Text Classification

+

Machine Learning Review

CS 287

Contents

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Sentiment

Good Sentences

- A thoughtful, provocative, insistently humanizing film.
- Occasionally melodramatic, it's also extremely effective.
- Guaranteed to move anyone who ever shook, rattled, or rolled.

Bad Sentences

- A sentimental mess that never rings true.
- This 100-minute movie only has about 25 minutes of decent material.
- Here, common sense flies out the window, along with the hail of bullets, none of which ever seem to hit Sascha.

Multiclass Sentiment

- ★★★★

  I visited The Abbey on several occasions on a visit to Cambridge and found it to be a solid, reliable and friendly place for a meal.
- ★★

  However, the food leaves something to be desired. A very obvious menu and average execution
- ★★★★★

  Fun, friendly neighborhood bar. Good drinks, good food, not too pricey. Great atmosphere!

Text Categorization

- Straightforward setup.
- Lots of practical applications:
  - Spam Filtering
  - Sentiment
  - Text Categorization
  - e-discovery
  - Twitter Mining
  - Author Identification
  - ...
- Introduces machine learning notation.

However, a relatively solved problem these days.

Contents

Preliminary Notation

- $\mathbf{b}, \mathbf{m}$; bold letters for vectors.
- $\mathbf{B}, \mathbf{M}$; bold capital letters for matrices.
- $\mathcal{B}, \mathcal{M}$; script-case for sets.
- B, M; capital letters for random variables.
- b_i, x_i; lower case for scalars or indexing into vectors.
- $\delta(i)$; one-hot vector at position i

$$\delta(2) = [0; 1; 0; \dots]$$

- $\mathbf{1}(x = y)$; indicator 1 if $x = y$, o.w. 0

Text Classification

1. Extract pertinent information from the sentence.
2. Use this to construct an input representation.
3. Classify this vector into an output class.

Input Representation:

- Conversion from text into a mathematical representation?
- Main focus of this class, representation of language
- Point in coming lectures: *sparse* vs. *dense* representations

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Sparse Features (Notation from YG)

- $\mathcal{F}$; a discrete set of features values.
- $f_1 \in \mathcal{F}, \ldots, f_k \in \mathcal{F}$; active features for input.

  For a given sentence, let $f_1, \ldots f_k$ be the relevant features.

  Typically $k << |\mathcal{F}|$.
- Sparse representation of the input defined as,

$$\mathbf{x} = \sum_{i=1}^{k} \delta(f_i)$$

- $\mathbf{x} \in \mathbb{R}^{1 \times d_{\mathrm{in}}}$; input representation

Features 1: Sparse Bag-of-Words Features

Representation is counts of input words,

- $\mathcal{F}$; the vocabulary of the language.
- $\mathbf{x} = \sum_i \delta(f_i)$

Example: Movie review input,

`A sentimental mess`

$$\mathbf{x} = \delta(\texttt{word:A}) + \delta(\texttt{word:sentimental}) + \delta(\texttt{word:mess})$$

$$\mathbf{x}^\top = \begin{bmatrix} 1 \\ \vdots \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ \vdots \\ 1 \\ 0 \end{bmatrix} = \begin{array}{l} \begin{bmatrix} 1 \\ \vdots \\ 1 \\ 1 \end{bmatrix} \end{array} \begin{array}{l} \texttt{word:A} \\ \vdots \\ \texttt{word:mess} \\ \texttt{word:sentimental} \end{array}$$

Features 2: Sparse Word Properties

Representation can use specific aspects of text.

- $\mathcal{F}$; Spelling, all-capitals, trigger words, etc.
- $\mathbf{x} = \sum_i \delta(f_i)$

Example: Spam Email

`Your diploma puts a UUNIVERSITY JOB PLACEMENT COUNSELOR at your disposal.`

$$\mathbf{x} = \delta(\texttt{misspelling}) + \delta(\texttt{allcapital}) + \delta(\texttt{trigger:diploma}) + \ldots$$

$$\mathbf{x}^\top = \begin{bmatrix} 0 \\ \vdots \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ \vdots \\ 0 \\ 0 \end{bmatrix} = \begin{array}{l} \begin{bmatrix} 1 \\ \vdots \\ 1 \\ 1 \end{bmatrix} \end{array} \begin{array}{l} \texttt{misspelling} \\ \vdots \\ \texttt{capital} \\ \texttt{word:diploma} \end{array}$$

Text Classification: Output Representation

1. Extract pertinent information from the sentence.
2. Use this to construct an input representation.
3. Classify this vector into an output class.

