+ Lista 15.10

- Mudanças de Variáveis.

$$\int_a^b f(x)\,dx = \int_a^b f(g(u))\,g'(u)\,du$$

- Jacobiano: $x = g(u,v)$, $y = h(u,v)$

$$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix}$$

$$\Delta A \approx \left|\frac{\partial(x,y)}{\partial(u,v)}\right| \Delta u\,\Delta v$$

- Mudança de variável numa integral dupla:

$$\iint_R f(x,y)\,dA = \iint_S f(x(u,v),y(u,v)) \left|\frac{\partial(x,y)}{\partial(u,v)}\right| du\,dv$$

- Integrais Triplas: $\iiint_R f(x,y,z)\,dV$

$$\iiint_S f(x(u,v,w),y(u,v,w),z(u,v,w)) \left|\frac{\partial(x,y,z)}{\partial(u,v,w)}\right| du\,dv\,dw$$

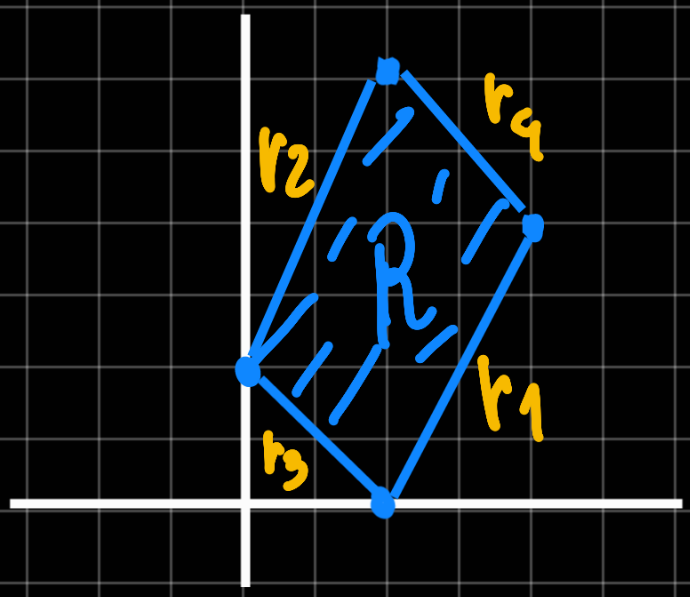
Exercícios:

R região no plano xy, T transformação que mapeia S no plano uv sobre R

11- R é limitado por:

$y = 2x - 1$, $y = 2x + 1$, $y = 1 - x$, $y = 3 - x$

r_1 r_2 r_3 r_4



Note que:

