"It is common sense to take a method and try it. If it fails, admit it frankly and try another. But above all, try something."

- Franklin D. Roosevelt

COHEN CHAP 6. ESTIMATION & T

For EDUC/PSY 6600

PROBLEMS WITH Z-TESTS

Often don't know σ^2

- Cannot compute SE_M

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{N}}$$

Can't s replace σ in SE_M and do z-test?

- Small samples – No, inaccurate results
- Large samples – Yes (>300 participants)

$$z = \frac{\bar{X} - \mu}{\frac{s}{\sqrt{N}}}$$

Small samples

- As N ↓, + skewness of sampling distribution of s^2 ↑
- As skewness ↑, s^2 underestimates σ^2
- As smaller s^2 is used in denominator of z-statistic equation, z will ↑, an overestimate
- ↑ risk of Type I error

Large samples

- s^2 unbiased estimate of σ^2 with large N
- σ is a constant
- s is NOT a constant
- Varies from sample to sample
- As N increases, $s \rightarrow \sigma$

THE T-DISTRIBUTION, "STUDENT'S T"

1908, William Gosset

- Guinness Brewing Company, England
- Invented t-test for small samples for brewing quality control

- Wrote paper using moniker "a student" discussing nature of SDM when using s^2 instead of σ^2

- Worked with Fisher, Neyman, Pearson, and Galton

STUDENT'S T & NORMAL (Z) DISTRIBUTIONS

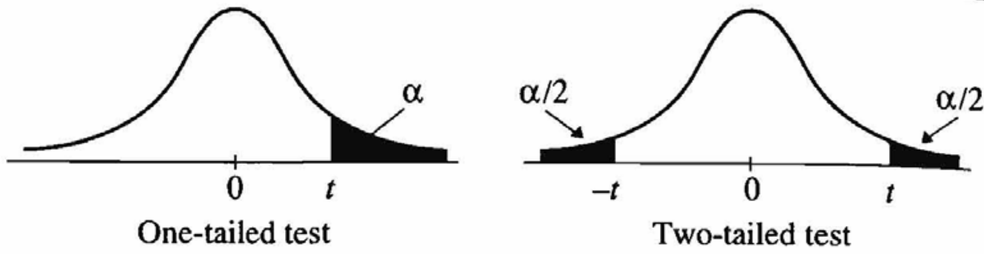
Similarities

- Follows mathematical function
- Symmetrical, continuous, bell-shaped
- Continues to ± infinity
- $M = 0$
- Area under curve = p(event[s])
- When N is large (≈ 300), $t = z$

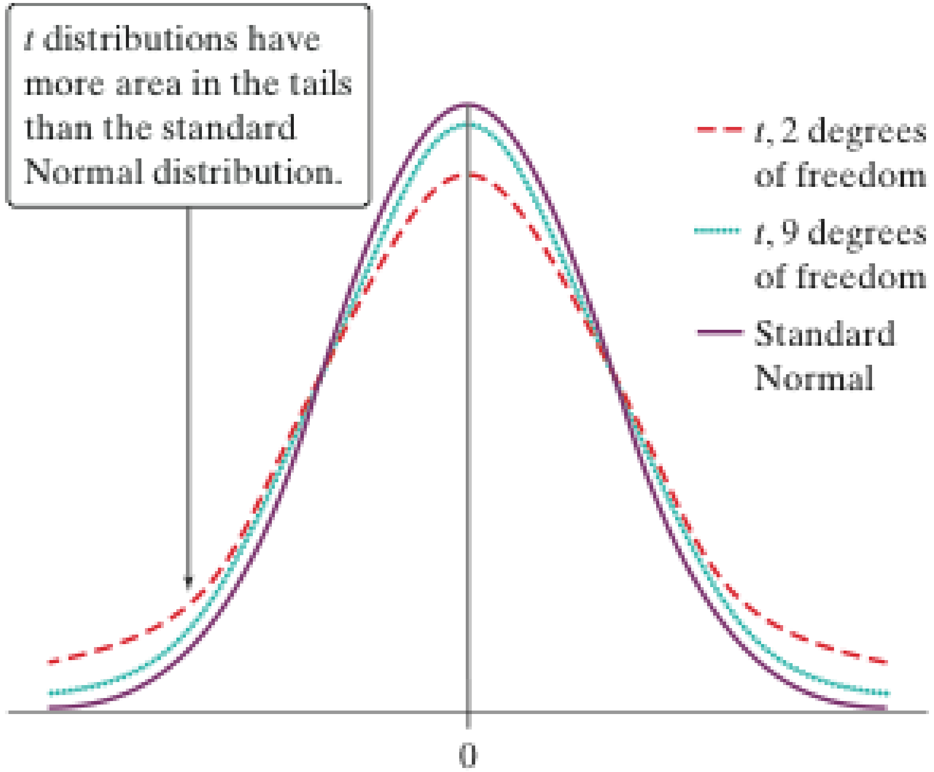
Differences

- Family of distributions
  - Different distribution for each N (or df)
- Larger area in tails (%) for any value of t corresponding to z
  - t_{crit} will be larger than z_{crit}, for a given α
- More difficult to reject H_0 w/ t-distribution
- $df = N - 1$
- As $df \uparrow$: critical value of $t \rightarrow z$