Output Representation:

- How do encode the output classes?
- We will use a one-hot output encoding.
- In future lectures, efficiency of output encoding.

Output Class Notation

- $\mathcal{C} = \{1, \ldots, d_{\text{out}}\}$; possible output classes
- $c \in \mathcal{C}$; always one true output class
- $\mathbf{y} = \delta(c) \in \mathbb{R}^{1 \times d_{\text{in}}}$; true one-hot output representation

Output Form: Binary Classification

Examples: spam/not-spam, good review/bad review, relevant/irrelevant document, many others.

- $d_{\text{out}} = 2$; two possible classes
- In our notation,

$$\begin{aligned} \textit{bad} \;\; c = 1 \quad \mathbf{y} &= \begin{bmatrix} 1 & 0 \end{bmatrix} \text{ vs.} \\ \textit{good} \;\; c = 2 \quad \mathbf{y} &= \begin{bmatrix} 0 & 1 \end{bmatrix} \end{aligned}$$

- Can also use a single output *sign* representation with $d_{\text{out}} = 1$

Output Form: Multiclass Classification

Examples: Yelp stars, etc.

- $d_{\text{out}} = 5$; for examples
- In our notation, one star, two star...

$$\begin{aligned} \star\ c = 1 \quad \mathbf{y} &= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \end{bmatrix} \text{ vs.} \\ \star\star\ c = 2 \quad \mathbf{y} &= \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \end{bmatrix} \ldots \end{aligned}$$

Examples: Word Prediction (Unit 3)

- $d_{\text{out}} > 100,000$;
- In our notation, $\mathcal{C}$ is vocabulary and each c is a word.

$$\begin{aligned} \textit{the}\ c = 1 \quad \mathbf{y} &= \begin{bmatrix} 1 & 0 & 0 & 0 & \ldots & 0 \end{bmatrix} \text{ vs.} \\ \textit{dog}\ c = 2 \quad \mathbf{y} &= \begin{bmatrix} 0 & 1 & 0 & 0 & \ldots & 0 \end{bmatrix} \ldots \end{aligned}$$

Evaluation

- Consider evaluating accuracy on outputs $\mathbf{y}_1, \dots, \mathbf{y}_n$.
- Given a predictions $\hat{c}_1 \dots \hat{c}_n$ we measure accuracy as,

$$\sum_{i=1}^{n} \frac{\mathbf{1}(\delta(\hat{c}_i) = \mathbf{y}_i)}{n}$$

- Simplest of several different metrics we will explore in the class.

Contents

Supervised Machine Learning

Let,

- $(\mathbf{x}_1, \mathbf{y}_1), \dots, (\mathbf{x}_n, \mathbf{y}_n)$; training data
- $\mathbf{x}_i \in \mathbb{R}^{1 \times d_{\text{in}}}$; input representations
- $\mathbf{y}_i \in \mathbb{R}^{1 \times d_{\text{out}}}$; true output representations (one-hot vectors)

Goal: Learn a classifier from input to output classes.

Note:

- $\mathbf{x}_i$ is an input vector $x_{i,j}$ is element of the vector, or just x_j when there is a clear single input .
- Practically, store design matrix $\mathbf{X} \in \mathbb{R}^{n \times d_{\text{in}}}$ and output classes.

Experimental Setup

- Data is split into three parts training, validation, and test.
- Experiments are all run on training and validation, test is final output.
- For assignments, full training and validation data, and only inputs for test.

For very small text classification data sets,

- Use K-fold cross-validation.
  1. Split into K folds (equal splits).
  2. For each fold, train on other K-1 folds, test on current fold.

