = \frac{s}{\sqrt{n}}$		$t = \frac{\bar{x} - \mu_0}{s / \sqrt{n}}$ $df = n - 1$	Use with a small sample or using the sample's SD instead of the population's $d = \bar{X} / SE_{\bar{x}}$

Ex) 1-Sample t Test: mean vs. *historic control*

A physician states that, in the past, the average number of times he saw each of his patients during the year **5**. However, he believes that his patients have visited him significantly **more frequently** during the past year. In order to validate this statement, he randomly selects **10** of his patients and determines the number of office visits during the past year. He obtains the values presented to the below.

9, 10, 8, 4, 8, 3, 0, 10, 15, 9

Do the data support his contention that the average number of times he has seen a patient in the last year is **different** that 5?

Step 1) State the Hypotheses

$$H_0 : \mu = 5$$

$$H_1 : \mu \neq 5$$

Step 2) Select the Statistical Test and Significance Level

- 1-sample t-Test for the Mean
- TWO tailed & $\alpha = .05$

Ex) 1-Sample t Test: mean vs. *historic control*

Step 3) Select the Sample and Collect the Data

```
x <- c(9, 10, 8, 4, 8, 3, 0, 10, 15, 9)  # compare to historic value: mu = 5
```

Sample Size (*N*)

```
length(x)
```

```
[1] 10
```

Sample Mean (*M*)

$$\bar{X} = \frac{\sum X}{N}$$

```
mean(x)
```

```
[1] 7.6
```

Sample Standard Deviation (*SD*)

```
sd(x)
```

```
[1] 4.247875
```

Standard Error (*SEM*)

$$\sigma_{\bar{X}} = \frac{s}{\sqrt{N}}$$

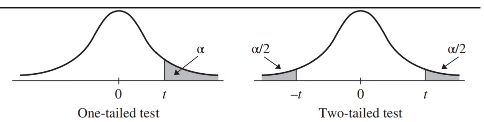
```
SE <-  sd(x) / sqrt(length(x))
SE
```

```
[1] 1.343296
```

Ex) 1-Sample t Test: mean vs. *historic control*

Step 4) Find the Region of Rejection

Table A.2
Critical Values of the *t* Distribution



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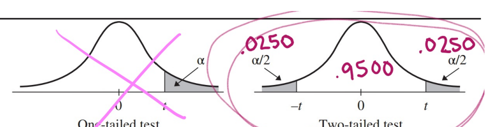

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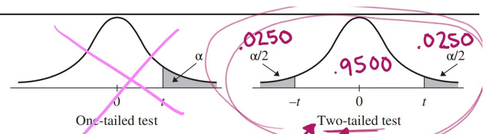

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df=9

"CRITICAL VALUE"

t_{CV}

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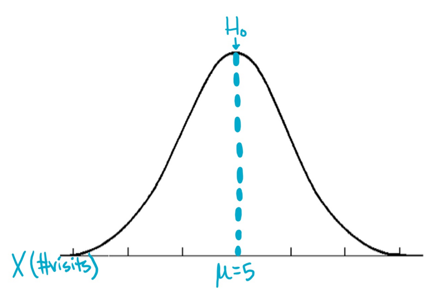
Ex) 1-Sample t Test: mean vs. *historic control*

Step 5) Calculate the Test Statistic



Ex) 1-Sample t Test: mean vs. *historic control*

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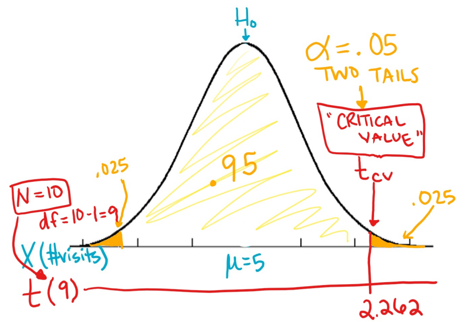


