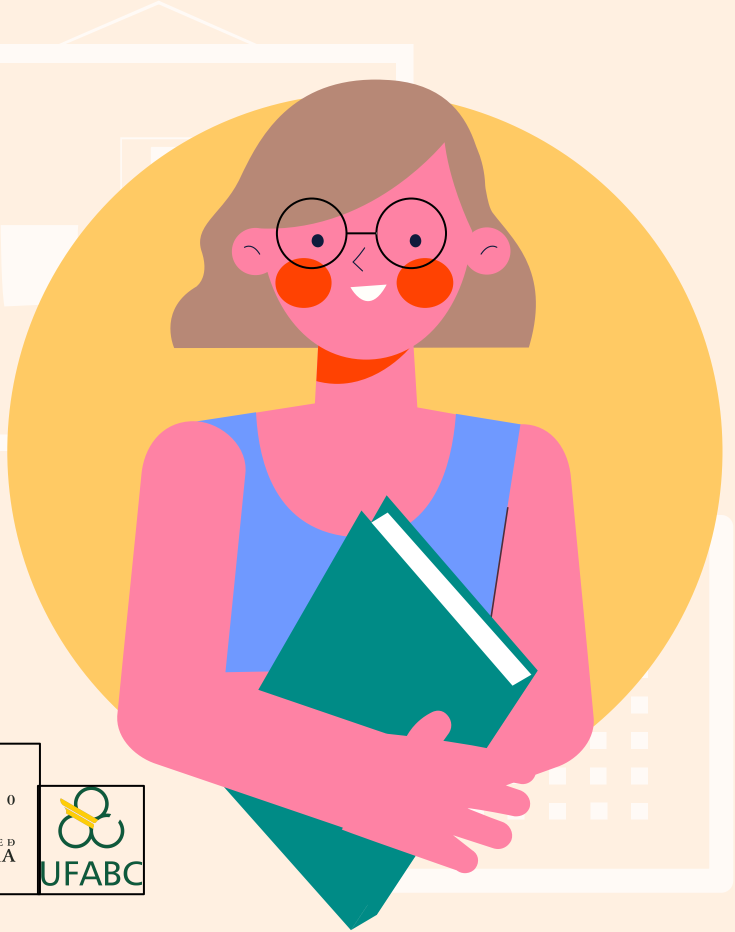

Progress update: 1-loop γ_m in IREG and DS

D. Carolina A. Perdomo
Advisors: Marcos S., Adriano C. and Brigitte H.

CAPES

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UFABC

Let's remember:

- 2-loop **γ_m-function** in QCD.

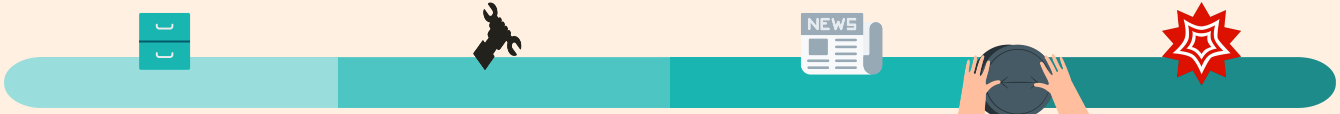
$$\gamma_m^{\overline{IREG}}\left(\Lambda^2\right) = 2\frac{\Lambda^2}{m^{\overline{IREG}}}\frac{\partial m^{\overline{IREG}}}{\partial \Lambda^2}$$
$$= -\beta^{\overline{IREG}}\left(\Lambda^2\right)\frac{\partial \ln Z_m^{\overline{IREG}}}{\partial g^{\overline{IREG}}}$$

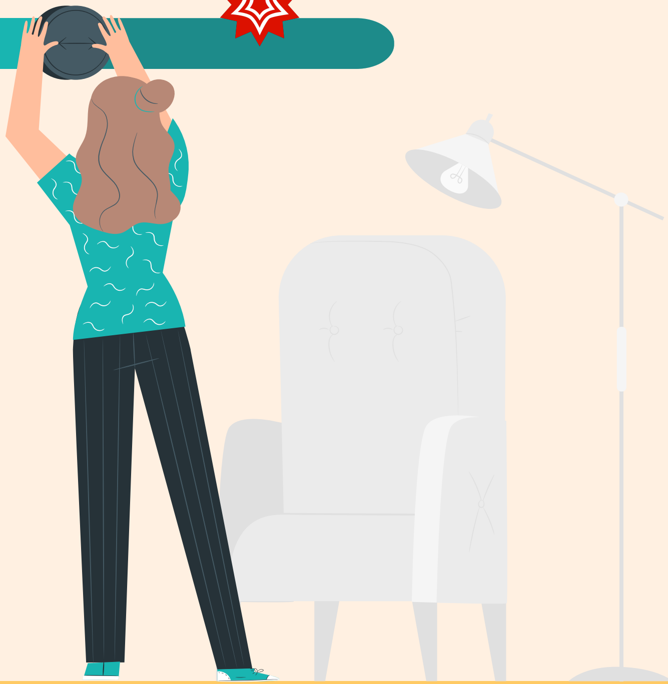
- Renormalization of QCD at 2-loop ➡️ **transition rules**.

Main Objective

ε-scalars ➡️ are they really necessary?
What is their role?

