# Competitive Programming Algorithms and Topics

# Ubiratan Neto

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5	Graph	_		9 #define mp make_tuple 10 #define mt make_tuple	
	5.1	Dynammic Connectivity - connected(u,v) query	1 /	11   #define DESYNC ios_base::sync_with_stdio(false); cin.tie(0); cout.tie(0)	
	5.2	Bellman Ford Shortest Path		12   #define pb push_back 13   #define vi vector <int></int>	
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	5.4	Kosaraju SCC	1 (	16 #define INF 1e18	
	5.5	Centroid Decomposition		17   #define ROOT 1 18   #define M 1000000007	
	5.6	·		19 const double PI = acos(-1);	
		Floyd Warshall Shortest Path		21 using namespace std;	
	5.7	Tarjan's Bridge/Articulations Algorithm		22   23   inline int mod(int n, int m) { int ret = n%m; if(ret < 0) ret += m; return	
	5.8	Max Flow Dinic's Algorithm		ret; }	
	5.9	HLD		25 int gcd(int a, int b) {	
	5.10	LCA	∠0	26   if(a == 0    b == 0) return 0;	

# 2. Data Structures

# 2.1. Dynamic Segment Tree

```
1  namespace DynamicSegmentTree{
     struct node {
       node *left, *right;
       //attributes of node
       node() {
         //initialize attributes
         left = NULL:
         right = NULL;
10
11
     };
12
13
     void combine(node *ans, node *left, node *right) {
14
       //combine operation
15
16
17
     void propagate(node * root, int 1, int r) {
18
       //check if exists lazy
19
20
       //apply lazy on node
21
22
        //propagate
       if(!root->left) root->left = new node();
23
24
       if(!root->right) root->right = new node();
25
26
       if(1 != r){
27
         //propagate operation
28
29
30
       //reset lazy
31
32
33
     void build(node *root, int 1, int r) {
34
       if(1 == r){
35
         //leaf operation
36
         return;
37
38
       int m = (1+r) >> 1;
39
       if(!root->left) root->left = new node();
40
       if(!root->right) root->right = new node();
41
       build(root->left, l, m);
42
       build(root->right, m+1, r);
43
       combine(root, root->left, root->right);
44
45
46
     void update(node *root, int 1, int r, int a, int b, int val) {
47
       propagate (root, 1, r);
48
       if(1 == a \&\& r == b) {
49
         //do lazy operation
50
         return;
51
52
       int m = (1+r) >> 1;
53
       if(!root->left) root->left = new node();
       if(!root->right) root->right = new node();
55
       if(b <= m) update(root->left, l, m, a, b, val);
       else if(m < a) update(root->right, m+1, r, a, b, val);
56
57
58
         update(root->left, 1, m, a, m, val);
59
         update(root->right, m+1, r, m+1, b, val);
60
61
       propagate (root, 1, r);
```

```
propagate(root->left, l, m);
62
63
       propagate(root->right, m+1, r);
64
       combine (root, root->left, root->right);
65
66
67
     node* guery(node *root, int 1, int r, int a, int b) {
68
       propagate (root, 1, r);
       if(1 == a \&\& r == b) {
69
70
         return root;
71
72
       int m = (1+r) >> 1;
73
       if(!root->left) root->left = new node();
       if(!root->right) root->right = new node();
74
75
       if (b <= m) return query(root->left, l, m, a, b);
76
       else if (m < a) return query (root->right, m+1, r, a, b);
77
       node *left = query(root->left, l,m,a,m);
78
       node *right = query(root->right, m+1, r, m+1, b);
79
       node *ans = new node();
80
       combine (ans, left, right);
81
       return ans:
82
83
84
```

# 2.2. Segment Tree

```
namespace SegmentTree{
2
3
     struct node{
        //attributes of node
       int lazv = 0;
       node() {}
     struct Tree{
10
       vector<node> st;
11
       Tree(){}
12
13
       Tree(int n) {
14
         st.resize(4*n);
15
16
17
       node combine (node a, node b) {
18
         node res;
19
         //combine operations
20
         return res;
21
22
23
       void propagate(int cur, int 1 , int r) {
24
          //return if there is no update
25
          //update tree using lazy node
26
         if(1 != r) {
27
            //propagate for left and right child
28
29
          //reset lazy node
30
31
32
       void build(int cur, int 1, int r) {
33
         if(1 == r) {
34
            //leaf operation
35
            return;
36
37
38
         int m = (1+r) >> 1;
```

```
build(2*cur, 1, m);
39
          build(2 \times \text{cur} + 1, m+1, r);
40
          st[cur] = combine(st[2*cur], st[2*cur+1]);
41
42
43
44
45
        void range_update(int cur, int 1, int r, int a, int b, long long val) {
          propagate(cur, 1, r);
46
47
          if(l == a \&\& r == b) {
48
            //lazy operation using val
49
            return;
50
51
52
          int m = (1+r)/2;
53
54
          if(b <= m) range_update(2*cur, 1, m, a, b, val);</pre>
55
          else if(m < a) range_update(2*cur+1, m+1, r, a, b, val);</pre>
56
57
            range_update(2*cur, 1, m, a, m, val);
58
            range_update(2*cur+1, m+1, r, m+1, b, val);
59
60
          propagate(cur, l , r);
61
62
          propagate (2*cur, 1, m);
63
          propagate (2*cur+1, m+1, r);
          st[cur] = combine(st[2*cur], st[2*cur+1]);
64
65
66
67
       node query(int cur, int 1, int r, int a, int b){
68
          propagate(cur, l, r);
69
          if(1 == a && r == b) return st[cur];
70
71
          int m = (1+r)/2;
72
          if(b <= m) return guery(2*cur, 1, m, a, b);
73
          else if (m < a) return query(2*cur+1, m+1, r, a, b);</pre>
74
          else {
75
            node left = query(2*cur, 1, m, a, m);
            node right = query (2*cur+1, m+1, r, m+1, b);
76
77
            node ans = combine(left, right);
78
            return ans:
79
80
81
     };
82
```

## 2.3. Fenwick Tree

```
struct BIT {
3
     vector<int> bit:
     BIT() {}
     int n:
     BIT(int n) {
10
       this->n = n;
       bit.resize(n+1);
11
12
13
     void update(int idx, int val){
14
15
       for (int i = idx; i \le n; i + = i\&-i) {
16
         bit[i]+=val;
```

```
18
19
20
     int prefix_query(int idx){
21
       int ans = 0;
22
        for (int i=idx; i>0; i -= i&-i) {
23
          ans += bit[i]:
2.4
25
       return ans;
2.6
27
28
     int query(int 1, int r){
29
        return prefix_query(r) - prefix_query(l-1);
30
31
32
     //int bit 0-1 it finds the index of k-th element active
     int kth(int k) {
33
34
       int cur = 0:
35
       int acc = 0;
36
        for(int i = 19; i >= 0; i--) {
37
         if (cur + (1 << i) > n) continue;
38
          if(acc + bit[cur + (1<<i)] < k) {
39
            cur += (1 << i);
40
            acc += bit[cur];
41
42
43
        return ++cur;
44
45
46
   };
```

## 2.4. Trie

```
namespace Trie{
2
3
     struct node {
4
       node *adj[SIZE_NODE];
5
          for(int i=0; i<SIZE_NODE; i++) adj[i] = NULL;</pre>
6
7
8
     };
9
10
     struct Tree {
11
12
       node *t;
13
14
        Tree(){
15
         t = new node();
16
17
18
        void add(){
19
         node *cur = t;
20
21
22
23
        int querv(){
24
          node *cur = t;
25
26
27
       void remove(){
28
          node *cur = t;
29
30
31
     };
```

```
32 |
33 | }
```

## 2.5. STL Ordered Set

```
//INCLUDES
  #include <ext/pb_ds/assoc_container.hpp>
3 #include <ext/pb_ds/tree_policy.hpp>
5 //NAMESPACE
6 using namespace __gnu_pbds;
8 typedef tree<
   int, //change for pair<int,int> to use like multiset
10 null type,
11 | less<int>, //change for pair<int, int> to use like multiset
12 rb tree tag,
13 | tree_order_statistics_node_update>
14 ordered_set;
15
16 //int differ = 0; for multiset
17
18 //ordered set myset; //declares a stl ordered set
19 //mvset.insert(1); //inserts
20 //myset.insert(make_pair(1, differ++)); //insertion for multiset
21 //myset.find by order(k)//returns an iterator to the k-th element (or
   //myset.order_of_key(x)//returns the number of elements strictly less than x
23 //myset.order_of_key(myset.lower_bound(make_pair(x, 0))) //for multisets
```

## 2.6. Convex Hull Trick

```
struct ConvexHullTrick {
     //max cht, suppose lines are added in crescent order of a
3
     ConvexHullTrick() {}
     struct line{
5
       int id, a, b;
       line() {}
6
7
       line(int id, int a, int b) : id(id), a(a), b(b) {}
8
9
     bool remove(line & a, line & b, line & c) {
10
       if((a.a - c.a) * (c.b - b.b) \le (b.a - c.a) * (c.b - a.b)) return true;
       else return false;
11
12
13
     vector<line> cht;
     void add(line & v) {
14
15
       if(cht.emptv()){
16
          cht.push_back(v);
17
18
       else {
19
         if(cht.back().a == v.a) return;
20
          while (cht.size() > 1 && remove (cht[cht.size()-2], cht.back(), v)) {
21
            cht.pop_back();
22
23
          cht.push_back(v);
24
25
26
27
     void preprocess_cht(vector< line > & v) {
28
       sort(v.begin(), v.end(), [](const line & a, const line & b){
29
         return (a.a < b.a || (a.a == b.a && a.b > b.b));
30
        });
31
       cht.clear();
```

```
for (int i=0; i<v.size(); i++) {</pre>
                                                                                                  void add(node * root, int 1, int r, line ln){
33
                                                                                                    if(!root->left) root->left = new node();
          add(v[i]);
                                                                                          34
34
                                                                                          35
                                                                                                    if(!root->right) root->right = new node();
35
                                                                                          36
                                                                                                    int m = (1+r) >> 1;
36
                                                                                          37
                                                                                                    bool left = ln.eval(1) < (root->ln).eval(1);
37
     int f(int i, int x) {
                                                                                          38
                                                                                                    bool mid = ln.eval(m) < (root->ln).eval(m);
38
        return cht[i].a*x + cht[i].b;
                                                                                          39
39
                                                                                          40
                                                                                                     if (mid) {
40
                                                                                          41
                                                                                                       swap(root->ln, ln);
41
     //return line index
                                                                                          42
42
     ii query(int x){
                                                                                          43
       if(cht.size() == 0) return ii(-INF,-INF);
if(cht.size() == 1) return ii(f(0, x), cht[0].id);
                                                                                                     if(1 == r) return;
43
                                                                                          44
44
                                                                                          45
                                                                                                     else if(left != mid) add(root->left, 1, m, ln);
        int l = 0, r = cht.size()-2;
45
                                                                                                     else add(root->right, m+1, r, ln);
                                                                                          46
                                                                                          47
46
        int ans= cht.size()-1;
        while(1 <= r){
47
                                                                                          48
48
         int m = (1+r)/2;
                                                                                          49
                                                                                                  int query(node * root, int 1, int r, int x){
         int y1 = f(m, x);
49
                                                                                          50
                                                                                                    if(!root->left) root->left = new node();
50
         int v2 = f(m+1, x);
                                                                                          51
                                                                                                    if(!root->right) root->right = new node();
         if (y1) >= y2) {
51
                                                                                          52
                                                                                                    int m = (1+r) >> 1;
52
            ans = m;
                                                                                          53
                                                                                                     if(l == r) return (root->ln).eval(x);
53
            r = m-1;
                                                                                          54
                                                                                                     else if(x < m) return min((root->ln).eval(x), query(root->left, l, m,
54
55
          else 1 = m+1:
                                                                                          55
                                                                                                     else return min((root->ln).eval(x), query(root->right, m+1, r, x));
56
                                                                                          56
57
        return ii(f(ans, x), cht[ans].id);
                                                                                          57
58
                                                                                          58
                                                                                                };
59
                                                                                          59
60
                                                                                          60
```

## 2.7. Lichao Segment Tree - Convex Hull Trick

32

```
namespace Lichao{
     //min lichao tree
     struct line {
       int a, b;
       line() {
         a = 0;
8
         b = INF:
9
       line(int a, int b) : a(a), b(b) {}
10
11
       int eval(int x){
12
         return a*x + b;
13
14
     };
15
16
     struct node {
17
       node * left, * right;
18
       line ln;
19
       node(){
20
         left = NULL;
21
         right = NULL;
22
23
     };
24
25
     struct Tree {
26
27
       node * root;
28
29
       Tree() {
30
         root = new node();
31
```

# 2.8. Lichao Segment Tree - Convex Hull Trick - Double Type

```
namespace Lichao{
     //min lichao tree working with doubles
     struct line {
       double a, b;
6
       line() {
         a = 0;
8
         b = INF;
9
10
        line (double a, double b) : a(a), b(b) {}
11
        double eval(double x) {
12
          return a*x + b;
13
14
     };
15
     struct node {
16
       node * left, * right;
17
       line ln;
18
19
       node(){
20
         left = NULL;
21
         right = NULL;
22
2.3
     };
25
     struct Tree {
26
27
       node * root;
28
29
       Tree() {
30
         root = new node();
31
32
```

```
void add(node * root, double 1, double r, line ln) {
34
         if(!root->left) root->left = new node();
35
         if(!root->right) root->right = new node();
36
         double m = (1+r)/2;
37
         bool left = ln.eval(l) < (root->ln).eval(l);
38
         bool mid = ln.eval(m) < (root->ln).eval(m);
39
40
         if (mid) {
           swap(root->ln, ln);
41
42
43
         if(abs(r-1) <= 1e-9) return;</pre>
44
45
         else if(left != mid) add(root->left, l, m, ln);
46
         else add(root->right, m, r, ln);
47
48
49
       double query(node * root, double 1, double r, double x) {
         if(!root->left) root->left = new node();
50
51
         if(!root->right) root->right = new node();
52
         double m = (1+r)/2.;
53
         if(abs(r-1) \le 1e-9) return (root->ln).eval(x);
54
          else if(x < m) return min((root->ln).eval(x), query(root->left, 1, m,
55
          else return min((root->ln).eval(x), query(root->right, m, r, x));
56
57
58
     };
59
```

# 2.9. Sparse Table

```
int spt[MAXN][LOGN];
   int e[MAXN];
   void spt_build(int *a, int n) {
4
        for (int i = 0; i < n; i++) {
            spt[i][0] = a[i];
6
7
8
9
       for (int i = 1; (1<<i) <= n; i++) {
10
            for (int j = 0; j+(1 << i) <= n; <math>j++) {
11
                spt[j][i] = min(spt[j][i-1], spt[j+(1<<(i-1))][i-1]);
12
13
14
15
       int k = 0:
16
        for (int j = 0; (1<<j) <= 2*n; j++) {
17
            for (; k \le n \&\& k < (1 << j); k++) {
18
                e[k] = j-1;
19
20
21
22
   int spt_rmq(int 1, int r) {
       return min(spt[l][e[sz]], spt[r-(1<<e[sz])+1][e[sz]]);
25
```

# 3. Uncategorized

# 3.1. Coordinate Compression

```
struct Compresser {
      vector<int> value;
      Compresser() {}
      Compresser(int n) {
8
        value.resize(n);
9
10
11
      void compress(vector<int> & v) {
12
        vector<int> tmp;
13
        set<int> s:
14
        for(int i=0; i<v.size(); i++) s.insert(v[i]);</pre>
15
        for(int x : s) tmp.pb(x);
16
        for(int i=0; i<v.size(); i++){</pre>
          int idx = lower_bound(tmp.begin(), tmp.end(), v[i]) - tmp.begin();
17
18
19
          v[i] = idx:
20
21
22
   } compresser;
```

