Dynamical Casimir Effect

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Review

Fifty Years of the Dynamical Casimir Effect

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Abstract: This is a digest of the main achievements in the wide area, called the Dynamical Casimir Effect nowadays, for the past 50 years, with the emphasis on results obtained after 2010.

Keywords: moving boundaries; photon generation from vacuum; quantum friction; effective Hamiltonians; parametric oscillator; atomic excitations; nonlinear optical materials; cavity and circuit QED; decoherence; entanglement

1. Introduction

Fifty years ago, in June 1969, G.T. Moore finished his PhD thesis (prepared under the guidance of H.N. Pendleton III and submitted to the Brandeis University), part of which was published in the next year as [1]. Using a simplified one-dimensional model, Moore showed that motions of ideal boundaries of a one-dimensional cavity could result in a generation of quanta of the electromagnetic field from the initial *vacuum quantum state*. A few years later, DeWitt [2] demonstrated that moving boundaries could induce particle creation from a vacuum in a single-mirror set-up. A more detailed study was performed by Fulling and Davies [3,4]. Thus, step by step, year by year, more and more authors followed this direction of research. In 1989, two names were suggested for such kinds of phenomena: *dynamic (or non-adiabatic) Casimir effect* [5] and *Nonstationary Casimir Effect* [6]. Being supported in part by the authority of Schwinger [7], the first name gradually acquired the overwhelming popularity, so that now we have an established direction in the theoretical and experimental physics, known under the general name *Dynamical Casimir Effect* (DCE). This area became rather large by now: more than 300 papers containing the words "dynamical Casimir" have been published already, including more than 100 publications during the past decade. Moore's paper [1] has been cited more than 400 times, and some authors use the name "Moore effect" instead of DCE (other names were "Mirror Induced Radiation" or "Motion Induced Radiation").

To combine different studies under the same "roof", it seems reasonable to assume the following definition of the Dynamical Casimir Effect: *Macroscopic phenomena caused by changes of vacuum quantum states of fields due to fast time variations of positions (or properties) of boundaries confining the fields (or other parameters)*. Such phenomena include, in particular, the modification of the Casimir force for moving boundaries. However, the most important manifestation is *the creation of the field quanta (photons) due to the motion of neutral boundaries*. The most important ingredients of the DCE are *quantum vacuum fluctuations* and *macroscopic manifestations*. The reference to vacuum fluctuations explains the appearance of Casimir's name (by analogy with the famous *static* Casimir effect, which is also considered frequently as a manifestation of quantum vacuum fluctuations), although Casimir himself did not write anything on this subject. Therefore, the DCE can be considered as the specific subfield of a much bigger physical area, known nowadays under the name *Casimir Physics*. This whole area is outside the present study, so that we give only a few references to the relevant reviews and books [8–24].

In turn, the subject of the DCE can be divided in several sub-areas. In the strict (narrow) sense, one can think about the "single mirror DCE" or "cavity DCE". In the most wide sense, the DCE can be related to the *amplification of quantum (vacuum) fluctuations in macroscopic systems*, and many authors

Fifty years of Dynamical Casimir Effect, V. Dodonov, Physics 2020, 2, 67-104, doi: 10.3390/physics2010007.

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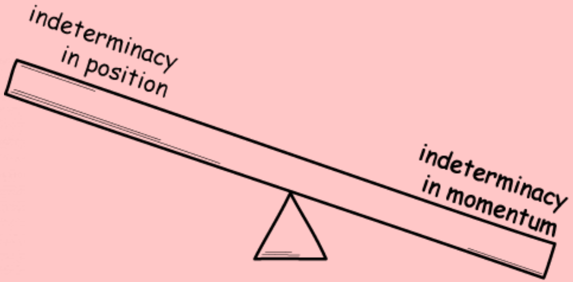
Overview

The Dynamical Casimir Effect (DCE)

Three basic “ideas” to keep in mind

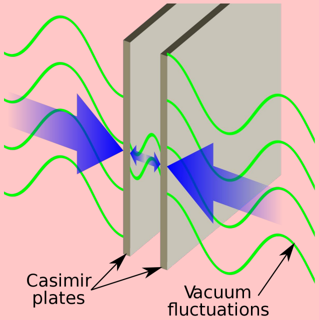
Quantum fluctuation

Around 1925 QM confirms the existence of Vacuum Fluctuations (by Heisenberg, Dirac and many others).



Uncertainty principle

$$\Delta x \, \Delta p \geqslant \hbar/2$$



Static Casimir Effect

Simplest setup: two parallel perfectly conducting plates at finite distance R.

