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May the four be with you:
Novel IR-subtraction methods to tackle NNLO calculations

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Abstract

NNLO

In this report, we present a discussion about different frameworks to perform precise higher-order computations for high-energy physics. These approaches implement novel strategies to deal with infrared and ultraviolet singularities in quantum field theories. A special emphasis is devoted to the local cancellation of these singularities, which can enhance the efficiency of computations and lead to discover novel mathematical properties in quantum field theories.

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- **Title:** May the four be with you: Novel IR-subtraction methods to tackle NNLO calculations.
- December 4, 2020
- arXiv:2012.02567v1

Context & Summary

Motivations

The calculation of multi-loop Feynman integrals which constitutes a challenger due to the present of singularities.

The Big Picture

The calculation of relevant HEP observables in view of future experimental data.

Schemes

They consider different regularization techniques. In order to analyse their features, they consider the NNLO correction to

$$\gamma^* \to q\bar{q}$$

Pre-print structure

(contents of the presentation)

01

Introduction

Motivation

02

IREG

4-dim implementation
at NNLO

03

NNLO in DS

Special emphasis in
DRED

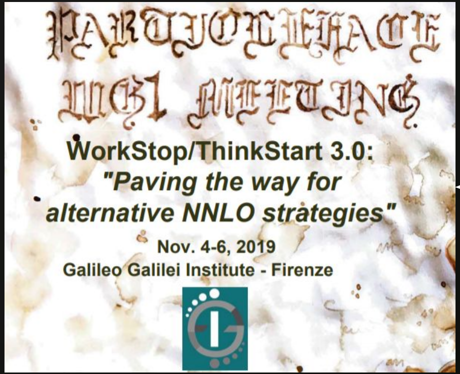
04

Outlook

+ and - of the schemes
Closing discussion

Disclaimer

For more references:



https://indico.ific.uv.es/event/3737/contributions/

And of course:

To d, or not to d: recent developments and comparisons of regularization schemes

C. Gnendiger[1,a], A. Signer[1,2], D. Stöckinger[3], A. Broggio[4], A. L. Cherchiglia[5], F. Driencourt-Mangin[6], A. R. Fazio[7], B. Hiller[8], P. Mastrolia[9,10], T. Peraro[11], R. Pittau[12], G. M. Pruna[1], G. Rodrigo[6], M. Sampaio[13], G. Sborlini[6,14,15], W. J. Torres Bobadilla[6,9,10], F. Tramontano[16,17], Y. Ulrich[1,2], A. Visconti[1,2]

Collider Physics at the Precision Frontier

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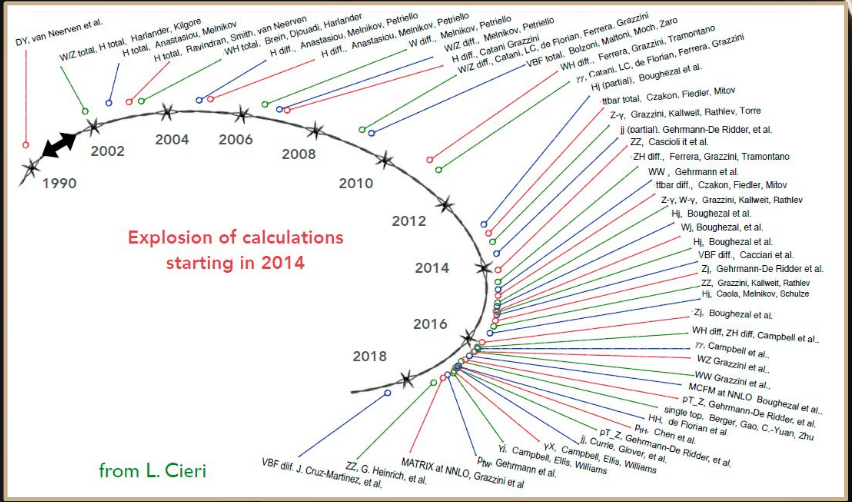
01
Introduction

Introduction

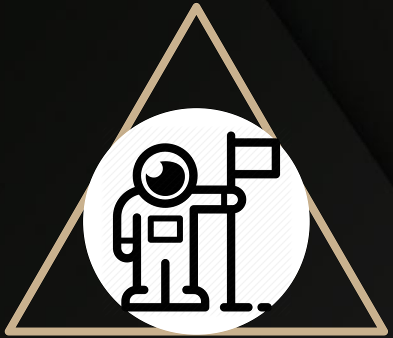
- NLO & NNLO
- Proper procedures to deal with IR-div and UV-div .
- $\gamma^* \to q\bar{q}$
- 4-dim vs d-dim implementations.



02
IREG

Non-dimensional scheme

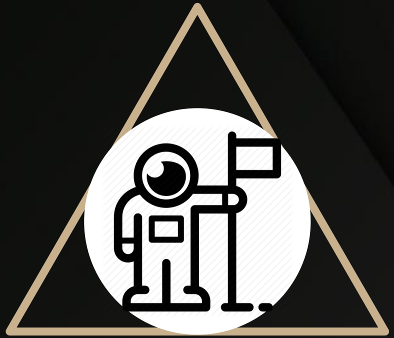
Implicit Regularization



IREG

It stays in the physical dimension!

- Alternative to traditional dimensional schemes.
- Momentum space.
- **UV-div** as BDI (basic divergent integrals).
- **UV-div** do not depend on physical parameters.
- Parametrization of regularization dependent terms as surface terms.



IREG

It stays in the physical dimension!

Implicit Regularization

- BDIs comply with the **Bogoliubov-Parasiuk-Hepp -Zimmerman** (BPHZ) program.
- Counterterms can be cast as BDI, without explicit evaluation.

The Rules of IREG

$$I_{log}^{\nu_1\nu_2}(\mu^2) \equiv \int_k \frac{k_{\nu_1} k_{\nu_2}}{(k^2-\mu^2)^3}$$

$$I_{log}(\mu^2) \equiv \int_k \frac{1}{(k^2-\mu^2)^2}$$

$$k^\mu k^\nu \cancel{\to} \frac{1}{d} k^2$$

1

Usual Dirac algebra.

2

External momenta.

$$\frac{1}{(k-p)^2-\mu^2} = \frac{1}{k^2-\mu^2} - \frac{p^2-2p\cdot k}{(k^2-\mu^2)[(k-p)^2-\mu^2]}$$

3

BDI

4

The μ^2 dependence by introducing a scale Λ^2.

$$I_{log}(\mu^2) = I_{log}(\Lambda^2) - \frac{i}{(4\pi)^2} \ln \frac{\Lambda^2}{\mu^2}$$

But what happens at 2-loops?

At 2-loop order at similar program can be devised

$$\mathcal{A} = \int_{k_1,k_2} G(p_1,\ldots,p_L,k_1,k_2)H_1(p_1,\ldots,p_L,k_1)H_2(p_1,\ldots,p_L,k_2)$$

3 regimes:

- k1->∞ ; k2->fixed.
- k1->fixed ; k2->∞
- k1->∞ ; k2->∞

$$\frac{1}{(k-p)^2-\mu^2} = \frac{1}{k^2-\mu^2} - \frac{p^2-2p\cdot k}{(k^2-\mu^2)[(k-p)^2-\mu^2]}$$

At 2-loop order at similar program can be devised

$$\mathcal{A} = \int_{k_1,k_2} G(p_1,\ldots,p_L,k_1,k_2)H_1(p_1,\ldots,p_L,k_1)H_2(p_1,\ldots,p_L,k_2)$$

$$\mathcal{A}_{k_1\to\infty} = \int_{k_2} \bar{H}_2(p_1,\ldots,p_L,k_2)\boxed{I_{log}(\lambda^2)}$$

$$\mathcal{A}_{k_2\to\infty} = \int_{k_1} \bar{H}_1(p_1,\ldots,p_L,k_1)\boxed{I_{log}(\lambda^2)}$$

$$\mathcal{A}_{k_1\to\infty,k_2\to\infty} = \mathcal{F}(p_1,\ldots,p_L)I_{log}(\lambda^2)$$

Further contributions to the 2-loop c.t

$$\bar{\mathcal{A}}_{k_1\to\infty} = \int_{k_2} \bar{H}_2(p_1,\ldots,p_L,k_2)\ln\left(-\frac{k_1^2-\mu^2}{\lambda^2}\right)$$
$$\bar{\mathcal{A}}_{k_2\to\infty} = \int_{k_1} \bar{H}_1(p_1,\ldots,p_L,k_1)\ln\left(-\frac{k_2^2-\mu^2}{\lambda^2}\right)$$

$$\Downarrow$$

$$I_{log}^{(2)}(\mu^2) \equiv \int_k \frac{1}{(k^2-\mu^2)^2}\ln\left(-\frac{k^2-\mu^2}{\lambda^2}\right)$$

BDI at higher order

$$I^{\nu_1\ldots\nu_m} = \int_{k_l} \frac{A^{\nu_1\ldots\nu_m}(k_l, q_i)}{\prod_i [(k_l - q_i)^2 - \mu^2]} \ln^{l-1}\left(-\frac{k_l^2 - \mu^2}{\lambda^2}\right)$$

$$I_{log}^{(l)}(\mu^2) \equiv \int_{k_l} \frac{1}{(k_l^2 - \mu^2)^2} \ln^{l-1}\left(-\frac{k_l^2 - \mu^2}{\lambda^2}\right),$$

$$I_{log}^{(l)\nu_1\cdots\nu_{2r}}(\mu^2) \equiv \int_{k_l} \frac{k_l^{\nu_1}\cdots k_l^{\nu_{2r}}}{(k_l^2 - \mu^2)^{r+1}} \ln^{l-1}\left(-\frac{k_l^2 - \mu^2}{\lambda^2}\right)$$

The divergent content can be expressed in terms of BDI in one loop momentum after performing n - 1 integrations.

Example: BDI at 2-loop

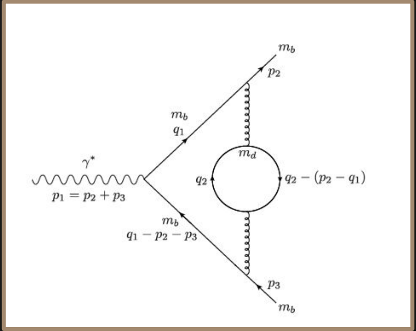
$$\mathcal{I} = \int_{k_l} G(k_l, p_i, \mu^2) \ln^{2-1}\left[-\frac{(k_l^2 - \mu^2)}{\Lambda^2}\right]$$

$$\int_k \frac{1}{(k+p)^2} \ln\left(-\frac{k^2}{\lambda^2}\right)$$

$$\int_k \frac{1}{k^2(k+p)^2} \ln\left(-\frac{k^2}{\lambda^2}\right)$$
$$\int_k \frac{k_\mu}{k^2(k+p)^2} \ln\left(-\frac{k^2}{\lambda^2}\right)$$
$$\int_k \frac{k_\mu k_\nu}{k^2(k+p)^2} \ln\left(-\frac{k^2}{\lambda^2}\right)$$

$$I_{log}^{(2)}(\mu^2) \equiv \int_{k_l} \frac{1}{(k_l^2 - \mu^2)^2} \ln^{2-1}\left(-\frac{k_l^2 - \mu^2}{\Lambda^2}\right),$$

A 2-loop example

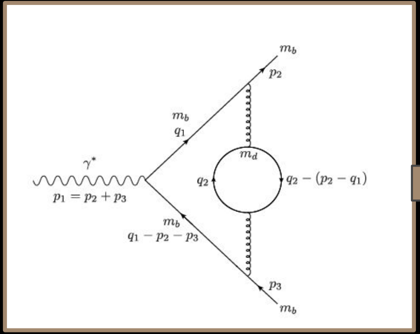


They evaluate the contribution that contains a quark loop of flavour d and external quarks of flavour b

The integration is performing according to IREG rules..

Virtual contributions

UV-div and IR-div.

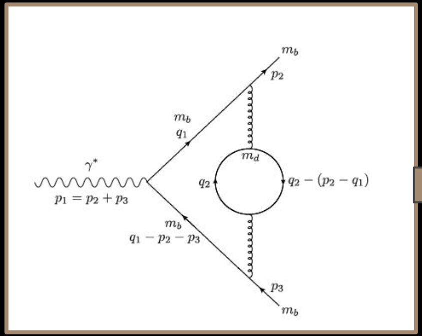


$$\int_q = \int d^4q/(2\pi)^4$$

$$\mathcal{M}^{\mu}_{ab} = \int_{q_1,q_2} \frac{C_F\, Q\, e\, g_s^4\, \delta_{ab} \times \bar{u}(p_2,m_b) N^{\mu}(m_d,m_b,p_2,p_3,q_1,q_2) v(p_3,m_b)}{((q_1+p_2)^2-m_b^2)(q_1^2-\mu^2)^2[(q_1-p_3)^2-m_b^2](q_2^2-m_d^2)[(q_2+q_1)^2-m_d^2]}$$

$$\begin{aligned}
N^{\mu} \;=\; & 4m_d^2[-2\not{q}_1(p_2-p_3+q_1)^{\mu}+\gamma^{\mu}(-2p_2\cdot p_3+2p_2\cdot q_1-2p_3\cdot q_1+q_1^2)+m_b(\not{q}_1\gamma^{\mu}+\gamma^{\mu}\not{q}_1)]+\\
+\; & 4\gamma^{\mu}\not{q}_1\not{q}_2(p_2\cdot q_1+2p_2\cdot q_2)-8\gamma^{\mu}(p_2\cdot q_1)(p_3\cdot q_2)-4\not{q}_2\not{q}_1\gamma^{\mu}(p_3\cdot q_1+2p_3\cdot q_2)-\\
-\; & 4\gamma^{\mu}(p_2\cdot q_2)(2p_3\cdot q_1+4p_3\cdot q_2-q_1^2)-2q_1^2(\not{q}_1\gamma^{\mu}\not{q}_2+\not{q}_2\gamma^{\mu}\not{q}_1+2\gamma^{\mu}p_3\cdot q_2)+8\not{q}_1(p_2-p_3)^{\mu}q_1\cdot q_2+\\
+\; & 8\not{q}_1 q_1^{\mu}q_1\cdot q_2-8\gamma^{\mu}(p_2\cdot q_1)(q_1\cdot q_2)+8\gamma^{\mu}(p_2\cdot p_3+p_3\cdot q_1)(q_1\cdot q_2)-4\gamma^{\mu}q_1^2 q_1\cdot q_2+8\not{q}_2\times\\
\times\; & ((q_1-q_2)^{\mu}q_1^2+2q_1^{\mu}q_1\cdot q_2)+8\not{q}_1 q_2^2(p_2-p_3)^{\mu}-8\gamma^{\mu}q_2^2(p_2\cdot q_1+p_2\cdot p_3+p_3\cdot q_1)-\\
-\; & 4m_b(\not{q}_1\gamma^{\mu}+\gamma^{\mu}\not{q}_1)(q_2^2+q_1\cdot q_2),
\end{aligned} \tag{4.18}$$

Now the integration over q_2 can be performed following the IREG rules.

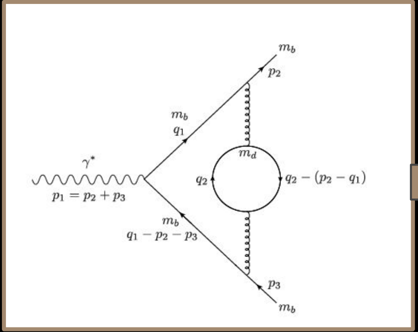


$$\mathcal{M}^{\mu}_{ab} = -\frac{2}{9} Q\, e\, C_F\, g_s^4\, \delta_{ab} \times \bar{u}(p_2, m_b) \Bigg[\underbrace{\int_{q_1} \boxed{\mathcal{I}^{\mu}(p_2, p_3, m_b, m_d, q_1, \mu, \lambda)}}_{\Sigma^{\mu}} \Bigg] v(p_3, m_b)$$

$$\mathcal{I}^{\mu} = \Big([q_1^2(-4p_2 \cdot p_3 - 2p_3 \cdot q_1 + q_1^2) + 2p_2 \cdot q_1 (2p_3 \cdot q_1 + q_1^2)]\gamma^{\mu} + 4q_1^2[m_b q_1^{\mu} - (p_2 - p_3 + q_1)^{\mu} \not{q}_1] \Big) \times$$
$$\left[\frac{-3\, I_{log}(\lambda^2) + 3\, b\, \boxed{Z_0(q_1^2, m_d^2, \lambda^2)} + b}{[(q_1+p_2)^2 - m_b^2](q_1^2 - \mu^2)^2[(q_1-p_3)^2 - m_b^2]} + \frac{6\, b\, m_d^2\, Z_0(q_1^2, m_d^2, m_d^2)}{[(q_1+p_2)^2 - m_b^2](q_1^2 - \mu^2)^3[(q_1-p_3)^2 - m_b^2]} \right].$$

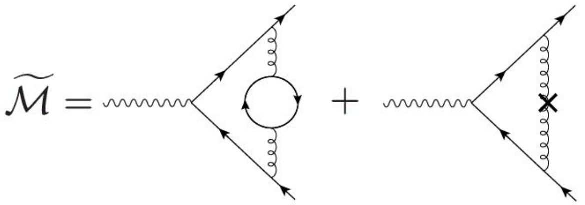
$$Z_0(p^2, m_1^2, m_2^2) \equiv \int_0^1 dx\; \ln\left(\frac{p^2 x(x-1) + m_1^2}{m_2^2} \right)$$

Finite contribution.

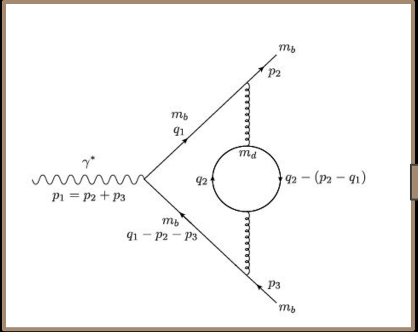


$$\mathcal{M}^{\mu}_{ab} = -\frac{2}{9} Q\, e\, C_F\, g_s^4\, \delta_{ab} \times \bar{u}(p_2, m_b) \Bigg[\underbrace{\int_{q_1} \boxed{\mathcal{I}^{\mu}(p_2, p_3, m_b, m_d, q_1, \mu, \lambda)}}_{\Sigma^{\mu}} \Bigg] v(p_3, m_b)$$

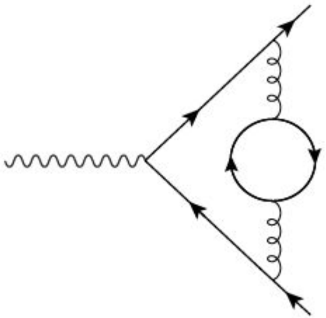
$$\mathcal{I}^{\mu} = \Big([q_1^2(-4p_2 \cdot p_3 - 2p_3 \cdot q_1 + q_1^2) + 2p_2 \cdot q_1 (2p_3 \cdot q_1 + q_1^2)]\gamma^{\mu} + 4q_1^2[m_b q_1^{\mu} - (p_2 - p_3 + q_1)^{\mu} \not{q}_1] \Big) \times$$
$$\left[\frac{-3\, \cancel{I_{log}(\lambda^2)} + 3\, b\, Z_0(q_1^2, m_d^2, \boxed{\lambda^2}) + b}{[(q_1 + p_2)^2 - m_b^2](q_1^2 - \mu^2)^2[(q_1 - p_3)^2 - m_b^2]} + \frac{6\, b\, m_d^2\, Z_0(q_1^2, m_d^2, \boxed{m_d^2})}{[(q_1 + p_2)^2 - m_b^2](q_1^2 - \mu^2)^3[(q_1 - p_3)^2 - m_b^2]} \right].$$



Diagrams entering in the UV renormalisation of scattering process $\gamma^* \to b\bar{b}$.



$\widetilde{\mathcal{M}} =$



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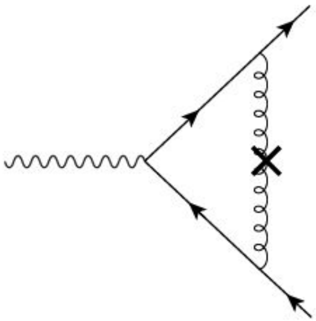


$$\widetilde{\mathcal{I}}^{\mu} = \Big([q_1^2(-4p_2\cdot p_3 - 2p_3\cdot q_1 + q_1^2) + 2p_2\cdot q_1(2p_3\cdot q_1 + q_1^2)]\gamma^{\mu} + 4q_1^2[m_b q_1^{\mu} - (p_2 - p_3 + q_1)^{\mu}\not{q}_1] \Big) \times$$

$$\left[\frac{3\,b\,Z_0(q_1^2, m_d^2, \lambda^2) + b}{[(q_1+p_2)^2 - m_b^2](q_1^2-\mu^2)^2[(q_1-p_3)^2 - m_b^2]} + \frac{6\,b\,m_d^2\,Z_0(q_1^2, m_d^2, m_d^2)}{[(q_1+p_2)^2 - m_b^2](q_1^2-\mu^2)^3[(q_1-p_3)^2 - m_b^2]} \right]$$

Virtual UV-div and virtual IR-div.

There is not an overlap of them.

Virtual Contributions: UV-part

$$\widetilde{\Sigma}^{\mu}\Big|_{UV} = \int_{q_1} \left(q_1^4\gamma^\mu - 4q_1^2 q_1^\mu \not{q}_1\right)\left[\frac{3\,b\,\boxed{Z_0(q_1^2, m_d^2, \lambda^2)} + b}{[(q_1+p_2)^2 - m_b^2](q_1^2-\mu^2)^2[(q_1-p_3)^2 - m_b^2]}\right]$$

$$q_1 \gg m_d$$

$$Z_0(p^2, m_1^2, m_2^2) \equiv \int_0^1 dx\ \ln\left(\frac{p^2 x(x-1) + m_1^2}{m_2^2}\right)$$

$$\widetilde{\Sigma}^{\mu}\Big|_{UV} = \int_{q_1} \left[\gamma^\mu - \frac{4q_1^\mu \not{q}_1}{(q_1^2-\mu^2)}\right]\left[\frac{3\,b\,\ln\left(-\frac{q_1^2-\mu^2}{\lambda^2}\right) - 5b}{[(q_1+p_2)^2 - m_b^2][(q_1-p_3)^2 - m_b^2]}\right]$$

Virtual Contributions: UV-part

$$\widetilde{\Sigma}^{\mu}\Big|_{UV} = \int_{q_1} \left[\gamma^{\mu} - \frac{4 q_1^{\mu} \not{q}_1}{(q_1^2 - \mu^2)}\right] \left[\frac{3\, b\, \ln\left(-\frac{q_1^2 - \mu^2}{\lambda^2}\right) - 5b}{[(q_1 + p_2)^2 - m_b^2][(q_1 - p_3)^2 - m_b^2]}\right]$$

Usual IREG rules

$$\widetilde{\Sigma}^{\mu}\Big|_{UV} = b\gamma^{\mu}\left[3 I_{log}^{(2)}(\lambda^2) - 5 I_{log}(\lambda^2)\right] - 4b\gamma^{\mu}\left[\frac{3}{4} I_{log}^{(2)}(\lambda^2) + \frac{3}{8} I_{log}(\lambda^2) - \frac{5}{4} I_{log}(\lambda^2)\right]$$

$$\mathcal{M}^{\mu}_{ab} = -\frac{2}{9} Q\, e\, C_F\, g_s^4\, \delta_{ab} \times \bar{u}(p_2, m_b) \left[\underbrace{\int_{q_1} \mathcal{I}^{\mu}(p_2, p_3, m_b, m_d, q_1, \mu, \lambda)}\right] v(p_3,$$

Virtual Contributions: UV-part

$$\mathcal{M}^{\mu}_{ab}\Big|^{2\,\text{loop}\,\text{ct}}_{UV} = Q\,e\,C_F\,g_s^4\,\delta_{ab}\,\bar{u}(p_2, m_b)\gamma^{\mu}v(p_3, m_b)\;\times\frac{b}{3}\,I_{log}(\lambda^2)$$

Virtual Contributions: IR-part

$\widetilde{\mathcal{I}}^{\mu}$

IR-part

Integrals	IR divergences
$\int_{q_1} \frac{p_2\cdot p_3}{(q_1^2-\mu^2)[(p_2+q_1)^2-m_b^2][(q_1-p_3)^2-m_b^2]}$	$\frac{-(s-2m_b^2)}{4\sqrt{sS}}\mathcal{F}(s,S,\mu)$
$\int_{q_1} \frac{(p_2\cdot q_1)(p_3\cdot q_1)}{(q_1^2-\mu^2)^2[(p_2+q_1)^2-m_b^2][(q_1-p_3)^2-m_b^2]}$	$-\frac{1}{4}\ln\left(\frac{m_b^2}{\mu^2}\right)$
$\int_{q_1} \frac{p_2\cdot p_3 Z_0(q_1^2,m_d^2,\lambda^2)}{(q_1^2-\mu^2)[(p_2+q_1)^2-m_b^2][(q_1-p_3)^2-m_b^2]}$	$\frac{-(s-2m_b^2)}{4\sqrt{sS}}\ln\left(\frac{m_d^2}{\lambda^2}\right)\mathcal{F}(s,S,\mu)$
$\int_{q_1} \frac{(p_2\cdot q_1)(p_3\cdot q_1)Z_0(q_1^2,m_d^2,\lambda^2)}{(q_1^2-\mu^2)^2[(p_2+q_1)^2-m_b^2][(q_1-p_3)^2-m_b^2]}$	$-\frac{1}{4}\ln\left(\frac{m_b^2}{\mu^2}\right)\ln\left(\frac{m_d^2}{\lambda^2}\right)$
$\int_{q_1} \frac{q_1^2 Z_0(q_1^2,m_d^2,m_d^2)}{(q_1^2-\mu^2)^2[(p_2+q_1)^2-m_b^2][(q_1-p_3)^2-m_b^2]}$	$\frac{-\ln(m_d^2/m_b^2)}{2\sqrt{sS}}\mathcal{F}(s,S,\mu)$
$\int_{q_1} \frac{(p_2\cdot q_1)Z_0(q_1^2,m_d^2,m_d^2)}{(q_1^2-\mu^2)^2[(p_2+q_1)^2-m_b^2][(q_1-p_3)^2-m_b^2]}$	$\frac{\ln(m_d^2/m_b^2)}{4m_b^2 sS}\left[m_b^2\sqrt{sS}\,\mathcal{F}(s,S,\mu)+sS\ln\left(\frac{m_b^2}{\mu^2}\right)\right]$
$\int_{q_1} \frac{(p_3\cdot q_1)Z_0(q_1^2,m_d^2,m_d^2)}{(q_1^2-\mu^2)^2[(p_2+q_1)^2-m_b^2][(q_1-p_3)^2-m_b^2]}$	$-\frac{\ln(m_d^2/m_b^2)}{4m_b^2 sS}\left[m_b^2\sqrt{sS}\,\mathcal{F}(s,S,\mu)+sS\ln\left(\frac{m_b^2}{\mu^2}\right)\right]$
$\int_{q_1} q_1^\mu \not{q}_1 Z_0(q_1^2,m_d^2,m_d^2)\ \times$ $\times\frac{1}{(q_1^2-\mu^2)^2[(p_2+q_1)^2-m_b^2][(q_1-p_3)^2-m_b^2]}$	$\frac{-\ln(m_d^2/m_b^2)}{4m_b sS^2}\Big\{m_b S\sqrt{sS}\ \mathcal{F}(s,S,\mu)\gamma^\mu+$ $+\,2(p_2^\mu-p_3^\mu)\left[m_b^2\sqrt{sS}\ \mathcal{F}(s,S,\mu)+sS\ln\left(\frac{m_b^2}{\mu^2}\right)\right]\Big\}$
$\int_{q_1} \frac{q_1^\mu Z_0(q_1^2,m_d^2,m_d^2)}{(q_1^2-\mu^2)^2[(p_2+q_1)^2-m_b^2][(q_1-p_3)^2-m_b^2]}$	$\frac{(p_2-p_3)^\mu\ln(m_d^2/m_b^2)}{2m_b^2 sS^2}\left[m_b^2\sqrt{sS}\ \mathcal{F}(s,S,\mu)+sS\ln\left(\frac{m_b^2}{\mu^2}\right)\right]$
$\int_{q_1} \frac{(q_1\cdot p_2)(q_1\cdot p_3)Z_0(q_1^2,m_d^2,m_d^2)}{(q_1^2-\mu^2)^3[(p_2+q_1)^2-m_b^2][(q_1-p_3)^2-m_b^2]}$	$\frac{\ln(m_d^2/m_b^2)\mathcal{F}(s,S,\mu)}{8\sqrt{sS}}+\left(\frac{\ln(m_d^2/m_b^2)}{4m_b^2}+\frac{1}{24m_d^2}\right)\ln\left(\frac{m_b^2}{\mu^2}\right)$

Table 1: We have denoted $s\equiv(p_2+p_3)^2=2(p_2\cdot p_3+m_b^2)$, $S\equiv s-4m_b^2$, $\mathcal{F}(s,S,\mu)\equiv\ln^2\left(\frac{2\mu^2}{\sqrt{sS}-S}\right)-\ln^2\left(\frac{-2\mu^2}{\sqrt{sS}+S}\right)$ and $Z_0(p^2,m_1^2,m_2^2)$ is defined as in equation $\boxed{4.21}$.

Virtual Contributions: IR-part

$$\mathcal{M}^{\mu}_{ab}\Big|^{2\,\mathrm{loop}}_{IR} = Q\, e\, C_F\, g_s^4\, \delta_{ab}\, \bar{u}(p_2, m_b)[\,(p_2 - p_3)^{\mu}\, \boxed{\mathcal{A}} + \gamma^{\mu}\, \boxed{\mathcal{B}}\,]v(p_3, m_b)$$

$$\mathcal{A} \;=\; \frac{16\, b\, m_d^2}{3\, S} \ln\left(\frac{m_d^2}{m_b^2}\right)\left[-\frac{m_b}{\sqrt{s\, S}}\mathcal{F}(s, S, \mu) - \frac{1}{m_b}\ln\left(\frac{m_b^2}{\mu^2}\right)\right]$$

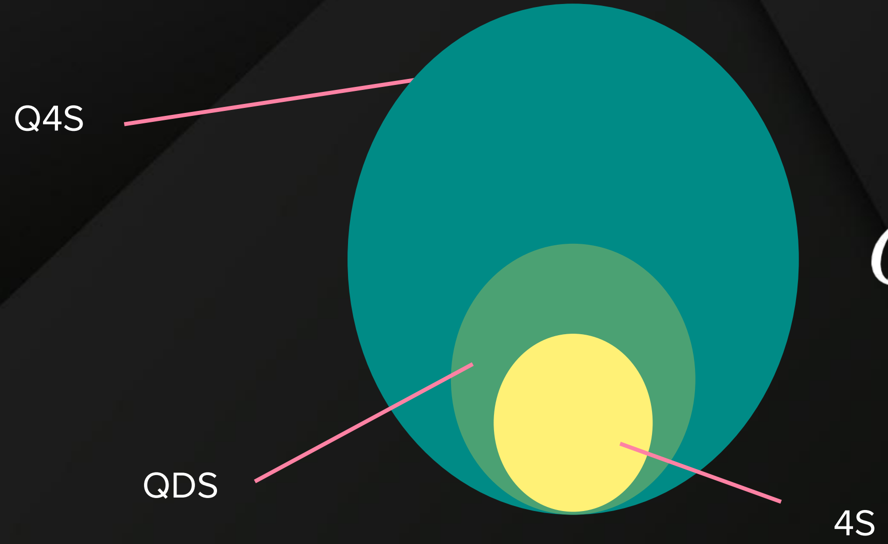
$$\begin{aligned}
\mathcal{B} \;=\;& \frac{2\, b}{3}\Bigg[\ln\left(\frac{m_b^2}{\mu^2}\right)\ln\left(\frac{m_d^2}{\lambda^2}\right) - \frac{4\, m_d^2}{m_b^2}\ln\left(\frac{m_d^2}{m_b^2}\right)\ln\left(\frac{m_b^2}{\mu^2}\right)\\
&+ \frac{2\, m_b^2}{3\sqrt{s\, S}}\left(3\ln\left(\frac{m_d^2}{\lambda^2}\right) + 1\right)\mathcal{F}(s, S, \mu) - \frac{1}{3}\sqrt{\frac{s}{S}}\left(3\ln\left(\frac{m_d^2}{\lambda^2}\right) + 1\right)\mathcal{F}(s, S, \mu)\\
&- 4\frac{m_d^2}{\sqrt{s\, S}}\ln\left(\frac{m_d^2}{m_b^2}\right)\mathcal{F}(s, S, \mu)\Bigg].
\end{aligned}$$

It is going to be cancel adding the real contribution.

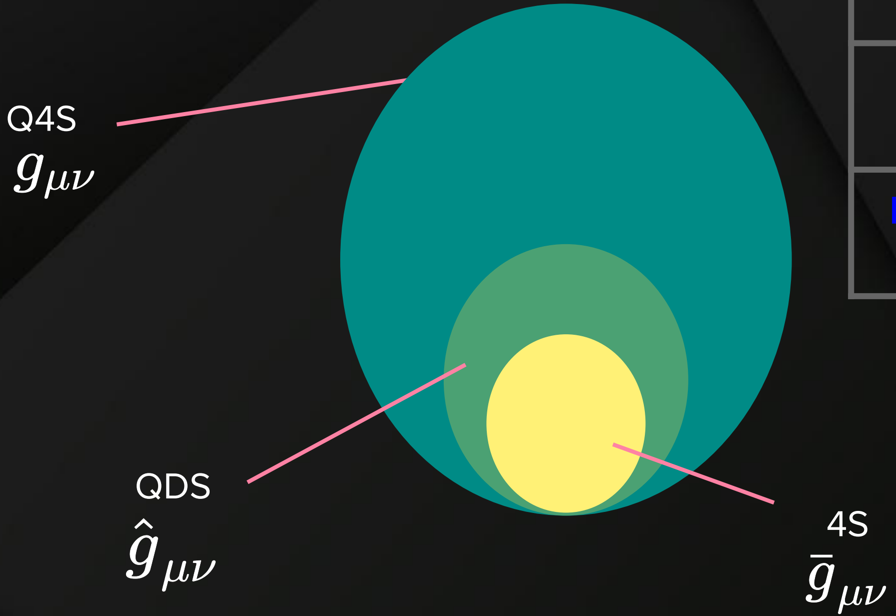
03

NNLO in DS

Novel DS



$$Q4S \supset QDS \supset 4S$$



$$Q4S \supset QDS \supset 4S$$

	DReg	DRED
Internal gluon	$\hat{g}_{\mu\nu}$	$g_{\mu\nu}$
External gluon	$\hat{g}_{\mu\nu}$	$g_{\mu\nu}$

$$g_{\mu\nu} = \hat{g}_{\mu\nu} + \underbrace{\tilde{g}_{\mu\nu}}_{2\varepsilon}$$

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	CDR	HV	FDH	DRED
internal gluon	$\hat{g}^{\mu\nu}$	$\hat{g}^{\mu\nu}$	$g^{\mu\nu}$	$g^{\mu\nu}$
external gluon	$\hat{g}^{\mu\nu}$	$\bar{g}^{\mu\nu}$	$\bar{g}^{\mu\nu}$	$g^{\mu\nu}$

Since the contribution of ϵ-scalars drops out for physical observables it is, of course, possible to leave them out from the very beginning. This is nothing but computing in CDR. However, including ϵ-scalars sometimes offers advantages, as it is (from a technical point of view) equivalent to performing the algebra in four dimensions. We reiterate the statement that introducing ϵ-scalars in diagrams and counterterms is nothing but a consistent procedure (also beyond leading order) to technically implement the often made instruction to "perform the algebra in four dimensions".[1]

LO

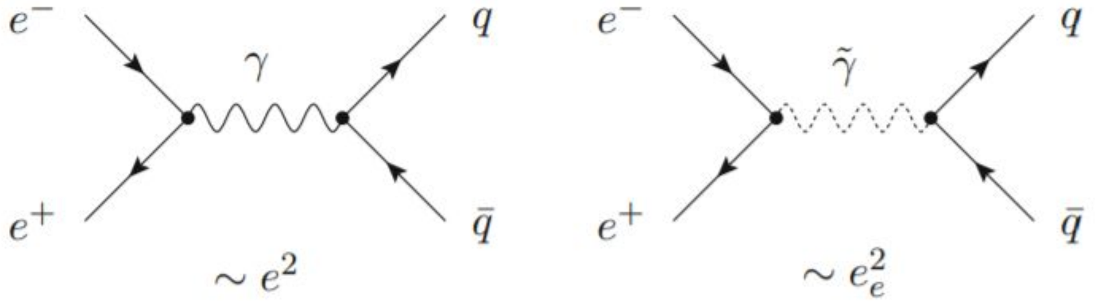

Fig. 1 Diagrams contributing to the process $e^+e^- \to \gamma^* \to q\bar{q}$ at the tree level. The interaction is mediated by a quasi d-dimensional photon γ (left) and a quasi n_ϵ-dimensional ϵ-scalar photon $\tilde{\gamma}$ (right), respectively. The right diagram only exists in DRED

$\omega^{(0)} \equiv Q_q^2 N_c/(3s)$

$$M_{\mathrm{DRED}}^{(0)} = M_{\mathrm{DRED}}^{(0,\gamma)} + M_{\mathrm{DRED}}^{(0,\tilde{\gamma})} = \omega^{(0)} \left[e^4 (d-2) + e_e^4 n_\epsilon \right].$$

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NLO (virtual corrections)

$$\omega^{(0)}\left(\frac{e^4}{4\pi}\right)(1-\epsilon)\,\Phi_2(\epsilon)$$

where: $$\Phi_2(\epsilon) = \left(\frac{4\pi}{s}\right)^{\epsilon}\frac{\Gamma(1-\epsilon)}{\Gamma(2-2\epsilon)} = 1 + \mathcal{O}(\epsilon)$$

$$\sigma^{(v)}_{\mathrm{CDR}} = \sigma^{(0)}_{\mathrm{CDR}}\left(\frac{\alpha_s}{4\pi}\right)C_F\left\{-\frac{4}{\epsilon^2}-\frac{6}{\epsilon}-16+\frac{7\pi^2}{3}+\mathcal{O}(\epsilon)\right\}$$

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NLO (virtual corrections)

Eur. Phys. J. C (2020) 80:215

$$\sigma^{(v)}_{\mathrm{DRED}} = \sigma^{(v,\gamma)}_{\mathrm{DRED}} + \sigma^{(v,\tilde{\gamma})}_{\mathrm{DRED}}$$

$$\omega^{(0)}\left(\frac{e^4}{4\pi}\right)(1-\epsilon)\,\Phi_2(\epsilon)$$

$$\sigma^{(v,\gamma)}_{\mathrm{DRED}} = \sigma^{(0,\tilde{\gamma})}_{\mathrm{DRED}}\left(\frac{\alpha_s}{4\pi}\right)C_F\left\{\boxed{-}\frac{4}{\epsilon^2}\boxed{-}\frac{6}{\epsilon} - 14 + \frac{7\pi^2}{3} + \epsilon\left[-30 + \frac{7\pi^2}{2} + \frac{28\zeta(3)}{3}\right] + \mathcal{O}(\epsilon^2)\right\},$$

$$\omega^{(0)}\left(\frac{e_e^4}{4\pi}\right)\left(\frac{n_\epsilon}{2}\right)\Phi_2(\epsilon)$$

$$\sigma^{(v,\tilde{\gamma})}_{\mathrm{DRED}} = \sigma^{(0,\tilde{\gamma})}_{\mathrm{DRED}}\left(\frac{\alpha_s}{4\pi}\right)C_F\left\{-\frac{4}{\epsilon^2} - \frac{6}{\epsilon} - 10 + \frac{7\pi^2}{3} + \epsilon\left[-16 + \frac{7\pi^2}{3} + \frac{28\zeta(3)}{3}\right] + \mathcal{O}(\epsilon^2)\right\}$$

Where: $$\Phi_2(\epsilon) = \left(\frac{4\pi}{s}\right)^{\epsilon}\frac{\Gamma(1-\epsilon)}{\Gamma(2-2\epsilon)} = 1 + \mathcal{O}(\epsilon)$$

NLO (virtual corrections)

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$$\sigma^{(v)}_{\mathrm{DRED}} = \sigma^{(v,\gamma)}_{\mathrm{DRED}} + \sigma^{(v,\tilde{\gamma})}_{\mathrm{DRED}}$$

$\omega^{(0)}\left(\frac{e^4}{4\pi}\right)(1-\epsilon)\,\Phi_2(\epsilon)$

$$\sigma^{(v,\gamma)}_{\mathrm{DRED}} = \sigma^{(0,\gamma)}_{\mathrm{DRED}}\left(\frac{\alpha_s}{4\pi}\right)C_F\left\{\boxed{-}\frac{4}{\epsilon^2}\boxed{-}\frac{6}{\epsilon} - 14 + \frac{7\pi^2}{3} + \epsilon\left[-30 + \frac{7\pi^2}{2} + \frac{28\zeta(3)}{3}\right] + \mathcal{O}(\epsilon^2)\right\},$$

$\omega^{(0)}\left(\frac{e_e^4}{4\pi}\right)\left(\frac{n_\epsilon}{2}\right)\Phi_2(\epsilon)$

$$\sigma^{(v,\tilde{\gamma})}_{\mathrm{DRED}} = \sigma^{(0,\tilde{\gamma})}_{\mathrm{DRED}}\left(\frac{\alpha_s}{4\pi}\right)C_F\left\{-\frac{4}{\epsilon^2} - \frac{6}{\epsilon} - 10 + \frac{7\pi^2}{3} + \epsilon\left[-16 + \frac{7\pi^2}{3} + \frac{28\zeta(3)}{3}\right] + \mathcal{O}(\epsilon^2)\right\}$$

With other DS: $\Phi_2(\epsilon) = \left(\frac{4\pi}{s}\right)^\epsilon \frac{\Gamma(1-\epsilon)}{\Gamma(2-2\epsilon)} = 1 + \mathcal{O}(\epsilon)$

$$\sigma^{(v)}_{\mathrm{FDH}} = \sigma^{(0)}\,C_F\,\Phi_2(\epsilon)\,c_\Gamma(\epsilon)\,s^{-\epsilon}\left\{a_s\left[\boxed{-}\frac{4}{\epsilon^2}\boxed{-}\frac{6}{\epsilon} - 16 + 2\pi^2 + \mathcal{O}(\epsilon)\right] + a_e\left[+\frac{n_\epsilon}{\epsilon} + \mathcal{O}(\epsilon^0)\right]\right\},$$

$\sigma^{(0)} = e^4/(4\pi)\,Q_q N_c/(3s)$

$(4\pi)^\epsilon \frac{\Gamma(1+\epsilon)\Gamma^2(1-\epsilon)}{\Gamma(1-2\epsilon)}$

NLO (real corrections)

$$\sigma^{(r)}_{\text{CDR}} = \sigma^{(0)}_{\text{CDR}} \left(\frac{\alpha_s}{4\pi}\right)^2 C_F \left\{\frac{4}{\epsilon^2} + \frac{6}{\epsilon} + 19 - \frac{7\pi^2}{3} + \mathcal{O}(\epsilon)\right\}$$

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$$\sigma^{(r)}_{\text{DRED}} = \sigma^{(0)}_{\text{DRED}} \left(\frac{\alpha_s}{4\pi}\right)^2 C_F \left\{\frac{4}{\epsilon^2} + \frac{6}{\epsilon} + 17 - \frac{7\pi^2}{3} + \mathcal{O}(\epsilon)\right\}$$

With other DS:

$$\sigma^{(r)}_{\text{FDH}} = \sigma^{(0)} C_F \Phi_3(\epsilon) \left\{a_s\left[\boxed{+}\frac{4}{\epsilon^2}\boxed{+}\frac{6}{\epsilon}+19-2\pi^2+\mathcal{O}(\epsilon)\right]+a_e\left[-\frac{n_\epsilon}{\epsilon}+\mathcal{O}(\epsilon^0)\right]\right\},$$

Combine virtual and real corrections!

$$\sigma^{(v)}_{\mathrm{DS}} + \sigma^{(r)}_{\mathrm{DS}}\Big|_{\epsilon\to 0}$$

CDR, FDH and DRED are consistent (if done properly) and yield the same results for physical observables.

$$\sigma^{(1)} = \sigma^{(0)} + \sigma^{(v)}_{\mathrm{FDH}} + \sigma^{(r)}_{\mathrm{FDH}}\Big|_{d\to 4} = \sigma^{(0)}\Big[1 + a_s\, 3\, C_F\Big]$$

NNLO

$$\sigma_{\text{FDH}}^{\text{NNLO}} = \sigma_{\text{FDH}}^{(vv)} + \sigma_{\text{FDH}}^{(rr)} + \sigma_{\text{FDH}}^{(rv)}$$

$$\sigma_{\text{FDH}}^{(vv)}(N_F) = \sigma^{(0)}\,\Phi_2(\epsilon)\,a_s^2\,C_F N_F\left[-\frac{2}{\epsilon^3} - \frac{8}{9\epsilon^2} + \frac{1}{\epsilon}\left(\frac{92}{27} + \frac{\pi^2}{3}\right) + \frac{1921}{81} - \frac{91\pi^2}{27} + \frac{4}{9}\zeta_3\right].$$

$$\sigma_{\text{FDH}}^{(rv)}(N_F) = \sigma^{(0)}\,\Phi_2(\epsilon)\,a_s^2\,C_F N_F\left[\frac{8}{3\epsilon^3} + \frac{4}{\epsilon^2} + \frac{1}{\epsilon}\left(\frac{32}{3} - \frac{14\pi^2}{9}\right) + \frac{94}{3} - \frac{7\pi^2}{3} - \frac{200}{9}\zeta_3\right], \qquad (2.21)$$

$$\sigma_{\text{FDH}}^{(rr)}(N_F) = \sigma^{(0)}\,\Phi_2(\epsilon)\,a_s^2\,C_F N_F\left[-\frac{2}{3\epsilon^3} - \frac{28}{9\epsilon^2} + \frac{1}{\epsilon}\left(-\frac{380}{27} + \frac{11\pi^2}{9}\right) - \frac{5350}{81} + \frac{154\pi^2}{27} + \frac{268}{9}\zeta_3\right].$$

$$\sigma_{\text{FDH}}^{(vv)}(N_F) + \sigma_{\text{FDH}}^{(rv)}(N_F) + \sigma_{\text{FDH}}^{(rr)}(N_F) = \sigma^{(0)}\,a_s^2\,C_F N_F\left[-11 + 8\zeta_3\right]$$

04
Outlook

IREG vs DS

DS

FDH and DRED

- + The evaluation of the Lorentz algebra is significantly simpler than in conventional dimensional regularisation. For an NNLO computation in DRED this is particularly true for double-real contributions and for integrated counterterms of subtraction methods since the $\mathcal{O}(\epsilon)$ terms of the matrix elements are not required.
- + FDH is more amenable to methods that rely on strictly four-dimensional objects like the spinor-helicity formalism and unitarity. Similar to completely four-dimensional regularisation approaches, however, this is not true for the treatment of γ_5.
- + As $\mathcal{O}(\epsilon)$ terms cannot contain any physical information, FDH and DRED might help to improve the conceptual understanding of regularisation and of subtraction methods. Both schemes constitute the most promising candidates to find links between dimensional regularisation and strictly four-dimensional approaches like FDU, FDR, and IREG.

IREG

IREG

- + Fully four-dimensional scheme for momentum integration and Clifford algebra. Although implicit four dimensional schemes such as IREG share the same problems with the γ_5 matrix a well-defined procedure can avoid inconsistencies as shown in [7, 70].
- + The UV content of a given Feynman integral can be cast in terms of a well-defined set of basic divergent integrals which do not need to be evaluated. From the viewpoint of anomalies in perturbation theory it is a useful scheme.
- + Generalisation to L-loops is straightforward and compatible with local subtraction theorems such as the BHPZ scheme and the Bogolyubov recursion formula.

IREG vs DS

DS

- The evaluation of (master) integrals is not affected. Compared to CDR we still need the same loop and phase-space integrals.
- The UV renormalisation is slightly more complicated than in CDR. The procedure, however, is standardised and well understood. For an NNLO computation in FDH or DRED, the evanescent renormalisation constants at most have to be known at one-loop order.

IREG

- Although IR and UV divergences are clearly separated in a gauge invariant way, and no extra fields are needed in the Lagrangian (such as epsilon-scalar fields) compatibility with factorisation theorems are yet to be studied beyond leading order.
- Although a diagrammatic all order proof of gauge invariance in abelian models can be constructed for IREG, a general all order proof for the non-abelian case need to be constructed based on quantum action principles.

Closing Discussion

- About theoretical errors.
- Chiral theories.
- The future

One of the main topics of this workshop regarded the subtleties that appear when extending a theory to d dimensions, whatever d means (as in the context of DREG). However, there are problems that also arise when $d = 4$: this is the case of γ^5. Of course, in the standard four-dimensional space-time there is a well established recipe to mathematically define γ^5. Moreover, in any even-dimensional Minkowskian manifold analogous objects to γ^5 are properly defined.

Regularisation involving γ_5 is problematic. In dimensional schemes the problems are well-known (see e.g. the review [224]), and recent references have focused on comparing different γ_5-prescriptions up to the two-loop level [6] and on determining gauge invariance-restoring counterterms for the Breitenlohner/Maison/'t Hooft/Veltman prescription of γ_5 [225]. Quite surprisingly, non-dimensional schemes are not exempted of issues in the presence of γ_5 [70, 71]. The reason boils down to requiring very basic properties such as shift invariance and numerator-denominator consistency to be respected, showing that virtually any regularization scheme will need to deal with γ_5-problems [7].

Therefore, consistent definition of γ_5, together with full understanding of its properties with respect to symmetries, gauge invariance and anomaly cancellation, is crucial for higher-order calculations. This is especially important in the context of high-precision predictions taking into account electroweak corrections.

- *How to re-define a QFT in such a way that no distinction among real and virtual corrections is done?*
- Even if we manage to combine the real and virtual contributions from the very beginning, still threshold singularities might survive. *How to tackle them and develop efficient techniques to integrate through thresholds?*

Thank You

Do you have any questions?