### Text Classification

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Machine Learning Review

CS 287

#### Contents

#### Text Classification

Preliminaries: Machine Learning for NLP

Features and Preprocessing

Output

#### Classification

Linear Models

Linear Model 1: Naive Bayes

Linear Model 2: Multiclass Logistic Regression

Linear Model 3: Multiclass Hinge-Loss

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### Sentiment

#### Good Sentences

- A thoughtful, provocative, insistently humanizing film.
- Occasionally melodramatic, it's also extremely effective.
- Guaranteed to move anyone who ever shook, rattled, or rolled.

#### **Bad Sentences**

- ▶ A sentimental mess that never rings true.
- ► This 100-minute movie only has about 25 minutes of decent material.
- Here, common sense flies out the window, along with the hail of bullets, none of which ever seem to hit Sascha.

### Multiclass Sentiment

\* \* \* \*\*

I visited The Abbey on several occasions on a visit to Cambridge and found it to be a solid, reliable and friendly place for a meal.

However, the food leaves something to be desired. A very obvious menu and average execution

\*\*\*\*\*
Fun, friendly neighborhood bar. Good drinks, good food, not too pricey. Great atmosphere!

### Text Categorization

- Straightforward setup.
- Lots of practical applications:
  - Spam Filtering
  - Sentiment
  - Text Categorization
  - e-discovery
  - ► Twitter Mining
  - Author Identification
  - **>**
- Introduces machine learning notation.

However, a relatively solved problem these days.

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# **Preliminary Notation**

- **b**, **m**; bold letters for vectors.
- ▶ B, M; bold capital letters for matrices.
- $\triangleright$   $\mathcal{B}$ ,  $\mathcal{M}$ ; script-case for sets.
- ▶ *B*, *M*; capital letters for random variables.
- $\triangleright$   $b_i$ ,  $x_i$ ; lower case for scalars or indexing into vectors.
- $lackbox{\delta}(i)$ ; one-hot vector at position i

$$\delta(2) = [0; 1; 0; \dots]$$

▶  $\mathbf{1}(x = y)$ ; indicator 1 if x = y, o.w. 0

### Text Classification

- 1. Extract pertinent information from the sentence.
- 2. Use this to construct an input representation.
- 3. Classify this vector into an output class.

#### Input Representation:

- Conversion from text into a mathematical representation?
- Main focus of this class, representation of language
- ▶ Point in coming lectures: *sparse* vs. *dense* representations

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# Sparse Features (Notation from YG)

- F; a discrete set of features values.
- ▶  $f_1 \in \mathcal{F}$ ,...,  $f_k \in \mathcal{F}$ ; active features for input.

For a given sentence, let  $f_1, \ldots f_k$  be the relevant features. Typically  $k << |\mathcal{F}|$ .

Sparse representation of the input defined as,

$$\mathbf{x} = \sum_{i=1}^{k} \delta(f_i)$$

 $\mathbf{x} \in \mathbb{R}^{1 \times d_{\mathrm{in}}}$ ; input representation

## Features 1: Sparse Bag-of-Words Features

Representation is counts of input words,

- $ightharpoonup \mathcal{F}$ ; the vocabulary of the language.
- $\mathbf{x} = \sum_{i} \delta(f_i)$

Example: Movie review input,

A sentimental mess

$$\mathbf{x} = \delta(\texttt{word:A}) + \delta(\texttt{word:sentimental}) + \delta(\texttt{word:mess})$$

$$\mathbf{x}^{\top} = \begin{bmatrix} 1 \\ \vdots \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ \vdots \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ \vdots \\ \vdots \\ 1 \\ \text{word:mess} \\ 1 \\ \text{word:sentimental} \end{bmatrix}$$

# Features 2: Sparse Word Properties

Representation can use specific aspects of text.

- ▶ F; Spelling, all-capitals, trigger words, etc.
- $\mathbf{x} = \sum_{i} \delta(f_i)$

Example: Spam Email

Your diploma puts a UUNIVERSITY JOB PLACEMENT COUNSELOR at your disposal.

$$\mathbf{x} = \delta(\mathtt{misspelling}) + \delta(\mathtt{allcapital}) + \delta(\mathtt{trigger:diploma}) + \dots$$

$$\mathbf{x}^\top = \begin{bmatrix} 0 \\ \vdots \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ \vdots \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ \vdots \\ 1 \\ \text{capital} \\ 1 \text{ word:diploma} \end{bmatrix}$$

### Text Classification: Output Representation

- 1. Extract pertinent information from the sentence.
- 2. Use this to construct an input representation.
- 3. Classify this vector into an output class.