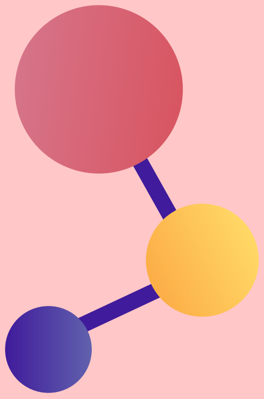
Mathematics. — *On the attraction between two perfectly conducting plates*. By H. B. G. CASIMIR.

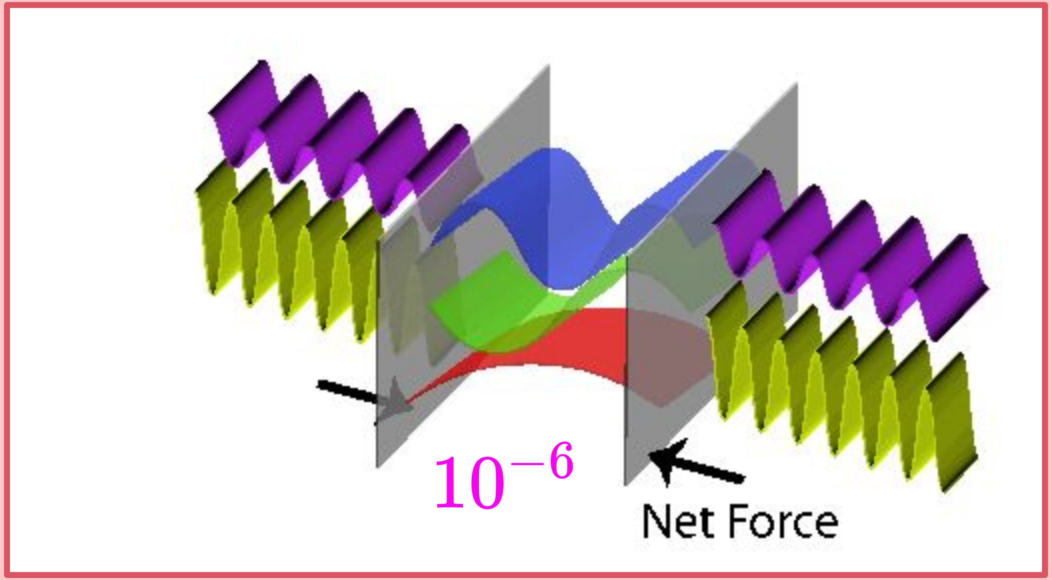
(Communicated at the meeting of May 29, 1948.)

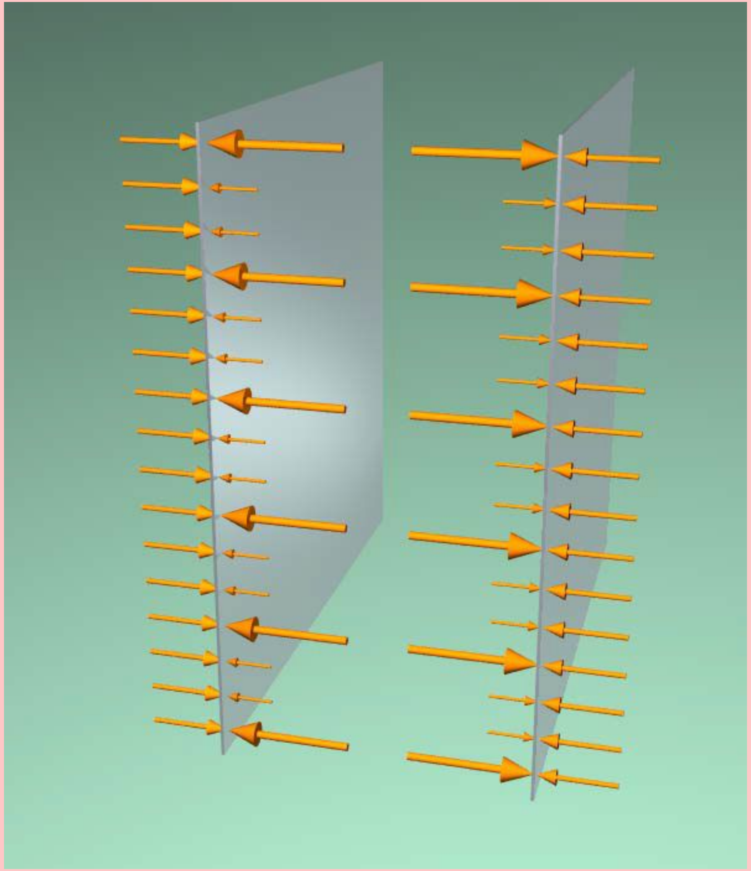
What is the DCE?

