# Language Modeling

+

Feed-Forward Networks 3

CS 287

### Review: LM ML Setup

Multi-class prediction problem,

$$(\mathbf{x}_1,\mathbf{y}_1),\ldots,(\mathbf{x}_n,\mathbf{y}_n)$$

- ▶ **y**<sub>i</sub>; the one-hot next word
- $ightharpoonup \mathbf{x}_i$ ; representation of the prefix  $(w_1, \ldots, w_{t-1})$

#### **Challenges:**

- How do you represent input?
- Smoothing is crucially important.
- Output space is very large (next class)

## Review: Perplexity

Previously, used accuracy as a metric.

Language modeling uses of version average negative log-likelihood

▶ For test data  $\bar{w}_1, \ldots, \bar{w}_n$ 

▶

$$NLL = -\frac{1}{n} \sum_{i=1}^{n} \log p(w_i | w_1, ..., w_{i-1})$$

Actually report *perplexity*,

$$perp = \exp(-\frac{1}{n} \sum_{i=1}^{n} \log p(w_i | w_1, \dots, w_{i-1}))$$

Requires modeling full distribution as opposed to argmax (hinge-loss)

# Review: Interpolation (Jelinek-Mercer Smoothing)

Can write recursively,

$$p_{interp}(w|c) = \lambda p_{ML}(w|c) + (1 - \lambda)p_{interp}(w|c')$$

Ensure that  $\lambda$  form convex combination

$$0 \le \lambda \le 1$$

How do you learn conjunction combinations?

### Quiz

Assume we have seen the following training sentences,

- a tractor drove slow
- the red tractor drove fast
- ▶ the parrot flew fast
- the parrot flew slow
- the tractor slowed down

Compute  $p_{ML}$  for bigrams and use them to estimate whether *parrot* or *tractor* fit better in the following contexts.

- 1. the red \_\_\_ ?
- 2. the \_\_\_ ?
- 3. the \_\_\_ drove?

### Answer I

а	tractor	1
the	red	<u>1</u>
the	parrot	$\frac{1}{2}$
the	tractor	$\frac{1}{4}$
red	tractor	1
tractor	drove	<u>2</u> 3
tractor	slowed	$\frac{1}{3}$
parrot	flew	1
•••		

### Answer II

- ▶ the red tractor
- ▶ the parrot
- ▶ the tractor drove

### Today's Class

$$p(w_i|w_{i-n+1},\ldots w_{i-1};\theta)$$

- Estimate this directly as a neural network.
- ▶ Two types of models, neural network and log-bilinear.
- ▶ Efficient methods for approximated estimation.

#### Intuition: NGram Issues

In training we might see,

the arizona corporations commission authorized

But at test we see,

the colorado businesses organization \_\_\_

- ▶ Does this training example help here?
  - Not really. No count overlap.
- Does backoff help here?
  - Maybe, if we have seen organization.
  - Mostly get nothing from the earlier words.

#### Intuition: NGram Issues

In training we might see,

the arizona corporations commission authorized

But at test we see,

the colorado businesses organization \_\_\_

- Does this training example help here?
  - Not really. No count overlap.
- ▶ Does backoff help here?
  - Maybe, if we have seen organization.
  - Mostly get nothing from the earlier words

#### Intuition: NGram Issues

In training we might see,

the arizona corporations commission authorized

But at test we see,

the colorado businesses organization \_\_\_\_

- ▶ Does this training example help here?
  - Not really. No count overlap.
- Does backoff help here?
  - Maybe, if we have seen organization.
  - Mostly get nothing from the earlier words.

### Goal

- ▶ Learn representations that share properties between similar words.
- Particularly helpful for unseen contexts.
- Not a silver bullet, e.g. proper nouns

the eagles play the arizona diamondbacks

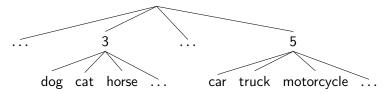
Whereas at test we might see,

the eagles play the colorado \_\_\_

(We will discuss this issue more for in MT)

### Baseline: Class-Based Language Models

Groups words into classes based on word-context.