Linear Models for Classification

Linear model,

$$\hat{\mathbf{y}} = f(\mathbf{x}\mathbf{W} + \mathbf{b})$$

- $\mathbf{W} \in \mathbb{R}^{d_{\text{in}} \times d_{\text{out}}}, \mathbf{b} \in \mathbb{R}^{1 \times d_{\text{out}}}$; model parameters
- $f : \mathbb{R}^{d_{\text{out}}} \mapsto \mathbb{R}^{d_{\text{out}}}$; activation function
- Sometimes $\mathbf{z} = \mathbf{x}\mathbf{W} + \mathbf{b}$ informally "score" vector.
- Note $\mathbf{z}$ and $\hat{\mathbf{y}}$ are not one-hot.

Class prediction,

$$\hat{c} = \operatorname*{arg\,max}_{i \in \mathcal{C}} \hat{y}_i = \operatorname*{arg\,max}_{i \in \mathcal{C}} (\mathbf{x}\mathbf{W} + \mathbf{b})_i$$

Interpreting Linear Models

Parameters give scores to possible outputs,

- $W_{f,i}$ is the score for sparse feature f under class i
- b_i is a prior score for class i
- $\hat{y}_i$ is the total score for class i
- $\hat{c}$ is highest scoring class under the linear model.

Example:

- For single feature score,

$$[\beta_1, \beta_2] = \delta(\texttt{word:dreadful})\mathbf{W},$$

Expect $\beta_1 > \beta_2$ (assuming 2 is class *good*).

Probabilistic Linear Models

Can estimate a linear model probabilistically,

- Let output be a random variable Y, with sample space $\mathcal{C}$.
- Representation be a random vector X.
- (Simplified frequentist representation)
- Interested in estimating parameters θ,

$$P(Y|X;\theta)$$

Informally we use $p(\mathbf{y} = \delta(c)|\mathbf{x})$ for $P(Y = c|X = \mathbf{x})$.

Generative Model. Joint Log-Likelihood as Loss

- $(\mathbf{x}_1, \mathbf{y}_1), \ldots, (\mathbf{x}_n, \mathbf{y}_n)$; supervised data
- Select parameters to maximize likelihood of training data.

$$\mathcal{L}(\theta) = -\sum_{i=1}^{n} \log p(\mathbf{x}_i, \mathbf{y}_i; \theta)$$

  For linear models $\theta = (\mathbf{W}, \mathbf{b})$

- Do this by minimizing negative log-likelihood (NLL).

$$\arg\min_{\theta} \mathcal{L}(\theta)$$

Multinomial Naive Bayes

Reminder, joint probability chain rule,

$$p(\mathbf{x}, \mathbf{y}) = p(\mathbf{x}|\mathbf{y})p(\mathbf{y})$$

For a sparse features, with observed classes we can write as,

$$\begin{aligned}
p(\mathbf{x}, \mathbf{y}) &= p(x_{f_1} = 1, \ldots, x_{f_k} = 1 | \mathbf{y} = \delta(c))p(\mathbf{y} = \delta(c)) = \\
&= \prod_{i=1}^{k} p(x_{f_i} = 1 | x_{f_1} = 1, \ldots, x_{f_{i-1}} = 1, \mathbf{y} = \delta(c))p(\mathbf{y} = \delta(c)) \approx \\
&= \prod_{i=1}^{k} p(x_{f_i} = 1 | \mathbf{y})p(\mathbf{y})
\end{aligned}$$

First is by chain-rule, second is by independence assumption.

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First is by chain-rule, second is by independence assumption.

Estimating Multinomial Distributions

Let S be a random variable with sample space $\mathcal{S}$ and we have observations $s_1, \ldots, s_n$,

- $P(S = s; \theta) = \mathrm{Cat}(s; \theta)$; parameterized as a multinomial distribution.
- Minimizing NLL for multinomial (MLE) for data has a closed-form.

$$P(S = s; \theta) = \mathrm{Cat}(s; \theta) = \sum_{i=1}^{n} \frac{\mathbf{1}(s_i = s)}{n}$$

- Exercise: Derive this by minimizing $\mathcal{L}$.
- Also called categorical or multinoulli (in Murphy).

