Physics @ UFABC

PRACTICAL APPROACHES TO GAUGE THEORIES: RENORMALIZATION GROUP FUNCTIONS AT TWO-LOOP WITHIN 4D IMPLICIT REGULARIZATION

IREG

Who’s presenting?

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IREG GROUP PROFILES

OUTLINE

01 INTRODUCTION
Motivations,
review and main
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02 THEORY
Renormalization
and regularization.

03 METHODOLOGY
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to 2-loop calculations.

04 RESULTS
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and γ-function within IREG.

05 CONCLUSIONS
Final discussions,
perspectives and
conclusions.

MAIN REFERENCES

List of relevant publications and articles in this research.

Play

More references

Main References

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Two-loop renormalisation of gauge theories in 4D implicit regularisation and connections to dimensional methods

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Abstract We compute the two-loop β-function of scalar and spinorial quantum electrodynamics as well as pure Yang–Mills and quantum chromodynamics using the background field method in a fully quadridimensional setup using implicit regularization (IREG). Moreover, a thorough comparison with dimensional approaches such as conventional dimensional regularization (CDR) and dimensional reduction (DRED) is presented. Subtleties related to Lorentz algebra contractions/symmetric integrations inside divergent integrals as well as renormalisation schemes are carefully discussed within IREG where the renormalisation constants are fully defined as basic divergent integrals to arbitrary loop order. Moreover, we confirm the hypothesis that momentum routing invariance in the loops of Feynman diagrams implemented via setting well-defined surface terms to zero deliver non-abelian gauge invariant amplitudes within IREG just as it has been proven for abelian theories.

1 Motivations

Unravelling physics beyond the standard model (SM) has entreated theoretical predictions for particle physics precision observables beyond next-to-leading-order (NLO). Such predictions rely on involved Feynman diagram calculations to evaluate scattering amplitudes both in the SM and its extensions. Theoretical models beyond the SM (BSM) can be constructed, for instance, as an extension in the Higgs sector by either changing the number of scalar multiplets or considering the Higgs boson as a composite particle – the so called Composite Higgs Models [1,2]. Supersymmetric and dark matter extensions have also been considered in order to explain SM deviations from experimental results [3] in electroweak precision observables (EWPO) which are known with an accuracy at the per cent level or better [4–6]. On the other hand, precise measurements and calculations of known particles and interactions are just as important to validate, redress, or refute new models. Also, in order to evade from unphysical scale dependence at low order, higher order terms are needed to smooth out such dependence in the resulting, more accurate, predictions. For example, a full N^3LO calculation for QCD corrections to gluon-fusion Higgs boson production was performed in [7] at center-of-mass energy 13 TeV. The considerably low residual theoretical uncertainty ($\approx$ 5–6%) and small sensitivity to scale variation ($\approx$ 2%) superseded earlier results below N^3LO. Because experimental uncertainties are expected to drop below the accuracy of theoretical data, as expected from future experimental measurements at the future circular collider (FCC-e^-e^+) [8], QCD theoretical uncertainties ought to be reduced at many levels so physics BSM can be ultimately ascertained.

Ultraviolet (UV) and infrared (IR) divergences are ubiquitous beyond leading order in S-matrix calculations and must be judiciously removed in order to automated computation codes for the evaluation of Feynman amplitudes. As a by-product of such subtractions, there remain residual dependencies on renormalisation (λ_R) and factorisation (λ_F) scales in the perturbative series that describes a physical observable. The dependence on such scales is expected to diminish after higher terms are taken into account and, at a given order, may in principle be minimised to yield a result the least sensitive to variations in the unphysical parameters [9]. However, the problem of scale setting has been studied extensively and there is no consensus on a procedure valid in general. For a recent account see [10].

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Dimensional regularization vs methods in fixed dimension with and without γ_5

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ABSTRACT: We study the Lorentz and Dirac algebra, including the antisymmetric ϵ tensor and the γ_5 matrix, in implicit gauge-invariant regularization/renormalization methods defined in fixed integer dimensions. They include constrained differential, implicit and four-dimensional renormalization. We find that these fixed-dimension methods face the same difficulties as the different versions of dimensional regularization. We propose a consistent procedure in these methods, similar to the consistent version of regularization by dimensional reduction.

KEYWORDS: Renormalization Regularization and Renormalons, Anomalies in Field and String Theories, Field Theories in Lower Dimensions

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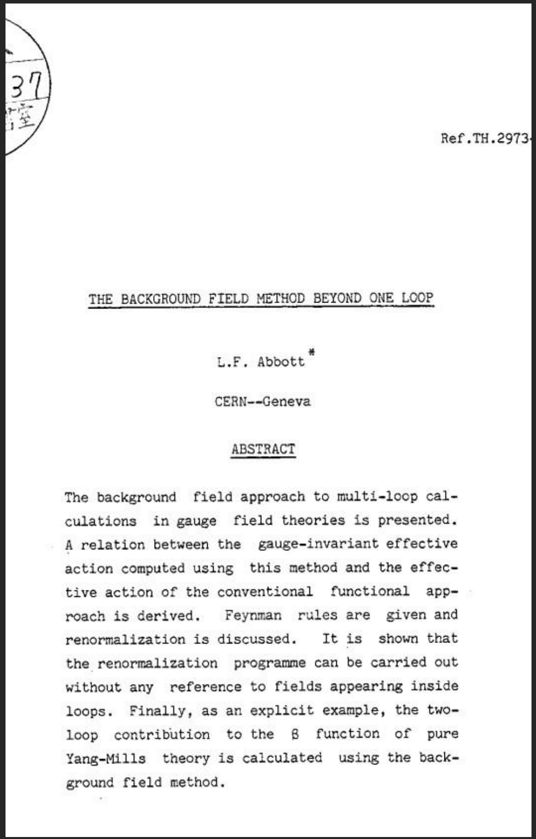

Ref.TH.2973

THE BACKGROUND FIELD METHOD BEYOND ONE LOOP

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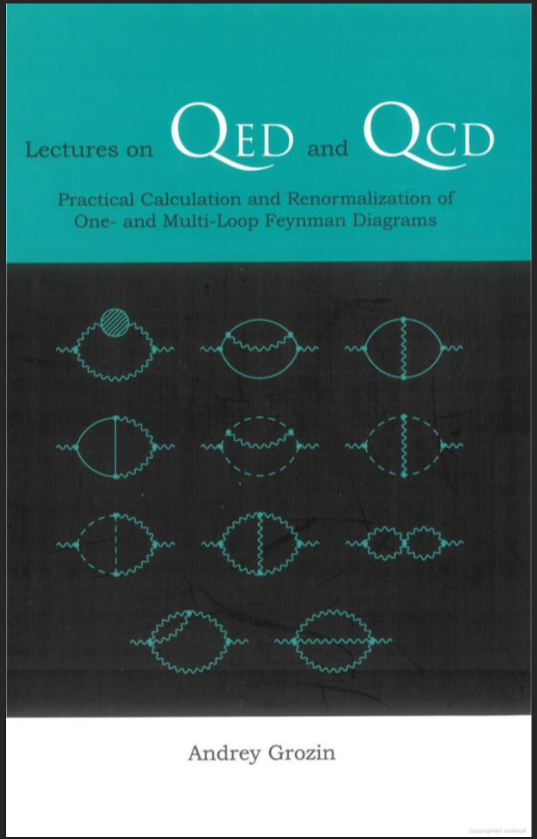
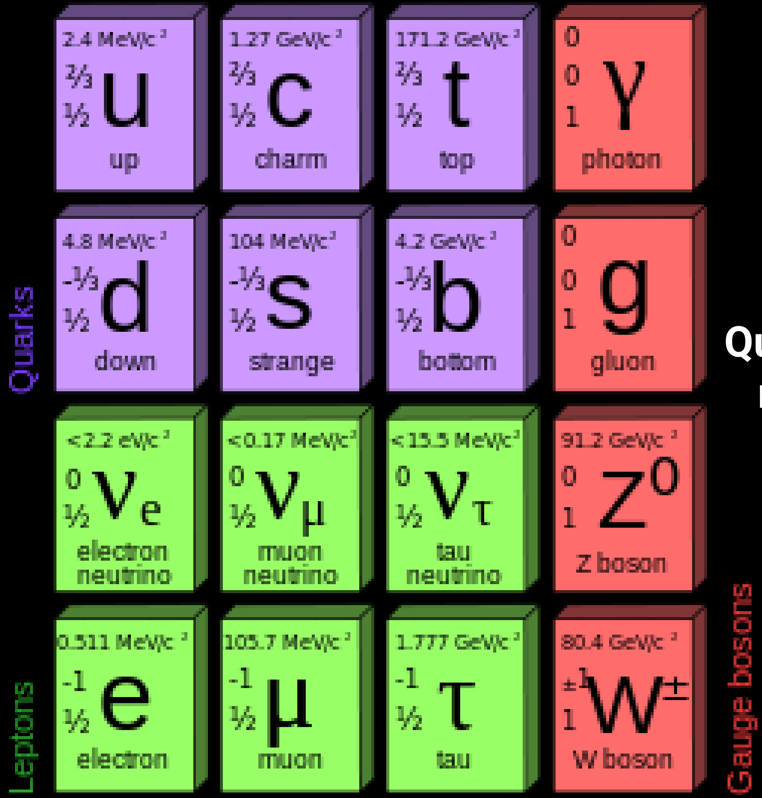
ABSTRACT

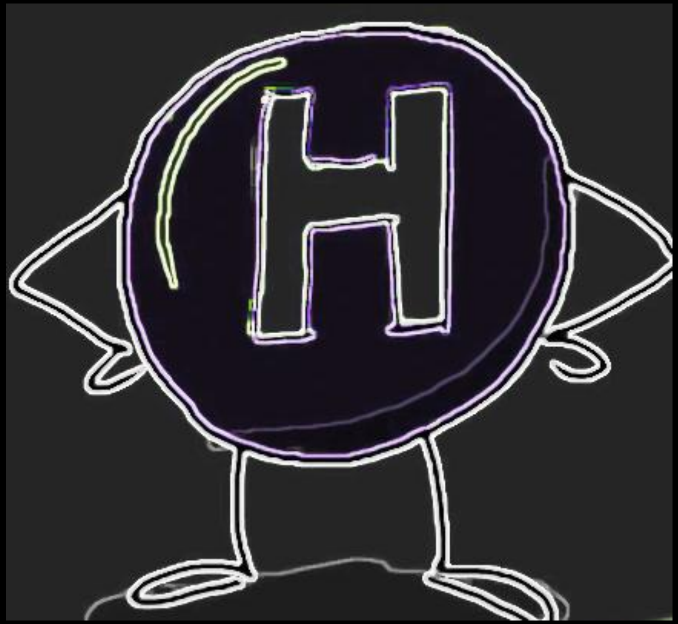
The background field approach to multi-loop calculations in gauge field theories is presented. A relation between the gauge-invariant effective action computed using this method and the effective action of the conventional functional approach is derived. Feynman rules are given and renormalization is discussed. It is shown that the renormalization programme can be carried out without any reference to fields appearing inside loops. Finally, as an explicit example, the two-loop contribution to the β function of pure Yang-Mills theory is calculated using the background field method.

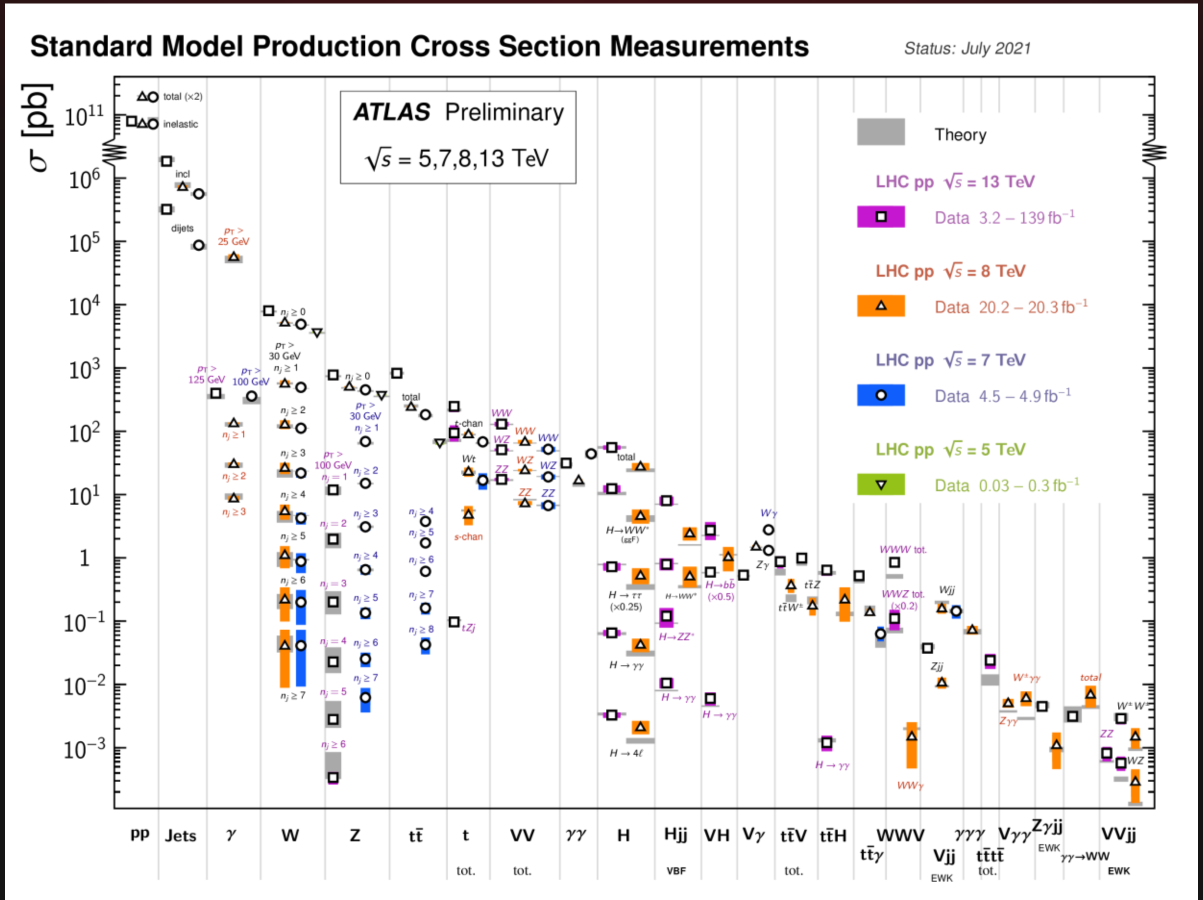




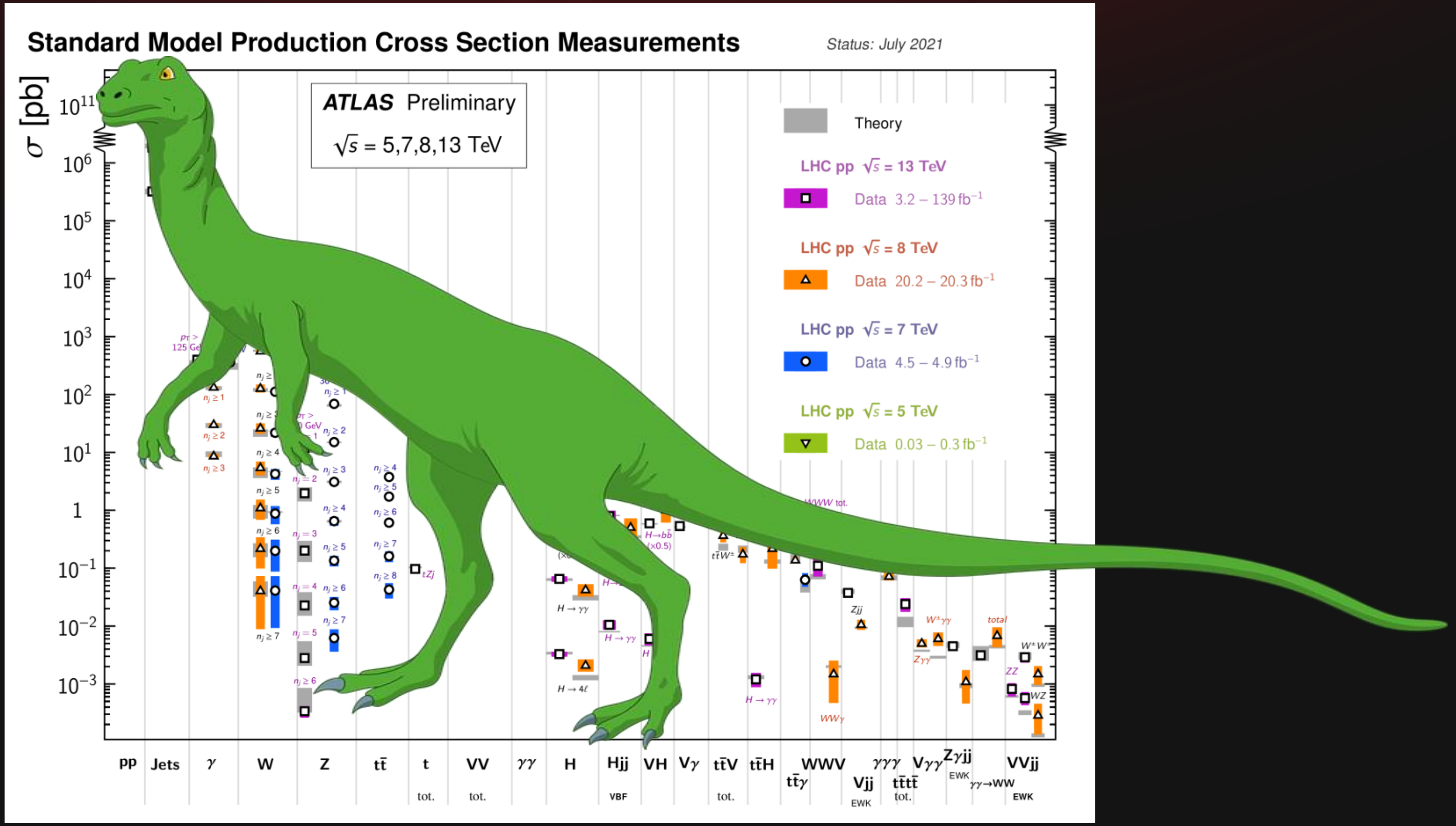
01 INTRODUCTION

Quantum Field Theory (QFT) is, until now, one of the most successful paradigms from the experimental and theoretical perspective, being the **Standard Model of particles (SM)** one of the most representative examples of it.





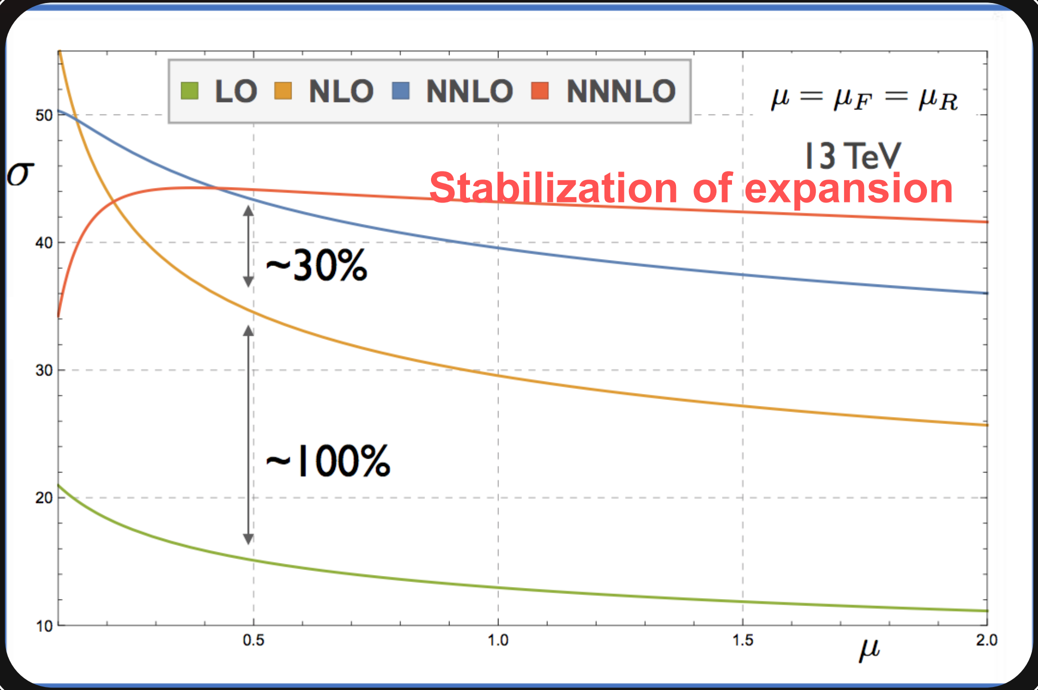
LHC incredibly successful at 7 , 8 & 13 TeV.



