Lista 4

Seção 10.4 : 7, 17, 23, 25, 29, 33, 45, 47, 51

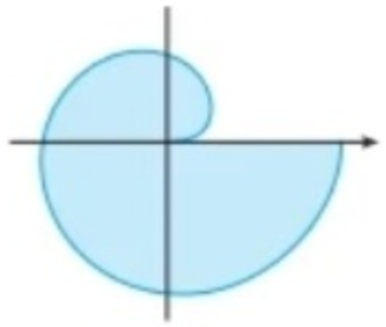
+ Monitoria_ Cal. 2

~ Fórmulas "úteis"

- Área: $A = \int_a^b \frac{r^2}{2} \, d\theta$

- Comprimento do Arco: $L = \int_a^b \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} \, d\theta$

7.



$r=\sqrt{\theta}$

Tome

$$A=\int_0^{2\pi}\frac{r^2}{2}\,d\theta=\int_0^{2\pi}\frac{\theta}{2}\,d\theta=$$

$0\leq\theta<2\pi$

$$\frac{1}{2}\,\frac{\theta^2}{2}\Bigg|_0^{2\pi}\rightarrow\frac{\theta^2}{4}\Bigg|_0^{2\pi}\rightarrow\frac{(2\pi)^2}{4}=\pi$$

17–21 Encontre a área da região dentro de um laço da curva.

17. $r = 4\cos 3\theta$

18. $r^2 = 4\cos 2\theta$

19. $r = \operatorname{sen} 4\theta$

20. $r^2 = \operatorname{sen} 2\theta$

21. $r = 1 + 2\operatorname{sen}\theta$ (laço interno)

Quando $r=0 \rightarrow 4\cos 3\theta=0 \leftrightarrow$

$\cos 3\theta=0 \rightarrow \cos 3\theta=\cos\frac{\pi}{2}$

devemos ter:

$$\begin{cases} 3\theta=\frac{\pi}{2}\rightarrow\theta=\frac{\pi}{6} \\ \text{ou} \\ 3\theta+\frac{\pi}{2}=2\pi\rightarrow 3\theta=\frac{3\pi}{2}\rightarrow\theta=\frac{\pi}{2} \end{cases}$$

$$A=\int\frac{r^2}{2}\,d\theta\rightarrow\int_{-\pi/6}^{\pi/6}\frac{(4\cos 3\theta)^2}{2}\,d\theta=8\int_{-\pi/6}^{\pi/6}\cos^2 3\theta\,d\theta$$

$$\left.\begin{array}{l}\cos 2x = 2\cos^2 x - 1 \\ \cos 6\theta = 2\cos^2 3\theta - 1\end{array}\right\} \cos^2 3\theta = \frac{1+\cos 6\theta}{2}$$

daí

$$8\int \left(\frac{1+\cos 6\theta}{2}\right) d\theta = 4\left(\int 1\, d\theta + \int \cos 6\theta\, d\theta\right)$$

θ $\qquad$ $\frac{\operatorname{sen} 6\theta}{6}$

$$4\left(\theta + \frac{\operatorname{sen} 6\theta}{6}\right)\Bigg|_{-\pi/6}^{\pi/6} =$$

23–28 Encontre a área da região que está dentro da primeira curva e fora da segunda curva.

23. $r = 4 \operatorname{sen} \theta, \quad r = 2$ $\qquad$ 24. $r = 1 - \operatorname{sen} \theta, \quad r = 1$

25. $r = 2 \cos \theta, \quad r = 1$

26. $r = 1 + \cos \theta, \quad r = 2 - \cos \theta$

27. $r = 3 \cos \theta, \quad r = 1 + \cos \theta$

23) $4 \operatorname{sen}\theta = 2 \rightarrow \operatorname{sen}\theta = \frac{1}{2}$