The last time we talked: project stages





01. Literature

γ_m at 1-loop in QCD (without the BFM)

02. BFM

Compute γ_m at 1-loop in QCD (with the BFM)

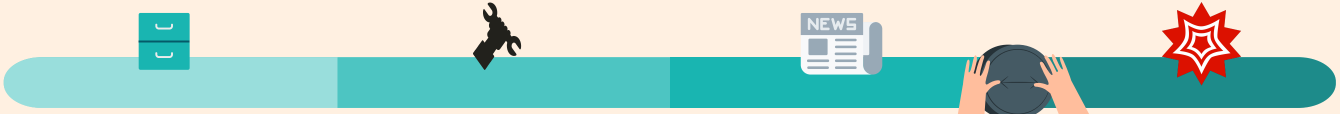
03. 2–loop

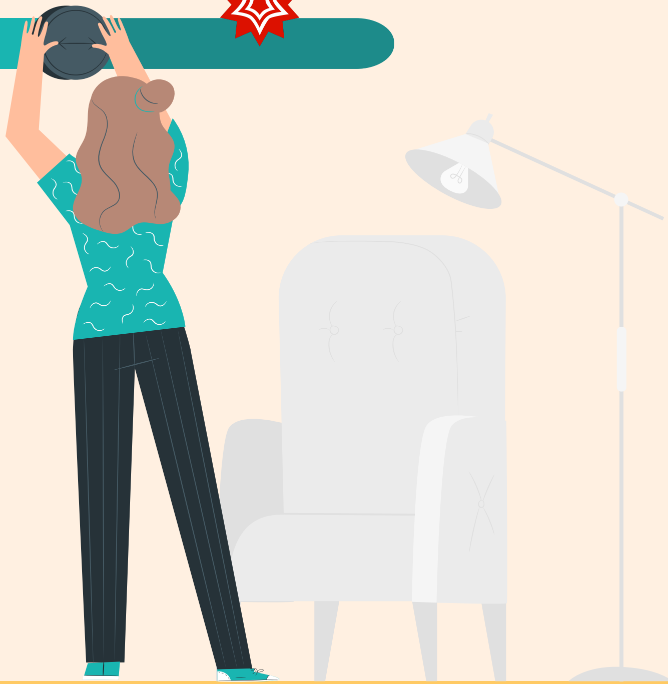
Compute γ_m at 2-loop in QCD with IREG.

04. Mathematica

Adapt the computation to Mathematica.

The last time we talked: project stages





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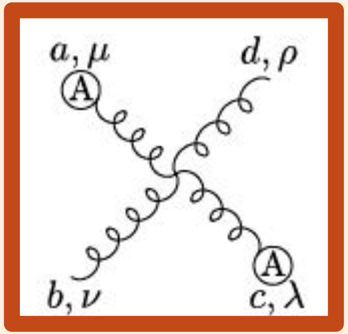
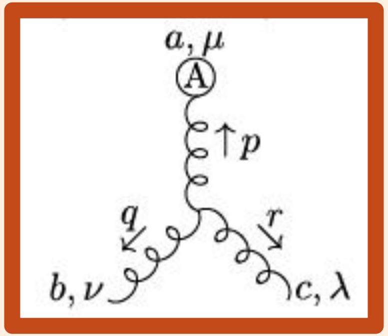
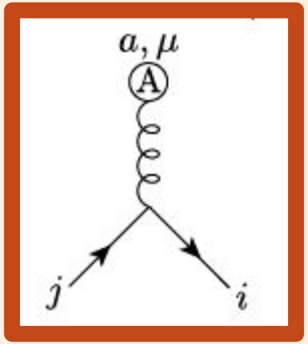

a, μ
d, ρ
A
A
b, ν
c, λ


a, μ
A
$\uparrow p$
q
r
b, ν
c, λ


a, μ
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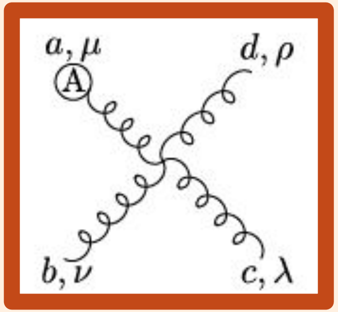
a, μ
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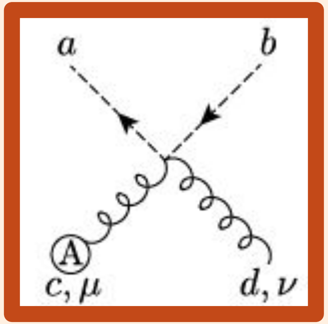
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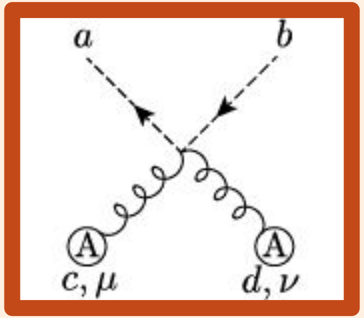
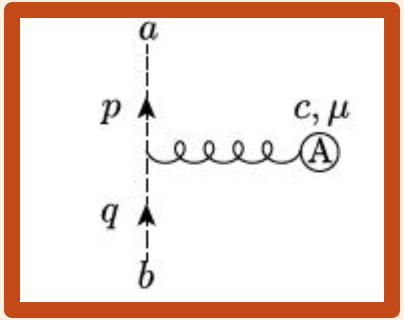
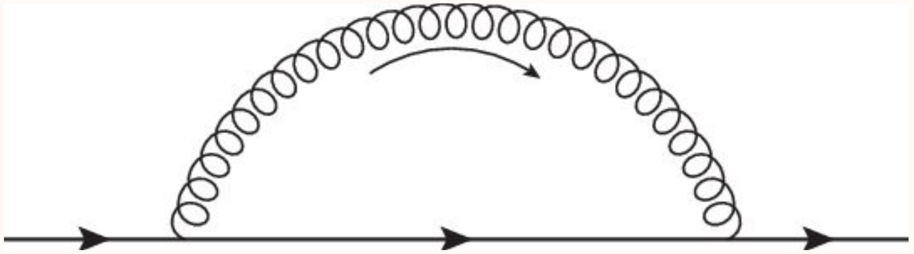