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Multinomial Naive Bayes

- Both $p(\mathbf{y})$ and $p(\mathbf{x}|\mathbf{y})$ are parameterized as multinomials.
- Fit first as,

$$p(\mathbf{y} = \delta(c)) = \sum_{i=1}^{n} \frac{1(\mathbf{y}_i = c)}{n}$$

- Fit second using count matrix $\mathbf{F}$,
  - Let

$$F_{f,c} = \sum_{i=1}^{n} \mathbf{1}(\mathbf{y}_i = c)\mathbf{1}(x_{i,f} = 1) \text{ forall } c \in \mathcal{C}, f \in \mathcal{F}$$

  - Then,

$$p(x_f = 1|\mathbf{y} = \delta(c)) = \frac{F_{f,c}}{\sum_{f' \in \mathcal{F}} F_{f',c}}$$

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Alternative: Multivariate Bernoulli Naive Bayes

- Both $p(\mathbf{y})$ is multinomial as above and $p(x_f|\mathbf{y})$ is Bernoulli over each features .
- Fit class as Categorical,

$$p(\mathbf{y} = \delta(c)) = \sum_{i=1}^{n} \frac{1(\mathbf{y}_i = c)}{n}$$

- Fit features using count matrix $\mathbf{F}$,
  - Let

$$F_{f,c} = \sum_{i=1}^{n} \mathbf{1}(\mathbf{y}_i = c)\mathbf{1}(x_{i,f} = 1) \text{ forall } c \in \mathcal{C}, f \in \mathcal{F}$$

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  - Then,
    $$p(x_f|\mathbf{y} = \delta(c)) = \frac{F_{f,c}}{\sum_{i=1}^{n} \mathbf{1}(\mathbf{y}_i = c)}$$

Getting a Conditional Distribution

- Generative models estimates of $P(X, Y)$, we want $P(Y|X)$.
- Bayes Rule,

$$p(\mathbf{y}|\mathbf{x}) = \frac{p(\mathbf{x}|\mathbf{y})p(\mathbf{y})}{p(\mathbf{x})}$$

- In log-space,

$$\log p(\mathbf{y}|\mathbf{x}) = \log p(\mathbf{x}|\mathbf{y}) + \log p(\mathbf{y}) - \log p(\mathbf{x})$$

Prediction with Naive Bayes

- For prediction, last term is constant, so

$$\arg\max_{c} \log p(\mathbf{y} = \delta(c)|\mathbf{x}) = \log p(\mathbf{x}|\mathbf{y} = \delta(c)) + \log p(\mathbf{y} = \delta(c))$$

- Can write as linear model,

$$W_{f,c} = \log p(x_f = 1|\mathbf{y} = c) \text{ for all } c \in \mathcal{C}, f \in \mathcal{F}$$

$$b_c = \log p(\mathbf{y} = \delta(c)) \text{ for all } c \in \mathcal{C}$$

Getting a Conditional Distribution

- What if we want conditional probabilities?

$$p(\mathbf{y}|\mathbf{x}) = \frac{p(\mathbf{x}|\mathbf{y})p(\mathbf{y})}{p(\mathbf{x})} = \frac{p(\mathbf{x}|\mathbf{y})p(\mathbf{y})}{\sum_{c' \in \mathcal{C}} p(\mathbf{x}, \mathbf{y} = \delta(c'))}$$

- Denominator is acquired by renormalizing,

$$p(\mathbf{y}|\mathbf{x}) \propto p(\mathbf{x}|\mathbf{y})p(\mathbf{y})$$

Practical Aspects: Calculating Log-Sum-Exp

- Because of numerical issues, calculate in log-space,

$$f(\mathbf{z}) = \log p(\mathbf{y} = \delta(c)|\mathbf{x}) = \log z_c - \log \sum_{c' \in \mathcal{C}} \exp(z_{c'})$$