$\operatorname{sen}\theta = \operatorname{sen}\frac{\pi}{6}$

$$\begin{cases}\theta = \frac{\pi}{6} \\ \text{ou} \\ \theta + \frac{\pi}{6} = \pi \rightarrow \theta = \frac{5\pi}{6}\end{cases}$$

$$A = \int_0^{\pi/6} \frac{1}{2}\left[(4\,\text{sen}\,\theta)^2 - 2^2\right] d\theta \rightarrow$$

$$\frac{1}{2}\left(16 \int_0^{\pi/6} \text{sen}^2\theta \, d\theta - \int \cancel{4 \, d\theta}^{\,4\theta} \right) \star$$

$$\cos 2\theta = 1 - 2\,\text{sen}^2\theta \Rightarrow \text{sen}^2\theta = \frac{\cos 2\theta - 1}{2}$$

$$\int \text{sen}^2\theta \, d\theta = \int \frac{\cos 2\theta - 1}{2} \, d\theta$$

$$\frac{16}{2} \int (\cancel{\cos 2\theta}_{\frac{\text{sen}\,2\theta}{2}} - \cancel{1}^{\,\theta}) \, d\theta = 8\left(\frac{\text{sen}\,2\theta}{2} - \theta\right)$$

$$4\,\text{sen}\,2\theta - 8\theta$$

Em $\star$ temos: $\frac{1}{2}(4\,\text{sen}\,2\theta - 8\theta - 4\theta)\,d\theta$

$$2\,\text{sen}\,2\theta - 12\theta \Big|_0^{\pi/6} =$$

25) $2\cos\theta = 1 \rightarrow \cos\theta = 1/2$

$$\cos\theta = \cos\pi/3 \begin{cases} \theta = \frac{\pi}{3} \\ \theta + \frac{\pi}{3} = 2\pi \rightarrow \theta = \frac{5\pi}{6} \end{cases}$$

$$A = \int_0^{\frac{\pi}{3}} \left[\frac{(2\cos\theta)^2 - 1^2}{2}\right] d\theta \Rightarrow$$

$$A = \int_0^{\frac{\pi}{3}} \frac{4\cos^2\theta - 1}{2} d\theta \rightarrow$$

$\cos 2\theta = 2\cos^2\theta - 1$

$4\cos^2\theta = 2\cos 2\theta + 2$

$$A = \int_0^{\frac{\pi}{3}} \frac{2\cos 2\theta + 2 - 1}{2} d\theta = \int_0^{\pi/3} \frac{2\cos 2\theta + 1}{2} d\theta$$

$$A = \int \frac{2\cos 2\theta}{2} d\theta + \int \frac{1}{2} d\theta \rightarrow$$

$\frac{\operatorname{sen} 2\theta}{2}$ $\quad \frac{\theta}{2}$ $\rightarrow$

$$A = \frac{1}{2}(\operatorname{sen} 2\theta + \theta) \Big|_0^{\pi/2}$$

29–34 Encontre a área da região que está dentro de ambas as curvas.

29. $r = 3\,\text{sen}\,\theta$, $r = 3\cos\theta$

$3\,\text{sen}\,\theta = 3\cos\theta \rightarrow \text{sen}\,\theta = \cos\theta \rightarrow$

$\dfrac{\text{sen}\,\theta}{\cos\theta} = 1 \rightarrow \text{tang}\,\theta = 1$

$\theta = \pi/4$ ou $\theta = \dfrac{5\pi}{4} \rightarrow$

$$A = 4 \cdot 2 \cdot \int_0^{\pi/4} \frac{(3\,\text{sen}\,\theta)^2}{2}\, d\theta \rightarrow$$

$$A = 8 \int_0^{\pi/4} \frac{9}{2}\,\text{sen}^2\theta\, d\theta \rightarrow 36 \int_0^{\pi/4} \text{sen}^2\theta\, d\theta$$

$\cos 2\theta = 1 - 2\,\text{sen}^2\theta \rightarrow \text{sen}^2\theta = \dfrac{1 - \cos 2\theta}{2}$

$$36 \int_0^{\pi/4} \left(\frac{1 - \cos 2\theta}{2}\right) d\theta \rightarrow$$

$$18 \left(\int 1\, d\theta - \int \cos 2\theta\, d\theta \right) \rightarrow$$

θ $\qquad \dfrac{\text{sen}\,2\theta}{2}$

$$18 \left(\theta - \frac{\text{sen}\,2\theta}{2} \right) \Big|_0^{\pi/4} = \ldots$$

0

$$2\,\text{sen}\,2\theta = 1 \rightarrow \text{Sen}\,2\theta = \frac{1}{2} \rightarrow$$