a
b
A
A
c, μ
d, ν


a
p
c, μ
A
q
b

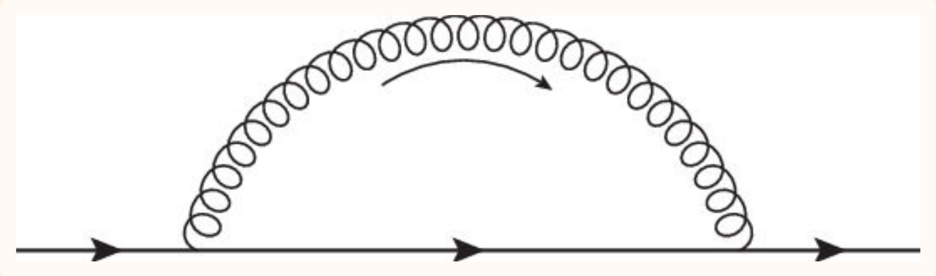
We don´t need to use the BFM for this diagram



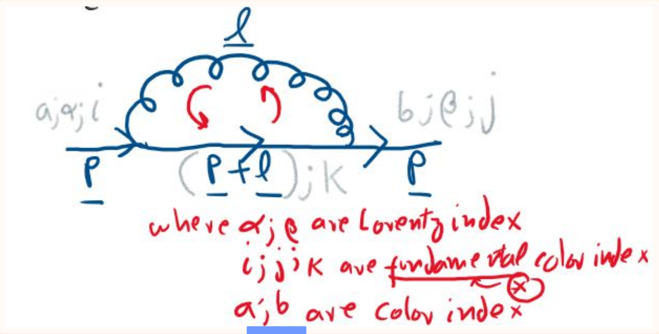
But...

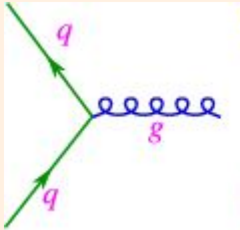
Change of plans

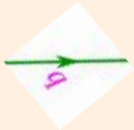
We decided it was worth doing γ_m in DRED and DReg to 1-loop also!



The usual: Feynman Rules







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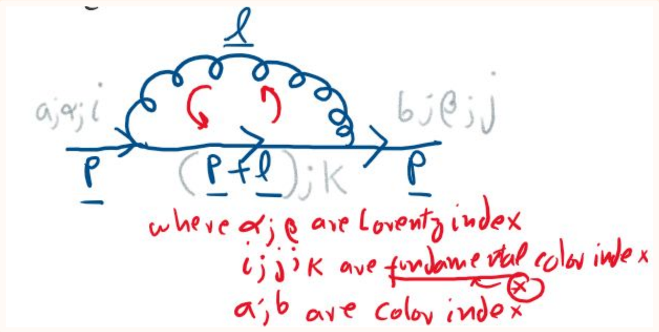
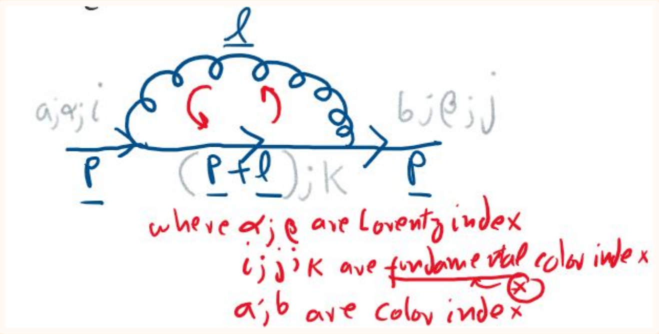


$$+\sigma_{ij}\left(p,m\right)=-ig_sC_2\left(r\right)\times\left[\int_l\frac{\gamma^\beta\gamma^\mu p_\mu\gamma_\beta}{D}+\int_1\frac{\gamma^\beta\gamma^\mu l_\mu\gamma_\beta}{D}+m\int_l\frac{\gamma^\beta\gamma_\beta}{D}\right]$$

$$\int_l=\int_{-\infty}^{+\infty}\frac{d^4l}{(2\pi)^4}$$

$$D=l^2\left[(p+l)^2-m^2\right]$$

The usual: Feynman Rules



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$$I=\int_l\frac{1}{D}=\int_l\frac{1}{(l^2)[(p+l)^2-m^2]}$$

$$I_\mu=\int_l\frac{l_\mu}{D}=\int_l\frac{l_\mu}{(l^2)[(p+l)^2-m^2]}$$

1-loop γ_m

01

IREG

02

DReg

03

DRED

01. γ_m in IREG

Our method.

