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Neural Networks 2

CS 287

### Review: Bilinear Model

Bilinear model,

$$\hat{\mathbf{y}} = f((\mathbf{x}^0 \mathbf{W}^0) \mathbf{W}^1 + \mathbf{b})$$

- $\mathbf{x}^0 \in \mathbb{R}^{1 imes d_0}$  start with one-hot.
- $ightharpoonup \mathbf{W}^0 \in \mathbb{R}^{d_0 imes d_{\mathrm{in}}}, \ d_0 = |\mathcal{F}|$
- $lackbox{W}^1 \in \mathbb{R}^{d_{
  m in} imes d_{
  m out}}$  ,  $\mathbf{b} \in \mathbb{R}^{1 imes d_{
  m out}}$  ; model parameters

#### Notes:

- Bilinear parameter interaction.
- $ightharpoonup d_0 >> d_{
  m in}$ , e.g.  $d_0 = 10000$ ,  $d_{
  m in} = 50$

### Review: Bilinear Model: Intuition

$$(\mathbf{x}^0\mathbf{W}^0)\mathbf{W}^1 + \mathbf{b}$$

$$\begin{bmatrix} w_{1,1}^1 & \cdots & w_{0,d_{\mathrm{out}}}^1 \\ \cdots & \cdots & \cdots \\ w_{d_{\mathrm{in}},0}^1 & \cdots & w_{d_{\mathrm{in}},d_{\mathrm{out}}}^1 \end{bmatrix}$$

### Review: Window Model

### **Goal:** predict $t_5$ .

Windowed word model.

$$w_1 \ w_2 \ [w_3 \ w_4 \ w_5 \ w_6 \ w_7] \ w_8$$

- ► w<sub>3</sub>, w<sub>4</sub>; left context
- ▶ *w*<sub>5</sub>; Word of interest
- $\triangleright$   $w_6$ ,  $w_7$ ; right context
- $d_{\text{win}}$ ; size of window ( $d_{\text{win}} = 5$ )

### Review: Dense Windowed BoW Features

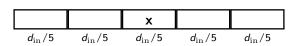
- $ightharpoonup f_1, \ldots, f_{d_{win}}$  are words in window
- ▶ Input representation is the concatenation of embeddings

$$\boldsymbol{x} = [v(f_1) \ v(f_2) \ \dots \ v(f_{d_{\min}})]$$

Example: Tagging

$$w_1 \ w_2 \ [w_3 \ w_4 \ w_5 \ w_6 \ w_7] \ w_8$$

$$\mathbf{x} = [v(w_3) \ v(w_4) \ v(w_5) \ v(w_6) \ v(w_7)]$$



Rows of  $W^1$  encode position specific weights.

### Quiz

We are doing tagging with a windowed bilinear model with hinge-loss and no capitalization features. The model has  $d_{\rm win}=5$ ,  $d_{\rm in}=50$ ,  $d_{\rm out}=40$ , and vocabulary size 10000.

We are given the input window:

The dog walked to the

Unfortunately we incorrectly classify walked as NN as opposed to VP, in a bilinear model with a hinge-loss .

What is the maximum number of parameters that receive a non-zero gradient?