- Various factorization methods for estimating with count-based approaches.
- ▶ However, assumes a hard-clustering, often estimated separately.

### Contents

Neural Language Models

Noise Contrastive Estimation

### Recall: Word Embeddings

▶ Embeddings give multi-dimensional representation of words.

► Ex: Closest by cosine similarity

	texas	0.932968706025
arizona	florida	0.932696958878
	kansas	0.914805968271
	colorado	0.904197441085
	minnesota	0.863925347525
	carolina	0.862697751337
	utah	0.861915722889
	miami	0.842350326527
	oregon	0.842065064748

► Gives a multi-clustering over words.

## Feed-Forward Neural NNLM (Bengio, 2003)

- $\triangleright$   $w_{i-n+1}, \dots w_{i-1}$  are input embedding representations
- ▶ *w<sub>i</sub>* is an output embedded representation
- Model simultaneously learns,
  - input word representations
  - output word representations
  - conjunctions of input words (through NLM, no n-gram features)

### Feed-Forward Neural Representation

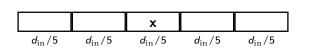
- $\triangleright p(w_i|w_{i-n+1},\ldots w_{i-1};\theta)$
- $f_1, \ldots, f_{d_{\min}}$  are words in window
- ▶ Input representation is the concatenation of embeddings

$$\mathbf{x} = [v(f_1) \ v(f_2) \ \dots \ v(f_{d_{\min}})]$$

Example: NNLM  $(d_{\text{win}} = 5)$ 

$$[w_3 \ w_4 \ w_5 \ w_6 \ w_7] \ w_8$$

$$\mathbf{x} = [v(w_3) \ v(w_4) \ v(w_5) \ v(w_6) \ v(w_7)]$$



# A Neural Probabilistic Language Model (Bengio, 2003)

One hidden layer multi-layer perceptron architecture,

$$NN_{MLP1}(\mathbf{x}) = \tanh(\mathbf{x}\mathbf{W}^1 + \mathbf{b}^1)\mathbf{W}^2 + \mathbf{b}^2$$

Neural network architecture on top of concat.

$$\hat{\mathbf{y}} = \operatorname{softmax}(\mathit{NN}_{\mathit{MLP1}}(\mathbf{x}))$$

Best model uses  $d_{\rm in}=30$ ,  $d_{\rm hid}=100$ .

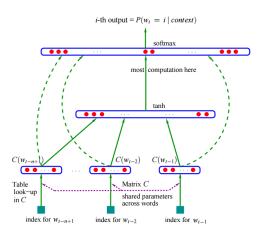
## A Neural Probabilistic Language Model

Optional, direct connection layers,

$$\mathit{NN}_{\mathit{DMLP1}}(\mathbf{x}) = [\tanh(\mathbf{x}\mathbf{W}^1 + \mathbf{b}^1), \mathbf{x}]W^2 + \mathbf{b}^2$$

- ullet  $\mathbf{W}^1 \in \mathbb{R}^{d_{\mathrm{in}} \times d_{\mathrm{hid}}}$ ,  $\mathbf{b}^1 \in \mathbb{R}^{1 \times d_{\mathrm{hid}}}$ ; first affine transformation
- ullet  $\mathbf{W}^2 \in \mathbb{R}^{(d_{ ext{hid}}+d_{ ext{in}}) imes d_{ ext{out}}}$ ,  $\mathbf{b}^2 \in \mathbb{R}^{1 imes d_{ ext{out}}}$ ; second affine transformation

# A Neural Probabilistic Language Model (Bengio, 2003)



Dashed-lines show the optional direct connections, C = v.

