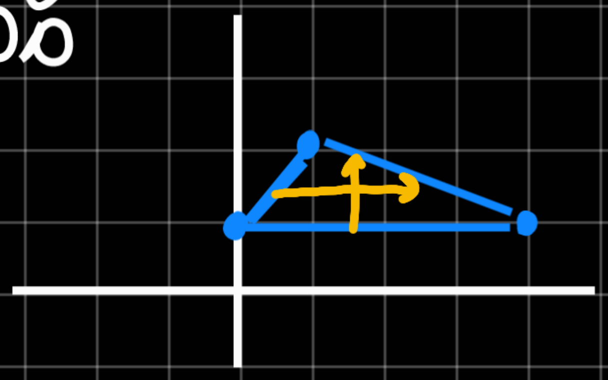
+ Lista 12: SECÇÃO 15,3

Integral Dupla $\iint f(x,y)\,dA$

19) $\iint y^2 dA$ Nos pontos $(0,1)$, $(1,2)$, $(4,1)$

Região



Retas que passam por $(0,1)$ e $(1,2)$

$y - y_0 = m(x - x_0)$: $m = \frac{\Delta y}{\Delta x}$

$m = \frac{2-1}{1-0} = 1 \rightarrow y - 1 = 1(x - 0)$

$y = x + 1 \rightarrow x = y - 1$

Reta que passa por $(1,2)$ e $(4,1)$

$m = \frac{1-2}{4-1} = \frac{-1}{3} \Rightarrow y - 2 = -\frac{1}{3}(x - 1) \rightarrow$

$y - 2 = -\frac{x}{3} + \frac{1}{3} \rightarrow y = -\frac{x}{3} + \frac{1}{3} + 2 \rightarrow y = -\frac{x}{3} + \frac{7}{3}$

$x = 7 - 3y$

Limites de integração

$1 < y < 2$

$y - 1 < x < 7 - 3y \rightarrow \int_1^2 \int_{y-1}^{7-3y} y^2 \, dy \, dx$

em $x \rightarrow \int y^2 dx \rightarrow y^2 x \Big|_{y-1}^{7-3y}$

$-4y^3 + 8y^2$

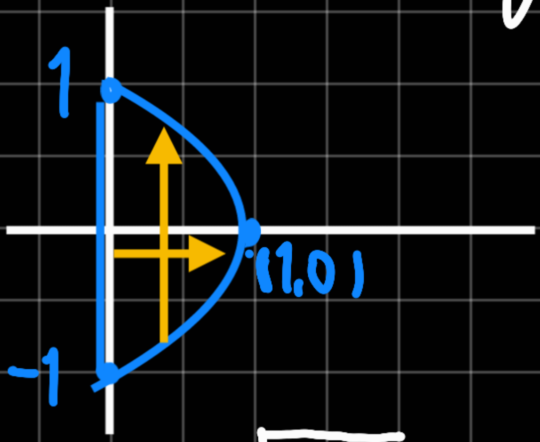
Em y $\int_0^1 -4y^3+8y^2\,dy \Rightarrow -\frac{4y^4}{4}+\frac{8y^3}{3}\Big|_0^2$

$-2^4+8\cdot\frac{2^3}{3} \Rightarrow -16+\frac{64}{3} = \frac{11}{3}$ //

20) $\iint xy^2\,dA$, $x=0$ e $x=\sqrt{1-y^2}$

$x=0 \Rightarrow 0=\sqrt{1-y^2}$

$y^2-1=0 \Rightarrow y^2=1 \Rightarrow y=-1$ ou $y=1$



$0 < x < \sqrt{1-y^2}$

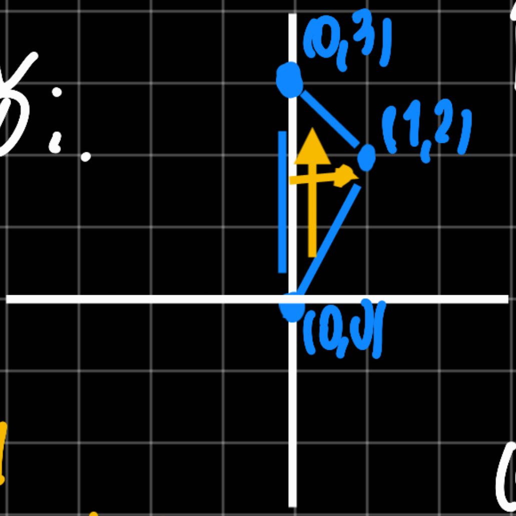
$-1 < y < 1$

$\int_{-1}^{1}\int_0^{\sqrt{1-y^2}} xy^2\,dy\,dx \Rightarrow \frac{2}{15}$ //

Resolvi usando Julia

22) $\iint 2xy\,dA$ $(0,0), (1,2), (0,3)$

Região:



