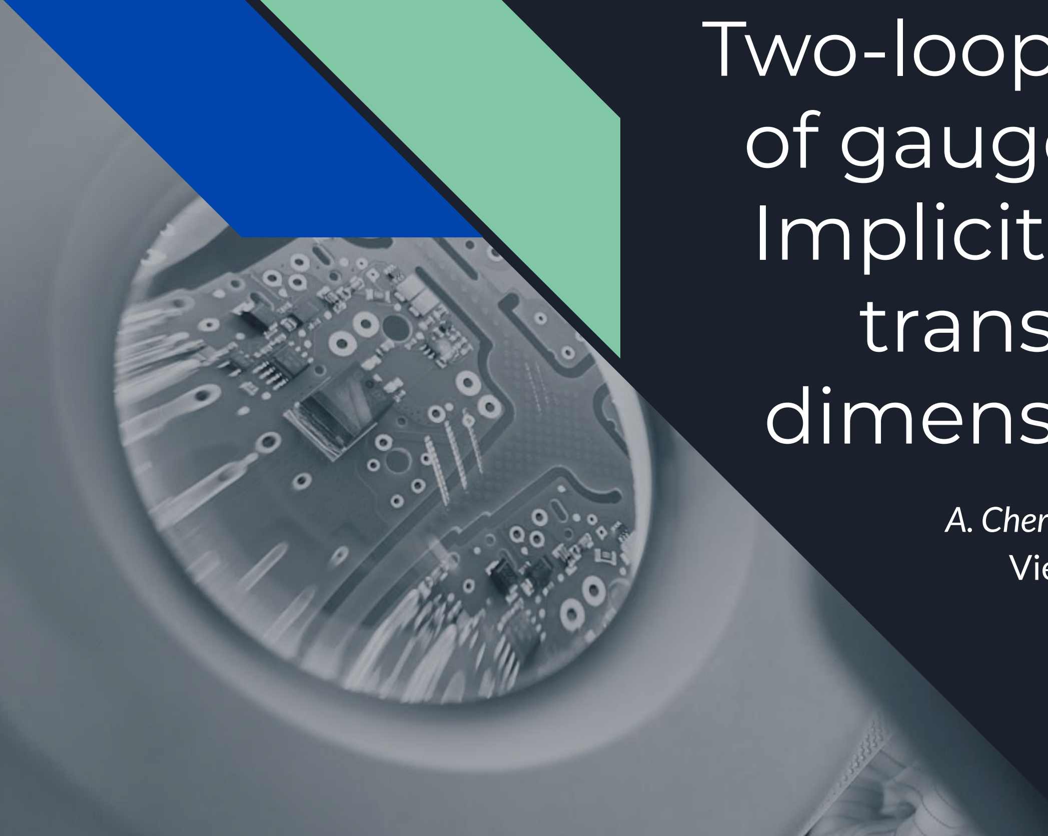
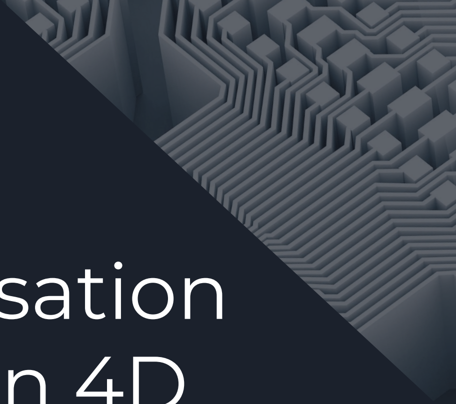




Two-loop renormalisation of gauge theories in 4D Implicit Regularisation: transition rules to dimensional methods.

A. Cherchiglia, D. C. Arias-Perdomo, A. R. Vieira, M. Sampaio and B. Hiller

arXiv:2006.10951 [hep-ph]