where for naive Bayes

$$\mathbf{z} = \mathbf{x}\mathbf{W} + \mathbf{b}$$

- However hard to calculate,

$$\log \sum_{c' \in \mathcal{C}} \exp(\hat{z}_{c'})$$

- Instead

$$\log \sum_{c' \in \mathcal{C}} \exp(\hat{y}_{c'} - M) + M$$

where $M = \max_{c' \in \mathcal{C}} \hat{z}_{c'}$

Practical Aspects: Calculating Log-Sum-Exp

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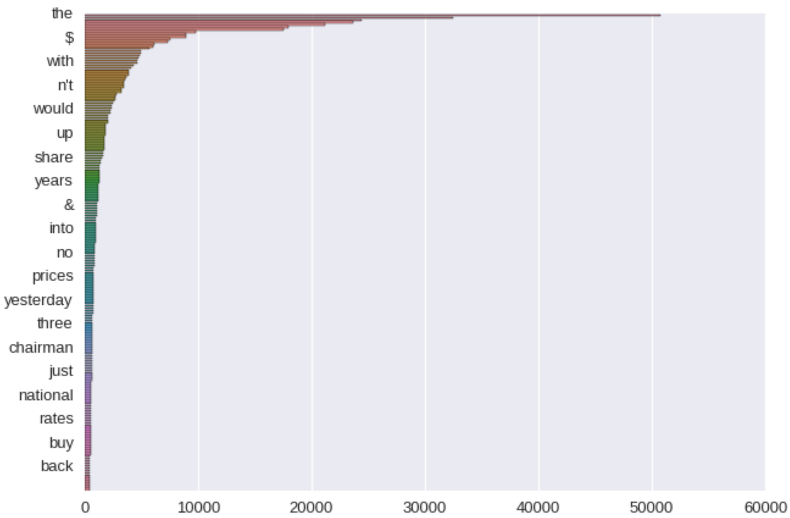
Digression: Zipf's Law



Laplace Smoothing

Method for handling the long tail of words by distributing mass,

- Add a value of α to each element in the sample space before normalization.

$$\theta_s = \frac{\alpha + \sum_{i=1}^{n} \mathbf{1}(s_i = s)}{\alpha|\mathcal{S}| + n}$$

- (Similar to Dirichlet prior in a Bayesian interpretation.)

For naive Bayes:

$$\hat{\mathbf{F}} = \alpha + F$$

Laplace Smoothing

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Naive Bayes In Practice

- Very fast to train
- Relatively interpretable.
- Performs quite well on small datasets

Method	RT-s	MPQA	CR	Subj.
MNB-uni	77.9	85.3	79.8	**92.6**
MNB-bi	**79.0**	**86.3**	80.0	<u>**93.6**</u>
SVM-uni	76.2	86.1	79.0	90.8
SVM-bi	77.7	<u>**86.7**</u>	80.8	91.7
NBSVM-uni	**78.1**	85.3	80.5	92.4
NBSVM-bi	<u>**79.4**</u>	**86.3**	<u>**81.8**</u>	**93.2**
RAE	76.8	85.7	–	–
RAE-pretrain	77.7	**86.4**	–	–
Voting-w/Rev.	63.1	81.7	74.2	–

(RT-S [movie review], CR [customer reports], MPQA [opinion polarity], SUBJ [subjectivity])

Multiclass Logisitic Regression

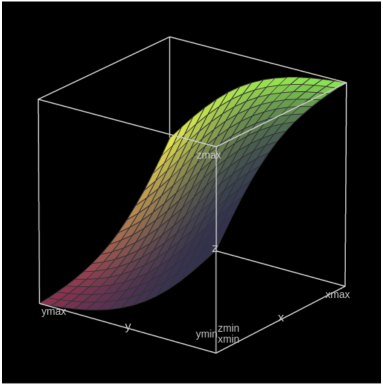
Alternative parametrization of probabilistic model.

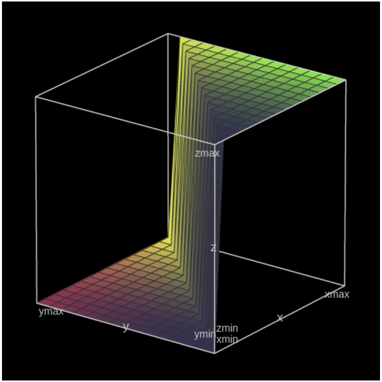
Use a softmax to force a distribution,

$$\text{softmax}(\mathbf{z}) = \frac{\exp(\mathbf{z})}{\sum_{c \in \mathcal{C}} \exp(z_c)}$$

- Exercise: Confirm always gives a distribution.
- Denominator known as *partition* function (we'll see many times).