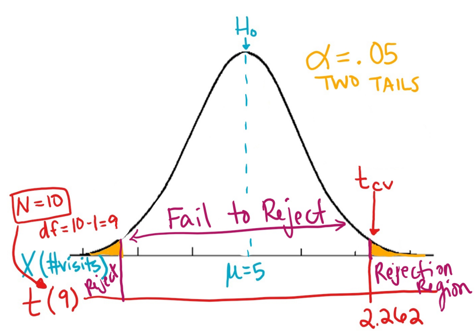
Ex) 1-Sample t Test: mean vs. *historic control*

Step 5) Calculate the Test Statistic

Observed t-score (*t*)

```
t <- (mean(x) - 5)/ SE
t
```

```
[1] 1.935538
```



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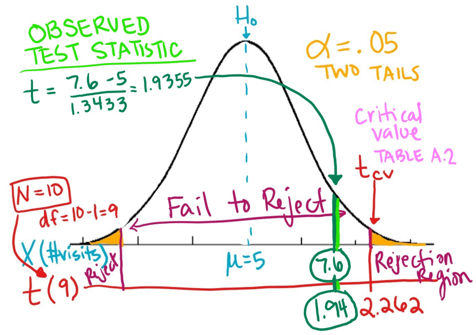

Ex) 1-Sample t Test: mean vs. *historic control*

Calculate the p-value *Only can get a rough range with this table*

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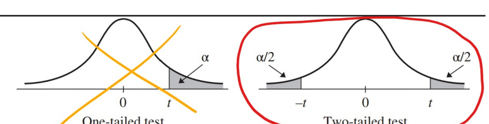

observed test statistic
t=1.94

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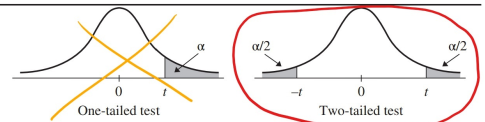

observed test statistic
$t = 1.94$

p-val
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Do the data support his contention that the average number of times he has seen a patient in the last year is different that 5?

Step 6) State the Conclusion *APA format in context*

> Even though this sample of ten patients has a mean of **7.60** visits per year, this could be due to sampling variance and does not provide evidence patients have **changed** the mean number of visit per year, $t(9) = 1.94$, $.05 < p < .10$.

*Note: Using the t **table**, we can only get a rough idea of the p-value, but when we use **software** we will get a p-vlaue with lots of decimal places.*

Confidence Intervals

Statistics are point estimates of a *population parameter* with **with error**

How **close** is estimate to population parameter?

- Confidence interval (CI) around point estimate *(Range of values)*
  - Upper limit: UL or UCL
  - Lower limit: LL or LCL

Confidence Intervals express our **confidence** or **uncertainty** in a sample statistic's ability to estimate the population parameter based on the *width*, which depends on both the sample's spread (SEM) and critical balue $(z_{CV} \text{ or } t_{cv})$

- Both are function of N
  - Larger $N \rightarrow$ Narrower CI

- More confident that sample point estimate (statistic) approximates population parameter
  - Narrow CI: Less confidence, more precision *(less error)*
  - Wide CI: More confidence, less precision *(more error)*