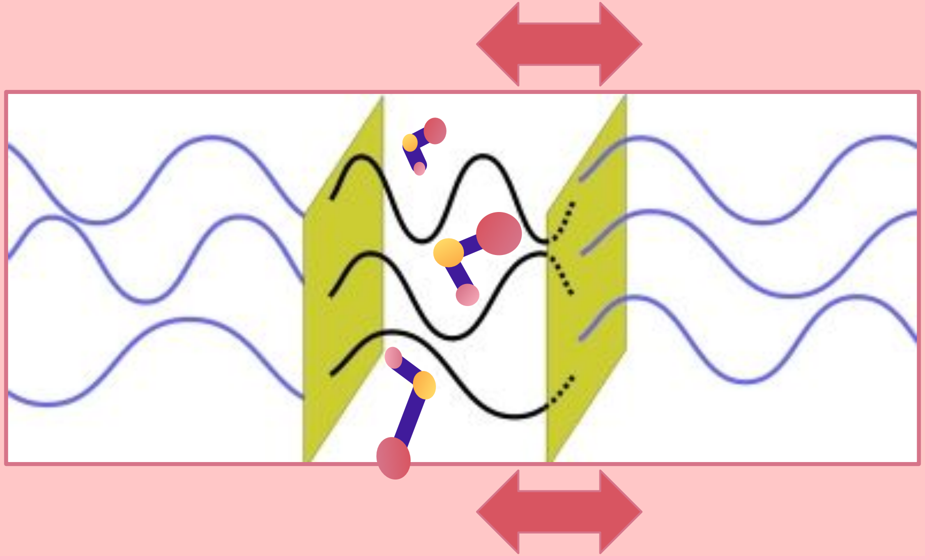
"Macroscopic phenomena caused by changes of **vacuum** quantum states of fields due to **fast time variations** of positions (or **properties**) of **boundaries** confining the fields."

- The modification of the Casimir force for moving boundaries.




10^{-6}
Net Force





By Giuseppe Ruoso - Les Houches - 09/06/2005

What is the DCE?

“Macroscopic phenomena caused by changes of **vacuum** quantum states of fields due to **fast time variations** of positions (or **properties**) of **boundaries** confining the fields.”

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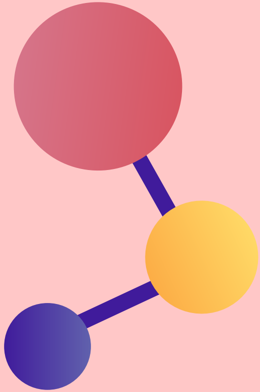
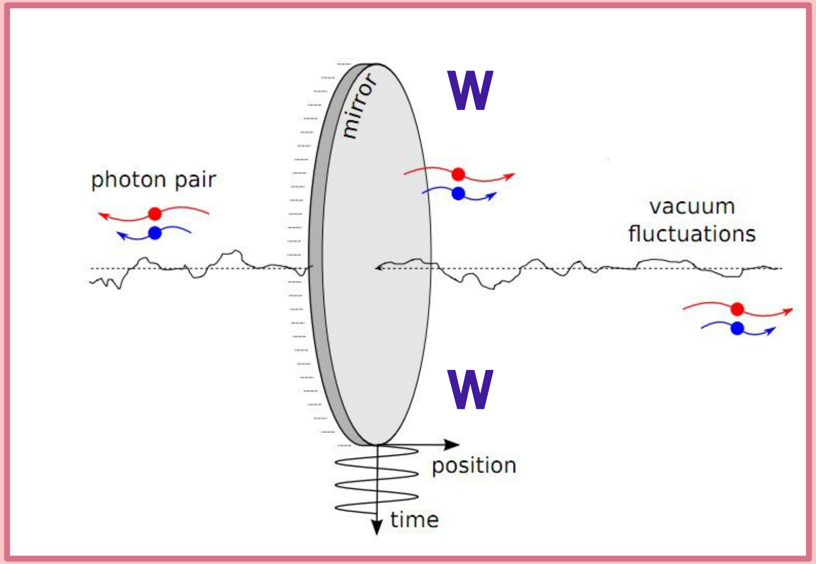
“However, the most important manifestation is the creation of the field quanta (photons) due to the motion of neutral boundaries.”




mirror
W
photon pair
vacuum
fluctuations
W
position
time

Who was the first?