LHC incredibly successful at 7 , 8 & 13 TeV.

HIGGS PRODUCTION AT N³LO

[Anastasiou, Duhr, Dulat, Herzog, Mistlberger (2018) JHEP 1905 (2019) 080]



PARTONS AND THE CROSS-SECTION

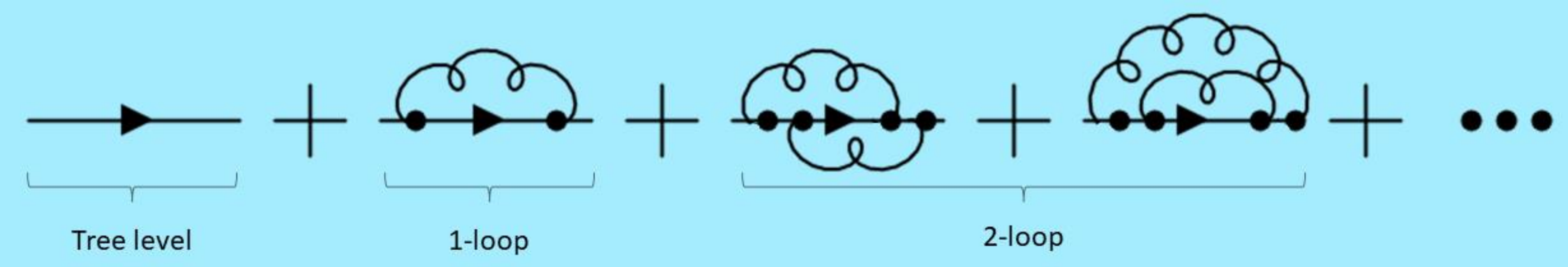
$$\sigma(p_1 p_2 \to X) = \underbrace{\sum_{ij} \int dx_1 \int dx_2 f_i^{p_1}(x_1) f_j^{p_2}(x_2)}_{\text{Non-perturbative parton distributions}} \times \underbrace{\hat{\sigma}(ij \to X)}_{\text{Perturbative parton cross-section}}$$

$$\hat{\sigma} \propto |A|^2$$

$$A = \alpha_s^n \left(A^{\text{tree}} + \frac{\alpha_s}{2\pi} A^{1-\text{loop}} + \left(\frac{\alpha_s}{2\pi}\right)^2 A^{2-\text{loop}} + \dots \right)$$

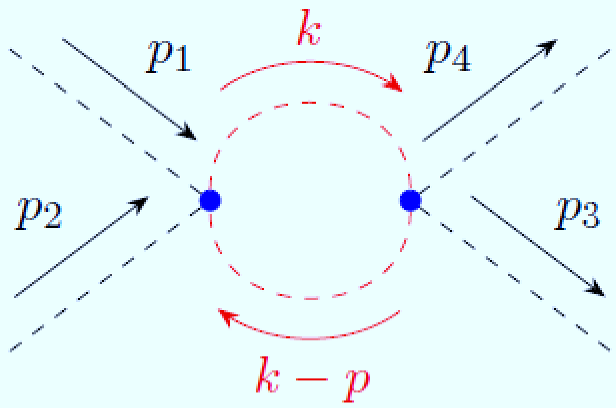
PARTONS AND THE CROSS-SECTION

$$A = \alpha_s^n \left(A^{\text{tree}} + \frac{\alpha_s}{2\pi} A^{1-\text{loop}} + \left(\frac{\alpha_s}{2\pi}\right)^2 A^{2-\text{loop}} + \cdots \right)$$



∞ in QFT

$$\mathcal{L}_{\phi^4} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}m^2\phi^2 - \underbrace{\frac{g}{4!}\phi^4}_{\text{interaction term}},$$



$$= (-ig)^2 \underbrace{\frac{1}{2}}_{\text{symmetry factor}} \int_k \frac{i^2}{(k^2-m^2)[(k-p)^2-m^2]}$$

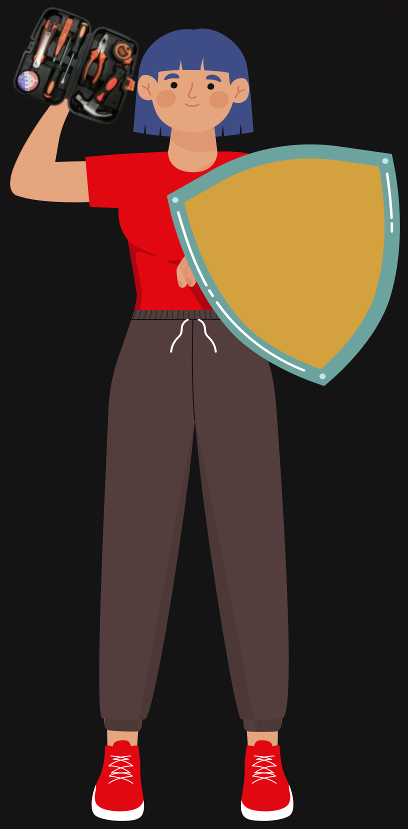
Φ4-Theory

 in QFT

UV-div

They appear when the momentum of the loop goes to infinity.

THE PERTURBATIVE KEY TOOLKIT FOR PRECISION



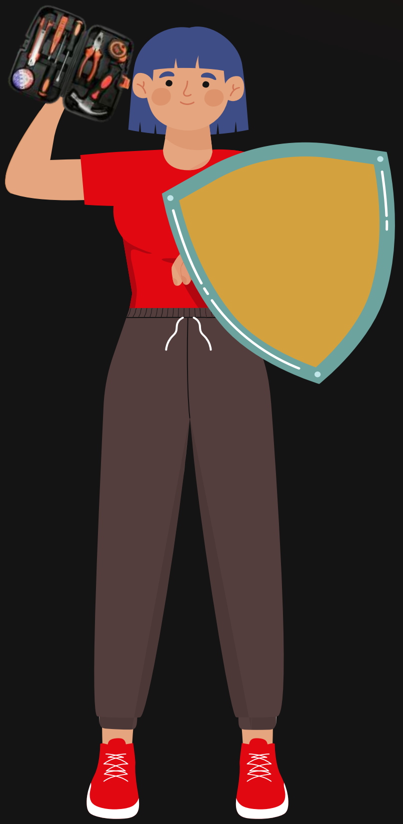
AUTOMATION

LO, NLO, N^{2}LO, N^{3}LO…
calculations in SM and BSM.

REGULARIZATION

DS: traditional dimensional regularization schemes.

THE PERTURBATIVE KEY TOOLKIT FOR PRECISION



AUTOMATION

LO, NLO, N^2LO, N^3LO… calculations in SM and BSM.

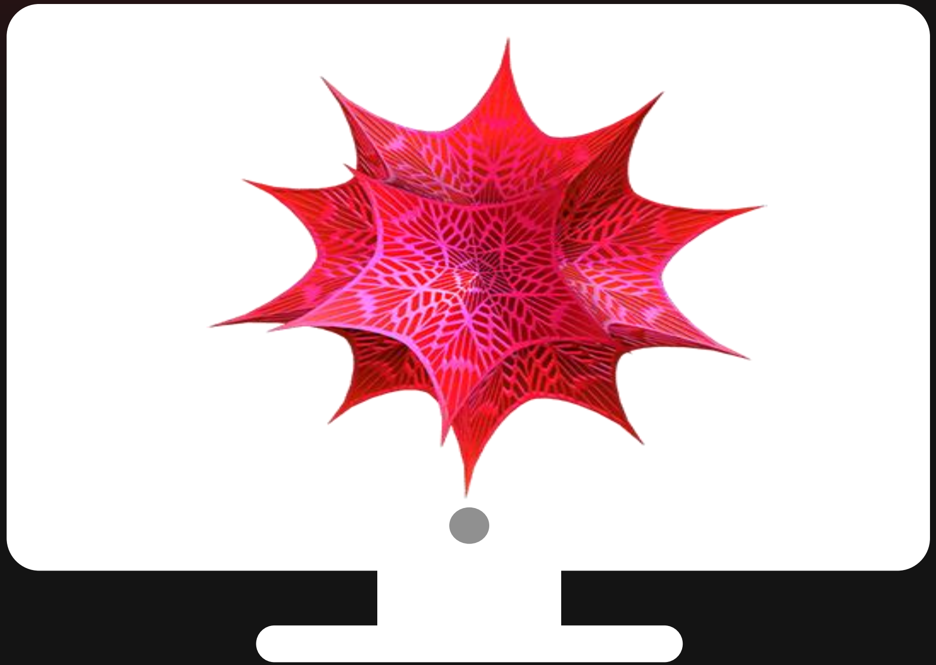
REGULARIZATION

IREG: Implicit Regularization

4-dimensions

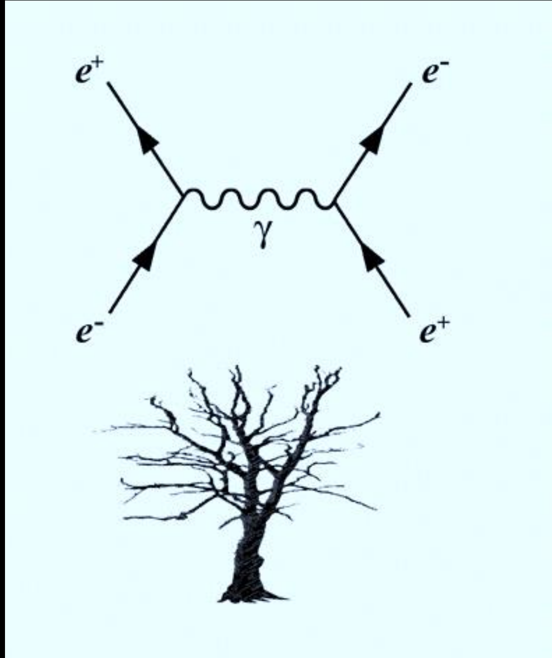
Objectives

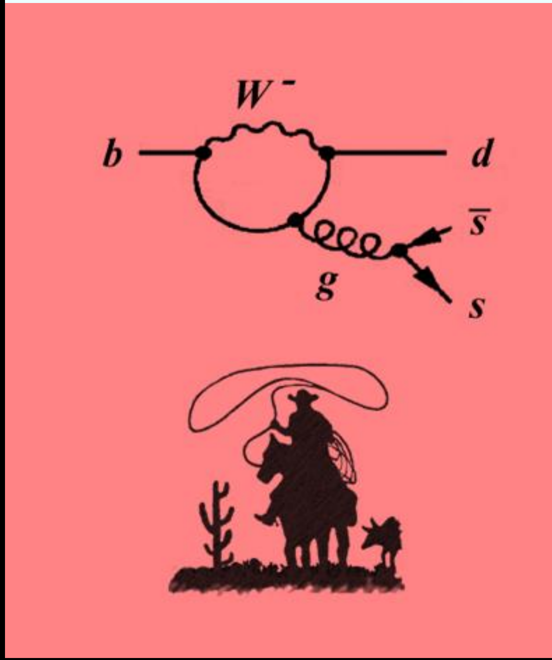
- N^2LO techniques within IREG.
- Tools with FeynArts and FeynCalc based on IREG.
- Gauge theories.
- β-function of pure Yang-Mills, QED and QCD in a mass-independent subtraction scheme.
- γ-function of QED at 2-loop within IREG.



02 THEORY

- Renormalization.
- Renormalization schemes.
- Renormalization group equations.
- Regularization.
- **Regularization schemes:** Implicit regularization (IREG) VS Dimensional Methods (DS).





ABOUT REGULARIZATION AND RENORMALIZATION

1

REGULARIZATION

Mathematical procedure.

2

REGULARIZATION SCHEMES

Different types of regularization techniques: CDR, DRED, and IREG.

3

RENORMALIZATION

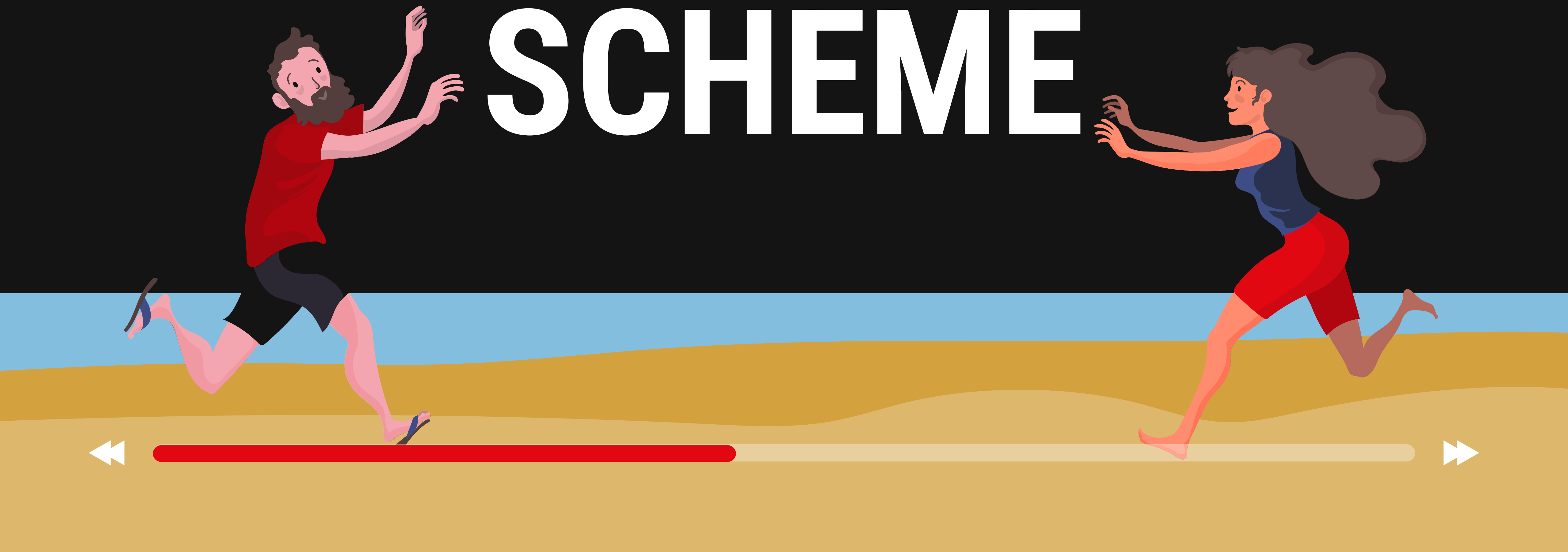
The technique to understand the nature of the divergence. It is related to the **physics**.

4

COUNTERTERMS

Technique by which regularized infinities will be canceled. They are **arbitrary** terms.

RENORMALIZATION SCHEME

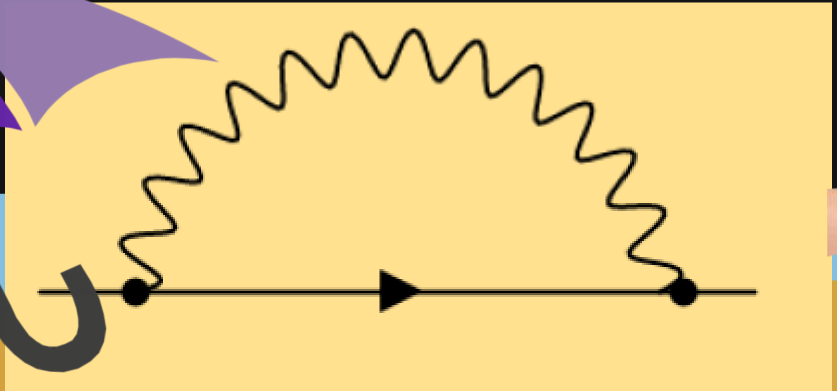


QED

QCD

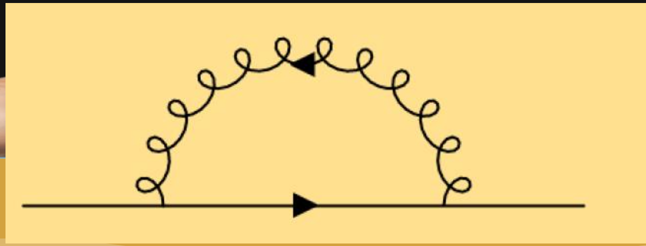
$$\mathcal{L}_0^{QED} = \bar{\psi}_0(i\not{D} - m)\psi_0 - \frac{1}{4}F_{0\mu\nu}F_0^{\mu\nu} + \Delta\mathcal{L} = -\frac{1}{2a_0}(\partial_\mu Q_0^\mu)^2$$

$$\boxed{\psi_0 = \sqrt{Z_2}\psi,} \quad Q_{0\mu} = \sqrt{Z_3}Q_\mu, \quad e_0 = Z_e e, \quad a_0 = Z_A a, \quad m_0 = Z_m m.$$



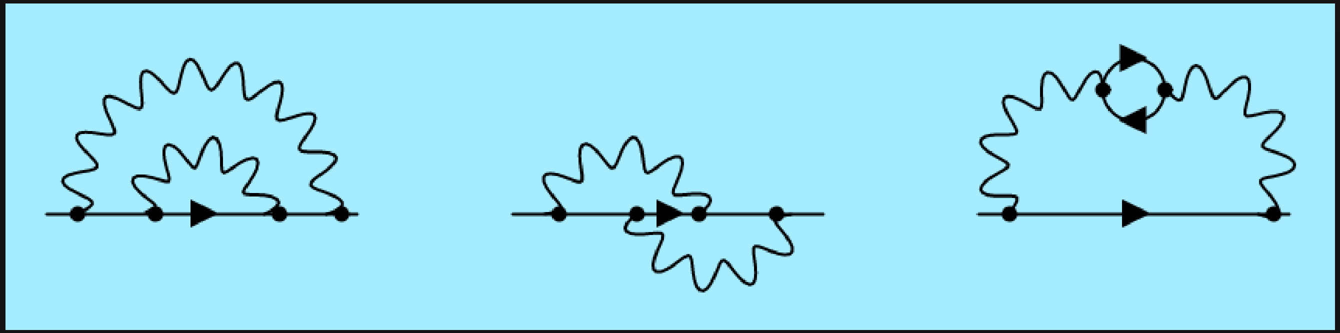
$$\mathcal{L}_0^{QCD} = -\frac{1}{4}F^a_{0\mu\nu}F_0^{a\mu\nu} - \frac{1}{2\xi}\partial^\mu Q^a_{0\mu}\partial^\nu Q^a_{0\nu} + \bar{\Psi}^i_0(i\not{D}^{ij}_\mu - m_0\delta^{ij})\Psi^j_0 - \partial^\mu\bar{c}^a_0 D^{ab}_\mu c^b_0$$

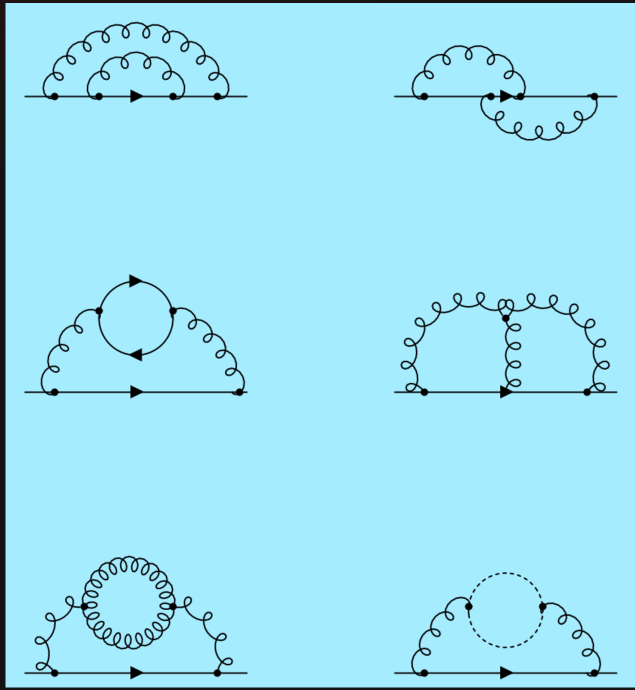
$$\begin{aligned} Q^a_{0\mu} &= Z_3^{1/2}Q^a_\mu, \\ c^a_0 &= \tilde{Z}_3^{1/2}c^a, \\ &\boxed{\Psi_0 = Z_2^{1/2}\Psi,} \\ g_0 &= Z_g g, \\ m_0 &= Z_m m. \end{aligned}$$