THE T-TABLE



	Level of Significance for One-Tailed Test					
	.10	.05	.025	.01	.005	.0005
	Level of Significance for Two-Tailed Test					
df	.20	.10	.05	.02	.01	.001
1	3.078	6.314	12.706	31.821	63.657	636.620
2	1.886	2.920	4.303	6.965	9.925	31.599
3	1.638	2.353	3.182	4.541	5.841	12.924
4	1.533	2.132	2.776	3.747	4.604	8.610
5	1.476	2.015	2.571	3.365	4.032	6.869
6	1.440	1.943	2.447	3.143	3.707	5.959
7	1.415	1.895	2.365	2.998	3.499	5.408
8	1.397	1.860	2.306	2.896	3.355	5.041
9	1.383	1.833	2.262	2.821	3.250	4.781
10	1.372	1.812	2.228	2.764	3.169	4.587
11	1.363	1.796	2.201	2.718	3.106	4.437
12	1.356	1.782	2.179	2.681	3.055	4.318
13	1.350	1.771	2.160	2.650	3.012	4.221
14	1.345	1.761	2.145	2.624	2.977	4.140
15	1.341	1.753	2.131	2.602	2.947	4.073



CALCULATING THE T-STATISTIC

- Interval/ratio data (ordinal *okay*: ≥ 10-16 values)
- Like z-, t-statistic represents a *SD* score (# of *SEs* $\bar{X}$ deviates from μ)
- When σ is known, t-statistic is sometimes computed (rather than z-statistic) if N is small
- Estimate pop. SE_M with sample data:

$$t = \frac{\bar{X} - \mu}{s_{\bar{X}}} = \frac{\bar{X} - \mu}{\frac{s}{\sqrt{N}}} = \frac{\bar{X} - \mu}{\sqrt{\frac{s^2}{N}}}, \; df = N - 1$$

- Estimated SE_M is amount observed mean <u>may</u> have deviated from true or population value

ASSUMPTIONS (SAME AS Z TESTS)

1. Sample was drawn at random (at least as representative as possible)

   Nothing can be done to fix NON-representative samples!

   Can not statistically test

2. SD of the sampled population = SD of the comparison population

   Very hard to judge

   Can not statistically test

3. Variables have a normal distribution

   Not as important if the sample is large (Central Limit Theorem)

   IF the sample is far from normal &/or small n, might want to transform variables

   Look at plots: histogram, boxplot, & QQ plot (straight 45° line) ← sensitive to outliers!!!

   Skewness & Kurtosis: Divided value by its *SE* & > ± 2 indicates issues

   Shapiro-Wilks test (small N): $p < .05$ → not normal

   Kolmogorov-Smirnov test (large N):

EXAMPLE: 1-SAMPLE T-TEST

A physician states that, in the past, the average number of times he saw each of his patients during the year was 5. However, he believes that his patients have visited him significantly more frequently during the past year. In order to validate this statement, he randomly selects 10 of his patients and determines the # of office visits during the past year. He obtains the values presented to the below.

9, 10, 8, 4, 8, 3, 0, 10, 15, 9

Do the data support his contention that the average number of times he has seen a patient in the last year

is different than 5?

CONFIDENCE INTERVALS

Statistics are ***point estimates***, or population parameters, with **error**

How close is estimate to pop. parameter?

