Neural Networks

$+$

Word Embeddings

CS 287

Contents

Tagging

Sentiment

Multiclass Sentiment

Tagging

- Straightforward setup.
- Lots of practical applications:
  - Spam Filtering
  - Sentiment
  - Text Categorization
  - e-discovery
  - Twitter Mining
  - Author Identification
  - ...
- Introduces machine learning notation.

However, a relatively solved problem these days.

Dataset: Penn Treebank

Penn Treebank,

- Central dataset in NLP.
- 1M word tokens, collected from Wall Street Journal.
- Annotated with syntactic structure.

Contents

Sentence Tagging

- $w_1, \ldots, w_n$; sentence words
- $t_1, \ldots, t_n$; sentence tags
- $\mathcal{C}$; output class, set of tags.

Window Model

Goal: predict t_5.

- Windowed word model.

$$w_1\, w_2 \left[w_3\, w_4\, w_5\, w_6\, w_7 \right] w_8$$

- w_3, w_4; left context
- w_6, w_7; right context

Boundary Cases

Goal: predict t_2.

$$\left[< s > w_1 w_2 w_3 w_4\right] w_5 w_6 w_7 w_8$$

Goal: predict t_8.

$$w_1 w_2 w_3 w_4 w_5 \left[w_6 w_7 w_8 < /s >< /s >\right]$$

Symbols $< s >$ and $< /s >$ represent boundary padding.

Review: Features

- $\mathcal{F}$; a discrete set of features types.
- $f_1 \in \mathcal{F}, \ldots, f_k \in \mathcal{F}$; active features for input.

  For a given sentence, let $f_1, \ldots f_k$ be the relevant features.
  Typically $k << |\mathcal{F}|$.

- Sparse representation of the input defined as,

$$\mathbf{x} = \sum_{i=1}^{k} \delta(f_i)$$

- $\mathbf{x} \in \mathbb{R}^{1 \times d_{\mathrm{in}}}$; input representation

Tag Features 1: Sparse Bag-of-Words

Representation is counts of input words,

- $\mathcal{F}$; the vocabulary of the language.
- $\mathbf{x} = \sum_i \delta(f_i)$

Example: ,

```
of New York-based Lorillard Corp.
```

$\mathbf{x} = \delta(\texttt{word:0:York-based}) + \delta(\texttt{word:-2:of}) + \delta(\texttt{word:-1:New}) + \delta(\texttt{wor}$

Tag Features 2: Sparse Word Properties

Representation based on word properties

- $\mathcal{F}$;

prefix(1)	prefix(2)	prefix(3)
suffix(1)	suffix(2)	suffix(3)
isFirstCapital	hasHyphen	endsInS
isAllCapital	hasDigit	isAllDigits
word		

- $\mathbf{x} = \sum_i \delta(f_i)$

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$\mathbf{x} = \delta(\texttt{word:0:York-based}) + \delta(\texttt{isFirstCapital:0:York-based}) + \delta(\texttt{s}$

The Role of Features

- Recall Zipf's law.
- Many words are ..
- Can capture patterns. example.

How much does this matter?

graph of tagging.

Sparse Tagging Model

- Create training data,

$$(\mathbf{x}_1, \mathbf{y}_1), \ldots, (\mathbf{x}_n, \mathbf{y}_n)$$

- Each $\mathbf{x}_i$ includes features of window.
- Each $\mathbf{y}_i$ is the one-hot tag encoding.
- Prediction accuracy is measured identically.

Naive Bayes/Logistic Regression for Tagging

- Setup is identical to text classification.
-

$$\hat{\mathbf{y}} = \mathbf{x}\mathbf{W} + \mathbf{b}$$

Contents

Non-Linear Models for Classification

- Neural network represent any non-linear classifier, for example

$$NN_1 = \tanh(\mathbf{x}\mathbf{W}^1 + \mathbf{b}^1))$$

$$= NN_1\mathbf{W}^2 + \mathbf{b}^2)$$

- Where $\mathbf{W}^1 \in \mathbb{R}^{d_{\mathrm{in}} \times dmid}$, $\mathbf{b}^1 \in \mathbb{R}^{1 \times dmid}$
- $\mathbf{W}^2 \in \mathbb{R}^{dmid \times dout}$, $\mathbf{b}^2 \in \mathbb{R}^{1 \times d_{\mathrm{out}}}$

Obligatory Diagram

Dense Features

Tag Features 2: Just Words

Representation can use specific aspects of text.