QED

QCD





RENORMALIZATION GROUP EQUATIONS

Group of dimensionless parameters that tells us about the scale dependence of them in a renormalizable field theory.

$$\beta = \lambda \frac{\partial}{\partial \lambda} g;$$
$$\gamma = \frac{\lambda}{2} \frac{\partial \ln Z_\psi}{\partial \lambda};$$
$$\gamma_m = \lambda \frac{\partial \ln Z_m}{\partial \lambda}.$$

$$A = Z_\phi - 1;$$
$$B = Z_m Z_\phi - 1;$$
$$C = Z_g Z_\phi^{3/2} - 1.$$

$$\mathcal{L}_{\phi^3_R} = \underbrace{\frac{1}{2}(\partial_\mu \phi_R)^2 - \frac{1}{2} m_R^2 \phi_R^2}_{\text{free lagrangian}} - \underbrace{\frac{g_R}{3!}\phi_R^3}_{\text{interaction term}} + \underbrace{\frac{1}{2} A (\partial_\mu \phi_R)^2 - \frac{1}{2} B m_R^2 \phi_R^2 - \frac{g_R}{3!} C \phi_R^3}_{\text{counterterms lagrangian}}$$

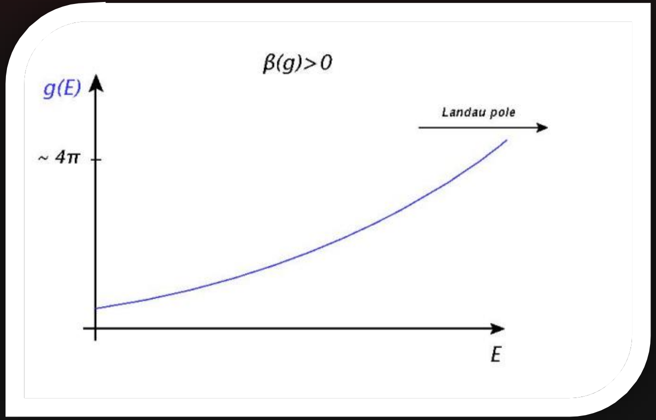
RENORMALIZATION GROUP EQUATIONS

β-function

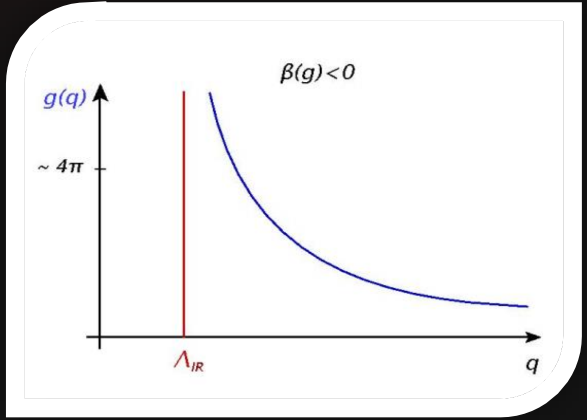
$$\beta = \lambda \frac{\partial}{\partial \lambda} g_R$$

γ-function

$$\gamma = \frac{\lambda}{2} \frac{\partial \ln Z_\psi}{\partial \lambda}$$



QED



QCD

THE RENORMALIZATION PROGRAM IN THE BACKGROUND FIELD METHOD

BFM is a technique for quantizing gauge field theories without losing explicit gauge invariance.

Quantities with no direct physical interpretation will not be gauge invariant.

$\mathcal{L} \Rightarrow$ Feynman Rules.

$\mathcal{L} \Rightarrow$ Compute physical quantities.

Which are gauge invariant and independent of the particular gauge chosen.

The BFM approach arranges things so that gauge invariance in $\mathcal{L}$ is still present once gauge fixing and ghost terms have been added.

$\text{BFM} \rightarrow \mathcal{L} \Rightarrow$ Green's Functions.

$\text{BFM} \rightarrow \mathcal{L} \Rightarrow$ Divergent counterterms.

Gauge invariant

THE RENORMALIZATION PROGRAM IN THE BACKGROUND FIELD METHOD

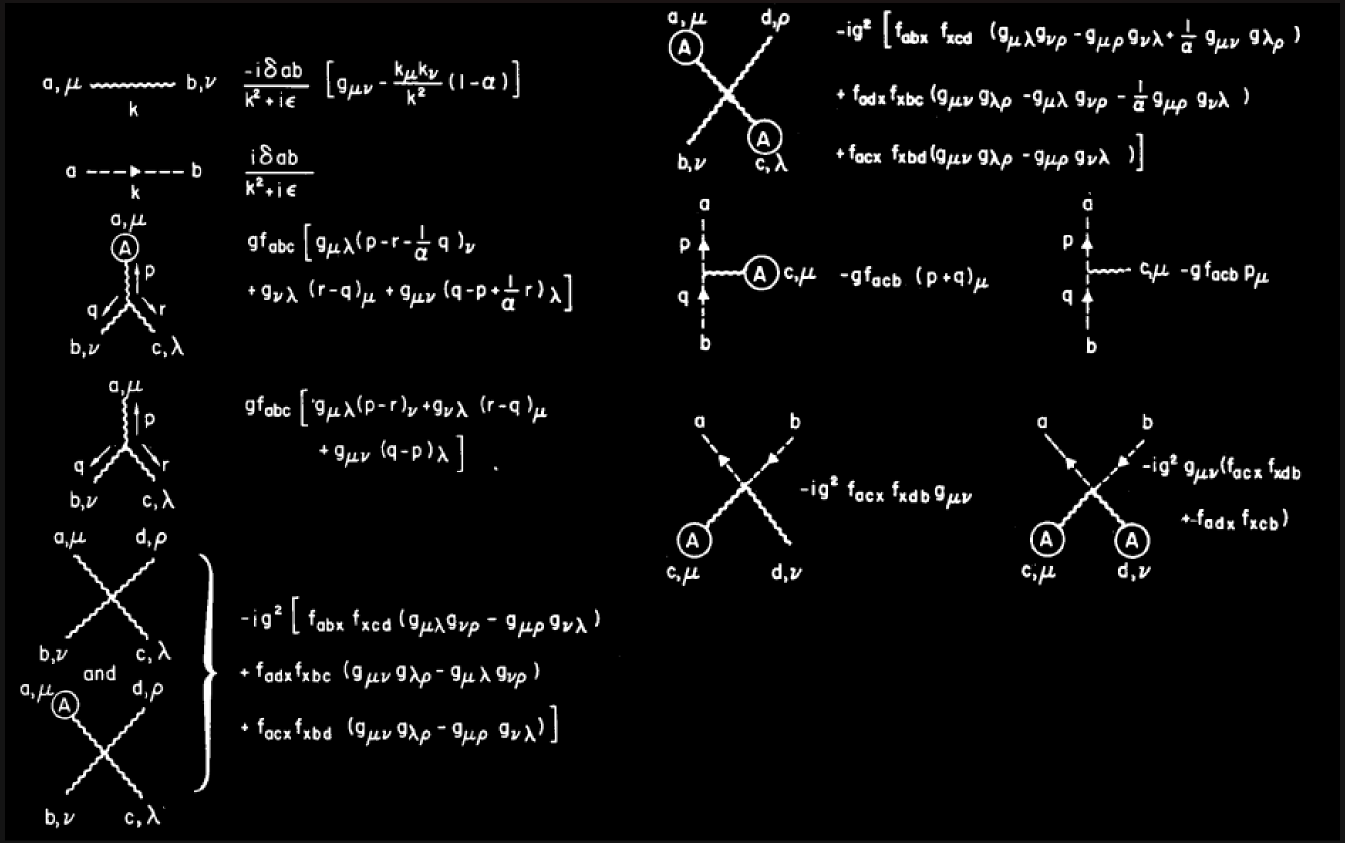
The approach of the BFM is to do a “field-shifting”: the BF is added to que quantum field in the action (S=QF+BF). After this, the method allows to a fix a gauge and evaluate the quantum corrections without breaking the background gauge symmetry.

$$Z(J) = \int \mathcal{D}_Q \det\left(\frac{\delta G^a}{\delta w^b}\right) \exp\left[i(S^{YM}(Q) - \frac{1}{2\alpha} G\cdot G + J\cdot Q)\right]$$

BFM

$$\tilde{Z}(J,\hat{A}) = \int \mathcal{D}_Q \det\left(\frac{\delta \tilde{G}^a}{\delta w^b}\right) \exp^{i(S^{YM}(Q+\hat{A}) - 1/2\alpha \tilde{G}\cdot\tilde{G} + J\cdot Q}$$

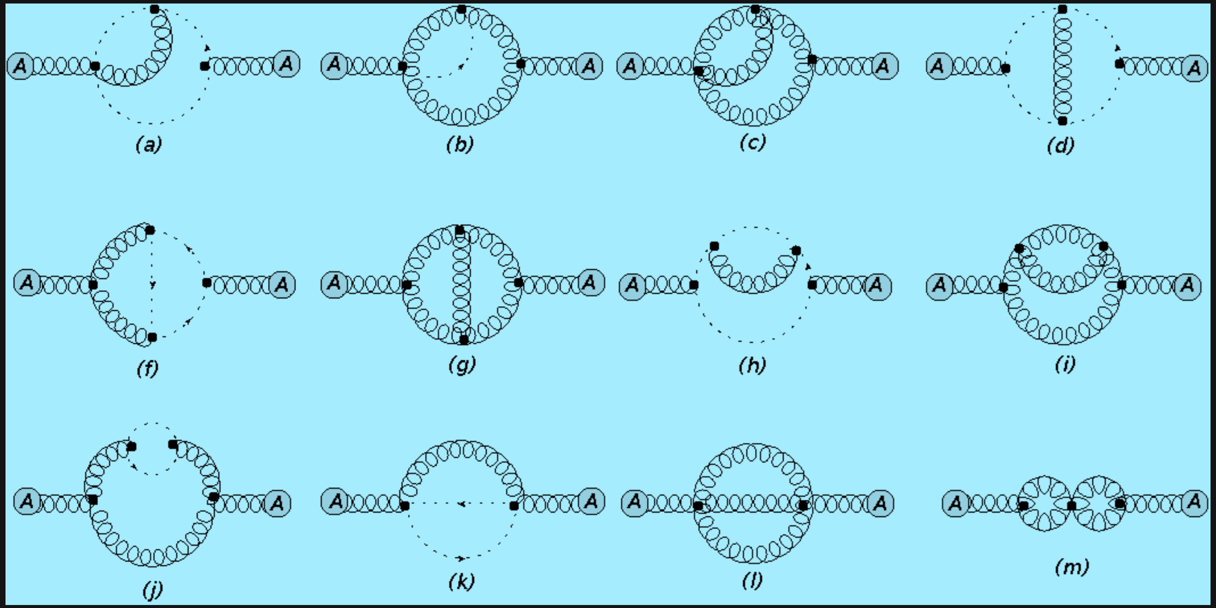
THE RENORMALIZATION PROGRAM IN THE BACKGROUND FIELD METHOD



THE RENORMALIZATION PROGRAM IN THE BACKGROUND FIELD METHOD

We need the 2-point functions as the Abelian theories!

$$Z_g = Z_{\hat{A}}^{-1/2}$$



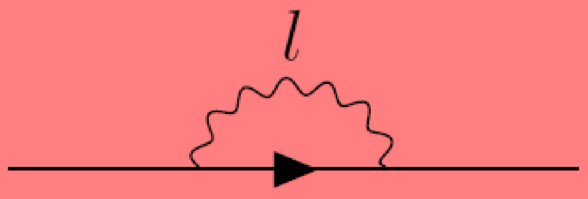
Pure Yang-Mills at 2-loop.

IMPLICIT REGULARIZATION (IREG)

- Alternative to traditional dimensional schemes.
- Momentum space.
- UV-div as basic divergent integrals.
- UV-div do not depend on physical parameters.

IMPLICIT REGULARIZATION (IREG)-EXAMPLE

$$\int_l = \int_{-\infty}^{+\infty} \frac{d^4 l}{(2\pi)^4}$$



$$= -i\Sigma^{(1)}(p,m) = \boxed{\int_l} (-ie\gamma^{\beta}) \frac{(-ig_{\alpha\beta})}{l^2} (-ie\gamma^{\alpha}) i \frac{(\not{p} + \not{l}) + m}{(p+l)^2 - m^2}$$

$$\gamma^{\beta} \not{p} \gamma_{\beta} = -2\not{p},$$
$$\gamma^{\xi}\gamma_{\xi} = 4\mathbb{1}.$$

WHEN DO WE KNOW THAT A DIAGRAM MUST BE REGULARIZED?

$$D = l^2[(p+l)^2 - m^2]$$

$$\Sigma^{(1)}(p,m) = ie^2 2\not{p}\int_l \frac{1}{\boxed{D}} + ie^2 2\gamma^\xi \int_l \frac{l_\xi}{D} - ie^2 4m \int_l \frac{1}{D}$$

$$\int_l \frac{l_\xi}{D} \to \Delta = 5 - 4 = 1,$$
$$\int_l \frac{1}{D} \to \Delta = 4 - 4 = 0.$$

Superficial degree of divergence Δ in a Feynman diagram for UV-div.

IREG-THE FICTITIOUS MASS

$$
\begin{aligned}
\Sigma^{(1)}(p,m) = & \boxed{\lim_{\mu^2\to 0}} ie^2 2\not{p} \int_l \frac{1}{(l^2 \boxed{-\mu^2})[(p+l)^2 - m^2 \boxed{-\mu^2}]} \\
& + \boxed{\lim_{\mu^2\to 0}} ie^2 2\gamma^{\xi} \int_l \frac{l_{\xi}}{(l^2 \boxed{-\mu^2})[(p+l)^2 - m^2 \boxed{-\mu^2}} \\
& - \boxed{\lim_{\mu^2\to 0}} ie^2 4m \int_l \frac{1}{(l^2 \boxed{-\mu^2})[(p+l)^2 - m^2 \boxed{-\mu^2}},
\end{aligned}
$$

IREG-THE RECURSIVE ALGEBRAIC IDENTITY

$$\frac{1}{[(p+l)^2-m^2-\mu^2]} = \frac{1}{(l^2-\mu^2)} - \frac{p^2+2p\cdot l-m^2}{(l^2-\mu^2)[(p+l)^2-m^2-\mu^2]}$$

$$I_\xi = \lim_{\mu^2\to 0}\int_l \frac{l_\xi}{(l^2-\mu^2)[(p+l)^2-m^2-\mu^2]} = I_\xi\Big|_{div} + I_\xi\Big|_{finite},$$

$$I = \lim_{\mu^2\to 0}\int_l \frac{1}{(l^2-\mu^2)[(p+l)^2-m^2-\mu^2]} = I\Big|_{div} + I\Big|_{finite}.$$

IREG-EXAMPLE

$$\frac{1}{[(p+l)^2-m^2-\mu^2]}=\frac{1}{(l^2-\mu^2)}-\frac{p^2+2p\cdot l-m^2}{(l^2-\mu^2)[(p+l)^2-m^2-\mu^2]}$$

$$I_\xi\Big|_{div}=-2p^\alpha\lim_{\mu^2\to 0}\int_l\frac{l_\xi l_\alpha}{(l^2-\mu^2)^3}$$

$$I\Big|_{div}=\lim_{\mu^2\to 0}\int_l\frac{1}{(l^2-\mu^2)^2}$$

IREG-THE BASIC DIVERGENT INTEGRALS

The objective of IREG will be to write the UV-div in terms of the Basic Divergent Integrals (BDI)

$$I\Big|_{div} = \lim_{\mu^2 \to 0} \int_l \frac{1}{(l^2-\mu^2)^2} \quad \Longleftrightarrow \quad I_{log}(\mu^2) \equiv \int_k \frac{1}{(k^2-\mu^2)^2}$$

$$I_\xi\Big|_{div} = -2p^\alpha \lim_{\mu^2 \to 0} \int_l \frac{l_\xi l_\alpha}{(l^2-\mu^2)^3} \quad \Longleftrightarrow \quad I_{log}^{\nu_1\nu_2}(\mu^2) \equiv \int_k \frac{k_{\nu_1} k_{\nu_2}}{(k^2-\mu^2)^3}$$

IREG-SURFACE TERMS

$$I_\xi\Big|_{div} = -2p^\alpha \lim_{\mu^2\to 0}\int_l \frac{l_\xi l_\alpha}{(l^2-\mu^2)^3}$$

$$\Longleftrightarrow$$

$$I_{log}^{\nu_1\nu_2}(\mu^2) \equiv \int_k \frac{k_{\nu_1}k_{\nu_2}}{(k^2-\mu^2)^3}$$

$$\begin{aligned}\int_l \frac{\partial}{\partial l_\alpha}\frac{l_\xi}{(l^2-\mu^2)^2} &= \int_l \frac{g_{\xi\alpha}}{(l^2-m^2)^2} - 4\int_l \frac{l_\xi l_\alpha}{(l^2-\mu^2)^3} \\ &= g_{\xi\alpha} I_{log}(\mu^2) - 4 I_{log}^{\xi\alpha}(\mu^2) \\ &\equiv \Upsilon_0^{(1)\xi\alpha},\end{aligned}$$

$$I_{log}^{\xi\alpha}(\mu^2) = \frac{g_{\xi\alpha}}{4} I_{log}(\mu^2) - \frac{1}{4}\underbrace{\int_l \frac{\partial}{\partial l_\alpha}\frac{l_\xi}{(l^2-\mu^2)^2}}_{\text{Surface Term}}.$$

IREG-INFRARED DIVERGENCE TERMS

$$I_{log}(\mu^2) \equiv \int_k \frac{1}{(k^2-\mu^2)^2}$$

IREG-INFRARED DIVERGENCE TERMS

$$I_{log}(\mu^2) \equiv \int_k \frac{1}{(k^2-\mu^2)^2}$$

$$\lim_{\mu^2 \to 0}$$

$$I_{log}(\mu^2) = I_{log}(\lambda^2) + \frac{i}{(4\pi)^2} \ln \frac{\lambda^2}{\mu^2}$$

Scale relations.

IREG-LINEAR INTEGRAL RESULT

$$
\begin{aligned}
I_\xi &= I_\xi\Big|_{div} + I_\xi\Big|_{finite} \\
&= -2p^\alpha\left[\frac{g_{\xi\alpha}}{4}I_{log}(\mu^2) - \frac{1}{4}\Upsilon_0^{(1)\mu\nu}\right] + I_\xi\Big|_{finite} \\
&= -\frac{p_\xi}{2}I_{log}(\mu^2) + 0 + I_\xi\Big|_{finite} \\
&= -\frac{p_\xi}{2}I_{log}(\lambda^2) - \frac{p_\xi}{2}b\ln\left(\frac{\lambda^2}{\mu^2}\right) \\
&- p^2\lim_{\mu^2\to 0}\int_l \frac{l_\xi}{(l^2-\mu^2)^3} + m^2\lim_{\mu^2\to 0}\int_l \frac{l_\xi}{(l^2-\mu^2)^3} \\
&+ \lim_{\mu^2\to 0}\int_l \frac{l_\xi(p^2+2p\cdot l-m^2)^2}{(l^2-\mu^2)^3[(p+l)^2-m^2-\mu^2]}.
\end{aligned}
$$

IREG-FINAL RESULT FOR THE ELECTRON PROPAGATOR AT 1-LOOP

$$\Sigma^{(1)}(p,m) = +\, ie^2(\not{p} - 4m)I_{log}(\lambda^2) + ie^2(\not{p} - 4m)b\ln\left(\frac{\lambda^2}{\mu^2}\right)$$

$$- ie^2 4m2b - ie^2 4m \lim_{\mu^2\to 0} \ln\left(\frac{\mu^2}{m^2}\right) b - ie^2 2\left(-\not{p}\frac{3}{2}b - \not{p}\frac{1}{2}b \lim_{\mu^2\to 0} \ln\left(\frac{\mu^2}{m^2}\right)\right)$$

$$= \underbrace{+ie^2(\not{p} - 4m)I_{log}(\lambda^2)}_{\text{Divergent}}$$

The μ^2 dependence dropped out, as it should!

$$- ie^2\not{p}\left[-3b - b\ln\left(\frac{\lambda^2}{m^2}\right)\right] - ie^2(-4m)\left[-2b - b\ln\left(\frac{\lambda^2}{m^2}\right)\right].$$

TRADITIONAL DIMENSIONAL SCHEMES (DS)

- DS are based on analytical continuations of the space: 4 -> d dimensions.

