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Neural Networks 2

CS 287

Review: Bilinear Model

Bilinear model,

$$\hat{\mathbf{y}} = f((\mathbf{x}^0 \mathbf{W}^0) \mathbf{W}^1 + \mathbf{b})$$

- $\mathbf{x}^0 \in \mathbb{R}^{1 imes d_0}$ start with one-hot.
- $ightharpoonup \mathbf{W}^0 \in \mathbb{R}^{d_0 imes d_{\mathrm{in}}}, \ d_0 = |\mathcal{F}|$
- $lackbox{W}^1 \in \mathbb{R}^{d_{
 m in} imes d_{
 m out}}$, $\mathbf{b} \in \mathbb{R}^{1 imes d_{
 m out}}$; model parameters

Notes:

- Bilinear parameter interaction.
- $ightharpoonup d_0 >> d_{
 m in}$, e.g. $d_0 = 10000$, $d_{
 m in} = 50$

Review: Bilinear Model: Intuition

$$(\mathbf{x}^0\mathbf{W}^0)\mathbf{W}^1 + \mathbf{b}$$

$$\begin{bmatrix} w_{1,1}^1 & \cdots & w_{0,d_{\mathrm{out}}}^1 \\ \cdots & \cdots & \cdots \\ w_{d_{\mathrm{in}},0}^1 & \cdots & w_{d_{\mathrm{in}},d_{\mathrm{out}}}^1 \end{bmatrix}$$

Review: Window Model

Goal: predict t_5 .

Windowed word model.

$$w_1 \ w_2 \ [w_3 \ w_4 \ w_5 \ w_6 \ w_7] \ w_8$$

- ► w₃, w₄; left context
- ▶ *w*₅; Word of interest
- \triangleright w_6 , w_7 ; right context
- d_{win} ; size of window ($d_{\text{win}} = 5$)

Review: Dense Windowed BoW Features

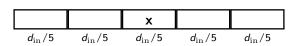
- $ightharpoonup f_1, \ldots, f_{d_{win}}$ are words in window
- ▶ Input representation is the concatenation of embeddings

$$\boldsymbol{x} = [v(f_1) \ v(f_2) \ \dots \ v(f_{d_{\min}})]$$

Example: Tagging

$$w_1 \ w_2 \ [w_3 \ w_4 \ w_5 \ w_6 \ w_7] \ w_8$$

$$\mathbf{x} = [v(w_3) \ v(w_4) \ v(w_5) \ v(w_6) \ v(w_7)]$$



Rows of W^1 encode position specific weights.

Quiz

We are doing tagging with a windowed bilinear model with hinge-loss and no capitalization features. The model has $d_{\rm win}=5$, $d_{\rm in}=50$, $d_{\rm out}=40$, and vocabulary size 10000.

We are given the input window:

The dog walked to the

Unfortunately we incorrectly classify walked as NN as opposed to VP, in a bilinear model with a hinge-loss .

What is the maximum number of parameters that receive a non-zero gradient?