ABSTRACT

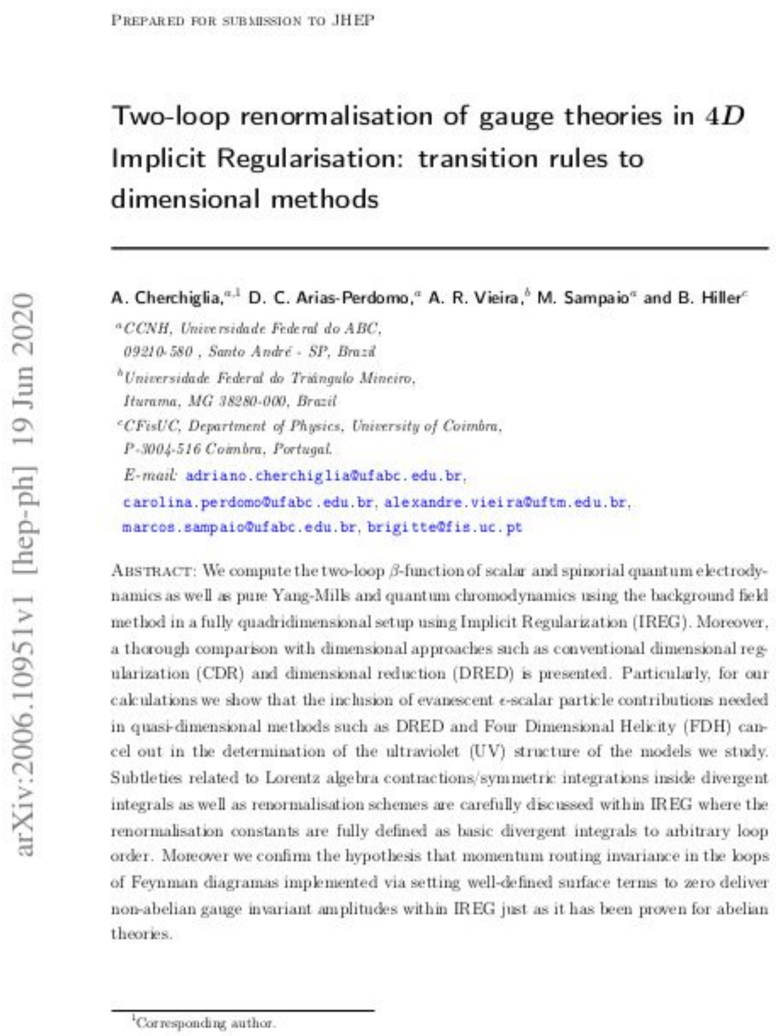
- We compute the **2-loop β-function** of scalar and spinorial **QED** as well as pure **YM** and **QCD** using the **background field method** (BFM) in a fully setup using **Implicit Regularization** (IREG).
- A thorough comparison with dimensional approaches such as **conventional dimensional regularization (CDR)** and **dimensional reduction (DRED)** is presented.
- Subtleties related to Lorentz algebra contractions/symmetric integrations inside divergent integrals as well as renormalisation schemes are carefully discussed within IREG.
- We confirm the hypothesis that **momentum routing invariance** (MRI) in the loops of Feynman diagrams implemented via setting well-defined **surface terms** to zero deliver non-abelian gauge invariant amplitudes within IREG.

Contents

Motivations

Unravelling physics **beyond the standard model** (BSM) has requested theoretical prediction for particle physics precision observables beyond **next-to-leading-order** (NLO). Such predictions rely on involved Feynman diagram calculations to evaluate scattering amplitudes both in the SM and its extensions.

On the other hand, precise measurements and calculations of known particles and interactions are just as important to validate, redress or refute new models.



Prepared for submission to JHEP

Two-loop renormalisation of gauge theories in $4D$ Implicit Regularisation: transition rules to dimensional methods

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Abstract: We compute the two-loop β-function of scalar and spinorial quantum electrodynamics as well as pure Yang-Mills and quantum chromodynamics using the background field method in a fully quadridimensional setup using Implicit Regularization (IREG). Moreover, a thorough comparison with dimensional approaches such as conventional dimensional regularization (CDR) and dimensional reduction (DRED) is presented. Particularly, for our calculations we show that the inclusion of evanescent ϵ-scalar particle contributions needed in quasi-dimensional methods such as DRED and Four Dimensional Helicity (FDH) cancel out in the determination of the ultraviolet (UV) structure of the models we study. Subtleties related to Lorentz algebra contractions/symmetric integrations inside divergent integrals as well as renormalisation schemes are carefully discussed within IREG where the renormalisation constants are fully defined as basic divergent integrals to arbitrary loop order. Moreover we confirm the hypothesis that momentum routing invariance in the loops of Feynman diagramas implemented via setting well-defined surface terms to zero deliver non-abelian gauge invariant amplitudes within IREG just as it has been proven for abelian theories.

[1]Corresponding author.

Understanding the difficulties

01

Ultraviolet (**UV-div**) and **infrared** (**IR-div**) divergences are all-over beyond leading order in S-matrix calculations and must be wisely removed in order to automated computation codes for the evaluation of Feynman amplitudes.

02

A general cross section in QCD usually includes short and long-distance behaviour and thus it is not computable directly in perturbation theory.

$$\mathcal{M}_n\left(\frac{\mu}{\lambda_{IR}}, \frac{p_i}{\lambda_{UV}}, \alpha_s(\lambda_{UV})\right) = Z\left(\frac{\mu}{\lambda_{IR}}, \frac{p_i}{\lambda_{IR}}, \alpha_s(\lambda_{IR})\right) H_n\left(\frac{p_i}{\lambda_{UV}}, \frac{\lambda_{IR}}{\lambda_{UV}}, \alpha_s(\lambda_{UV})\right), \quad (1.1)$$

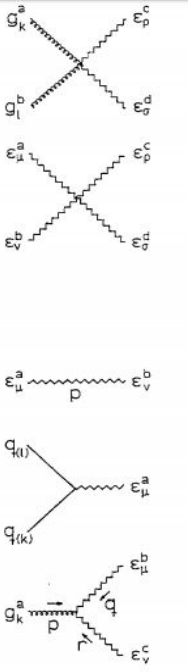
- In order to tackle the problems discussed above, the choice of a **regularisation scheme** for UV-div or IR-div in Feynman amplitudes matters.
- Novel schemes have been proposed aiming at improving **conventional dimensional regularisation (CDR)**. For example: **dimensional reduction (DRED)**.
- Modifications on CDR can lead to additional terms at the Lagrangian level, such as the ε-scalar particles.