r_1: $y - 2x = -1$

r_2: $y - 2x = 1$

Tomo $y - 2x = u$

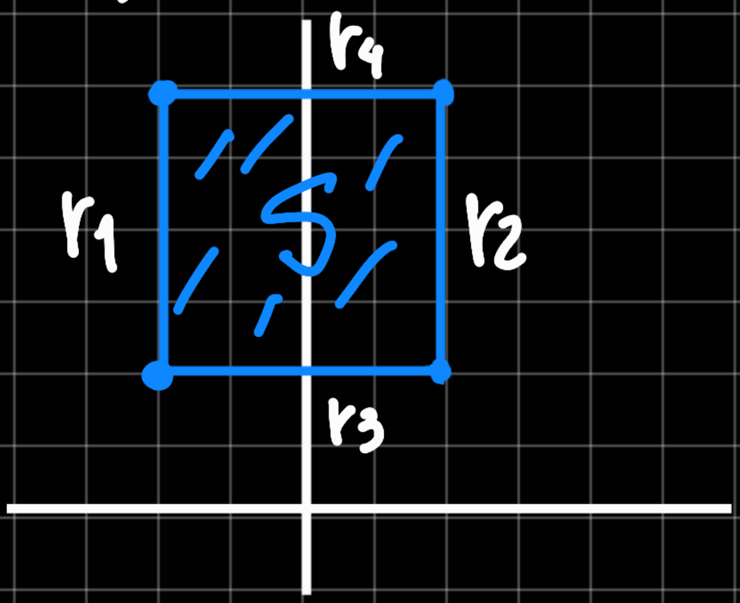
r_1: $u = -1$ e r_2: $u = 1$

r_3: $y + x = 1$ tomo $y + x = v$ $\rightarrow$ r_3: $v = 1$

r_4: $y + x = 3$ r_4: $v = 3$

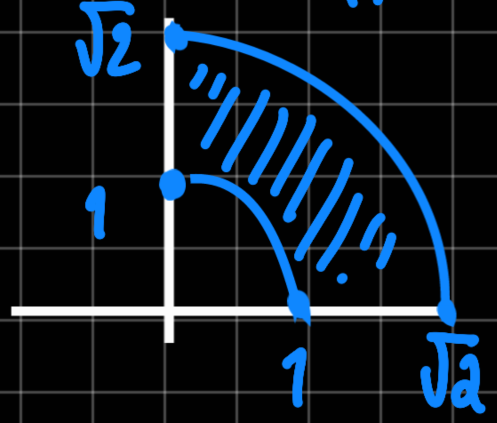
Daí

Temos:



Como: $u = y - 2x$, $v = x + y$ $\rightarrow$ $x = \frac{v - u}{3}$; $y = \frac{u + 2v}{3}$

13) $x^2+y^2=1$; $x^2+y^2=2$ no 1º quad

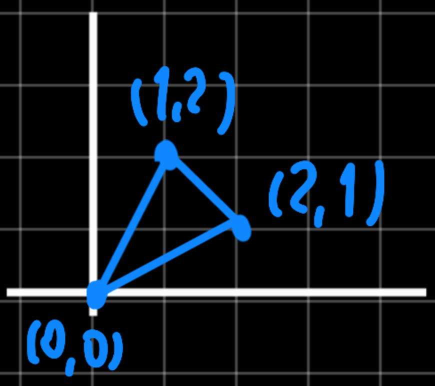


$$\left.\begin{array}{l} 1<r<\sqrt{2} \\ 0<\theta<\pi/2 \end{array}\right)$$

Tome $x=u\cos v$; $y=u\,\text{sen}\,v$

$1<u<\sqrt{2}$ e $0<v<\pi/2$

15) $\displaystyle\iint_R (x-3y)\,dA$



$x=2u+v$

$y=u+2v$

$\dfrac{dx}{du}=2$; $\dfrac{dx}{dv}=1$; $\dfrac{dy}{du}=1$; $\dfrac{dy}{dv}=2 \Rightarrow$

$$J=\begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix}=4-1=3$$

$x-3y=(2u+v)-3(u+2v) \Rightarrow$

$2u+v-3u-6v=-u-5v$

retas:

r_1: $(0,0)\to(2,1)$: $y=x/2$: $(u+2v)=\dfrac{(2u+v)}{2}$

$\cancel{2u}+4v=\cancel{2u}+v \rightarrow v=0$

r_2: $(0,0\to(1,2)$: $y=2x$: $u+2v=2(2u+v) \rightarrow$

$\cancel{2u}+2v=\cancel{4u}+2v \rightarrow u=0$

$r3: (1,2) \to (2,1) \therefore x+y=3$

$(u+2v)+(2u+v)=3 \to 3u+3v=3 \therefore u+v=1$

Quando $u=0 \to v=1$ e $v=0 \to u=1$

Daí: $0<u<1$ e $0<v<1$ $\}$ $u=1-v$

Integrando: $\int_0^1 \int_0^{1-v} (-u-5v).(3)\,du\,dv$

Em u: $\int_0^{1-v} (-3)(u+5v)\,du = -3$

16) $\iint_R (4x+8y)\,dA$ $R \therefore (-1,3), (1,-3)$
$(3,-1), (1,5)$

$x = \frac{1}{4}(u-v)$ e $y = \frac{(v-3u)}{4}$

. Jacobiano

$\frac{dx}{du} = \frac{1}{4}$; $\frac{dx}{dv} = -\frac{1}{4}$; $\frac{dy}{du} = \frac{-3}{4}$ e $\frac{dy}{dv} = \frac{1}{4}$

$$J = \begin{vmatrix} \frac{dx}{du} & \frac{dy}{dv} \\ \frac{dy}{du} & \frac{dy}{dv} \end{vmatrix} = \begin{vmatrix} 1/4 & \frac{1}{4} \\ -3/4 & 1/4 \end{vmatrix} = \frac{1}{16} + \frac{3}{16} = \frac{4}{16} = \frac{1}{4}$$