- In DS, UV-div manifest as poles: 1/ε

$$\int_{-\infty}^{+\infty} \frac{d^4 k_{[4]}}{(2\pi)^4} \to \underbrace{\mu^{4-d}}_{\text{scale}} \int_{-\infty}^{+\infty} \frac{d^d k_{[d]}}{(2\pi)^d}$$

CORRESPONDENCES AT 1-LOOP

UV-div

DS

$$\underbrace{\frac{1}{\epsilon}}_{\text{Poles when } \varepsilon \to 0}$$

Poles when ε -> 0

UV-div

IREG

$$\underbrace{I_{log}(\mu^2) \equiv \int_k \frac{1}{(k^2-\mu^2)^2}}$$

Basic divergent integrals

CORRESPONDENCES AT 1-LOOP

UV-div

DS

Same!

UV-div

IREG

$$I = \int_k \frac{1}{k^2(k-p)^2} \overset{\text{IREG}}{=} I_{log}(\lambda^2) - b\ln\left[-\frac{p^2}{\lambda^2}\right] + 2b$$

d= 4- 2ε

$$I^d_{log}(\lambda^2) = b\left[\frac{1}{\epsilon} - \gamma_E + \ln(4\pi) + \ln\left(\frac{\mu_{DR}^2}{\lambda^2}\right)\right]$$

$$I\Big|_d = b\left[\frac{1}{\epsilon} - \gamma_E + \ln(4\pi) - \ln\left[-\frac{p^2}{\mu_{DR}^2}\right] + 2\right]$$

CORRESPONDENCES AT n-LOOPS

UV-div

DS

$$\mathcal{J}_d = b^n \left[\frac{1}{(4\pi)^{-\epsilon}} \left(-\frac{\mu_{DR}^2}{p^2} \right)^{\epsilon} \frac{\Gamma(\epsilon)\Gamma^2(1-\epsilon)}{\Gamma(2-2\epsilon)} \right]^n$$

$$\neq$$

UV-div

IREG

$$J = \int_{k_1} \frac{1}{k_1^2 (k_1 - p)^2} \cdots \int_{k_n} \frac{1}{k_n^2 (k_n - p)^2}$$

$$J^{\mathrm{IREG}} = \left[I_{log}(\lambda^2) - b \ln \left[-\frac{p^2}{\lambda^2} \right] + 2b \right]^n$$

d= 4- 2ε

$$\frac{b}{(4\pi)^{-\epsilon}} \left(\frac{\mu_{DR}^2}{\lambda^2} \right)^{\epsilon} \Gamma(\epsilon)$$

$$\mathcal{J}^{\mathrm{IREG}}\Big|_d = b^n \left[\frac{1}{(4\pi)^{-\epsilon}} \left(-\frac{\mu_{DR}^2}{p^2} \right)^{\epsilon} \Gamma(\epsilon) + 2 \right]^n$$

03 METHODOLOGY

The calculation of amplitudes at higher order is essential in QFT. In this direction, it is practical to develop routines for automated calculations.

Play

Algorithms

ALGORITHM: β-FUNCTION

01

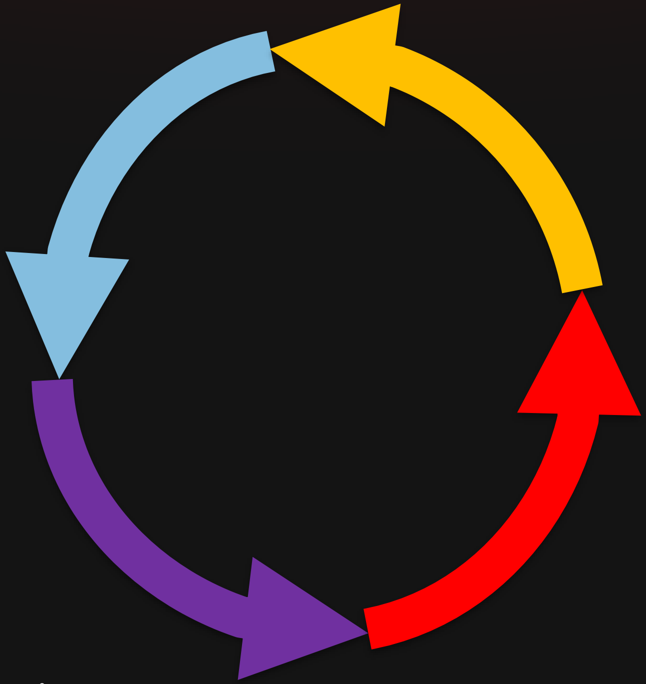
FEYNARTS&FORMCALC

We create the topologies and the amplitudes. We do the contractions.

02

REGULARIZATION

We make the necessary simplifications and reductions for the integrals. This depends on the regularization scheme.



03

INTEGRAL EVALUATION

According to the regularization method used.

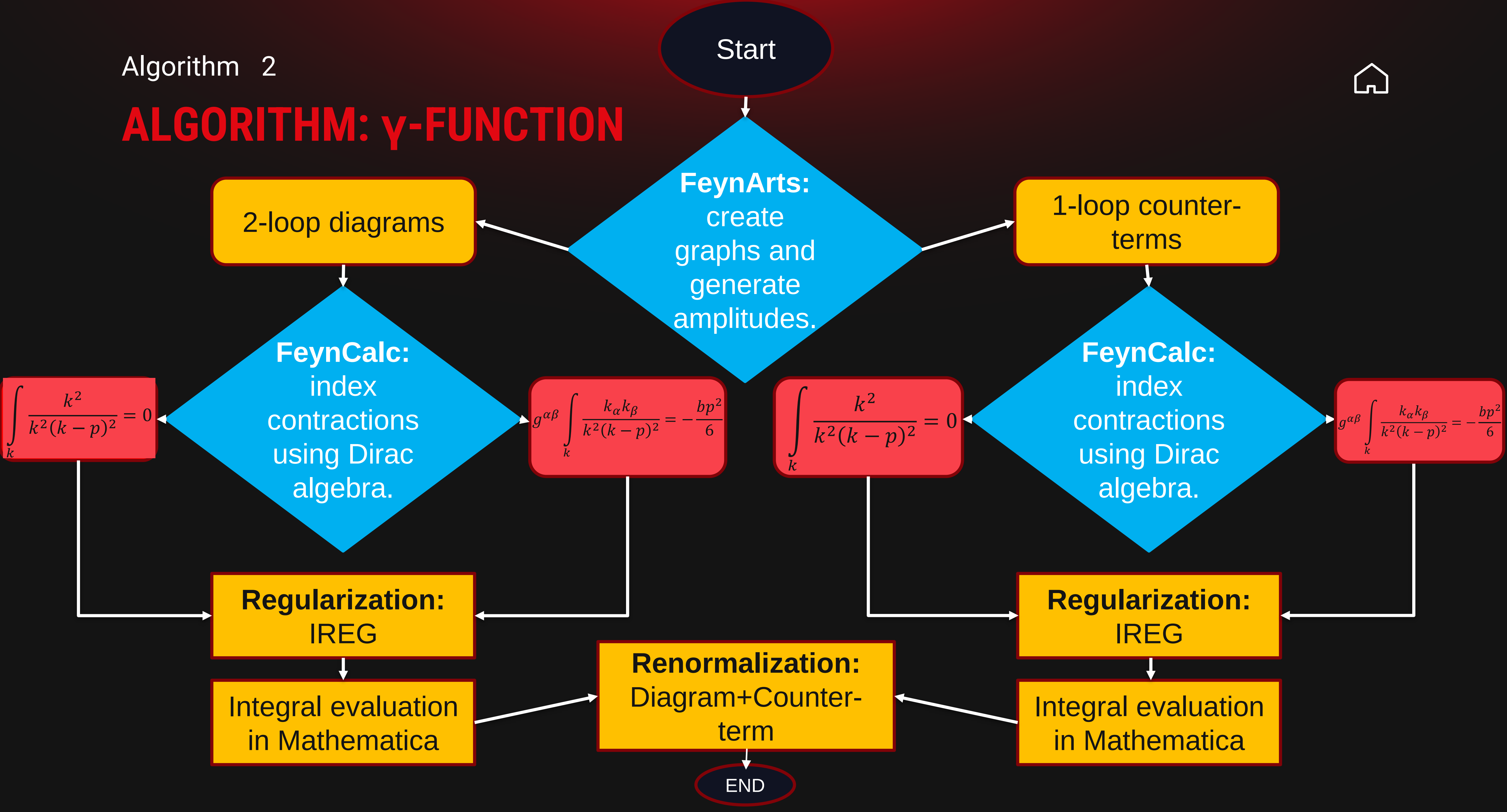
04

β-FUNCTION

Renormalization.

Algorithm 2

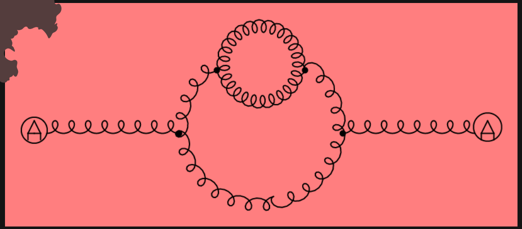
ALGORITHM: γ-FUNCTION



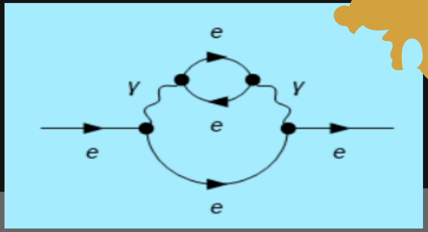
GAUGE THEORIES WITH IREG AT 2-LOOP

2-loop approach examples when:

- We have a gluonic loop.



- We have a fermionic loop.

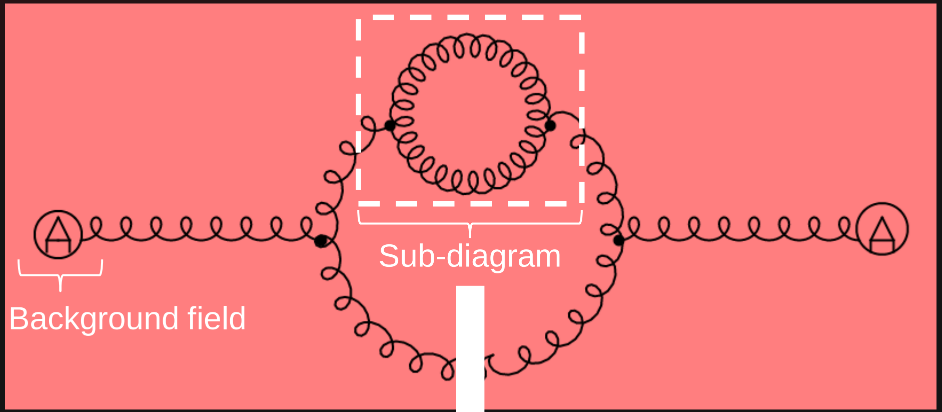


SUBTLETIES

- **Stage 1:** Feynman Rules. **Don't evaluate the integral of the subdiagram.** Perform Contractions and simplifications.

$$\int_k \frac{k_\lambda k_\sigma}{k^4(k-p)^2}\int_l \frac{l^2}{l^2(l-k)^2} \propto \int_k \frac{k_\lambda k_\sigma}{k^4(k-p)^2}\int_l \frac{1}{(l-k)^2}$$

- **Stage 2:** Regularization!



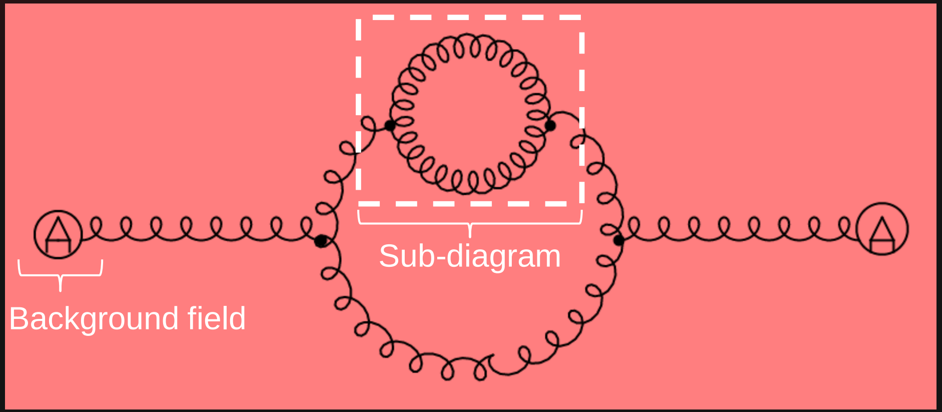
$$-g^2 C_A \delta^{mn}\frac{1}{2}\int_l \frac{1}{l^2(l-k)^2}[2k_\nu k_\mu + 5k_\mu l_\nu + k_\nu l_\mu - 10 l_\nu l_\mu - g^{\nu\mu}(5k^2+2l^2-2k\cdot l)]$$

SUBTLETIES

- **Stage 1:** Feynman Rules. **Don't evaluate the integral of the subdiagram.** Perform Contractions and simplifications.

$$\int_k \frac{k_\lambda k_\sigma}{k^4(k-p)^2} \int_l \frac{l^2}{l^2(l-k)^2} \propto \int_k \frac{k_\lambda k_\sigma}{k^4(k-p)^2} \int_l \frac{1}{(l-k)^2}$$

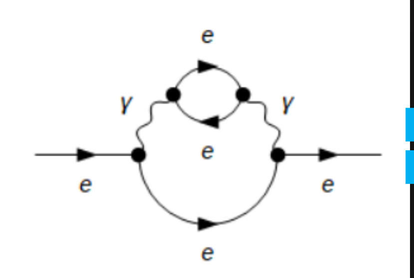
- **Stage 2:** Regularization!



$$\Pi^{\lambda\sigma}_{ls} = +ig^4 C_A^2 \delta^{sl} \int_k \frac{1}{k^4(k-p)^2} \times \xi_{\mu\nu\lambda\sigma} \times \left[\frac{1}{2}\int_l \frac{1}{l^2(l-k)^2}(2k_\nu k_\mu + 5k_\mu l_\nu + k_\nu l_\mu - 10 l_\nu l_\mu - g^{\nu\mu}(5k^2 + 2l^2 - 2k\cdot l))\right]$$

SUBTLETIES

SUBTLETIES

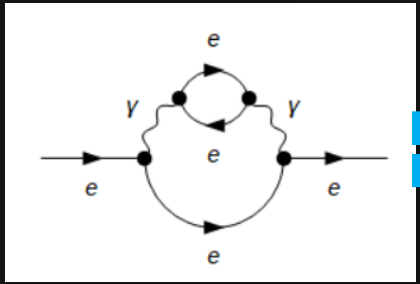


$$= \int_l \int_k \frac{i\mathrm{e}^2\gamma^\beta.\left(\not{l}+\not{p}\right).\gamma^\nu \mathrm{tr}\left(\mathrm{e}^2\left(-\left(-\not{k}\right).\gamma^\nu.\left(\not{l}-\not{k}\right).\gamma^\beta\right)\right)}{(l+p)^2k^2l^4(l-k)^2}$$

$$\mathrm{tr}\left[\mathrm{e}^2\left(-\left(-\left(\gamma\cdot k\right)\right).\gamma^\nu.\left(\gamma\cdot\left(l-k\right)\right).\gamma^\beta\right)\right)$$

$$= 4\mathrm{e}^2k^2g^{\beta\nu} - 4\mathrm{e}^2g^{\beta\nu}\left(k\cdot l\right) - 8\mathrm{e}^2k^\beta k^\nu + 4\mathrm{e}^2k^\beta l^\nu + 4\mathrm{e}^2k^\nu l^\beta$$

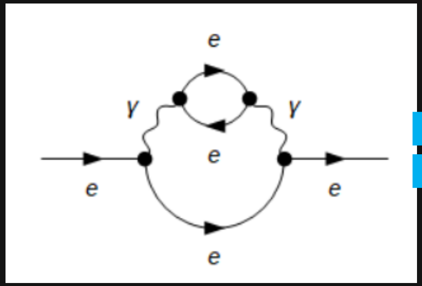
SUBTLETIES



$$= \int_l \int_k \frac{ie^2\gamma^\beta.\left(\not{l}+\not{p}\right).\gamma^\nu \mathrm{tr}\left(e^2\left(-\left(-\not{k}\right).\gamma^\nu.\left(\not{l}-\not{k}\right).\gamma^\beta\right)\right)}{(l+p)^2k^2l^4(l-k)^2}$$

$$\begin{aligned}
&e^2\gamma^\beta.\left(\gamma\cdot(l+p)\right).\gamma^\nu\left[4e^2k^2g^{\beta\nu}-4e^2g^{\beta\nu}\left(k\cdot l\right)-8e^2k^\beta k^\nu+4e^2k^\beta l^\nu+4e^2k^\nu l^\beta\right]\\
&=-e^4 8\gamma^\beta\left(\gamma\cdot l\right)\gamma^\nu k_\beta k_\nu+4e^4\gamma^\beta\left(\gamma\cdot l\right)\gamma^\nu k^2 g_{\beta\nu}+4e^4\gamma^\beta\left(\gamma\cdot l\right)\gamma^\nu k_\nu l_\beta\\
&+4e^4\gamma^\beta\left(\gamma\cdot l\right)\gamma^\nu k_\beta l_\nu-4e^4\gamma^\beta\left(\gamma\cdot l\right)\gamma^\nu\left(k\cdot l\right)g_{\beta\nu}-8e^4\gamma^\beta\left(\gamma\cdot p\right)\gamma^\nu k_\beta k_\nu\\
&+4e^4\gamma^\beta\left(\gamma\cdot p\right)\gamma^\nu k^2 g_{\beta\nu}+4e^4\gamma^\beta\left(\gamma\cdot p\right)\gamma^\nu k_\nu l_\beta+4e^4\gamma^\beta\left(\gamma\cdot p\right)\gamma^\nu k_\beta l_\nu\\
&-4e^4\gamma^\beta\left(\gamma\cdot p\right)\gamma^\nu\left(k\cdot l\right)g_{\beta\nu}.
\end{aligned}$$

SUBTLETIES



$$= \int_l \int_k \frac{ie^2\gamma^\beta.\left(\not{l} + \not{p}\right).\gamma^\nu \mathrm{tr}\left(e^2\left(-\left(-\not{k}\right).\gamma^\nu.\left(\not{l}-\not{k}\right).\gamma^\beta\right)\right)}{(l+p)^2k^2l^4(l-k)^2}$$

$$\begin{aligned}
-8e^4\gamma^\beta\left(\gamma\cdot l\right)\gamma^\nu k_\beta k_\nu &= -8e^4\gamma^\beta\gamma^\alpha l_\alpha k_\beta\gamma^\nu k_\nu\\
&= -8e^4\left(2g^{\alpha\beta}-\gamma^\alpha\gamma^\beta\right)l_\alpha k_\beta\gamma^\nu k_\nu\\
&= -16e^4\left(l\cdot k\right)\left(\gamma\cdot k\right)+8e^4\gamma^\alpha l_\alpha\gamma^\beta k_\beta\gamma^\nu k_\nu\\
&= -16e^4\left(l\cdot k\right)\left(\gamma\cdot k\right)+8e^4\left(\gamma\cdot l\right)\frac{1}{2}\left(\gamma^\beta\gamma^\nu+\gamma^\nu\gamma^\beta\right)k_\nu k_\beta\\
&= -16e^4\left(l\cdot k\right)\left(\gamma\cdot k\right)+8e^4\left(\gamma\cdot l\right)g^{\beta\nu}k_\beta k_\nu\\
&= -16e^4\left(l\cdot k\right)\left(\gamma\cdot k\right)+8e^4\left(\gamma\cdot l\right)k^2.
\end{aligned}$$

$$\int_k \frac{k^2}{k^2(k-p)^2} = 0$$

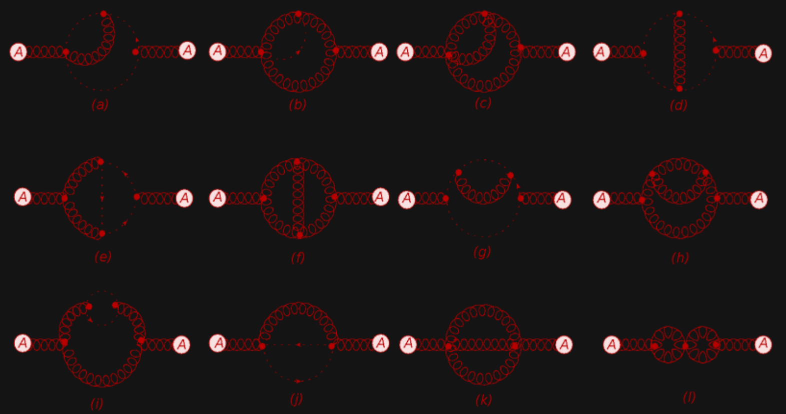
$$g^{\alpha\beta}\int_k \frac{k_\alpha k_\beta}{k^2(k-p)^2} = -\frac{bp^2}{6}$$

04 RESULTS

We calculate within IREG the β-function of YM, QED and, QCD; Also, the γ-function for QED at 2-loops, considering the dependence on the gauge parameter.