# A Neural Probabilistic Language Model

	n	c	h	m	direct	mix	train.	valid.	test.
MLP1	5		50	60	yes	no	182	284	268
MLP2	5		50	60	yes	yes		275	257
MLP3	5		0	60	yes	no	201	327	310
MLP4	5		0	60	yes	yes		286	272
MLP5	5		50	30	yes	no	209	296	279
MLP6	5		50	30	yes	yes		273	259
MLP7	3		50	30	yes	no	210	309	293
MLP8	3		50	30	yes	yes		284	270
MLP9	5		100	30	no	no	175	280	276
MLP10	5		100	30	no	yes		265	252
Del. Int.	3						31	352	336
Kneser-Ney back-off	3							334	323
Kneser-Ney back-off	4							332	321
Kneser-Ney back-off	5							332	321
class-based back-off	3	150						348	334
class-based back-off	3	200						354	340
class-based back-off	3	500						326	312
class-based back-off	3	1000						335	319
class-based back-off	3	2000						343	326
class-based back-off	4	500						327	312
class-based back-off	5	500						327	312

#### **Parameters**

- ▶ Bengio NNLM has  $d_{\rm hid} = 100$ ,  $d_{\rm win} = 5$ ,  $d_{\rm in} = 5 \times 50$
- ▶ In-Class: How many parameters does it have? How does this compare to Kneser-Ney smoothing?

#### Historical Note

- Bengio et al notes that many of these aspects predate the work
- ► Furthermore proposes many of the ideas that Collobert et al. and word2vec implement and scale
- Around this time, very few NLP papers on NN, most-cited papers are about conditional random fields (CRFs).

# Log-Bilinear Language Model (Mnih & Hinton, 2007)

Slightly different input representation. Now let:

$$\mathbf{x} = \sum_{i=1}^{a_{\min}} v(f_i) \mathbf{C}_i$$

▶ Instead of concatenating, weight each  $v(f_i)$  by position-specific weight matrix  $\mathbf{C}_i$ .

Then use:

$$\hat{\mathbf{y}} = \operatorname{softmax}(\mathbf{x}\mathbf{W}^1 + \mathbf{b})$$

- Note no tanh layer.
- ▶ **W**<sup>1</sup> can use input embeddings too, or not (Mnih and Teh, 2012)
- ▶ Can be faster to use, and in some cases simpler.

## Comparison

Both count-based models and feed-forward NNLMs are Markovian language models,

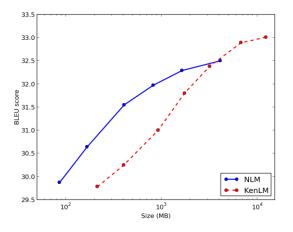
#### Comparison:

- Training Speed: ngrams are much faster (more coming)
- ▶ Usage Speed: ngrams very fast, NN can be fast with some tricks.
- ▶ Memory: NN models can be much smaller (but there are big ones)
- ► Accuracy: Comparable for small data, NN does better with more.

#### Advantages of NN model

- Can be trained end-to-end.
- Does not require smoothing methods.

## Translation Performance (and Blunsom, 2015)



### Contents

Neural Language Models

Noise Contrastive Estimation

### Review: Softmax Issues

Use a softmax to force a distribution,

$$\mathsf{softmax}(\mathbf{z}) = \frac{\mathsf{exp}(\mathbf{z})}{\sum_{c \in \mathcal{C}} \mathsf{exp}(z_c)}$$

$$\log \operatorname{softmax}(\mathbf{z}) = \mathbf{z} - \log \sum_{c \in \mathcal{C}} \exp(z_c)$$

- **Issue:** class C is huge.
- For C&W, 100,000, for word2vec 1,000,000 types
- ▶ Note largest dataset is 6 billion words

### **Unnormalized Scores**

Recall the score defined as (dropping bias)

$$z = tanh(xW^1)W^2$$

Unnormalized score of each word before soft-max,

$$z_j = \tanh(\mathbf{x}\mathbf{W}^1)\mathbf{W}^2_{*,j}$$

for any  $j \in \{1, \dots d_{\text{out}}\}$ 

Note: can be computed efficiently  $\mathit{O}(1)$  versus  $\mathit{O}(\mathit{d}_{\mathrm{out}}).$ 