$$\mathscr{A}_4 = \underbrace{\mathscr{A}_d}_{4\text{-}2\epsilon} + \underbrace{\mathscr{A}_N}_{2\epsilon}$$

$$\mathcal{L}^{(4)} = \mathcal{L}^{(d)} + \mathcal{L}^{(2\epsilon)}$$

$$\begin{aligned}\mathscr{L}^{(\varepsilon)} = & -\tfrac{1}{2}(\partial^i \mathscr{G}^\rho_a)^2 + g f_{abc}(\partial^i \mathscr{G}^\rho_a) G_{bi} \mathscr{G}_{c\rho} - g (T_a)_{kl} \mathscr{G}^\rho_a \bar{\psi}_k \gamma_\rho \psi_l \\ & -\tfrac{1}{2} g^2 f_{abc} f_{ade} G^i_b \mathscr{G}^\rho_c G_{di} \mathscr{G}_{e\rho} \\ & -\tfrac{1}{4} g^2 f_{abc} f_{ade} \mathscr{G}^\rho_b \mathscr{G}^\sigma_c \mathscr{G}_{d\rho} \mathscr{G}_{e\sigma}. \qquad (2.8)\end{aligned}$$

Körner, J.G., Tung, M.M. Dimensional reduction methods in QCD. *Z. Phys. C - Particles and Fields* 64, 255–265 (1994). https://doi.org/10.1007/BF01557396



$$-ig_s^2 (f^{ace} f^{bde} + f^{ade} f^{bce})\, g^{(N)}_{kl} g^{(\varepsilon)}_{\rho\sigma}$$

$g^{(N)} g^{(N)} \varepsilon\varepsilon$ vertex

$$-ig_s^2 f^{abe} f^{cde} (g^{(\varepsilon)}_{\mu\rho} g^{(\varepsilon)}_{\nu\sigma} - g^{(\varepsilon)}_{\mu\sigma} g^{(\varepsilon)}_{\nu\rho})$$
$$-ig_s^2 f^{ace} f^{bde} (g^{(\varepsilon)}_{\mu\nu} g^{(\varepsilon)}_{\rho\sigma} - g^{(\varepsilon)}_{\mu\sigma} g^{(\varepsilon)}_{\nu\rho})$$
$$-ig_s^2 f^{ade} f^{cbe} (g^{(\varepsilon)}_{\mu\rho} g^{(\varepsilon)}_{\nu\sigma} - g^{(\varepsilon)}_{\mu\nu} g^{(\varepsilon)}_{\sigma\rho})$$

$\varepsilon\varepsilon\varepsilon\varepsilon$ vertex

$$-i \frac{g^{(\varepsilon)}_{\mu\nu}}{p^2} \delta^{ab}$$

ε propagator

$$-ig_s \gamma^{(\varepsilon)}_\mu (T^a)_{kl}$$

$q\bar{q}\varepsilon$ vertex

$$-g_s f^{abc} (q-r)_k\, g^{(\varepsilon)}_{\mu\nu}$$

$g^{(N)} \varepsilon\varepsilon$ vertex

Fig. 1. Feynman rules for ε-scalars (all momenta flow into the vertices)

Alternatively, some schemes that operate only on the physical dimension of the underlying model have been constructed. **Implicit Regularization (IRED)** is one of these schemes.

Pre-print outline

In this contribution we focus on the UV renormalisation of scalar/spinorial QED and QCD to 2-loop order within IREG. Working entirely in four dimensions, we calculate the **β-functions** using the **background field method (BFM)** in a **minimal subtraction scheme** of the **basic divergent integrals (BDI's)** in internal momenta.

Contents

Survey of regularisation schemes and IREG rules

$$\int_{-\infty}^{+\infty} \frac{d^4 k_{[4]}}{(2\pi)^4} \to \underbrace{\mu^{4-d}}_{\text{scale}} \int_{-\infty}^{+\infty} \frac{d^d k_{[d]}}{(2\pi)^d},$$

- We analytically continue the integrals into d dimensions, where d = 4 − 2ε.
- Some variants of CDR such as DRED have been developed.
- In dimensional methods both UV and IR infinities appear as poles **1/ε.**
- For an efficient computational code to evaluate Feynman amplitudes beyond leading order, UV-div and IR-div ought to be subtracted by a scheme that respects **unitarity, causality and symmetries.**

Eur. Phys. J. C (2017) 77:471
DOI 10.1140/epjc/s10052-017-5023-2
THE EUROPEAN PHYSICAL JOURNAL C
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Regular Article - Theoretical Physics

To *d*, or not to *d*: recent developments and comparisons of regularization schemes

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Received: 5 May 2017 / Accepted: 24 June 2017 / Published online: 14 July 2017

	DReg	DRED
Internal gluon	$\hat{g}_{\mu\nu}$	$g^{\mu\nu}$
External gluon	$\hat{g}_{\mu\nu}$	$g^{\mu\nu}$

Treatment of internal and external gluons in dimensional methods

The rules of IREG

- Satisfies Lorentz invariance, locality and unitarity.
- Systematisation to arbitrary loop order.