#### **Output Representation:**

- ▶ How do encode the output classes?
- ▶ We will use a one-hot output encoding.
- ▶ In future lectures, efficiency of output encoding.

### **Output Class Notation**

- $ightharpoonup \mathcal{C} = \{1, \ldots, d_{\text{out}}\};$  possible output classes
- $ightharpoonup c \in \mathcal{C}$ ; always one true output class
- ullet  $\mathbf{y} = \delta(c) \in \mathbb{R}^{1 imes d_{\mathrm{in}}};$  true one-hot output representation

# Output Form: Binary Classification

Examples: spam/not-spam, good review/bad review, relevant/irrelevant document, many others.

- $d_{\text{out}} = 2$ ; two possible classes
- In our notation,

bad 
$$c = 1$$
  $\mathbf{y} = \begin{bmatrix} 1 & 0 \end{bmatrix}$  vs. good  $c = 2$   $\mathbf{y} = \begin{bmatrix} 0 & 1 \end{bmatrix}$ 

ightharpoonup Can also use a single output sign representation with  $d_{\mathrm{out}}=1$ 

## Output Form: Multiclass Classification

Examples: Yelp stars, etc.

- $d_{\text{out}} = 5$ ; for examples
- ▶ In our notation, one star, two star...

$$\star c = 1$$
  $\mathbf{y} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \end{bmatrix}$  vs.  
 $\star \star c = 2$   $\mathbf{y} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \end{bmatrix} \dots$ 

Examples: Word Prediction (Unit 3)

- ►  $d_{\text{out}} > 100,000$ ;
- ▶ In our notation, C is vocabulary and each c is a word.

the 
$$c = 1$$
  $\mathbf{y} = \begin{bmatrix} 1 & 0 & 0 & 0 & \dots & 0 \end{bmatrix}$  vs.  $\log c = 2$   $\mathbf{y} = \begin{bmatrix} 0 & 1 & 0 & 0 & \dots & 0 \end{bmatrix}$ ...

### **Evaluation**

- ▶ Consider evaluating accuracy on outputs  $y_1, ..., y_n$ .
- ▶ Given a predictions  $\hat{c_1} \dots \hat{c_n}$  we measure accuracy as,

$$\sum_{i=1}^{n} \frac{\mathbf{1}(\delta(\hat{c}_i) = \mathbf{y}_i)}{n}$$

▶ Simplest of several different metrics we will explore in the class.

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# Supervised Machine Learning

### Let,

- $(\mathbf{x}_1, \mathbf{y}_1), \dots, (\mathbf{x}_n, \mathbf{y}_n);$  training data
- $\mathbf{x}_i \in \mathbb{R}^{1 \times d_{\mathrm{in}}}$ ; input representations
- $\mathbf{y}_i \in \mathbb{R}^{1 imes d_{ ext{out}}}$ ; true output representations (one-hot vectors)

Goal: Learn a classifier from input to output classes.

#### Note:

- $ightharpoonup \mathbf{x}_i$  is an input vector  $x_{i,j}$  is element of the vector, or just  $x_j$  when there is a clear single input .
- lacktriangle Practically, store design matrix  $old X \in \mathbb{R}^{n imes d_{\mathrm{in}}}$  and output classes.

### Experimental Setup

- Data is split into three parts training, validation, and test.
- Experiments are all run on training and validation, test is final output.
- ► For assignments, full training and validation data, and only inputs for test.

For very small text classification data sets,

- Use K-fold cross-validation.
  - 1. Split into K folds (equal splits).
  - 2. For each fold, train on other K-1 folds, test on current fold.

### Linear Models for Classification

Linear model,

$$\hat{\mathbf{y}} = f(\mathbf{xW} + \mathbf{b})$$

- ullet  $\mathbf{W} \in \mathbb{R}^{d_{\mathrm{in}} \times d_{\mathrm{out}}}, \mathbf{b} \in \mathbb{R}^{1 \times d_{\mathrm{out}}};$  model parameters
- $f: \mathbb{R}^{d_{\mathrm{out}}} \mapsto \mathbb{R}^{d_{\mathrm{out}}}$ ; activation function
- ▶ Sometimes  $\mathbf{z} = \mathbf{x}\mathbf{W} + \mathbf{b}$  informally "score" vector.
- ► Note **z** and **ŷ** are not one-hot.