G. T. **Moore**, Quantum theory of electromagnetic field in a variable-length **one-dim**ensional cavity,
J. Math. Phys. 11, 2679 (1970)

The same physics under different names

- SCE.
- Single mirror DCE.
- Cavity DCE.
- Quantum circuits with time-dependent parameters.

02 DCE sub-areas

1-dim and 3-dim Models with Moving Boundaries

One-Dimensional Models with Moving Boundaries: classic fields

Consider a massless scalar field A(x,t) in 1-dim space (or dimension 1+1). The Hamiltonian for A(x,t) in a non-stationary cavity is:

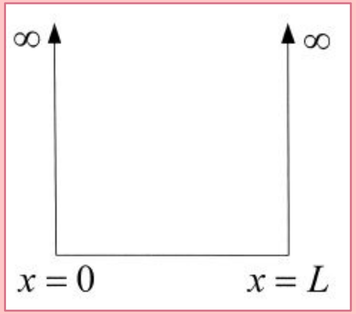
$$\left[\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2}\right] A(x,t) = 0$$

$$0 < x < L(t)$$

- We impose the following boundary conditions which simulate the presence of the plates: $A(0,t) = A(L(t),t) = 0$
- The solutions was obtaining for: $L(t) = L_0(1 + \alpha t)$

One-Dimensional Models with Moving Boundaries: quantum fields

In scalar EM, the field A(x,t) is confined between two ideal surfaces.



- Coordinates: X = 0; X= L(t) > 0
- With both walls at rest for t<0, you can write the initial field: $A(x,t<0) = \sum_{n=0}^{\infty} c_n \sin\left(\frac{n\pi x}{L_0}\right) \exp(-i\omega_n t)$ $\omega_n = \frac{n\pi}{L_0}$

What happens with $A(x,t)$ when $t>0$?

A(x,t) for t>0: Moore's approach

$$\left[\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2}\right] A(x,t) = 0$$

1. The wave equation in the single spatial dimension has a solution of the form: **A(x,t) = f(x-t)+g(t+x)**
2. Moore has found a complete set of solutions to it.
3. These solutions satisfy the initial condition of the field **A(x,t<0)** and the **Dirichlet boundary conditions**.

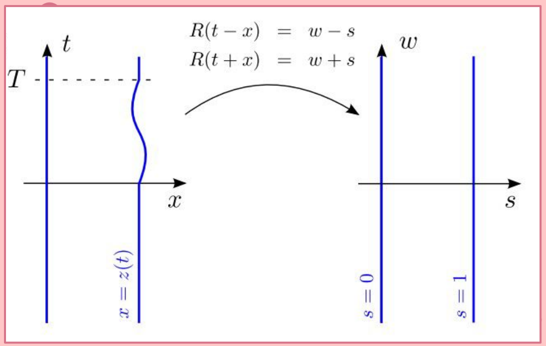
$$A_n(x,t) = C_n \left[\exp[-i\pi n R(t-x)] - \exp[-i\pi n R(t+x)]\right]$$

4. Where **R(ξ)** satisfy the functional equation: **R(t + L(t)) – R(t – L(t)) = 2**.
5. For an arbitrary non-relativistic law of motion, one can find the solution for **R(ξ)** in the form of the expansion over subsequent time derivatives of the wall displacement.

$$R(\xi) = \xi\lambda(\xi) - \tfrac{1}{2}\xi^2\dot{\lambda}(\xi) + \ldots;\ \lambda(\xi) \equiv L^{-1}(\xi).$$

6. Several exact solutions to the Moore eq. were found with the aid of the "**inverse**" method.

Generation of Quanta inside the 1D Cavity with Moving Boundary



Field operator $\hat{A}(x,t)$ at $t<0$ in the in the Heisenberg representation with both the plates at rest at $x_{left} = 0$ and $x_{right} = L_0$:

$$\hat{A}_{in} = 2\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} \sin \frac{n\pi x}{L_0} \hat{b}_n \exp(-i\omega_n t) + h.c.$$

The H operator:

$$\hat{H} = \sum_{n=1}^{\infty} w_n (\hat{b}_n^\dagger \hat{b}_n - \tfrac{1}{2})$$

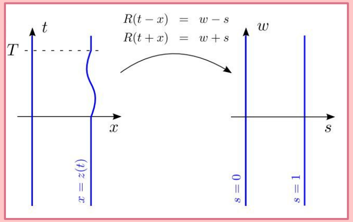
- For t>0:

$$\hat{A}(x,t) = 2\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} \left[\hat{b}_n \Psi^n(x,t)\right] + h.c.$$