International Journal of Modern Physics A
Vol. 26, No. 15 (2011) 2591–2635

DOI: 10.1142/S0217751X11053419

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SYSTEMATIC IMPLEMENTATION OF IMPLICIT REGULARIZATION FOR MULTILOOP FEYNMAN DIAGRAMS

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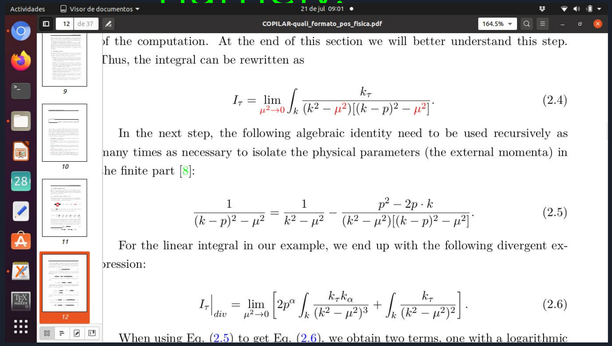
Received 27 January 2011
Revised 22 March 2011

Implicit Regularization (IReg) is a candidate to become an invariant framework in momentum space to perform Feynman diagram calculations to arbitrary loop order. In this work we present a systematic implementation of our method that automatically displays the terms to be subtracted by Bogoliubov's recursion formula. Therefore, we achieve a twofold objective: we show that the IReg program respects unitarity, locality and Lorentz invariance and we show that our method is consistent since we are able to display the divergent content of a multiloop amplitude in a well-defined set of basic divergent integrals in one-loop momentum only which is the essence of IReg. Moreover, we conjecture that momentum routing invariance in the loops, which has been shown to be connected with gauge symmetry, is a fundamental symmetry of any Feynman diagram in a renormalizable quantum field theory.

1. Perform the usual Dirac algebra in the physical dimension. Care must be exercised in avoiding symmetric integration in divergent amplitudes, namely,



of the computation. At the end of this section we will better understand this step. Thus, the integral can be rewritten as

$$I_\tau = \lim_{\mu^2 \to 0} \int_k \frac{k_\tau}{(k^2-\mu^2)[(k-p)^2-\mu^2]}. \tag{2.4}$$

In the next step, the following algebraic identity need to be used recursively as many times as necessary to isolate the physical parameters (the external momenta) in the finite part [8]:

$$\frac{1}{(k-p)^2-\mu^2} = \frac{1}{k^2-\mu^2} - \frac{p^2-2p\cdot k}{(k^2-\mu^2)[(k-p)^2-\mu^2]}. \tag{2.5}$$

For the linear integral in our example, we end up with the following divergent expression:

$$I_\tau\Big|_{div} = \lim_{\mu^2\to 0}\left[2p^\alpha \int_k \frac{k_\tau k_\alpha}{(k^2-\mu^2)^3} + \int_k \frac{k_\tau}{(k^2-\mu^2)^2}\right]. \tag{2.6}$$

When using Eq. (2.5) to get Eq. (2.6), we obtain two terms, one with a logarithmic

Notation

$$\int_k \equiv \int d^4k/(2\pi)^4$$

$$\int_k k^{\mu_1}\cdots k^{\mu_{2m}} f(k^2) = \frac{1}{(2m)!}\int_k g^{\{\mu_1\mu_2}\cdots g^{\mu_{2m-1}\mu_{2m}\}} f(k^2)$$

Examples:

$$k^\mu k^\nu \to \frac{1}{d}k^2$$

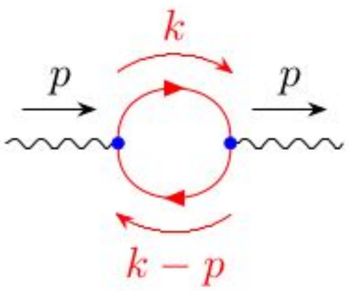
$$k^\mu k^\nu k^\rho k^\sigma \to \frac{1}{d(d+2)}(k^2)^2(g^{\mu\nu}g^{\rho\sigma} + g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho})$$

2. Remove external momenta and define BDI.

$$\frac{1}{(k-p)^2-\mu^2} = \frac{1}{k^2-\mu^2} - \frac{p^2-2p\cdot k}{(k^2-\mu^2)[(k-p)^2-\mu^2]}.$$

$$I_{log}(\mu^2) \equiv \int_k \frac{1}{(k^2-\mu^2)^2},$$

$$I_{quad}(\mu^2) \equiv \int_k \frac{1}{(k^2-\mu^2)},$$



How can we understand this? See this example where D = (k²)(k − p)².