Class prediction,

$$\hat{c} = \argmax_{i \in \mathcal{C}} \hat{y_i} = \argmax_{i \in \mathcal{C}} (\mathbf{xW} + \mathbf{b})_i$$

### Interpreting Linear Models

Parameters give scores to possible outputs,

- $\triangleright$   $W_{f,i}$  is the score for sparse feature f under class i
- $\triangleright$   $b_i$  is a prior score for class i
- $\triangleright$   $\hat{y}_i$  is the total score for class i
- ightharpoonup  $\hat{c}$  is highest scoring class under the linear model.

#### Example:

► For single feature score,

$$[eta_1,eta_2]=oldsymbol{\delta}( exttt{word:dreadful})oldsymbol{W}$$
,

Expect  $\beta_1 > \beta_2$  (assuming 2 is class *good*).

### Probabilistic Linear Models

Can estimate a linear model probabilistically,

- Let output be a random variable Y, with sample space C.
- Representation be a random vector X.
- ► (Simplified frequentist representation)
- ▶ Interested in estimating parameters  $\theta$ ,

$$P(Y|X;\theta)$$

Informally we use  $p(\mathbf{y} = \delta(c)|\mathbf{x})$  for  $P(Y = c|X = \mathbf{x})$ .

## Generative Model. Joint Log-Likelihood as Loss

- $(\mathbf{x}_1, \mathbf{y}_1), \dots, (\mathbf{x}_n, \mathbf{y}_n);$  supervised data
- ▶ Select parameters to maximize likelihood of training data.

$$\mathcal{L}(\theta) = -\sum_{i=1}^{n} \log p(\mathbf{x}_i, \mathbf{y}_i; \theta)$$

For linear models  $\theta = (\mathbf{W}, \mathbf{b})$ 

▶ Do this by minimizing negative log-likelihood (NLL).

$$\arg\min_{\boldsymbol{\theta}} \mathcal{L}(\boldsymbol{\theta})$$

# Multinomial Naive Bayes

Reminder, joint probability chain rule,

$$\rho(\mathbf{x},\mathbf{y}) = \rho(\mathbf{x}|\mathbf{y})\rho(\mathbf{y})$$

For a sparse features, with observed classes we can write as,

$$\begin{split} \rho(\mathbf{x}, \mathbf{y}) &= \rho(x_{f_1} = 1, \dots, x_{f_k} = 1 | \mathbf{y} = \delta(c)) \rho(\mathbf{y} = \delta(c)) = \\ &= \prod_{i=1}^k \rho(x_{f_i} = 1 | x_{f_1} = 1, \dots, x_{f_{i-1}} = 1, \mathbf{y} = \delta(c)) \rho(\mathbf{y} = \delta(c)) \approx \\ &= \prod_{i=1}^k \rho(x_{f_i} = 1 | \mathbf{y}) \rho(\mathbf{y}) \end{split}$$

First is by chain-rule, second is by independence assumption.

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# **Estimating Multinomial Distributions**

Let S be a random variable with sample space S and we have observations  $s_1, \ldots, s_n$ ,

- ▶  $P(S = s; \theta) = \text{Cat}(s; \theta)$ ; parameterized as a multinomial distribution.
- ▶ Minimizing NLL for multinomial (MLE) for data has a closed-form.

$$P(S=s;\theta) = \operatorname{Cat}(s;\theta) = \sum_{i=1}^{n} \frac{\mathbf{1}(s_i=s)}{n}$$

- ightharpoonup Exercise: Derive this by minimizing  $\mathcal{L}$ .
- Also called categorical or multinoulli (in Murphy).

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# Multinomial Naive Bayes

- ▶ Both p(y) and p(x|y) are parameterized as multinomials.
- ► Fit first as,

$$p(\mathbf{y} = \delta(c)) = \sum_{i=1}^{n} \frac{1(\mathbf{y}_i = c)}{n}$$

- ► Fit second using count matrix **F** ,
  - Let

$$F_{f,c} = \sum_{i=1}^{n} \mathbf{1}(\mathbf{y}_i = c) \mathbf{1}(x_{i,f} = 1) \text{ for all } c \in \mathcal{C}, f \in \mathcal{F}$$

► Then

$$p(x_f = 1 | \mathbf{y} = \delta(c)) = \frac{F_{f,c}}{\sum_{c \in T} F_{f',c}}$$

# Multinomial Naive Bayes

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## Alternative: Multivariate Bernoulli Naive Bayes

- ▶ Both  $p(\mathbf{y})$  is multinomial as above and  $p(x_f|\mathbf{y})$  is Bernoulli over each features .
- ► Fit class as Categorical,

$$p(\mathbf{y} = \delta(c)) = \sum_{i=1}^{n} \frac{1(\mathbf{y}_i = c)}{n}$$

- ► Fit features using count matrix **F**,
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► Then,