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 \begin{bmatrix} w_{0,1}^{1} & \dots & w_{0,d_{\mathrm{in}}}^{0} \\ w_{the,1}^{0} & \dots & w_{the,d_{\mathrm{in}}}^{0} \\ \vdots & & & & & \\ w_{dog,1}^{0} & \dots & w_{dog,d_{\mathrm{in}}}^{0} \\ \vdots & & & & & \\ \vdots & & & & & \\ w_{walked,1}^{0} & \dots & w_{walked,d_{\mathrm{in}}}^{0} \\ \vdots & & & & & \\ \vdots & & & & & \\ w_{to,1}^{0} & \dots & w_{to,d_{\mathrm{in}}}^{0} \\ \vdots & & & & & \\ w_{to,1}^{0} & \dots & w_{to,d_{\mathrm{in}}}^{0} \\ \vdots & & & & & \\ w_{te,1}^{0} & \dots & w_{the,d_{\mathrm{in}}}^{0} \\ \vdots & & & & & \\ w_{do,1}^{0} & \dots & w_{do,d_{\mathrm{in}}}^{0} \end{bmatrix} \begin{bmatrix} w_{1,1}^{1} & \dots & w_{1,NN}^{1} & \dots & w_{1,VP}^{1} & w_{0,d_{\mathrm{out}}}^{1} \\ \vdots & & & & & \\ w_{din}^{1}, NN & \dots & w_{din}^{1}, NN & \dots & w_{din}^{1}, NN & \dots & w_{din}^{1}, NN \\ \vdots & & & & & \\ w_{do,1}^{0} & \dots & w_{do,d_{\mathrm{in}}}^{0} \end{bmatrix}
```

 $\mathbf{W}^0 = 5 \times d_{\rm in}$ $\mathbf{W}^1 = d_{\rm in} \times 2$

Consider the following windowed model, and assume for now a linear model.

$$[w_1 \text{ the } w_3 w_4 w_5]$$

- ▶ What information do we have about the tag of w_3 ?
- ▶ What weight should the features values associated with the in position w₂ take?

Next Consider the following windowed model, and assume for now a linear model.

$$[w_1 \ w_2 \ w_3 \ dog \ w_5]$$

- ▶ What information do we have about the tag of w_3 ?
- ▶ What weight should the features values associated with dog in position w_4 take?

Now finally consider the following windowed model, and assume for now a linear model.

$$[w_1 \text{ the } w_3 \text{ dog } w_5]$$

- ▶ What information do we have about the tag of w_3 ?
- What weight would we want if we combined both the features values?

Contents

Neural Networks

Backpropagation



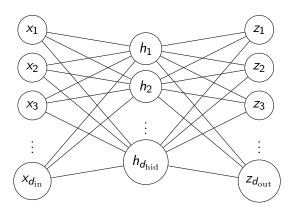
Neural Network

One-layer multi-layer perceptron architecture,

$$NN_{MLP1}(\mathbf{x}) = g(\mathbf{x}\mathbf{W}^1 + \mathbf{b}^1)W^2 + \mathbf{b}^2$$

- **xW** + **b**; perceptron
- **x** is the dense representation in $\mathbb{R}^{1 \times d_{\mathrm{in}}}$
- ullet $\mathbf{W}^1 \in \mathbb{R}^{d_{
 m in} imes d_{
 m hid}}$, $\mathbf{b}^1 \in \mathbb{R}^{1 imes d_{
 m hid}}$; first affine transformation
- $m{W}^2 \in \mathbb{R}^{d_{ ext{hid}} imes d_{ ext{out}}}$, $m{b}^2 \in \mathbb{R}^{1 imes d_{ ext{out}}}$; second affine transformation
- $ightharpoonup g: \mathbb{R}^{d_{ ext{hid}} imes d_{ ext{hid}}}$ is an activation non-linearity (often pointwise)
- $g(\mathbf{xW}^1 + \mathbf{b}^1)$ is the hidden layer

Schematic



Non-Linearities: 0/1

0/1 function:

$$0/1(t) = \mathbf{1}(t > 0)$$



- $ightharpoonup 01((xW^1 + b^1)_i)$
- Intuition: On, if above a threshold

Exercise

Input layer to NN_{MLP1} is the sparse indicator features of the word at each position.

Design a network to recognize

$$[w_1 \text{ the } w_3 \text{ dog } w_5]$$

▶ Design a network to recognize where w_2 is not the

$$[w_1 \ w_2 \ w_3 \ dog \ w_5]$$

Feature Conjunctions

Many NLP tasks require conjunctive features, examples

- Sequence-based taggers look at last two-part of speech tags.
- Chinese part-of-speech taggers look at first character and last tag.
- Higher-level models (parses) look at tags of words and distances apart (example)

For some natural language tasks, conjunctions are painstakingly hard.

