Facts of life with γ_5:

Treatment of γ_5 in Dimensionally-Regularized Chiral Yang-Mills Theory. What can we learn from it?

Main reference: Bélusca-Maïto, H., Ilakovac, A., Madjor-Božinović, M. and **Stöckinger**, D. *Dimensional regularization and Breitenlohner-Maison/'t Hooft-Veltman scheme for γ5 applied to chiral YM theories: full one-loop counterterm and RGE structure. J. High Energ. Phys.* 2020, 24 (2020). https://doi.org/10.1007/JHEP08(2020)024

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Context & Motivations of main Ref.

- **Motivations of the authors:** The existence of chiral fermions is a fundamental fact of nature. There are numerous schemes in DReg for treating γ_5. In the reference, they study the application of a DS at the one-loop level in a massless chiral Yang-Mills theory.
- **The Big Picture:** Application to chiral gauge theories such as the electroweak Standard Model.
- **Problem with γ_5:** Regularisation involving γ_5 is problematic. In DS starting from the standard SM-Lagrangian and using γ_5, which does not anticommute with the other Dirac matrices γ_μ leads to "spurious anomalies" which violate chiral symmetry and hence gauge invariance.
- **Scheme they proposed for treating chiral theories:** Propose using algebraic renormalization techniques in *Dimensional Regularization (DReg)* with the *'t Hooft-Veltman-Breitenlohner-Maison scheme (BMHV)*.
- **Objectives:** Apply BMHV scheme to general chiral gauge theories without compromises and work out its properties in detail. Obtain a treatment of γ_5 and other related intrinsically 4-dimensional Lorentz objects that is consistent by construction at any loop order.
- **What they obtained?:** They obtain a symmetry-restoring counterterm to restore the breaking of the gauge and BRST invariance by the BMHV (the main complication of the method).

- **Disclaimer (what this presentation/discussion is about):** the Right-Handed(R) model and the BRST invariance of the R-model in 4-dim (section 3 of the paper).

Outline

01 **Introduction: $\gamma 5$ in 4-dim**

02 The R-Model in 4-dim

03 BRST invariance in 4-dim

04 Last comments: BMHV vs IREG

Introduction

Realistic models in 4D contains chiral fermions.

Problems do appear when attempting to extend the definition of genuinely intrinsically 4-dimensional objects, like $\gamma 5$ or Levi-Civita symbol. These two objects appear in **chiral theories**.

Dimensional method

DReg works for bosonic fields and also 4-component fermions, and also for γµ matrices.

DReg formally extends a 4-dim space-time in d=4-2ε dimensions.

Metric:

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \hat{g}_{\mu\nu}$$

Evanescent object

$$\{\gamma^\mu,\gamma^\nu\} = 2g^{\mu\nu}\mathbb{1}\,, \qquad \{\gamma^\mu,\bar{\gamma}^\nu\} = \{\bar{\gamma}^\mu,\bar{\gamma}^\nu\} = 2\bar{g}^{\mu\nu}\mathbb{1}$$
$$\{\bar{\gamma}^\mu,\hat{\gamma}^\nu\} = 0\,, \qquad \{\gamma^\mu,\hat{\gamma}^\nu\} = \{\hat{\gamma}^\mu,\hat{\gamma}^\nu\} = 2\hat{g}^{\mu\nu}\mathbb{1}$$

$\mathrm{Tr}\,\gamma^\mu = 0$ $\quad$ $\mathrm{Tr}\,\bar{\gamma}^\mu = 0$ $\quad$ $\mathrm{Tr}\,\hat{\gamma}^\mu = 0$

What about γ_5 in DReg?

Inconsistent if $\gamma 5$ anticommute.

$$\mathrm{Tr}(\gamma_5\gamma_{\mu_1}\cdots\gamma_{\mu_4})$$

$$\propto$$

$$(d-4)\,\mathrm{Tr}(\gamma_5\gamma_{\mu_1}\cdots\gamma_{\mu_4}) \xrightarrow{d\to 4} 0$$

Instead of

$$4i\epsilon_{\mu_1\mu_2\mu_3\mu_4}$$

[For more detailed information, check:
Bruque, A.M., **Cherchiglia, A.L.** & Pérez-Victoria, M. *Dimensional regularization vs methods in fixed dimension with and without γ5. J. High Energ. Phys.* 2018, 109 (2018). https://doi.org/10.1007/JHEP08(2018)109]

So, how to get consistency?