THE TWO-LOOP GAUGE β-FUNCTION FOR ABELIAN AND NON-ABELIAN THEORIES

Pure Yang-Mills



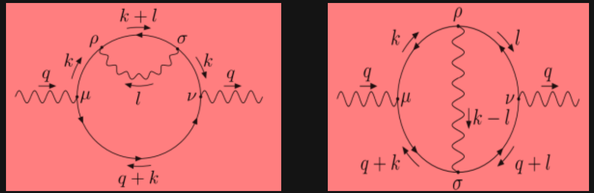
$$\frac{i g_s^4 C_A^2 \delta^{ab}}{(4\pi)^4}\left[A g_{\mu\nu} p^2 - B p_\mu p_\nu\right]$$

β

DETINITION

$$\beta = -g_R\left[\beta_0\left(\frac{\tilde{g}_R}{4\pi}\right)^2 + \beta_1\left(\frac{\tilde{g}_R}{4\pi}\right)^4\right]$$

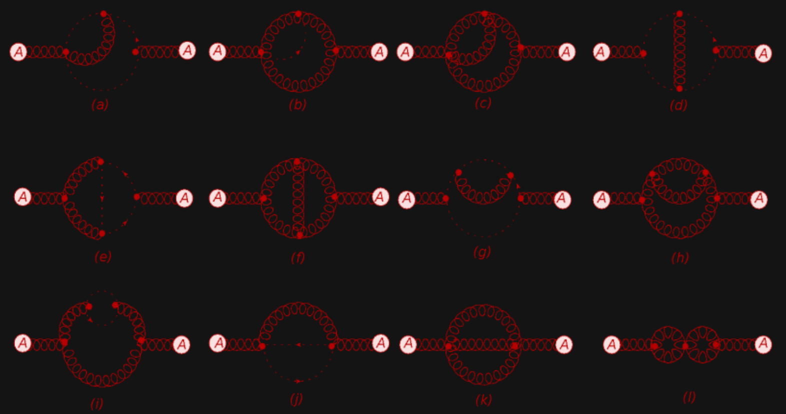
QED



$$A_{\mu\nu} = \frac{i g_e^4}{(4\pi)^4}\left[A g_{\mu\nu} p^2 - B p_\mu p_\nu\right]$$

THE TWO-LOOP GAUGE β-FUNCTION FOR ABELIAN AND NON-ABELIAN THEORIES

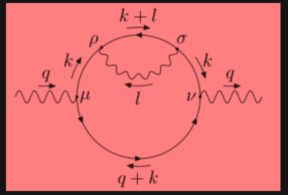
Pure Yang-Mills

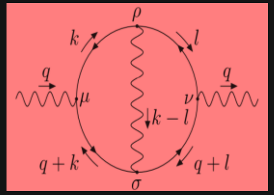


$$\frac{i g_s^4 C_A^2 \delta^{ab}}{(4\pi)^4}\left[A g_{\mu\nu} p^2 - B p_\mu p_\nu\right]$$

β

QED





$$\mathcal{A}_{\mu\nu} = \frac{i g_e^4}{(4\pi)^4}\left[A g_{\mu\nu} p^2 - B p_\mu p_\nu\right]$$

COEFFICIENTS

$$\beta = -g_R\left[\beta_0\left(\frac{\tilde{g}_R}{4\pi}\right)^2 + \beta_1\left(\frac{\tilde{g}_R}{4\pi}\right)^4\right]$$

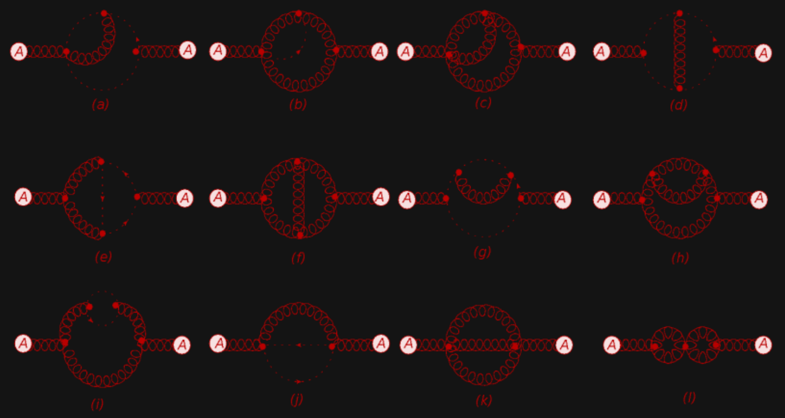
$$\beta_0|_{\text{IREG}} = \frac{11 C_A}{3} \qquad \beta_1|_{\text{IREG}} = \frac{34 C_A^2}{3}$$

$$\beta_0|_{\text{IREG}} = -\frac{4}{3} \qquad \beta_1|_{\text{IREG}} = -4$$

… same as DS!

THE TWO-LOOP GAUGE β-FUNCTION FOR ABELIAN AND NON-ABELIAN THEORIES

QCD



β

COEFFICIENTS

$$\beta = -g_R\left[\beta_0\left(\frac{\tilde{g}_R}{4\pi}\right)^2 + \beta_1\left(\frac{\tilde{g}_R}{4\pi}\right)^4\right]$$

$$\frac{i g_s^4 n_f}{(4\pi)^4}\left[A g_{\mu\nu}p^2 - B p_\mu p_\nu\right]$$

$$\beta_0|_{\text{IREG}} = 11 - \frac{2}{3}n_f$$

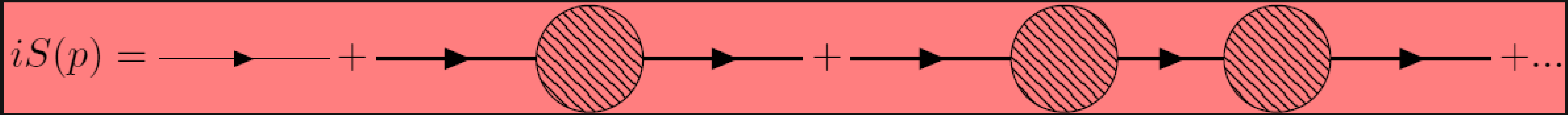
$$\beta_1|_{\text{IREG}} = 102 - \frac{38}{3}n_f$$

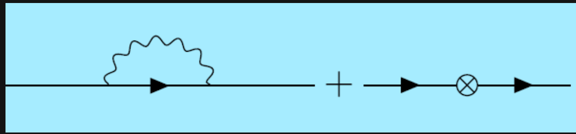
… same as DS!

THE ANOMALOUS DIMENSION AT 2-LOOP WITHIN IREG

From the QED lagrangian:

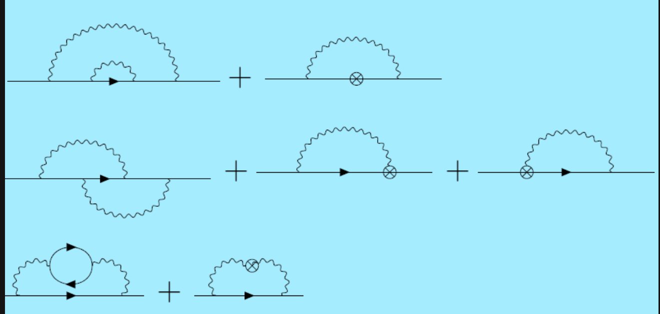
$$\bar{\Psi}\not{D}\Psi \to \bar{\Psi}_r\not{D}\Psi_r + (Z_\Psi - 1)\bar{\Psi}_r\not{D}\Psi_r$$





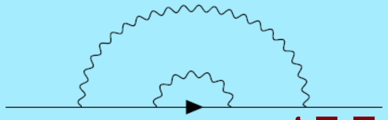
$$\Sigma_v(p^2)\Big|_{div}\not{p} + (Z_\Psi - 1)\not{p} + finite$$

$$Z_\Psi = 1 + \delta_\Psi^{(1)} = 1 + ie^2 I_{log}(\lambda^2)$$

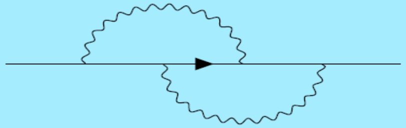


THE ANOMALOUS DIMENSION AT 2-LOOP WITHIN IREG

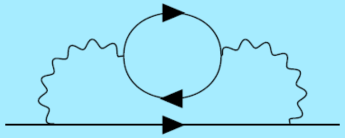
DIAGRAMS



$$= -\frac{1}{2} i e^4 \left(2b I_{\log}^{(2)}(\lambda^2) + \boxed{2b I_{\log}(\lambda^2)\log\left(-\frac{p^2}{\lambda^2}\right)} - 7b I_{\log}(\lambda^2) - 2 I_{\log}^2(\lambda^2)\right)\not{p} + finite$$

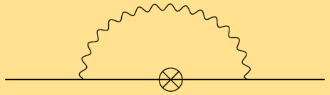


$$= i e^4 \left(2b I_{\log}^{(2)}(\lambda^2) + 2b I_{\log}(\lambda^2)\log\left(-\frac{p^2}{\lambda^2}\right) - 7b I_{\log}(\lambda^2) - 2 I_{\log}^2(\lambda^2)\right)\not{p} + finite$$

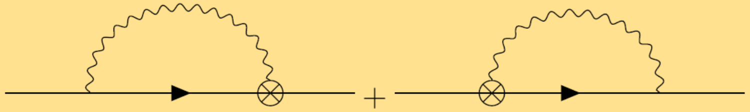


$$= 2ibe^4 I_{\log}(\lambda^2)\not{p} + finite$$

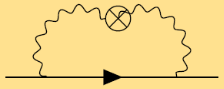
COUNTERTERMS



$$= e^4 \delta_\psi^{(1)} \left(b\log\left(-\frac{p^2}{\lambda^2}\right) - 2b - I_{\log}(\lambda^2)\right)\not{p}$$



$$= 2e^4 \delta_\psi^{(1)} \left(b\log\left(-\frac{p^2}{\lambda^2}\right) - 2b - I_{\log}(\lambda^2)\right)\not{p}$$



$$= -e^4 b \delta_A^{(1)} \not{p}.$$

THE ANOMALOUS DIMENSION AT 2-LOOP WITHIN IREG

$$\Sigma^{(2)}(p)\Big|_{div} = -e^4 b \not{p} I_{\log}^{(2)}(\lambda^2) + \frac{5}{6} e^4 b \not{p} I_{\log}(\lambda^2)$$

$$Z_\Psi\Big|_{IREG} = 1 + ie^2 I_{\log}(\lambda^2) + e^4 b \left[-I_{\log}^{(2)}(\lambda^2) + \frac{5}{6} I_{\log}(\lambda^2) \right]$$

THE ANOMALOUS DIMENSION AT 2-LOOP WITHIN IREG

$$Z_\Psi\Big|_{IREG} = 1 + ie^2 I_{\log}(\lambda^2) + e^4 b\left[\underbrace{-I^{(2)}_{\log}(\lambda^2)}_{\int \frac{d^4k}{(2\pi)^4}\frac{1}{(k^2-\lambda^2)^2}\ln\left[-\frac{k^2}{\lambda^2}\right]} + \frac{5}{6} I_{\log}(\lambda^2)\right]$$

In d-dim

$$\frac{b}{(4\pi)^{-2\epsilon}}\left(\frac{\mu_{DR}^2}{\lambda^2}\right)^{2\epsilon}\Gamma(\epsilon)\left[\Gamma(\epsilon) - \frac{\Gamma(2-4\epsilon)\Gamma(2\epsilon)}{\Gamma(2-3\epsilon)}\right]$$

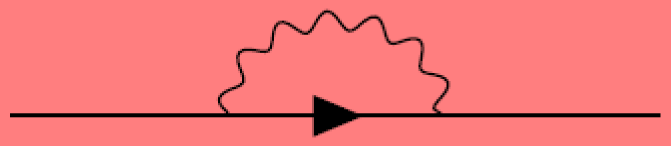
$$Z_\Psi\Big|_{CDR} = 1 - a\frac{\alpha}{4\pi\epsilon} + \left[\frac{a^2}{2} + \left(n_f + \frac{3}{4}\right)\epsilon\right]\left(\frac{\alpha}{4\pi\epsilon}\right)^2$$

✓ the terms in ϵ^{-2} can be reproduced

THE ANOMALOUS DIMENSION AT 2-LOOP WITHIN IREG

$$Z_\Psi = 1 + \boxed{a} A_1 g^2 + A_2 g^4$$

We need to consider the gauge parameter dependece!



$$= \Sigma(p) = e^2 \boxed{a_0} I_{\log}(\lambda^2)\gamma \cdot p$$

THE ANOMALOUS DIMENSION AT 2-LOOP WITHIN IREG-RESULT

$$\gamma = \frac{\lambda}{2}\frac{\partial \ln Z_\psi}{\partial \lambda}$$

$$Z_\Psi = 1 + aA_1 g^2 + A_2 g^4$$

$$\ln(Z_\Psi) = aA_1 g^2 + \left(A_2 - \frac{A_1^2}{2}\right) g^4 + O(g^6)$$

$$\gamma_1 = \frac{\lambda}{2} a \frac{\partial}{\partial \lambda} A_1$$

$$\gamma_2 = a\beta_1 A_1 + \frac{\lambda}{2}\frac{\partial}{\partial \lambda}\left(A_2 - \frac{a^2 A_1^2}{2}\right) + \frac{1}{2}\gamma_A^{(1)} A_1$$

THE ANOMALOUS DIMENSION AT 2-LOOP WITHIN IREG-RESULT

$$\gamma = \frac{\lambda}{2}\frac{\partial \ln Z_\psi}{\partial \lambda}$$

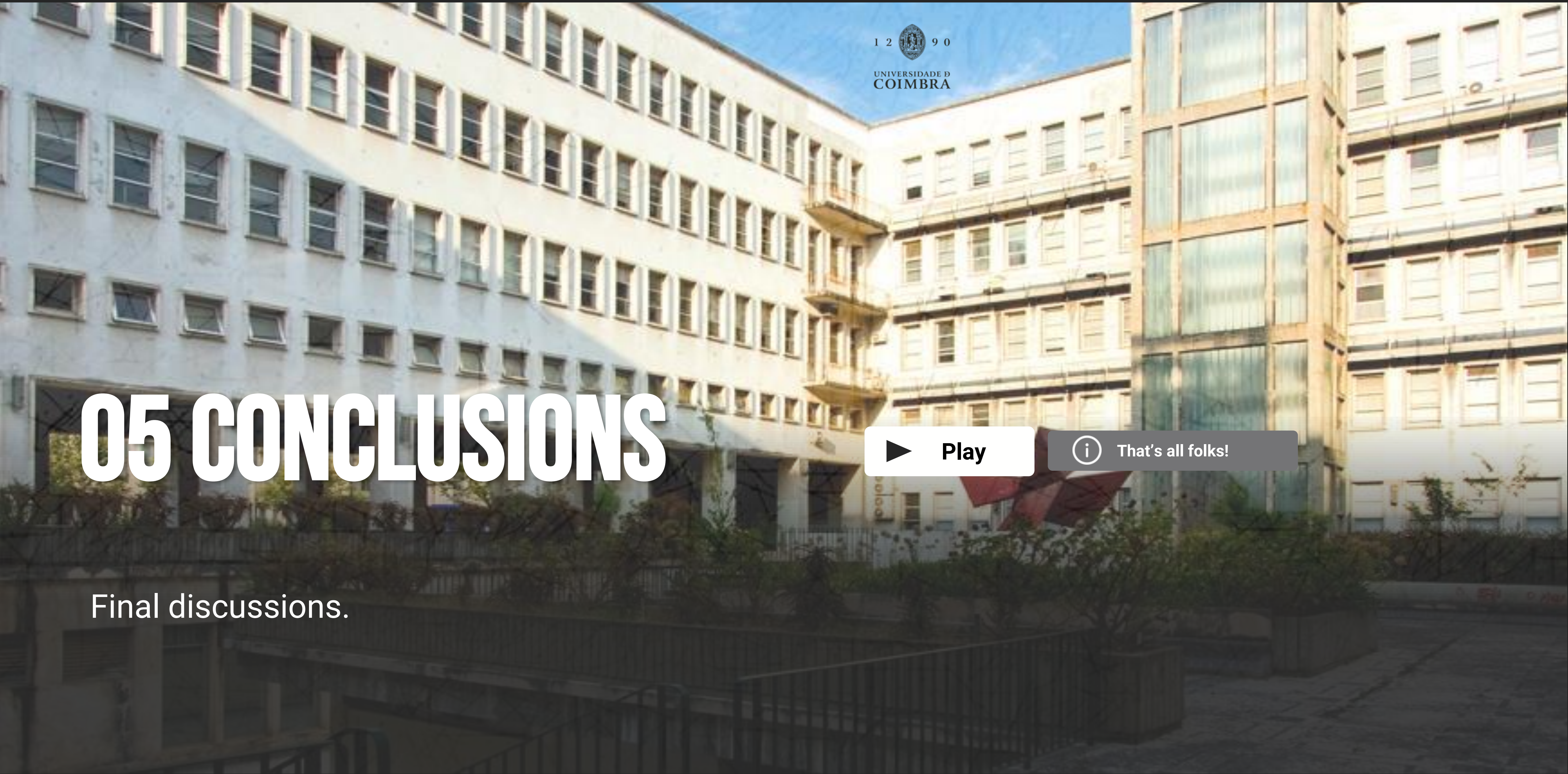
$$\gamma_1 = \frac{\lambda}{2} a \frac{\partial}{\partial \lambda} A_1$$

$$\gamma_2 = a\beta_1 A_1 + \frac{\lambda}{2}\frac{\partial}{\partial \lambda}\left(A_2 - \frac{a^2 A_1^2}{2}\right) + \frac{1}{2}\gamma_A^{(1)} A_1$$

$$\gamma_1 e^2 = \frac{e^2}{16\pi^2}$$

$$\gamma_2 e^4 = -\frac{1}{3}\frac{e^4}{2(4\pi)^4}$$

IREG 05 Conclusions Supported by: CAPES Carolina Perdomo



05 CONCLUSIONS

Play

That's all folks!

Final discussions.

PERSPECTIVES: THE ANOMALOUS MASS DIMENSION AT 2-LOOP

1

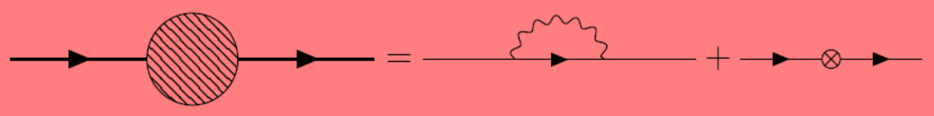
To pursue the IREG program to apply it to precision calculations, we need to evaluate the anomalous dimensions for massive gauge theories.

2

$$\gamma_m = \lambda \frac{\partial \ln Z_m}{\partial \lambda}$$

3

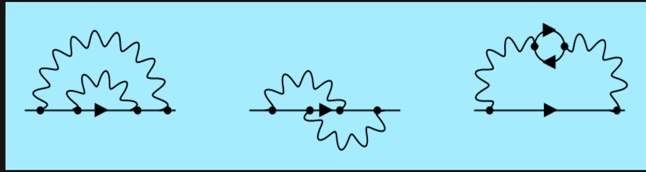
At 1-loop:



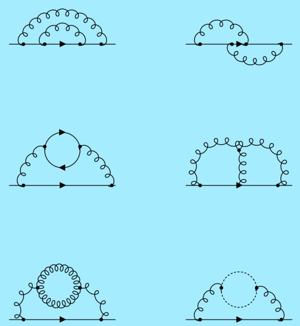
$$= \Sigma^{(1)}(p,m) + \delta_\Psi^{(1)} \not{p} - (\delta_m^{(1)} + \delta_\Psi^{(1)})m$$

4

Same relevant diagrams:



QED



QCD

CONCLUSIONS

1

The approach we used to handle the UV-div is through the technique of IREG.

2

We calculated the β-function to 2-loop order for YM, QED and QCD in a quadri-dimensional framework.

