# QED’s fermion correction at 2-loop with IREG: brainstorming

Carolina Perdomo

CAPES

UFABC

1290 UNIVERSIDADE D COIMBRA

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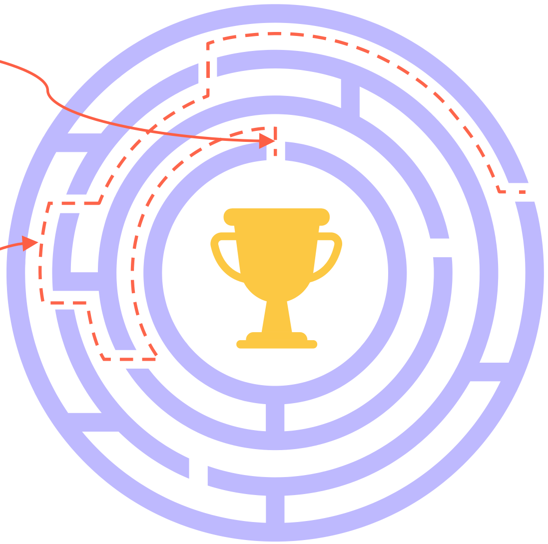
# 01-Review

# Quick Recap

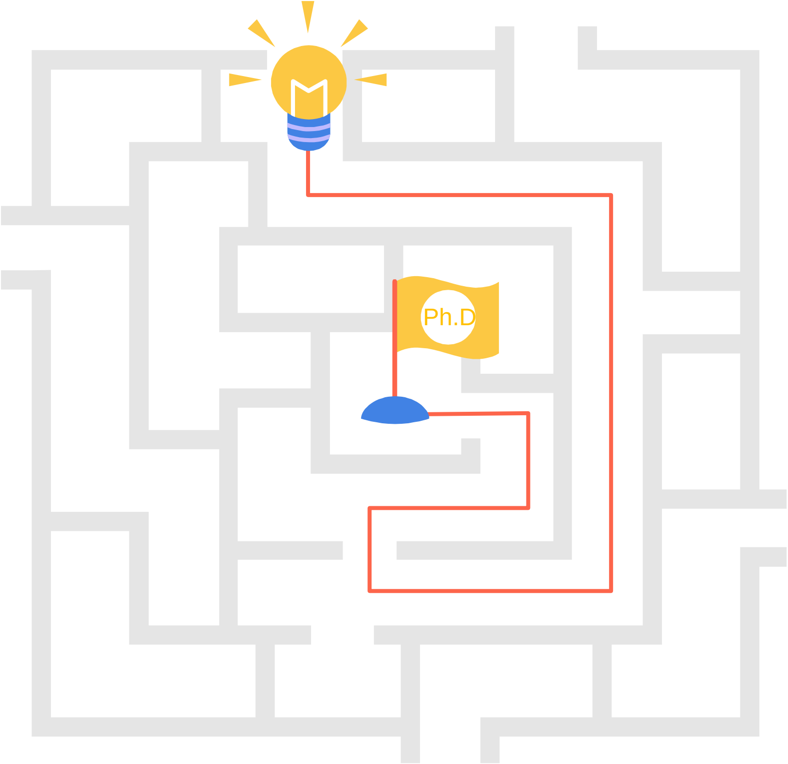
**2-loop $\gamma_m$-function in QCD**

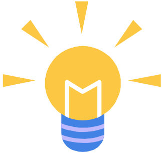
$$\gamma_m^{\overline{IREG}}\left(\Lambda^2\right) = 2\frac{\Lambda^2}{m^{\overline{IREG}}}\frac{\partial m^{\overline{IREG}}}{\partial \Lambda^2}$$
$$= -\beta^{\overline{IREG}}\left(\Lambda^2\right)\frac{\partial \ln Z_m^{\overline{IREG}}}{\partial g^{\overline{IREG}}}$$