Reta que passa por $(1,2)$ e $(0,3)$ $\therefore y=-x+3$

Reta que passa por $(0,0)$ e $(1,2)$: $y=2x$

$0 < Y < 1$

$2x < x < -x+3$

$\int_0^1\int_{2x}^{-x+3} 2xy\,dy\,dx = \frac{7}{4}$ //

Faça a conta eu confio no seu Potencial...

23) Volume da Região $x-2y+z=1$

$x+y=1$ e $x^2+y=1$

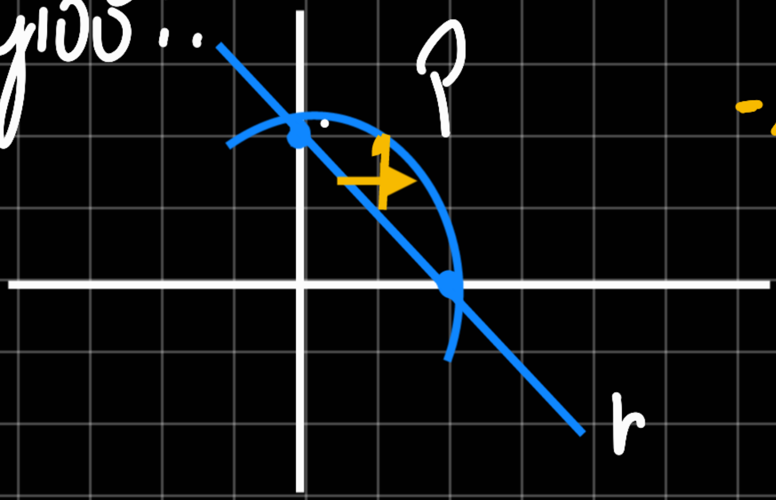
Do enunciado $z=1+2y-x$

r: $y=-x+1$ e $y=1-x^2$ P

No encontro: $-x+1=1-x^2 \Rightarrow x^2=x$

$x=0$ ou $x=1$

Região:



$0<x<1$

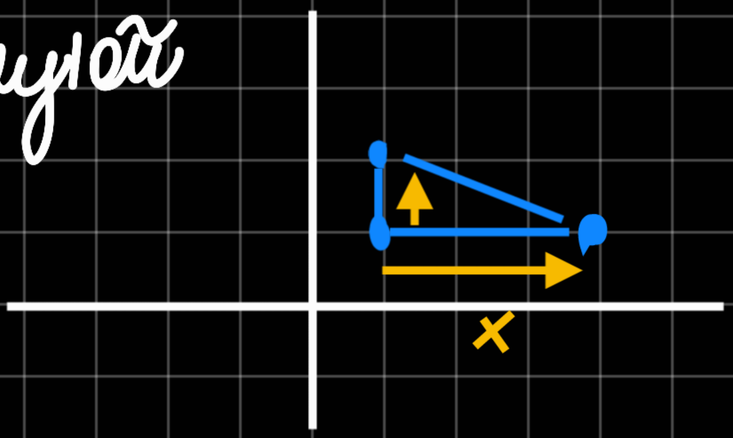
$-x+1<y<1-x^2$

$$\int_0^1\int_{1-x}^{1-x} 1+2y-x \, dxdy$$

$= \frac{17}{60}$ também fiz no Julia

25) $z=xy$ Triangulo $(1,1), (4,1)$ $(1,2)$

Região



Retas: $(1,2)-(4,1)$

$y-2=\frac{1-2}{4-1}(x-1)$

$y=\frac{-x+7}{3}$ (Já fizemos

$1<x<4$

$1<y<\frac{-x+7}{3} \rightarrow \int_1^4\int_1^{\frac{7-x}{3}} xy\,dxdy = \frac{31}{8}$

