Lista 8.2

Regra da Cadeia:

Se $y = f(x)$ e $x = g(t)$, $\left\{ \boxed{\dfrac{dy}{dt} = \dfrac{dy}{dx} \cdot \dfrac{dx}{dt}} \right.$

Se $Z = f(x,y)$ com $x = f(t)$ e $y = g(t)$ então: $\left\{ \boxed{\dfrac{dz}{dt} = \dfrac{dz}{dx} \cdot \dfrac{dx}{dt} + \dfrac{dz}{dy} \dfrac{dy}{dt}} \right.$

Se $Z = f(x,y)$ com $x = f(s,t)$ e $y = g(s,t)$ então:

$$\boxed{\frac{dz}{ds} = \frac{dz}{dx} \cdot \frac{dx}{ds} + \frac{dz}{dy} \cdot \frac{dy}{ds}} \quad e \quad \boxed{\frac{dz}{dt} = \frac{dz}{dx} \frac{dx}{dt} + \frac{dz}{dy} \cdot \frac{dy}{dt}}$$

Derivada Implícita:

$F(x, f(x))$, $y = f(x)$

$$\boxed{\frac{\partial y}{\partial x} = -\frac{\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial y}} = \frac{-F_x}{F_y}}$$

21–26 Utilize a Regra da Cadeia para determinar as derivadas parciais indicadas.

21. $z = x^2 + xy^3, \quad x = uv^2 + w^3, \quad y = u + ve^w;$

$\dfrac{\partial z}{\partial u}, \dfrac{\partial z}{\partial v}, \dfrac{\partial z}{\partial w}$ quando $u = 2, \quad v = 1, \quad w = 0$

22. $u = \sqrt{r^2 + s^2}, \quad r = y + x\cos t, \quad s = x + y \operatorname{sen} t;$

$\dfrac{\partial u}{\partial x}, \dfrac{\partial u}{\partial y}, \dfrac{\partial u}{\partial t}$ quando $x = 1, \quad y = 2, \quad t = 0$

23. $w = xy + yz + zx, \quad x = r\cos\theta, \quad y = r \operatorname{sen}\theta, \quad z = r\theta;$

$\dfrac{\partial w}{\partial r}, \dfrac{\partial w}{\partial \theta}$ quando $r = 2, \theta = \pi/2$

24. $P = \sqrt{u^2 + v^2 + w^2}, \quad u = xe^y, \quad v = ye^x, \quad w = e^{xy};$

$\dfrac{\partial P}{\partial x}, \dfrac{\partial P}{\partial y}$ quando $x = 0, \quad y = 2$

25. $N = \dfrac{p + q}{p + r}, \quad p = u + vw, \quad q = v + uw, \quad r = w + uv;$

$\dfrac{\partial N}{\partial u}, \dfrac{\partial N}{\partial v}, \dfrac{\partial N}{\partial w}$ quando $u = 2, \quad v = 3, \quad w = 4$

21) $Z = x^2 + xy^3$, $x = uv^2 + w^3$

$y = u + ve^w$

$u = 2$; $v = 1$ e $w = 0$ → $x = 2.1 + 0 \therefore x = 2$

$y = 2 + 1.e^0 \therefore y = 2$

$$\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x} \cdot \frac{dx}{du} + \frac{\partial z}{\partial y} \cdot \frac{dy}{du}$$

Subst: tems

$$\frac{dz}{dx} = 2x + y^3, \quad \frac{dx}{du} = v^2$$

$$\frac{dz}{dy} = 3xy^2; \quad \frac{dy}{du} = 1$$

$$(2x + y^3) \cdot v^2 + 3xy^2 \cdot 1 \rightarrow$$

$$(2.2 + 2^3).1 + 3.2.2^2.1 \rightarrow (4 + 8) + 24 = 36$$

$$\frac{\partial z}{\partial v} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial v} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial v}$$

$\frac{\partial x}{\partial v} = 2uv$ e $\frac{\partial y}{\partial v} = e^{w}$ subst

$$\frac{\partial z}{\partial v} = (2x+y^3).2uv + 3xy^2.e^{w}$$

$$(2.2+2^3).2.2.1 + 3.2.2^2.e^0 =$$

$$(4+8).4 + 24 = 48+24 = 72$$

$$\frac{\partial z}{\partial w} = \frac{\partial z}{\partial x}.\frac{\partial x}{\partial w} + \frac{\partial z}{\partial y}.\frac{\partial y}{\partial w} \Big|$$ com