3

We have obtained Z_A by conducting the subtraction of subdivergences within IREG and compared with CDR and DRED.

4

Evaluating the BDI's of IREG in 4-2ε dimensions at the end of the calculation does not yield the same residues for the DS poles of arbitrary orders.

CONCLUSIONS

5

We calculate the γ-function to 2-loop order for QED.

6

We developed two algorithms to automatize the computation of the divergent amplitudes in Mathematica.

7

We study the subtleties of the calculation in the presence of gluonic and fermionic loops within IREG.

THANKS!

Do you have any questions?

https://www.linkedin.com/in/c-arias-perdomo/

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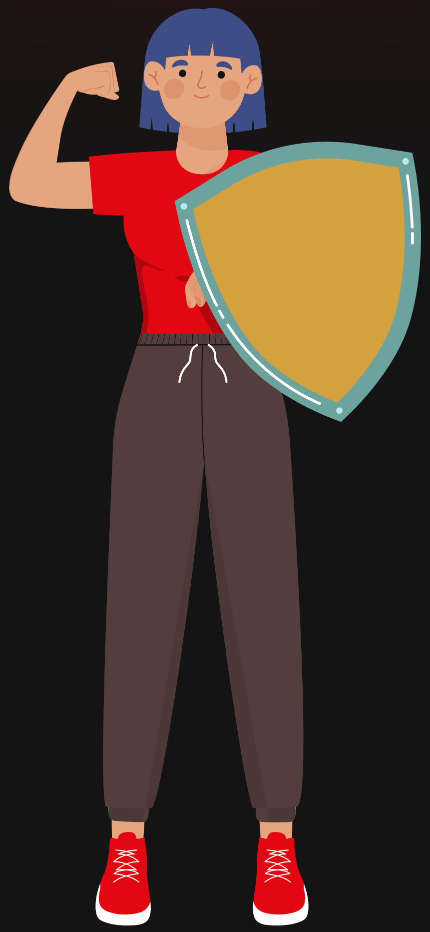
BACKUP SLIDES



More references:

MAIN REFERENCES

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THE EUROPEAN PHYSICAL JOURNAL C

Regular Article - Theoretical Physics

To d, or not to d: recent developments and comparisons of regularization schemes

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Abstract We give an introduction to several regularization schemes that deal with ultraviolet and infrared singularities appearing in higher-order computations in quantum field theories. Comparing the computation of simple quantities in the various schemes, we point out similarities and differences between them.

Contents

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Springer

It’s complex people:

Understanding the difficulties

01

Ultraviolet (UV-div) and **infrared** (IR-div) divergences are all-over beyond leading order in S-matrix calculations and must be wisely removed in order to automated computation codes for the evaluation of Feynman amplitudes.

02

A general cross-section in QCD usually includes short and long-distance behavior, and thus it is not computable directly in perturbation theory.

$$\mathcal{M}_n\left(\frac{\mu}{\lambda_{IR}},\frac{p_i}{\lambda_{UV}},\alpha_s(\lambda_{UV})\right)=Z\left(\frac{\mu}{\lambda_{IR}},\frac{p_i}{\lambda_{IR}},\alpha_s(\lambda_{IR})\right)H_n\left(\frac{p_i}{\lambda_{UV}},\frac{\lambda_{IR}}{\lambda_{UV}},\alpha_s(\lambda_{UV})\right)$$

Quadratic terms:

QUADRATIC TERMS AND IREG

$$0 = \left[\int d^n k\,(f(k+p) - f(k))\right]^R = p_\nu \left[\int d^n k \frac{\partial}{\partial k_\nu} f(k)\right]^R + \mathcal{O}(p^2)$$

$$[aF + bG]^R = a[F]^R + b[G]^R$$

$$\begin{aligned} f_{\mu\nu} &= \int d^4k \frac{\partial}{\partial k_\mu} \frac{k_\nu}{(k^2 - m^2)} \\ &= \int d^4k \left(\frac{g_{\mu\nu}}{k^2 - m^2} - 2\frac{k_\mu k_\nu}{(k^2 - m^2)^2} \right) \end{aligned}$$

$$I_{\mu\nu} = \int d^4k \frac{k_\mu k_\nu}{(k^2 - m^2)^2}$$

$$\begin{aligned} [I_{\mu\nu}]^R &= \frac{1}{2} g_{\mu\nu} \left[\int d^4k \frac{1}{k^2 - m^2}\right]^R \\ &= \frac{1}{2} g_{\mu\nu} \left(\left[\int d^4k \frac{k^2}{(k^2 - m^2)^2}\right]^R - \left[\int d^4k \frac{m^2}{(k^2 - m^2)^2}\right]^R \right) \\ &= \frac{1}{2} g_{\mu\nu} \left([I_{\alpha\alpha}]^R - \left[\int d^4k \frac{m^2}{(k^2 - m^2)^2}\right]^R \right). \end{aligned}$$

$$g_{\mu\nu}[I_{\mu\nu}]^R = [g_{\mu\nu} I_{\mu\nu}]^R - \left[\int d^4k \frac{m^2}{(k^2 - m^2)^2}\right]^R$$

Phi-4:

$$\mathcal{L}_{\phi^4} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}m^2\phi^2 - \underbrace{\frac{g}{4!}\phi^4}_{\text{interaction term}},$$

$$\phi_0 = Z_\phi^{1/2}\phi_R$$

$$m_0^2 = Z_m m_R^2$$

$$g_0 = Z_g g_R$$

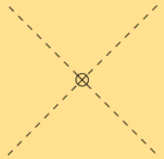
$$\mathcal{L}_{\phi_R^4} = \frac{1}{2}(\partial_\mu Z_\phi^{1/2}\phi_R)^2 - \frac{1}{2}Z_m m_R^2 (Z_\phi^{1/2}\phi_R)^2 - \frac{Z_g g_R}{4!}(Z_\phi^{1/2}\phi_R)^4$$

$$\mathcal{L}_{\phi^4_R} = \frac{1}{2}(\partial_\mu Z_\phi^{1/2}\phi_R)^2 - \frac{1}{2}Z_m m_R^2 (Z_\phi^{1/2}\phi_R)^2 - \frac{Z_g g_R}{4!}(Z_\phi^{1/2}\phi_R)^4$$

$$\left.\begin{aligned} Z_\phi &= 1 + A \\ Z_\phi Z_m &= 1 + B \\ Z_\phi^2 Z_g &= 1 + C \end{aligned}\right\} \text{Counterterms}$$

$$\mathcal{L}_{\phi^4_R} = \underbrace{\frac{1}{2}(\partial_\mu\phi_R)^2 - \frac{1}{2}m_R^2\phi_R^2}_{\text{free Lagrangian}} - \underbrace{\frac{g_R}{4!}\phi_R^4}_{\text{interaction term}} + \underbrace{\frac{1}{2}A(\partial_\mu\phi_R)^2 - \frac{1}{2}Bm_R^2\phi_R^2 - \frac{g_R}{4!}C\phi_R^4}_{\text{counterterms Lagrangian}}$$

$$- - - - \otimes - - - - - = i(Ap^2 - Bm_R^2)$$



$$= -ig_R C$$

RGF:

There are many other ways to measure α_s, such as from the hadronic decay rate of the τ lepton, from deep inelastic scattering, lattice calculations, multijet rates, event shapes, etc. In each of these measurements, α_s is extracted from physical quantities. However, α_s is only defined within some regularization and subtraction scheme, so some convention must be chosen to make comparisons between these extractions useful. In particular, since α_s

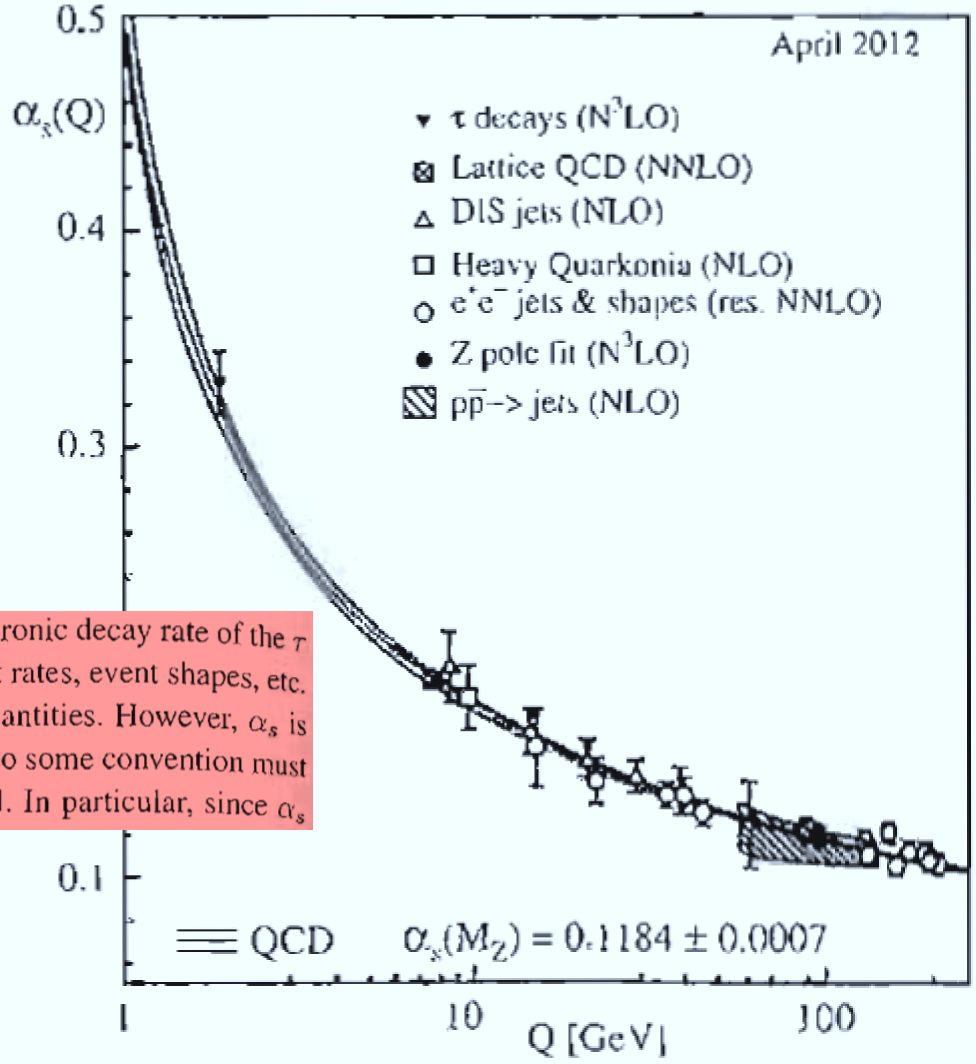


Fig. 26.1 Running coupling and data. The best fit value for the $\overline{\text{MS}}$ strong coupling constant is $\alpha_s(m_Z) = 0.1184 \pm 0.0007$. Image from [Particle Data Group (Beringer *et al.*), 2012].

The renormalization group equations

Coupling constant

- Green's functions for the bare ϕ⁴ Lagrangian: $$\Gamma_0^{(n)}(p_1,\ldots,p_n;m_0,g_0)$$

- Green's functions for the ϕ⁴ counterterm Lagrangian: $$\Gamma^{(n)}(p_1,\ldots,p_n;m_R(\lambda),g_R(\lambda),\lambda)$$

Scaled parameter

Still, they are the same theory.

- Both of them are the same Green's function related by:

$$\underbrace{\Gamma_0^{(n)}(p_1,\ldots,p_n;m_0,g_0)}_{\text{independent of the scaled parameter}} = \underbrace{Z_\Gamma^{-1}}_{\text{Renor. const.}}\underbrace{\Gamma^{(n)}(p_1,\ldots,p_n;m_R(\lambda),g_R(\lambda),\lambda)}_{\text{dependent of the scaled parameter}}$$

$$\underbrace{\Gamma_0^{(n)}(p_1,\ldots,p_n;m_0,g_0)}_{\text{independent of the scaled parameter}} \Rightarrow \lambda\frac{d\Gamma_0^{(n)}}{d\lambda}=0$$

$$\lambda\frac{d}{d\lambda}[Z_\Gamma^{-1}\Gamma^{(n)}(p_1,\ldots,p_n;m_R(\lambda),g_R(\lambda),\lambda)]=0$$

$$Z_\Gamma^{-1}=\left[-\lambda\frac{d\ln Z_\Gamma}{d\lambda}+\lambda\frac{\partial g_i}{\partial\lambda}\frac{\partial}{\partial g_i}+\lambda\frac{\partial m_i}{\partial\lambda}\frac{\partial}{\partial m_i}+\lambda\frac{\partial}{\partial\lambda}\right]\Gamma^{(n)}=0$$

β-function γm-function

$$\boxed{Z_\Gamma^{-1} = \left[-\lambda\frac{d\ln Z_\Gamma}{d\lambda} + \lambda\frac{\partial g_i}{\partial\lambda}\frac{\partial}{\partial g_i} + \lambda\frac{\partial m_i}{\partial\lambda}\frac{\partial}{\partial m_i} + \lambda\frac{\partial}{\partial\lambda}\right]\Gamma^{(n)} = 0}$$

These partial derivatives equations are the **renormalization group equations** for renormalized Green functions. The whole group simply tell us about the scale dependence of the parameters in a renormalizable field theory.

THE RENORMALIZATION PROGRAM IN THE BACKGROUND FIELD METHOD

β-function

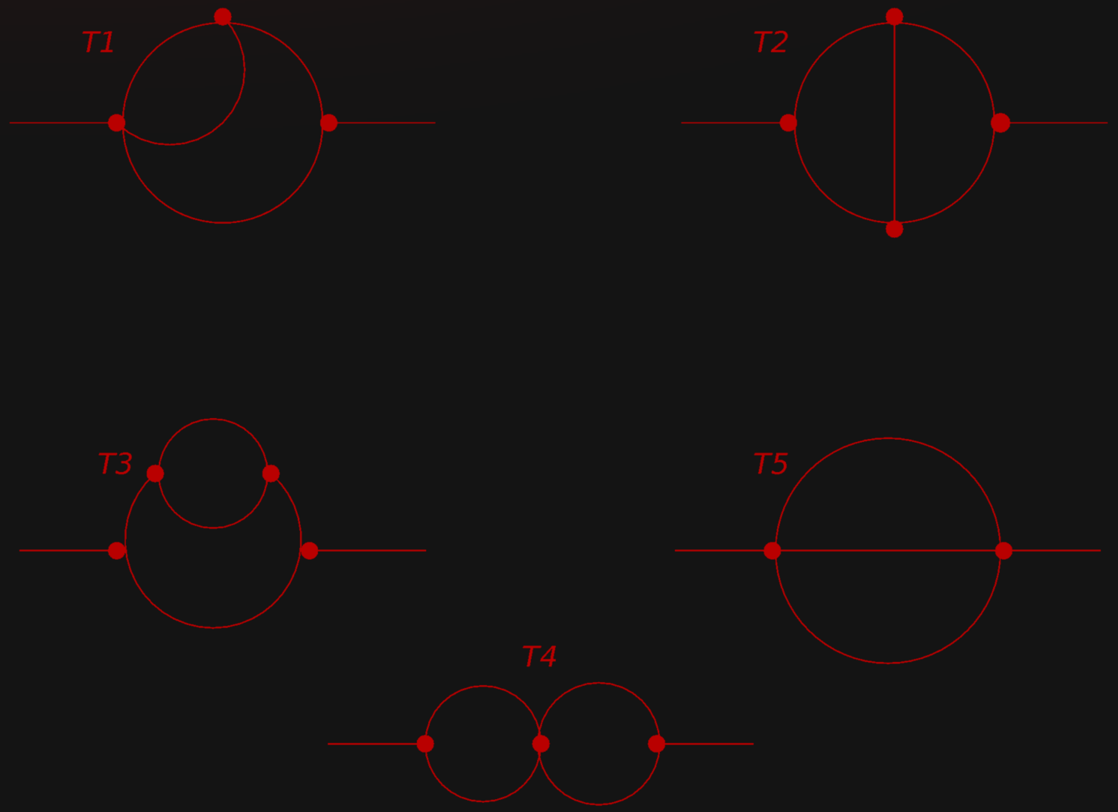
$$\beta = \lambda \frac{\partial}{\partial \lambda} g_R$$

The approach of the BFM is to do a “field-shifting”: the BF is added to que quantum field in the action (S=QF+BF). After this, the method allows to a fix a gauge and evaluate the quantum corrections without breaking the background gauge symmetry.

THE RENORMALIZATION PROGRAM IN THE BACKGROUND FIELD METHOD

We need the 2-point functions as the Abelian theories!

$$\boxed{Z_g = Z_{\hat{A}}^{-1/2}}$$



THE RENORMALIZATION GROUP FUNCTIONS IN IREG AND DS

β-FUNCTION

$$\beta = \lambda \frac{\partial}{\partial \lambda} g_R$$

$$Z_A = 1 + A_1 \tilde{g_R}^2 + A_2 \tilde{g_R}^4$$

$$\beta = \frac{g_R}{2} \lambda \frac{\partial}{\partial \lambda} \left[A_1 \tilde{g_R}^2 + \left(A_2 - \frac{A_1^2}{2} \tilde{g_R}^4 \right) \right]$$

THE RENORMALIZATION GROUP FUNCTIONS IN IREG AND DS

β-FUNCTION IREG

$$\beta = -g_R\left[-\frac{g_R^2}{2}\lambda\frac{\partial}{\partial\lambda}A_1 - \frac{g_R^4}{2}\lambda\frac{\partial}{\partial\lambda}A_2\right]$$

β –FUNCTION DS

$$\beta = \frac{d-4}{2}g_R\left[A_1\tilde{g}_R^2 + 2A_2\tilde{g}_R^4\right]$$

THE RENORMALIZATION GROUP FUNCTIONS IN IREG AND DS

γ-FUNCTION

$$\gamma = \frac{\lambda}{2}\frac{\partial \ln Z_\psi}{\partial \lambda}$$

We will talk about it in a few slides.