- Where the function Ψ satisfy the initial conditions of the system:

$$\sum_{n=0}^{\infty} c_n \sin\left(\frac{n\pi x}{L_0}\right) \exp(-i\omega_n t)$$

- 0 < t < T: $\hat{b}_m ; \hat{b}_m^\dagger$ and t>T: $\hat{a}_m ; \hat{a}_m^\dagger$



We can related the operators by a Bogoliubov transformation:

$$\hat{a}_m = \sum_{n=1}^{\infty} (\hat{b}_n \alpha_{nm} + \hat{b}_n^\dagger \beta_{nm}^*), m = 1, 2, \ldots$$

$$N_m \equiv < in|\hat{a}_m^\dagger \hat{a}_m|in >= \sum_n |\beta_{nm}|$$

Particle production rate or distribution

Extra comment: about regularization

What happens if I want the energy of the field only between the plates, $0 < x < L$? What happens if I want the vacuum energy?

$$< 0|\hat{H}|0 >$$

- The energy is divergent.

You actually can have a formal expression for the zero-point energy. If you want to give it a mathematical sense, you have to resort to a regularization scheme

Expansions over the Instantaneous Basis

This is another approach to study the generation of quanta at 1D.

- Remember:

$$\hat{A}_{in} = 2\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} \sin \frac{n\pi x}{L_0} \hat{b}_n \exp(-i\omega_n t) + h.c.$$

$$\hat{A}(x,t) = 2\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} \left[\hat{b}_n \Psi^n(x,t)\right] + h.c.$$

- We expand each function ψ (x, t) in a series with respect to the *instantaneous basis*:

$$\Psi^{(n)}(x, t > 0) = \sum_{k=1}^{\infty} Q_k^n(t) \sqrt{\frac{L_0}{L(t)}} \sin\left(\frac{\pi k[x - u(t9]}{L(t)}\right)$$

$$Q_k^{(n)}(0) = \delta_{kn},\ \dot{Q}_k^{(n)}(0) = -i\omega_n\delta_{kn},\ k,n = 1,2,\ldots.$$

This way we satisfy automatically the boundary (and initial) conditions. Then, we can arrive at an infinite set of coupled differential eq.

$$\ddot{Q}_k^{(n)} + \omega_k^2(t) Q_k^{(n)} = 2\sum_{j=1}^{\infty} g_{kj}(t)\dot{Q}_j^{(n)} + \sum_{j=1}^{\infty} \dot{g}_{kj}(t) Q_j^{(n)} + \mathcal{O}\left(g_{kj}^2\right)$$

Writing

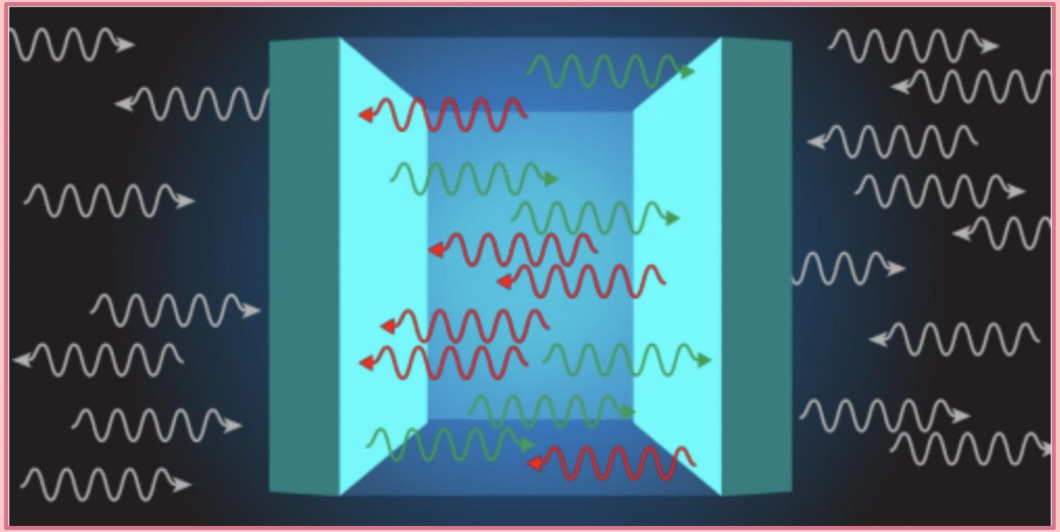
$$Q_k^{(n)}(t)\rho_k^{(n)}(t)e^{-ik\omega_1(1+\delta)t} - \rho_{-k}^{(n)}(t)e^{ik\omega_1(1+\delta)t} \tag{30}$$