Why is it called the softmax?





$$\text{softmax}([x \ \ y]) = \frac{\exp(x)}{\exp(x) + \exp(y)}$$

$$\arg\max([x \ \ y]) = \mathbf{1}(x > y)$$

Multiclass Logistic Regression

$$\hat{\mathbf{y}} = f(\mathbf{x}\mathbf{W} + \mathbf{b})$$

Directly estimate the conditional distribution (discriminative)

$$\log p(\mathbf{y} = c | \mathbf{x}; \theta) = \hat{y} = \log \operatorname{softmax}(\mathbf{z}) = \frac{\exp(z_c)}{\sum_{c'} \exp(z_{c'})}$$

- where $\mathbf{z} = \mathbf{x}\mathbf{W} + \mathbf{b}$
- $\mathbf{W} \in \mathbb{R}^{d_{\text{in}} \times d_{\text{out}}}, \mathbf{b} \in \mathbb{R}^{1 \times d_{\text{out}}}$; model parameters

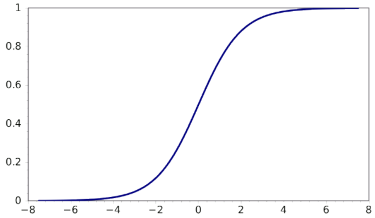
Special Case: Logistic Regression

For binary classification:

$$\begin{aligned}\text{softmax}([z_1\ z_2]) &= \frac{\exp(z_1)}{\exp(z_1)+\exp(z_2)} \\ &= \frac{1}{1+\exp(-(z_1-z_2))} = \sigma(z_1-z_2)\end{aligned}$$

Logistic sigmoid function:

$$\sigma(t) = \frac{1}{1+\exp(-t)}$$



A Model with Many Names

- Multinomial Logistic Regression
- Log-Linear Model (particularly in NLP)
- Softmax Regression
- Max-Entropy (MaxEnt)

Fitting Parameters

Recall probabilistic objective is:

$$\mathcal{L}(\theta) = -\sum_{i=1}^{n} \log p(\mathbf{y}_i | \mathbf{x}_i; \theta) = \sum_{i=1}^{n} L_{cross-entropy}(\mathbf{y}_i, \hat{\mathbf{y}}_i)$$

4 And the distribution is parameterized as a softmax,

$$\begin{aligned} L_{cross-entropy}(\mathbf{y}, \hat{\mathbf{y}}) &= -\log p(\mathbf{y} = c | \mathbf{x}; \theta) \\ &= \log \text{softmax}(\mathbf{z})_c \\ &= \hat{z}_c - \log \sum_{c' \in \mathcal{C}} \exp(z_{c'}) \end{aligned}$$

However, this is much harder to minimize, **no closed form**.

Symbolic Gradients

- Partials of $L(y, \hat{y})$

$$\frac{\partial L(y, \hat{y})}{\partial \hat{y}_j} = \frac{\mathbf{1}(y_j = 1)}{\hat{y}_j}$$

- Partials of $\hat{\mathbf{y}} = \text{softmax}(\mathbf{z})$

$$\frac{\partial \hat{y}_j}{\partial z_i} = \begin{cases} \hat{y}_i(1 - \hat{y}_i) & i = j \\ -\hat{y}_i \hat{y}_j & i \neq j \end{cases}$$

- Partials of $\mathbf{z} = \mathbf{x}\mathbf{W} + \mathbf{b}$

$$\frac{\partial z_i}{\partial b_{i'}} = \mathbf{1}(i = i') \quad \frac{\partial z_i}{\partial W_{f,i'}} = \mathbf{1}(i = i')$$

Homework: Compute these for yourself.