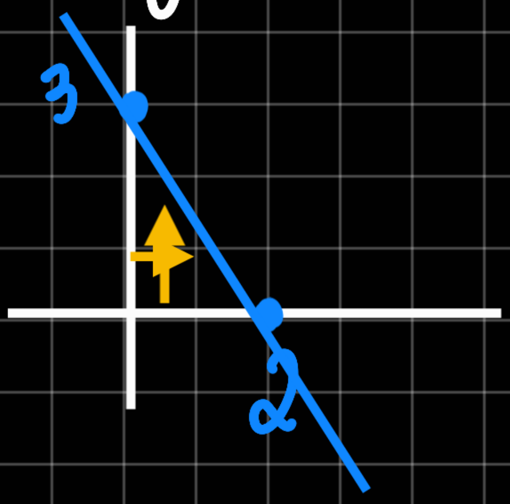
27) Planos Coordenados e $3x+2y+z=6$

$(0,0,0)$

$z = 6-3x-2y$

$0 = 6-3x-2y$

$2y = -3x+6 \rightarrow y = -\frac{3x}{2}+3$



$0 < x < 2$

$0 < y < -\frac{3x}{2}+3$

Você consegue

$$\int_0^2\int_0^{-\frac{3x}{2}+3}(6-3x-2y)\,dy\,dx = 6$$

31) $x^2+y^2=1$

$y=z,\ x=0,\ z=0$

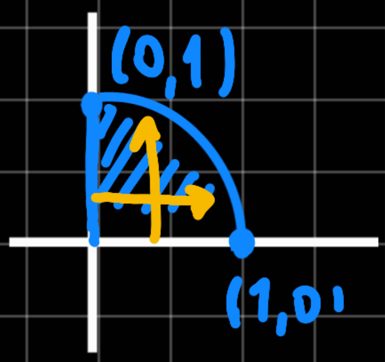
1º Octante

$x=0 \rightarrow y^2=1$

$y=-1$ ou $+1$

$z=y=0 \rightarrow x^2=1$

$x=\pm 1$



$0 < x < 1$

$0 < y < \sqrt{1-x^2}$

$0 < y < 1$

$0 < x < \sqrt{1-y^2}$

$$\int_0^1\int_0^{\sqrt{1-x^2}}(y-0)\,dx\,dy$$

Em y: $\left.\frac{y^2}{2}\right|_0^{\sqrt{1-x^2}} = \frac{1-x^2}{2}$

em x: $\int_0^1 \frac{1-x^2}{2} = \left.\frac{x}{2}-\frac{x^3}{6}\right|_0^1 = \frac{1}{2}-\frac{1}{6}$

$\frac{3-1}{6} = \frac{2}{6} = \frac{1}{3}$

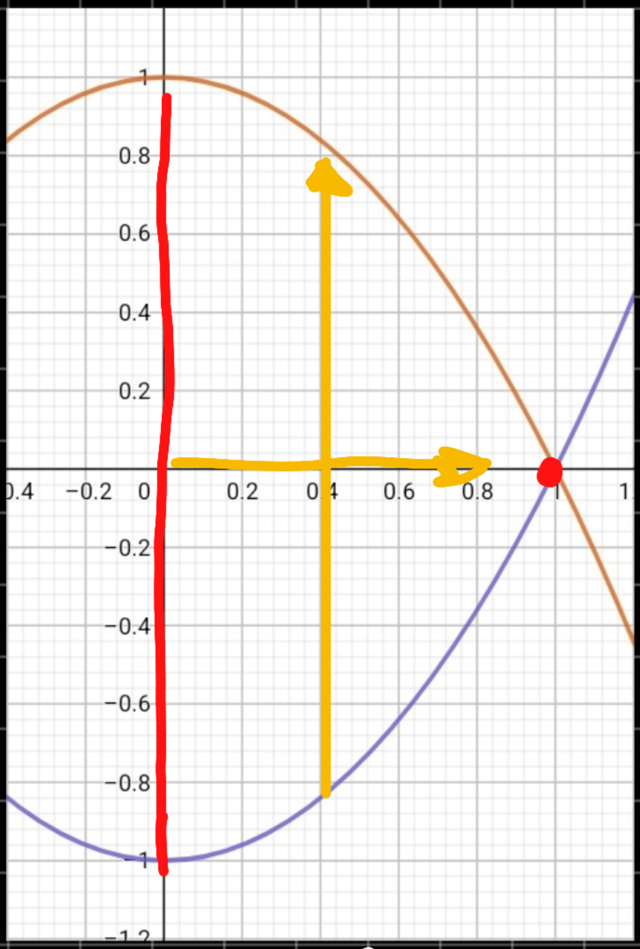
35) $y = 1 - x^2$, $y = x^2 - 1$ Plano $x + y + z = 2$

$2x + 2y - z + 10 = 0$

Interseção:

$1 - x^2 = x^2 - 1 \rightarrow 2x^2 = 2 \Rightarrow x^2 = 1 \therefore x = \pm 1$

Plano: $z = 2 - x - y$ e $z = 2x + 2y + 10$



$-1 < x < 1$

$1 - x^2 < y < x^2 - 1$

$$\int_{-1}^{1}\int_{1-x^2}^{x^2-1} (2x + 2x + 10) - (2 - x - y)\, dA$$

$$\int_{-1}^{1}\int_{1-x^2}^{x^2-1} 3x + 3y + 8\, dx\, dy$$

em y: $\displaystyle\int_{1-x^2}^{x^2-1} 3x + 3y + 8\, dy = 3xy + \frac{3y^2}{2} + 8y \Big|_{1-x^2}^{x^2-1}$

Conta... Muita Conta Chegamos $\frac{64}{3}$

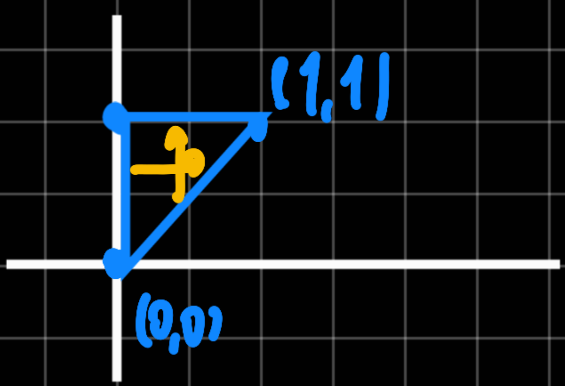
• Ordem de Integração

43) $\displaystyle\int_0^1 \int_0^y f(x,y)\, dx\, dy \xrightarrow[\text{Grafico}]{}$ $0 < x < y$, $0 < y < 1$

y x

$0 < x < y < 1$

No encontro $x = y = 0$ ou 1.