't Hooft - Veltman - Breitenlohner - Maison scheme (BMHV scheme)

- Scheme proved to be axiomatically consistent at all orders by Breitenlohner and Maison [Breitenlohner,Maison1975, Breitenlohner,Maison1977].
- Together with MS(bar) subtraction (subtracting the poles, possibly with some finite part)->Dimensional renormalization (DimRen).

So, how to get consistency?

The BMHV scheme for $\gamma5$, defines $\gamma5$ to be anticommuting with Dirac matrices in the 4-dimensional subspace, and commuting in the (2ε)-dimensional subspace:

$$\{\gamma_5, \bar{\gamma}^\mu\} = 0\,, \quad [\gamma_5, \hat{\gamma}^\mu] = 0\,,$$

$$\{\gamma_5, \gamma^\mu\} = \{\gamma_5, \hat{\gamma}^\mu\} = 2\gamma_5\hat{\gamma}^\mu\,, \quad [\gamma_5, \gamma^\mu] = [\gamma_5, \bar{\gamma}^\mu] = 2\gamma_5\bar{\gamma}^\mu$$

Otherwise $\gamma5$ keeps its usual 4-dimensional behaviour

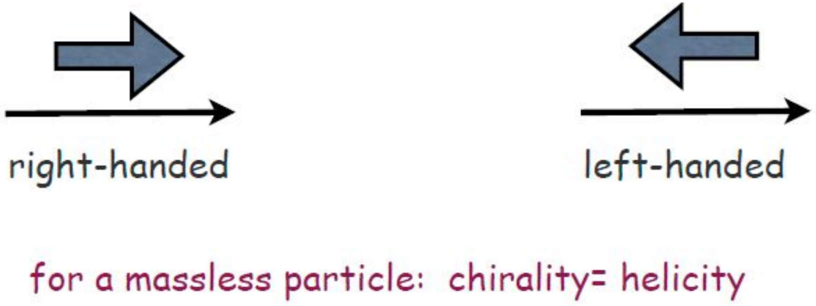
$$\gamma_5 = \frac{-i}{4!}\epsilon_{\mu\nu\rho\sigma}\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma$$

$$\gamma_5^2 = \mathbb{1}$$

The Right-Handed Model ® in 4-dim

The R-model in 4-dim

- Model with generic gauge group $\mathcal{G}$ (usually SU(N)) with right-handed (RH) fermions in right (R) rep. of $\mathcal{G}$ and scalars in S rep. of $\mathcal{G}$, both coupling to gauge bosons.
- Originally defined in 4 dimensions, using either Weyl, or Dirac fermions with projectors.



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$$\mathbb{P}_{R/L} = (1 \pm \gamma_5)/2$$

$$\mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{fermions}} + \mathcal{L}_{\text{scalars}} + \mathcal{L}_{\text{Yukawa}}$$

DEFINING S_0

$$\mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{fermions}} + \mathcal{L}_{\text{scalars}} + \mathcal{L}_{\text{Yukawa}}$$

$$F^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g f^{abc} G^b_\mu G^c_\nu$$

$$\mathcal{L}_{\text{gauge}} = \frac{-1}{4} F^a_{\mu\nu} F^{a\,\mu\nu}$$

Weyl spinor
(2 components)

$$D_{ij\,\mu} = \partial_\mu \delta_{ij} - i g G^a_\mu T^a_{R\,ij}$$

$$\mathcal{L}_{\text{fermions}} = i\xi \overbrace{\sigma^\mu}^{\text{Matrix}} D_\mu \bar{\xi}$$

$$\mathcal{L}_{\text{scalars}} = \overbrace{\frac{1}{2}(D_\mu \Phi^m)^2}^{\text{Dynamics}} - \overbrace{\frac{\lambda^{mnop}}{4!} \Phi_m \Phi_n \Phi_o \Phi_p}^{\text{It will contribute in the perturbative part (?)}}$$