$$\Pi_{\tau\nu}(k) = -4e^2 \left[2\underbrace{\int_k \frac{k_\tau k_\nu}{D}}_{\Delta=2} - p_\tau \underbrace{\int_k \frac{k_\nu}{D}}_{\Delta=1} - p_\nu \underbrace{\int_k \frac{k_\tau}{D}}_{\Delta=1} - g_{\tau\nu} \underbrace{\int_k \frac{k^2}{D}}_{\Delta=2} + g_{\tau\nu} p^\sigma \underbrace{\int_k \frac{k_\sigma}{D}}_{\Delta=1} + g_{\tau\nu} \underbrace{\int_k \frac{1}{D}}_{\Delta=0} \right]$$

$$I = \int_k \frac{1}{D} = \int_k \frac{1}{(k^2)(k-p)^2} = \lim_{\mu^2 \to 0} \int_k \frac{1}{(k^2-\mu^2)[(k-p)^2-\mu^2]},$$

$$\frac{1}{(k-p)^2-\mu^2} = \frac{1}{k^2-\mu^2} - \frac{p^2-2p\cdot k}{(k^2-\mu^2)[(k-p)^2-\mu^2]}.$$

$$I_{log}(\mu^2) \equiv \int_k \frac{1}{(k^2-\mu^2)^2}$$

$$I = I\Big|_{div} + I\Big|_{finite} = I_{log}(\mu^2) - \lim_{\mu^2 \to 0} \int_k \frac{(p^2-2pk)}{(k^2-\mu^2)^2[(k-p)^2-\mu^2]}.$$

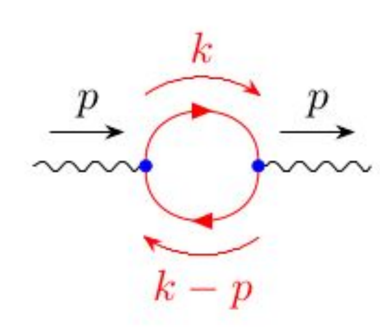
$$I = \int_k \frac{1}{D} = \int_k \frac{1}{(k^2)(k-p)^2}$$

$$I_\tau = \int_k \frac{k_\tau}{D} = \int_k \frac{k_\tau}{(k^2)(k-p)^2}$$

?

3. BDI’s with Lorentz indices may be written as linear combinations of BDI’s without Lorentz indices (with the same superficial degree of divergence) plus well defined **surface terms (ST’s)**.

Another example:



From Eur. Phys. J. C 77, 471 (2017) where D = (k²)(k − p) ².

$$\Pi_{\tau\nu}(k) = -4e^2 \left[2 \underbrace{\int_k \frac{k_\tau k_\nu}{D}}_{\Delta=2} - p_\tau \underbrace{\int_k \frac{k_\nu}{D}}_{\Delta=1} - p_\nu \underbrace{\int_k \frac{k_\tau}{D}}_{\Delta=1} - g_{\tau\nu} \underbrace{\int_k \frac{k^2}{D}}_{\Delta=2} + g_{\tau\nu} p^\sigma \underbrace{\int_k \frac{k_\sigma}{D}}_{\Delta=1} + g_{\tau\nu} \underbrace{\int_k \frac{1}{D}}_{\Delta=0} \right]$$

$$I_\tau = \lim_{\mu^2 \to 0} \int_k \frac{k_\tau}{(k^2 - \mu^2)[(k-p)^2 - \mu^2]}.$$

$$I_\tau = \lim_{\mu^2 \to 0} \int_k \frac{k_\tau}{(k^2 - \mu^2)[(k-p)^2 - \mu^2]}.$$

$$\frac{1}{(k-p)^2 - \mu^2} = \frac{1}{k^2 - \mu^2} - \frac{p^2 - 2p \cdot k}{(k^2 - \mu^2)[(k-p)^2 - \mu^2]}.$$

$$I_{log}^{\nu_1 \nu_2}(\mu^2) \equiv \int_k \frac{k_{\nu_1} k_{\nu_2}}{(k^2 - \mu^2)^3}$$

Surface term (ST)

$$I_\tau \Big|_{div} = \lim_{\mu^2 \to 0} \left[2p^\alpha \int_k \frac{k_\tau k_\alpha}{(k^2 - \mu^2)^3} + \int_k \frac{k_\tau}{(k^2 - \mu^2)^2} \right]$$

$$+$$

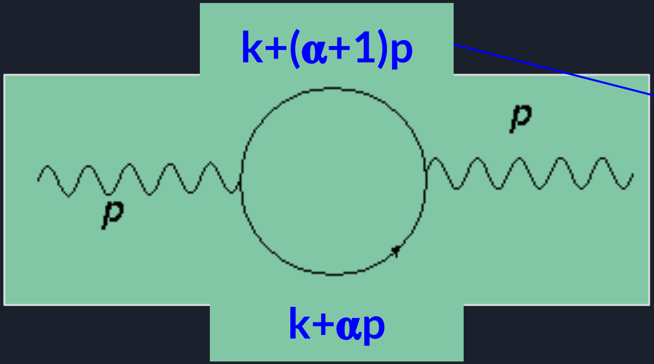
$$I_\tau \Big|_{finite} = -p^2 \lim_{\mu^2 \to 0} \int_k \frac{k_\tau}{(k^2 - \mu^2)^3} + \lim_{\mu^2 \to 0} \int_k k_\tau \frac{p^2 - 2pk}{(k^2 - \mu^2)^3[(k-p)^2 - \mu^2]}$$

$$\Upsilon_0^{(1)\mu\nu} = \int_k \frac{\partial}{\partial k_\mu} \frac{k^\nu}{(k^2-\mu^2)^2} = 4\left[\frac{g_{\mu\nu}}{4} I_{log}(\mu^2) - I_{log}^{\mu\nu}(\mu^2)\right], \quad (2.4)$$

$$\Upsilon_2^{(1)\mu\nu} = \int_k \frac{\partial}{\partial k_\mu} \frac{k^\nu}{(k^2-\mu^2)} = 2\left[\frac{g_{\mu\nu}}{2} I_{quad}(\mu^2) - I_{quad}^{\mu\nu}(\mu^2)\right]. \quad (2.5)$$