# 3.2. Longest Increasing Subsequence

```
/* Use upper_bound to swap to longest non decreasing subsequence */
    struct LIS{
 3
     vector<int> seq;
     vector< ii > pointer;
      int sz;
8
      LIS() {}
1.0
     LIS(int n) {
11
        seq.resize(n+1);
12
        pointer.resize(n);
13
14
15
      void calculate(vector<int> & v) {
16
        int n = v.size();
        vector<int> aux(n+1);
17
18
        for(int i=1; i<=n; i++) {</pre>
19
          seq[i] = INT_MAX;
          aux[i] = -1;
20
21
22
        seq[0] = INT_MIN;
23
        aux[0] = -1;
24
        for(int i=0; i<n; i++) {</pre>
2.5
          int index = lower_bound(seq.begin(), seq.end(), v[i]) - seq.begin();
26
27
          if(seq[index+1] > v[i]){
28
            seq[index+1] = min(seq[index+1], v[i]);
29
            aux[index+1] = i;
30
31
          pointer[i] = ii(index+1, aux[index]);
32
33
        for(int i=n; i>=0; i--) {
34
          if(seq[i] != INT MAX) {
```

```
sz = i:
36
             break;
37
38
39
40
    };
```

## 3.3. LIS 2D

```
struct LIS2D{
2
     struct node {
       node *left, *right;
5
       int mx = (int) 1e18+1;
       node() {
        mx = (int) 1e18+1;
         left = NULL:
8
9
         right = NULL;
10
11
     };
12
13
     LIS2D() {}
14
15
     vector<node *> lis;
16
     int L,R,size;
17
18
     void combine(node *ans, node *left, node *right){
19
       if(left && right) ans->mx = min(left->mx, right->mx);
20
       else if(left) ans->mx = left->mx;
21
       else if(right) ans->mx = right->mx;
22
       else ans->mx = (int)1e18+1;
23
24
25
     void update(node *root, int 1, int r, int idx, int val){
26
       if(1 == r){
27
         root->mx = min(root->mx, val);
28
         return:
29
30
       int m = (1+r) >> 1;
31
       if(idx <= m) {
32
         if(!root->left) root->left = new node();
33
         update(root->left, l, m, idx, val);
34
35
36
         if(!root->right) root->right = new node();
37
         update(root->right, m+1, r, idx, val);
38
39
       combine(root, root->left, root->right);
40
41
42
     int query(node *root, int 1, int r, int a, int b) {
43
       if(l == a && r == b) {
44
         return root->mx;
45
46
       int m = (1+r) >> 1;
47
       if(b <= m){
         if(!root->left) return (int)1e18+1;
48
49
         else return query(root->left, l, m, a, b);
50
51
       else if (m < a) {
52
         if(!root->right) return (int)1e18+1;
53
         else return query(root->right, m+1, r, a, b);
54
55
       int left = (int) le18+1;
```

```
int right = (int)1e18+1;
57
        if(root->left) left = query(root->left, l,m,a,m);
58
        if(root->right) right = query(root->right, m+1, r, m+1, b);
59
        return min(left, right);
60
      bool check(int id, int x, int v) {
        int val = query(lis[id], L, R, L, x-1);
        return val < v;
      void calculate(vector< ii > & v) {
        int n = v.size();
        lis.resize(n+1);
        set<int> sx;
        vector<int> aux;
        for(int i=0; i<n; i++) {</pre>
          sx.insert(v[i].ff);
        for(int x : sx) aux.pb(x);
76
        L = -1, R = sx.size();
77
        for(int i=0; i<n; i++) {</pre>
78
          v[i].ff = lower_bound(aux.begin(), aux.end(), v[i].ff) - aux.begin();
79
80
        for(int i=0; i<=n; i++) {
81
          lis[i] = new node();
82
83
        update(lis[0], L, R, L, -(int)1e18-1);
84
        size = 0:
85
        for(ii par : v) {
          int x = par.ff, y = par.ss;
          int 1 = 0, r = n-1;
          int ans = 0;
          while(1 <= r)</pre>
            int m = (1+r) >> 1;
            if(check(m, x, v)){
              ans = m:
              1 = m+1;
            else r = m-1:
          size = max(size, ans+1);
          update(lis[ans+1], L, R, x, y);
99
100
     }
102
104 | int32_t main() {
105
     int n;
      scanf("%d", &n);
106
107
      vector< ii > v(n);
      set<int> sx;
108
109
      vector<int> aux;
      for(int i=0; i<n; i++) {
110
111
        scanf("%d%d", &v[i].ff, &v[i].ss);
      LIS2D lis2d:
114
      lis2d.calculate(v);
      printf("%d\n", lis2d.size);
116 }
```

## 3.4. Inversion Count - Merge Sort

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64 6.5 66

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```
1 | int mergesort_count (vector<int> & v) {
     vector<int> a,b;
      if(v.size() == 1) return 0;
      for(int i=0; i<v.size()/2; i++) a.push_back(v[i]);</pre>
     for(int i=v.size()/2; i<v.size(); i++) b.push_back(v[i]);</pre>
     int ans = 0;
      ans += mergesort_count(a);
     ans += mergesort_count(b);
     a.push_back(LLONG_MAX);
1.0
     b.push back (LLONG MAX);
     int x = 0, y = 0;
for(int i=0; i<v.size(); i++) {</pre>
11
12
13
        if(a[x] <= b[v]) {
14
          v[i] = a[x++];
15
16
        else
17
         v[i] = b[y++];
18
          ans += a.size() - x -1;
19
20
21
     return ans;
22
```

## 3.5. Mo's Decomposition

```
namespace Mos {
2
3
     int sqr;
4
     struct query{
       int id, l, r, ans;
        bool operator<(const query & b) const {</pre>
          if(l/sqr != b.l/sqr) return l/sqr < b.l/sqr;</pre>
9
          return (1/sqr) % 2 ? r > b.r : r < b.r;
10
11
     };
12
13
     struct QueryDecomposition {
14
15
       vector<query> q;
16
17
        QueryDecomposition(int n, int ng) {
18
          q.resize(nq);
19
          sqr = (int)sqrt(n);
20
21
22
       void read() {
23
24
25
26
        void add(int idx) {
27
28
        }
29
30
        void remove(int idx){
31
32
33
34
        int answer_query() {
35
36
37
38
        void calculate(){
39
          sort(q.begin(), q.end());
```

```
int 1 = 0, r = -1;
          for(int i=0; i<q.size(); i++){</pre>
41
42
            while(q[i].l < l) add(--1);
43
            while(r < q[i].r) add(++r);
44
            while (q[i].l > l) remove (l++);
45
            while (r > q[i].r) remove (r--);
46
            q[i].ans = answer_query();
47
48
49
50
        void print(){
          sort(q.begin(), q.end(), [](const query & a, const query & b){
51
52
            return a.id < b.id;</pre>
53
54
55
          for(query x : q){
56
            cout << x.ans << endl;
57
58
59
     };
60
61
```

- 4. Math and Number Theory
- 4.1. Diophantine Equations + CRT

```
namespace NT {
     int GCD(int a, int b) {
        if(a == 0) return b;
5
        else return GCD (b%a, a);
6
7
8
     tuple<int,int> ExtendedEuclidean(int a, int b) {
9
        //solves \ ax+by = gcd(a,b)
10
        //careful when a or b equal to 0
11
       if(a == 0) return make_tuple(0,1);
       int x,y;
12
1.3
        tie(x,y) = ExtendedEuclidean(b%a, a);
14
        return make_tuple(y - (b/a) *x, x);
15
16
     bool FailDiophantine = false;
17
18
19
      tuple<int,int> Diophantine(int a, int b, int c) {
       FailDiophantine = false;
20
21
        //finds a solution for ax+by = c
22
        //given a solution (x,y), all solutions have the form (x +
        m*(b/gcd(a,b)), y - m*(a/(gcd(a,b))), multiplied by (c/g)
23
24
       int q = GCD(a,b);
25
        if(q == 0 || c%q != 0) {
26
2.7
         FailDiophantine = true;
28
          return make_tuple(0,0);
29
30
31
        int x, y;
32
33
        tie(x,y) = ExtendedEuclidean(a, b);
34
        int s1 = x*(c/q), s2 = y*(c/q);
35
36
        //shifts solution
```

```
38
         int ans = -1;
 39
         while(1 <= r){
 40
           int m = (1+r) >> 1;
 41
           if (s2 + m*(a/q) >= 0) {
 42
            ans = m;
 43
             r = m-1:
 44
           else 1 = m+1;
 45
 46
 47
         if (ans !=-1) {
 48
          s1 = s1 - ans*(b/q);
 49
           s2 = s2 + ans*(a/q);
 50
 51
 52
        1 = 0, r = 1e9;
 53
         ans = -1;
 54
         while(1 <= r){
 55
          int m = (1+r) >> 1;
 56
          if(s1 + m*(a/q) >= 0){
 57
            ans = m;
 58
             r = m-1;
 59
 60
           else 1 = m+1:
 61
         if (ans !=-1) {
 62
 63
           s1 = s1 + ans*(b/q);
           s2 = s2 - ans * (a/q);
 64
 65
 66
 67
         1 = 0, r = 1e9;
 68
         ans = -1:
 69
         while(1 <= r){
 70
          int m = (1+r) >> 1;
 71
           if (s1 - m*(a/q) \le s2 + m*(b/q)) {
 72
             ans = m;
 73
             r = m-1:
 74
 75
           else l = m+1;
 76
 77
         if (ans !=-1) {
 78
          s1 = s1 - ans * (b/q);
 79
           s2 = s2 + ans*(a/q);
 80
 81
 82
         return make_tuple(s1, s2);
 83
 84
 85
      bool FailCRT = false;
 86
 87
       tuple<int, int> CRT(vector<int> & a, vector<int> & n) {
 88
         FailCRT = false:
 89
         for(int i=0; i<a.size(); i++) a[i] = mod(a[i], n[i]);</pre>
 90
         int ans = a[0]:
 91
         int modulo = n[0];
 92
 93
         for(int i=1; i<a.size(); i++){</pre>
 94
           int x,y;
 95
           tie(x,y) = ExtendedEuclidean(modulo, n[i]);
 96
           int g = GCD(modulo, n[i]);
 97
 98
           if(q == 0 || (a[i] - ans) %q != 0) {
 99
            FailCRT = true;
             return make_tuple(0,0);
100
101
```

int 1 = 0, r = 1e9;

# 4.2. Discrete Logarithm - Shanks Baby-Step Giant-Step

```
/* Baby-Step Giant-Step Shank's Algorithm */
3 namespace NT {
     int discrete_log(int a, int b, int p) {
       a %= p, b %= p;
7
       if(b == 1) return 0;
8
10
       int cnt = 0, t = 1;
11
        for(int g = gcd(a, p); g != 1; g = gcd(a, p)){
12
         if(b % g) return -1;
13
14
          p /= q, b /= q, t = t * a / q % p;
15
          cnt++;
16
17
          if(b == t) return cnt;
1.8
19
20
        map<int, int> hash;
21
        int m = (sqrt(p) + 1);
22
        int base = b;
23
        for(int i = 0; i != m; ++i){
24
25
         hash[base] = i;
26
          base = base * a % p;
27
28
29
       base = 1;
        for(int i=0; i<m; i++) {</pre>
30
31
          base = (base*a)%p;
32
33
34
       int cur = t;
35
        for(int i = 1; i <= m + 1; ++i) {</pre>
36
          cur = cur * base % p;
37
          if(hash.count(cur)) return i * m - hash[cur] + cnt;
38
39
       return -1;
40
41
42 }
```

## 4.3. Binomial Coefficient DP

```
/* Dynammic Programming for Binomial Coefficient Calculation */
   /* Using Stiefel Rule C(n, k) = C(n-1, k) + C(n-1, k-1) */
   int binomial(int n ,int k) {
     int c[n+10][k + 10];
     memset(c, 0 , sizeof c);
     c[0][0] = 1;
     for(int i = 1;i<=n;i++){</pre>
9
       for (int j = min(i, k); j>0; j--) {
         c[i][j] = c[i-1][j] + c[i-1][j-1];
10
11
12
1.3
     return c[n][k];
14
```

## 4.4. Erathostenes Sieve + Logn Prime Factorization

```
/* Erasthostenes Sieve Implementation + Euler's Totient */
   /* Calculate primes from 2 to N */
   /* lf[i] stores the lowest prime factor of i(logn factorization) */
   namespace NT {
     const int MAX_N = 1123456;
     bitset<MAX_N> prime;
     vector<int> primes;
10
     int lf[MAX N];
11
     int totient[MAX_N];
12
13
     void Sieve(int n) {
14
       for(int i=0; i<=n; i++) lf[i] = i;
15
       prime.set();
16
17
       prime[0] = false;
18
       prime[1] = false;
       for(int p = 2; p*p <= n; p++) {
19
20
         if(prime[p]){
21
            for(int i=p*p; i<=n; i+=p){
22
              prime[i] = false;
23
              lf[i] = min(lf[i], p);
24
25
26
27
        for (int i=2; i<=n; i++) if (prime[i]) primes.pb(i);</pre>
28
29
30
     void EulerTotient(int n) {
31
       for(int i=0; i<=n; i++) totient[i] = i;</pre>
32
       for(int p = 2; p <= n; p++) {
33
         if(totient[p] == p){
34
           totient[p] = p-1;
35
            for(int i=p+p; i<=n; i+=p){
36
             totient[i] = (totient[i]/p) * (p-1);
37
38
39
40
41
42
   };
```

# 4.5. Segmented Sieve

```
/* Segmented Erathostenes Sieve */
    /* Needs primes up to sqrt(N) - Use normal sieve to get them */
    namespace NT {
      const int MAX N = 1123456;
     bitset<MAX_N> prime;
     vector<int> primes;
9
      vector<int> seg_primes;
10
11
     void Sieve(int n) {
12
       prime.set();
1.3
        prime[0] = false;
14
        prime[1] = false;
15
        for (int p = 2; p*p \le n; p++) {
          if(prime[p]){
16
            for (int i=p*p; i<=n; i+=p) {</pre>
17
18
              prime[i] = false;
19
20
21
22
        for(int i=2; i<=n; i++) if(prime[i]) primes.pb(i);</pre>
23
24
25
      void SegmentedSieve(int 1, int r){
26
       prime.set();
27
        seg primes.clear();
28
        for(int p : primes) {
2.9
          int start = 1 - 1%p - p;
30
          while(start < 1) start += p;</pre>
31
          if(p == start) start += p;
          for(int i = start; i<=r; i+=p) {</pre>
32
33
            prime[i-l] = false;
3.4
35
36
        for (int i=0; i<r-l+1; i++) {</pre>
37
          if(prime[i] && l+i > 1) {
            seg_primes.pb(l+i);
38
39
40
41
42
43
```

# 4.6. Matrix Exponentiation

```
/* Matrix Exponentiation Implementation */
2
3
   struct Matrix{
     vector< vector<int> > m;
4
     Matrix() {}
     Matrix(int 1, int c) {
7
       m.resize(l, vector<int>(c));
8
9
10
     Matrix operator * (Matrix b) const {
       Matrix c(m.size(), b.m[0].size());
11
        for (int i = 0; i<m.size(); i++) {</pre>
12
13
          for(int j = 0; j<b.m[0].size(); j++) {</pre>
14
            for(int k = 0; k<b.m.size(); k++){</pre>
15
              c.m[i][j] += (m[i][k]*b.m[k][j]);
16
17
18
19
        return c;
20
21
22
23
     Matrix exp(int k) {
24
       if(k == 1) return *this;
25
       Matrix c = (*this).exp(k/2);
26
       c = c*c;
27
       if(k%2) c = c*(*this);
28
        return c;
29
30
   };
```

## 4.7. Fast Fourier Transform - Recursive and Iterative

```
/* Fast Fourier Transform Implementation */
   /* Complex numbers implemented by hand */
   /* Poly needs to have degree of next power of 2 (result poly has size
       next_pot2(2*n) */
   /* Uses Roots of Unity (Z^n = 1, divide and conquer strategy)
   /* Inverse FFT only changes to the conjugate of Primitive Root of Unity */
   /* Remember to use round to get integer value of Coefficients of Poly C */
   /* Iterative FFT is way faster (bit reversal idea + straightforward conquer
       for each block of each size) */
   /* std::complex doubles the execution time */
10
   namespace FFT{
11
12
     struct Complex{
13
       double a, b;
14
15
       Complex (double a, double b) : a(a), b(b) {}
16
17
       Complex(): a(0), b(0) {}
18
19
       Complex conjugate() const {
20
         return Complex(a, -b);
21
22
23
       double size2() const {
24
         return a*a + b*b;
25
26
27
       void operator=(const Complex & b) {
```