Right-handed Weyl fermion

$$\mathcal{L}_{\text{Yukawa}} = -\frac{(Y_R)^m_{ij}}{2} \Phi_m \bar{\xi}_i \bar{\xi}_j + \text{h.c.}$$

Weyl spinor -> Dirac spinor in d-dim

$$\bar{\xi} \to \mathbb{P}_{\mathrm{R}}\psi \equiv \psi_R \quad \overline{\psi_R} = \overline{\psi}_L \equiv \overline{\psi}\mathbb{P}_{\mathrm{L}}$$

$$\mathcal{L}_{\text{fermions}} = i\overline{\psi_R}_i \not{D}^{ij}\psi_{Rj} = i\overline{\psi_R}_i \not{\partial}\psi_{Ri} + gT^a_{Rij}\overline{\psi_R}_i \not{G}^a \psi_{Rj}$$

$$\mathcal{L}_{\text{Yukawa}} = -\frac{(Y_R)^m_{ij}}{2}\Phi_m \overline{\psi_{Ri}}^C \psi_{Rj} - \frac{(Y_R)^{m\,*}_{ij}}{2}\Phi_m \overline{\psi_{Ri}}\psi_{Rj}^C$$

BRST invariance in 4-dim

Gauge fixing and ghost terms

$$
\begin{aligned}
sG^a_\mu &= D^{ab}_\mu c^b = \partial_\mu c^a + g f^{abc} G^b_\mu c^c \\
s\psi_i &= s\psi_{Ri} = i c^a g T^a_{Rij} \psi_{Rj}\,, \\
s\overline{\psi}_i &= s\overline{\psi_{R}}_i = +i\overline{\psi}_{Rj} c^a g T^a_{Rji}\,, \\
s\psi_{Li} &= 0\,, \\
s\overline{\psi}_{Li} &= 0\,, \\
s\Phi_m &= i c^a g \theta^a_{mn} \Phi_n\,.
\end{aligned}
$$

There remains a residual symmetry even after fixing the gauge: BRST

$$
\begin{aligned}
sc^a &= -\frac{1}{2} g f^{abc} c^b c^c \equiv i g c^2 \\
s\bar{c}^a &= B^a\,, \\
sB^a &= 0\,.
\end{aligned}
$$

$$\boxed{\mathcal{L}_{\text{ghost}} = \partial^\mu \bar{c}_a \cdot D^{ab}_\mu c_b \equiv -\bar{c}_a \partial^\mu D^{ab}_\mu c_b}$$

$$\boxed{\mathcal{L}_{\text{g-fix}} = \frac{\xi}{2} B^a B_a + B^a \partial^\mu G^a_\mu}$$

Any BRST-invariant term built from the existing fields can be added to S_0

$$S_0^{(4D)} = \int \mathrm{d}^4 x \, (\mathcal{L}_{\mathsf{YM}}^{(4D)} + \mathcal{L}_{\Psi}^{(4D)} + \mathcal{L}_{\Phi}^{(4D)} + \mathcal{L}_{\mathsf{Yuk}}^{(4D)} + \mathcal{L}_{\mathsf{gh}}^{(4D)} + \mathcal{L}_{\mathsf{g\text{-}fix}}^{(4D)})$$

$$\boxed{\begin{aligned}
\mathcal{L}_{\mathsf{YM}}^{(4D)} &= \frac{-1}{4} F_{\mu\nu}^{a} F^{a\,\mu\nu} \, , \; \mathcal{L}_{\Phi}^{(4D)} = \frac{1}{2}(D_\mu \Phi^m)^2 - \frac{\lambda_H^{mnop}}{4!} \Phi_m \Phi_n \Phi_o \Phi_p \, , \\
\mathcal{L}_{\Psi}^{(4D)} &= i \overline{\Psi}_i \not{\partial} \mathbb{P}_{\mathsf{R}} \Psi_i + g_S T_{R\,ij}^{\;a} \overline{\Psi}_i \not{G}^a \mathbb{P}_{\mathsf{R}} \Psi_j \equiv i \overline{\Psi}_i \not{D}_R^{ij} \Psi_j \, , \\
\mathcal{L}_{\mathsf{Yuk}}^{(4D)} &= -(Y_R)_{ij}^m \Phi_m \overline{\Psi}_i^C \mathbb{P}_{\mathsf{R}} \Psi_j + \mathsf{h.c.} \, , \\
\mathcal{L}_{\mathsf{gh}}^{(4D)} &= \partial_\mu \bar{c}_a \cdot D^{ab\,\mu} c_b \, , \; \mathcal{L}_{\mathsf{g\text{-}fix}}^{(4D)} = \frac{\xi}{2} B^a B_a + B^a \partial^\mu G_\mu^a \, .
\end{aligned}}$$

!