ST's: Arbitrary Regularization dependent hidden parameter of Feynman Diagram Calculations

Can they be fixed? Momentum Routing Invariance in a Feynman Diagram



α

Arbitrary Routing

MRI ⟷ **ST=0** ⟷ **Gauge Invariance**

Momentum Routing Invariance in Extended QED: Assuring Gauge Invariance Beyond Tree Level

A. R. Vieira[a,b] *, A. L. Cherchiglia[c] †, and Marcos Sampaio[a] ‡

4. An arbitrary positive (renormalisation group) mass scale λ appears via regularisation independent identities.

$$\boxed{I_{log}(\mu^2) = I_{log}(\lambda^2) - \frac{i}{(4\pi)^2} \ln \frac{\lambda^2}{\mu^2}}$$

$$i/(4\pi)^2 = b.$$

Deduction:

$$I_{log}(\mu^2) \equiv \int_k \frac{1}{(k^2-\mu^2)^2}$$

$$\frac{d}{d\mu^2} I_{log}(\mu^2) = \int_k \frac{2}{(k^2-\mu^2)^3}$$

$$\frac{d}{d\mu^2} I_{log}(\mu^2) = -\frac{b}{\mu^2}$$

5. At 2-loop order the divergent content can be expressed in terms of BDI in one loop momentum after performing 1 integration.

$$\int_k \frac{1}{(k+p)^2} \ln\left(-\frac{k^2}{\lambda^2}\right)$$

$$\mathcal{I} = \int_{k_l} G(k_l, p_i, \mu^2) \ln^{2-1}\left[-\frac{(k_l^2-\mu^2)}{\Lambda^2}\right]$$

$$\int_k \frac{1}{k^2(k+p)^2} \ln\left(-\frac{k^2}{\lambda^2}\right)$$

$$\int_k \frac{k_\mu}{k^2(k+p)^2} \ln\left(-\frac{k^2}{\lambda^2}\right)$$

$$\int_k \frac{k_\mu k_\nu}{k^2(k+p)^2} \ln\left(-\frac{k^2}{\lambda^2}\right)$$

6. The BDIs are cast as:

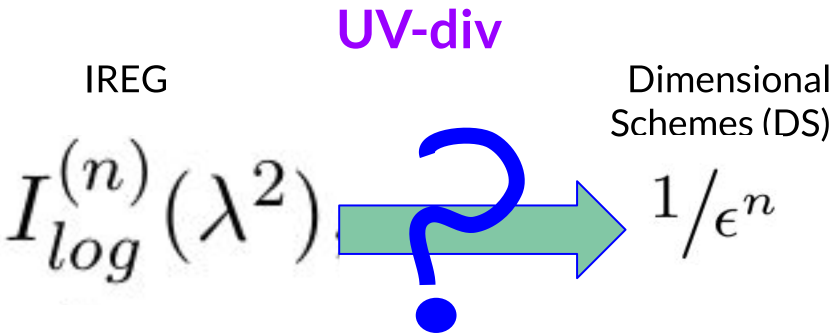
$$I_{log}^{(2)}(\mu^2) \equiv \int_{k_l} \frac{1}{(k_l^2-\mu^2)^2} \ln^{2-1}\left(-\frac{k_l^2-\mu^2}{\Lambda^2}\right),$$

$$I_{log}^{(2)\nu_1\cdots\nu_{2r}}(\mu^2) \equiv \int_{k_l} \frac{k_l^{\nu_1}\cdots k_l^{\nu_{2r}}}{(k_l^2-\mu^2)^{r+1}} \ln^{2-1}\left(-\frac{k_l^2-\mu^2}{\Lambda^2}\right),$$

$$I_{quad}^{(2)}(\mu^2) \equiv \int_{k_l} \frac{1}{(k_l^2-\mu^2)} \ln^{l-1}\left(-\frac{k_l^2-\mu^2}{\lambda^2}\right).$$

Correspondence among IREG and dimensional methods

- We analyse to which extent it is possible to recover results for amplitudes evaluated by dimensional methods once the result in IREG is known.



$$I = \int_k \frac{1}{D} = \int_k \frac{1}{(k^2)(k-p)^2}$$

Example at 1-loop order

$$\overset{\text{IREG}}{=} I_{log}(\lambda^2) - b\ln\left[-\frac{p^2}{\lambda^2}\right] + 2b$$

$$I^d_{log}(\lambda^2) = (\mu_{DR})^{4-d}\int \frac{d^dk}{(2\pi)^d}\frac{1}{(k^2-\lambda^2)^2}$$

$$= b\left[\frac{1}{\epsilon} - \gamma_E + \ln(4\pi) + \ln\left(\frac{\mu^2_{DR}}{\lambda^2}\right)\right]$$

$$I\Big|_d = b\left[\frac{1}{\epsilon} - \gamma_E + \ln(4\pi) - \ln\left[-\frac{p^2}{\mu^2_{DR}}\right] + 2\right]$$

Same result as DS!