#### Coherence

- Saw similar idea earlier for ranking embedding.
- ▶ Idea: Learn to distinguish coherent n-grams from corruption.
- Want to discriminate correct next words from other choices.

```
[ the dog walks ]
[ the dog house ]
[ the dog cats ]
[ the dog skips ]
```

### Warm-Up

Imagine we have a new dataset,

$$((\mathbf{x}_1,\mathbf{y}_1),\mathbf{d}_1),\ldots,((\mathbf{x}_n,\mathbf{y}_n),\mathbf{d}_n),$$

- ▶ **x**; representation of context  $w_{i-n+1}, \ldots w_{i-1}$
- $\triangleright$  **y**; a possible  $w_i$
- $\triangleright$  d; 1 if **y** is correct, 0 otherwise

Objective is based on predicted  $\hat{d}$ :

$$\mathcal{L}(\theta) = \sum_{i} L_{crossentropy}(d_i, \hat{d}_i)$$

# Warm-Up: Binary Classification

How do we score  $(\mathbf{x}_i, \mathbf{y}_i = \delta(c))$ ?

Could use unnormalized score,

$$z_c = \tanh(\mathbf{x}\mathbf{W}^1)\mathbf{W}_{*,c}^2$$

Becomes softmax regression/non-linear logistic regression,

$$\hat{d} = \sigma(z_c)$$

- Much faster
- But does not help us train LM.

### Implementation

Standard MLP language model, (only takes in  $\mathbf{x}$ )

$$\mathbf{x} \Rightarrow \mathbf{W}^1 \Rightarrow \mathsf{tanh} \Rightarrow \mathbf{W}^2 \Rightarrow \mathsf{softmax}$$

Computing binary (takes in  $\mathbf{x}$  and  $\mathbf{y}$ )

$$\hat{d} = \sigma(z_c)$$

$$\mathbf{x} \Rightarrow \mathbf{W}^1 \Rightarrow \tanh \Rightarrow \overset{\cdot}{\mathbf{W}^2_{*,w}(\mathrm{Lookup})} \Rightarrow \sigma$$

#### Probabilistic model,

- ▶ Introduce random variable D
- ▶ If D = 1 produce true sample
- ▶ If D = 0 produce sample from a noise distribution.
- ► Hyperparameter *K* is ratio of noise

$$p(D=1) = \frac{1}{K+1}$$
$$p(D=0) = \frac{K}{K+1}$$

For a given  $\mathbf{x}$ ,  $\mathbf{y}$ ,

$$\begin{split} \rho(D=1|\mathbf{x},\mathbf{y}) &= \frac{p(\mathbf{y}|D=1,\mathbf{x})p(D=1|\mathbf{x})}{\sum_{d}p(|\mathbf{y}|D=d,\mathbf{x})p(D=d|\mathbf{x})} \\ &= \frac{p(\mathbf{y}|D=1,\mathbf{x})p(D=1|\mathbf{x})}{p(\mathbf{x}|D=0)p(D=0|\mathbf{x}) + p(\mathbf{y}|D=1,\mathbf{x})p(D=1|\mathbf{x})} \end{split}$$

Plug-in the noise distribution and hyperparameters,

$$\begin{split} \rho(D=1|\mathbf{x},\mathbf{y}) &= \frac{\frac{1}{K+1}\rho(\mathbf{y}|D=1,\mathbf{x})}{\frac{1}{K+1}\rho(\mathbf{y}|D=1,\mathbf{x}) + \frac{K}{K+1}\rho(\mathbf{y}|D=0,\mathbf{x})} \\ &= \frac{\rho(\mathbf{y}|D=1,\mathbf{x})}{\rho(\mathbf{y}|D=1,\mathbf{x}) + K\rho(\mathbf{y}|D=0,\mathbf{x})} \\ &= \sigma(\log\rho(\mathbf{y}|D=1,\mathbf{x}) - \log(K\rho(\mathbf{y}|D=0,\mathbf{x}))) \end{split}$$