1. Symmetric integration must be avoided (source of error).

$$k^{\mu} k^{\nu} \cancel{\to} \frac{1}{d} k^{2}$$

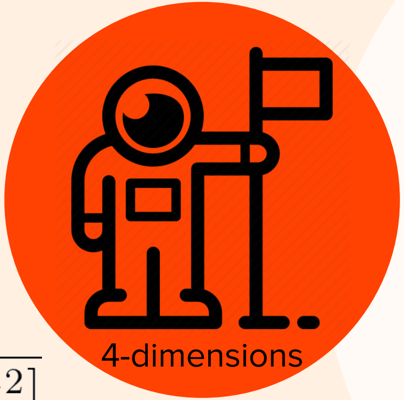
2. Fictitious mass μ^2 to avoid spurious IR-div.

3. External momenta.

$$\frac{1}{(k-p)^2-\mu^2} = \frac{1}{k^2-\mu^2} - \frac{p^2-2p\cdot k}{(k^2-\mu^2)[(k-p)^2-\mu^2]}$$

4. Divergent part in terms of BDI's which may be written as linear combinations of scalar BDIs plus ST.

5. The μ^2 dependence by introducing a scale Λ^2.



IREG

$$\boxed{+\sigma_{ij}\left(p,m\right)=-ig_sC_2\left(r\right)\times\left[\int_l\frac{\gamma^\beta\gamma^\mu p_\mu\gamma_\beta}{D}+\int_1\frac{\gamma^\beta\gamma^\mu l_\mu\gamma_\beta}{D}+m\int_l\frac{\gamma^\beta\gamma_\beta}{D}\right]}$$

Dirac Algebra

$$\boxed{\gamma^\beta\gamma^\mu p_\mu\gamma_\beta=-2\gamma^\mu p_\mu}$$

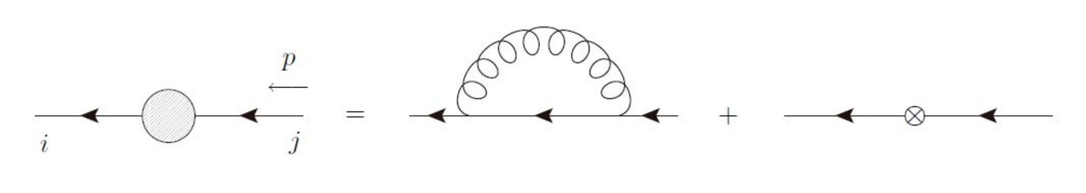
$$\boxed{\gamma^\beta\gamma_\beta=4I_{4\times4}}$$

$$\sigma_{ij}\left(p,m\right)=-ig_s^2C_2\left(r\right)\left(\gamma^\mu p_\mu-4m\right)\lim_{\mu^2\to0}Ilog(\mu^2)+ig_s^2C_2\left(r\right)\lim_{\mu^2\to0}g^{\mu\nu}\Gamma^{(0)}_{\mu\nu}\gamma^\mu p_\mu+$$ Finite terms

IREG

$$\sigma_{ij}(p,m) = -ig_s^2 C_2(r)(\gamma^\mu p_\mu - 4m)\lim_{\mu^2\to 0} Ilog(\mu^2) + ig_s^2 C_2(r)\lim_{\mu^2\to 0} g^{\mu\nu}\Gamma^{(0)}_{\mu\nu}\gamma^\mu p_\mu +$$ Finite terms

$$I_{log}(\mu^2) = I_{log}(\Lambda^2) - \frac{i}{(4\pi)^2}\ln\frac{\Lambda^2}{\mu^2}$$



$$\Sigma(p) = \sigma_{ij}(p,m) + \delta_2\gamma^\mu p_\mu - (\delta_m + \delta_2)m$$

IREG

$$\boxed{\gamma_m^{\overline{IREG}}(\Lambda^2) = 2\frac{\Lambda^2}{m^{\overline{IREG}}}\frac{\partial m^{\overline{IREG}}}{\partial\Lambda^2}}$$

$$Z_m = 1 + \delta_m = 1 + 3ig_s^2 C_2(r)\, Ilog(\Lambda^2)$$

$$\boxed{\gamma_m^{\overline{IREG}}(\Lambda^2) = \frac{6g_s^2}{(4\pi)^2}C_2(r) + \dots}$$

Sampaio, M., Scarpelli, A.P.B., Ottoni, J.E. *et al.* Implicit Regularization and Renormalization of QCD. *Int J Theor Phys* 45, 436–457 (2006). https://doi.org/10.1007/s10773-006-9045-z

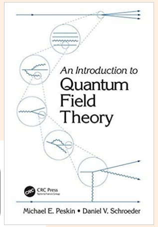
02. γ_m in DReg

Traditional Dimensional Method.

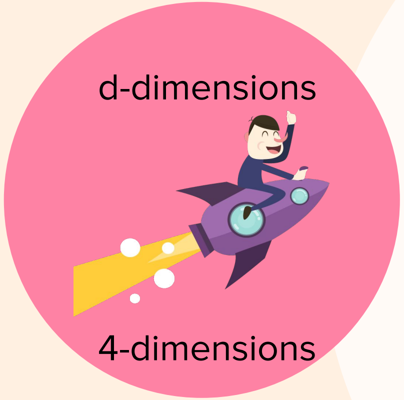
Before we try to interpret this result, let us summarize the calculation methods we used. The techniques are common to all loop calculations:

1. Draw the diagram(s) and write down the amplitude.
2. Introduce Feynman parameters to combine the denominators of the propagators.
3. Complete the square in the new denominator by shifting to a new loop momentum variable, ℓ.
4. Write the numerator in terms of ℓ. Drop odd powers of ℓ, and rewrite even powers using identities like (6.46).
5. Perform the momentum integral by means of a Wick rotation and four-dimensional spherical coordinates.