**Transition rules between IREG and the DS**

# The practical objective: the 2-loop fermion correction of QCD



## Challenge

Six 2-loops diagrams + 5 counter terms = more than 59 mass integrals to do

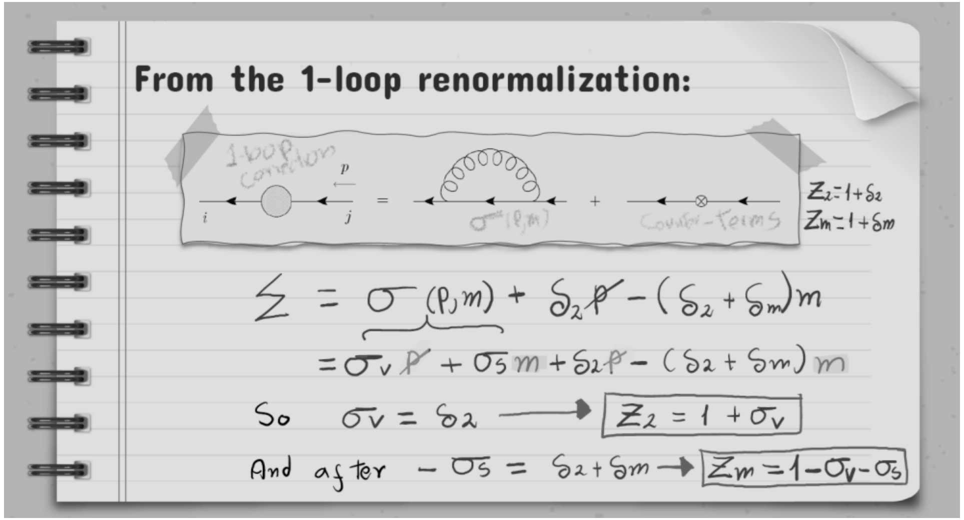
**Luckily**

(So far) most integrals have the same “shape” and repeat themselves

## Solution

Study the abelian case first (control case) and divide the work in the case with mass and without mass.