- $\mathcal{F}$; Spelling, all-capitals, trigger words, etc.
- $\mathbf{x} = \sum_i \delta(f_i)$

Example: Spam Email

`Your diploma puts a UUNIVERSITY JOB PLACEMENT COUNSELOR at your disposal.`

$$\mathbf{x} = v(\texttt{misspelling}) + v(\texttt{allcapital}) + v(\texttt{trigger:diploma}) + \dots$$

$$\mathbf{x}^\top = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ \vdots \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} = \begin{array}{l} \left[\begin{array}{c} 1 \\ \vdots \\ 1 \\ 1 \end{array}\right] \end{array} \begin{array}{l} \texttt{misspelling} \\ \vdots \\ \texttt{capital} \\ \texttt{word:diploma} \end{array}$$

Text Classification: Output Representation

1. Extract pertinent information from the sentence.
2. Use this to construct an input representation.
3. Classify this vector into an output class.

Output Representation:

- How do encode the output classes?
- We will use a one-hot output encoding.
- In future lectures, efficiency of output encoding.

Output Class Notation

- $\mathcal{C} = \{1, \ldots, d_{\text{out}}\}$; possible output classes
- $c \in \mathcal{C}$; always one true output class
- $\mathbf{y} = \delta(c) \in \mathbb{R}^{1 \times d_{\text{in}}}$; true one-hot output representation

Output Form: Binary Classification

Examples: spam/not-spam, good review/bad review, relevant/irrelevant document, many others.

- $d_{\text{out}} = 2$; two possible classes
- In our notation,

$$\begin{aligned} \textit{bad}\;\; c = 1 \quad \mathbf{y} &= \begin{bmatrix} 1 & 0 \end{bmatrix} \text{ vs.} \\ \textit{good}\;\; c = 2 \quad \mathbf{y} &= \begin{bmatrix} 0 & 1 \end{bmatrix} \end{aligned}$$

- Can also use a single output *sign* representation with $d_{\text{out}} = 1$

Output Form: Multiclass Classification

Examples: Yelp stars, etc.

- $d_{\text{out}} = 5$; for examples
- In our notation, one star, two star...

$$\begin{aligned} \star\ c = 1 \quad \mathbf{y} &= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \end{bmatrix} \text{ vs.} \\ \star\star\ c = 2 \quad \mathbf{y} &= \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \end{bmatrix} \ldots \end{aligned}$$

Examples: Word Prediction (Unit 3)

- $d_{\text{out}} > 100,000$;
- In our notation, $\mathcal{C}$ is vocabulary and each c is a word.

$$\begin{aligned} \textit{the}\ c = 1 \quad \mathbf{y} &= \begin{bmatrix} 1 & 0 & 0 & 0 & \ldots & 0 \end{bmatrix} \text{ vs.} \\ \textit{dog}\ c = 2 \quad \mathbf{y} &= \begin{bmatrix} 0 & 1 & 0 & 0 & \ldots & 0 \end{bmatrix} \ldots \end{aligned}$$

Evaluation

- Consider evaluating accuracy on outputs $\mathbf{y}_1, \ldots, \mathbf{y}_n$.
- Given a decisions $\hat{c}_1 \ldots \hat{c}_n$ we measure accuracy as,

$$\sum_{i=1}^{n} \frac{\mathbf{1}(\delta(\hat{c}_i) = \mathbf{y}_i)}{n}$$

- Simplest of several different metrics we will explore in the class.