The momentum integral in the last step will often be divergent. In that case we must define (or *regularize*) the integral using the Pauli-Villars prescription or some other device.



$$\int_{-\infty}^{+\infty} \frac{d^4 k_{[4]}}{(2\pi)^4} \to \underbrace{\mu^{4-d}}_{\text{scale}} \int_{-\infty}^{+\infty} \frac{d^d k_{[d]}}{(2\pi)^d}$$



DReg

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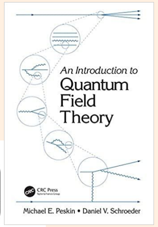
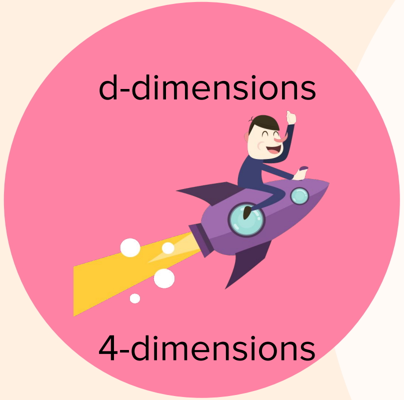
5. ~~Perform the momentum integral by means of a Wick rotation and four-dimensional spherical coordinates.~~

Alternatively, one can use the following table of d-dimensional integrals in Minkowski space:

$$\int \frac{d^d\ell}{(2\pi)^d} \frac{1}{(\ell^2-\Delta)^n} = \frac{(-1)^n i}{(4\pi)^{d/2}} \frac{\Gamma(n-\frac{d}{2})}{\Gamma(n)} \left(\frac{1}{\Delta}\right)^{n-\frac{d}{2}} \tag{A.44}$$

$$\int \frac{d^d\ell}{(2\pi)^d} \frac{\ell^2}{(\ell^2-\Delta)^n} = \frac{(-1)^{n-1} i}{(4\pi)^{d/2}} \frac{d}{2} \frac{\Gamma(n-\frac{d}{2}-1)}{\Gamma(n)} \left(\frac{1}{\Delta}\right)^{n-\frac{d}{2}-1} \tag{A.45}$$





DReg

$$
+\sigma_{ij}\left(p,m\right)=-ig_{s}C_{2}\left(r\right)\times\left[\int_{l}\frac{\gamma^{\beta}\gamma^{\mu}p_{\mu}\gamma_{\beta}}{D}+\int_{1}\frac{\gamma^{\beta}\gamma^{\mu}l_{\mu}\gamma_{\beta}}{D}+m\int_{l}\frac{\gamma^{\beta}\gamma_{\beta}}{D}\right]
$$

Dirac Algebra

$$
\boxed{\gamma^{\beta}\gamma^{\mu}p_{\mu}\gamma_{\beta}=(2-d)\gamma^{\mu}p_{\mu}}
$$

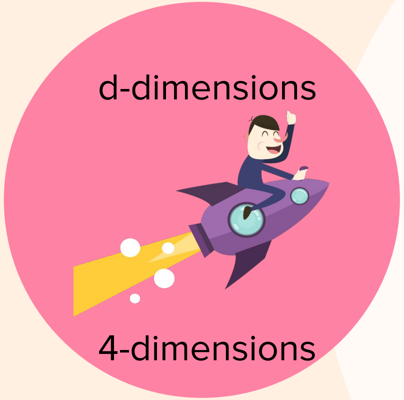
$$
\boxed{\gamma^{\beta}\gamma_{\beta}=d}
$$

$$
\sigma_{ij}\left(p,m\right)=\frac{g_{s}^{2}}{\left(4\pi\right)^{2}}C_{2}\left(r\right)\{\gamma^{n}p_{\mu}
$$

$$
\left[1-\frac{1}{\varepsilon}+\gamma_{E}-\ln\left(\frac{4\pi\mu_{DS}^{2}}{m^{2}}\right)+2\int_{0}^{1}(1-x)\ln\left(\frac{1}{x+\left(\frac{xp}{m}\right)^{2}}\right)dx\right]
$$

$$
+m
$$

$$
\left[-2+\frac{4}{\varepsilon}-4\gamma_{E}+4(n\left(\frac{4\pi\mu_{DS}^{2}}{m^{2}}\right)-4\int_{0}^{1}\ln\left(\frac{1}{x+\left(\frac{xp}{m}\right)^{2}}\right)dx\right]\}
$$