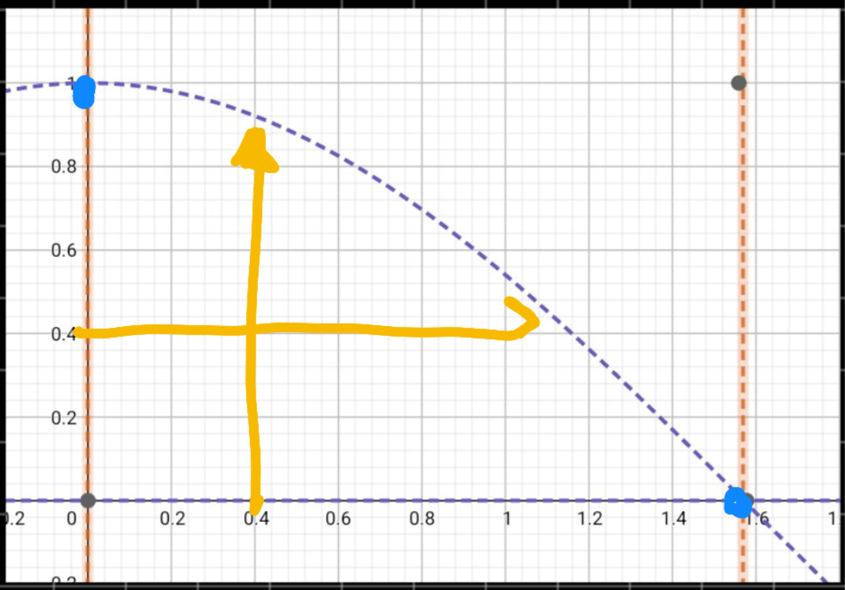


$0 < x < 1$
$x < y < 1$ → $\int_0^1 \int_x^1 f(x,y)\,dA$

Mudando:
$0 < y < 1$
$0 < x < \arccos y$

45) $\int_0^{\pi/2} \int_0^{\cos x} f(x,y)\,dA$

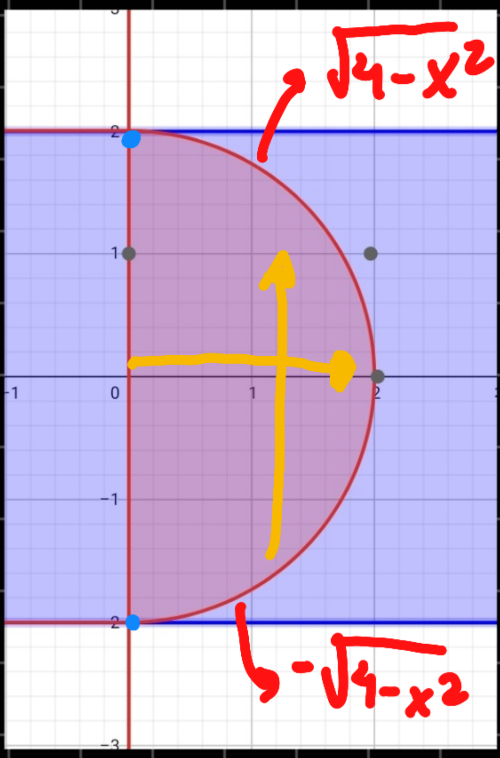
Gráfico
$0 < y < \cos x$ →
$0 < x < \frac{\pi}{2}$



$y < \cos x$
↳ $x < \arccos y$

decrescente Nesse Intervalo !!! → $\int_0^1 \int_0^{\arccos y} f(x,y)\,dA$

46) $\int_{-2}^{2} \int_0^{\sqrt{4-y^2}} f(x,y)\,dA$

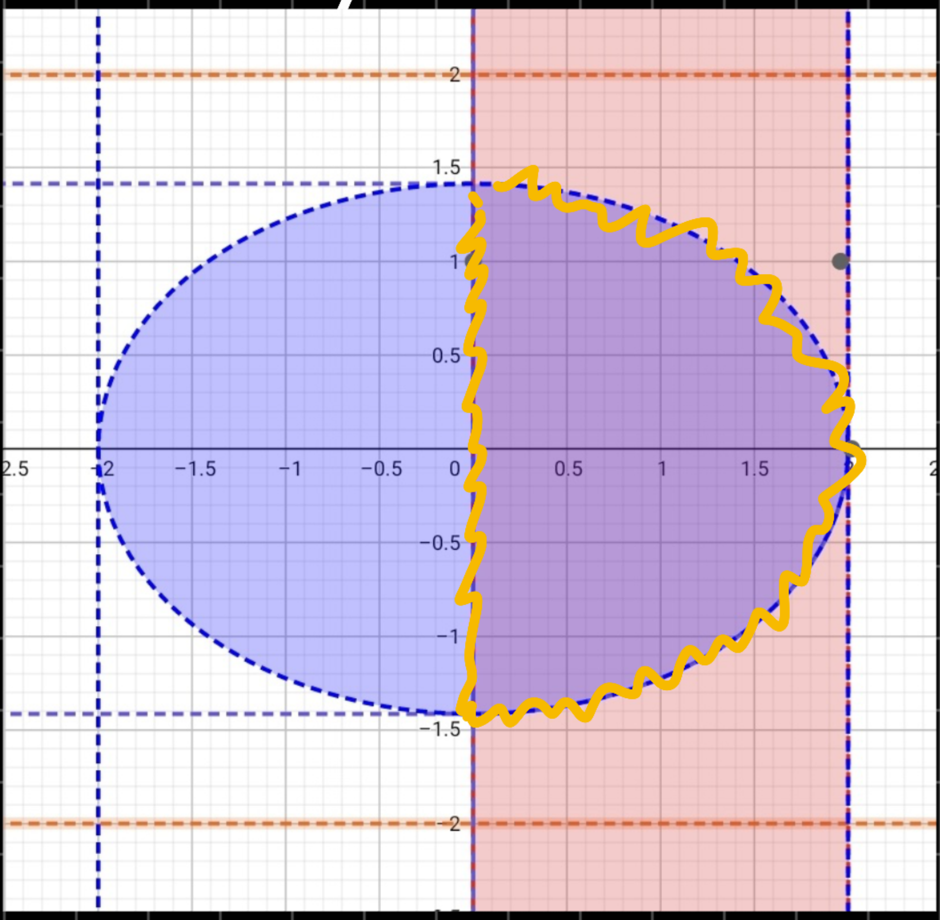


$0 < x < \sqrt{4-y^2}$
$-2 < y < 2$

No encontro
$x = \sqrt{4-y^2} \rightarrow x^2 = 4 - y^2 \rightarrow y^2 = 4 - x^2$
$y^2 = 4 - x^2 \rightarrow y = \pm\sqrt{4-x^2}$