Contents

Supervised Machine Learning

Let,

- $(\mathbf{x}_1, \mathbf{y}_1), \ldots, (\mathbf{x}_n, \mathbf{y}_n)$; training data
- $\mathbf{x}_i \in \mathbb{R}^{1 \times d_{\text{in}}}$; input representations
- $\mathbf{y}_i \in \mathbb{R}^{1 \times d_{\text{out}}}$; gold output representations (one-hot vectors)

Goal: Learn a classifier from input to output classes.

Note:

- $\mathbf{x}_i$ is an input vector $x_{i,j}$ is element of the vector, or just x_j when there is a clear single input .
- Practically, store design matrix $\mathbf{X} \in \mathbb{R}^{n \times d_{\text{in}}}$ and output classes.

Experimental Setup

- Data is split into three parts training, validation, and test.
- Experiments are all run on training and validation, test is final output.
- For assignments, full training and validation data, and only inputs for test.

For very small text classification data sets,

- Use K-fold cross-validation.
  1. Split into K folds (equal splits).
  2. For each fold, train on other K-1 folds, test on current fold.

Linear Models for Classification

Linear model,

$$\hat{\mathbf{y}} = \mathbf{x}\mathbf{W} + \mathbf{b}$$

- $\mathbf{W} \in \mathbb{R}^{d_{\text{in}} \times d_{\text{out}}}, \mathbf{b} \in \mathbb{R}^{1 \times d_{\text{out}}}$; model parameters
- Note $\hat{\mathbf{y}}$ is **not** one-hot, informally "score" vector.

Class decision,

$$\hat{c} = \operatorname*{arg\,max}_{i \in \mathcal{C}} \hat{y}_i$$

Interpreting Linear Models

Parameters give scores to possible outputs,

- $W_{f,i}$ is the score for sparse feature f under class i
- b_i is a prior score for class i
- $\hat{y}_i$ is the total score for class i
- $\hat{c}$ is highest scoring class under the linear model.

Example:

- For single feature score,

$$[\beta_1, \beta_2] = \delta(\texttt{word:dreadful})\mathbf{W},$$

Expect $\beta_2 > \beta_1$ (assuming 2 is class *good*).

Probabilistic Linear Models

Can estimate a linear model probabilistically ,

- Let output be a random variable Y, with sample space $\mathcal{C}$.
- Representation be a random vector X.
- Interested in estimating parameters θ of,

$$P(Y|X;\theta)$$

Informally we use $p(\mathbf{y} = c|\mathbf{x})$ for $P(Y = c|X = \mathbf{x})$.

Log-Likelihood as Loss

- $(\mathbf{x}_1, \mathbf{y}_1), \ldots, (\mathbf{x}_n, \mathbf{y}_n)$; supervised data
- Select parameters to maximize likelihood of training data.

$$\mathcal{L}(\theta) = -\sum_{i=1}^{n} \log p(\mathbf{y}_i | \mathbf{x}_i; \theta)$$

  For linear models $\theta = (\mathbf{W}, \mathbf{b})$
- Do this by minimizing negative log-likelihood (NLL).

$$\arg\min_{\theta} \mathcal{L}(\theta)$$

Naive Bayes 1: Probabilistic Factorization

Reminder, Bayes Rule

$$p(\mathbf{y}|\mathbf{x}) = \frac{p(\mathbf{x}|\mathbf{y})p(\mathbf{y})}{p(\mathbf{x})}$$

Can be instead written (with $\propto$ as normalizing factor)

$$p(\mathbf{y}|\mathbf{x}) \propto p(\mathbf{x}|\mathbf{y})p(\mathbf{y})$$

For NLL, $p(\mathbf{x})$ doesn't matter, estimate $p(\mathbf{x}|\mathbf{y})$ and $p(\mathbf{y})$.

