

Journal Club de Partículas e Campos

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Collider Physics at the Precision Frontier

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Abstract

The precision frontier in collider physics is being pushed at impressive speed, from both the experimental and the theoretical side. The aim of this review is to give an overview of recent developments in precision calculations within the Standard Model of particle physics, in particular in the Higgs sector. While the first part focuses on phenomenological results, the second part reviews some of the techniques which allowed the rapid progress in the field of precision calculations. The focus is on analytic and semi-numerical techniques for multi-loop amplitudes, however fully numerical methods as well as subtraction schemes for infrared divergent real radiation beyond NLO are also briefly described. Title:

Collider Physics at the Precision Frontier

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[arXiv:2009.00516[hep-ph]]

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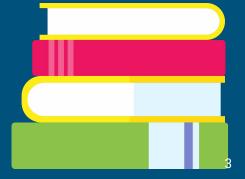
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Summary

This **review** do an overview of recent developments in precision calculations:

- Higgs phenomenological results.
- Techniques in the field of precision calculations: analytical and numerical methods.
- Subtraction schemes for IR-div.



Pre-print structure (contents of the presentation)

1. Introduction

Motivations and a brief history.

2. Current status of precision Higgs phenomenology

- N3L0 corrections.
- Higgs production in gluon fusion.
- Tools for differential NNLO predictions.

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Pre-print structure (contents of the presentation)

3. Overview of modern techniques for loop amplitudes.

- Multi-loop amplitudes: glossary, amplitude representation and reduction.
- Analytic methods vs Numerical methods.
- 4. Schemes to isolate IR-div.

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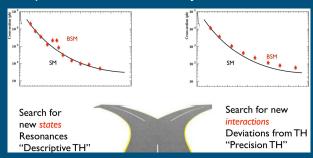
Disclaimer





Introduction

- Exploring the Higgs sector is one of the main pillars in the planning process of the future collider physics program.
- The key is precision!
- A lot of progress has been achieved towards the goal of describing hadron collider processes consistently at NLO.



Les Houches 2019: Physics at TeV Colliders Standard Model Working Group Report

Conveners

Higgs physics: SM issues

D. de Florian (Theory), M. Donegà (CMS), M. Dührssen-Debling (ATLAS), S. Jones (Theory)

SM: Loops and Multilegs

J. Bendavid (CMS), A. Huss (Theory), J. Huston (ATLAS), S. Kallweit (Theory), D. Maître (Theory), S. Marzani (Jets contact), B. Nachman (Jets contact, ATLAS)

Tools and Monte Carlos
V. Ciulli (CMS), S. Prestel (Theory), E. Re (Theory)

Abstract

2020

Mar

Xiv:2003.01700v1

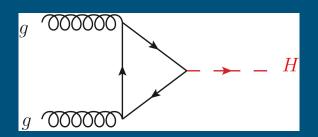
This Report summarizes the proceedings of the 2019 Les Houches workshop on Physics at TeV Colliders. Session 1 dealt with (1) new developments for high precision Standard Model calculations, (II) the sensitivity of parton distribution functions to the experimental inputs, (III) new developments in jet substructure techniques and a detailed examination of gluon fragmentation at the LHC, (IV) issues in the theoretical description of the production of Standard Model Higgs bosons and how to relate experimental measurements, and (V) Monte Carlo event generator studies relating to PDF evolution and comparisons of important processes at the LHC.

Acknowledgements

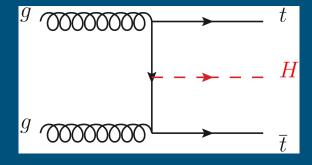
We would like to thank the organizers (N. Berger, F. Boudjema, C. Delaunay, M. Delmastro, B. Fuks, S. Gascon, M. H. Genest, P. Gras, J. P. Guillet, B. Herrmann, S. Kraml, N. Makovec, G. Moreau, E. Re) and the Les Houches staff for the stimulating environment always present at Les Houches. We thank the Formation Permanente du CNRS, the IDEX Université Grenoble Alpes, the Université Savoie Mont Blanc, LAPP and LAPTh for support.