```
this->a = b.a;
    this->b = b.b;
  Complex operator+(const Complex & y) const {
    return Complex(a + v.a, b + v.b);
  Complex operator-(const Complex & y) const {
    return Complex(a - y.a, b - y.b);
  Complex operator*(const Complex & y) const {
    return Complex(a*y.a - b*y.b, a*y.b + b*y.a);
  Complex operator/(const double & x) const {
    return Complex(a/x, b/x);
  Complex operator/(const Complex & y) const {
    return (*this) * (y.conjugate()/y.size2());
} ;
struct Polv{
 vector<Complex> c:
 Poly() {}
 Poly(int n) {
    int sz = (31 - \_builtin\_clz(n)%32) + 1;
    c.resize((1 << (sz-1) == n ? n : (1 << sz)) << 1);
  int size() const{
    return (int)c.size();
inline Complex PrimitiveRootOfUnity(int n) {
  const double PI = acos(-1);
  return Complex(cos(2*PI/(double)n), sin(2*PI/(double)n));
inline Complex InversePrimitiveRootOfUnity(int n) {
  const double PI = acos(-1);
  return Complex(cos(-2*PI/(double)n), sin(-2*PI/(double)n));
void DFT(Polv & A, bool inverse) {
 int n = A.size();
 int lq = 0;
 while(n > 0) lg++, n>>=1;
 n = A.size();
 lq-=2;
  for(int i=0; i<n; i++) {</pre>
    int i = 0;
    for(int b=0; b <= lg; b++) {
      if(i & (1 << b)) j |= (1 << (lg - b));
    if(i < j) swap(A.c[i], A.c[j]);</pre>
  for(int len=2; len <= n; len <<= 1) {</pre>
    Complex w;
    if(inverse) w = InversePrimitiveRootOfUnity(len);
    else w = PrimitiveRootOfUnity(len);
```

28

29

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5.8

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87

88

89

90

91

92

```
94
           for(int i=0; i<n; i+=len) {</pre>
 95
             Complex x(1,0);
 96
             for(int j=0; j<len/2; j++) {</pre>
 97
               Complex u = A.c[i+j], v = x*A.c[i+j+len/2];
 98
                A.c[i+j] = u + v;
 99
                A.c[i+j+len/2] = u - v;
100
                x = x * w;
101
102
103
104
105
        if(inverse) for(int i=0; i<n; i++) A.c[i] = A.c[i]/n;
106
107
108
      /* Skipable */
109
      Poly RecursiveFFT (Poly A, int n, Complex w) {
110
        if(n == 1) return A;
111
112
        Poly A_even (n/2), A_odd (n/2);
113
         for(int i=0; i<n; i+=2){
114
115
          A even.c[i/2] = A.c[i];
           A_{odd.c[i/2]} = A.c[i+1];
116
117
118
119
         Poly F_even = RecursiveFFT(A_even, n/2, w*w);
         Poly F_odd = RecursiveFFT(A_odd, n/2, w*w);
120
121
        Poly F(n);
122
        Complex x(1, 0);
123
124
         for (int i=0; i<n/2; i++) {</pre>
125
          F.c[i] = F even.c[i] + x*F odd.c[i];
126
           F.c[i + n/2] = F even.c[i] - x*F odd.c[i];
127
           x = x * w;
128
129
130
         return F;
131
132
      /* Skipable */
133
134
      Poly Convolution (Poly & F_A, Poly & F_B) {
135
        Poly F_C(F_A.size()>>1);
136
         for(int i=0; i<F_A.size(); i++) F_C.c[i] = F_A.c[i]*F_B.c[i];</pre>
137
         return F C;
138
139
      Poly multiply (Poly & A, Poly & B) {
140
141
        DFT(A, false);
142
143
        DFT(B, false);
144
145
        Poly C = Convolution(A, B);
146
147
        DFT(C, true);
148
149
        return C:
150
151
152
```

## 4.8. Count Divisors in cbrt(n)

```
1 namespace NT{
```

```
int CountDivisors(int x) {
5
        int ans = 1;
6
        for(int i=2; i*i*i <= x; i++) {</pre>
7
          int cnt = 1;
8
          while (x\%i == 0) {
9
            cnt++;
            x/=i;
10
11
12
          ans *= cnt;
13
14
        if(PrimalityTest(x,15)) ans*=2;
15
        else if((int)sqrt(x)*(int)sqrt(x) == x && PrimalityTest((int)sqrt(x),
16
        15)) ans*=3;
17
        else if (x != 1) ans *=4;
18
19
        return ans;
20
21
22 }
```

# 4.9. Count Prime Factors in cbrt(n)

```
1 namespace NT {
      int CountPrimeFactors(int x) {
5
        int ans = 0:
6
        for(int i=2; i*i*i <= x; i++) {</pre>
7
          while (x\%i == 0) {
8
            ans++;
9
            x/=i;
1.0
11
12
13
        if(PrimalityTest(x, 10)) ans++;
14
        else if((int)sgrt(x)*(int)sgrt(x) == x && PrimalityTest((int)sgrt(x),
        10)) ans+=2;
15
        else if(x != 1) ans+=2;
16
17
        return ans;
18
19
20
21
```

# 4.10. Shank's Baby Step Giant Step

```
/* Baby-Step Giant-Step Shank's Algorithm */
namespace NT {
    int discrete_log(int a, int b, int p) {
        a %= p, b %= p;
        if(b == 1) return 0;
        int cnt = 0, t = 1;
        for(int g = gcd(a, p); g != 1; g = gcd(a, p)) {
            if(b % g) return -1;
        }
}
```

```
p /= q, b /= q, t = t * a / q % p;
15
          cnt++;
16
17
         if(b == t) return cnt;
18
19
20
       map<int, int> hash;
21
        int m = (sqrt(p) + 1);
22
        int base = b;
2.3
24
        for(int i = 0; i != m; ++i){
25
         hash[base] = i;
26
         base = base * a % p;
27
28
29
       base = 1;
       for (int i=0; i<m; i++) {</pre>
30
31
         base = (base*a)%p;
32
33
34
        int cur = t;
35
        for (int i = 1; i \le m + 1; ++i) {
36
          cur = cur * base % p;
37
          if (hash.count(cur)) return i * m - hash[cur] + cnt;
38
39
       return -1;
40
41
42
```

# 4.11. Diophantine Equations and CRT

```
namespace NT {
2
3
     int GCD(int a, int b) {
4
       if(a == 0) return b;
5
       else return GCD (b%a, a);
6
7
8
     tuple<int,int> ExtendedEuclidean(int a, int b) {
       //solves ax+by = gcd(a,b)
9
10
        //careful when a or b equal to 0
11
       if(a == 0) return make_tuple(0,1);
12
13
       tie(x,v) = ExtendedEuclidean(b%a, a);
14
       return make_tuple(y - (b/a)*x, x);
15
16
17
     bool FailDiophantine = false;
18
19
     tuple<int, int> Diophantine(int a, int b, int c) {
20
       FailDiophantine = false:
21
       //finds a solution for ax+by = c
22
       //given a solution (x,y), all solutions have the form (x +
       m*(b/qcd(a,b)), y - m*(a/(qcd(a,b))), multiplied by (c/q)
23
24
       int q = GCD(a,b);
25
26
       if(q == 0 || c%q != 0) {
27
         FailDiophantine = true;
28
         return make tuple(0,0);
29
30
31
       int x,y;
```

```
tie(x, y) = ExtendedEuclidean(a, b);
 int s1 = x*(c/q), s2 = y*(c/q);
  //shifts solution
  int 1 = 0, r = 1e9;
  int ans = -1;
 while(1 <= r){
   int m = (1+r) >> 1;
    if (s2 + m*(a/g) >= 0) {
     ans = m;
      r = m-1:
    else 1 = m+1;
 if (ans !=-1) {
   s1 = s1 - ans*(b/g);
    s2 = s2 + ans * (a/q);
 1 = 0, r = 1e9;
 ans = -1;
  while(1 <= r){
   int m = (1+r) >> 1;
    if (s1 + m*(a/q) >= 0) {
     ans = m;
      r = m-1;
    else 1 = m+1;
  if (ans != -1) {
   s1 = s1 + ans*(b/q);
    s2 = s2 - ans*(a/q);
 1 = 0, r = 1e9;
 ans = -1;
 while(1 <= r){
    int m = (1+r) >> 1;
    if (s1 - m*(a/q) \le s2 + m*(b/q)) {
      ans = m:
     r = m-1;
    else 1 = m+1:
  if (ans !=-1) {
    s1 = s1 - ans*(b/q);
    s2 = s2 + ans*(a/q);
 return make_tuple(s1, s2);
bool FailCRT = false:
tuple<int,int> CRT(vector<int> & a, vector<int> & n) {
 FailCRT = false:
  for(int i=0; i<a.size(); i++) a[i] = mod(a[i], n[i]);</pre>
 int ans = a[0];
 int modulo = n[0];
  for(int i=1; i<a.size(); i++){</pre>
    int x,y;
    tie(x,y) = ExtendedEuclidean(modulo, n[i]);
    int g = GCD (modulo, n[i]);
```

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96

```
98
           if(q == 0 || (a[i] - ans) %q != 0) {
 99
            FailCRT = true;
100
             return make_tuple(0,0);
101
102
           ans = mod(ans + (x*(a[i] - ans)/q)%(n[i]/q) * modulo, modulo*n[i]/q);
103
104
          modulo = modulo * n[i]/q;
105
106
107
        return make_tuple(ans, modulo);
108
109
110
```

# 4.12. Miller Rabin Primality Test

```
namespace NT {
2
3
     int mulmod(int a, int b, int c) {
4
        int x = 0, y=a%c;
5
        while(b > 0) {
          if(b%2 == 1){
            x = (x+y) %c;
10
11
12
          y = (y*2) %c;
          b /= \bar{2};
13
14
15
16
        return x%c;
17
18
19
     int expmod(int a, int k, int p) {
20
       if(k == 0) return 1;
21
       if(k == 1) return a;
22
23
       int aux = expmod(a, k/2, p);
24
        aux = mulmod(aux, aux, p);
25
26
       if(k%2) aux = mulmod(aux, a, p);
27
       return aux;
28
29
     bool PrimalityTest(int p, int iterations) {
30
31
        //Miller Rabin Primality Test
32
        mt19937 mt_rand(time(0));
33
34
       if(p < 2) return false;</pre>
35
       if(p == 2) return true;
36
        if(p%2 == 0) return false;
37
38
        int fixed s = p-1;
39
        while (fixed_s^{\circ}2 == 0) fixed_s /= 2;
40
41
        for(int iter = 0; iter < iterations; iter++) {</pre>
42
43
          int s = fixed_s;
44
45
          int a = mt_rand()%(p-1) + 1;
46
          int b = expmod(a, s, p);
47
```

```
while(s != p-1 && b != 1 && b != p-1) {
49
            b = mulmod(b, b, p);
50
            s *= 2;
51
52
53
          if (b != p-1 \&\& s%2 == 0) return false;
54
55
56
57
        return true;
58
59
60
61
```

## 4.13. Sum of Divisors in a Range

```
1 namespace NT {
       2
                                                  int SumOfDivisors(int a, int b) {
                                                                 int m = sqrt(b);
     5
                                                                    int s = 0;
                                                                     for (int f = 1; f <= m; f++) {</pre>
                                                                                      int x = (b/f) * (b/f) - max(m, (a-1)/f) * max(m, (a-1)/f) + (b/f) - 
                                                                                   s += f * (b/f - (a-1)/f);
       8
     9
                                                                                     s += x/2;
10
11
                                                                    return s;
12
13
14
```

# 5. String Algorithms

# 5.1. KMP Failure Function + String Matching

```
/* Knuth - Morris - Pratt Algorithm */
   struct KMP {
3
    vector<int> pi;
4
6
     vector<int> matches;
8
     KMP() {}
10
     void calculate(string t) {
11
       int n = t.size();
12
       pi.resize(n);
13
       pi[0] = 0;
14
        for(int i = 1; i < n; i++) {
15
         pi[i] = pi[i-1];
         while(pi[i] > 0 && t[i] != t[pi[i]]) pi[i] = pi[pi[i]-1];
16
         if(t[i] == t[pi[i]]) pi[i]++;
17
18
19
20
     void matching(string s) {
21
22
       int j = 0;
23
       int n = s.size();
24
        for(int i=0; i<n; i++) {</pre>
25
         while(j > 0 \&\& s[i] != t[j]) j = pi[j-1];
26
         if(s[i] == t[j]) j++;
```

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 $s[sa[x+1]+lcp[x]]){$ 

## 5.2. Z-Function

```
/* Z-function */
    /* Calculate the size K of the largest substring which is a prefix */
3
   struct ZFunction{
     vector<int> z;
     ZFunction() {}
     void calculate(string t) {
10
11
       int n = t.size();
       z.resize(n);
12
13
       z[0] = 0;
       int 1 = 0, r = 0;
14
15
       for (int i=1; i<n; i++) {</pre>
16
          if(i > r){
17
           1 = i;
18
            r = i;
19
20
          z[i] = min(z[i-1], r-i+1);
          while (i + z[i] < n \& \& t[i + z[i]] == t[z[i]]) z[i]++;
21
22
          if(i + z[i] > r) {
           1 = i;
23
24
           r = i + z[i]-1;
25
26
27
28
29
```

# 5.3. Suffix Array + Linear Sort

```
/* Suffix Array using Counting Sort Implementation */
   /* rnk is inverse of sa arrav */
   /* aux arrays are needed for sorting step */
   /* inverse sorting (using rotating arrays and blocks of power of 2) */
4
   /* rmq data structure needed for calculating lcp of two non adjacent
       suffixes sorted */
   struct SuffixArray{
9
     vector<int> rnk,tmp,sa, sa_aux, lcp, pot, sp[22];
1.0
11
     int block, n;
12
13
     string s;
14
15
     SuffixArray() {}
16
     SuffixArray(string t){
17
18
       s = t;
19
       n = t.size();
```

```
rnk.resize(n+1);
  for(int i=0; i<22; i++) sp[i].resize(n+1);</pre>
  pot.resize(n+1);
  tmp.resize(max(257LL, n+1));
  sa.resize(n+1);
  sa_aux.resize(n+1);
 lcp.resize(n+1);
 block = 0;
bool suffixcmp(int i, int j){
  if(rnk[i] != rnk[j]) return rnk[i] < rnk[j];</pre>
  i+=block, j+=block;
  i%=n;
  j%=n;
  return rnk[i] < rnk[j];</pre>
void suffixSort(int MAX VAL){
  for(int i=0; i<=MAX_VAL; i++) tmp[i] = 0;</pre>
  for(int i=0; i<n; i++) tmp[rnk[i]]++;</pre>
  for(int i=1; i<=MAX VAL; i++) tmp[i] += tmp[i-1];</pre>
  for (int i = n-1; i >= 0; i--) {
      int aux = sa[i]-block;
      aux%=n;
      if(aux < 0) aux+=n;
      sa_aux[--tmp[rnk[aux]]] = aux;
  for(int i=0; i<n; i++) sa[i] = sa_aux[i];</pre>
  tmp[0] = 0;
  for (int i=1; i < n; i++) tmp[i] = tmp[i-1] + suffix cmp(sa[i-1], sa[i]);
  for(int i=0; i<n; i++) rnk[sa[i]] = tmp[i];</pre>
void calculate() {
  s+='\0';
  for(int i=0; i<n; i++) {</pre>
    sa[i] = i;
    rnk[i] = s[i];
    tmp[i] = 0;
  suffixSort (256);
  block = 1:
  while (tmp[n-1] != n-1) {
    suffixSort(tmp[n-1]);
    block*=2;
  for(int i=0; i<n-1; i++) sa[i] = sa[i+1];</pre>
  n--;
  tmp[0] = 0;
  for (int i=1; i < n; i++) tmp[i] = tmp[i-1] + suffix cmp(sa[i-1], sa[i]);
  for(int i=0; i<n; i++) rnk[sa[i]] = tmp[i];</pre>
  s.pop_back();
  sa.pop_back();
void calculate_lcp() {
  int last = 0;
  for(int i=0; i<n; i++) {</pre>
    if(rnk[i] == n-1) continue;
    int x = rnk[i];
    lcp[x] = max(0LL, last-1);
    while(sa[x] + lcp[x] < n \&\& sa[x+1] + lcp[x] < n \&\& s[sa[x]+lcp[x]] ==
```