For a sparse model, with observed classes we can write as,

$$p(x_{f_1} = 1, \ldots, x_{f_k} = 1|\mathbf{y} = c)p(\mathbf{y} = c)$$

Naive Bayes 2: Independence Assumption

$$p(x_{f_1} = 1, \ldots, x_{f_k} = 1 | \mathbf{y} = c) p(\mathbf{y} = c) =$$

$$\prod_{i=1}^{k} p(x_{f_i} = 1 | x_{f_1} = 1, \ldots, x_{f_{i-1}} = 1, \mathbf{y} = c) p(\mathbf{y} = c) \approx$$

$$\prod_{i=1}^{k} p(x_{f_i} | \mathbf{y}) p(\mathbf{y})$$

First is by chain-rule, second is by assumption.

Multinomial Model

Brief aside,

- $P(S;\theta)$; parameterized as a multinomial distribution.
- Minimizing NLL for multinomial for data has a closed-form.

$$P(S = s;\theta) = \theta_s = \sum_{i=1}^{n} \frac{\mathbf{1}(s_i = s)}{n}$$

- Exercise: Derive this by minimizing $\mathcal{L}$.

Multinomial Naive Bayes

- Both $p(\mathbf{y})$ and $p(\mathbf{x}|\mathbf{y})$ are parameterized as multinomials.
- Fit first as,

$$p(\mathbf{y} = c) = \sum_{i=1}^{n} \frac{1(\mathbf{y}_i = c)}{n}$$

- Fit second using count matrix $\mathbf{F}$,
  - Let

$$F_{f,c} = \sum_{i=1}^{n} \mathbf{1}(\mathbf{y}_i = c)\mathbf{1}(x_{i,f} = 1) \text{ forall } c \in \mathcal{C}, f \in \mathcal{F}$$

  - Then,

$$p(x_f = 1|\mathbf{y} = c) = \frac{F_{f,c}}{\sum_{f' \in \mathcal{F}} F_{f',c}}$$

How does this become a linear classifier?

$$W_{f,c} = \log p(x_f = 1|\mathbf{y} = c)$$

$$b_c = \log p(\mathbf{y} = c) \text{ forall } c \in \mathcal{Y}$$

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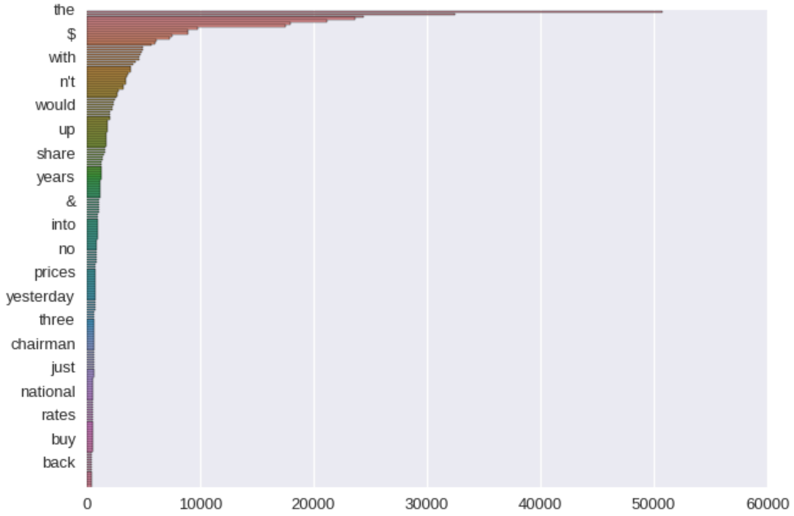
Digression: Zipf's Law



Laplacian Smoothing

Method for handling the long tail of words by distributing mass,

- Add a value of α to each element in the sample space before normalization.

$$\theta_s = \frac{\alpha + \sum_{i=1}^{n} \mathbf{1}(s_i = s)}{\alpha|\mathcal{S}| + n}$$

- (Similar to Dirichlet prior in a Bayesian interpretation.)