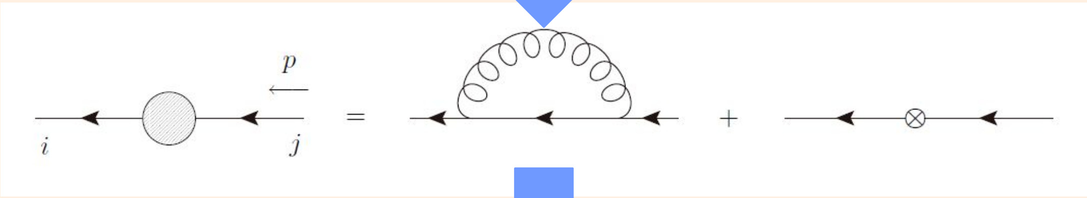
DReg

$$\sigma_{ij}(p,m) = \frac{g_s^2}{(4\pi)^2} C_2(r)\{\gamma^n p_\mu$$

$$\left[1 - \frac{1}{\varepsilon} + \gamma_E - \ln\left(\frac{4\pi\mu_{DS}^2}{m^2}\right) + 2\int_0^1 (1-x)\ln\left(\frac{1}{x + \left(\frac{xp}{m}\right)^2}\right) dx\right]$$

$$+m$$

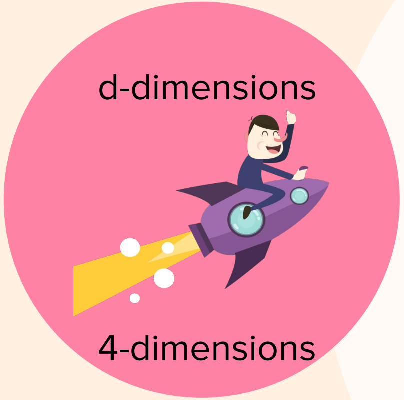
$$\left[-2 + \frac{4}{\varepsilon} - 4\gamma_E + 4\ln\left(\frac{4\pi\mu_{DS}^2}{m^2}\right) - 4\int_0^1 \ln\left(\frac{1}{x + \left(\frac{xp}{m}\right)^2}\right) dx\right]\}$$



$$Z_m^{\overline{MS}} = 1 - \frac{g_s^2}{(4\pi)^2} C_2(r)\{-1 + \frac{3}{\varepsilon} - 3\gamma_E + 3\ln\left(\frac{4\pi\mu_{DS}^2}{m^2}\right) +$$

$$2\int_0^1 (1-x)\ln\left(\frac{1}{x + \left(\frac{xp}{m}\right)^2}\right) dx\}$$

$$\boxed{\gamma_m^{\overline{MS}}(\mu^2) = \frac{6 g_s^2}{(4\pi)^2} C_2(r) + \ldots}$$

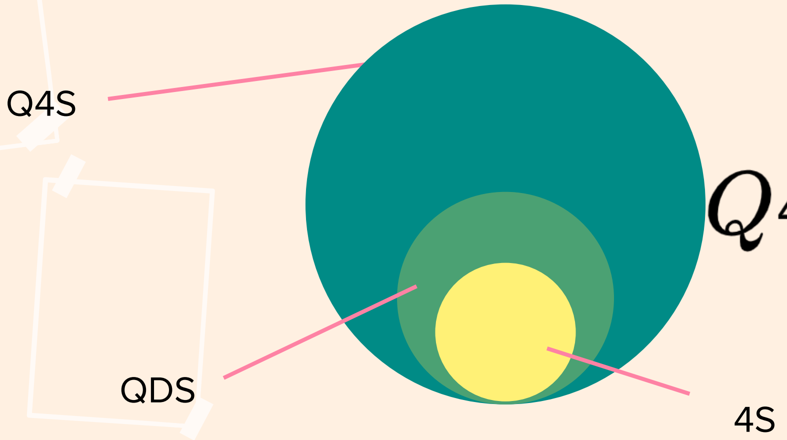


DReg

03. γ_m in DRED

Alternative DS.

Dimensional Schemes



$$Q4S \supset QDS \supset 4S$$

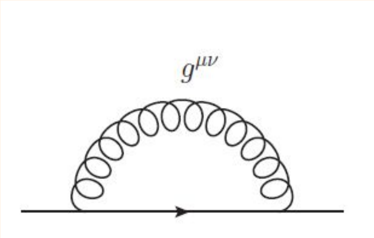
$$\mathscr{A}_4 = \underbrace{\mathscr{A}_d}_{4-2\varepsilon} + \underbrace{\mathscr{A}_N}_{2\varepsilon}$$

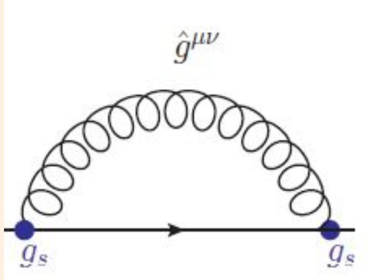
$$g_{\mu\nu} = \hat{g}_{\mu\nu} + \underbrace{\tilde{g}_{\mu\nu}}_{2\varepsilon}$$

$$\Longrightarrow \quad \mathscr{L}_4 = \mathscr{L}_d + \mathscr{L}_{2\epsilon}$$