# Why the massless case also?

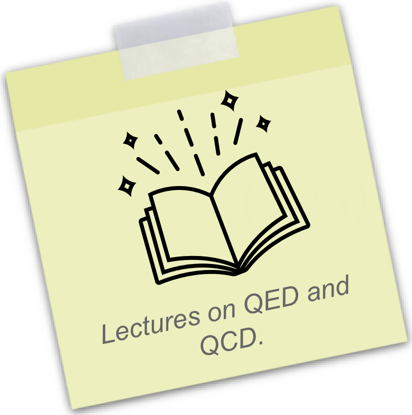


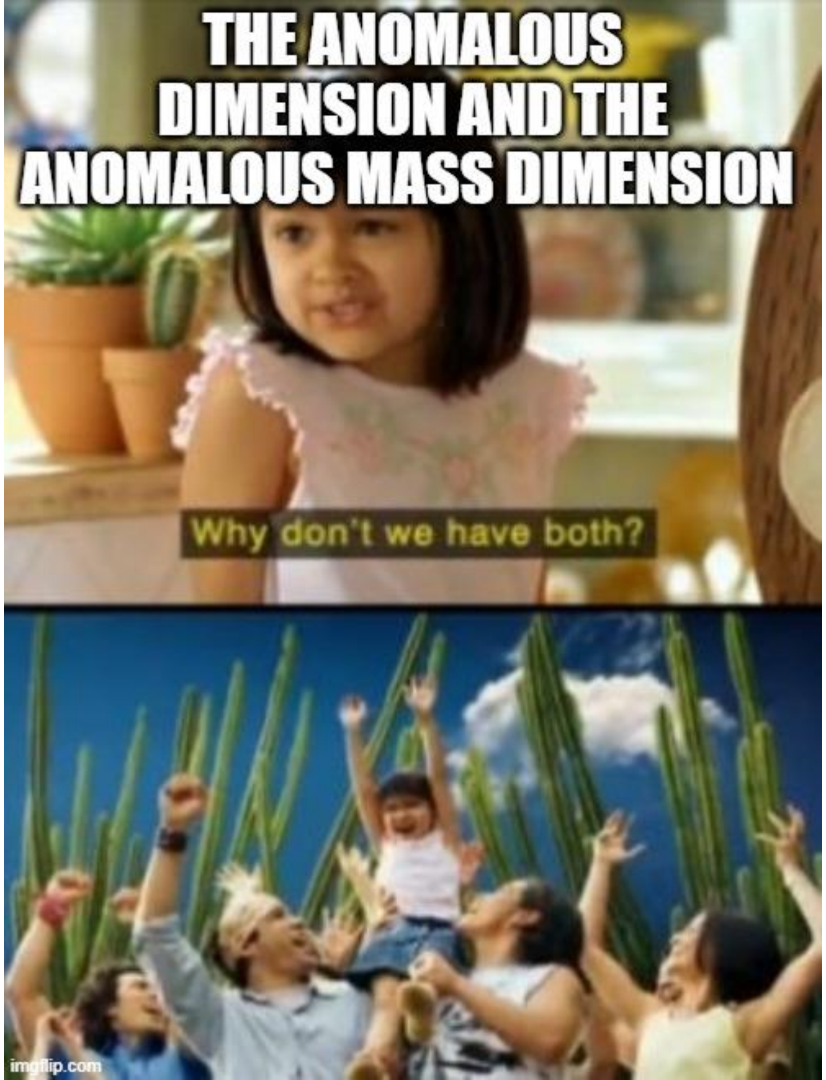

“The first of them [the renormalization constant of the field] is the same as in the massless case.”

# The anomalous dimensión (of the field)

the anomalous dimension is defined by

$$\gamma_A(\alpha(\mu)) = \frac{d \log Z_A(\alpha(\mu))}{d \log \mu} .$$




THE ANOMALOUS DIMENSION AND THE ANOMALOUS MASS DIMENSION
Why don't we have both?
imgflip.com

# Workplan

01

**Massless QED**
γ-function

02

**Massless QCD**
γ-function

03

**Mass QED**
γm-function

04

**Mass QCD**
γm-function

# Workplan


01

**Massless QED**

γ-function

02

**Massless QCD**

γ-function

03

**Mass QED**

γm-function

04

**Mass QCD**

γm-function

# Review: 1-loop

**A**

**Relevant diagram**

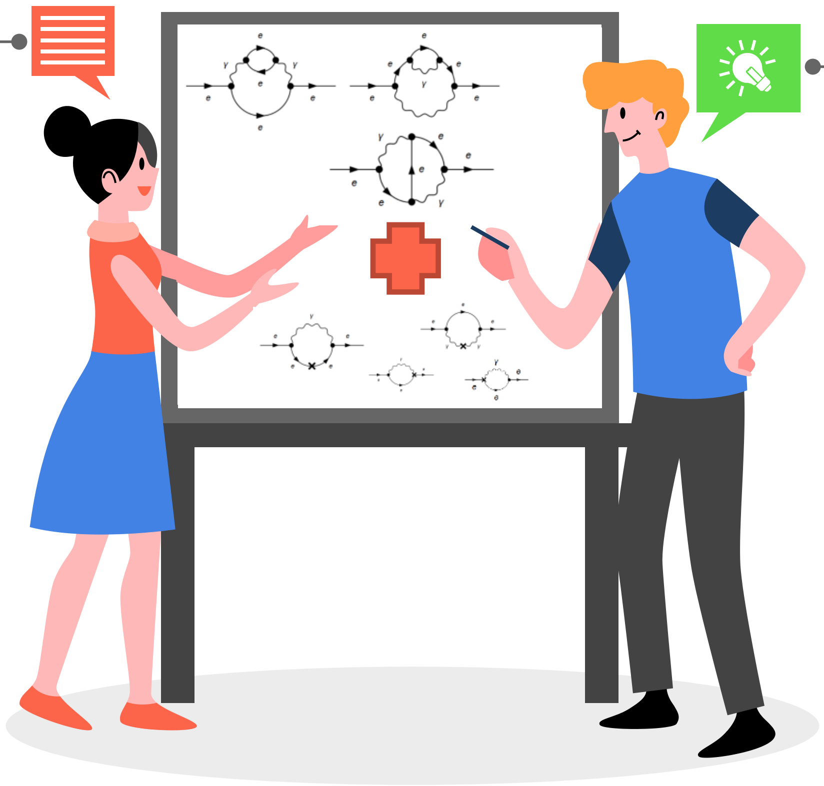
Just 1.

**B**

**Relevant counter-term**

Just 1 also.

# Review: 2-loop



**A**

## Relevant diagrams

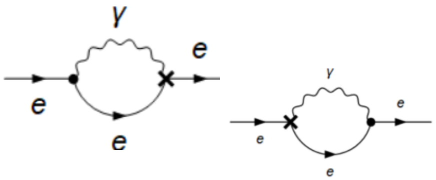
Three 2-loop diagrams.