Precision Higgs phenomenology

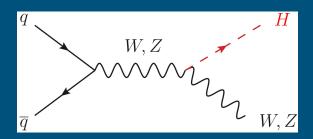
Higgs boson production channels



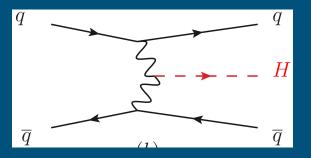
Gluon-fusion



ttH production

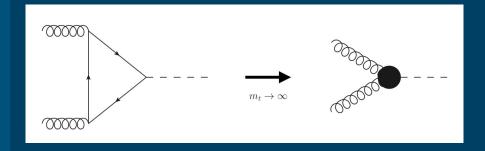


Higgsstrahlung



Vector-boson fusion

N3L0 corrections



approximations: $m_t \rightarrow \infty$ limit: "Heavy top limit" (HTL)

N3L0 corrections

2.1. No Corrections and beyond

The predictions for hadron collider cross sections beyond NNLO available to date are the N³LO corrections to Higgs production in gluon fusion in the HTL, for the inclusive case in the threshold approximation [16, 17] and exact [18], as well as differential in rapidity [19, 20]. Very recently, the N⁴LO soft and virtual corrections to inclusive Higgs production have been calculated [21].

Z, Higgs production in bottom quark fusion is also known to N³LO [22, [23], including matching the 4- and 5-flavour schemes to third order in the strong coupling [24].

Inclusive Higgs boson production in vector boson fusion (VBF) at N³LO has been calculated in Ref. [25], Higgs boson pair production in VBF in Ref. [26], both based on the projection-to-Born method [27, [28]. N³LO corrections to Higgs boson pair production in gluon fusion in the HTL have been calculated in Refs. [29H3I].

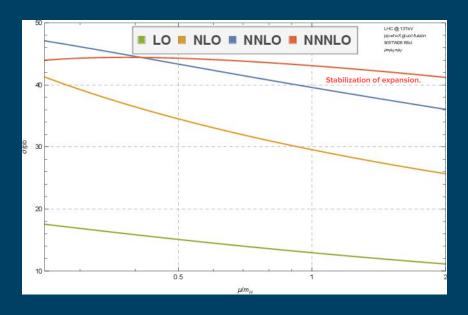
in Refs. [29-31].

The Drell-Yan cross section to third order in the strong coupling has been presented in Ref. [32], charged current Drell-Yan production at N³LO in Ref. [33].

N³LO corrections to jet production in deep inelastic scattering [34] as well as charged current DIS [35] also have been calculated using the projection-to-Born method.

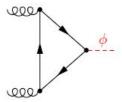
It should also be mentioned that, apart from hadron collider cross sections, more inclusive quantities calculated at N³LO or beyond are known since some time, such as deep inelastic structure functions [36] or the Higgs decay to hadrons, which is available at N⁴LO [37]-[42]. For more details about multi-loop results we refer to Section [3.2.3]

N3L0 corrections



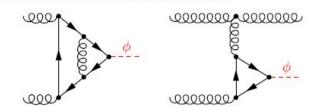
Anastasiou, Duhr, Dulat, Herzog, Mistlberger (2018) JHEP 1905 (2019) 080

Gluon fusion: Leading order (LO) calculation: involves quark loops in the SM

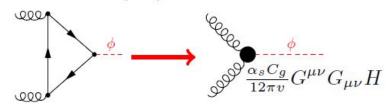


Next-to-leading order (NLO) QCD: add also real contributions

From: Stefan Liebler (DESY) 09/2016



Next-to-NLO (NNLO) QCD: known in "heavy top-limit" $m_H \ll 2m_t$

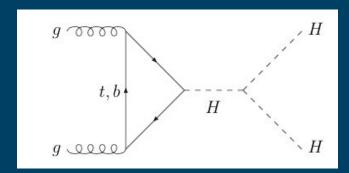


Next-to-NNLO (N3LO) QCD: soft expansion

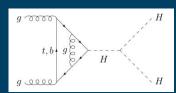
EXAMPLE: Higgs Boson Pair production via Gluon fusion

$$\sigma_{\rm NLO}(pp \to HH + X) = \sigma_{\rm LO} + \Delta \sigma_{\rm virt} + \Delta \sigma_{gg} + \Delta \sigma_{gq} + \Delta \sigma_{q\bar{q}},$$