Final tree-level action in 4 dimensions

$$S_0^{(4D)} = \int \mathrm{d}^4 x \, (\mathcal{L}_{\mathsf{YM}}^{(4D)} + \mathcal{L}_{\Psi}^{(4D)} + \mathcal{L}_{\Phi}^{(4D)} + \mathcal{L}_{\mathsf{Yuk}}^{(4D)} + \mathcal{L}_{\mathsf{gh}}^{(4D)} + \mathcal{L}_{\mathsf{g\text{-}fix}}^{(4D)})$$

$$\mathcal{L}_{\mathsf{YM}}^{(4D)} = \frac{-1}{4} F^a_{\mu\nu} F^{a\,\mu\nu} \,, \; \mathcal{L}_{\Phi}^{(4D)} = \frac{1}{2}(D_\mu \Phi^m)^2 - \frac{\lambda_H^{mnop}}{4!} \Phi_m \Phi_n \Phi_o \Phi_p \,,$$

$$\boxed{\mathcal{L}_{\Psi}^{(4D)} = i\overline{\Psi}_i \partial\!\!\!/ \mathbb{P}_{\mathsf{R}} \Psi_i + g_S T_{R\,ij}^{\,a} \overline{\Psi}_i G\!\!\!/^a \mathbb{P}_{\mathsf{R}} \Psi_j \equiv i\overline{\Psi}_i D\!\!\!\!/_R^{ij} \Psi_j} \,,$$

$$\boxed{\mathcal{L}_{\mathsf{Yuk}}^{(4D)} = -(Y_R)^m_{ij} \Phi_m \overline{\Psi}_i^C \mathbb{P}_{\mathsf{R}} \Psi_j + \mathsf{h.c.}} \,,$$

$$\mathcal{L}_{\mathsf{gh}}^{(4D)} = \partial_\mu \bar{c}_a \cdot D^{ab\,\mu} c_b \,, \; \mathcal{L}_{\mathsf{g\text{-}fix}}^{(4D)} = \frac{\xi}{2} B^a B_a + B^a \partial^\mu G^a_\mu \,.$$

Final tree-level action in 4 dimensions

Last comments: BMHV vs IREG

R-model action So in d-dim

$$\mathcal{L}^{(4D)}_{\text{fermions}} = i\overline{\psi_{R_i}}\not{D}^{ij}\psi_{R_j} = \boxed{i\overline{\psi_{R_i}}\not{\partial}\psi_{R_i}} + \boxed{gT^a_{R_{ij}}\overline{\psi_{R_i}}\not{G}^a\psi_{R_j}} \xrightarrow{\text{d-dim}} ?$$

D-dim extension

Trivially done for bosonic fields.

Problem 1

Kinetic term projects only the purely 4-dim derivative→a purely 4-dim propagator

$$i\overline{\psi}_i\mathbb{P}_{\mathrm{L}}\not{\partial}\mathbb{P}_{\mathrm{R}}\psi_i$$

Problem 2

How to promote the interaction term?

$$\overline{\Psi}\not{G}\Psi$$

Contrast

Fermionic fields need some care.

Solution

Consider a Dirac fermion and use the fully d-dim covariant term:

$$i\overline{\Psi}_i\not{\partial}\Psi_i$$

Solution

$$\mathbb{P}_{\mathrm{L}}\gamma^\mu \neq \gamma^\mu\mathbb{P}_{\mathrm{R}}$$

(inequivalent choices)

Use the interaction term that makes calculations the most simple.

$$\overline{\psi}_i\gamma^\mu\mathbb{P}_{\mathrm{R}}\psi_j \quad \overline{\psi}_i\mathbb{P}_{\mathrm{L}}\gamma^\mu\psi_j \quad \boxed{\overline{\psi}_i\mathbb{P}_{\mathrm{L}}\gamma^\mu\mathbb{P}_{\mathrm{R}}\psi_j}$$

NO unique way of extending the model to d-dim

!