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► Then,

$$p(x_f|\mathbf{y} = \delta(c)) = \frac{F_{f,c}}{\sum_{i=1}^{n} \mathbf{1}(\mathbf{y}_i = c)}$$

# Getting a Conditional Distribution

- ▶ Generative models estimates of P(X, Y), we want P(Y|X).
- ► Bayes Rule,

$$p(\mathbf{y}|\mathbf{x}) = \frac{p(\mathbf{x}|\mathbf{y})p(\mathbf{y})}{p(\mathbf{x})}$$

▶ In log-space,

$$\log p(\mathbf{y}|\mathbf{x}) = \log p(\mathbf{x}|\mathbf{y}) + \log p(\mathbf{y}) - \log p(\mathbf{x})$$

## Prediction with Naive Bayes

▶ For prediction, last term is constant, so

$$\operatorname*{arg\,max}\limits_{c}\log p(\mathbf{y}=\delta(c)|\mathbf{x})=\log p(\mathbf{x}|\mathbf{y}=\delta(c))+\log p(\mathbf{y}=\delta(c))$$

Can write as linear model,

$$W_{f,c} = \log p(x_f = 1 | \mathbf{y} = c) \text{ for all } c \in \mathcal{C}, f \in \mathcal{F}$$
  $b_c = \log p(\mathbf{y} = \delta(c)) \text{ for all } c \in \mathcal{C}$ 

# Getting a Conditional Distribution

What if we want conditional probabilities?

$$p(\mathbf{y}|\mathbf{x}) = \frac{p(\mathbf{x}|\mathbf{y})p(\mathbf{y})}{p(\mathbf{x})} = \frac{p(\mathbf{x}|\mathbf{y})p(\mathbf{y})}{\sum_{c' \in \mathcal{C}} p(\mathbf{x}, \mathbf{y} = \delta(c'))}$$

Denominator is acquired by renormalizing,

$$p(\mathbf{y}|\mathbf{x}) \propto p(\mathbf{x}|\mathbf{y})p(\mathbf{y})$$

## Practical Aspects: Calculating Log-Sum-Exp

Because of numerical issues, calculate in log-space,

$$f(\mathbf{z}) = \log p(\mathbf{y} = \delta(c)|\mathbf{x}) = \log z_c - \log \sum_{c' \in \mathcal{C}} \exp(z_{c'})$$

where for naive Bayes

$$z = xW + b$$

► However hard to calculate,

$$\log \sum_{c' \in \mathcal{C}} \exp(\hat{z}_{c'})$$

Instead

$$\log \sum_{c' \in C} \exp(\hat{y}_{c'} - M) + M$$

where  $M = \max_{c' \in C} \hat{z}_{c'}$ 

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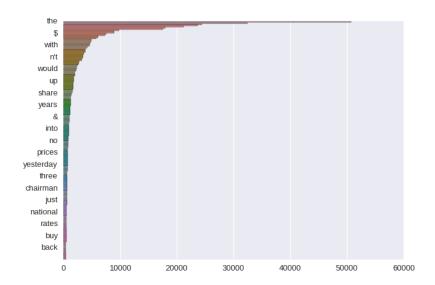
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## Digression: Zipf's Law



# Laplace Smoothing

Method for handling the long tail of words by distributing mass,

▶ Add a value of  $\alpha$  to each element in the sample space before normalization.

$$\theta_s = \frac{\alpha + \sum_{i=1}^n \mathbf{1}(s_i = s)}{\alpha |\mathcal{S}| + n}$$

► (Similar to Dirichlet prior in a Bayesian interpretation.)

For naive Bayes:

$$\hat{\mathbf{F}} = \alpha + F$$

# Laplace Smoothing

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### Naive Bayes In Practice

- Very fast to train
- Relatively interpretable.
- Performs quite well on small datasets

Method	RT-s	MPQA	CR	Subj.
MNB-uni	77.9	85.3	79.8	92.6
MNB-bi	79.0	86.3	80.0	93.6
SVM-uni	76.2	86.1	79.0	90.8
SVM-bi	77.7	<u>86.7</u>	80.8	91.7
NBSVM-uni	78.1	85.3	80.5	92.4
NBSVM-bi	<u>79.4</u>	86.3	<u>81.8</u>	93.2
RAE	76.8	85.7	_	_
RAE-pretrain	77.7	86.4	_	_
Voting-w/Rev.	63.1	81.7	74.2	_

(RT-S [movie review], CR [customer reports], MPQA [opinion polarity], SUBJ [subjectivity])