```
lcp[x]++;
 85
 86
           last = lcp[x];
 87
 88
 89
 90
       void build_lcp_table() {
 91
         int k = 0;
 92
         for (int j = 0; (1<<j) <= 2*n; j++) {
 93
           for(; k \le n \&\& k < (1 << \dot{j}); k++) {
 94
               pot[k] = j-1;
 95
 96
 97
         for (int i=0; i<n; i++) {</pre>
 98
           sp[0][i] = lcp[i];
 99
100
         for (int i = 1; (1<<i) <= n; i++) {</pre>
101
           for (int j = 0; j+(1 << i) <= n; <math>j++) {
102
             sp[i][j] = min(sp[i-1][j], sp[i-1][j+(1<<(i-1))]);
103
104
105
106
       int query_lcp(int x, int y) {
107
         if(x = y) return n - x;
108
109
         if(rnk[x] > rnk[y]) swap(x,y);
110
         int l = rnk[x], r = rnk[y]-1;
         return min(sp[pot[r-l+1]][1], sp[pot[r-l+1]][r-(1LL<<pot[r-l+1])+1]);
111
112
113
114
       int number_of_substrings() {
115
         int ans = n - sa[0];
116
         for (int i=0; i<n-1; i++) {</pre>
117
           int length = n - sa[i+1];
           ans += length - lcp[i];
118
119
120
         return ans;
121
122
123
```

#### 5.4. Rolling Hash

```
namespace Hash{
2
3
     int B1, B2, M1, M2;
4
     void init(){
       B1 = rand() %65536;
       B2 = rand() %65536;
       M1 = 1000000007;
9
       M2 = 1000000009;
10
11
12
     struct RollingHash{
13
14
       vector< ii > hash;
15
       vector< ii > base;
16
17
       RollingHash() {}
18
19
       void calculate(string s) {
20
         int n = s.size();
21
         hash.resize(n+1); base.resize(n+1);
```

```
base[0] = ii(1, 1);
23
          hash[0] = ii(0, 0);
24
          for (int i=1; i<=n; i++) {</pre>
25
           int val = (int)(s[i-1]);
26
           base[i] = ii \pmod{base[i-1].ff*B1, M1}, \mod{base[i-1].ss*B2, M2};
27
           hash[i] = ii (mod(hash[i-1].ff*B1 + val, M1), mod(hash[i-1].ss*B2 +
        val, M2));
28
29
3.0
31
       ii query(int 1, int r){
32
         ii ret;
33
          ret.ff = mod(hash[r].ff - hash[l-1].ff*base[r-l+1].ff, M1);
          ret.ss = mod(hash[r].ss - hash[l-1].ss*base[r-l+1].ss, M2);
34
35
          return ret;
36
37
38
     };
39
40
```

## 5.5. Aho-Corasick

```
map<char, int> *nxt;
   int *slinks;
   vector<int> *dlinks;
   void aho_build(const vector<string>& words) {
       int len_words = 1;
7
        for(const string& w : words) {
            len_words += w.size();
8
9
10
       nxt = new map<char, int>[len words];
11
       dlinks = new vector<int>[len words];
12
       int root = 0, fre = 1;
13
        for(int i = 0; i < words.size(); i++) {</pre>
14
            const string& w = words[i];
15
           int cur = root;
            for(const char& c : w) {
16
17
                if (nxt[cur].count(c) == 0) {
18
                    nxt[cur][c] = fre++;
19
                cur = nxt[cur][c];
20
21
22
            dlinks[cur].push_back(i);
23
24
25
        slinks = new int[len_words];
26
        slinks[0] = -1;
27
        queue<int> q;
28
        for(const pair<char, int>& ch : nxt[root]) {
29
            slinks[ch.second] = root;
30
           q.push (ch.second);
31
32
       while(!q.empty()) {
33
            const int cur = q.front();
34
35
            for(const pair<char, int>& ch : nxt[cur]) {
36
                int sl = slinks[cur];
37
                while(sl != root && nxt[sl].count(ch.first) == 0)
38
                    sl = slinks[sl];
39
                if (nxt[sl].count(ch.first) != 0)
40
                    sl = nxt[sl][ch.first];
41
                slinks[ch.second] = sl;
```

9

10

11

12

1.3

14

1.5

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64 65 66

67

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69

70

```
copy(dlinks[sl].begin(), dlinks[sl].end(),
42
        back inserter(dlinks[ch.second]));
43
                q.push (ch.second);
44
45
46
47
   vector< vector<int> > aho matches(const vector<string>& words, const string&
        text) {
48
        int root = 0;
       int cur = root;
49
       vector< vector<int> > matches(text.size());
50
51
        // vector< vector<int> > matches(words.size());
52
        for(int i = 0; i < text.size(); i++) {</pre>
53
            while(cur != root && nxt[cur].count(text[i]) == 0)
54
                cur = slinks[cur];
55
            if(nxt[cur].count(text[i]) != 0)
56
                cur = nxt[cur][text[i]];
57
58
            // returns matching words per position in text
59
            for(int w id : dlinks[cur]) {
                matches[i-words[w_id].size()+1].push_back(w_id);
60
61
62
            // // returns matching positions per word
63
64
            // for(int w_id : dlinks[cur]) {
65
                   matches[w_id].push_back(i-words[w_id].size()+1);
66
67
68
        return matches;
69
70
71
   int32_t main() {
72
       vector<string> words;
73
       words.push_back("he");
74
       words.push_back("hers");
75
       words.push back("his");
76
       words.push_back("she");
77
        string text = "heishers sheishis hihershe!";
78
        aho_build(words);
79
       vector< vector<int> > matches = aho_matches(words, text);
80
81
       for(int i = 0; i < matches.size(); i++) {</pre>
82
            cout << i;
83
            for(int id : matches[i]) {
84
                cout << " " << words[id];
85
86
            cout << endl;
87
        // for(int i = 0; i < matches.size(); i++) {</pre>
88
89
               cout << words[i];</pre>
90
               for(int p : matches[i]) {
91
                   cout << " " << p;
92
93
               cout << endl:
94
95
```

#### 5.6. Suffix Automata - Tested ??

```
#include <bits/stdc++.h>
using namespace std;
struct SuffixAutomaton {
```

```
vector< map<char, int> > nxt;
vector<int> slink;
vector<int> len;
int lstr:
int root;
vector<bool> is_terminal;
int slen;
vector<vector<int>> slink tree;
//vector<int> terminals;
SuffixAutomaton(const string& s) {
 slen = s.size();
  // add root
  nxt.push_back(map<char,int>());
 len.push_back(0);
  slink.push_back(-1);
  is_terminal.push_back(false);
 lstr = root = 0;
  for(int i = 0; i < s.size(); i++) {</pre>
    // add r
    nxt.push back(map<char, int>());
    len.push back(i+1);
    slink.push_back(0);
    is_terminal.push_back(false);
    int r = nxt.size()-1;
    // Find p (longest suffix of last r with edge with new character)
    int p = lstr;
    while (p \ge 0 \&\& nxt[p].count(s[i]) == 0) {
     // Add edge with new character
     nxt[p][s[i]] = r;
     p = slink[p];
    if (p !=-1) {
      // There is an suffix of last r that has edge to new character
      int q = nxt[p][s[i]];
      if(len[p] + 1 == len[q]) {
        // the longest suffix of new r is the logest of class q
        // There is no need to split
        slink[r] = q;
      } else {
        // Need to split
        // Add q'. New class that longest sufix of r and q.
        nxt.push_back(nxt[q]); // Copy from q
        len.push_back(len[p]+1);
        slink.push_back(slink[q]); // Copy from q
        is_terminal.push_back(false);
        int ql = nxt.size()-1;
        slink[q] = ql;
        slink[r] = ql;
        // q' will have every suffix of p that was previously conected to q
        while (p >= 0 && nxt[p][s[i]] == q) {
         nxt[p][s[i]] = ql;
         p = slink[p];
    lstr = r;
    if(i == s.size()-1) {
     p = r;
      while (p >= 0) {
        //terminals.push_back(p);
```

```
is_terminal[p] = true;
 72
               p = slink[p];
 73
 74
 75
 76
 77
 78
      bool is_substr(const string& s) {
 79
         int cur = root;
         for(int i = 0; i < s.size(); i++) {</pre>
 80
 81
           if(nxt[cur].count(s[i]) == 0) return false;
 82
           cur = nxt[cur][s[i]];
 83
 84
         return true;
 85
 86
      bool is_suffix(const string& s) {
 87
        int cur = root;
         for(int i = 0; i < s.size(); i++) {</pre>
 88
 89
           if(nxt[cur].count(s[i]) == 0) return false;
 90
           cur = nxt[cur][s[i]];
 91
 92
         if(is terminal[cur]) return true;
 93
        return false:
 94
 95
      void dfs_num_substr(int v, int *dp) {
 96
 97
         dp[v] = 1;
 98
         for(pair<char, int> ad : nxt[v]) {
 99
           if(dp[ad.second] == -1)
100
             dfs_num_substr(ad.second, dp);
101
           dp[v] += dp[ad.second];
102
103
104
      int num substr()
105
         int dp[nxt.size()];
106
         memset (dp, -1, sizeof dp);
107
         dfs_num_substr(root, dp);
108
        return dp[root]-1; // Remove empty substring
109
110
111
      void dfs_num_matches(int v, int *dp) {
        dp[v] = 0;
112
        if(is_terminal[v]) dp[v] = 1;
113
114
         for(pair<char, int> ad : nxt[v]) {
115
           if(dp[ad.second] == -1)
116
             dfs_num_matches(ad.second, dp);
117
           dp[v] += dp[ad.second];
118
119
120
      int num_matches(const string& s) {
121
        int cur = root;
122
         for(int i = 0; i < s.size(); i++) {</pre>
123
           if(nxt[cur].count(s[i]) == 0) return 0;
124
           cur = nxt[cur][s[i]];
125
126
         int dp[nxt.size()];
127
         memset (dp, -1, sizeof dp);
128
         dfs_num_matches(cur, dp);
129
         return dp[cur];
130
131
132
      void dfs_first_match(int v, int *dp) {
133
        dp[v] = 0;
134
         if(is_terminal[v]) dp[v] = 1;
         for(pair<char, int> ad : nxt[v]) {
```

```
if(dp[ad.second] == -1) {
136
137
             dfs first match (ad.second, dp);
138
             dp[v] = max(dp[v], dp[ad.second]+1);
139
140
141
142
      int first_match(const string& s) {
143
         int cur = root;
         for(int i = 0; i < s.size(); i++) {</pre>
144
145
           if(nxt[cur].count(s[i]) == 0) return -1;
146
           cur = nxt[cur][s[i]];
147
148
         int dp[nxt.size()];
149
         memset (dp, -1, sizeof dp);
        dfs_first_match(cur, dp);
150
151
         return slen-(dp[cur]-1)-s.size();
152
153
154
      void dfs all matches(int v, vector<int>& ans) {
155
         //cout << v << endl:
156
         if(slink tree[v].size()==0)
157
           ans.push back(len[v]);
158
         for(int ad : slink_tree[v])
159
           dfs_all_matches(ad, ans);
160
161
162
      vector<int> all_matches(const string& s) {
163
         slink_tree = vector<vector<int>> (slink.size());
164
         for(int i=0;i<slink.size();i++) {</pre>
165
           if(slink[i] >= 0) slink_tree[slink[i]].push_back(i);
166
167
         int cur = root;
         for(int i = 0; i < s.size(); i++) {</pre>
168
169
           if(nxt[cur].count(s[i]) == 0) return vector<int>();
170
           cur = nxt[cur][s[i]];
171
172
        vector<int> ans;
173
         dfs_all_matches(cur, ans);
174
         for(int i = 0; i < ans.size(); i++) {</pre>
           ans[i] -= s.size();
175
176
177
          // Last one is not valid
178
         return ans:
179
180
    };
181
182 | int main() {
183
      string s;
184
      cin >> s;
185
      SuffixAutomaton sa(s);
186
      cout << sa.num_substr() << endl;</pre>
187
     // cout << sa.terminals.size() << endl;</pre>
188
     // for(int ter: sa.terminals) cout << ter << " ";
189
     // cout << endl:
190
      int T;
191
      cin >> T;
192
      string w:
193
      while (T--)
194
        cin >> w;
195
        cout << sa.is_substr(w) << endl;</pre>
196
        cout << sa.is_suffix(w) << endl;</pre>
197
        cout << sa.num_matches(w) << endl;</pre>
198
        cout << sa.first_match(w) << endl;</pre>
199
         SuffixAutomaton sb(s+"$"+w);
200
         vector<int> matches = sb.all_matches(w);
```

```
201 | for(int i : matches) cout << i << " ";
202 | cout << endl;
203 |
204 |}
```

# 6. Geometry

## 6.1. 2D Structures

```
/////// Geometry Structures
   namespace Geo2D {
     struct Point {
       int x,y;
       Point(){
         x = 0;
         y = 0;
10
11
12
       Point(int x, int y) : x(x), y(y) {}
13
14
       Point (Point a, Point b) {
15
16
         x = b.x - a.x;
17
         y = b.y - a.y;
18
19
20
       Point operator+(const Point b) const{
21
         return Point(x + b.x, y + b.y);
22
23
24
       Point operator-(const Point b) const{
25
         return Point(x - b.x, y - b.y);
26
27
       int operator*(const Point b) const{
28
29
         return (x*b.x + y*b.y);
30
31
       int operator^(const Point b) const{
32
33
         return x*b.y - y*b.x;
34
35
36
       Point scale(int n) {
37
         return Point(x*n, y*n);
38
39
       void operator=(const Point b) {
40
41
         x = b.x;
         y = b.y;
42
43
44
       bool operator==(const Point b) {
45
         return x == b.x && y == b.y;
46
47
48
49
       double distanceTo(Point b){
50
         return sqrt ((x - b.x) * (x - b.x) + (y - b.y) * (y - b.y));
51
52
       int squareDistanceTo(Point b){
53
54
         return (x - b.x) * (x - b.x) + (y - b.y) * (y - b.y);
55
56
57
       bool operator<(const Point & p) const{</pre>
58
         return tie(x,y) < tie(p.x, p.y);</pre>
59
60
```

```
double size() {
 62
           return sqrt(x*x + v*v);
 63
 64
 65
         int squareSize(){
 66
           return x*x + y*y;
 67
 68
         //Only with double type
 69
 70
         Point normalize() {
 71
           return Point((double)x/size(), (double)y/size());
 72
 73
 74
         void rotate(double ang) {
 75
           double xx = x, yy = y;
 76
           x = xx*cos(ang) + yy*-sin(ang);
 77
           y = xx*sin(ang) + yy*cos(ang);
 78
 79
 80
       };
 81
       struct Line {
 82
 83
         Point p, q;
 84
         Point v;
 85
         Point normal;
 86
 87
         int a, b, c;
 88
 89
         Line() {
 90
         p = Point();
 91
           q = Point();
 92
          v = Point():
 93
           normal = Point();
 94
           a = 0;
 95
          b = 0:
 96
           c = 0;
 97
 98
 99
         Line(int aa, int bb, int cc) {
100
          a = aa;
101
          b = bb;
          c = cc;
102
           normal = Point(a,b);
103
           v = Point(-normal.y, normal.x);
           p = Point();
106
           q = Point();
107
108
109
         void operator=(const Line 1) {
110
          a = 1.a;
          b = 1.b;
111
112
          c = 1.c;
           p = 1.p;
113
114
           q = 1.q;
115
           \vec{v} = 1.\vec{v};
           normal = l.normal;
116
117
118
119
         Line(Point r, Point s) {
120
           p = r;
121
           q = s;
122
           v = Point(r, s);
123
           normal = Point(-v.y, v.x);
           a = -v.y;
124
125
           b = v.x;
```