What would happen if we use IREG?

$$\mathcal{L}^{(4D)}_{\text{fermions}} = i\overline{\psi_R}_i \not{D}^{ij} \psi_{Rj} = \boxed{i\overline{\psi_R}_i \not{\partial} \psi_{Ri}} + \boxed{g T^a_{Rij} \overline{\psi_R}_i \not{G}^a \psi_{Rj}}$$

D-dim extension problems

1

Kinetic term in S_0 is chiral.

$$i\overline{\psi}_i \mathbb{P}_L \not{\partial} \mathbb{P}_R \psi_i$$

2

Interaction term

$$\overline{\Psi} \not{G} \Psi$$

Carolina & Yuri speculations

Purely 4-dim propagator

$$\Delta(p) = \mathbb{P}_R i\not{p} \mathbb{P}_L / \bar{p}^2$$

Regularized loop diagrams still possible (?)

In 4-dim:

$$\gamma_\mu \mathbb{P}_R = \mathbb{P}_L \gamma_\mu = \mathbb{P}_L \gamma_\mu \mathbb{P}_R$$

We won't have differences by evanescent terms.

!

Cause and Effect-BMHV

Cause 1

$$i\overline{\psi_{R_i}}\partial\!\!\!/\psi_{R_i}$$

Effect 1

$$i\overline{\psi}_i\partial\!\!\!/\psi_i$$

Cause 2

$$gT^a_{R_{ij}}\overline{\psi_{R_i}}G\!\!\!/^a\psi_{R_j}$$

Effect 2

$$g_S T^a_{R_{ij}}\overline{\Psi}_i \mathbb{P}_\mathrm{L} G\!\!\!/^a \mathbb{P}_\mathrm{R} \Psi_j$$

Cause and Effect-BMHV (re-write)

$$\mathcal{L}_{\text{fermions}} = \mathcal{L}_{\text{fermions,inv}} + \mathcal{L}_{\text{fermions,evan}}$$

$$\mathcal{L}_{\text{fermions,inv}} = i\overline{\psi}_i \overline{\partial\!\!\!/} \psi_i + g T_{R_{ij}}^{a} \overline{\psi_{R_i}} \overline{G\!\!\!/}^{a} \psi_{R_j}$$

$$\boxed{\mathcal{L}_{\text{fermions,evan}} = i\overline{\psi}_i \widehat{\partial\!\!\!/} \psi_i} \,.$$

$$\mathcal{L}_{\text{fermions,evan}} = i\overline{\psi_{L}}_i \widehat{\partial\!\!\!/} \psi_{R_i} + i\overline{\psi_{R}}_i \widehat{\partial\!\!\!/} \psi_{L_i} \,, \tag{3.32}$$

which highlights the fact that it mixes left- and right-chiral fields which have different gauge transformation properties. This causes the breaking of gauge and BRST invariance — the central difficulty of the BMHV scheme.

In IREG, as we'll work in 4D the model is BRST-invariant at tree-level.

The rest of the model is straightforwardly extended to d dimensions: we define the d-

Problem vs Solution

If this is the problem...

The model is not BRST-invariant in d-dimensions due to the evanescent part. This breaking generates some Feynman Rules.

... This is the solution

Determine the symmetry restoring counterterms required in the BMHV scheme at 1-loop

...But then again, this is not the objective of this presentation.

Thanks

Do you have any questions?