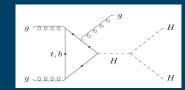
 σ_{LO} :



 $\Delta \sigma_{\text{virt}}$:



 $\Delta \sigma_{ij}$:



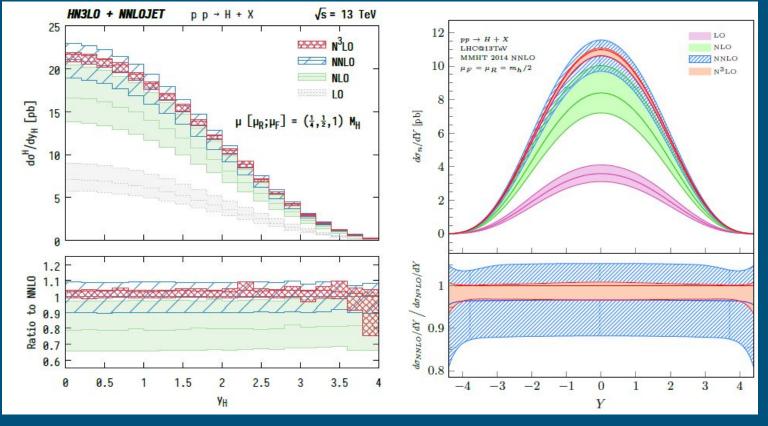
Triangular diagrams

Current Status

- Virtual & real (N)NLO QCD corrections in large top mass limit (HTL): ~100% (Dawson,Diffinaler,Spira, de Florian, Mazzitell
- The NNLO_{HTL} corrections have been implemented in different programs.
- New expansion/extrapolation
 methods (Sinher Maier Rauli)

Current Status

 Exact results for the 3-loop form factor with a single massive quark and otherwise light quarks have been presented. (Czakon, Miggetledt)



[Cieri, Chen, Gehrmann, Glover, Huss, arXiv: 1807.11501]

[Dulat,BM,Pelloni,arXiv:1810.09462]

Higgs boson rapidity distribution at N3LO

| $\delta(\text{scale})$ | $\delta(\text{PDF-TH})$ | $\delta(\mathrm{EW})$ | $\delta(t,b,c)$ | $\delta(1/m_t)$ | $\delta(\text{PDF})$ | $\delta(\alpha_s)$ |
|------------------------|-------------------------|------------------------|------------------------|------------------------|----------------------|------------------------|
| +0.10 pb -1.15 pb | $\pm 0.56~\mathrm{pb}$ | $\pm 0.49~\mathrm{pb}$ | $\pm 0.40~\mathrm{pb}$ | $\pm 0.49~\mathrm{pb}$ | \pm 0.89 pb | +1.25 pb -1.26 pb |
| $^{+0.21\%}_{-2.37\%}$ | $\pm 1.16\%$ | ±1% | $\pm 0.83\%$ | ±1% | $\pm 1.85\%$ | $^{+2.59\%}_{-2.62\%}$ |

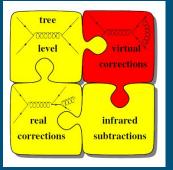
Status of the theory uncertainties on Higgs boson production in gluon fusion at $\sqrt{s} = 13$ TeV.