γm-FUNCTION

$$\gamma_m^{\overline{DS}}(\mu) = \frac{\mu}{m^{\overline{DS}}}\frac{\partial m^{\overline{DS}}}{\partial \mu} = -\beta^{\overline{DS}}(\mu)\frac{\partial \ln Z_m^{\overline{DS}}}{\partial g^{\overline{DS}}}$$

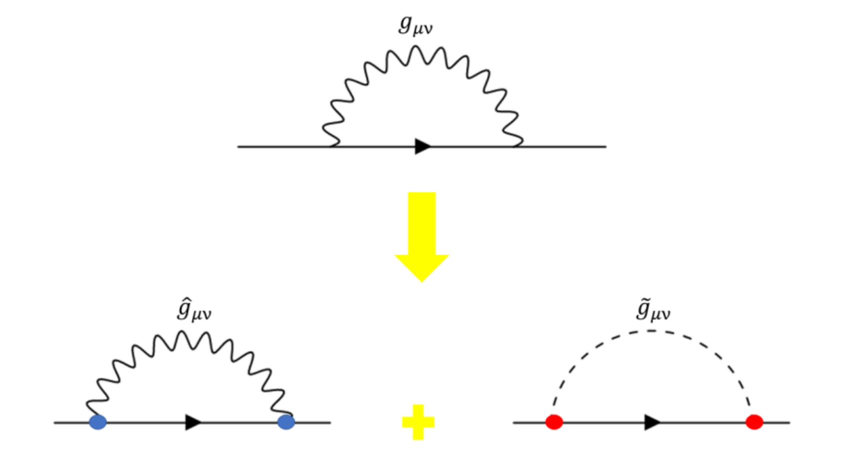
$$\gamma_m^{\overline{IREG}}(\lambda^2) = 2\frac{\lambda^2}{m^{\overline{IREG}}}\frac{\partial m^{\overline{IREG}}}{\partial \lambda^2} = -\beta^{\overline{IREG}}(\lambda^2)\frac{\partial \ln Z_m^{\overline{IREG}}}{\partial g^{\overline{IREG}}}$$

DRED:

TRADITIONAL DIMENSIONAL SCHEMES (DS)

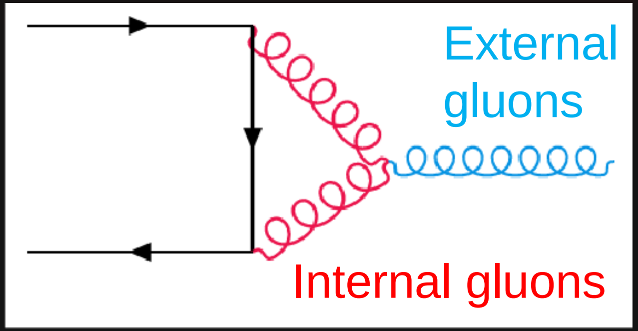
- Dimensional regularization (CDR) analytically continues the integral into d=4-2ε.
- Alternative schemes to CDR have been developed, such as dimensional reduction (DRED).
- In DRED, we split the boson fields ¨4¨ into a boson space “d” plus a boson space N=2ε.

$$\mathcal{L}^{(4)} = \mathcal{L}^{(d)} + \mathcal{L}^{(2\epsilon)}$$

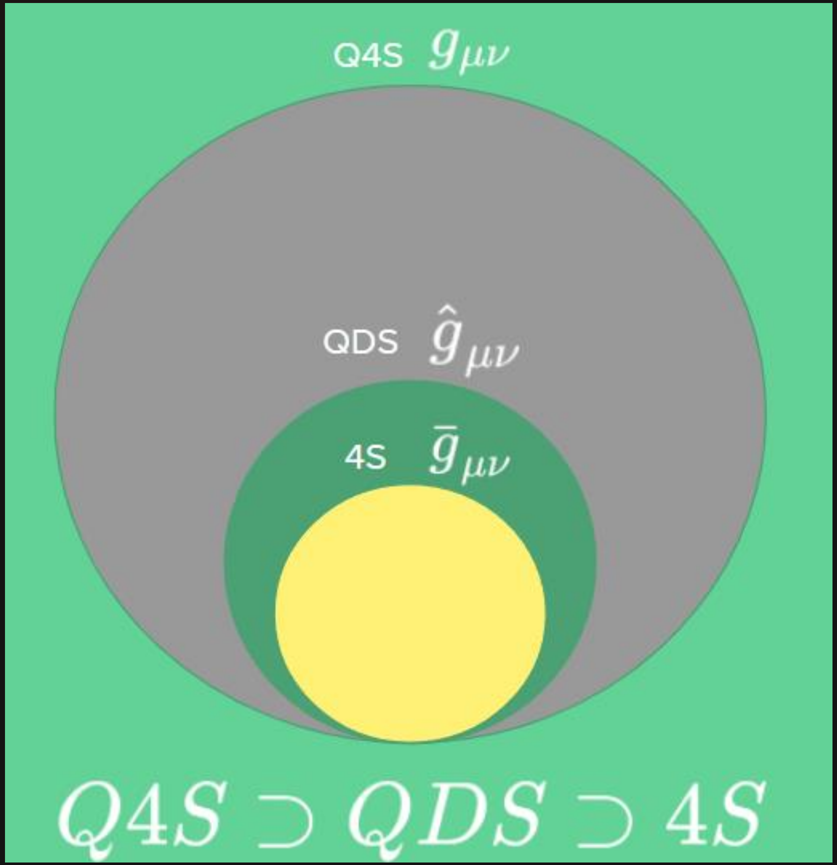


TRADITIONAL DIMENSIONAL SCHEMES (DS)

$$Q4S = \underbrace{QDS}_{4\text{-}2\epsilon} + \underbrace{Q(2\epsilon)S}_{2\epsilon},$$



	CDR	DRED
Internal gluon	$\hat{g}_{\mu\nu}$	$g^{\mu\nu}$
External gluon	$\hat{g}_{\mu\nu}$	$g^{\mu\nu}$



Dimensional Reduction (DRED)

- Modifications on CDR can lead to additional terms at the Lagrangian level, such as the **ε-scalar** particles.

$$\mathscr{A}_4 = \underbrace{\mathscr{A}_d}_{4\text{-}2\epsilon} + \underbrace{\mathscr{A}_N}_{2\epsilon} \Rightarrow \mathcal{L}^{(4)} = \mathcal{L}^{(d)} + \mathcal{L}^{(2\epsilon)}$$

$$\begin{aligned}\mathscr{L}^{(\varepsilon)} = & -\tfrac{1}{2}(\partial^i \mathscr{G}^\rho_a)^2 + g f_{abc}(\partial^i \mathscr{G}^\rho_a) G_{bi} \mathscr{G}_{c\rho} - g (T_a)_{kl} \mathscr{G}^\rho_a \bar{\psi}_k \gamma_\rho \psi_l \\ & -\tfrac{1}{2} g^2 f_{abc} f_{ade} G^i_b \mathscr{G}^\rho_c G_{di} \mathscr{G}_{e\rho} \\ & -\tfrac{1}{4} g^2 f_{abc} f_{ade} \mathscr{G}^\rho_b \mathscr{G}^\sigma_c \mathscr{G}_{d\rho} \mathscr{G}_{e\sigma}. \end{aligned} \qquad (2.8)$$

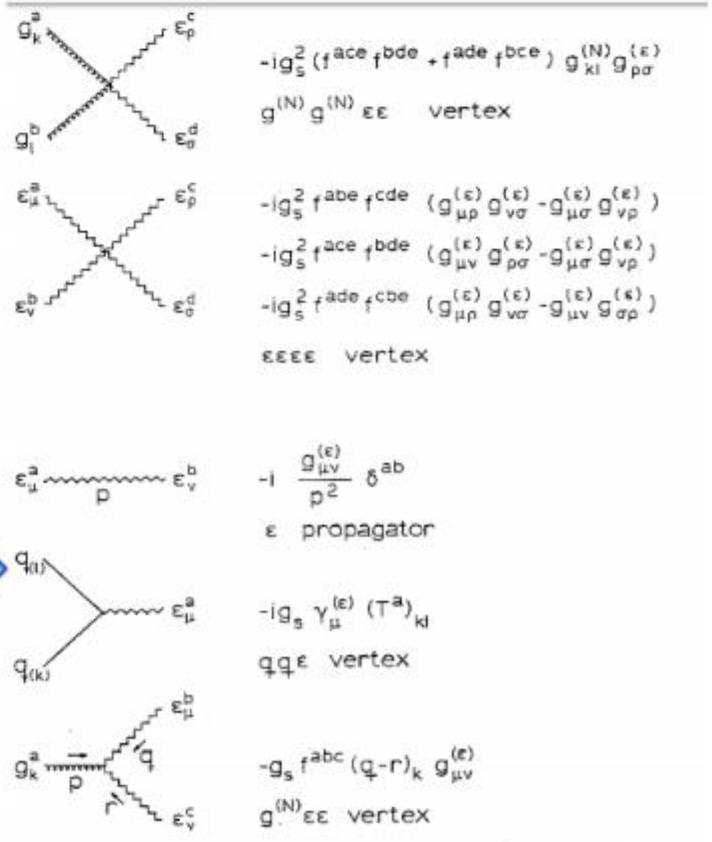


Fig. 1. Feynman rules for ε-scalars (all momenta flow into the vertices)

Körner, J.G., Tung, M.M. Dimensional reduction methods in QCD. *Z. Phys. C - Particles and Fields* 64, 255–265 (1994). https://doi.org/10.1007/BF01557396

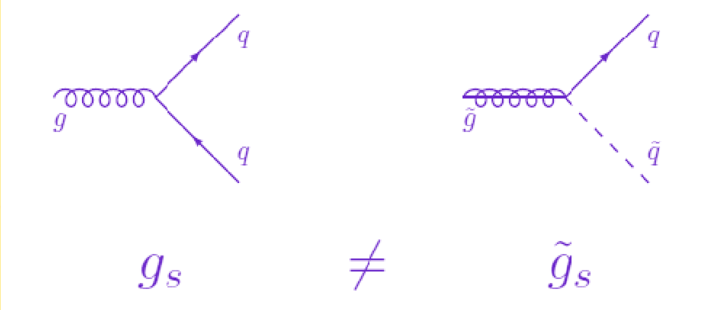
Reason

[S.P. Martin and M.T. Vaughn, Phys. Lett. B 318 331 (1993)]

$N_{spin1} = D$

$N_{spin1/2} = 2^\wedge(D/2)$

$$\boxed{Z_g \neq \tilde{Z}_g}$$



BFM:

THE RENORMALIZATION PROGRAM IN THE BACKGROUND FIELD METHOD

- We redefine a new set of variables:

$$\hat{A}_o = Z_{\hat{A}} \hat{A}_r; \quad g_o = Z_g g_r; \quad \alpha_o = Z_\alpha \alpha_r$$

- Q field doesn't need to be renormalized (only appears in loops).

- Infinities in the action must take the gauge invariant form of a divergent constant <u>times</u> the gauge field strength tensor.

$$F^a_{\mu\nu} F^{a\mu\nu} = Z_A^{1/2} [\partial_\mu A^a_\nu - \partial_\nu A^a_\mu + \underbrace{Z_g Z_{\hat{A}}^{1/2}}_{\boxed{Z_g = Z_{\hat{A}}^{-1/2}}} g f^{abc} A^b_\mu A^c_\nu]$$

Superficial degree of divergences:

Superficial degree of UV-div divergence

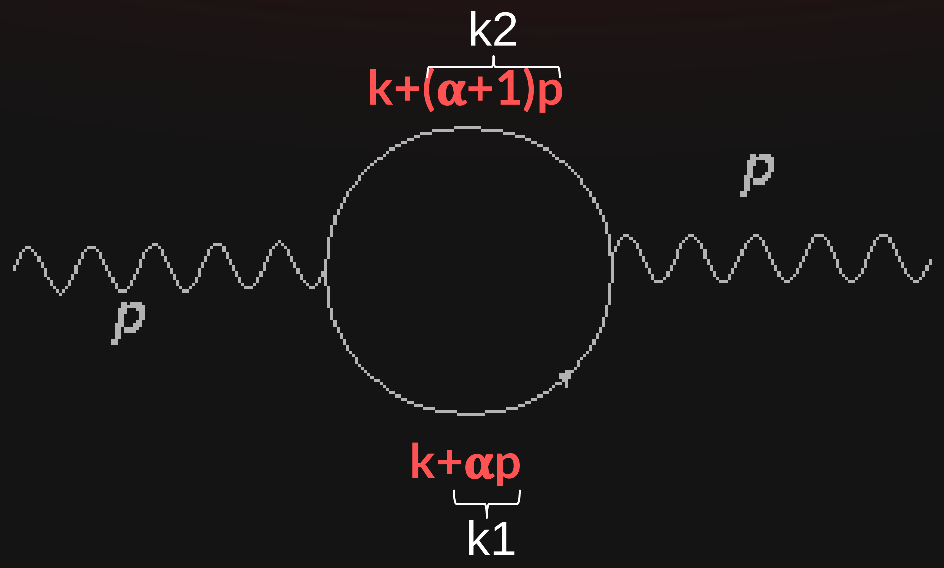
$$\Delta = dl - 2p$$

$$l = \underbrace{p}_{p = \frac{nv - N}{2}} - v + 1$$

$$\Delta = d + \left[n \left(\frac{d-2}{2} \right) - d \right] v - \left(\frac{d-2}{2} \right) N$$

Momentum routing invariance:

IREG-MOMENTUM ROUTING INVARIANCE IN A FEYNMAN DIAGRAM



α

Arbitrary Routing

MRI $\longleftrightarrow$ ST=0 $\longleftrightarrow$ Gauge invariance

Momentum Routing Invariance and Gauge Invariance

$\boxed{k_1 - k_2 = p}$

Final result:

$$\Pi_{\mu\nu} = \tilde{\Pi}_{\mu\nu} + 4\Big(\boxed{\Upsilon^{2}_{\mu\nu}} - \frac{1}{2}(k_1^2 + k_2^2)\boxed{\Upsilon^{0}_{\mu\nu}} + \frac{1}{3}(k_1^\alpha k_1^\beta + k_2^\alpha k_2^\beta + k_1^\alpha k_2^\beta)\boxed{\Upsilon^{0}_{\mu\nu\alpha\beta}}$$

$$- (k_1 + k_2)^\alpha (k_1 + k_2)_\mu \boxed{\Upsilon^{0}_{\nu\alpha}} - \frac{1}{2}(k_1^\alpha k_1^\beta + k_2^\alpha k_2^\beta) g_{\mu\nu} \boxed{\Upsilon^{0}_{\alpha\beta}}\Big) \quad \text{where}$$

$\boxed{\Upsilon = 0 \quad \mathrm{MRI}}$

It depends explicitly of k1-k2 -> routing invariance

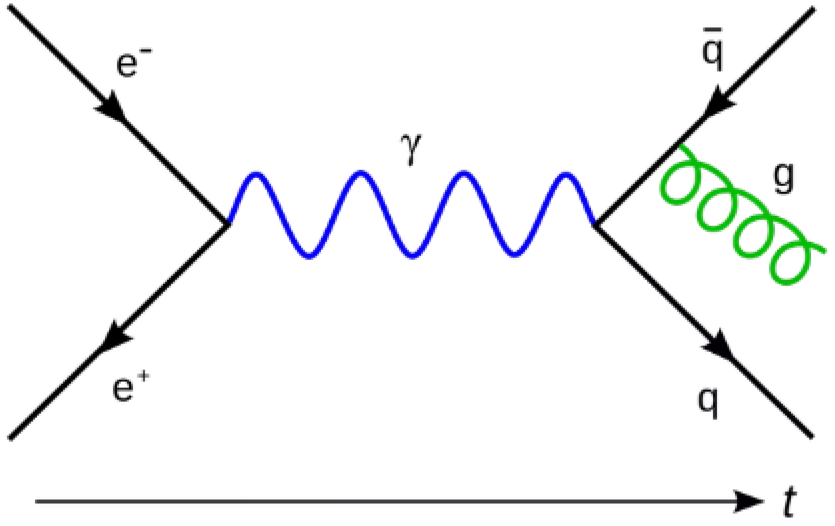
$$\tilde{\Pi}_{\mu\nu} = \frac{4}{3}\left((k_1 - k_2)^2 g_{\mu\nu} - (k_1 - k_2)_\mu (k_1 - k_2)_\nu\right) \underbrace{\left(I_{log}(\mu^2) - \frac{i}{(4\pi)^2}\left(\frac{5}{3} + \ln\frac{-(k_1-k_2)^2}{\mu^2}\right)\right)}_{\text{IR diverg. eliminated}}$$

Scale relation

Ward Identity: $$(k_1 - k_2)_\mu \Pi_{\mu\nu} = 0$$ **(massless photon)**

IR-div:

IR-div: example



$$\boxed{\int d\Phi_n |\mu|^2}$$

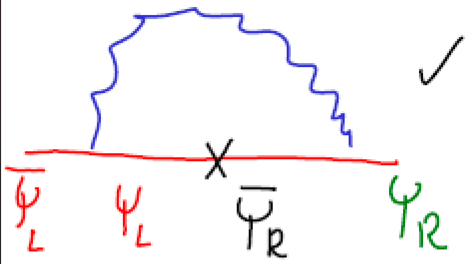
- The gluon emitted in the process has a very low energy, so low that it will not be detected in the detector.
- Even upgrading the detector, it still has an energy limit that it cannot detect.
- At the level of theoretical calculation, this physical limit is not being considered. If so, you would be able to detect any gluon at any energy, no matter how low.
- These divergences are removed in the observables automatically. We don't need to renormalize them.

Mass insertion:

The mass insertion

The electron mass anomalous dimension can be found in a similar way. We can calculate $\Sigma_S(p^2)$ at two loops (neglecting m^2) by retaining m_0 in the numerator of a single electron propagator in Fig. 4.13 and setting $m_0 \to 0$ in all the other places. This single m_0 has to be somewhere along the electron line which goes through all the diagrams, not in the electron loop in the first diagram: we need one helicity flip of the external electron, and one helicity flip in a loop yields zero contribution. Then we extract Z_m from (2.86). It must be gauge-invariant, because $m(\mu)$ is gauge-invariant. The result is

– Grozin



✓

$\bar\psi \not{A} \psi + \underline{m}\, \bar\psi \psi$

$\psi \to \psi_L + \psi_R$

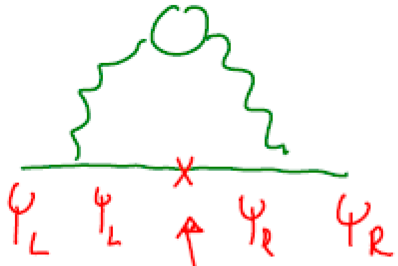
$\bar\psi_L \psi_R$

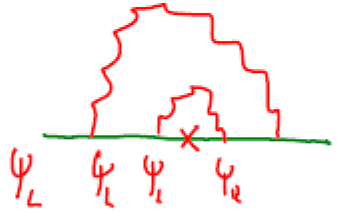
$\bar{Q}_L H e_R$

$\{ g \bar\psi_L \not{A} \psi_L + g \bar\psi_R \not{A} \psi_R$

P_L , P_R

$m \bar\psi_L \psi_R + m \bar\psi_R \psi_L$





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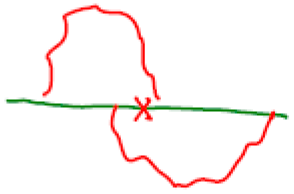

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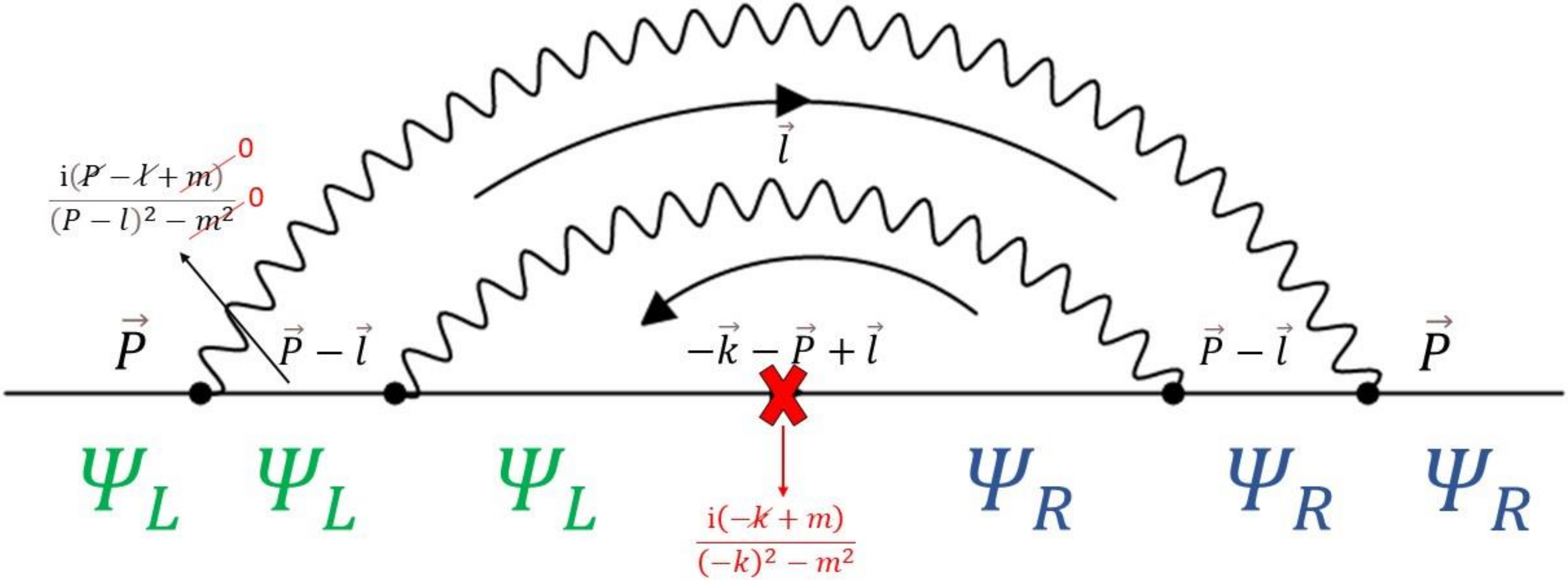
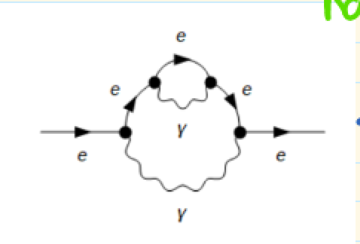
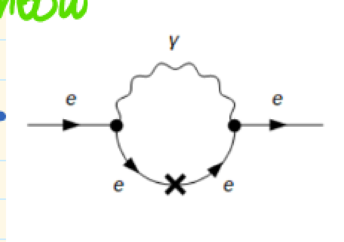

$\frac{i(\cancel{P} - \cancel{l} + \cancel{m}^{0})}{(P-l)^2 - \cancel{m^2}^{0}}$
$\vec{l}$
$\vec{P}$
$\vec{P} - \vec{l}$
$-\vec{k} - \vec{P} + \vec{l}$
$\vec{P} - \vec{l}$
$\vec{P}$
Ψ_L
Ψ_L
Ψ_L
$\frac{i(-\cancel{k} + m)}{(-k)^2 - m^2}$
Ψ_R
Ψ_R
Ψ_R

Almost there!