```
126
           c = -(a*p.x + b*p.y);
127
128
         void flip_sign() {
129
130
           a = -a, b = -b, c = -c;
131
132
133
         void normalize() {
           if(a < 0) flip_sign();
134
135
           else if(a == 0 && b < 0) flip_sign();
136
           else if(a == 0 && b == 0 && c < 0) flip_sign();
           int g = max(a, max(b,c));
137
138
           if(a != 0) g = gcd(q, a); if(b != 0) g = gcd(q, b); if(c != 0) g =
         gcd(g,c);
139
           if (q > 0) a/=q, b/=q, c/=q;
140
141
142
         bool operator<(const Line & 1) const{</pre>
143
           return tie(a,b,c) < tie(1.a, 1.b, 1.c);
144
145
146
      };
147
       struct Circle{
148
149
        Point c;
         double r;
150
151
         Circle() {}
152
         Circle(Point center, double radius) : c(center), r(radius) {}
153
154
         bool operator=(Circle circ) {
155
          c = circ.c;
156
           r = circ.r;
157
158
         pair<Point, Point> getTangentPoints(Point p) {
159
160
           //p needs to be outside the circle
           double d = p.distanceTo(c);
161
           double ang = asin(1.*r/d);
162
163
           Point v1(p, c);
           v1.rotate(ang);
164
           Point v2(p, c);
165
166
           v2.rotate(-ang);
           v1 = v1.scale(sqrt(d*d - r*r)/d);
167
           v2 = v2.scale(sgrt(d*d - r*r)/d);
168
169
           Point p1(v1.x + p.x, v1.y + p.y);
170
           Point p2(v2.x + p.x, v2.y + p.y);
171
           return make_pair(p1,p2);
172
173
174
         double sectorArea(double ang) {
175
           return (ang*r*r)/2.;
176
177
178
         double arcLength(double ang) {
179
           return ang*r;
180
181
         double sectorArea (Point p1, Point p2) {
182
183
           double h = p1.distanceTo(p2);
184
           double ang = acos(1. - h*h/r*r);
185
           return sectorArea(ang);
186
187
188
         double arcLength (Point p1, Point p2) {
189
           double h = p1.distanceTo(p2);
```

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102

103

104

105

106

107

```
190
          double ang = acos(1. - (h*h)/(2*r*r));
191
          return arcLength (ang);
192
193
194
        bool inside(const Point & p) {
195
          if(Point(c,p).size() + EPS < r) return true;</pre>
196
          else if(r + EPS < Point(c,p).size()) return false;</pre>
197
          else return true:
198
199
200
      };
201
202
203
    //////// End of Geometry Structures
204
```

## 6.2. 2D Geometry Functions

```
/////// Geometry Algorithms
3
   namespace Geo2D {
4
5
     double distancePointLine(Point p, Line 1) {
       if(l.normal.squareSize() == 0) return INF;
7
       return (double) (1.*abs(l.a*p.x + l.b*p.y + l.c))/l.normal.size();
8
9
     double distancePointSegment(Point p, Line 1) {
10
11
       int dot1 = Point(l.p, p)*Point(l.p, l.g);
12
       int dot2 = Point(1.q, p) *Point(1.q, 1.p);
13
       if(dot1 >= 0 && dot2 >= 0) return distancePointLine(p, 1);
14
15
       else return min(p.distanceTo(l.p), p.distanceTo(l.q));
16
17
18
     double distancePointRay(Point p, Line 1) {
       int dot = Point(l.p, p)*l.v;
19
2.0
       if (dot >= 0) return distancePointLine(p, 1);
21
       else return p.distanceTo(l.p);
22
23
24
     Point closestPointInSegment (Point p, Line s) {
25
       //returns closest point from p in segment s
26
       Point u = s.v.normalize();
       Point w(s.p, p);
27
28
       Point res = u.scale(u*w);
29
       if(u*w < 0 || u*w > s.p.distanceTo(s.q)){
30
         if (p.distanceTo(s.p) < p.distanceTo(s.q)) return s.p;</pre>
31
         else return s.g:
32
33
       else return Point(s.p.x + res.x, s.p.y + res.y);
34
3.5
     Point intersectionSegmentSegment(Line s1, Line s2) {
36
37
       //Assumes that intersection exists
38
       //Assuming that endpoints are ordered by x
39
       if (s1.p.x > s1.q.x) swap(s1.p, s1.q);
40
       if (s2.p.x > s2.q.x) swap(s2.p, s2.q);
41
42
       if(abs(s1.v^s2.v) <= EPS){
43
44
         //parallel segments
```

```
Point v1(s2.p, s1.p);
    if(s1.p.x == s1.q.x \&\& s2.p.x == s2.q.x \&\& s1.p.x == s2.p.x)
     Point ansl, ansr:
      if(s1.p.y > s1.q.y) swap(s1.p, s1.q);
     if(s2.p.y > s2.q.y) swap(s2.p, s2.q);
     if(s1.p.y <= s2.p.y) ansl = s2.p;
      else ansl = s1.p;
     if(s2.q.y <= s1.q.y) ansr = s2.q;
      else ansr = s1.q;
      if(ansl.x == ansr.x && ansl.y == ansr.y){
        //cout << ansr.x << " " << ansr.y << endl;
        return Point(ansr.x, ansr.y);
      else {
        if(ansl.x == ansr.x && ansl.y > ansr.y) swap(ansl, ansr);
        //cout << ansl.x << " " << ansl.y << endl << ansr.x << " " <<
  ansr.y << endl:
        return Point(INF, INF);
    else if (abs(s1.v^v1) <= EPS) {
     Point ansl, ansr;
      if(s1.p.x \le s2.p.x) ansl = s2.p;
      else ansl = s1.p;
     if(s2.q.x \le s1.q.x) ansr = s2.q;
      else ansr = s1.q;
      if(ansl.x == ansr.x && ansl.y == ansr.y) {
       //cout << ansr.x << " " << ansr.y << endl;
        return Point(ansr.x, ansr.y);
        if(ansl.x == ansr.x && ansl.y > ansr.y) swap(ansl, ansr);
        //cout << ansl.x << " " << ansl.y << endl << ansr.x << " " <<
  ansr.v << endl;
        return Point(INF, INF);
  else {
    //general case
    int a1 = s1.q.y - s1.p.y;
    int b1 = s1.p.x - s1.q.x;
    int c1 = a1 \times s1.p.x + b1 \times s1.p.y;
    int a2 = s2.q.y - s2.p.y;
    int b2 = s2.p.x - s2.q.x;
    int c2 = a2*s2.p.x + b2*s2.p.y;
    int det = a1*b2 - a2*b1;
    double x = (double) (b2*c1 - b1*c2) / (double) det*1.;
    double y = (double) (a1*c2 - a2*c1) / (double) det*1.;
    //cout << x << " " << y << endl;
    return Point(x, v);
double distanceSegmentSegment(Line 11, Line 12) {
 if(l1.p == 12.p && l1.q == 12.q) return 0;
  if (11.q == 12.p && 11.p == 12.q) return 0;
 if((11.v^12.v) != 0){
    Line r1(11.p, 11.q);
    Line r2(11.q, 11.p);
    Line r3(12.p, 12.q);
```

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221

222

223

224

```
108
          Line r4(12.q, 12.p);
109
110
          int cross1 = (Point(r3.p, r1.p)^r3.v);
111
          int cross2 = (Point(r3.p, r1.g)^r3.v);
          if(cross2 < cross1) swap(cross1, cross2);</pre>
112
113
          bool ok1 = (cross1 <= 0 && cross2 >= 0) || (distancePointLine(r1.p,
114
         r3) > distancePointLine(r1.q, r3));
115
          cross1 = (Point(r1.p, r3.p)^r1.v);
116
          cross2 = (Point(r1.p, r3.q)^r1.v);
117
118
          if(cross2 < cross1) swap(cross1, cross2);</pre>
119
120
          bool ok2 = (cross1 <= 0 && cross2 >= 0) || (distancePointLine(r3.p.
         r1) > distancePointLine(r3.g, r1));
121
122
          cross1 = (Point(r3.p, r2.p)^r3.v);
123
          cross2 = (Point(r3.p, r2.q)^r3.v);
          if(cross2 < cross1) swap(cross1, cross2);</pre>
124
125
          bool ok3 = (cross1 <= 0 && cross2 >= 0) || (distancePointLine(r2.p,
126
         r3) > distancePointLine(r2.q, r3));
127
128
          cross1 = (Point(r2.p, r3.p)^r2.v);
129
          cross2 = (Point(r2.p, r3.q)^r2.v);
130
          if(cross2 < cross1) swap(cross1, cross2);</pre>
131
          bool ok4 = (cross1 <= 0 && cross2 >= 0) || (distancePointLine(r3.p,
132
         r2) > distancePointLine(r3.q, r2));
133
          cross1 = (Point(r4.p, r1.p)^r4.v);
134
135
          cross2 = (Point(r4.p, r1.q)^r4.v);
136
          if(cross2 < cross1) swap(cross1, cross2);</pre>
137
138
          bool ok5 = (cross1 <= 0 && cross2 >= 0) || (distancePointLine(r1.p,
         r4) > distancePointLine(r1.q, r4));
139
140
          cross1 = (Point(r1.p, r4.p)^r1.v);
141
          cross2 = (Point(r1.p, r4.q)^r1.v);
142
          if(cross2 < cross1) swap(cross1, cross2);</pre>
143
144
          bool ok6 = (cross1 <= 0 && cross2 >= 0) || (distancePointLine(r4.p,
         r1) > distancePointLine(r4.q, r1));
145
146
          cross1 = (Point(r4.p, r2.p)^r4.v);
147
          cross2 = (Point(r4.p, r2.q)^r4.v);
          if(cross2 < cross1) swap(cross1, cross2);</pre>
148
149
150
          bool ok7 = (cross1 <= 0 && cross2 >= 0) || (distancePointLine(r2.p,
         r4) > distancePointLine(r2.g, r4));
151
          cross1 = (Point(r2.p, r4.p)^r2.v);
152
153
          cross2 = (Point(r2.p, r4.q)^r2.v);
154
          if(cross2 < cross1) swap(cross1, cross2);</pre>
155
          bool ok8 = (cross1 <= 0 && cross2 >= 0) || (distancePointLine(r4.p,
156
         r2) > distancePointLine(r4.q, r2));
157
158
          if (ok1 && ok2 && ok3 && ok4 && ok5 && ok6 && ok7 && ok8) return 0;
159
160
161
162
         double ans = distancePointSegment(11.p, 12);
        ans = min(ans, distancePointSegment(11.q, 12));
```

```
ans = min(ans, distancePointSegment(12.p, 11));
 ans = min(ans, distancePointSegment(12.g, 11));
 return ans:
double distanceSegmentRay(Line s, Line r) {
 if((s.v^r.v) != 0){
    Line r1(s.p, s.q);
    Line r2(s.q, s.p);
    int cross1 = (Point(r.p, r1.p)^r.v);
    int cross2 = (Point(r.p, r1.q)^r.v);
    if(cross2 < cross1) swap(cross1, cross2);</pre>
    bool ok1 = (cross1 <= 0 && cross2 >= 0) || (distancePointLine(r1.p, r)
  > distancePointLine(r1.q, r));
    cross1 = (Point(r1.p, r.p)^r1.v);
    cross2 = (Point(r1.p, r.q)^r1.v);
    if(cross2 < cross1) swap(cross1, cross2);</pre>
    bool ok2 = (cross1 \le 0 \&\& cross2 \ge 0) \mid | (distancePointLine(r.p, r1))
  > distancePointLine(r.q, r1));
    cross1 = (Point(r.p, r2.p)^r.v);
    cross2 = (Point(r.p, r2.q)^r.v);
    if(cross2 < cross1) swap(cross1, cross2);</pre>
    bool ok3 = (cross1 <= 0 && cross2 >= 0) || (distancePointLine(r2.p, r)
  > distancePointLine(r2.q, r));
    cross1 = (Point(r2.p, r.p)^r2.v);
    cross2 = (Point(r2.p, r.q)^r2.v);
    if(cross2 < cross1) swap(cross1, cross2);</pre>
    bool ok4 = (cross1 \le 0 \&\& cross2 \ge 0) \mid | (distancePointLine(r.p, r2))
  > distancePointLine(r.q, r2));
    if (ok1 && ok2 && ok3 && ok4) return 0;
  double ans = INF;
  int dot = Point(s.p, r.p)*Point(r.p, s.q);
  if(dot >= 0) ans = min(ans, distancePointLine(r.p, s));
  else ans = min(ans, min(r.p.distanceTo(s.p), r.p.distanceTo(s.q)));
 dot = Point(r.p, s.p) *r.v;
 if (dot >= 0) ans = min(ans, distancePointLine(s.p, r));
 else ans = min(ans, r.p.distanceTo(s.p));
 dot = Point(r.p, s.q) *r.v;
 if(dot >= 0) ans = min(ans, distancePointLine(s.q, r));
 else ans = min(ans, r.p.distanceTo(s.q));
  return ans;
double distanceSegmentLine(Line s, Line 1) {
 if((s.v^l.v) == 0){
   return distancePointLine(s.p, 1);
 int cross1 = (Point(l.p, s.p)^l.v);
```

```
int cross2 = (Point(l.p, s.q)^l.v);
                                                                                       286
        if(cross2 < cross1) swap(cross1, cross2);</pre>
226
                                                                                       287
227
        if(cross1 <= 0 && cross2 >= 0) return 0;
228
        else return min(distancePointLine(s.p, 1), distancePointLine(s.q,1));
                                                                                       288
229
                                                                                       289
230
                                                                                       290
231
                                                                                                = c2.r;
      double distanceLineRay(Line 1, Line r) {
                                                                                        291
232
        if((1.v^r.v) == 0)
                                                                                                double r1sqr = c1.r*c1.r;
233
                                                                                       292
2.34
          return distancePointLine(r.p, 1);
                                                                                       293
                                                                                                double r2sqr = c2.r*c2.r;
                                                                                                double dsqr = d*d;
235
                                                                                       294
                                                                                       295
236
237
        int cross1 = (Point(l.p, r.p)^l.v);
                                                                                       296
        int cross2 = (Point(l.p, r.q)^l.v);
238
                                                                                       297
        if(cross2 < cross1) swap(cross1, cross2);</pre>
239
                                                                                       298
        if((cross1 <= 0 && cross2 >= 0) || (distancePointLine(r.p, 1) >
240
                                                                                       299
         distancePointLine(r.q, 1))) return 0;
                                                                                       300
        return distancePointLine(r.p, 1);
                                                                                       301
                                                                                                return area1 + area2;
241
242
                                                                                       302
                                                                                       303
243
      double distanceLineLine(Line 11, Line 12){
                                                                                       304
244
        if((11.v^12.v) == 0){
                                                                                       305
245
                                                                                       306
                                                                                                //NOT TESTED!!!!!!!!
246
           return distancePointLine(11.p, 12);
                                                                                       307
                                                                                                //no intersection
247
                                                                                       308
248
        else return 0;
                                                                                       309
249
                                                                                       310
250
                                                                                                //one intersection
251
      double distanceRayRay(Line r1, Line r2) {
                                                                                       311
        if((r1.v^r2.v) != 0){
                                                                                       312
252
253
                                                                                       313
                                                                                                  vector<Point> ret;
254
           int cross1 = (Point(r1.p, r2.p)^r1.v);
                                                                                       314
                                                                                                  ret.pb(Point(x0, y0));
           int cross2 = (Point(r1.p, r2.q)^r1.v);
2.5.5
                                                                                       315
                                                                                                  return ret:
256
          if(cross2 < cross1) swap(cross1, cross2);</pre>
                                                                                       316
          bool ok1 = (cross1 <= 0 && cross2 >= 0) || (distancePointLine(r2.p,
                                                                                       317
257
                                                                                                //general case
         r1) > distancePointLine(r2.g, r1));
                                                                                       318
                                                                                       319
2.58
                                                                                       320
259
           cross1 = (Point(r2.p, r1.p)^r2.v);
                                                                                       321
           cross2 = (Point(r2.p, r1.q)^r2.v);
260
                                                                                       322
261
          if(cross2 < cross1) swap(cross1, cross2);</pre>
                                                                                       323
262
                                                                                       324
263
          bool ok2 = (cross1 <= 0 && cross2 >= 0) || (distancePointLine(r1.p.
                                                                                       325
                                                                                                vector<Point> ret;
         r2) > distancePointLine(r1.g, r2));
                                                                                       326
                                                                                                ret.pb(p1); ret.pb(p2);
264
265
          if(ok1 && ok2) return 0;
                                                                                        327
                                                                                                return ret:
266
                                                                                       328
                                                                                       329
267
                                                                                       330
268
                                                                                       331
                                                                                                //NOT TESTED!!!!!!!!
269
         double ans = INF:
                                                                                       332
        int dot = Point(r2.p, r1.p) *r2.v;
270
                                                                                       333
                                                                                                Point translation = c1.c;
271
        if(dot >= 0) ans = min(ans, distancePointLine(r1.p, r2));
                                                                                        334
                                                                                                c1.c = Point(0,0);
272
        else ans = min(ans, r2.p.distanceTo(r1.p));
                                                                                        335
                                                                                                c2.c = c2.c - translation;
273
                                                                                       336
274
        dot = Point(r1.p, r2.p)*r1.v;
                                                                                       337
275
        if (dot >= 0) ans = min(ans, distancePointLine(r2.p, r1));
                                                                                       338
                                                                                                if(c1.c == c2.c){
276
        else ans = min(ans, r1.p.distanceTo(r2.p));
                                                                                       339
277
278
        return ans;
                                                                                       340
                                                                                                  if(c1.r == c2.r){
279
280
                                                                                       341
                                                                                                    vector<Point> ret;
                                                                                       342
                                                                                                    ret.pb(Point());
281
                                                                                       343
                                                                                                    ret.pb(Point());
282
      double circleCircleIntersection(Circle c1, Circle c2){
                                                                                       344
                                                                                                    ret.pb(Point());
283
                                                                                       345
                                                                                                    return ret:
284
        if((c1.r+c2.r)*(c1.r+c2.r) \le (c2.c.x-c1.c.x)*(c2.c.x-c1.c.x) +
                                                                                       346
         (c2.c.y-c1.c.y)*(c2.c.y-c1.c.y))
                                                                                       347
                                                                                                  else return vector<Point>();
          return 0:
```