| process | name | Refs. | remarks |
|-------------------------------------|------------------|-----------------|---|
| pp 	o H | FEHIPRO | [373, 47] | t - and b -quark masses up to $\mathcal{O}(\alpha_s^3)$ |
| pp 	o H | HNNLO | [80, 81, 374] | t - and b -quark masses up to $\mathcal{O}(\alpha_s^3)$ |
| pp 	o H | Нот | [381, 382] | p_T -dist., NNLL resummation |
| $pp \to \gamma \gamma$ | 2γ NNLO | [375, 383] | also available in MCFM [384] |
| t 	o W(l u) b | NNTOPDEC | [385] | https://nntopdec.hepforge.org/ |
| Drell-Yan | FEWZ | [377, 378, 386] | |
| Drell-Yan | DYNNLO | [379] | |
| Drell-Yan | DYTURBO | [380, 387, 379] | merge of DYNNLO, DYRES, DYQT |
| | | | https://dyturbo.hepforge.org/ |
| $pp \rightarrow VH$ | MCFM | [137] | |
| VBF H prod. | PROVBFH | [28, 25, 175] | |
| VBF HH prod. | ProVbfHH | [26, 345, 175] | |
| colour singlet $2 \to 1, 2$ | MATRIX+OPENLOOPS | [388-391] | https://matrix.hepforge.org |
| | | | incl. NLO EW corrections for $VV^{(\prime)}$ |
| colour singlet $2 \to 1, 2$ | MATRIX+RADISH | [392] | incl. q_T resummations |
| colour singlet $2 \rightarrow 1, 2$ | MCFM | [202, 171] | https://mcfm.fnal.gov/ |

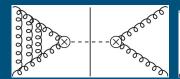
Tools for differential NNLO predictions

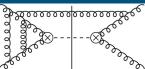
Now, let's go to main part: techniques for loop amplitudes

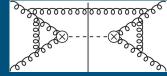
Motivations

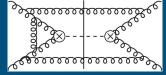


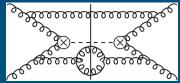
- Scattering amplitudes are the core of any perturbative calculation of a physical quantity relevant to particle interactions and collider experiments.
- NLO automation are in a rather mature state.
- Lots of Feynman diagrams at NNLO... and even more at N3LO.
- Example: diagramatic contributions at N3LO for the gluon-fusion.











 Combination of loop corrections and real emissions computed using Feynman diagrams is the only way for analytic computations at N3LO at this point.

Motivations (other difficulties)

Old Ultraviolet (UV-div) and infrared (IR-div) divergences are all-over beyond leading order in S-matrix calculations and must be wisely removed in order to automated computation codes for the evaluation of Feynman amplitudes.

Vocabulary

Integral family:

$$F(\nu_1 \dots \nu_n) = \int \prod_{l=1}^{L} \frac{\mathrm{d}^D k_l}{i\pi^{\frac{D}{2}}} \prod_{j=1}^{n} \frac{1}{P_j^{\nu_j}(\{k\}, \{p\}, m_j^2)}$$

very different at two loops (and beyond)!

Master integrals:

$$=\sum_i C_4^i + \sum_i C_3^i + \sum_i C_2^i - \cdots + \mathcal{R}$$
 rational part

Amplitude calculations

Workflow

- 1. Amplitude generation, for example in terms of Feynman diagrams.
- 2. Bringing the amplitude to a convenient form.
- 3. Reduction to master integrals.
- 4. Calculation of the master integrals.
- 5. Evaluation of the amplitude.

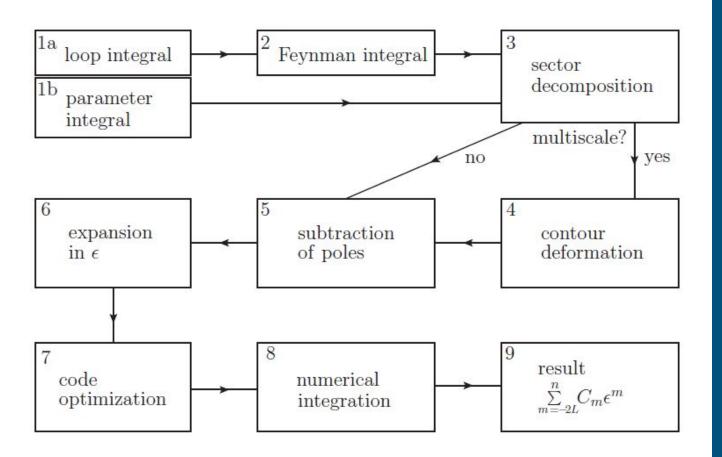


Figure 8: Workflow of the program pySecDec.