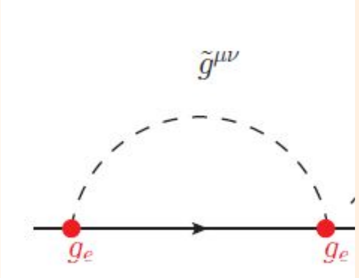
New Feynman Rules

Using DRED

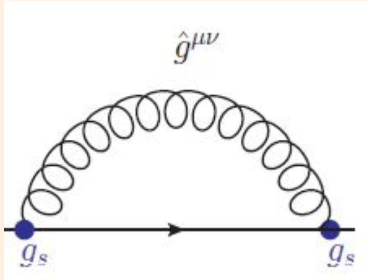


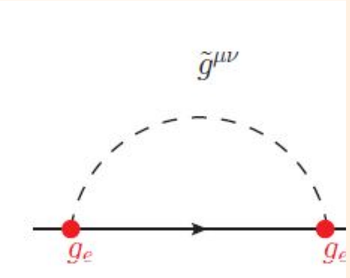


g_s



g_e





g_s

g_e

Dirac Algebra

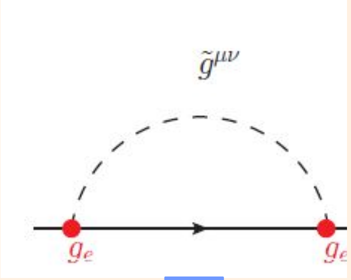
$$\widehat{\gamma}^{\beta}\widehat{\gamma}^{\mu}\widehat{p}_{\mu}\widehat{\gamma}_{\beta} = (2-d)\,\widehat{\gamma}^{\mu}\widehat{p}_{\mu}$$

$$\widehat{\gamma}^{\mu}\widehat{\gamma}_{\mu} = d$$

$$\tilde{\gamma}^{\beta}\widehat{\gamma}^{\mu}\widehat{p}_{\mu}\tilde{\gamma}_{\beta} = -N_{\epsilon}\widehat{\gamma}^{\mu}\widehat{p}_{\mu}$$

$$\tilde{\gamma}^{\mu}\tilde{\gamma}_{\mu} = \underbrace{N_{\epsilon}}_{2\varepsilon}$$

Same as DReg!



$$\tilde{\gamma}^{\beta}\widehat{\gamma}^{\mu}\widehat{p}_{\mu}\tilde{\gamma}_{\beta} = -N_{\epsilon}\widehat{\gamma}^{\mu}\widehat{p}_{\mu}$$
$$\tilde{\gamma}^{\mu}\tilde{\gamma}_{\mu} = N_{\epsilon}$$

$$\tilde{\sigma_{ij}}(p,m) = -\frac{g_e^2}{(4\pi)^2}C_2(r)\{\gamma^n p_\mu$$

Finite terms

1

$$\left[\frac{1}{2}\boxed{N_\epsilon\frac{1}{\varepsilon}} - \frac{1}{2}N_\epsilon\gamma_E + \frac{1}{2}N_\epsilon\ln\left(\frac{4\pi\mu_{DS}^2}{m^2}\right) - 2N_\epsilon\int_0^1(1-x)\ln\left(\frac{1}{x+\left(\frac{xp}{m}\right)^2}\right)dx\right]$$
$$+m$$

2

$$\left[\boxed{\frac{1}{\epsilon}N_\epsilon} - N_\epsilon\gamma_E + N_\epsilon\ln\left(\frac{4\pi\mu_{DS}^2}{m^2}\right) + N_\epsilon\int_0^1\ln\left(\frac{1}{x+\left(\frac{xp}{m}\right)^2}\right)dx\right]\}$$

That means γ_m in DRED is the same as DReg at 1-loop.

Summary

$$\gamma_m^{\overline{IREG}}(\Lambda^2) = 2\frac{\Lambda^2}{m^{\overline{IREG}}}\frac{\partial m^{\overline{IREG}}}{\partial \Lambda^2}$$

01

IREG

$$\gamma_m^{\overline{IREG}}(\Lambda^2) = \frac{6g_s^2}{(4\pi)^2}C_2(r) + \ldots$$

02

DReg

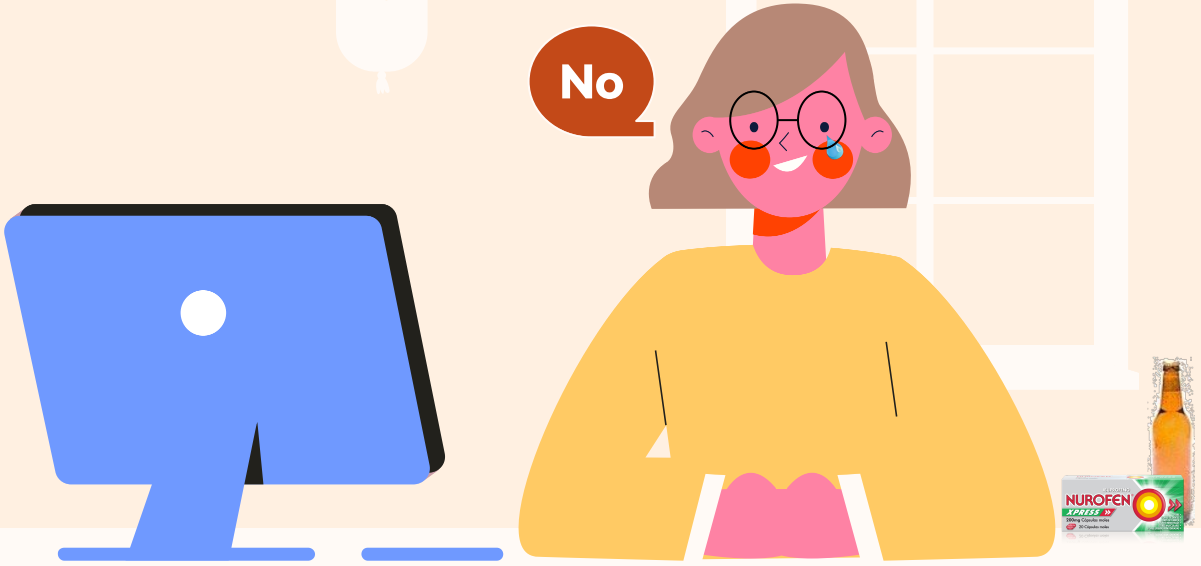
$$\gamma_m^{\overline{MS}}(\mu^2) = \frac{6g_s^2}{(4\pi)^2}C_2(r) + \ldots$$