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$$\frac{\partial \hat{y}_j}{\partial z_i} = \begin{cases} \hat{y}_i(1 - \hat{y}_i) & i = j \\ -\hat{y}_i \hat{y}_j & i \neq j \end{cases}$$

- Partials of $\mathbf{z} = \mathbf{x}\mathbf{W} + \mathbf{b}$

$$\frac{\partial z_i}{\partial b_{i'}} = \mathbf{1}(i = i') \quad \frac{\partial z_i}{\partial W_{f,i'}} = \mathbf{1}(i = i')$$

Homework: Compute these for yourself.

Review: Chain Rule

Assume we have a function and a loss:

$$f : \mathbb{R}^m \to \mathbb{R}^n \quad L : \mathbb{R}^n \to \mathbb{R}$$

Then

$$\frac{\partial L(f(\mathbf{x}))}{\partial x_i} = \sum_{j=1}^{n} \frac{\partial f(\mathbf{x})_j}{\partial x_i} \frac{\partial L(f(\mathbf{x}))}{\partial f(\mathbf{x})_j}$$

For Softmax regression:

$$\frac{\partial L(y, \hat{y})}{\partial z_i} = \sum_j \frac{\partial \hat{y}_j}{\partial z_i} \frac{\mathbf{1}(y_j = 1)}{\hat{y}_j} = \begin{cases} 1 - \hat{y}_i & y_i = 1 \\ -\hat{y}_j & ow. \end{cases}$$

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Minimizing Gradients in Practice

Consider one example $(\mathbf{x}, \mathbf{y})$, we compute forward and then backward,

1. Compute scores $\mathbf{z} = \mathbf{x}\mathbf{W} + \mathbf{b}$
2. Compute softmax of scores, $\hat{\mathbf{y}} = \text{softmax}(\mathbf{z})$
3. Compute loss of scores, $L(\mathbf{y}, \hat{\mathbf{y}})$
4. Compute gradient $\frac{\partial L(y, \hat{y})}{\partial \hat{y}_j}$.
5. Compute gradient $\frac{\partial L(y, \hat{y})}{\partial z_i}$.
6. Compute gradient of $\mathbf{b}$ for all $i' \in \mathcal{C}$ and $\mathbf{W}$ for all $i' \in \mathcal{C}, f \in \mathcal{F}$,

$$\frac{\partial L}{\partial b_i'} = \frac{\partial L}{\partial z_i'} \qquad \frac{\partial L}{\partial W_{f,i'}} = \frac{\partial L}{\partial z_i'}$$

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Gradient-Based Optimization: SGD

procedure SGD
    while training criterion is not met **do**
        Sample a training example $\mathbf{x}_i, \mathbf{y}_i$
        Compute the loss $L(\hat{\mathbf{y}}_i, \mathbf{y}_i; \theta)$
        Compute gradients $\hat{\mathbf{g}}$ of $L(\hat{\mathbf{y}}_i, \mathbf{y}_i; \theta)$ with respect to θ
        $\theta \leftarrow \theta + \eta_k \hat{\mathbf{g}}$
    end while
    return θ
end procedure

Gradient-Based Optimization: Minibatch SGD

while training criterion is not met **do**

  Sample a minibatch of m examples $(\mathbf{x}_1, \mathbf{y}_1), \ldots, (\mathbf{x}_m, \mathbf{y}_m)$

  $\hat{\mathbf{g}} \leftarrow 0$

  for $i = 1 \text{ to } m$ **do**

    Compute the loss $L(\hat{\mathbf{y}}_i, \mathbf{y}_i; \theta)$

    Compute gradients $\mathbf{g}'$ of $L(\hat{\mathbf{y}}_i, \mathbf{y}_i; \theta)$ with respect to θ

    $\hat{\mathbf{g}} \leftarrow \hat{\mathbf{g}} + \frac{1}{m}\mathbf{g}'$

  end for

  $\theta \leftarrow \theta + \eta_k \hat{\mathbf{g}}$

end while

return θ

Softmax Notes: Regularization

$$\mathcal{L}(\theta) = -\sum_{i=1}^{n} L(\hat{\mathbf{y}}, \mathbf{y}) + ||\theta||_2^2$$

Softmax Notes: Calculating Log-Sum-Exp

- Calculating $\log \sum_{c' \in \mathcal{C}} \exp(\hat{y}_{c'}$ directly numerical issues.
- Instead $\log \sum_{c' \in \mathcal{C}} \exp(\hat{y}_{c'} - M) + M$ where $M = \max_{c' \in \mathcal{C}} \hat{y}_c'$

Pros and Cons of Logistic Regression

- Less strong independence assumption.
- Can be very effective with good features.
- Still yields a probability distribution.
- Fitting parameters is more difficult.