STAGE 1: FeynArts&FormCalc

Contractions

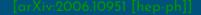


STAGE 2: Regularization



Integral reduction

STAGE 3: Integral Evaluation





Amplitude calculations: workflow

Amplitude representation

$$\mathcal{A}^{\mu_1...\mu_n} = \sum \text{Feynman diagrams} \text{ (algebraic expressions)}$$

decompose into linearly independent Lorentz structures $T_j^{\mu_1\dots\mu_n}$ multiplying form factors F_j : $\mathcal{A}^{\mu_1\dots\mu_n}=\sum F_j\,T_j^{\mu_1\dots\mu_n}$

Amplitude reductions

$$F_j = \sum c_k^{(j)} I_k^{(j)}$$

And then, what comes next?

The calculation of the Master Integrals! (easy to say, not so easy to do)

Calculation of the Master integrals

• differential equations very successful for analytic evaluations

Kotikov '91; Remiddi '97, Gehrmann, Remiddi '00, Henn '13 ...

integration of Mellin-Barnes representation
 analytic as well as numerical prominent examples

V.A. Smirnov '99; Tausk '99; Czakon '05; Dubovyk, Freitas, Gluza, Riemann, Usovitsch '16, '19

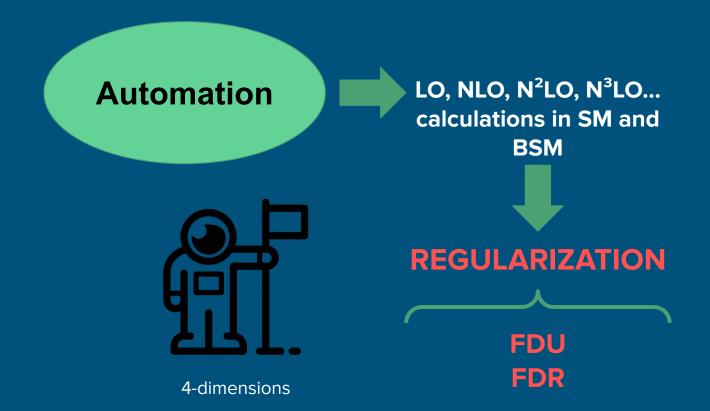
direct evaluation in momentum space
 only possible numerically, needs cancellation of singularities in integrand
 Soper '99; Gong, Soper, Nagy '09; Weinzierl, Reuschle et al. '10-'20;
 Rodrigo, Sborlini, Driencourt-Mangin, Torres-Bobadilla et al. '08-'20; Capatti, Hirschi, Kernmaschah, Ruijl '19

 direct evaluation in Feynman parameter space for more complicated integrals analytic solution impractical

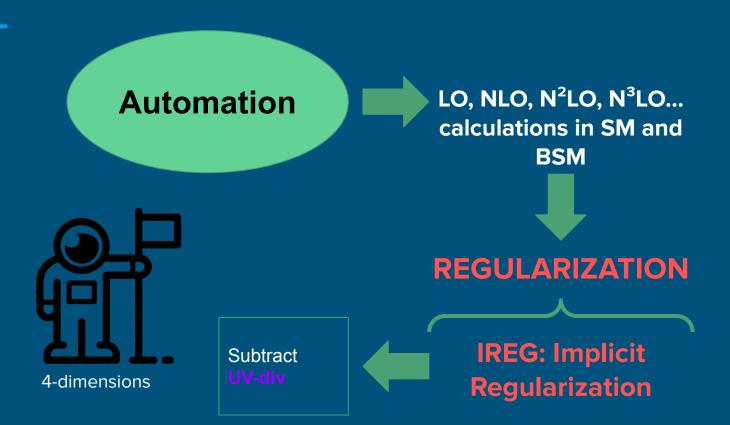
Passarino, Uccirati et al '01-'18; De Doncker, Fujimoto, Kurihara, et al. '04-'19, Volkov '19

sector decomposition: Hepp '66; Denner, Roth '96; Binoth, GH '00; Bogner, Weinzierl '07; Smirnov et al. '08-'16; Anastasiou et al '08; Carter, GH '10; Borowka, GH, Jahn, Jones, Kerner, Schlenk, Zirke '12-'20