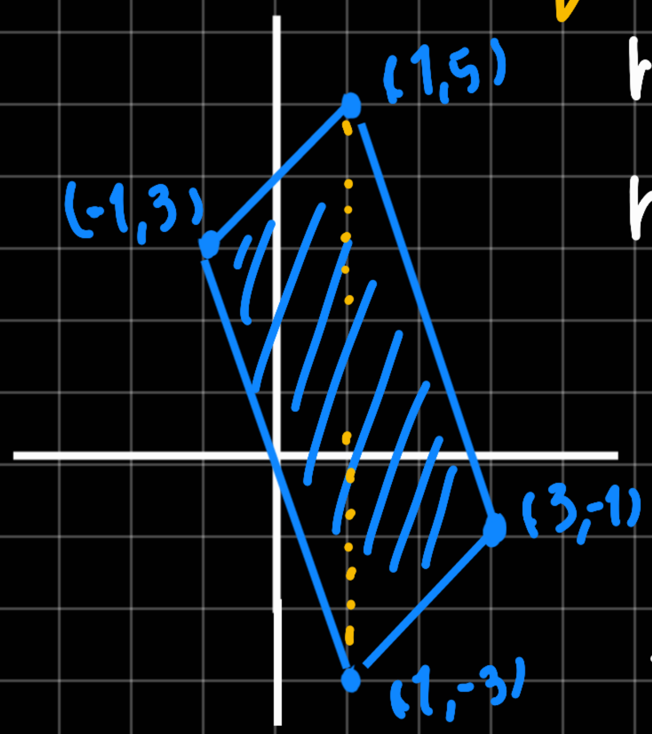
. Mudança de variável

$$\left.\begin{array}{l} 4x = u-v \\ 4y = v-3u \end{array}\right\} \to (4x+8y) =$$

$(u-v)+2(v-3u) \to$

$u-v+2v-6u = v-5u$

• Intervalos das Regiões



Retas:

r1: $(-1,3) \to (1,5)$: $y = x+4$

$\overbrace{x-y}^{} = -4$ $\quad u=4$

r2: $(1,5) \to (3,-1)$

$\therefore \overbrace{3x+y}^{v} = 8 \therefore$ $\quad v=8$

r3: $(1,-3) \to (3,-1)$

$x - y = 4 \to$ $\quad u=4$

r4: $(1,-3) \to (-1,3) \therefore y = -3x \therefore y+3x=0$

$v=0$ $\qquad u=-4 \to u=4$

• **Região:** $v=0$ e $v=8$

• **Integrando:** $\int_0^8 \int_{-4}^4 (3v - 5u) \cdot \frac{1}{4}\, du\, dv$

em u: $\int_{-4}^4 (3v - 5u)\, du = 3vu - 5\frac{u^2}{2} \Big|_{-4}^{4}$

$\left(3 \cdot v \cdot 4 - 5 \cdot \frac{4^2}{2}\right) - \left(3v(-4) - 5 \cdot \frac{(-4)^2}{2}\right) \to$

$12v - 40 + 12v + 40 = 24v$

Em v: $\frac{1}{4}\int_0^8 24v\, dv \therefore \frac{24v^2}{2} \Big|_0^8 = \frac{12v^2}{4}$

$3 \cdot 8^2 = 3 \cdot 64 = \underline{192}$

19) $\iint_R xy\,dA$ $\left\{ \begin{array}{l} y = x \\ y = 3x \end{array} \right\}$ $\begin{array}{l} xy = 1 \\ xy = 3 \end{array}$ e $x = \frac{u}{v}$; $y = v$

$\frac{dx}{du} = \frac{1}{v}$; $\frac{dy}{du} = 0$; $\frac{dx}{dv} = \frac{-u}{v^2}$; $\frac{dy}{dv} = 1$

• Jacobi:. $J = \begin{vmatrix} 1/v & -u/v^2 \\ 0 & 1 \end{vmatrix} = \frac{1}{v}$