For naive Bayes:

$$\hat{\mathbf{F}} = \alpha + F$$

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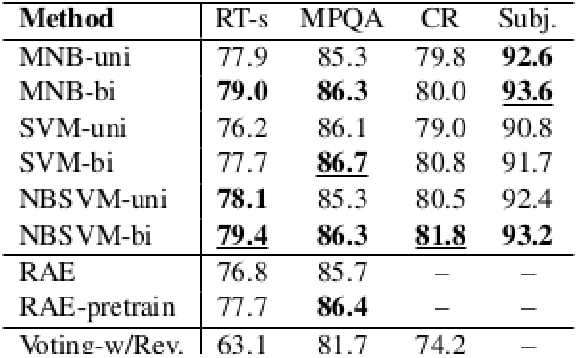
Naive Bayes In Practice

- Very fast to train
- Relatively interpretable.
- Performs quite well on small datasets **?**



Method	RT-s	MPQA	CR	Subj.
MNB-uni	77.9	85.3	79.8	**92.6**
MNB-bi	**79.0**	**86.3**	80.0	**93.6**
SVM-uni	76.2	86.1	79.0	90.8
SVM-bi	77.7	**86.7**	80.8	91.7
NBSVM-uni	**78.1**	85.3	80.5	92.4
NBSVM-bi	**79.4**	**86.3**	**81.8**	**93.2**
RAE	76.8	85.7	–	–
RAE-pretrain	77.7	**86.4**	–	–
Voting-w/Rev.	63.1	81.7	74.2	–

(RT-S [movie review], CR [customer reports], MPQA [opinion polarity], SUBJ [subjectivity])

Multiclass Logisitic Regression

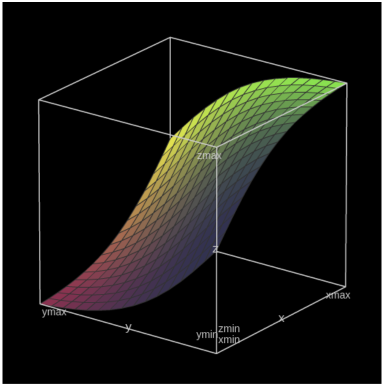
Alternative parametrization of probabilistic model.

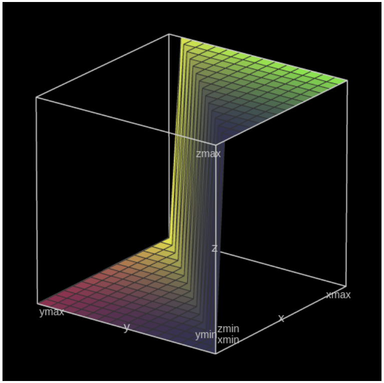
Use a softmax to force a distribution,

$$\text{softmax}(\mathbf{z}) = \frac{\exp(\mathbf{z})}{\displaystyle\sum_{c \in \mathcal{C}} \exp(z_c)}$$

- Exercise: Confirm always gives a distribution.
- Denominator known as *partition* function (we'll see many times).

Why is it called the softmax?





$$\mathrm{softmax}([x\ \ y]) = \frac{\exp(x)}{\exp(x) + \exp(y)}$$

$$\arg\max([x\ \ y]) = \mathbf{1}(x > y)$$

Multiclass logistic regression

$$\mathbf{z} = \mathbf{x}\mathbf{W} + \mathbf{b}$$

$$p(\mathbf{y} = c|\mathbf{x}; \theta) = \hat{y} = \text{softmax}(\mathbf{z}) = \frac{\exp(z_c)}{\sum_{c'} \exp(z_{c'})}$$

- $\mathbf{W} \in \mathbb{R}^{d_{\text{in}} \times d_{\text{out}}}$, $\mathbf{b} \in \mathbb{R}^{1 \times d_{\text{out}}}$: model parameters

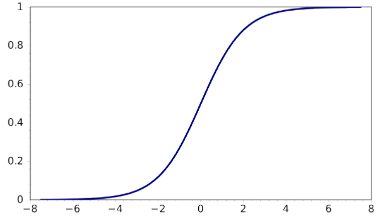
Special Case: Logistic Regression

For binary classification:

$$\begin{aligned}\text{softmax}([z_1 \;\; z_2]) &= \frac{\exp(z_1)}{\exp(z_1)+\exp(z_2)} \\ &= \frac{1}{1+\exp(-(z_1-z_2))} = \sigma(z_1-z_2)\end{aligned}$$