Numerical methods: 4 dimensional methods



Numerical methods: 4 dimensional methods



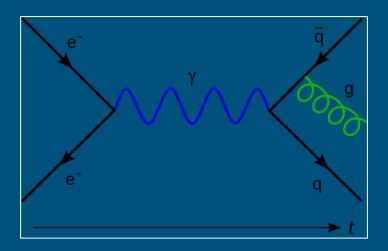


| | analytic | numerical |
|-------------------------------------|-----------------------|----------------------------|
| pole cancellation | exact | with numerical uncertainty |
| control of integrable singularities | analytic continuation | less straightforward |
| fast evaluation | yes | depends |
| extension to more scales/loops | difficult | promising |
| automation | difficult | less difficult |

Strong and weak points of analytic versus numerical evaluations of loop integrals

Last Part: IR-div

IR-div: example



- The gluon emitted in the process has a very low energy, so low that it will not be detected in the detector.
- Even upgrading the detector, it still has an energy limit that it cannot detect.
- At the level of theoretical calculation, this physical limit is not being considered. If so, you would be able to detect any gluon at any energy, no matter how low.
- These divergences are removed in the observables automatically. We don't need to renormalize them. 37

IR-div: types

two types: (a) soft, (b) collinear

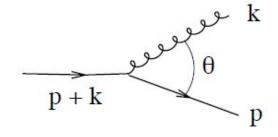
Consider the emission of a gluon from a hard quark:

$$p = E(1, 0, 0, 1)$$

$$k = \omega(1, 0, \sin \theta, \cos \theta)$$

$$(p+k)^2 = 2E\,\omega\,(1-\cos\theta)$$

will go to zero if the gluon becomes soft $(\omega \to 0)$ or if quark and gluon become collinear $(\theta \to 0)$



Real radiation: Schemes to isolate IR-div

- antenna subtraction analytically integrated subtraction terms [Gehrmann-DeRidder, Gehrmann, Glover '05]
- qt "subtraction" slicing, (colourless final states)
 [Catani, Grazzini '07]
- N-jettiness slicing
 [Gaunt, Stahlhofen, Tackmann, Walsh '15]

 [Boughezal, Focke, Liu, Petriello '15]
- sector-improved residue subtraction numerically integrated [Czakon, Heymes, Mitov '10; Czakon, Heymes '14] [Boughezal et al. '11] subtraction terms
- nested subtraction [Caola, Melnikov, Röntsch '17, '19-]
- projection to Born/ structure function approach only special kinematics
 [Goa, Li, Zhu '12] [Brucherseifer, Caola, Melnikov '14] [Cacciari, Dreyer, Karlberg, Salam, Zanderighi '15]
- colorful only final state colour so far [Del Duca, Somogyi, Trocsanyi et al '05-]
- local analytic sector subtraction [Magnea, Maina, Pellicioli, Signorile, Torielli, Uccurati '19-]

Real radiation: Schemes to isolate IR-div

| method | analytic integration |
|--|----------------------|
| | of subtraction terms |
| subtraction | |
| antenna subtraction [948, 975–981, 949] | yes |
| sector-improved residue subtraction [952–955] | no |
| nested soft-collinear subtraction [203, 964–969] | yes |
| ColorFulNNLO [982–991] | partial |
| projection to Born [27, 972, 28, 25] | yes |
| local analytic subtraction [970, 971] | yes |
| slicing | |
| q_T [80, 379] | yes |
| N-jettiness [923–925] | yes |
| geometric subtraction [973] | yes |

Summary

- 2->3 processes at NNLO certainly form one of the current frontiers, posing challenges from both the loop amplitude as well as the real radiation side.
- Numerical methods scale better with increasing numbers of mass scales, however they have other challenges, and tackling those has also seen much progress in the last few years.
- All this is part of a bigger picture, which is the preparation for the future of high energy physics.

Thanks!