31. $r = \text{sen}\,2\theta$, $r = \cos 2\theta$

32. $r = 3 + 2\cos\theta$, $r = 3 + 2\,\text{sen}\,\theta$

33. $r = 2\,\text{sen}\,\theta$, $r = \ldots$

$$\text{Sen}\,2\theta = \text{Sen}\,\frac{\pi}{3} \rightarrow 2\theta = \frac{\pi}{3} \rightarrow \theta = \frac{\pi}{6}$$

$$A = 6 \cdot \int_0^{\frac{\pi}{6}} \frac{2\,\text{sen}\,2\theta}{2}\,d\theta \rightarrow 6\int \cancel{\text{sen}}\,2\theta\,d\theta$$

$$-\frac{\cos 2\theta}{2}$$

$$-6\,\frac{\cos 2\theta}{2}\Big|_0^{\frac{\pi}{2}} \rightarrow -3\cos 2\theta\Big|_0^{\frac{\pi}{6}}$$

$$-3\,\cancel{\cos}^{\,1/2}\,\frac{\pi}{3} + 3\,\cancel{\cos}^{\,1}\,0 \rightarrow -\frac{3}{2} + 3 = \frac{3}{2}$$

3

$$dr = -2\,\text{sen}\,\theta\,d\theta$$

$$L = \int_a^b \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\,d\theta =$$

45–48 Calcule o comprimento exato da curva polar.

45. $r = 2\cos\theta$, $0 \leq \theta \leq \pi$ **46.** $r = 5^\theta$, $0 \leq \theta \leq 2\pi$

47. $r = \theta^2$, $0 \leq \theta \leq 2\pi$ **48.** $r = 2(1 + \cos\theta)$

$$L = \int_0^{\pi} \sqrt{(2\cos\theta)^2 + (-2\,\text{sen}\,\theta)^2}\,d\theta$$

$$L = \int_0^{\pi} \sqrt{4(\cos^2\theta + \text{sen}^2\theta)}\,d\theta = \int_0^{\pi} 2\,d\theta = 2\theta\Big|_0^{\pi}$$

47- $r=\theta^2 \rightarrow \frac{dr}{d\theta}=2\theta$

$$L=\int_0^{\pi}\sqrt{(\theta^2)^2+(2\theta)^2}\,d\theta \rightarrow \int_0^{\pi}\sqrt{\theta^4+4\theta^2}\,d\theta$$

$$\int_0^{\pi}\theta\sqrt{\theta^2+4}\,d\theta$$

$u=\theta^2+4$

$du=2\theta d\theta$

$$\int_a^b\sqrt{u}\,du=\int_a^b u^{1/2}du=\frac{2u^{3/2}}{3}\Big|_a^b$$

$$\frac{2}{3}(\theta^2+4)\Big|_0^{\pi} \rightarrow \frac{2}{3}(\pi^2+4)-\frac{2}{3}(0+4)$$

$$\boxed{\frac{2\pi^2}{3}}$$

c)

51–54 Use uma calculadora ou um computador para encontrar o comprimento do laço, com precisão de quatro casas decimais. Se necessário, use um gráfico para determinar o intervalo de parâmetro.

51. Uma volta na curva $r = \cos 2\theta$

52. $r = \operatorname{tg}\theta$, $\pi/6 \le \theta \le \pi/3$

53. $r = \operatorname{sen}(6 \operatorname{sen}\theta)$

$r = \cos 2\theta \rightarrow dr = -2\operatorname{sen}(2\theta)\,d\theta$

$$L = \int_a^b \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta \Rightarrow$$

$$\int_a^b \sqrt{(\cos 2\theta)^2 + (-2\operatorname{sen}(2\theta))^2}\, d\theta \Rightarrow$$

$$\int_a^b \sqrt{\cos^2 2\theta + 4\operatorname{sen}^2(2\theta)}\, d\theta$$

$\operatorname{sen}^2 x + \cos^2 x = 1 \rightarrow$

$$\underbrace{\underbrace{\cos^2(2\theta) + \operatorname{sen}^2(2\theta)}_{1} + 3\operatorname{sen}^2(2\theta)}_{1 + 3\operatorname{sen}^2(2\theta)}$$

$$\int_a^b \sqrt{1 + 3\operatorname{sen}^2(2\theta)}\, d\theta$$

Use uma Calculadora!