Can we extended this approach to higher orders?

Example at n-loop order

$$\mathcal{J} = \int_{k_1} \frac{1}{k_1^2(k_1-p)^2} \cdots \int_{k_n} \frac{1}{k_n^2(k_n-p)^2}.$$

$$= \left[I_{log}(\lambda^2) - b\ln\left[-\frac{p^2}{\lambda^2}\right] + 2b \right]^n.$$

$$I_{log}^d(\lambda^2) = (\mu_{DR})^{4-d} \int \frac{d^dk}{(2\pi)^d} \frac{1}{(k^2-\lambda^2)^2}$$

$$= \frac{b}{(4\pi)^{-\epsilon}} \left(\frac{\mu_{DR}^2}{\lambda^2}\right)^{\epsilon} \Gamma(\epsilon)$$

and

$$\ln\left(-\frac{p^2}{\lambda^2}\right) = \lim_{\epsilon \downarrow 0} \frac{\Gamma(\epsilon)}{(4\pi)^{-\epsilon}} \left[\left(\frac{\mu_{DR}^2}{\lambda^2}\right)^{\epsilon} - \left(-\frac{\mu_{DR}^2}{p^2}\right)^{\epsilon} \right]$$

$$\boxed{\mathcal{J}^{\mathrm{IREG}}\Big|_d = b^n \left[\frac{1}{(4\pi)^{-\epsilon}} \left(-\frac{\mu_{DR}^2}{p^2}\right)^{\epsilon} \Gamma(\epsilon) + 2 \right]^n}$$

Only agree in

ϵ^{-n} and ϵ^{-n+1}

Commercial Pause
Doubts?

Contents

Practical approach to multi-loop calculation

It is to be useful to develop a minimal set of rules. We propose, regarding multi-loop calculations:

- ST's=0.
- Avoid symmetric integration.
- Apply Feynman rules to the **whole** multi-loop diagram.
- Perform contractions (in four-dimensions) and simplify numerator against denominator if possible.

$$g_{\mu\nu}\int_k k^\mu k^\nu f(k)\Big|_{\mathrm{IREG}} \neq \int_k k^2 f(k)\Big|_{\mathrm{IREG}}$$

$$\int_{k,q}\frac{k^2}{k^2q^2(k-q)^2}\Big|_{\mathrm{IREG}} = \int_{k,q}\frac{1}{q^2(k-q)^2}\Big|_{\mathrm{IREG}}$$

- However, we don't manipulate the numerator to enforce cancellations.

$$\int_{k,q}\frac{k.p}{k^2(k-p)^2q^2(k-q)^2}\Big|_{\mathrm{IREG}} \neq \int_{k,q}\frac{k^2+p^2-(k-p)^2}{2k^2(k-p)^2q^2(k-q)^2}\Big|_{\mathrm{IREG}}.$$

After these, the rules sketched for IREG can be applied.

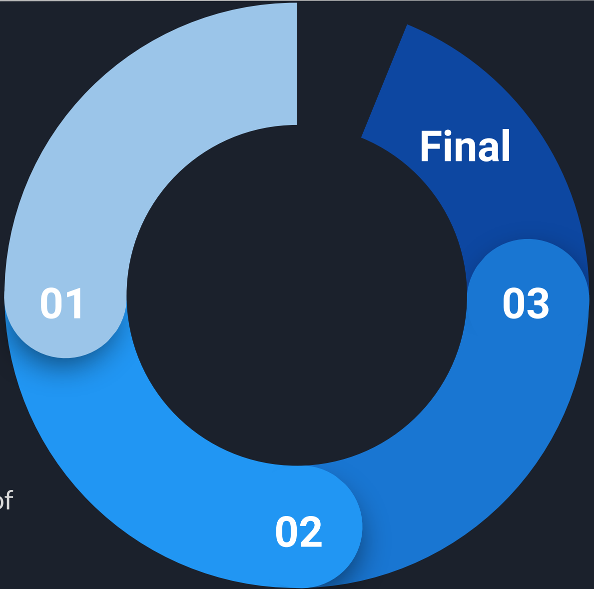
Two-loop applications in gauge theories
Algorithm

FeynArts&FormCalc

We create the topologies, compute the amplitudes and do the contractions.

Regularization

We make the necessary simplifications and reductions for the integrals. This depends of the regularization scheme,



β-function

Integral evaluation

According to the regularization method used.