and using the method of slowly varying amplitudes [110], one can derive an infinite system of coupled ordinary differential equations for the coefficients $\rho_k^{(n)}(t)$ [109]:

$$\frac{\mathrm{d}}{\mathrm{d}\tau}\rho_k^{(n)}(-1)^p\left[(k+p)\rho_{k+p}^{(n)} - (k-p)\rho_{k-p}^{(n)}\right] + 2i\gamma k\rho_k^{(n)}, \tag{31}$$

$$\tau = \varepsilon\omega_1 t/2, \qquad \gamma = \delta/\varepsilon.$$

Three-Dimensional Models with Moving Boundaries



Effective Hamiltonian Approach

Suppose that the set of Maxwell's eq. in a medium with time-independent parameters and boundaries can be reduced to:

$$\hat{K}(\{L\})\mathbf{F}_\alpha(\mathbf{r},\{L\}) = w_\alpha^2(\{L\})\mathbf{F}_\alpha(\mathbf{r},\{L\})$$

Then for {L(t)} and the Dirichlet boundary conditions:

$$\mathbf{F}(\mathbf{r},t) = \sum_\alpha q_\alpha(t)\mathbf{F}_\alpha(\mathbf{r};\{L(t)\})$$

$$H = \frac{1}{2}\sum_\alpha \left[p_\alpha^2 + \omega_\alpha^2(L(t))q_\alpha^2\right] + \frac{\dot{L}(t)}{L(t)}\sum_{\alpha\neq\beta} p_\alpha m_{\alpha\beta} q_\beta,$$

03
Last comments

Circuits!

4. General Parametric DCE

4.1. Circuit DCE

In view of great difficulties in observing weak manifestations of the DCE, several authors came to the idea of simulating (modeling) this effect in more simple arrangements. Probably, the simplest possibility is to use some electrical circuits [309]. The idea to use quantum resonant oscillatory contours or Josephson junctions with time-dependent parameters (capacitance, inductance, magnetic flux, critical current, etc.) was put forward by Man'ko many years ago [310]. More elaborated proposals in the same direction were presented in [311–315]. The idea to use a superconducting coplanar waveguide in combination with a Josephson junction was developed in [316–319], and the experiments were reported in [320–323]. Their success is related to the possibility of achieving the effective velocity of the boundary up to 25% of the light velocity in vacuum (although the analogy with the motion of real boundaries is not perfect). A detailed review of this approach was given in [31]. The experiments [320–323] were performed in the frequency interval of a few GHz. Their arrangements can be considered as realizations of the *one-dimensional* models with time-dependent parameters. The experiment [320] was performed with an open strip-line waveguide, where the photon generation was achieved due to periodical fast changes of the boundary conditions on one side of the line. Although that boundary conditions can be interpreted as equivalent to a high effective velocity of an oscillating boundary, such an interpretation has a limited range of validity. The second experiment [322] used the parametric resonance excitation of quanta due to changes of the effective speed of light. This arrangement can be considered as a one-dimensional system with periodically varying distributed parameters. The results of that experiments stimulated many theoretical papers, suggesting further improvements of the experimental schemes [324–337]. The circuit QED with "artificial atoms" (qubits) was the subject of studies [338–350]. The most recent review on parametric effects in circuit QED can be found in [351].

Observation of the Dynamical Casimir Effect in a Superconducting Circuit

C.M. Wilson[1],G. Johansson[1], A. Pourkabirian[1], J.R. Johansson[2], T. Duty[3], F. Nori[2,4] & P. Delsing[1]

[1] *Depart. of Microtechnology and Nanoscience, Chalmers University of Technology, Göteborg, Sweden*

[2] *Advanced Science Institute, RIKEN, Wako-shi, Saitama 351-0198, Japan*

[3] *University of New South Wales, Sydney, NSW, 2052 Australia and*

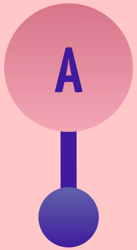
[4] *University of Michigan, Ann Arbor, MI 48109, USA*

(Dated: May 25, 2011)