Logistic sigmoid function:

$$\sigma(t) = \frac{1}{1+\exp(-t)}$$



A Model with Many Names

- Multinomial Logistic Regression
- Log-Linear Model (particularly in NLP)
- Softmax Regression
- Max-Entropy (MaxEnt)

Fitting Parameters

Recall probabilistic objective is:

$$\mathcal{L}(\theta) = -\sum_{i=1}^{n} \log p(\mathbf{y}_i | \mathbf{x}_i; \theta) = \sum_{i=1}^{n} L_{cross-entropy}(\mathbf{y}_i, \hat{\mathbf{y}}_i)$$

4 And the distribution is parameterized as a softmax,

$$\begin{aligned} L_{cross-entropy}(\mathbf{y}, \hat{\mathbf{y}}) &= -\log p(\mathbf{y} = c | \mathbf{x}; \theta) \\ &= \log \text{softmax}(\mathbf{z})_c \\ &= \hat{z}_c - \log \sum_{c' \in \mathcal{C}} \exp(z_{c'}) \end{aligned}$$

However, this is much harder to minimize, **no closed form**.

Symbolic Gradients

- Partials of $L(y, \hat{y})$

$$\frac{\partial L(y, \hat{y})}{\partial \hat{y}_j} = \frac{\mathbf{1}(y_j = 1)}{\hat{y}_j}$$

- Partials of $\hat{\mathbf{y}} = \text{softmax}(\mathbf{z})$

$$\frac{\partial \hat{y}_j}{\partial z_i} = \begin{cases} \hat{y}_i(1 - \hat{y}_i) & i = j \\ -\hat{y}_i \hat{y}_j & i \neq j \end{cases}$$

- Partials of $\mathbf{z} = \mathbf{x}\mathbf{W} + \mathbf{b}$

$$\frac{\partial z_i}{\partial b_{i'}} = \mathbf{1}(i = i') \quad \frac{\partial z_i}{\partial W_{f,i'}} = \mathbf{1}(i = i')$$

Homework: Compute these for yourself.

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Homework: Compute these for yourself.

Review: Chain Rule

Assume we have a function and a loss:

$$f : \mathbb{R}^m \to \mathbb{R}^n \quad L : \mathbb{R}^n \to \mathbb{R}$$

Then

$$\frac{\partial L(f(\mathbf{x}))}{\partial x_i} = \sum_{j=1}^{n} \frac{\partial f(\mathbf{x})_j}{\partial x_i} \frac{\partial L(f(\mathbf{x}))}{\partial f(\mathbf{x})_j}$$

For Softmax regression:

$$\frac{\partial L(y, \hat{y})}{\partial z_i} = \sum_j \frac{\partial \hat{y}_j}{\partial z_i} \frac{\mathbf{1}(y_j = 1)}{\hat{y}_j} = \begin{cases} 1 - \hat{y}_i & y_i = 1 \\ -\hat{y}_j & ow. \end{cases}$$

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Minimizing Gradients in Practice

Consider one example $(\mathbf{x}, \mathbf{y})$, we compute forward and then backward,

1. Compute scores $\mathbf{z} = \mathbf{x}\mathbf{W} + \mathbf{b}$
2. Compute softmax of scores, $\hat{\mathbf{y}} = \text{softmax}(\mathbf{z})$
3. Compute loss of scores, $L(\mathbf{y}, \hat{\mathbf{y}})$
4. Compute gradient $\frac{\partial L(y,\hat{y})}{\partial \hat{y}_j}$.
5. Compute gradient $\frac{\partial L(y,\hat{y})}{\partial z_i}$.
6. Compute gradient of $\mathbf{b}$ for all $i' \in \mathcal{C}$ and $\mathbf{W}$ for all $i' \in \mathcal{C}, f \in \mathcal{F}$,

$$\frac{\partial L}{\partial b_i'} = \frac{\partial L}{\partial z_i'} \qquad \frac{\partial L}{\partial W_{f,i'}} = \frac{\partial L}{\partial z_i'}$$