$\frac{\partial x}{\partial w} = 3w^2$ e $\frac{\partial y}{\partial w} = w.ve^{w}$ logo

$$(2x+y^3).3w^2 + 3xy^2.(w.ve^{w})$$

$$(2.2+2^3).3.0 + 3.2.2^2.0.1.e^0 = 0$$

23) $w = xy + yz + zx$

$x = r\cos\theta$, $y = r\operatorname{sen}\theta$, $z = r\theta$

$r=2$ e $\theta = \pi/2$

$x = 2.\cos(\frac{\pi}{2}) = 0$; $y = 2\operatorname{sen}(\frac{\pi}{2}) = 2$

$z = 2.\frac{\pi}{2} = \pi$

$\frac{dx}{dr} = \cos\theta$, $\frac{dy}{dr} = \operatorname{sen}\theta$; $\frac{dz}{dr} = \theta$

$$\frac{\partial w}{\partial r} = \frac{\partial w}{\partial x} \cdot \frac{\partial x}{\partial r} + \frac{\partial w}{\partial y} \cdot \frac{\partial z}{\partial r} + \frac{\partial w}{\partial z} \cdot \frac{dz}{dr}$$

$$\frac{\partial w}{\partial x} = y + z \; ; \; \frac{\partial w}{\partial z} = y + x \; ; \; \frac{\partial w}{\partial y} = x + y$$

Substi.:

$$(y+z)\cos\theta + (x+z)\operatorname{sen}\theta + (y+x)\theta$$

$$(2+\pi)\cos\frac{\pi}{2} + (0+\pi)\cdot\operatorname{sen}\frac{\pi}{2} + (2+0)\cdot\frac{\pi}{2}$$

$$\pi + \pi = 2\pi$$

$$\frac{\partial x}{\partial \theta} = -r\operatorname{sen}\theta, \; \frac{\partial y}{d\theta} = r\cos\theta \; , \; \frac{\partial z}{\partial \theta} = r$$

Subst.:

$$(y+z)\cdot(-r\operatorname{sen}\theta) + (x+z)\,r\cos\theta + (y+x)\cdot r$$

$$(2+\pi)\cdot(-2) + (0+\pi)\cdot 0 + (2+0)\cdot 2 = -2\pi$$

25- $N = \dfrac{p+q}{p+r}$ $\quad p = u + vw$

$q = v + uw$

$r = w + uv$

$$\frac{\partial N}{\partial u} = \frac{dN}{\partial p}\cdot\frac{\partial p}{\partial u} + \frac{\partial N}{\partial q}\cdot\frac{\partial q}{du} + \frac{\partial N}{dr}\cdot\frac{\partial r}{\partial u}$$

$$y = \frac{u}{v} \rightarrow y' = \frac{u'v - uv'}{v^2}$$

$$\frac{\partial N}{\partial p} = \frac{1(p+r)-(p+q).1}{(p+r)^2} \quad ; \quad dp$$

$$\frac{\partial N}{\partial q} = \frac{1.(p+r)-(p+q).0}{(p+r)^2} \quad ; \quad \frac{\partial N}{\partial r} = \frac{0(p+r)-(p+q)1}{(p+r)^2}$$

$$\frac{\partial p}{\partial u} = 1 \ , \quad \frac{\partial q}{\partial u} = w \quad ; \quad \frac{\partial r}{\partial u} = v$$

Subst.: $\frac{(r-q)+(p+r)w-(p+q)v}{(p+r)^2}$

$$\frac{\partial N}{\partial v} = \frac{\partial N}{\partial p}\cdot\frac{\partial p}{\partial v} + \frac{\partial N}{\partial q}\frac{\partial q}{\partial v} + \frac{\partial N}{\partial r}\cdot\frac{\partial r}{\partial v}$$

$$\frac{\partial N}{\partial p} = \frac{r-q}{(p+r)^2} \quad ; \quad \frac{\partial N}{\partial q} = \frac{p+r}{(p+r)^2} \quad e \quad \frac{\partial N}{\partial r} = \frac{-(p+q)}{(p+r)^2}$$

$$\frac{\partial p}{\partial v} = w, \quad \frac{\partial q}{\partial v} = 1 \quad e \quad \frac{\partial r}{\partial v} = u$$