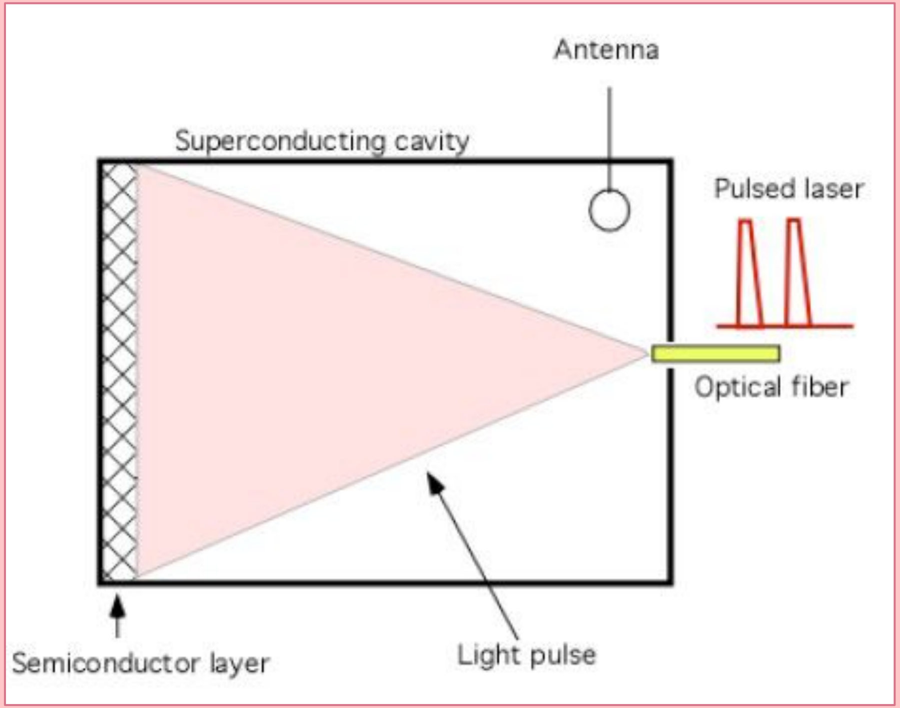
PACS numbers:

Interesting problems

Interaction between fields and detectors



Simulations



Simulations with Semiconductor Slabs

Padua experiment “MIR”

Simulations

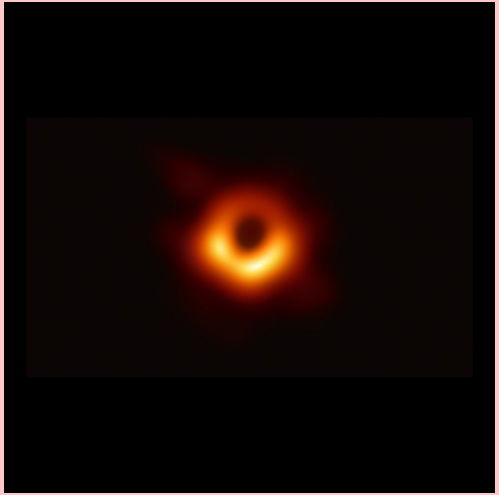
Simulations with Linear and Nonlinear Optical Materials

5.3. Simulations with Linear and Nonlinear Optical Materials

The main mechanism of the DCE is amplification of vacuum fluctuations due to fast variations of instantaneous eigenfrequences of the field modes. These variations can happen either due to change of real dimensions of the cavity confining the field (DCE in narrow sense) or due to changes of the effective (optical) length, when dielectric properties of the medium inside the cavity depend on time (DCE in wide sense). This second possibility was investigated theoretically by many authors [96,275,437–460]. The main problem is how to realize fast variations of the dielectric permeability in real experiments. The idea to use laser beams passing through a material with nonlinear optical properties was considered in [5,429,438–440,448], and concrete experimental schemes were proposed in [461–464]. Although those schemes are rather different (Dezael and Lambrecht [461] and Hizhnyakov [463] considered nonlinear crystals with the second-order nonlinearity, whereas Faccio and Carusotto [462] proposed to exploit the third-order Kerr effect), their common feature is the prediction of generation of infrared [461,462] or even visible [463] photons, whose frequency is of the same order as the frequency of the laser beam. The experiment based on the suggestion [462] was realized recently [465]. A simulation of the DCE in photonic lattices or photonic crystals was suggested in papers [466,467].