- ▶ NNs: Capacity to learn conjunctions and feature combinations.
- ► Also possible with other convex models such as SVMs

Feature Conjunctions

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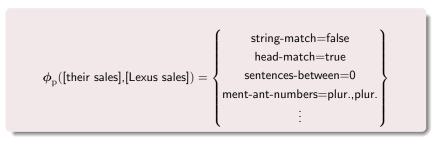
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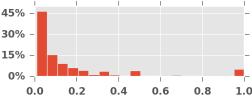
- ▶ NNs: Capacity to learn conjunctions and feature combinations.
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Simple Antecedent/Pairwise Features Not Discriminative

E.g., is [Lexus sales] the antecedent of [their sales]?

Common pairwise features: String/Head Match, Sentences
 Between, Mention-Antecedent Numbers/Heads/Genders, etc.





Dealing with the Feature Problem

Finding discriminative features is a major challenge for coreference systems [Fernandes et al. 2012; Durrett and Klein 2013]

- Typical to define (or search for) feature conjunction-schemes to improve predictive performance [Fernandes et al. 2012; Durrett and Klein 2013; Björkelund and Kuhn 2014]. For instance:
 - string-match $(x,y) \land \mathsf{type}(x) \land \mathsf{type}(y)$ [Durrett and Klein 2013], where

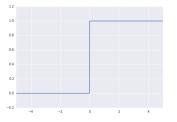
$$\mathsf{type}(x) = \begin{cases} \mathsf{Nom.} & \text{if } x \text{ is nominal} \\ \mathsf{Prop.} & \text{if } x \text{ is proper} \\ \mathsf{citation\text{-}form}(x) & \text{if } x \text{ is pronominal} \end{cases}$$

- substring-match(head $(x),y) \land$ substring-match $(x,\text{head}(y)) \land$ coarse-type $(y) \land$ coarse-type(x) [Björkelund and Kuhn 2014]
- Not just a problem for Mention Ranking systems!

Non-Linearities: 0/1

0/1 function:

$$0/1(t) = \mathbf{1}(t > 0)$$





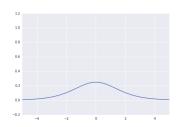
▶ Issue: No gradient anywhere

Non-Linear Functions: Sigmoid

Logistic sigmoid function:

$$\sigma(t) = \frac{1}{1 + \exp(-t)}$$





- $\boldsymbol{\triangleright} \ \sigma((\mathbf{xW}^1 + \mathbf{b}^1)_i)$
- ▶ Intuition: Each hidden dimension ("neuron") is result of logistic regression.

Other Non-Linearities: ReLU

Rectified Linear Unit:

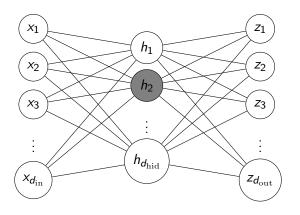
$$ReLU(t) = max\{0, t\}$$





- ▶ Intuition: Each hidden-unit gives activation margin
- ▶ No gradient (saturation) when below 0.

Saturation: Intuition

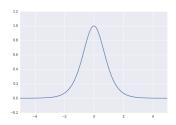


Other Non-Linearities: Tanh

Hyperbolic Tangeant:

$$\tanh(t) = \frac{\exp(t) - \exp(-t)}{\exp(t) + \exp(-t)}$$





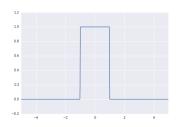
▶ Intuition: Similar to sigmoid, but range between 0 and -1.

Other Non-Linearities: Hard Tanh

Hyperbolic Tangeant:

$$\operatorname{hardtanh}(t) = \begin{cases} -1 & t < -1 \\ t & -1 \le t \le 1 \\ 1 & t > 1 \end{cases}$$



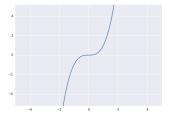


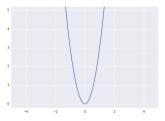
▶ Intuition: Similar to sigmoid, but range between 0 and -1.

Other Non-Linearities: Cube

Cube non-linearity (directly encourage parameter interaction):

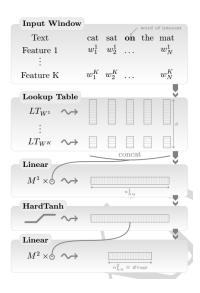
$$cube(t) = t^3$$





▶ Intuition: Directly encourage higher-order interactions.

Tagging from Scratch



Function Approximator

MLP1 is a universal approximator

Can approximate with any desired non-zero amount of error a family of functions that include all continuous functions on a closed and bounded subset of \mathbb{R}^n , and any function mapping from any finite dimensional discrete space to another (YG)

Caveats:

- Does not give size of hidden layer.
- Does not specify how hard this is to learn.