Subst.: $\frac{(r-q)}{(p+r)^2}w + \frac{(p+r)}{(p+r)^2}.1 - \frac{(p+q)}{(p+r^2)}.u$

$$\frac{(r-q)w+(p+r)-(p+q)u}{(p+r)^2}$$

Novamente

$$\frac{\partial N}{\partial w} = \frac{\partial N}{\partial p}\cdot\frac{\partial p}{\partial w} + \frac{\partial N}{\partial q}\cdot\frac{\partial q}{\partial w} + \frac{\partial N}{\partial r}\cdot\frac{\partial r}{\partial w}$$

$$\frac{\partial N}{\partial p} = \frac{(r-q)}{(p+r)^2} \;;\; \frac{\partial N}{\partial q} = \frac{\cancel{(p+r)}}{(p+r)^2} = \frac{1}{(p+r)}$$

$$\frac{\partial N}{\partial r} = \frac{-(p+q)}{(p+r)^2} \quad e \quad \frac{\partial p}{\partial w} = v,\; \frac{\partial q}{\partial w} = u;\; \frac{\partial p}{\partial w} = 1$$

Subst: $\frac{(r-q)}{(p+r)^2}\, v + \frac{1}{(p+r)^2}\cdot u - \frac{(p+q)}{(p+r)^2}\cdot 1 =$

$$\frac{(r-q)\,v + (p+r)\,u - (p+q)}{(p+r)^2}$$

Terminei a Conta Por favor...

35. A temperatura em um ponto (x, y) é $T(x, y)$, medida em graus Celsius. Um inseto rasteja, de modo que sua posição após t segundos é dada por $x = \sqrt{1 + t}$, $y = 2 + \frac{1}{3}t$, onde x e y são medidos em centímetros. A função da temperatura satisfaz $T_x(2, 3) = 4$ e $T_y(2, 3) = 3$. Quão rápido a temperatura aumenta no caminho do inseto depois de três segundos?

35- $T(x,y)$ $x = \sqrt{1+t}$, $y = 2 + \frac{t}{3}$

$T_x(2,3) = 4$, $T_y(2,3) = 3$: $t = 3$ segundos

$$\frac{dT}{dt} = \frac{\partial T}{\partial x} \cdot \frac{\partial x}{\partial t} + \frac{\partial T}{\partial y} \cdot \frac{dy}{dt}$$

$\frac{dT}{dx} = T_x(x,y)$; $\frac{dT}{dy} = T_y(x,y)$, $t = 3 \Rightarrow x = \sqrt{1+3} = 2$

$y = 2 + \frac{3}{3} = 3$

$\frac{dx}{dt} = \frac{1}{2\sqrt{1+t}} = \frac{1}{2\sqrt{1+3}} = \frac{1}{4}$ $\frac{dy}{dt} = \frac{1}{3}$

Subs:

$$\frac{dT}{dt} = T_x(x,y) \cdot \frac{dx}{dt} + T_y(x,y)\frac{dy}{dt} \rightarrow$$ uni

$$\frac{dT}{dt} = 4 \cdot \frac{1}{4} + 3 \cdot \frac{1}{3} = 1 + 1 = 2 \quad 2°C/s$$

39. O comprimento ℓ, a largura w e a altura h de uma caixa variam com o tempo. Em um determinado momento, as dimensões são $\ell = 1$ m e $w = h = 2$ m, ℓ e w estão aumentando em uma taxa de 2 m/s enquanto h está decrescendo em uma taxa de 3 m/s. Nesse instante, encontre as taxas em que as seguintes quantidades estão variando.

(a) O volume

(b) A área da superfície

(c) O comprimento da diagonal

$V = l \cdot h \cdot w$

$l = 1m$, $h = w = 2m$

$\frac{dl}{dt} = \frac{\partial w}{\partial t} = 2\, m/s$; $\frac{\partial h}{\partial t} = -3\, m/s$

a) $V = l.h.w \Rightarrow$

$$\frac{dV}{dt} = \frac{\partial V}{\partial l}\cdot\frac{\partial l}{dt} + \frac{\partial V}{\partial h}\cdot\frac{dh}{\partial t} + \frac{\partial V}{\partial w}\cdot\frac{dw}{dt}$$