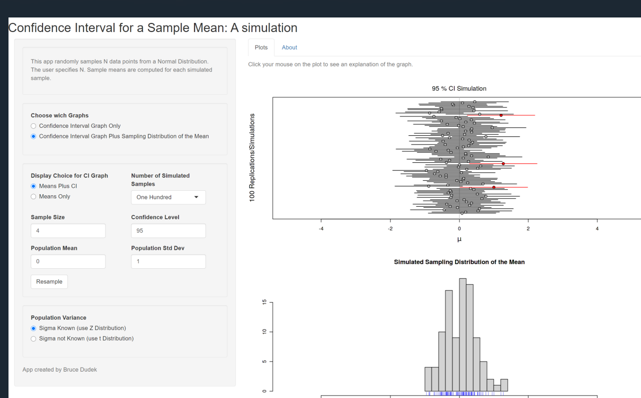
Confidence Intervals - Introduction

Calculating the Confidence interval for a mean using a formula - statistics help

Confidence Intervals: Crash Course Statistics #20

Interactive Shiny App

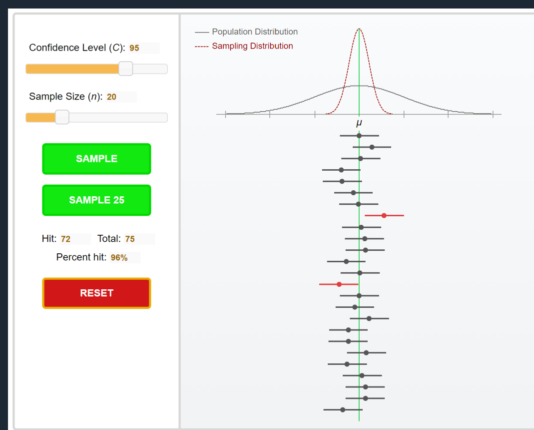
Confidence Intervals for a Sample Mean, A Simulation link



> University at Albany Psychology Department, Psychology Department, by Bruce Dudek

Interactive HTML App

Statistical Applet: Confidence Intervals link



> BFW-Bedford, Freeman & Worth Publishing, by Digital First

The Formula for all Confidence Intervals

Every Confidence Interval has two parts:

- Point Estimate Value = our best guess
- Margin of Error = plus-or-minus uncertainty

$$\text{Point Estimate Value} \quad \pm \quad \text{Margin of Error}$$

Every Margin of Error has two parts:

- Critical Value = look up on a table for a certain % confidence
- Standard Error for the Estimate = measure of spread

$$\text{Point Estimate Value} \quad \pm \quad \text{Critical Value} \quad \times \quad \text{Standard Error for the Estimate}$$

Known population *SD* &/or Large *N*

$$\bar{x} \ \pm \ z_{CV} \times \frac{\sigma}{\sqrt{N}}$$

Unknown population *SD* &/or Small *N*

$$\bar{x} \ \pm \ t_{CV} \times \frac{s}{\sqrt{N}}$$

Steps to Construct a Confidence Interval

1. Select your random sample size
2. Select the **Level of Confidence**
   - Generally 95% *(can be 80, 90, or even 99%)*
3. Select random sample and collect data
4. Find the **Critical Value**
   - Based on $\alpha = 1 - Conf$ & # of tails
   - Default: 95% $(\alpha = .05)$ and 2 tails
5. Calculate the Interval **End Points**

$$Est \pm CV_{Est} \times SE_{Est}$$

Narrow CI:

- large smaple
- Lower %

Wider CI:

- smaller sample
- Higher %

95% CI with z score

$$\bar{x} \pm 1.96 \times \frac{\sigma}{\sqrt{N}}$$

99% CI with z score

$$\bar{x} \pm 2.58 \times \frac{\sigma}{\sqrt{N}}$$

Formula Sheet

One-Sample Tests

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Estimate	C.V.	SE$_{estimate}$	Hypothesis	Test-statistic (df)	Notes
$\bar{x}$ Sample mean	± z*	$SE_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$	$H_0: \mu_x = \mu_0$ $H_a: \mu_x[\neq><]\mu_0$	$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$	Use if you know the population SD or when sample is very large
	± t*	$SE_{\bar{x}} = \frac{s}{\sqrt{n}}$		$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$ $df = n - 1$	Use with a small sample or using the sample's SD instead of the population's $d = \bar{X}/SE_{\bar{x}}$

Ex) Confidence Interval for a Mean

A physician states that, in the past, the average number of times he saw each of his patients during the year **5**. However, he believes that his patients have visited him significantly **more frequently** during the past year. In order to validate this statement, he randomly selects **10** of his patients and determines the number of office visits during the past year. He obtains the values presented to the below.

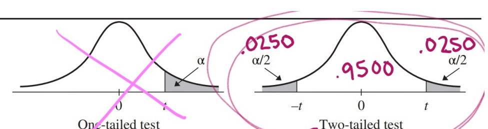
9, 10, 8, 4, 8, 3, 0, 10, 15, 9

Construct a 95% confidence interval for the mean number of visits per patient.

Ex) Confidence Interval for a Mean

Find the Critical Value: for 95%, use α = .05, but ALWAYS use TWO_TAILED for confidence intervals!