The Background Field Method

$$S^{YM}(Q) = -{}^1/_4 \int d^4x (F^a_{\mu\nu})^2$$

$$Z(J) = \int \mathcal{D}_Q \det\left(\frac{\delta G^a}{\delta w^b}\right) \exp^{i(S^{YM}(Q) - {}^1/_{2\alpha} G\cdot G + J\cdot Q},$$

$$G \cdot G \equiv \int d^4x G^a G^a.$$

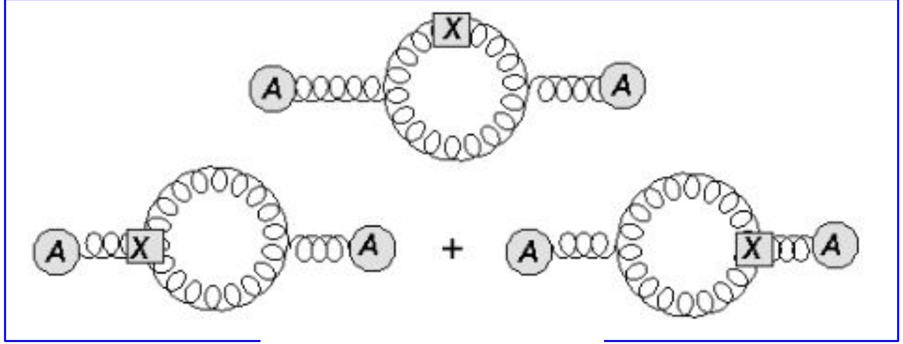
$$J \cdot Q \equiv \int d^4x J^a_\mu Q^a_\mu$$

$$\tilde{Z}(J, A) = \int \mathcal{D}_Q \det\left(\frac{\delta \tilde{G}^a}{\delta w^b}\right) \exp^{i(S^{YM}(Q+A) - {}^1/_{2\alpha} \tilde{G}\cdot\tilde{G} + J\cdot Q}$$

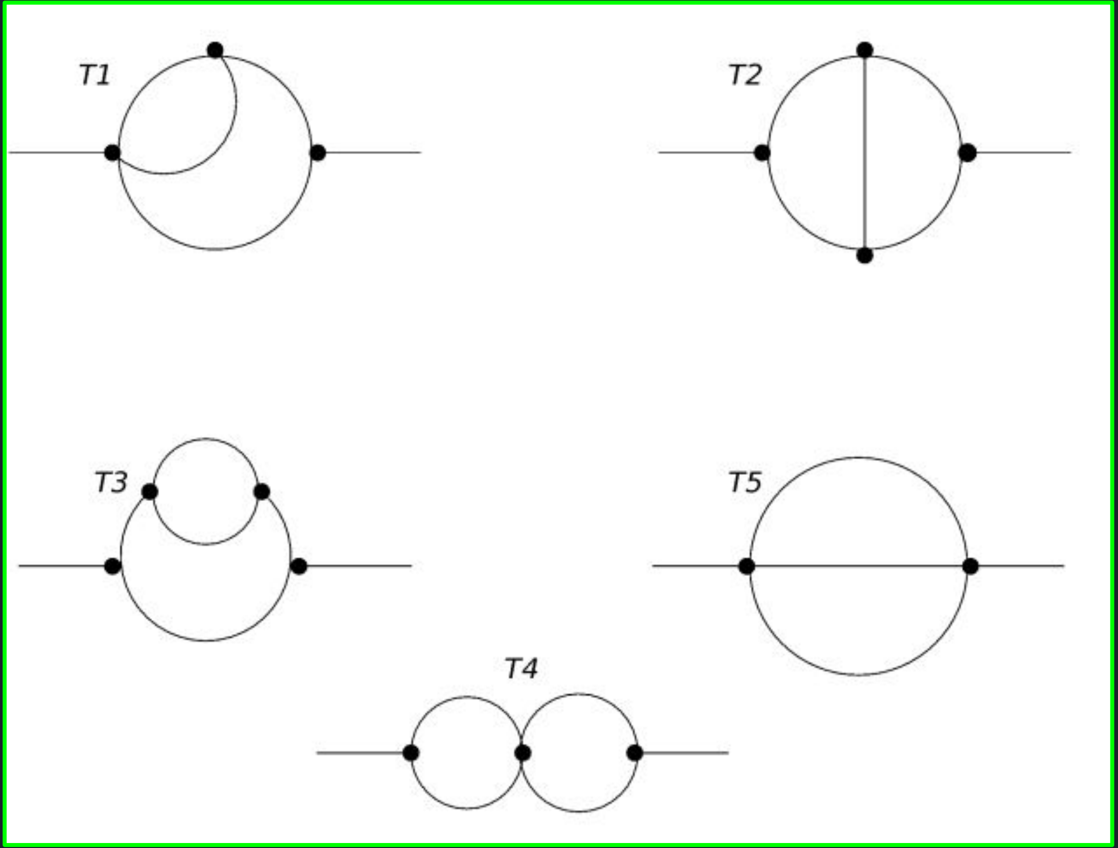
BFM renormalization

$$\boxed{Z_g = Z_{\hat{A}}^{-1/2}}$$

$$\hat{A}_o = Z_{\hat{A}} \hat{A}_r; \quad g_o = Z_g g_r; \quad \alpha_o = Z_\alpha \alpha_r,$$



$$Z_\alpha = (1 + \delta_\alpha)$$



In this contribution we will only deal with two-point functions