Similar models make will be the main focus of this class.

Other Loss Functions

What if we just try to directly find $\mathbf{W}$ and $\mathbf{b}$?

$$\hat{\mathbf{y}} = \mathbf{x}\mathbf{W} + \mathbf{b}$$

- No longer a probabilistic interpretation.
- Just try to find parameters that fit training data.

Hinge Loss

$$\mathcal{L}(\theta) = \sum_{i=1}^{n} L_{hinge}(\hat{\mathbf{y}}, \mathbf{y})$$

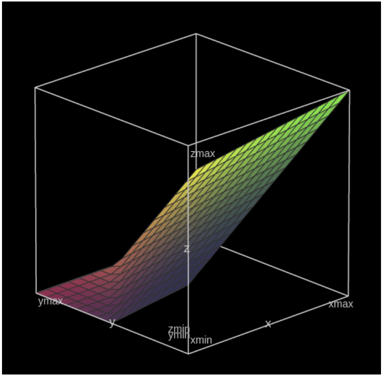
$$L(\hat{\mathbf{y}}, \mathbf{y}) = \max\{0, 1 - (\hat{y}_c + \hat{y}_{c'})\}$$

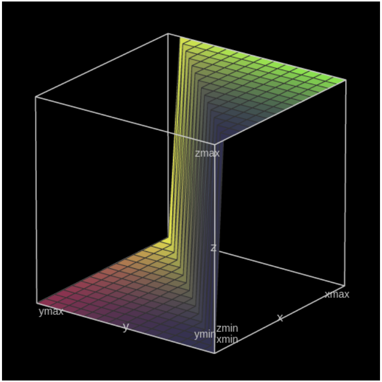
Where

- Let c be defined as true class $y_{i,c} = 1$
- Let c' be defined as the highest scoring non-true class

$$c' = \arg\max_{i \in \mathcal{C} \setminus \{c\}} \hat{y}_i$$

Hinge Loss





$$hinge(\hat{\mathbf{y}}) = \mathbf{1}(\max\{0, 1 - (y - x)\})$$

$$\arg\max([x \;\; y]) = \mathbf{1}(x > y)$$

Symbolic Gradients

- Let c be defined as true class $y_{i,c} = 1$
- Let c' be defined as the highest scoring non-true class

$$c' = \arg\max_{i \in \mathcal{C} \setminus \{c\}} \hat{y}_i$$

Much simpler than logistic regression.

- Partials of $L(y, \hat{y})$

$$\frac{\partial L(y, k\hat{y})}{\partial \hat{y}_j} = \mathbf{1}(j = c) - \mathbf{1}(j = c')$$

Notes: Hinge Loss: Regularization

- Many different names,
  - Margin Classifier
  - Multiclass Hinge
  - Linear SVM
- Important to use regularization.

$$\mathcal{L}(\theta) = -\sum_{i=1}^{n} L(\hat{\mathbf{y}}, \mathbf{y}) + ||\theta||_2^2$$

- Can be much more efficient to train than LR. (No partition).

Results: Longer Reviews

Our results	RT-2k	IMDB	Subj.
MNB-uni	83.45	83.55	**92.58**
MNB-bi	85.85	86.59	**93.56**
SVM-uni	86.25	86.95	90.84
SVM-bi	87.40	**89.16**	91.74
NBSVM-uni	87.80	88.29	92.40
NBSVM-bi	**89.45**	**91.22**	**93.18**
BoW (bnc)	85.45	87.8	87.77
BoW (bΔt′c)	85.8	88.23	85.65
LDA	66.7	67.42	66.65
Full+BoW	87.85	88.33	88.45
Full+Unlab'd+BoW	**88.9**	88.89	88.13

IMDB (longer movie review), Subj (longer subjectivity)

- NBSVM is hinge-loss interpolated with Naive Bayes.