03

DRED

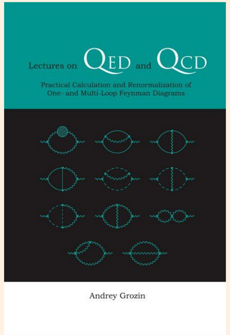
$$\gamma_m^{\overline{DRED}}(\mu^2) = \frac{6g_s^2}{(4\pi)^2}C_2(r) + \ldots$$

Will that be the case at 2-loop?

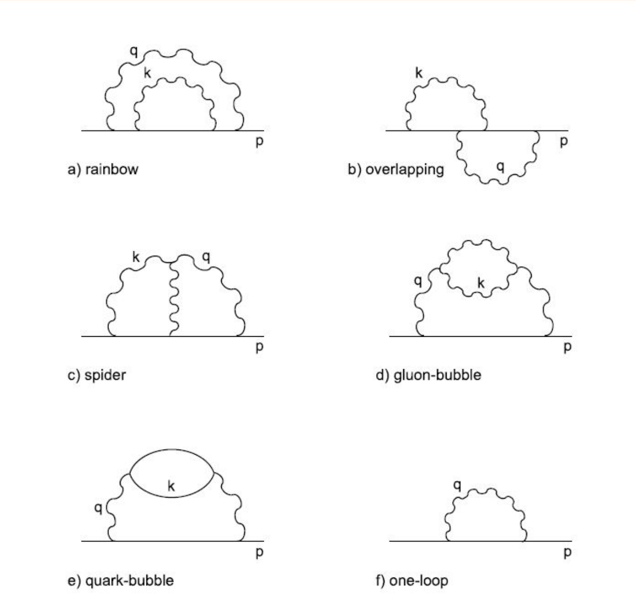


2-loop: Perspectives and things to do

How is γm at 2-loop?
We need to know the
consistency
relationships.



2-loop: Perspectives and things to do

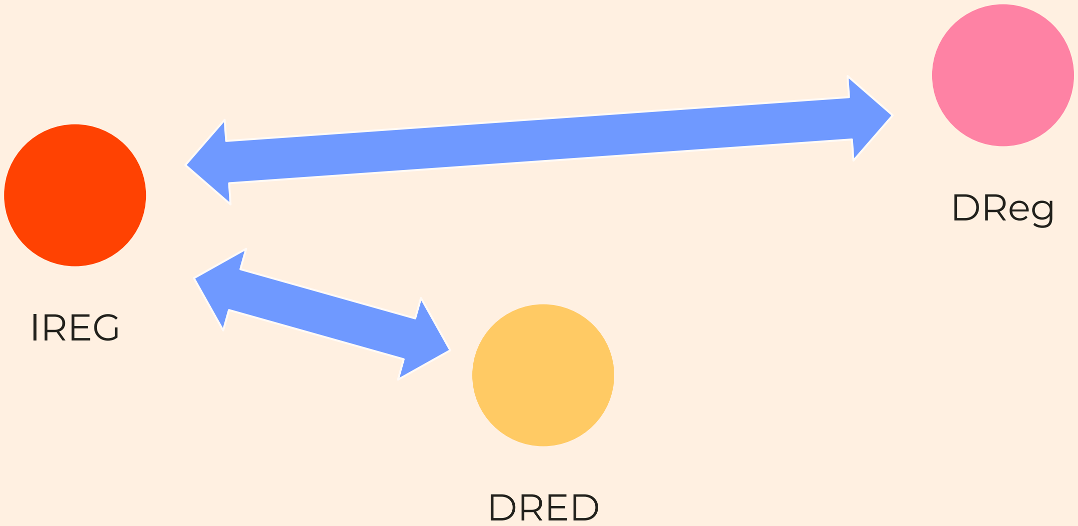


+ Evanescent diagrams

Automate

It is necessary.

Transition rules

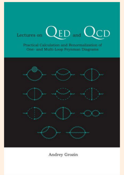


New Project Stages

Literature

01

γ_m at 2-loop: consistency relationships.



2-loops computations

02

Transition rules

03

?

Next Week

00 — 1-loop finite terms

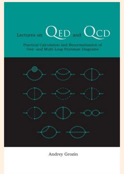
$$m^{\overline{\mathrm{DR}}} = m^{\overline{\mathrm{MS}}}\left[1 - \frac{\alpha_e}{\pi}\frac{1}{4}C_F + \ldots\right.$$

[Robert V. Harlander *et al* JHEP09(2006)053]

How is this for IREG?

01 — Literature

γ_m at 2-loop: consistency relationships.



02 — 2-loops computations

03 — Transition rules

?

Thanks

Does anyone have any questions?

(10 min)