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4. Compute gradient $\frac{\partial L(y,\hat{y})}{\partial \hat{y}_j}$.
5. Compute gradient $\frac{\partial L(y,\hat{y})}{\partial z_i}$.
6. Compute gradient of $\mathbf{b}$ for all $i' \in \mathcal{C}$ and $\mathbf{W}$ for all $i' \in \mathcal{C}, f \in \mathcal{F}$,

$$\frac{\partial L}{\partial b'_i} = \frac{\partial L}{\partial z'_i} \qquad \frac{\partial L}{\partial W_{f,i'}} = \frac{\partial L}{\partial z'_i}$$

Gradient-Based Optimization: SGD

```
procedure SGD
    while training criterion is not met do
        Sample a training example $\mathbf{x}_i, \mathbf{y}_i$
        Compute the loss $L(\hat{\mathbf{y}}_i, \mathbf{y}_i; \theta)$
        Compute gradients $\hat{\mathbf{g}}$ of $L(\hat{\mathbf{y}}_i, \mathbf{y}_i; \theta)$ with respect to $\theta$
        $\theta \leftarrow \theta + \eta_k \hat{\mathbf{g}}$
    end while
    return $\theta$
end procedure
```

Gradient-Based Optimization: Minibatch SGD

```
**while** training criterion is not met **do**
    Sample a minibatch of $m$ examples $(\mathbf{x}_1, \mathbf{y}_1), \ldots, (\mathbf{x}_m, \mathbf{y}_m)$
    $\hat{\mathbf{g}} \leftarrow 0$
    **for** $i = 1 \text{ to } m$ **do**
        Compute the loss $L(\hat{\mathbf{y}}_i, \mathbf{y}_i; \theta)$
        Compute gradients $\mathbf{g}'$ of $L(\hat{\mathbf{y}}_i, \mathbf{y}_i; \theta)$ with respect to $\theta$
        $\hat{\mathbf{g}} \leftarrow \hat{\mathbf{g}} + \frac{1}{m}\mathbf{g}'$
    **end for**
    $\theta \leftarrow \theta + \eta_k \hat{\mathbf{g}}$
**end while**
**return** $\theta$
```

Softmax Notes: Regularization

$$\mathcal{L}(\theta) = -\sum_{i=1}^{n} L(\hat{\mathbf{y}}, \mathbf{y}) + ||\theta||_2^2$$

Softmax Notes: Calculating Log-Sum-Exp

- Calculating $\log \sum_{c' \in \mathcal{C}} \exp(\hat{y}_{c'}$ directly numerical issues.
- Instead $\log \sum_{c' \in \mathcal{C}} \exp(\hat{y}_{c'} - M) + M$ where $M = \max_{c' \in \mathcal{C}} \hat{y}_c'$

Pros and Cons of Logistic Regression

- Less strong independence assumption.
- Can be very effective with good features.
- Still yields a probability distribution.
- Fitting parameters is more difficult.

Similar models make will be the main focus of this class.

Other Loss Functions

What if we just try to directly find $\mathbf{W}$ and $\mathbf{b}$?

$$\hat{\mathbf{y}} = \mathbf{x}\mathbf{W} + \mathbf{b}$$

- No longer a probabilistic interpretation.
- Just try to find parameters that fit training data.