Table A.2
Critical Values of the t Distribution



N=10

df=9

"CRITICAL VALUE"

t_{CV}

	Level of Significance for One-Tailed Test					
	~~.10~~	~~.05~~	~~.025~~	~~.01~~	~~.005~~	~~.0005~~
	Level of Significance for Two-Tailed Test					
df	**.20**	**.10**	**.05**	**.02**	**.01**	**.001**
1	3.078	6.314	12.706	31.821	63.657	636.620
2	1.886	2.920	4.303	6.965	9.925	31.599
3	1.638	2.353	3.182	4.541	5.841	12.924
4	1.533	2.132	2.776	3.747	4.604	8.610
5	1.476	2.015	2.571	3.365	4.032	6.869
6	1.440	1.943	2.447	3.143	3.707	5.959
7	1.415	1.895	2.365	2.998	3.499	5.408
8	1.397	1.860	2.306	2.896	3.355	5.041
9	1.383	1.833	2.262	2.821	3.250	4.781
10	1.372	1.812	2.228	2.764	3.169	4.587
11	1.363	1.796	2.201	2.718	3.106	4.437
12	1.356	1.782	2.179	2.681	3.055	4.318
13	1.350	1.771	2.160	2.650	3.012	4.221
14	1.345	1.761	2.145	2.624	2.977	4.140
15	1.341	1.753	2.131	2.602	2.947	4.073
16	1.337	1.746	2.120	2.583	2.921	4.015

Ex) Confidence Interval for a Mean

```
x <- c(9, 10, 8, 4, 8, 3, 0, 10, 15, 9)
```

Sample Size (*N*)

```
length(x)
```

```
[1] 10
```

Sample Mean (*M*)

```
mean(x)
```

```
[1] 7.6
```

Sample Standard Deviation (*SD*)

```
sd(x)
```

```
[1] 4.247875
```

Standard Error: **Standard Error (*SEM*)**

```
SE <-  sd(x) / sqrt(length(x))
SE
```

```
[1] 1.343296
```

Confidence Interval (t_CV = 2.262)

```
mean(x) - 2.262 * SE
```

```
[1] 4.561464
```

```
mean(x) + 2.262 * SE
```

```
[1] 10.63854
```


Ex) Confidence Interval for a Mean

A physician states that, in the past, the average number of times he saw each of his patients during the year was 5. However, he believes that his patients have visited him significantly **more frequently** during the past year. In order to validate this statement, he randomly selects 10 of his patients and determines the number of office visits during the past year. He obtains the values presented to the below.

9, 10, 8, 4, 8, 3, 0, 10, 15, 9

Construct a 95% confidence interval for the mean number of visits per patient.

1. Best estimate

$$\bar{X} = \frac{\sum X}{n} = \frac{76}{10} = \mathbf{7.6}$$

2. Critical Value

df = n – 1 = 10 – 1 = 9

Always use TWO tails

→ Critical t = 2.262

3. Standard Error for the Estimate

Sample standard deviation, S = 4.25

$$SE_{mean} = s_{\bar{X}} = \frac{s_{sample}}{\sqrt{n}} = \frac{4.25}{\sqrt{10}} = 1.34$$

4. Put it together

$$Est \pm CV \times SE_{est} \rightarrow$$

$$7.60 \pm 2.262 \times 1.34$$

$$7.60 \pm 3.03$$

$$\boxed{\mathbf{4.57, 10.63}}$$

Estimating the Population Mean

Point estimate (M) is in the center of CI

Degree of confidence determined by α and corresponding critical value (CV)

- Commonly use 95% CI, so $\alpha = .05$
- Can also compute a .90, .99, or any size CI

z-distribution:
Known population variance or N is large (about 300)

$$\bar{x} \pm z_{cv} \times \frac{\sigma}{\sqrt{N}}$$

t -distribution:
Do not know population variance or N is small

$$\bar{x} \pm t_{cv} \times \frac{s}{\sqrt{N}}$$

is NOT the meaning of a 95% CI

There is **NOT** a 95% chance that the population M lies between the 2 CLs from your sample's CI !!!

Each random sample will have a different CI with different CLs and a different M value

IS the meaning of a 95% CI

95% of the CIs that could be constructed over repeated sampling will contain M Yours **MAY** be one of them

5% chance our sample's 95% CI does not contain μ
Related to **Type I Error**

APA Style Writeup

Z-test

(happens to be a statistically significant difference)

The hourly fee (M = $72) for our sample of current psychotherapists is significantly greater, z = 4.0, p < .001, than the 1960 hourly rate (M = $63, in current dollars).