- *Confidence interval* (*CI*) around *point estimate* (Range of values)
- Confidence limits (*CL*)
  - Values that bound *CI*
  - Upper limit: *UL* or *UCL*
  - Lower limit: *LL* or *LCL*

CI expresses our confidence in a statistic & the width depends on SE_M and t_{crit}

- Both are function of N
- Larger N → Smaller *CI*
  - More *confident* that sample *point estimate (statistic)* approximates population parameter

Narrow *CI* → Less confidence, more precision (less error)

Wide *CI* → More confidence, less precision (more error)

STEPS TO CONSTRUCT A CONFIDENCE INTERVAL

1) Select your random sample size
2) Select the Level of Confidence
   - Generally 95% (can by 80, 90, or even 99%)
3) Select random sample and collect data
4) Find the region of Rejection
   - Based on α & # of tails
5) Calculate the Interval

$$Est \pm CV \times SE_{est}$$

Narrow CI	**Wider CI**
Large N	*smaller N*
Lower %	*Higher %*

Example: 95% CI with z-score

$$\bar{X} \pm \mathbf{1.96} \times \frac{\sigma}{\sqrt{n}}$$

Example: 99% CI with z-score

$$\bar{X} \pm \mathbf{2.58} \times \frac{\sigma}{\sqrt{n}}$$

EXAMPLE: CONFIDENCE INTERVAL FOR THE MEAN

A physician states that, in the past, the average number of times he saw each of his patients during the year was 5. However, he believes that his patients have visited him significantly more frequently during the past year. In order to validate this statement, he randomly selects 10 of his patients and determines the # of office visits during the past year. He obtains the values presented to the below.

9, 10, 8, 4, 8, 3, 0, 10, 15, 9

Construct a 95% confidence interval for the mean number of visits per patient.

ESTIMATING THE POPULATION MEAN

- Point estimate (*M*) is in the center of *CI*

$$Est \pm CV \times SE_{est}$$

- Degree of confidence determined by α and corresponding t_{crit}
  - Common to use 95% *CI* ($\alpha = .05$)
  - Can also compute a .90, .99, or any size *CI*

- z-distribution: Known population variance or N is large (≈300)

$$\bar{X} \pm z_{crit} \times \frac{\sigma}{\sqrt{n}}$$

- t -distribution: Do not know population variance or N is small

$$\bar{X} \pm t_{crit} \times \frac{s}{\sqrt{n}}$$

NOT the meaning of a 95% *CI*

There is NOT a 95% chance that the population *M* lies between the 2 *CL*s from your sample's *CI* !!!

Each random sample will have a different *CI* with different *CL*s and a different *M* value

Meaning of a 95% *CI*

95% of the *CI*s that could be constructed over repeated sampling will contain *M*

Yours *MAY* be one of them

5% chance our sample's 95% *CI* does not contain μ

Related to Type I error

BOOTSTRAPPED CONFIDENCE INTERVALS

- Avoids assuming that your variable is normally distributed
- Computer-intensive…not by hand!
- Easy as pie for SPSS

IBM SPSS seems to have removed the bootstrap option from the basic software and offers it as an add on now???

Do NOT do chap 6 section C #4

- Basic Idea:
  1. Draw a random sample from your sample (with replacement) ← some may be chosen multiple times or no times
  2. compute this sample's mean, SD, and t-score
  3. repeat 1 & 2 lots of times, like 1,000+ (this is the not-by-hand part ;)
  4. Use the set of t-scores (1,000+ of them) & see where the original t-score falls in the distribution

APA: RESULTS OF A 1-SAMPLE Z-TEST

- Z-test (happens to be a statistically significant difference):

The hourly fee (M = \$72) for our sample of current psychotherapists is significantly greater, $z = 4.0$, $p < .001$, than the 1960 hourly rate (μ = \$63, in current dollars).

- T-test (happens to be quite reach .05 significance level):

Although the mean hourly fee for our sample of current psychotherapists was considerably higher (M = \$72, $SD = 22.5$) than the 1960 population mean (μ = \$63, in current dollars), this difference only approached statistical significance, $t(24) = 2.00$, $p = .06$.

SPSS: PERFORM A 1-SAMPLE T-TEST & CI

```
* t-test come with a confidence interval.

T-TEST
  /TESTVAL=50
  /VARIABLES=AGE
  /CRITERIA=CI(.95).


* change the confidenc elevel to 99%.

T-TEST
  /TESTVAL=50
  /VARIABLES=AGE
  /CRITERIA=CI(.99).
```

T-Test

One-Sample Statistics

	N	Mean	Std. Deviation	Std. Error Mean
AGE Patient's Incoming Age	25	59.64	12.932	2.586

One-Sample Test

	Test Value = 50					
					95% Confidence Interval of the Difference	
	t	df	Sig. (2-tailed)	Mean Difference	Lower	Upper
AGE Patient's Incoming Age	3.727	24	.001	9.640	4.30	14.98

One-Sample Test

	Test Value = 50					
					99% Confidence Interval of the Difference	
	t	df	Sig. (2-tailed)	Mean Difference	Lower	Upper
AGE Patient's Incoming Age	3.727	24	.001	9.640	2.41	16.87