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 \begin{bmatrix} w_{0,1}^{1} & \dots & w_{0,d_{\mathrm{in}}}^{0} \\ w_{the,1}^{0} & \dots & w_{the,d_{\mathrm{in}}}^{0} \\ \vdots & & & & & \\ w_{dog,1}^{0} & \dots & w_{dog,d_{\mathrm{in}}}^{0} \\ \vdots & & & & & \\ \vdots & & & & & \\ w_{walked,1}^{0} & \dots & w_{walked,d_{\mathrm{in}}}^{0} \\ \vdots & & & & & \\ \vdots & & & & & \\ w_{to,1}^{0} & \dots & w_{to,d_{\mathrm{in}}}^{0} \\ \vdots & & & & & \\ w_{to,1}^{0} & \dots & w_{to,d_{\mathrm{in}}}^{0} \\ \vdots & & & & & \\ w_{the,1}^{0} & \dots & w_{the,d_{\mathrm{in}}}^{0} \\ \vdots & & & & & \\ w_{do,1}^{0} & \dots & w_{do,d_{\mathrm{in}}}^{0} \end{bmatrix} \begin{bmatrix} w_{1,1}^{1} & \dots & w_{1,NN}^{1} & \dots & w_{1,VP}^{1} & w_{0,d_{\mathrm{out}}}^{1} \\ \vdots & & & & & \\ w_{din}^{1}, NN & \dots & w_{din}^{1}, NN & \dots & w_{din}^{1}, NP & w_{din}^{1}, d_{\mathrm{out}} \end{bmatrix}
```

 $\mathbf{W}^0 = 5 \times d_{\rm in}$  $\mathbf{W}^1 = d_{\rm in} \times 2$ 

Consider the following windowed model, and assume for now a linear model.

$$w_1$$
 the  $w_3$   $w_4$   $w_5$ 

- ▶ What information do we have about the tag of  $w_3$ ?
- ▶ What weight should the features values associated with the in position w<sub>2</sub> take?

Next Consider the following windowed model, and assume for now a linear model.

$$w_1$$
  $w_2$   $w_3$  dog  $w_5$ 

- ▶ What information do we have about the tag of  $w_3$ ?
- ▶ What weight should the features values associated with dog in position  $w_4$  take?

Now finally consider the following windowed model, and assume for now a linear model.

#### $w_1$ the $w_3$ dog $w_5$

- ▶ What information do we have about the tag of  $w_3$ ?
- What weight would we want if we combined both the features values?

## **Table**

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Backpropagation

Semi-Supervised Training



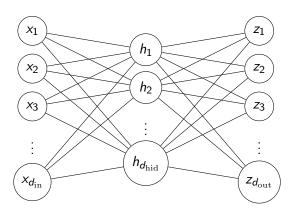
### **Neural Network**

One-layer multi-layer perceptron architecture,

$$NN_{MLP1}(\mathbf{x}) = g(\mathbf{x}\mathbf{W}^1 + \mathbf{b}^1)W^2 + \mathbf{b}^2$$

- **xW** + **b**; perceptron
- **x** is the dense representation in  $\mathbb{R}^{1 \times d_{\mathrm{in}}}$
- ullet  $\mathbf{W}^1 \in \mathbb{R}^{d_{
  m in} imes d_{
  m hid}}$ ,  $\mathbf{b}^1 \in \mathbb{R}^{1 imes d_{
  m hid}}$ ; first affine transformation
- $m{W}^2 \in \mathbb{R}^{d_{ ext{hid}} imes d_{ ext{out}}}$  ,  $m{b}^2 \in \mathbb{R}^{1 imes d_{ ext{out}}}$ ; second affine transformation
- $ightharpoonup g: \mathbb{R}^{d_{ ext{hid}} imes d_{ ext{hid}}}$  is an activation non-linearity (often pointwise)
- $g(\mathbf{xW}^1 + \mathbf{b}^1)$  is the hidden layer

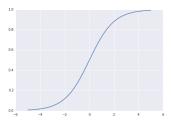
### Schematic



### Non-Linear Functions

Logistic sigmoid function:

$$\sigma(t) = \frac{1}{1 + \exp(-t)}$$



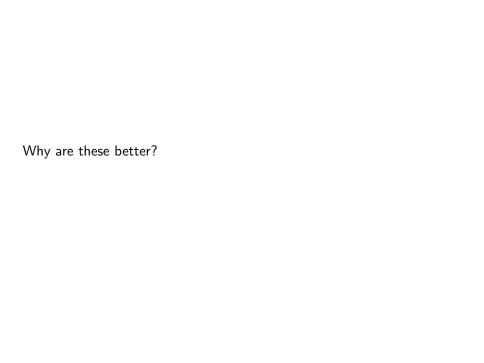
- $\boldsymbol{\sigma}((\mathbf{x}\mathbf{W}^1+\mathbf{b}^1)_i)$
- ▶ Intuition: Each hidden dimension ("neuron") is result of logistic regression.
- ▶ These probabilities are "features" for next layer.

# Feature Conjunctions

Consider the example  $\dots$ 

# Non-Convexity

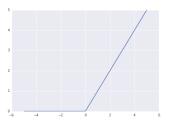




### Other Non-Linearities: ReLU

Rectified Linear Unit:

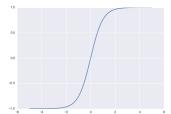
$$\mathsf{ReLU}(t) = \max\{0, t\}$$



Intuition:

### Saturation

### Saturation: Intuition



# Function Approximator

MLP1 is a universal approximator

# Deep Neural Networks (DNNs)

Can stack MLPs,

$$\begin{split} \mathit{NN}_\mathit{MLP1}(\mathbf{x}) &= g(\mathbf{x}\mathbf{W}^1 + \mathbf{b}^1)W^2 + \mathbf{b}^2 \\ \mathit{NN}_\mathit{MLP2}(\mathbf{x}) &= g(\mathit{NN}_\mathit{MLP1}(\mathbf{x})\mathbf{W}^1 + \mathbf{b}^1)W^2 + \mathbf{b}^2 \end{split}$$

Can have multiple hidden layers, etc.

# Other types of networks

Highway Network (one example)

$$NN_{MLP2}(\mathbf{x}) = g(NN_{MLP1}(\mathbf{x})\mathbf{W}^1 + \mathbf{b}^1)W^2 + \mathbf{b}^2$$

# Deep Neural Networks (DNNs)

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Neural Networks

 ${\sf Backpropagation}$ 

Semi-Supervised Training

Consider a vector-valued parameterized function  $f(\mathbf{x};)$  where

▶ 
$$f(\mathbf{x}) : \mathbb{R}^m \mapsto \mathbb{R}^n$$
; function

 $ightharpoonup \in \mathbb{R}^d$ ; function parameters

Consider a scalar-valued loss function  $L(\mathbf{x};)$  where

 $L(\mathbf{x}): \mathbb{R}^n \mapsto \mathbb{R}$ ; function

# Backpropagation

Forward Compute  $L(f(\ldots f()))$ 

Backward

$$\frac{\partial L}{\partial f(\dots f(x_i))} = \sum_{j=1}^m \frac{\partial f(\mathbf{x})_j}{\partial x_i} \frac{\partial L(f(\mathbf{x}))}{\partial f(\mathbf{x})_j}$$

# Torch Implementation



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# Torch Implementation



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### Torch Names

- ▶ x; input
- ▶  $f(\mathbf{x})$ ; self.output (saved on forward pass)
- $ightharpoonup \frac{\partial L}{x_i}$ ; self.gradInput
- $ightharpoonup \frac{\partial L}{f(\mathbf{x})_i}$ ; gradOutput
- ▶ ; gradWeight
- $ightharpoonup \frac{\partial L}{\partial t}$ ; gradWeight

### Max

### Max

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