```
if((c1.r-c2.r)*(c1.r-c2.r) >= (c2.c.x-c1.c.x)*(c2.c.x-c1.c.x) +
  (c2.c.y-c1.c.y) * (c2.c.y-c1.c.y)) {
    return PI*min(c1.r, c2.r)*min(c1.r, c2.r);
  double x1 = c1.c.x, x2 = c2.c.x, y1 = c1.c.y, y2 = c2.c.y, r1 = c1.r, r2
  double d = sqrt((x2-x1)*(x2-x1) + (y2-y1)*(y2-y1));
  double alpha1 = acos(((c1.r + c2.r) *(c1.r - c2.r) + dsqr)/(2.*d*r1)); double alpha2 = acos(((c2.r + c1.r) *(c2.r - c1.r) + dsqr)/(2.*d*r2));
  double area1 = r1sgr*(alpha1 - sin(alpha1)*cos(alpha1));
  double area2 = r2sqr*(alpha2 - sin(alpha2)*cos(alpha2));
vector<Point> intersectionLineCircle(Line 1, Circle circ){
  if((1.c*1.c)/(circ.r*circ.r) > 1.a*1.a + 1.b*1.b) return vector<Point>();
  double x0 = -1.a*1.c/(1.a*1.a+1.b*1.b), y0 = -1.b*1.c/(1.a*1.a+1.b*1.b);
  if(abs((l.c*l.c)/(circ.r*circ.r) - (l.a*l.a + l.b*l.b)) \le EPS){
  double d = circ.r*circ.r - (l.c*l.c)/(l.a*l.a+l.b*l.b);
  double mult = sqrt(d/(1.a*1.a+1.b*1.b));
  Point p1(x0 + 1.b*mult, y0 - 1.a*mult);
 Point p2(x0 - 1.b*mult, y0 + 1.a*mult);
vector<Point> intersectionCircleCircle(Circle c1, Circle c2){
  //translate first circle to origin
  //check if centers are equal
    //if radius are equal = infinite intersections (return 3 points to
  indicate), else = no intersection(empty)
```

```
349
350
351
        Line 1(-2*c2.c.x, -2*c2.c.y, c2.c.x*c2.c.x + c2.c.y*c2.c.y + c1.r*c1.r -
         c2.r*c2.r);
352
353
        vector<Point> ret = intersectionLineCircle(1, c1);
354
355
        for(Point & p : ret){
          p = p + translation;
356
357
358
359
        return ret;
360
361
362
      Point barycenter (Point & a, Point & b, Point & c, double pA, double pB,
         double pC) {
        Point ret = (a.scale(pA) + b.scale(pB) + c.scale(pC));
363
364
        ret.x /= (pA + pB + pC);
365
        ret.y /= (pA + pB + pC);
366
        return ret:
367
368
369
      Point circumcenter (Point & a, Point & b, Point & c) {
370
         double pA = Point(b,c).squareSize(), pB = Point(a,c).squareSize(), pC =
         Point(a,b).squareSize();
371
         return barycenter(a,b,c, pA*(pB+pC-pA), pB*(pC+pA-pB), pC*(pA+pB-pC));
372
373
374
      Point centroid (Point & a, Point & b, Point & c) {
375
        return barycenter(a,b,c,1,1,1);
376
377
378
      Point incenter (Point & a, Point & b, Point & c) {
379
         return barycenter(a,b,c, Point(b,c).size(), Point(a,c).size(),
         Point(a,b).size());
380
381
382
      Point excenter (Point & a, Point & b, Point & c) {
383
        return barycenter(a,b,c, -Point(b,c).size(), Point(a,c).size(),
         Point(a,b).size());
384
385
386
      Point orthocenter (Point & a. Point & b. Point & c) {
387
         double pA = Point(b,c).squareSize(), pB = Point(a,c).squareSize(), pC =
         Point(a,b).squareSize();
        return barycenter(a, b, c, (pA+pB-pC)*(pC+pA-pB), (pB+pC-pA)*(pA+pB-pC),
388
         (pC+pA-pB)*(pB+pC-pA));
389
390
391
      Circle minimumCircle(vector<Point> & v) {
392
        Circle circ(Point(0,0), 1e-14);
        random_shuffle(v.begin(), v.end());
393
394
        for (int i=0; i<v.size(); i++) {</pre>
395
          if(!circ.inside(v[i])){
396
             circ = Circle(v[i], 0);
397
             for(int j=0; j<i; j++) {</pre>
398
               if(!circ.inside(v[j])){
399
                 circ = Circle((v[i] + v[j]).scale(0.5), Point(v[i],
         v[j]).size()*0.5);
400
                 for (int k = 0; k < \dot{\gamma}; k++) {
401
                   if(!circ.inside(v[k])){
402
                     Point center = circumcenter(v[i], v[j], v[k]);
403
                     circ = Circle(center, Point(center, v[k]).size());
404
```

```
return circ;
long long ClosestPairOfPoints(vector<Point> &a) {
  //returns square of distance
  long long mid = a[a.size()/2].x;
 int n = a.size();
  vector<Point> 1;
 vector<Point> r;
 int i = 0;
  for(; i < a.size()/2; i++) l.push_back(a[i]);</pre>
  for(; i < a.size(); i++) r.push_back(a[i]);</pre>
 long long d = LLONG_MAX;
  if(l.size() > 1) {
     d = min(d, ClosestPairOfPoints(1));
  } if(r.size() > 1) {
    d = min(d, ClosestPairOfPoints(r));
 a.clear();
 vector<Point> 11;
 vector<Point> rr;
 int i = 0;
  i = 0;
  for(int k=0; k<n; k++){
    if(i < l.size() && j < r.size()){
      if(r[j].y <= l[i].y) {
        if((r[j].x - mid) * (r[j].x - mid) < d) {
          rr.push_back(r[j]);
        a.push_back(r[j++]);
        if((l[i].x - mid) * (l[i].x - mid) < d) {</pre>
          ll.push_back(l[i]);
        a.push_back(l[i++]);
    else if(i < l.size()){</pre>
      if((l[i].x - mid) * (l[i].x - mid) < d) {
        ll.push_back(l[i]);
      a.push_back(l[i++]);
    else
      if((r[j].x - mid) * (r[j].x - mid) < d) {
          rr.push_back(r[j]);
        a.push_back(r[j++]);
  for(int i = 0; i < ll.size(); i++) {</pre>
```

407

408

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410

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412

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461

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465

466

467

468 469

```
470
471
          int ini = 0, end = rr.size()-1;
472
          int j;
473
474
          while(ini < end) {</pre>
475
            j = (ini + end) / 2;
476
            if((rr[j].y - ll[i].y)*(rr[j].y - ll[i].y) > d && rr[j].y < ll[i].y)
477
              ini = j+1;
478
            else end = j;
479
480
481
          j = ini;
482
          for(; j < rr.size(); j++) {</pre>
483
            if((rr[j].y - ll[i].y)*(rr[j].y - ll[i].y) > d) break;
484
485
            long long cur = (ll[i].x - rr[j].x)*(ll[i].x - rr[j].x) + (ll[i].y)
          rr[j].y)*(ll[i].y - rr[j].y);
486
            if(cur < d) {
487
              d = cur;
488
489
490
491
        return d:
492
493
494
495
    /////// End of Geometry Algorithms
```

## 6.3. Convex Hull - Monotone Chain Algorithm

```
namespace Geo2D
3
     struct ConvexHull {
4
5
       vector< Point > points, lower, upper;
6
7
       ConvexHull(){}
8
9
       void calculate(vector<Point> v) {
10
          sort(v.begin(), v.end());
          for(int i=0; i<v.size(); i++){</pre>
11
12
            while(upper.size() >= 2 && (Point(upper[upper.size()-2],
        upper.back()) Point(upper.back(), v[i])) >= OLL) upper.pop_back();
13
           upper.push_back(v[i]);
14
          reverse(v.begin(), v.end());
15
16
          for(int i=0; i<v.size(); i++){</pre>
           while(lower.size() >= 2 && (Point(lower[lower.size()-2],
17
        lower.back())^Point(lower.back(), v[i])) >= OLL) lower.pop_back();
1.8
           lower.push_back(v[i]);
19
20
          for(int i=upper.size()-2; i>=0; i--) points.push_back(upper[i]);
21
          for(int i=lower.size()-2; i>=0; i--) points.push back(lower[i]);
22
          reverse(lower.begin(), lower.end());
23
24
25
       double area() {
          double area = points.back().x*points[0].y -
26
        points.back().y*points[0].x;
27
         for(int i=0; i<points.size()-1; i++){</pre>
28
           area += points[i].x*points[i+1].y - points[i].y*points[i+1].x;
29
30
          return area/2.:
31
32
33
       int area2(){
34
         int area2 = points.back().x*points[0].y - points.back().y*points[0].x;
35
          for(int i=0; i<points.size()-1; i++){
36
           area2 += points[i].x*points[i+1].y - points[i].y*points[i+1].x;
37
38
         return area2:
39
40
41
        double perimeter() {
42
          double val = Point(points[0], points.back()).size();
43
          for(int i=0; i<points.size()-1; i++){
44
           val += Point(points[i], points[i+1]).size();
45
46
          return val;
47
48
49
       bool insideHull(Point p) {
50
51
          auto it = lower_bound(lower.begin(), lower.end(), p);
          if(it != lower.end() && *it == p) return true;
52
          it = lower_bound(upper.begin(), upper.end(), p);
53
54
          if(it != upper.end() && *it == p) return true;
55
56
          if(p.x == upper[0].x){
57
           if(p.y > upper[0].y) {
58
               /upper
59
              if(upper[1].x != upper[0].x) return false;
60
              else if(p.y <= upper[1].y) return true;</pre>
```

```
62
             else {
 63
               //lower
 64
               if(lower[1].x != lower[0].x) return false;
 65
               else if(p.v >= lower[1].v) return true;
 66
 67
             return false:
 68
           Point v1.v2:
 69
 70
           //upper or lower
 71
           int ansu = -1, ansl = -1;
 72
           int l = 0, r = upper.size()-2;
           while(1 <= r){
 73
             int m = (1+r)>>1LL;
 74
 75
             if(upper[m].x < p.x && p.x <= upper[m+1].x) {
 76
               ansu = m;
 77
               break;
 78
 79
             else if (upper [m+1] . x < p.x) l = m+1;
 80
             else r = m-1;
 81
 82
           l = 0, r = lower.size()-2;
 83
           while(1 <= r){
             int m = (1+r) >> 1LL:
 84
 85
             if(lower[m].x < p.x && p.x <= lower[m+1].x) {
 86
               ansl = m;
 87
               break;
 88
             else if(lower[m+1].x < p.x) l = m+1;
 89
 90
             else r = m-1;
 91
 92
           if(ansu == -1 || ansl == -1) return false;
 93
           bool oku = false, okl = false;
           v1 = Point(upper[ansu], upper[ansu+1]);
 94
 95
           v2 = Point(upper[ansu], p);
 96
           oku = ((v1^v2) \le 0);
           v1 = Point(lower[ansl], lower[ansl+1]);
 97
 98
           v2 = Point(lower[ansl], p);
           okl = ((v1^v2) >= 0);
 99
100
           if(oku && okl) return true;
           else return false;
101
102
103
104
      };
105
106
```

## 6.4. Rotation in 3D

```
struct Vec3d{
     double x,y,z;
2
3
     Vec3d operator+(const Vec3d & rhs) const{
4
       return {x+rhs.x, y+rhs.y, z+rhs.z};
     Vec3d operator*(const double k) const {
7
       return {k*x, k*y, k*z};
8
9
     Vec3d operator-(const Vec3d & rhs) const{
10
       return *this + rhs*-1;
11
12
     Vec3d operator/(const double k) const {
13
       return \{x/k, y/k, z/k\};
14
15
     double operator* (const Vec3d & rhs) const{
```

```
return x*rhs.x+y*rhs.y+z*rhs.z;
16 l
17
18
     double norm_sq() { return (*this) * (*this); }
19
     double norm() { return sqrt(norm_sq()); }
20 };
21
22 Vec3d rotate(Vec3d p, Vec3d u /*unit vector*/, double ang) {
     double dot = p*u;
23
     double co = cos(ang);
24
     double si = sin(ang);
2.5
     double x = u.x*dot*(1-co) + p.x*co + (u.y*p.z-u.z*p.y)*si;
26
     double y = u.y*dot*(1-co) + p.y*co + (u.z*p.x-u.x*p.z)*si;
28
     double z = u.z*dot*(1-co) + p.z*co + (u.x*p.y-u.y*p.x)*si;
29
     return {x, y, z};
30 }
```

# 7. Graphs

# 7.1. Dynammic Connectivity - connected(u, v) query

```
/* Dynammic Connectivity Implementation */
   /* Uses Divide and Conquer Offline approach */
   /* Able to answer if two vertex <u, v> are connected */
   /* No multi-edges allowed */
   /* DSU + Rollback is used to backtrack merges */
   /* N is defined as the maximum graph size given by input */
8 #define N MAX INPUT
10 | int uf[N];
11 | int sz[N]:
12
13
  struct event {
14
    int op, u, v, l, r;
1.5
     event() {}
16
    event (int o, int a, int b, int x, int y) : op(o), u(a), v(b), l(x), r(y) {}
17
18
19 map< pair<int, int>, int > edge_to_l;
   stack< pair<int*,int> > hist;
   vector<event> events;
22
23
   int init(int n) {
24
    for(int i=0; i<=n; i++){
25
       uf[i] = i:
26
       sz[i] = 1;
27
28
29
   int find(int u) {
30
    if(uf[u] == u) return u;
     else return find(uf[u]);
33
34
35
   void merge(int u, int v) {
    int a = find(u);
37
     int b = find(v);
38
     if(a == b) return;
39
     if(sz[a] < sz[b]) {
40
       hist.push(make_pair(&uf[a], uf[a]));
41
       uf[a] = b;
42
       hist.push(make_pair(&sz[b], sz[b]));
43
       sz[b] += sz[a];
44
45
     else {
46
       hist.push(make_pair(&uf[b], uf[b]));
47
       hist.push(make_pair(&sz[a], sz[a]));
48
       uf[b] = a;
49
       sz[a] += sz[b];
50
51
52
   int snap() {
     return hist.size();
55
56
57
  void rollback(int t) {
     while(hist.size() > t){
59
       pair<int*, int> aux = hist.top();
60
       hist.pop();
       *aux.first = aux.second;
```

```
62 l
 63 }
 64
 65 void solve(int 1, int r){
 66
     if(1 == r){
 67
        if(events[1].op == 2) {
          if(find(events[1].u) == find(events[1].v)) cout << "YES" << endl;</pre>
 68
           else cout << "NO" << endl;</pre>
 69
70
71
        return;
72
73
74
      int m = (1+r)/2;
75
      //doing for [L, m]
76
      int t = snap();
77
      for(int i=1; i<=r; i++) {
78
        if(events[i].op == 0 || events[i].op == 1){
79
          if(events[i].1 <= 1 && m <= events[i].r) merge(events[i].u,</pre>
         events[i].v);
      solve(l, m);
      rollback(t);
85
      //doing for [m+1, R]
      t = snap();
87
      for(int i=1; i<=r; i++) {</pre>
        if(events[i].op == 0 || events[i].op == 1){
89
          if(events[i].1 <= m+1 && r <= events[i].r) merge(events[i].u,</pre>
         events[i].v);
90
91
92
      solve(m+1, r);
93
      rollback(t);
94
9.5
    void offline_process() {
96
      int n, q;
97
98
      cin >> n >> q; //number of vertex and queries
99
100
      for(int i=0; i<q; i++) {
101
        string op;
102
        int u, v;
103
        cin >> op >> u >> v; //add, remove or query for u, v
104
        if(u > v) swap(u,v);
105
        if(op == "add") {
106
          events.push_back(event(0, u, v, i, -1));
107
           edge to l[make pair(u,v)] = i;
108
109
         else if(op == "rem"){
110
          int l = edge_to_l[make_pair(u, v)];
111
           events.push_back(event(1, u, v, 1, i));
112
           events[l].r = i;
113
114
        else if(op == "conn") {
115
           events.push_back(event(2, u, v, -1, -1));
116
117
118
      for(int i=0; i<q; i++) {
119
        if(events[i].op == 0) {
120
          if(events[i].r == -1){
121
            events[i].r = events.size();
122
             events.push_back(event(1, events[i].u, events[i].v, events[i].l,
         events[i].r));
123
```

## 7.2. Bellman Ford Shortest Path

```
struct BellmanFord{
2
3
     struct edges {
       int u, v, weight;
       edges(int u , int v, int weight) :
       u(ū),
       v(v),
       weight(weight) {}
10
11
     vector<int> dist;
12
13
     bool cycle = false;
14
15
     BellmanFord(){}
16
     BellmanFord(int n) {
17
18
       dist.resize(n+1);
19
20
     void calculate(int source){
21
22
        for(int i=0; i<dist.size(); i++){</pre>
23
          dist[i] = INF;
24
25
       dist[source] = 0;
26
       for(int k=0; k<dist.size()-1; k++){
27
          for(int i=0; i<e.size(); i++){</pre>
28
            if(dist[e[i].v] > dist[e[i].u] + e[i].weight){
29
              dist[e[i].v] = dist[e[i].u] + e[i].weight;
30
31
32
33
       for (int i=0; i<e.size(); i++) {</pre>
34
         if(dist[e[i].v] > dist[e[i].u] + e[i].weight){
35
            cycle = true;
36
37
38
39
```

# 7.3. Eulerian Circuits/Paths

```
1 //Graph - Euler path
   //for undirected graph
   //circuit - 2 vertex with odd grades
5 //simple path - all vertex with even grades
   //this algorithm generates a circuit, if you need a path between u, v
7 //create a new edge u-v, compute circuit u..u, then delete the last u
9 //for directed graph
10 //circuit - all vertex needs enter grade = exit grade
11 //path - one vertex needs to have one more enter grade
12 //and the other needs to have one more exit grade
13 //this algorithm generates a circuit, if you need a path between u, v
14 //create a new edge u-v, considering that u have one more enter grade
15 //and v one more exit grade
17 | struct EulerianCircuit {
18
    vector< set<int> > adj;
19
     vector<int> walk;
20
     vector<int> deg;
21
22
     int s, t;
23
     EulerianCiruit();
24
25
     EulerianCircuit(int n){
26
       deg.resize(n+1);
27
       adj.resize(n+1);
28
29
30
     void undirected_euler(int u) {
31
       while(!adj[u].empty()){
32
         int v = *(--adj[u].end());
33
3.4
         adj[u].erase(v);
35
         adj[v].erase(adj[v].find(u));
36
37
         euler(v);
38
39
40
       walk.push_back(u);
41
42
43
     void directed_euler(int u) {
       while (!adj[u].empty()) {
44
45
         int v = *(--adj[u].end());
46
47
         adj[u].erase(v);
48
49
         euler(v);
50
51
52
       walk.push back(u);
53
54
55 };
```

## 7.4. Kosaraju SCC

```
struct SCC {
2
3
     vector< vector<int> > adj_t;
4
     vector< vector<int> > scc_adj;
     int comp;
     vector<bool> vis;
     vector<int> scc:
     stack<int> vertex:
10
     SCC() {}
11
12
     SCC(int n) {
        adj_t.resize(n+1, vector<int>());
13
14
        scc_adj.resize(n+1, vector<int>());
15
        comp = 0;
16
        vis.resize(n+1);
17
        scc.resize(n+1);
18
19
2.0
     void dfs(int u) {
21
        vis[u] = true;
22
        for (int i=0; i < adj[u].size(); i++) {</pre>
23
          int v = adi[u][i];
24
          if(!vis[v]) dfs(v);
25
26
        vertex.push(u);
27
28
29
     void dfst(int u, int comp) {
30
        scc[u] = comp;
31
        vis[u] = true;
32
        for (int i=0; i < adj_t[u].size(); i++) {</pre>
33
          int v = adj_t[u][i];
34
          if(!vis[v]) dfst(v,comp);
35
36
37
38
     void calculate() {
39
        int n = vis.size()-1;
40
41
        for (int i=0; i<=n; i++) {</pre>
42
          vis[i] = false;
43
44
45
        for (int i=1; i<=n; i++) {</pre>
          if(!vis[i]){
46
47
            dfs(i);
48
49
50
51
        for (int i=1; i<=n; i++) {</pre>
52
          for(int v : adj[i]){
53
            adj_t[v].pb(i);
54
55
56
57
        for(int i=1; i<=n; i++){
58
          vis[i] = false;
59
60
61
        while(!vertex.empty()){
62
         if(!vis[vertex.top()]){
63
            comp++;
```

```
64
            dfst(vertex.top(),comp);
65
66
          vertex.pop();
67
68
69
        //set< ii > edge_check; //eliminates duplicate edges (additional O(logn))
70
71
        for (int i=1; i<=n; i++) {</pre>
72
         for(int j=0; j<adj[i].size(); j++){</pre>
            int v = adj[i][j];
7.3
74
            if(scc[i] == scc[v]) continue;
            //if(edge_check.count(ii(scc[i], scc[v]))) continue; //eliminates
75
        duplicate edges (additional O(logn))
76
           scc_adj[scc[i]].push_back(scc[v]);
77
            //edge_check.insert(ii(scc[i], scc[v])); //eliminates duplicate
        edges (additional O(logn))
78
79
80
81
82 };
```

## 7.5. Centroid Decomposition

```
1 /* Centroid Decomposition Implementation */
2 /* c_p[] contains the centroid predecessor on centroid tree */
3 | /* removed[] says if the node was already selected as a centroid (limit the
        subtree search) */
   /* L[] contains the height of the vertex (from root) on centroid tree (Max
    /* N is equal to the maximum size of tree (given by statement) */
   struct CentroidDecomposition {
     vector<bool> removed;
     vector<int> L, subsz;
10
     vector<int> c_p;
11
12
     CentroidDecomposition() {}
13
14
     CentroidDecomposition(int n) {
15
        removed.resize(n+1);
16
       L.resize(n+1);
17
        c_p.resize(n+1);
18
        subsz.resize(n+1);
        for (int i=0; i<=n;i++) {</pre>
19
          c_p[i] = -1;
20
21
22
23
24
     void centroid_subsz(int u, int p) {
        subsz[u] = 1;
26
        for(int i=0; i<adj[u].size(); i++){</pre>
27
          int v = adj[u][i];
28
          if(v == p || removed[v]) continue;
29
          centroid subsz(v.u):
30
          subsz[u] += subsz[v];
31
32
33
34
     int find_centroid(int u, int p, int sub){
35
        for(int i=0; i<adj[u].size(); i++){</pre>
36
         int v = adj[u][i];
37
          if(v == p | removed[v]) continue;
38
         if(subsz[v] > subsz[sub]/2){
```

```
return find_centroid(v, u, sub);
40
41
42
       return u;
43
44
45
     void centroid_decomp(int u, int p, int r){
       centroid_subsz(u,-1);
46
47
       int centroid = find_centroid(u, -1, u);
       L[centroid] = r;
48
49
       c_p[centroid] = p;
        removed[centroid] = true;
50
51
52
       //problem pre-processing
53
54
       for(int i=0; i<adj[centroid].size(); i++){</pre>
55
         int v = adj[centroid][i];
56
         if(removed[v]) continue;
57
          centroid_decomp(v, centroid, r+1);
58
59
   };
```

# 7.6. Floyd Warshall Shortest Path

```
struct FloydWarshall {
2
3
     vector< vector<int> > > dist;
4
     FloydWarshall() {}
6
     FloydWarshall(int n) {
8
        dist.resize(n+1, vector< vector< int > >(n+1, vector<int>(n+1)));
10
11
     void relax(int i, int j, int k) {
12
        dist[k][i][j] = min(dist[k-1][i][j], dist[k-1][i][k] + dist[k-1][k][j]);
13
14
15
     void calculate() {
        for(int k=0; k<dist.size(); k++){</pre>
16
17
          for(int i=1; i<dist.size(); i++){</pre>
18
            for(int j=1; j<dist.size(); j++){</pre>
19
              if(i==j) dist[k][i][j] = 0;
20
              else dist[k][i][j] = INF;
21
22
23
24
        for(int k=1; k<dist.size(); k++){</pre>
25
          for(int i=1; i<dist.size(); i++) {</pre>
            for(int j=1; j<dist.size(); j++) {</pre>
26
27
              relax(i,j,k);
28
29
30
31
32
33
```

# 7.7. Tarjan's Bridge/Articulations Algorithm

```
//Graph - Tarjan Bridges Algorithm
    //calculate bridges, articulations and all connected components
   struct Tarian{
     int cont = 0;
     vector<int> st;
     vector<int> low:
     vector< ii > bridges;
     vector<bool> isArticulation;
10
11
12
     Tarjan() {}
1.3
     Tarjan(int n) {
14
15
        st.resize(n+1);
16
        low.resize(n+1);
       isArticulation.resize(n+1);
17
18
        cont = 0;
19
       bridges.clear();
20
21
22
     void calculate(int u, int p = -1) {
23
       st[u] = low[u] = ++cont;
24
        int son = 0;
25
        for(int i=0; i<adj[u].size(); i++){</pre>
26
         if(adj[u][i]==p){
27
           p = 0;
28
            continue;
2.9
          if(!st[adj[u][i]]){
30
31
            calculate(adj[u][i], u);
32
            low[u] = min(low[u], low[adj[u][i]]);
33
            if(low[adj[u][i]] >= st[u]) isArticulation[u] = true; //check
        articulation
34
35
            if(low[adj[u][i]] > st[u]) { //check if its a bridge
36
              bridges.push_back(ii(u, adj[u][i]));
37
38
39
            son++;
40
41
          else low[u] = min(low[u], st[adj[u][i]]);
42
43
44
       if (p == -1) {
45
         if(son > 1) isArticulation[u] = true;
46
          else isArticulation[u] = false;
47
48
49 };
```

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for(FlowEdge e : adj[i]){

## 7.8. Max Flow Dinic's Algorithm

```
struct Dinic {
2
3
     struct FlowEdge{
4
       int v, rev, c, cap;
       FlowEdge() {}
       FlowEdge(int v, int c, int cap, int rev) : v(v), c(c), cap(cap),
       rev(rev) {}
7
8
9
     vector< vector<FlowEdge> > adj;
10
     vector<int> level, used;
11
     int src, snk;
12
     int sz;
     int max flow;
13
14
     Dinic(){}
15
     Dinic(int n) {
       src = 0;
16
       snk = n+1;
17
18
       adj.resize(n+2, vector< FlowEdge >());
19
       level.resize(n+2);
20
       used.resize(n+2);
21
       sz = n+2;
22
       \max flow = 0;
23
24
25
     void add_edge(int u, int v, int c){
26
       int id1 = adj[u].size();
27
       int id2 = adj[v].size();
28
       adj[u].pb(FlowEdge(v, c, c, id2));
29
       adj[v].pb(FlowEdge(u, 0, 0, id1));
30
31
32
     void add to src(int v, int c){
33
       adj[src].pb(FlowEdge(v, c, c, -1));
34
35
36
     void add_to_snk(int u, int c){
37
       adj[u].pb(FlowEdge(snk, c, c, -1));
38
39
40
     bool bfs(){
       for (int i=0; i<sz; i++) {</pre>
41
42
         level[i] = -1;
43
44
45
       level[src] = 0;
46
       queue<int> q; q.push(src);
47
48
       while(!q.empty()){
         int cur = q.front();
49
50
          q.pop();
51
          for(FlowEdge e : adj[cur]) {
           if(level[e.v] == -1 && e.c > 0){
52
5.3
             level[e.v] = level[cur]+1;
              q.push(e.v);
54
55
56
57
58
59
       return (level[snk] == -1 ? false : true);
60
61
     int send_flow(int u, int flow){
```

```
if(u == snk) return flow;
  for(int &i = used[u]; i<adj[u].size(); i++) {</pre>
    FlowEdge &e = adj[u][i];
    if(level[u]+1 != level[e.v] || e.c <= 0) continue;</pre>
    int new_flow = min(flow, e.c);
    int adjusted_flow = send_flow(e.v, new_flow);
    if(adjusted_flow > 0) {
      e.c -= adjusted flow;
      if(e.rev != -1) adj[e.v][e.rev].c += adjusted_flow;
      return adjusted_flow;
  return 0;
void calculate(){
  if(src == snk) {max_flow = -1; return;} //not sure if needed
  max_flow = 0;
  while(bfs()){
    for(int i=0; i<sz; i++) used[i] = 0;</pre>
    while(int inc = send_flow(src, INF)) max_flow += inc;
vector< ii > mincut(){
  bool vis[sz];
  for(int i=0; i<sz; i++) vis[i] = false;</pre>
  queue<int> q:
  q.push(src);
  vis[src] = true;
  while(!q.empty()){
    int cur = q.front();
    q.pop();
    for(FlowEdge e : adj[cur]) {
      if(e.c > 0 && !vis[e.v]){
        q.push(e.v);
        vis[e.v] = true;
  vector< ii > cut;
  for(int i=1; i<=sz-2; i++) {</pre>
    if(!vis[i]) continue;
    for(FlowEdge e : adj[i]){
      if(1 \le e.v \&\& e.v \le sz-2 \&\& !vis[e.v] \&\& e.cap > 0 \&\& e.c == 0)
  cut.pb(ii(i, e.v));
  return cut;
vector< ii > min edge cover() {
  bool covered[sz];
  for(int i=0; i<sz; i++) covered[i] = false;</pre>
  vector< ii > edge_cover;
  for(int i=1; i<sz-1; i++) {</pre>
```

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101

```
if(e.cap == 0 || e.v > sz-2) continue;
127
128
             if(e.c == 0){
129
               edge_cover.pb(ii(i, e.v));
130
               covered[i] = true;
131
               covered[e.v] = true;
132
               break:
133
134
135
136
         for (int i=1; i<sz-1; i++) {</pre>
137
           for(FlowEdge e : adj[i]) {
138
             if(e.cap == 0 || e.v > sz-2) continue;
139
             if(e.c == 0) continue;
140
             if(!covered[i] || !covered[e.v]){
               edge_cover.pb(ii(i, e.v));
141
142
               covered[i] = true;
               covered[e.v] = true;
143
144
145
146
147
         return edge_cover;
148
149
150
    };
```

#### 7.9. HLD

```
struct HLD {
2
3
     struct node {
        //node values
       int val = 0; //sets neutral value for the needed operation
       node() {}
       node(int val) : val(val) {
9
         lazy = 0;
10
11
12
       node merge (node b) {
13
         node ret;
14
          //merge nodes
15
          return ret;
16
17
     };
18
19
20
     struct SegmentTree {
21
22
       vector<node> st;
23
24
       SegmentTree() {}
25
26
       void construct(int n) {
27
         st.resize(4*n);
28
29
       void propagate(int cur, int 1, int r){
30
31
          //check if exists operation
32
33
         //apply lazy
34
35
         if(1 != r){
36
           //propagate lazy
                                                                                        102
37
```

```
//reset lazv
  void build(int cur, int 1, int r){
    if(1 == r){
      //apply on leaf
      return;
    int m = (1+r) >> 1;
    build(2*cur, 1, m);
    build(2*cur+1, m+1, r);
    st[cur] = st[2*cur].merge(st[2*cur+1]);
  void update(int cur, int 1, int r, int a, int b, int val) {
    propagate(cur, l, r);
    if(b < 1 || r < a) return;
    if(a <= 1 && r <= b){
      //apply on lazy
      propagate(cur, 1, r);
      return;
    int m = (1+r) >> 1;
    update(2*cur, 1, m, a, b, val);
    update(2*cur+1, m+1, r, a, b, val);
    st[cur] = st[2*cur].merge(st[2*cur+1]);
  node query(int cur, int 1, int r, int a, int b){
    propagate(cur, 1, r);
    if(b < 1 || r < a) return node();
    if(a <= l && r <= b) return st[cur];</pre>
    int m = (1+r) >> 1;
    node lef = query(2*cur, 1, m, a, b);
    node riq = query(2*cur+1, m+1, r, a, b);
    return lef.merge(rig);
} st;
vector<int> L, P, ch, subsz, in, out;
int t;
HLD () {}
HLD(int n) {
 L.resize(n+1);
 P.resize(n+1);
  ch.resize(n+1);
  subsz.resize(n+1);
  in.resize(n+1);
  out.resize(n+1);
  st.construct(n+1);
  t = 0;
  for(int i=0; i<=n; i++) {</pre>
    ch[i] = i;
    P[i] = -1;
    L[i] = 0;
```

```
103
104
      void precalculate(int u, int p = -1) {
105
         subsz[u] = 1;
106
         for(int &v : adj[u]) {
107
          if(v == p) continue;
108
          P[v] = u;
109
          L[v] = L[u] + 1;
           precalculate(v, u);
110
           if(subsz[adj[u][0]] < subsz[v]) swap(adj[u][0], v);
111
           subsz[u] += subsz[v];
112
113
114
115
      void build(int u, int p = -1) {
116
117
        in[u] = ++t;
        for(int v : adj[u]){
118
119
          if(v == p) continue;
120
          if(adj[u][0] == v){
121
             ch[v] = ch[u];
122
           build(v, u);
123
124
125
        out[u] = t;
126
127
128
      void calculate(int root = 1) {
129
         precalculate(root);
130
         build(root);
131
132
      void build_ds() {
133
134
        st.build(1, 1, t);
135
136
137
      void path_update(int a, int b, int val, bool edge_update = false){
138
         while(ch[a] != ch[b]) {
139
           if(L[ch[b]] > L[ch[a]]) swap(a,b);
140
           st.update(1, 1, t, in[ch[a]], in[a], val);
141
           a = P[ch[a]];
142
143
        if(L[b] < L[a]) swap(a,b);
144
        if(in[a]+edge_update <= in[b]) st.update(1, 1, t, in[a]+edge_update,</pre>
         in[b], val);
145
146
147
      void node_update(int u, int val){
148
        st.update(1, 1, t, in[u], in[u], val);
149
150
151
      void edge_update(int u, int v, int val){
152
        if(L[u] > L[v]) swap(u, v);
153
        st.update(1, 1, t, in[v], in[v], val);
154
155
156
      void subtree_update(int u, int val, bool edge_update = false){
157
        if(in[u] + edge_update <= out[u]) st.update(1, 1, t, in[u] +</pre>
         edge_update, out[u], val);
158
159
160
      node path_query(int a, int b, bool edge_query = false) {
161
        node ans:
162
         while(ch[a] != ch[b]) {
163
          if(L[ch[b]] > L[ch[a]]) swap(a,b);
           ans = ans.merge(st.query(1, 1, t, in[ch[a]], in[a]));
164
165
           a = P[ch[a]];
```

```
166
167
        if(L[b] < L[a]) swap(a,b);
168
        if(in[a]+edge_query <= in[b]) ans = ans.merge(st.query(1, 1, t,</pre>
         in[a]+edge_query, in[b]));
169
         return ans;
170
171
172
      node node_query(int u){
173
         return st.query(1, 1, t, in[u], in[u]);
174
175
176
      node edge_guery(int u, int v){
177
        if(L[u] > L[v]) swap(u,v);
178
        return st.query(1, 1, t, in[v], in[v]);
179
180
181
      node subtree_query(int u, bool edge_query = false) {
182
        if(in[u] + edge_query <= out[u]) return st.query(1, 1, t, in[u] +</pre>
         edge_query, out[u]);
183
         else return node();
184
185
186 };
```

#### 7.10. LCA

```
struct LCA {
2
     int tempo;
     vector<int> st, ed, dad, anc[20], L;
4
     vector<bool> vis:
     LCA() {}
     LCA(int n) {
10
       tempo = 0:
11
        st.resize(n+1);
12
        ed.resize(n+1);
       dad.resize(n+1);
13
       L.resize(n+1);
14
        for(int i=0; i<20; i++) anc[i].resize(n+1);</pre>
15
16
       vis.resize(n+1);
17
       for(int i=0; i<=n; i++) vis[i] = false;</pre>
18
19
2.0
     void dfs(int u) {
21
       vis[u] = true;
22
        st[u] = tempo++;
23
        for (int i=0; i < adj[u].size(); i++) {</pre>
24
          int v = adj[u][i];
25
          if(!vis[v]){
26
            dad[v] = u;
27
            L[v] = L[u]+1;
28
            dfs(v);
29
30
31
       ed[u] = tempo++;
32
33
34
     bool is ancestor(int u, int v) {
35
       return st[u] <= st[v] && st[v] <= ed[u];</pre>
36
37
38
     int query(int u, int v){
39
       if(is_ancestor(u,v)) return u;
40
        for(int i=19; i>=0; i--){
41
          if(anc[i][u] == -1) continue;
42
          if(!is\_ancestor(anc[i][u],v)) u = anc[i][u];
43
44
       return dad[u]:
45
46
47
     int distance(int u, int v) {
48
        return L[u] + L[v] - 2*L[query(u,v)];
49
50
51
     void precalculate(){
52
        dad[1] = -1;
53
        L[1] = 0:
54
        dfs(1):
        for (int i=1; i<st.size(); i++) {</pre>
55
56
          anc[0][i] = dad[i];
57
58
        for (int i=1; i<20; i++) {</pre>
59
          for(int j=1; j<st.size(); j++) {</pre>
60
              if (anc[i-1][j] != -1){
61
                anc[i][j] = anc[i-1][anc[i-1][j]];
62
63
              else {
```

```
anc[i][j] = -1;
64
65
66
67
68
69
70 };
```

# 7.11. Block-Cut Tree

```
1 | vector<int> adi[N];
2 int vis[N];
3 int ini[N];
4 int bef[N]:
5 bool art[N]
6 | int num_bridges = 0;
   int T = 0:
9
   void dfs_tarjan(int v, int p) {
       vis[v] = 1;
10
11
       bef[v] = ini[v] = ++T;
        art[v] = false;
12
       int num_sub=0;
13
14
        for(int i = 0; i < adj[v].size(); i++) {</pre>
            int ad = adj[v][i];
15
            if (ad == p) continue;
16
            if(!vis[ad]) {
17
18
                dfs_tarjan(ad, v);
                if(bef[ad] > ini[v]) {
19
20
                    // Bridge
21
                    num_bridges++;
22
23
                if (bef[ad] >= ini[v] && p != -1) {
24
                    // v is an articulation
25
                    art[v] = true;
26
27
                num_sub++;
28
29
            bef[v] = min(bef[v], ini[ad]);
3.0
31
       if(p == -1 && num_sub > 1) {
32
            // Root is an articulation
33
            art[v] = true;
34
35
36
37
   int curId;
   vector<int> adjbct[2*112345];
38
39
40
   void dfs_block_cut(int v, int id) {
41
       vis[v] = 1:
42
       if(id != -1) {
43
            adjbct[v].pb(id);
44
            adjbct[id].pb(v);
45
46
        for(int i = 0; i < adj[v].size(); i++) {</pre>
            int ad = adj[v][i];
47
48
            if(!vis[ad]) {
49
                if(bef[ad] >= ini[v]) {
50
                    curId++;
51
                    adjbct[v].pb(curId);
52
                    adjbct[curId].pb(v);
53
                    dfs_block_cut(ad, curId);
54
                } else {
```

```
dfs_block_cut(ad, id);
56
57
58
59
60
61
   int32 t main() {
       num_bridges = 0;
62
       T = 0:
63
64
       memset(vis, 0, sizeof vis);
65
       for(int i = 1; i <= n; i++)
66
            dfs_tarjan(i, -1);
67
68
       curId = n;
       memset(vis, 0, sizeof vis);
69
70
       for(int i = 1; i <= n; i++)</pre>
71
            dfs_block_cut(i, -1);
72
```

## 7.12. Dominator Tree

```
template<typename T = int>
   struct LinkDsu{
     vector<int> r;
     vector<T> best;
     LinkDsu(int n = 0){
       r = vector<int>(n); iota(r.begin(), r.end(), 0);
       best = vector<T>(n);
8
     int find(int u) {
10
       if (r[u] == u)
11
12
         return u;
13
14
         int v = find(r[u]);
15
         if (best[r[u]] < best[u]) best[u] = best[r[u]];</pre>
16
         return r[u] = v;
17
18
19
20
     T eval(int u) { find(u); return best[u]; }
     void link(int p, int u) { r[u] = p;
21
22
     void set(int u, T x) { best[u] = x;
23
24
25
   struct DominatorTree{
     typedef vector<vector<int>> Graph;
27
     vector<int> semi, dom, parent, st, from;
28
     Graph succ, pred, bucket;
29
     int r, n, tempo;
30
31
     void dfs(int u, int p) {
32
       semi[u] = u;
33
       from[st[u] = tempo++] = u;
34
       parent[u] = p;
35
       for (int v : succ[u]) {
36
         pred[v].push_back(u);
37
         if (semi[v] == -1) { dfs(v, u); }
38
39
40
     void build() {
41
42
       n = succ.size();
43
       dom.assign(n, -1);
```

```
semi.assign(n, -1);
45
       parent.assign(n, -1);
46
       st.assign(n, 0);
        from.assign(n, -1);
47
48
       pred = Graph(n, vector<int>());
49
        bucket = Graph(n, vector<int>());
50
       LinkDsu<pair<int,int>> dsu(n);
       tempo = 0;
51
52
5.3
        dfs(r, r);
54
        for(int i = 0; i < n; i++) dsu.set(i, make_pair(st[i], i));</pre>
55
56
        for (int i = tempo - 1; i; i--) {
57
         int u = from[i];
58
         for (int v : pred[u]) {
59
           int w = dsu.eval(v).second;
60
           if (st[semi[w]] < st[semi[u]]) { semi[u] = semi[w]; }</pre>
61
62
          dsu.set(u, make_pair(st[semi[u]], u));
63
          bucket[semi[u]].push_back(u);
64
          dsu.link(parent[u], u);
65
          for(int v : bucket[parent[u]]) {
           int w = dsu.eval(v).second;
67
            dom[v] = semi[w] == parent[u] ? parent[u] : w;
68
69
          bucket[parent[u]].clear();
70
71
        for (int i = 1; i < tempo; i++) {</pre>
         int u = from[i];
72
73
          if (dom[u] != semi[u]) dom[u] = dom[dom[u]];
74
75
76
77
     DominatorTree(const Graph & g, int s) : succ(g), r(s) {
78
       build();
79
80 };
```

## 7.13. Minimum Path Cover Problem On DAG's

```
1 int32 t main() {
    DESYNC:
3
     int n.m:
      cin >> n >> m;
     Dinic dinic(n+n):
      for(int i=1; i<=n; i++) {</pre>
        dinic.add to src(i, 1);
8
        dinic.add_to_snk(i+n, 1);
9
10
11
      for(int i=0; i<m; i++) {</pre>
12
        int u, v;
13
        cin >> u >>v;
        dinic.add_edge(u,v+n,1);
14
1.5
16
17
      dinic.calculate();
18
      for(int i=1; i<=n; i++) {
        for(Dinic::FlowEdge e : dinic.adj[i]) {
19
          if (e.cap == 1 && e.c == 0 && 1 <= e.v && e.v-n <= n) {
20
21
            adj[i].pb(e.v-n);
22
            dq[e.v-n]++;
23
24
```

```
26
27
      for (int i=1; i<=n; i++) {</pre>
28
        if(dg[i] == 0){
29
          paths.pb(vector<int>());
30
          go(i, paths.size()-1);
31
32
33
34
      cout << paths.size() <<endl;</pre>
      for(int i=0; i<paths.size(); i++) {</pre>
        for(int v : paths[i]) cout << v << " ";</pre>
36
37
        cout << endl;
38
39
40
```

## 7.14. Stoer-Wagner Minimum Cut in Undirected Graphs

```
/* Initialization */
  int cost[n + 1][n + 1];
3 memset(cost, 0, sizeof cost);
4 while (m--) {
    int u, v, c;
     u = input.next();
     v = input.next();
     c = input.next();
     cost[u][v] = c;
10
     cost[v][u] = c;
11
12
   /* Stoer-Wagner: global minimum cut in undirected graphs */
13 | int min_cut = 1000000000;
14 bool added[n + 1];
15 int vertex_cost[n + 1];
16 | for (int vertices_count = n; vertices_count > 1; --vertices_count) {
17
     memset(added, 0, sizeof(added[0]) * (vertices_count + 1));
18
     memset(vertex_cost, 0, sizeof(vertex_cost[0]) * (vertices_count + 1));
19
     int s = -1, t = -1;
     for (int i = 1; i <= vertices_count; ++i) {</pre>
       int vert = 1;
22
       while (added[vert]) ++vert;
23
       for (int j = 1; j <= vertices_count; ++j)</pre>
         if (!added[j] && vertex_cost[j] > vertex_cost[vert]) vert = j;
24
25
       if (i == vertices_count - 1)
         s = vert;
26
27
       else if (i == vertices_count) {
28
         t = vert;
29
         min_cut = min(min_cut, vertex_cost[vert]);
30
31
       added[vert] = 1;
32
       for (int j = 1; j <= vertices_count; ++j) vertex_cost[j] +=</pre>
       cost[vert][j];
33
     for (int i = 1; i <= vertices_count; ++i) {</pre>
34
       cost[s][i] += cost[t][i];
35
36
       cost[i][s] += cost[i][t];
37
     for (int i = 1; i <= vertices_count; ++i) {</pre>
38
39
       cost[t][i] = cost[vertices count][i];
40
       cost[i][t] = cost[i][vertices_count];
41
42
43 printf("%d\n", min_cut);
```