T-test

(happens to not quite reach .05 significance level)

Although the mean hourly fee for our sample of current psychotherapists was considerably higher (*M* =$72, *SD* = $22.5) than the 1960 population mean (M = $63, in current dollars), this difference only approached statistical significance, *t*(24) = 2.00, *p* = .061.

Let's Apply This to the Cancer Dataset

Read in the Data

```
library(tidyverse)    # Loads several very helpful 'tidy' packages
library(haven)        # Read in SPSS datasets
library(furniture)    # Nice tables (by our own Tyson Barrett)
library(psych)        # Lots of nice tid-bits
```

```
cancer_raw <- haven::read_spss("cancer.sav")
```

And Clean It

```
cancer_clean <- cancer_raw %>%
  dplyr::rename_all(tolower) %>%
  dplyr::mutate(id = factor(id)) %>%
  dplyr::mutate(trt = factor(trt,
                             labels = c("Placebo",
                                        "Aloe Juice"))) %>%
  dplyr::mutate(stage = factor(stage))
```

The Cancer Dataset

Show 8 entries Search:

	id	trt	age	weighin	stage	totalcin	totalcw2	totalcw4	totalcw6
1	1	Placebo	52	124	2	6	6	6	7
2	5	Placebo	77	160	1	9	6	10	9
3	6	Placebo	60	136.5	4	7	9	17	19
4	9	Placebo	61	179.6	1	6	7	9	3
5	11	Placebo	59	175.8	2	6	7	16	13
6	15	Placebo	69	167.6	1	6	6	6	11
7	21	Placebo	67	186	1	6	11	11	10
8	26	Placebo	56	158	3	6	11	15	15

Showing 1 to 8 of 25 entries Previous 1 2 3 4 Next

1 sample t Test vs. Historic Control

> Do the patients weigh more than 165 pounds at intake, on average?

```
cancer_clean %>%
  dplyr::pull(weighin) %>%
  t.test(mu = 165)
```

```

	One Sample t-test

data:  .
t = 2.0765, df = 24, p-value = 0.04872
alternative hypothesis: true mean is not equal to 165
95 percent confidence interval:
 165.0807 191.4793
sample estimates:
mean of x
   178.28
```

> The patients in this study (N = 25) weigh **178.28** pounds on average, which is significantly more that **165** pounds, $t(24) = 2.08$, $p = .049$, 95% CI [165.08, 191.48].

...Change the Confidence Level

> Find a 99% confidence level for the population mean weight.

```
cancer_clean %>%
  dplyr::pull(weighin) %>%
  t.test(mu = 165,
         conf.level = 0.99)
```

```

	One Sample t-test

data:  .
t = 2.0765, df = 24, p-value = 0.04872
alternative hypothesis: true mean is not equal to 165
99 percent confidence interval:
 160.3927 196.1673
sample estimates:
mean of x 
   178.28 
```

> The patients in this study (N = 25) weigh **178.28** pounds on average, which is significantly more that **165** pounds, $t(24) = 2.08$, $p = .049$, 99% *CI* [160.39, 196.17].

...Restrict to a Subsample

> Do the patients with stage 3 & 4 cancer weigh more than 165 pounds at intake, on average?

```
cancer_clean %>%
  dplyr::filter(stage %in% c("3", "4")) %>%
  dplyr::pull(weighin) %>%
  t.test(mu = 165)
```

```

	One Sample t-test

data:  .
t = 0.82627, df = 5, p-value = 0.4463
alternative hypothesis: true mean is not equal to 165
95 percent confidence interval:
 137.0283 219.4717
sample estimates:
mean of x 
   178.25 
```

> The patients in this study with stage 3 or 4 cancer (n = 6) weigh **178.25** pounds on average, which is not significantly more that **165** pounds, $t(5) = 0.83$, $p = .446$, 95% *CI* [137.02, 219.47].

Questions?

Next Topic

Independent Samples t Tests for Means