Deep Neural Networks (DNNs)

Can stack MLPs, create deep fully connected networks,

$$\begin{split} \mathit{NN}_\mathit{MLP1}(\mathbf{x}) &= g(\mathbf{x}\mathbf{W}^1 + \mathbf{b}^1)W^2 + \mathbf{b}^2 \\ \mathit{NN}_\mathit{MLP2}(\mathbf{x}) &= g(\mathit{NN}_\mathit{MLP1}(\mathbf{x})\mathbf{W}^1 + \mathbf{b}^1)W^2 + \mathbf{b}^2 \end{split}$$

- ► Can have multiple hidden layers, etc.
- ▶ Benefit: may be able to find better function
- Known to be harder to train (although other approaches)

Other Layers

We will discuss many other neural network layers,

- convolutional
- attention-based
- gated layers
- **.** . . .

Highway Network

y : output from CharCNN

Multilayer Perceptron

$$z = g(Wy + b)$$

Highway Network

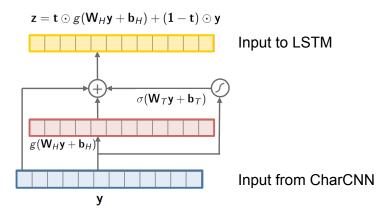
(Srivastava, Greff, and Schmidhuber 2015)

$$z = t \odot g(W_H y + b_H) + (1 - t) \odot y$$

 $\mathbf{W}_H, \mathbf{b}_H$: Affine transformation $\mathbf{t} = \sigma(\mathbf{W}_T\mathbf{y} + \mathbf{b}_T)$: transform gate $\mathbf{1} - \mathbf{t}$: carry gate

Hierarchical, adaptive composition of character *n*-grams.

Highway Network



Contents

Neural Networks

Backpropagation

Sequential Neural Network

Sequential neural networks consist of a series of composed functions, Consider a vector-valued parameterized functions f_1, \ldots, f_k where

- $f_i(\mathbf{x}; \boldsymbol{\theta}_i) : \mathbb{R}^{n_{i-1}} \mapsto \mathbb{R}^{n_i}$; function
- $m{ heta} \in \mathbb{R}^{d_i}$; function parameters

Consider a scalar-valued loss function $L(\mathbf{y}, \hat{\mathbf{y}})$ where

▶ $L(\mathbf{y}, *) : \mathbb{R}^{n_k} \mapsto \mathbb{R}$; loss for input

Backpropagation

Forward Step (f-prop):

Compute

$$L(f_k(\ldots f_1(\mathbf{x}^0)))$$

Saving intermediary values

$$f_i(\ldots f_1(\mathbf{x}^0))$$

Backward Step (b-prop):

$$\frac{\partial L}{\partial f_i(\dots f_1(\mathbf{x}^0))} = \sum_{j=1}^{n_i} \frac{\partial f_{i+1}(\dots f_1(\mathbf{x}^0))_j}{\partial f_i(\dots f_1(\mathbf{x}^0))} \frac{\partial L}{\partial f_{i+1}(\dots f_1(\mathbf{x}^0))_j}$$

$$\frac{\partial L}{\partial \theta_i} = \sum_{j=1}^{n_i} \frac{\partial f_{i+1}(\dots f_1(\mathbf{x}^0))_j}{\partial \theta_i} \frac{\partial L}{\partial f_{i+1}(\dots f_1(\mathbf{x}^0))_j}$$

Backpropagation

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Backpropagation

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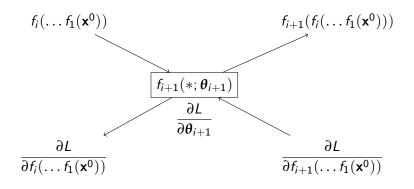
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Backpropagation: Data flow

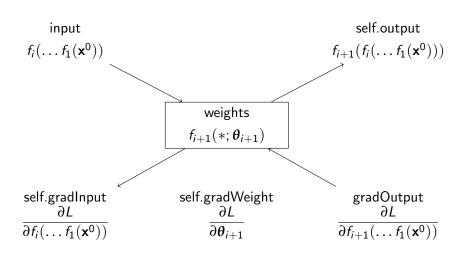


Torch Implementation

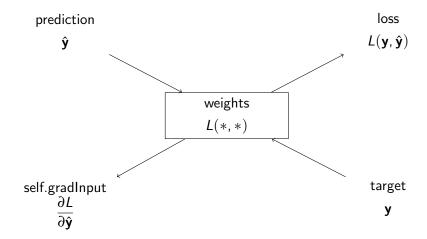
Torch uses declarative unit-based specification of NN

- Every function is a represented as a unit.
- ► Responsibilities:
 - 1. Expose any parameters θ_{i+1} as tensors
 - 2. Compute $f_{i+1}(\mathbf{x}, \theta_{i+1})$ (fprop)
 - 3. Compute any necessary state needed for bprop
 - 4. Compute chain-rule given $\frac{\partial L}{\partial f_{i+1}(...f_1(\mathbf{x}^0))}$ and $f_i(\dots f_1(\mathbf{x}^0))$
 - 5. Compute parameter gradient $\frac{\partial L}{\partial \theta_{i+1}}$
- Contract: forward will always be called before backward.

Torch Units



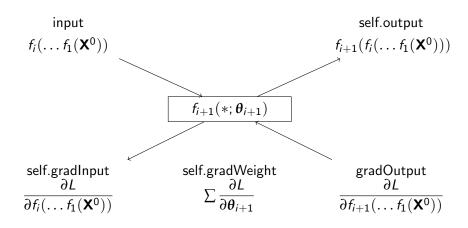
Loss Criterions



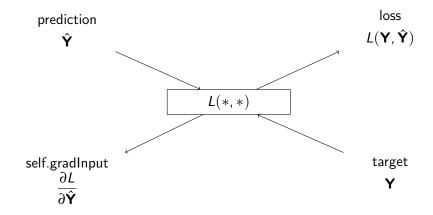
Torch Internals

 $[\mathsf{Web}]$

Torch Units: Batch



Loss Criterions



Today

- ▶ Benefits of neural networks
- ► Training neural networks

Next time: Pretraining and word embeddings