Mudando: $0 < x < 2$

$-\sqrt{4-x^2} < y < \sqrt{4+x^2}$

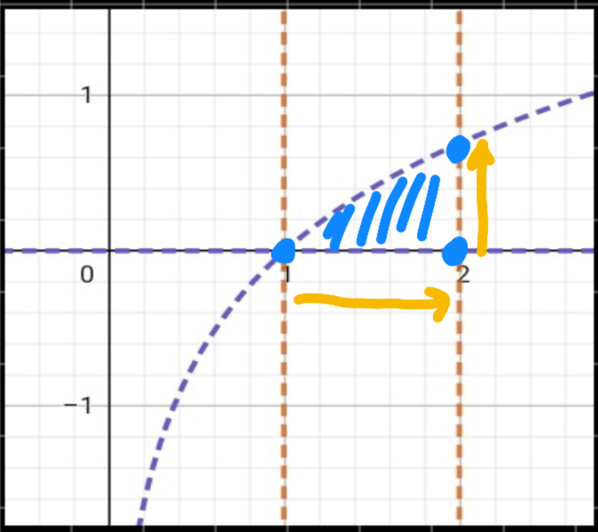


47) $\int_1^2 \int_0^{\ln x} f(x,y)\,dy\,dx$

$1 < x < 2$
$0 < y < \ln x$

$y < \ln x \rightarrow e^y < x \rightarrow$

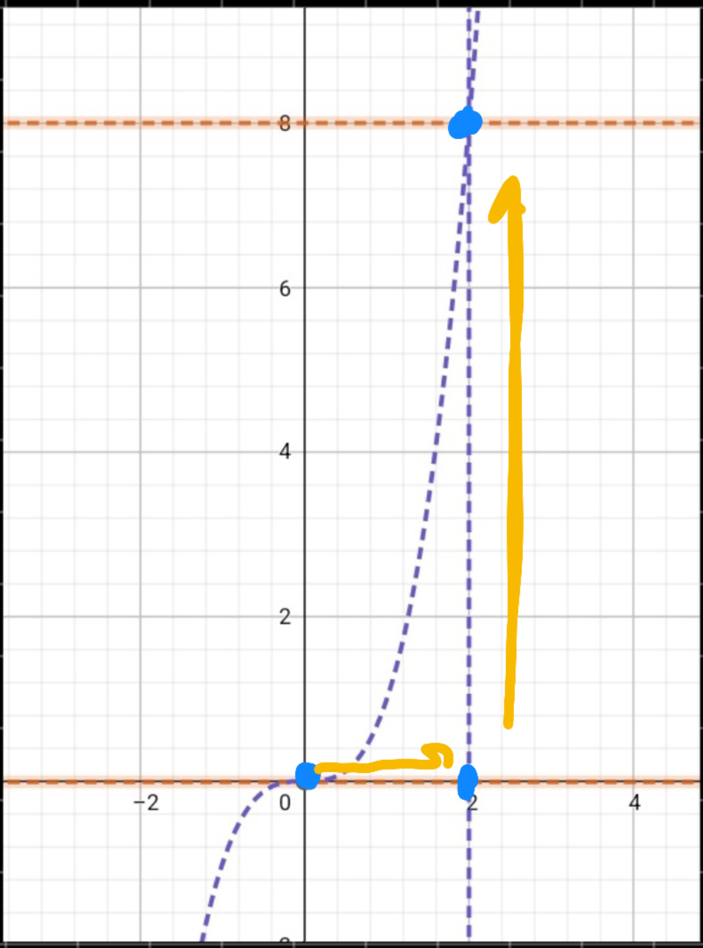
$e^y < x < 2$
$0 < y < \ln 2$



$\int_0^{\ln 2} \int_{e^y}^{2} f(x,y)\,dA$

54) $\int_0^8 \int_{\sqrt[3]{y}}^{2} e^{x^4}\,dx\,dy$

$0 < y < 8$
$\sqrt[3]{y} < x < 2$



Da região podemos ver facilmente que:

$0 < \sqrt[3]{y} < x < 2$

$\sqrt[3]{y} < x \to y < x^3$

$0 < y < x^3$
$0 < x < 2$

$\int_0^2 \int_0^{x^3} e^{x^4}\,dy\,dx \to$

em y: $\int_0^{x^3} e^{x^4}\,dy \to y e^{x^4} \Big|_0^{x^3} \cdot x^3 e^{x^4}$

em x: $\int_0^2 x^3 e^{x^4}\,dx$

$u = x^4 \to$
$du = 4x^3 dx$

$\int_0^2 e^u \frac{du}{4} \to \frac{e^u}{4} \to \frac{e^{x^4}}{4}\Big|_0^2$

$\frac{e^{2^4} - e^0}{4} \to \frac{e^{16} - 1}{4}$

1 .