$$\frac{dV}{dl} = hw \;;\; \frac{dV}{\partial h} = lw \;;\; \frac{dV}{dw} = lh$$

$1.2 = 2 \qquad 2.2 = 4 \qquad 1.2 = 2$

subst: $2.2 + 2.22 + 1.2.(-3) = 6 m^3/s$

b) $A = 2(lh + lw + hw)$

$$\frac{dS}{dt} = \frac{dS}{\partial l}\cdot\frac{\partial l}{dt} + \frac{\partial S}{\partial w}\cdot\frac{dw}{dt} + \frac{\partial S}{\partial h}\cdot\frac{\partial h}{\partial t} =$$

$$\frac{\partial S}{\partial l} = 2(h+w) \;;\; \frac{\partial S}{\partial w} = 2(h+l) \;;\; \frac{\partial S}{\partial h} = 2(l+w)$$

$2(2+2) = 8 \qquad 2(2+1) = 6 \qquad 2(1+2)\; 6$

subst: $8.2 + 6.2 + 6(-3) = 10 m^2/s$

c) $D^2 = l^2 + w^2 + h^2 \rightarrow D = \sqrt{1^2 + 2^2 + 2^2} = 3$

$$\frac{\partial D}{\partial t} = \frac{\partial D}{\partial l}\cdot\frac{\partial l}{\partial t} + \frac{\partial D}{\partial w}\cdot\frac{\partial w}{\partial t} + \frac{\partial D}{\partial h}\cdot\frac{dh}{\partial t}$$

usando Regra da Cadeia...

$$2) \frac{\partial D}{\partial t} = 2l\frac{\partial l}{\partial t} + 2w\cdot\frac{\partial w}{\partial t} + 2h\cdot\frac{\partial h}{\partial t}$$

$$3) \frac{\partial D}{\partial t} = 2\cdot 2\cdot 2 + 2\cdot 2\cdot 2 + 2\cdot 2\cdot(-3) = 0$$

$$\frac{\partial D}{\partial t} = 0 \text{ m/s}$$

45. Se $z = f(x, y)$, onde $x = r\cos\theta$ e $y = r \operatorname{sen}\theta$, (a) determine $\partial z/\partial r$ e $\partial z/\partial\theta$ e (b) mostre que

$$\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = \left(\frac{\partial z}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial z}{\partial \theta}\right)^2$$

$x = r\cos\theta$ $z = f(x,y)$

$y = r \operatorname{sen}\theta$

a) $\frac{\partial z}{\partial r}$ e $\frac{\partial z}{\partial \theta}$

$$\frac{\partial z}{\partial r} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial r} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial r}$$

$$\frac{\partial z}{\partial \theta} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial \theta} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial \theta}$$

$\frac{\partial x}{\partial r} = \cos\theta$

$\frac{\partial y}{\partial r} = \operatorname{sen}\theta$

$\frac{\partial x}{\partial \theta} = -r\operatorname{sen}\theta$

$\frac{\partial y}{\partial \theta} = r\cos\theta$

$$\frac{\partial z}{\partial r} = \frac{\partial z}{\partial x}\cdot\cos\theta + \frac{\partial z}{\partial x}\operatorname{sen}\theta$$

$$\frac{\partial z}{\partial \theta} = \frac{\partial z}{\partial x}\cdot -r\operatorname{sen}\theta + \frac{\partial z}{\partial y} r\cos\theta$$

b) Elevando as equações ao quadrado e somando as duas.

$$\left(\frac{\partial z}{\partial r}\right)^2 = \left(\frac{\partial z}{\partial x} \cdot \cos\theta\right)^2 + 2\frac{\partial z}{\partial x} \cdot \frac{\partial z}{\partial y} \cos\theta \cdot \operatorname{sen}\theta +$$

$$\left(\frac{\partial z}{\partial x} \operatorname{sen}\theta\right)^2$$

$$\left(\frac{\partial z}{\partial \theta}\right)^2 = \left(\frac{\partial z}{\partial x} \cdot r \operatorname{sen}\theta\right)^2 + 2\frac{\partial z}{\partial x} \cdot \frac{\partial z}{\partial y} r^2 \cos\theta \operatorname{sen}\theta$$

$$\left(\frac{\partial z}{\partial y} r \cos\theta\right)^2$$

Temos:

$$\left(\frac{\partial z}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial z}{\partial \theta}\right)^2 = (\cos^2\theta + \operatorname{sen}^2\theta)\left(\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2\right)$$

$$\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2$$