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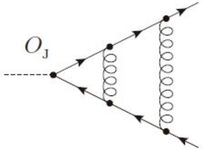
$$\boxed{\mathcal{L}_{\text{ext}} = \rho^{\mu}_{a} s G^{a}_{\mu} + \zeta_a s c^a + \bar{R}^i s \psi_{R_i} + R^i s \overline{\psi_{R_i}} + \mathcal{Y}^m s \Phi_m}$$

$$s\mathcal{J} = 0$$

$$\mathcal{J} = \rho^{\mu}_{a}, \zeta_a, R, \bar{R}, \mathcal{Y}^m$$

Pseudo-scalar form factors in FDH (from Signer's article-EXTRA)

- pseudoscalar form factor at two loops in FDH
  (i) γ_5^{BM}, e.g.



$$O_J \;=\; \partial_\mu\big(\overline{\psi}\,\gamma^\mu\gamma_5\,\psi\big) \;\to\; \varepsilon_{[4]}^{\mu\nu\rho\sigma}\Big\{\partial_\mu\big(\overline{\psi}\,\gamma_\nu\gamma_\rho\gamma_\sigma\,\psi\big)\Big\}_{[d]}$$

- as before: non-trivial operator renormalization: $O_{J,\text{ren.}} = \left(Z_{\overline{\text{MS}}}^{\text{BM}}\, Z_5^{\text{BM}}\right) O_{J,\text{bare}}$
  $\to$ results in FDH [Signer,CG '17]

$$Z_{\overline{\text{MS}}}^{\text{BM}} = 1+\left(\frac{\alpha_s}{4\pi}\right)C_F\frac{n_\epsilon}{\epsilon}+\left(\frac{\alpha_s}{4\pi}\right)^2\Big\{C_A C_F\Big[\frac{22}{3\epsilon}+n_\epsilon\Big(-\frac{1}{\epsilon^2}+\frac{11}{3\epsilon}\Big)+n_\epsilon^2\Big(\frac{1}{2\epsilon^2}+\frac{1}{4\epsilon}\Big)\Big]$$
$$+C_F^2\Big[n_\epsilon\Big(-\frac{1}{\epsilon^2}-\frac{4}{\epsilon}\Big)-\frac{3n_\epsilon^2}{4\epsilon}\Big]+C_F N_F\Big[\frac{5}{3\epsilon}+n_\epsilon\Big(\frac{1}{2\epsilon^2}-\frac{1}{4\epsilon}\Big)\Big]\Big\}+\mathcal{O}(\alpha_s^3)$$

$$Z_5^{\text{BM}} = 1+\left(\frac{\alpha_s}{4\pi}\right)\Big\{-4\,C_F\Big\}+\left(\frac{\alpha_s}{4\pi}\right)^2\Big\{22\,C_F^2-\frac{107}{9}\,C_A+\frac{31}{18}\,C_F N_F\Big\}+\mathcal{O}(\alpha_s^3)$$

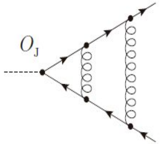
- Z_5^{BM} usually obtained by imposing

$$\partial_\mu\, j_5^\mu \;=\; 2m\, j_5 + \frac{\alpha_s}{4\pi}\,\varepsilon^{\mu\nu\rho\sigma}\,G^a_{\mu\nu}G^a_{\rho\sigma}$$

anomalous term of $\mathcal{O}(\alpha_s) \Rightarrow (L{+}1)$-loop calculation needed to obtain L-loop value of Z_5^{BM} (known up to two loops)

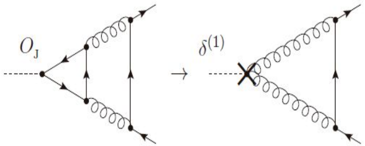
- pseudoscalar form factor at two loops in FDH
  (ii) $\gamma_5^{\text{AC}} \to$ distinguish two classes of diagrams

Type A:
γ_5 attached to external quark line, e.g.



- no γ_5-odd traces $\to$ use γ_5^{AC}
- then: trivial op. ren. $O_{J,\text{ren.}}^{\text{AC}} = O_{J,\text{bare}}^{\text{AC}}$

Type B:
γ_5 attached to quark loop, e.g.



- vanishes for γ_5^{AC} and standard trace
- however: value of anomaly known from before (γ_5^{BM}, FDF, ...)
- op.ren. effectively reduced by one order

Comparing this approach with γ_5^{BM}, the L-loop value of Z_5^{BM} can be obtained from a genuine L-loop calculation.

Gnendiger, 2018-Loops and Legs in Quantum Field Theory)