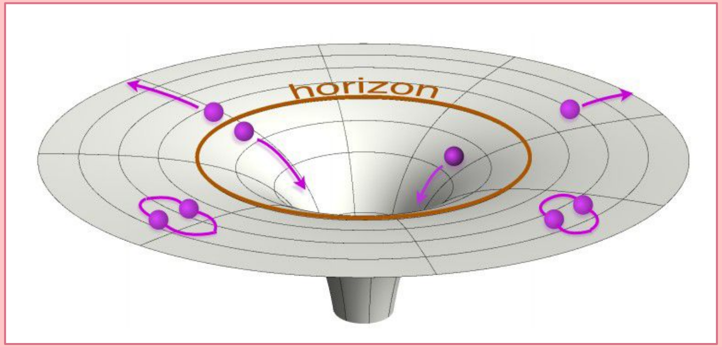
The evaluation of a possibility to obtain parametric amplification of the *microwave* vacuum field, using a reentrant cavity enclosing a nonlinear crystal with a strong third-order nonlinearity, whose refractive index is modulated by near infrared high-intensity laser pulses, was performed in [468,469]. Such a configuration seems to be more adequate for the simulation of the DCE because the effective time-varying length of the cavity is created by infrared quanta, whereas an antenna put inside the cavity can select microwave (RF) quanta only, whose frequency is five orders of magnitude smaller. This could help experimentalists to get rid of various spurious effects due to other possible mechanisms of photon generation.

DCE and Other Quantum Phenomena



Hawking Radiation

DCE and Other Quantum Phenomena



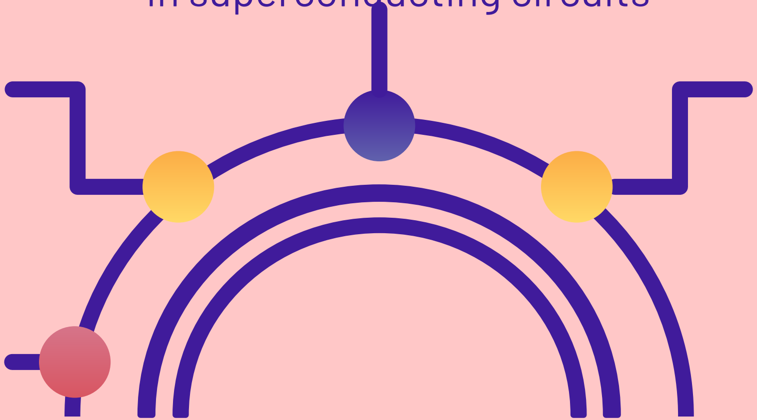
- Vacuum fluctuations at the event horizon results in the breaking up of pairs of virtual particles. One of the particles is trapped in the black hole, and the other escapes to infinity.
- An observer at rest far from the black hole sees a black-body radiation.

Summary

Manifestation of quantum vacuum energy

Circuits

Some proposals consider implementing some of these processes
in superconducting circuits



Challenge

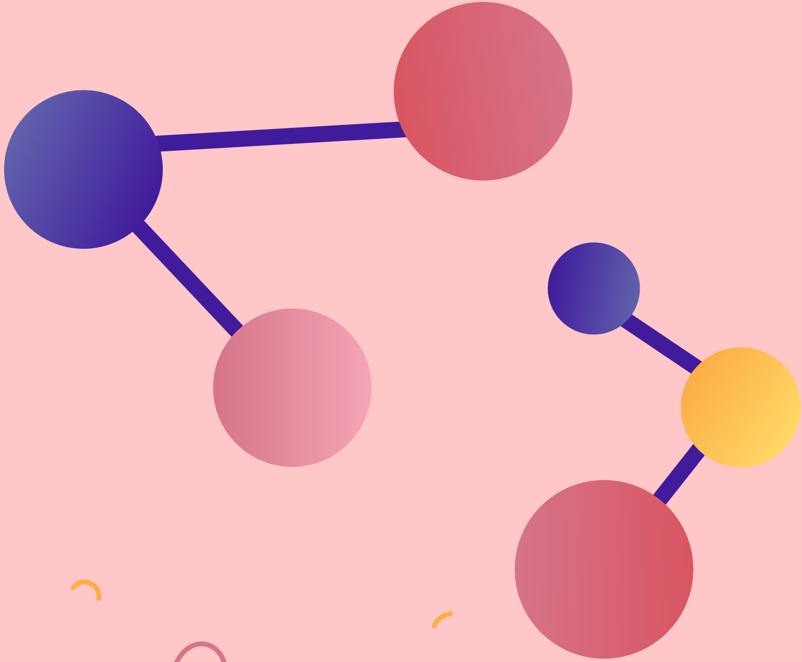
An observation of the “real” DCE in cavities with moving boundaries.

Interesting Model

From different sub-areas point of view.

Thanks

Does anyone have any questions?



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