Rainbow



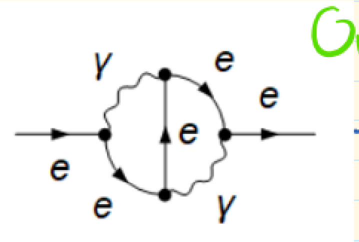
$+$



$$= -\frac{1}{2}\, i\, b\, \mathrm{e}^4 \left(\left(2\,\mathrm{Ilog2}\lambda 2 - 3\,\mathrm{Ilog}\lambda 2\right) \overline{\gamma}\cdot\overline{p} + 32\, m_e \left(\mathrm{Ilog2}\lambda 2 - 2\,\mathrm{Ilog}\lambda 2\right)\right)$$

$+$

Overlapped



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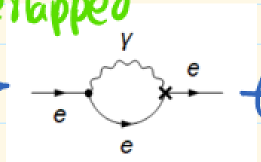

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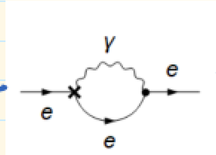
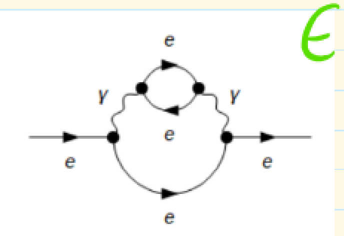


$$= i\, b\, \mathrm{e}^4 \left(\left(2\,\mathrm{Ilog2}\lambda 2 - 3\,\mathrm{Ilog}\lambda 2\right) \overline{\gamma}\cdot\overline{p} - 4\,\mathrm{Ilog}\lambda 2\, m_e\right)$$

$+$

Electron-Bubble



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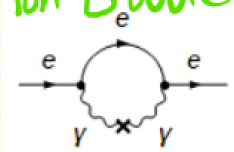


$$= \frac{2}{3}\, i\, b\, \mathrm{e}^4 \left(\mathrm{Ilog}\lambda 2\, \overline{\gamma}\cdot\overline{p} + 2\, m_e \left(3\,\mathrm{Ilog2}\lambda 2 - 5\,\mathrm{Ilog}\lambda 2\right)\right)$$

$$= ibe^4\left[I\log^{(2)}(\lambda^2) - \frac{5}{6} I\log(\lambda^2)\right]\not{p} + ibe^4\left[-12\, I\log^{(2)}(\lambda^2) + \frac{64}{3} I\log(\lambda^2)\right] m$$

Others:

Amplitude calculations

Workflow

1. Amplitude generation, for example in terms of Feynman diagrams.

2. Bringing the amplitude to a convenient form.

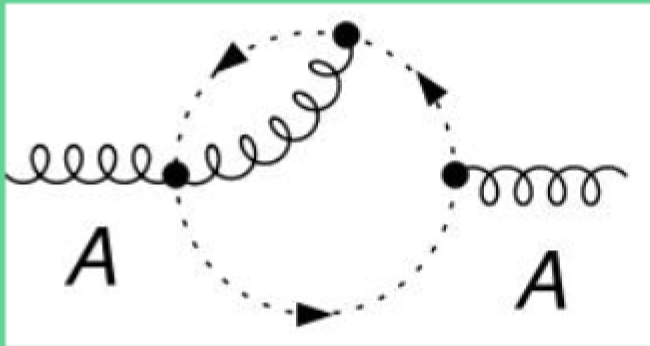
3. Reduction to master integrals.

4. Calculation of the master integrals.

5. Evaluation of the amplitude.

IREG

(a)



Apart from minus a sign and b, they have the same coefficient

Non-local term

$$\rho_{IREG} = I_{log}(\Lambda^2)\ln\left[-\frac{p^2}{\Lambda^2}\right]$$

$$\frac{ig_s^4 C_A^2 \delta^{ab}}{(4\pi)^4}\left(\boxed{\frac{1}{3b}} I_{log}^{(2)}(\Lambda^2) \boxed{-\frac{1}{3b^2}} I_{log}^2(\Lambda^2) + \boxed{\frac{1}{3b}}\rho_{IREG}\right)\left[g_{\mu\nu}p^2 - p_\mu p_\nu\right] +$$

$$\boxed{\left[\frac{-29}{18b} I_{log}(\Lambda^2) g_{\mu\nu}p^2 - \frac{-17}{18b} I_{log}(\Lambda^2) p_\mu p_\nu\right]}$$

Not transverse individually, just in the sum of all diagrams

Was this correlation expected? yes

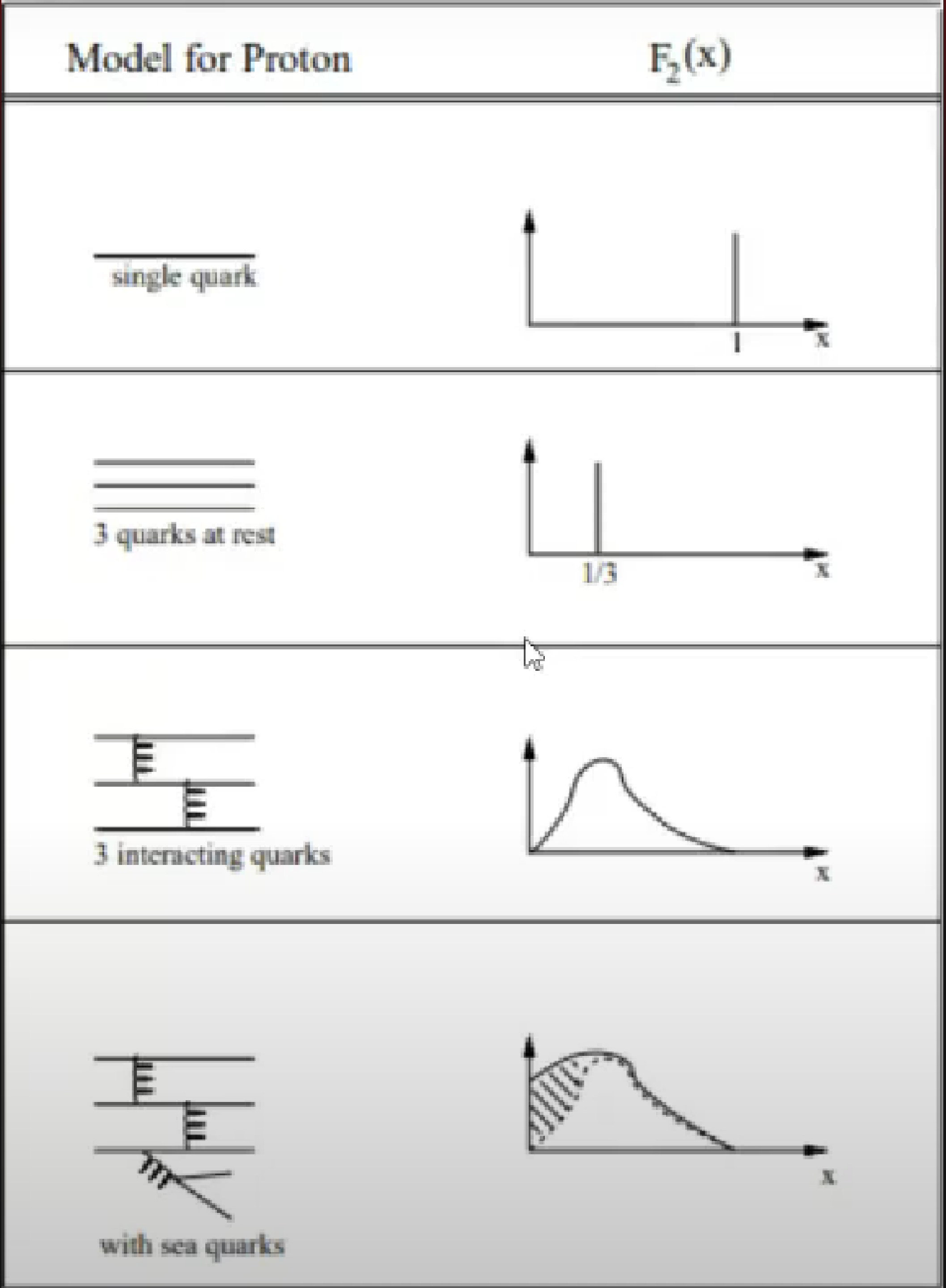
$$\frac{d}{d\mu^2} I_{log}(\mu^2) = \int_k \frac{2}{(k^2-\mu^2)^3}$$

$$\int_k \frac{d^dk}{(2\pi)^d} \frac{1}{(k^2+2kQ-M^2)^\alpha} = i\frac{(-1)^\alpha}{(4\pi)^{d/2}} \frac{\Gamma(\alpha - d/2)}{\Gamma(\alpha)} \frac{1}{(Q^2+M^2)^{\alpha - d/2}}$$

$$\frac{d}{d\mu^2} I_{log}(\mu^2) = -\frac{b}{\mu^2}$$

$$\int_{\lambda^2}^{\mu^2} dI_{log}(\mu^2) = -b \int_{\lambda^2}^{\mu^2} \frac{d(\mu^2)}{\mu^2}$$

$$I_{log}(\mu^2) = I_{log}(\lambda^2) + b \ln\left(\frac{\lambda^2}{\mu^2}\right)$$



Model for Proton
$F_2(x)$
single quark
1
x
3 quarks at rest
1/3
x
3 interacting quarks
x
with sea quarks
x

Klein-Gordon propagator

$$S_0 = \frac{1}{2}\int \frac{d^4k}{(2\pi)^4}\Big[-\widetilde{\varphi}(k)(k^2+m^2)\widetilde{\varphi}(-k) + \widetilde{J}(k)\widetilde{\varphi}(-k) + \widetilde{J}(-k)\widetilde{\varphi}(k)\Big], \quad (8.7)$$

$$\widetilde{\chi}(k) = \widetilde{\varphi}(k) - \frac{\widetilde{J}(k)}{k^2+m^2} \qquad \mathcal{D}\varphi = \mathcal{D}\chi$$

$$S_0 = \frac{1}{2}\int \frac{d^4k}{(2\pi)^4}\left[\frac{\widetilde{J}(k)\widetilde{J}(-k)}{k^2+m^2} - \widetilde{\chi}(k)(k^2+m^2)\widetilde{\chi}(-k)\right]$$

$$Z_0(0) = \langle 0|0\rangle_{J=0} = \int D\chi \exp\left[-\frac{i}{2}\int\frac{d^4k}{(2\pi)^4}\widetilde{\chi}(k)\left(k^2+m^2\right)\widetilde{\chi}(-k)\right] = 1 \quad \text{(no interactions)}$$

$$\boxed{\begin{aligned} Z_0(J) &= \exp\left[\frac{i}{2}\int\frac{d^4k}{(2\pi)^4}\,\frac{\widetilde{J}(k)\widetilde{J}(-k)}{k^2+m^2-i\epsilon}\right] \\ &= \exp\left[\frac{i}{2}\int d^4x\,d^4x'\,J(x)\Delta(x-x')J(x')\right]. \qquad (8.10)\end{aligned}}$$

$$\Delta(x-x') = \int\frac{d^4k}{(2\pi)^4}\,\frac{e^{ik(x-x')}}{k^2+m^2-i\epsilon}$$

Feynman propagator

Photon propagator

Path integral formalism

$$Z_0(J) = \int \mathcal{D}A \, e^{iS_0} \, ,$$

$$S_0 = \int d^4x \left[-\tfrac{1}{4} F^{\mu\nu} F_{\mu\nu} + J^\mu A_\mu \right] \, .$$

$$\widetilde{A}_\mu(k) = \int d^4x \, e^{-ik.x} A_\mu(x), \quad A_\mu(x) = \frac{1}{(2\pi)^4} \int d^4k \, e^{ik.x} \widetilde{A}_\mu(k)$$

$$S_0 = \frac{1}{2} \int \frac{d^4k}{(2\pi)^4} \Big[-\widetilde{A}_\mu(k) \big(\underbrace{k^2 g^{\mu\nu} - k^\mu k^\nu}_{k^2 P^{\mu\nu}} \big) \widetilde{A}_\nu(-k) + \widetilde{J}^\mu(k) \widetilde{A}_\mu(-k) + \widetilde{J}^\mu(-k) \widetilde{A}_\mu(k) \Big] .$$

Projection matrix:

...but $P^{\mu\nu} k_\nu = 0$, zero eigenvalue...not invertible

$$P^{\mu\nu}(k) \equiv g^{\mu\nu} - k^\mu k^\nu / k^2 \, .$$

The photon propagator

- The propagators determined by terms quadratic in the fields, using the Euler Lagrange equations.

$$\partial_\mu F^{\mu\nu} = \partial_\mu \partial^\mu A^\nu - \partial^\nu(\partial^\mu A_\mu) = j^\nu \equiv \left(g^{\nu\lambda}\partial^2 - \partial^\nu\partial^\lambda\right)A_\lambda$$

Choose as

$$-\frac{1}{\xi}\partial^\mu A_\mu$$

(gauge fixing)

Gauge ambiguity

$$A_\mu \to A_\mu + \partial_\mu \alpha \qquad \partial^\mu A_\mu \to \partial^\mu A_\mu + \partial^2 \alpha$$

i.e. with suitable "gauge" choice of α ("ξ" gauge) want to solve

$$\partial_\mu \partial^\mu A^\nu - (1-\frac{1}{\xi})\partial^\nu(\partial_\mu A^\mu) \equiv (g^{\nu\lambda}\partial^2 - (1-\frac{1}{\xi})\partial^\nu\partial^\lambda)A_\lambda = j^\nu$$

Now invertible … in momentum space the photon propagator is

$$\boxed{-i\left(g^{\mu\nu}p^2 - (1-\frac{1}{\xi})p^\mu p^\nu\right)^{-1} = \frac{i}{p^2}\left(-g_{\mu\nu} + (1-\xi)\frac{p_\mu p_\nu}{p^2}\right)}$$

$A_\mu(x)$ gauge field

↳ 4 "degrees of freedom"
(1° → 1 field)

$F_{\mu\nu}(x)$ Maxwel tensor

↳ gauge invariant under

$$\boxed{\delta F_{\mu\nu} = 0}$$

(☆) ⊙J⊙

$\delta A_\mu = \partial \lambda(x)$
(variation of gauge potential is the derivative of a gauge parameter)

$\mathcal{L}_M = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$

↳ describe only 2 propagating "degrees of freedom"
(the polarization of EM)

↳ also is a gauge invariant operator.

Ref: for what is gauge-> "Quantum Field Theory for the Gifted Amateur" (chapter 14)

Ref: for the degrees of freedom of QED Lagrangian ->

Gauge symmetry: "...a way to write down something in a covariant way which corrects the couting of the degrees of freedom..."

↓

(1) Hamiltonian approach. → breaks Lorentz invariance. Ref: Itzyson-Fuber
(2) the path integral approach.
(3) Effective action. → $\Gamma[A]$ (functional of the gauge field)

Ref: Abbot's first paper about the background field method.

[10:36]: this a crucial point also. → remember: the action is gauge invariant

$$Z[A] = \int \underbrace{\mathcal{D}_A}_{\text{measure in the configuration space}} e^{-S[A]}$$

You integrate over directions but there is not dependance in the direction of the action

"split"

$$\underbrace{\mathcal{D}A_{go}}_{\text{gauge orbit}} \underbrace{\mathcal{D}A_{\perp}}_{\text{orthogonal to gauge orbit}}$$

→ gives you something well define

the integrations here as S doesn't depend on the gauge orbit... gives an ∞ volume (that's a problem!)

one way of solving it: latice

How we find a converge way of doing previous integral? Let's add "converge factor"!

aka: let's add to the theory some "things" that makes the integral converge.
(we gonna remove that after)

$$Z[A] = \int \mathcal{D}A \; e^{-S[A] + S_{g.f}(A)}$$

$S_{g.f}$ → gauge fixing theory

]: But of course, you are adding something that modifies the theory, so you must do it in a clever way. He shows that here with an example.

Example: $S = \int d^4x \left(-\frac{1}{4} F_{\mu\nu} F^{\mu\nu}\right) = +\frac{1}{2} \int d^4x \left(A_\mu \Box A^\mu - \cancel{(\partial A)^2}\right)$

You could chose $S_{g.f}$ to cancel this term here and you end with something similar to the scalar theory.

Still you end in a problem when you do quantization

[17:57]: $[\dot{A}_\mu(\underline{x}, \tau), A_\nu(\underline{y}, \tau)] = i\hbar \underbrace{\eta_{\mu\nu}}_{\text{object not defenitively positive in Hilbert Space}} \delta^3(\underline{x} - \underline{y})$

→ Not defenitive posite norm to define your states (that is the problem).

[33:22]: Remember $(\star)$

$\circledast$?

Anti-derivation → derivation that anticommute whit fermionic fields. → $s[\text{fermionic field}] = -$

commuting

$$\delta A_\mu = \partial_\mu \lambda \xrightarrow[\text{the gauge parameter}]{\text{fermionize}} s A_\mu = \partial_\mu \underbrace{C}_{\text{anticommuting}} \rightarrow \text{Ghost}$$

analogous to (δ)

$\mathcal{J}^2 = 0$

$\mathcal{J}A\wedge A = F_A = 0$? (A)

(Gauge symmetry)

"So you see, I replaced the gauge transformation by something that is very similar. It's a gauge transformation where the gauge parameter becomes a fermionic quantity."

$$\boxed{s A_\mu = \partial_\mu C \quad \text{BRST transformation of the gauge fields}}$$

$$\int \mathcal{D}A_\mu \mathcal{D}\phi_i e^{i\int d^4x \mathcal{L}[A,\phi_i]} = \frac{1}{f(\xi)} \int \mathcal{D}\pi \mathcal{D}A_\mu \mathcal{D}\phi_i e^{i\int d^4x \mathcal{L}[A,\phi_i] - \frac{1}{2\xi}(\Box\pi - \partial_\mu A_\mu)^2}$$
$$= \left[\frac{1}{f(\xi)} \int \mathcal{D}\pi\right] \int \mathcal{D}A_\mu \mathcal{D}\phi_i e^{i\int d^4x \mathcal{L}[A,\phi_i] - \frac{1}{2\xi}(\partial_\mu A_\mu)^2},$$

$$f[A] = \int \mathcal{D}\pi \exp\left[-i\int d^4x \frac{1}{2\xi}\left(\partial_\mu A^a_\mu - \partial_\mu D_\mu \pi^a\right)^2\right], \tag{25.90}$$

so that

$$\begin{aligned}
&\int \mathcal{D}A_\mu \mathcal{D}\phi_i e^{i\int d^4x \mathcal{L}[A,\phi_i]} \\
&= \int \mathcal{D}\pi \mathcal{D}A_\mu \mathcal{D}\phi_i \frac{1}{f[A]} \exp\left(i\int d^4x \mathcal{L}[A,\phi_i] - \frac{1}{2\xi}\left(\partial_\mu A^a_\mu - \partial_\mu D_\mu \pi^a\right)^2\right) \\
&= \left[\int \mathcal{D}\pi\right] \int \mathcal{D}A_\mu \mathcal{D}\phi_i \frac{1}{f[A]} \exp\left(i\int d^4x \mathcal{L}[A,\phi_i] - \frac{1}{2\xi}\left(\partial_\mu A^a_\mu\right)^2\right),
\end{aligned} \tag{25.91}$$

Higgs Decay into $\bar{b}b$ quarks

$$\Gamma(H \to \bar{f}f) = \frac{G_F\, M_H}{4\sqrt{2}\pi}\, m_f^2(\mu)\, R^S(s = M_H^2, \mu)$$

decay width of the Higgs boson

R^S is the spectral density of the scalar correlator and is known to α_s^4 /P. Baikov, J. Kühn, K.Ch. (2006)/

$$\begin{aligned} R^S(s = M_H^2, \mu = M_H) &= 1 + 5.667\, a_s + 29.147\, a_s^2 + 41.758\, a_s^3 - 825.7\, a_s^4 \\ &= 1 + 0.2041 + 0.0379 + 0.0020 - 0.00140 \end{aligned}$$

where we set $a_s = \alpha_s/\pi = 0.0360$ (for the Higgs mass value $M_H = 125$ GeV and $\alpha_s(M_Z) = 0.118$)

$m_b(\mu = M_H)$ is to be obtained with RG running from $m_b(\mu = 10\,\text{GeV})$ and, thus, depends on β and γ_m: