Review: Bilinear Model

Bilinear model,

$$\hat{\mathbf{y}} = f((\mathbf{x}^0 \mathbf{W}^0)\mathbf{W}^1 + \mathbf{b})$$

- $\mathbf{x}^0 \in \mathbb{R}^{1 \times d_0}$ start with one-hot.
- $\mathbf{W}^0 \in \mathbb{R}^{d_0 \times d_{\mathrm{in}}}$, $d_0 = |\mathcal{F}|$
- $\mathbf{W}^1 \in \mathbb{R}^{d_{\mathrm{in}} \times d_{\mathrm{out}}}, \mathbf{b} \in \mathbb{R}^{1 \times d_{\mathrm{out}}}$; model parameters

Notes:

- Bilinear parameter interaction.
- $d_0 >> d_{\mathrm{in}}$, e.g. $d_0 = 10000, d_{\mathrm{in}} = 50$

Review: Bilinear Model: Intuition

$$(\mathbf{x}^0\mathbf{W}^0)\mathbf{W}^1 + \mathbf{b}$$

$$\begin{bmatrix} 0 & \ldots & {\color{red}1} & \ldots & 0 \end{bmatrix} \begin{bmatrix} w^0_{1,1} & \ldots & w^0_{0,d_{\mathrm{in}}} \\ & \vdots & \\ & \vdots & \\ & \vdots & \\ {\color{red}w^0_{k,1}} & {\color{red}\ldots} & {\color{red}w^0_{k,d_{\mathrm{in}}}} \\ & \vdots & \\ & \vdots & \\ & \vdots & \\ w^0_{d_0,1} & \ldots & w^0_{d_0,d_{\mathrm{in}}} \end{bmatrix} \begin{bmatrix} {\color{red}w^1_{1,1}} & \ldots & \ldots & {\color{red}w^1_{0,d_{\mathrm{out}}}} \\ & \ddots & \ddots & \\ {\color{red}w^1_{d_{\mathrm{in}},0}} & \ldots & \ldots & {\color{red}w^1_{d_{\mathrm{in}},d_{\mathrm{out}}}} \end{bmatrix}$$

Review: Window Model

Goal: predict t_5.

- Windowed word model.

$$w_1 \;\; w_2 \;\; [w_3 \;\; w_4 \;\; w_5 \;\; w_6 \;\; w_7] \;\; w_8$$

- w_3, w_4; left context
- w_5; Word of interest
- w_6, w_7; right context
- d_{win}; size of window ($d_{\mathrm{win}} = 5$)

Review: Dense Windowed BoW Features

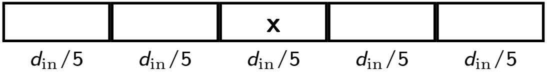
- $f_1, \ldots, f_{d_{\mathrm{win}}}$ are words in window
- Input representation is the concatenation of embeddings

$$\mathbf{x} = [v(f_1)\ v(f_2)\ \ldots\ v(f_{d_{\mathrm{win}}})]$$

Example: Tagging

$$w_1\ w_2\ [w_3\ w_4\ w_5\ w_6\ w_7]\ w_8$$

$$\mathbf{x} = [v(w_3)\ v(w_4)\ v(w_5)\ v(w_6)\ v(w_7)]$$



Rows of $\mathbf{W}^1$ encode position specific weights.

Quiz

We are doing tagging with a windowed bilinear model with hinge-loss and no capitalization features. The model has $d_{\mathrm{win}} = 5$, $d_{\mathrm{in}} = 50$, $d_{\mathrm{out}} = 40$, and vocabulary size 10000.

We are given the input window:

`The dog walked to the`

Unfortunately we incorrectly classify `walked` as `NN` as opposed to `VP`, in a bilinear model with a hinge-loss .

What is the maximum number of parameters that receive a non-zero gradient?

Answer:

$$
\begin{bmatrix} 1 & \dots & 1 & \dots & 1 & \dots & 1 & \dots & 1 \end{bmatrix}
\begin{bmatrix}
w^0_{1,1} & \cdots & w^0_{0,d_{\text{in}}} \\
w^0_{the,1} & \cdots & w^0_{the,d_{\text{in}}} \\
\vdots & & \\
w^0_{dog,1} & \cdots & w^0_{dog,d_{\text{in}}} \\
\vdots & & \\
w^0_{walked,1} & \cdots & w^0_{walked,d_{\text{in}}} \\
\vdots & & \\
w^0_{to,1} & \cdots & w^0_{to,d_{\text{in}}} \\
\vdots & & \\
w^0_{the,1} & \cdots & w^0_{the,d_{\text{in}}} \\
\vdots & & \\
w^0_{d_0,1} & \cdots & w^0_{d_0,d_{\text{in}}}
\end{bmatrix}
\begin{bmatrix}
w^1_{1,1} & \dots & w^1_{1,NN} & \dots & w^1_{1,VP} & \dots & w^1_{0,d_{\text{out}}} \\
 & \ddots & \ddots & & & & \\
w^1_{d_{\text{in}},0} & \dots & w^1_{d_{\text{in}},NN} & \dots & w^1_{d_{\text{in}},VP} & \dots & w^1_{d_{\text{in}},d_{\text{out}}}
\end{bmatrix}
$$

$$\mathbf{W}^0 = 5 \times d_{\text{in}}$$

$$\mathbf{W}^1 = d_{\text{in}} \times 2$$

Contents

Neural Network

One-layer multi-layer perceptron architecture,

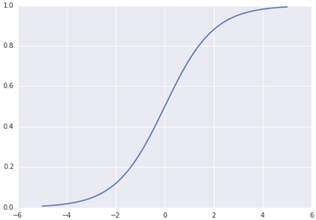
$$NN_{MLP1}(\mathbf{x}) = g(\mathbf{x}\mathbf{W}^1 + \mathbf{b}^1)W^2 + \mathbf{b}^2$$

- $\mathbf{x}\mathbf{W} + \mathbf{b}$; *perceptron*
- $\mathbf{x}$ is the dense representation in $\mathbb{R}^{1 \times d_{\text{in}}}$
- $\mathbf{W}^1 \in \mathbb{R}^{d_{\text{in}} \times d_{\text{hid}}}$, $\mathbf{b}^1 \in \mathbb{R}^{1 \times d_{\text{hid}}}$; first affine transformation
- $\mathbf{W}^2 \in \mathbb{R}^{d_{\text{hid}} \times d_{\text{out}}}$, $\mathbf{b}^2 \in \mathbb{R}^{1 \times d_{\text{out}}}$; second affine transformation
- $g : \mathbb{R}^{d_{\text{hid}} \times d_{\text{hid}}}$ is an *activation non-linearity* (often pointwise)
- $g(\mathbf{x}\mathbf{W}^1 + \mathbf{b}^1)$ is the *hidden layer*

Non-Linear Functions

Logistic sigmoid function:

$$\sigma(t) = \frac{1}{1 + \exp(-t)}$$



- Intuition: Each dimension of hidden-layer is the prob. under a logistic regression model.

Why are these better?

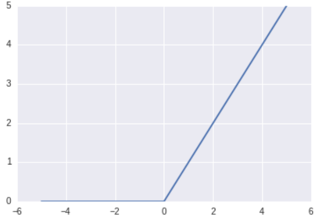
Function Approximator

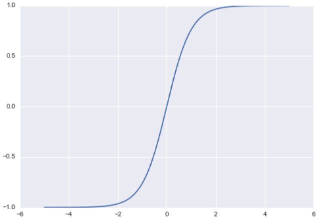
MLP1 is a universal approximator

Other Non-Linearities: ReLU

Rectified Linear Unit:

$$\mathrm{ReLU}(t) = \max\{0, t\}$$




1.0
0.5
0.0
−0.5
−1.0
−6
−4
−2
0
2
4
6

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