• Mudança de Variável: $x = \frac{u}{y} \to xy = u$

quando $x = y \to y^2 = u \therefore v^2 = u \to v = \sqrt{u}$

$y = 3x \to v = 3u \to v^2 = 3u$ $v = \sqrt{3u}$

$xy = 1 \to u = 1$ e $xy = 3 \to u = 3$

• Integral:. $\iint u \cdot \left|\frac{1}{v}\right| du\,dv = \int_1^3 \int_{\sqrt{u}}^{\sqrt{3u}} u \frac{1}{v}\, du\,dv$

em v:. $\int_{\sqrt{u}}^{\sqrt{3u}} u \cdot \frac{1}{v}\, dv = u(\ln v)\Big|_{\sqrt{u}}^{\sqrt{3u}}$

$u(\ln\sqrt{3u} - \ln\sqrt{u}) => u \ln(\sqrt{3})$

em u: $\int_1^3 u \ln\sqrt{3}\, du = \frac{u^2}{2} \ln(\sqrt{3})\Big|_1^3$

$\frac{\ln(\sqrt{3})}{2}(9-1) = 4\ln(\sqrt{3})$

26) $\iint_R \operatorname{Sen}(9x^2+4y^2)\,dA \qquad 9x^2+4y^2=1$

• Mudança de variável!

Tome $u=3x$ e $y=2v \Rightarrow u^2+v^2=1$

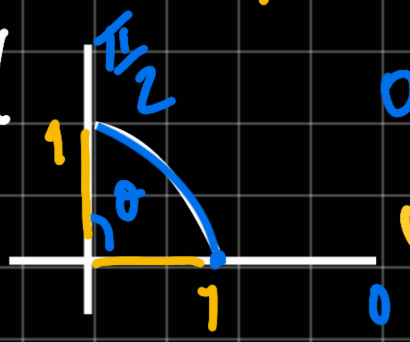
• Jacobiano

$$\begin{vmatrix} \frac{dx}{du} & \frac{dx}{dv} \\ \frac{dy}{du} & \frac{dy}{dv} \end{vmatrix} = \begin{vmatrix} 1/3 & 0 \\ 0 & 1/2 \end{vmatrix} = \frac{1}{6}$$

• Região: $u^2+v^2\leq 1$ com $u\geq 0$ e $v\geq 0$

usando coordenadas Polares: $u^2+v^2=1=r$

1° Quad



$0<\theta<\frac{\pi}{2}$

$0<r<1$ $\Rightarrow \iint_A f(c,s)\,r\,dr\,d\theta$

Assim $\int_0^{\frac{\pi}{2}}\int_0^1 \operatorname{Sen}(r^2)\cdot\frac{1}{6}\cdot r\,dr\,d\theta$

Faz $u=r^2 \Rightarrow du=2r\,dr \therefore r\,dr=\frac{du}{2}$

Em r: $\int_0^1 \operatorname{sen}(u)\frac{du}{2} = \frac{1}{2}(-\cos(u))\Big|_0^1$

$\frac{1}{2}(\cos 0-\cos 1) = \frac{1}{2}(1-\cos 1)$

Em θ $\int_0^{\frac{\pi}{2}} \frac{1}{6}\cdot\frac{1}{2}(1-\cos 1)\,d\theta = \frac{1}{12}(1-\cos 1)\,\theta\Big|_0^{\frac{\pi}{2}}$

$\frac{\pi}{24}(1-\cos 1)$

Extra: Secção 15.6

3) Área da Região $3x+2y+z=6$ no 1º Octante:

$z=6-2y-3x$

. Região: no Plano xy: $z=0 \Rightarrow 3x+2y=6$

Se $x=0 \Rightarrow y=3$ se $y=0 \Rightarrow x=2$

$0<x<2$ e $0<y<3$

Como adotar: $y=\frac{6-3x}{2} \Rightarrow y=3-\frac{3x}{2}$

assim: $0<x<2$
$0<y<3-\frac{3x}{2}$

Sua Área Será: $\iint z\,dA \Rightarrow \int_0^2\int_0^{3-\frac{3x}{2}} (6-2y-3x)\,dx\,dy$

em Y: $\int_0^{3-\frac{3x}{2}} (6-2y-3x)\,dY \Rightarrow 6y-\frac{2y^2}{2}-3xy \Big|_0^{3-\frac{3x}{2}}$

Fazendo os Contas no Julia Temos: $3\sqrt{14}$ //