Hinge Loss

$$\mathcal{L}(\theta) = \sum_{i=1}^{n} L_{hinge}(\hat{\mathbf{y}}, \mathbf{y})$$

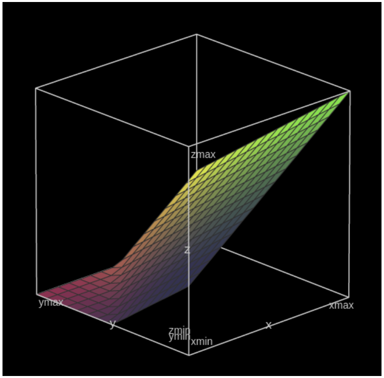
$$L(\hat{\mathbf{y}}, \mathbf{y}) = \max\{0, 1 - (\hat{y}_c + \hat{y}_{c'})\}$$

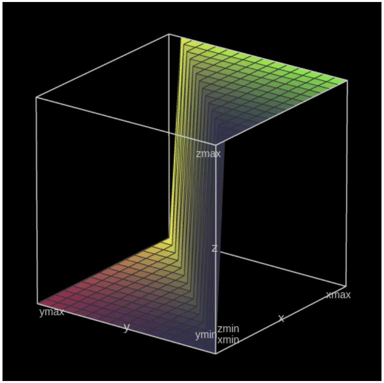
Where

- Let c be defined as gold class $y_{i,c} = 1$
- Let c' be defined as the highest scoring non-gold class

$$c' = \arg\max_{i \in \mathcal{C} \setminus \{c\}} \hat{y}_i$$

Hinge Loss





$hinge(\hat{\mathbf{y}}) = \mathbf{1}(\max\{0, 1-(y-x)\})$

$\arg\max([x\ y]) = \mathbf{1}(x > y)$

Symbolic Gradients

- Let c be defined as gold class $y_{i,c} = 1$
- Let c' be defined as the highest scoring non-gold class

$$c' = \operatorname*{arg\,max}_{i \in \mathcal{C} \setminus \{c\}} \hat{y}_i$$

Much simpler than logistic regression.

- Partials of $L(y, \hat{y})$

$$\frac{\partial L(y, k\hat{y})}{\partial \hat{y}_j} = \mathbf{1}(j = c) - \mathbf{1}(j = c')$$

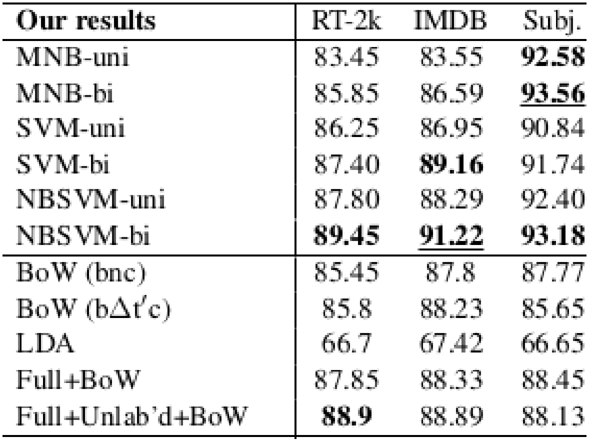
Notes: Hinge Loss: Regularization

- Many different names,
  - Margin Classifier
  - Multiclass Hinge
  - Linear SVM
- Important to use regularization.

$$\mathcal{L}(\theta) = -\sum_{i=1}^{n} L(\hat{\mathbf{y}}, \mathbf{y}) + ||\theta||_2^2$$

- Can be much more efficient to train than LR. (No partition).

Results: Longer Reviews



Our results	RT-2k	IMDB	Subj.
MNB-uni	83.45	83.55	**92.58**
MNB-bi	85.85	86.59	**<u>93.56</u>**
SVM-uni	86.25	86.95	90.84
SVM-bi	87.40	**89.16**	91.74
NBSVM-uni	87.80	88.29	92.40
NBSVM-bi	**89.45**	**<u>91.22</u>**	**93.18**
BoW (bnc)	85.45	87.8	87.77
BoW (bΔt′c)	85.8	88.23	85.65
LDA	66.7	67.42	66.65
Full+BoW	87.85	88.33	88.45
Full+Unlab'd+BoW	**88.9**	88.89	88.13

IMDB (longer movie review), Subj (longer subjectivity)

- NBSVM is hinge-loss interpolated with Naive Bayes.