With

$$p(D = 1 | \mathbf{x}, \mathbf{y}) = \sigma(\log p(\mathbf{y} | D = 1, \mathbf{x}) - \log(Kp(\mathbf{y} | D = 0, \mathbf{x})))$$

we the training objective for a corpus that has K noise samples  $s_{i,k}$  per example is:

$$\begin{split} \mathcal{L}(\theta) &= \sum_{i} \log p(D = 1 | \mathbf{x}_{i}, \mathbf{y}_{i}) + \sum_{k=1}^{K} \log p(D = 0 | \mathbf{x}_{i}, Y = s_{i,k}) \\ &= \sum_{i} \log \sigma \left( \log p(\mathbf{y}_{i} | D = 1, \mathbf{x}_{i}) - \log(Kp(\mathbf{y}_{i} | D = 0, \mathbf{x}_{i})) \right) \\ &+ \sum_{k=1}^{K} \log \left( 1 - \sigma \left( \log p(s_{i,k} | D = 1, \mathbf{x}_{i}) - \log(Kp(s_{i,k} | D = 0, \mathbf{x}_{i})) \right) \right) \end{split}$$

▶ In practice, sample  $s_{i,k}$  from unigram distribution

But we still have a problem:  $\mathcal{L}$  defined in terms of normalized distributions  $\log p(\mathbf{y}|D=1,\mathbf{x})$ 

#### Solution:

- ▶ instead of explicitly normalizing, estimate  $Z(\mathbf{x})$ , normalizing constant of each context  $\mathbf{x}$ , as a parameter (Gutmann & Hyvärinen, 2010)
- ▶ Mnih and Teh (2012) show that fixing  $Z(\mathbf{x}) = 1$  for all contexts works just as well
- ▶ So we can replace  $\log p(\mathbf{y} = \delta(c)|D=1,\mathbf{x})$  with  $z_c$ , as computed by our network

So we now have

$$\begin{split} \mathcal{L}(\theta) &= \sum_{i} \log \sigma(\mathbf{z}_{c_{i}} - \log(\mathit{Kp}_{\mathit{ML}}(c_{i}))) \\ &+ \sum_{k=1}^{\mathit{K}} \log(1 - \sigma(\mathbf{z}_{\mathit{s}_{i,k}} - \log(\mathit{Kp}_{\mathit{ML}}(\mathit{s}_{i,k})))) \end{split}$$

▶ Mnih and Teh (2012) show that gradient of  $\mathcal{L}$  approaches gradient of true language model's log-likelihood objective as  $k \to \infty$ .

### Implementation

- How do you efficiently compute z<sub>c</sub>?
   Need a lookup table (and dot-product) for output embeddings!
   (Not full matrix-vector product).
- How do you efficiently handle p<sub>ML</sub>(c)
  Also can be done with lookuptable and add.
- How do you handle sampling?
   Can precompute large number of samples (not example specific).
- How do you handle loss? Simply BinaryNLL Objective.

### Implementation

Standard MLP language model,

$$\mathbf{x} \Rightarrow \mathbf{W}^1 \Rightarrow \tanh \Rightarrow \mathbf{W}^2 \Rightarrow \text{softmax}$$

Computing  $\sigma(z_c - \log(Kp_{ML}(c)))$ ,

$$\mathbf{x}\Rightarrow\mathbf{W}^{1}\Rightarrow\tanh\Rightarrow\frac{\cdot}{\mathbf{W}_{*,w}^{2}(\mathrm{Lookup})}\Rightarrow\frac{-}{\mathit{Kp}_{\mathit{ML}}(c)(\mathrm{Lookup})}\Rightarrow\sigma$$

(Efficiency, compute first three layers only once for K+1)

## Using in Practice

Several options for test time,

- Use full softmax with learned parameters.
- ► Compute subset of scores and renormalize (homework) .
- Can sometimes just use treat unormalized params as being normalized (self-normalization)