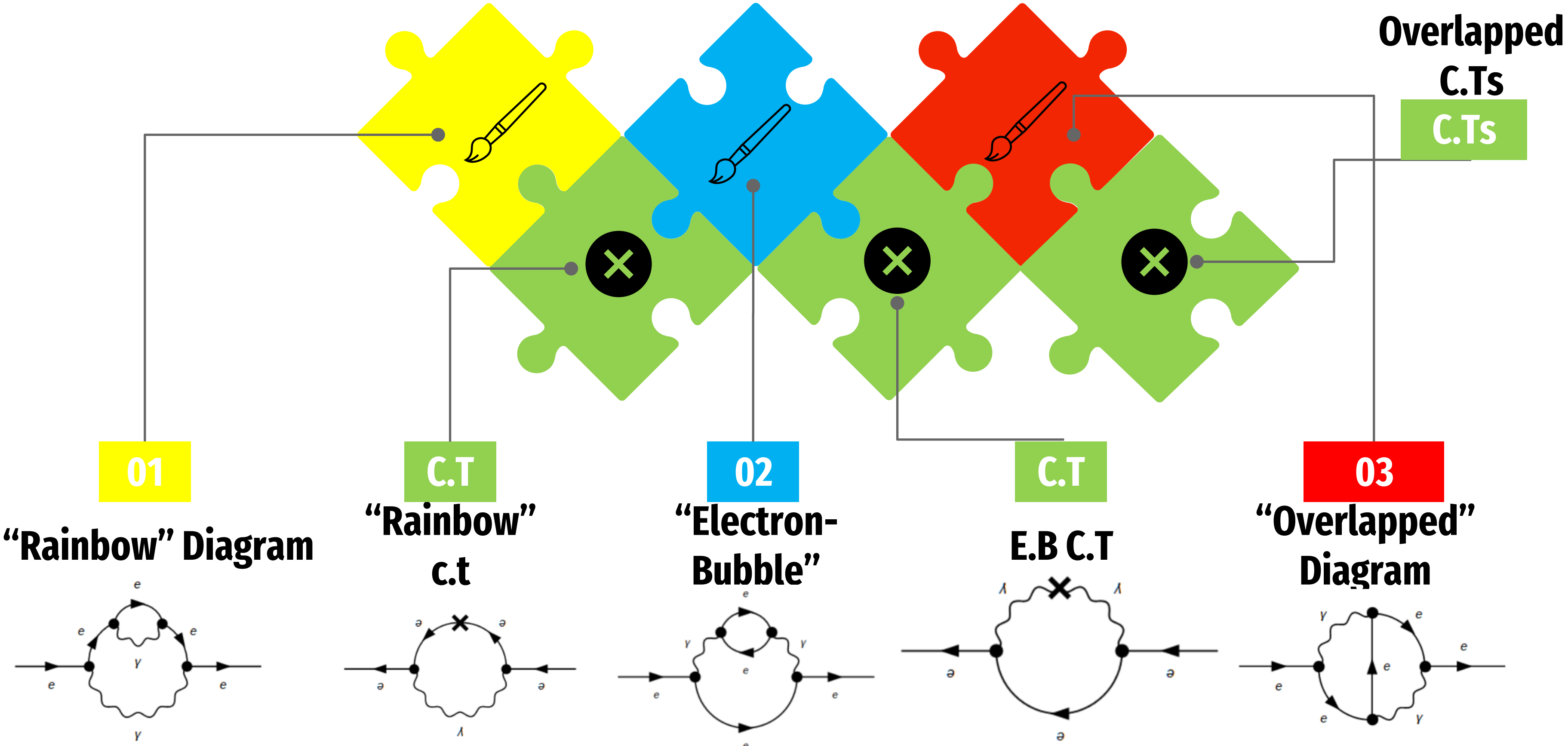
**B**

## Relevant counter-terms

Four 1-loop counter-terms.

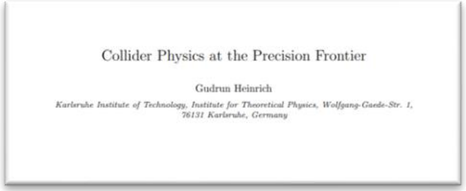
# The 2-loop fermion of QED



# Workflow

1. Amplitude generation.
2. Bringing the amplitude to a convenient form.
3. Reduction to master integrals.
4. Calculation of the MI.
5. Evaluation of the amplitude.



Collider Physics at the Precision Frontier

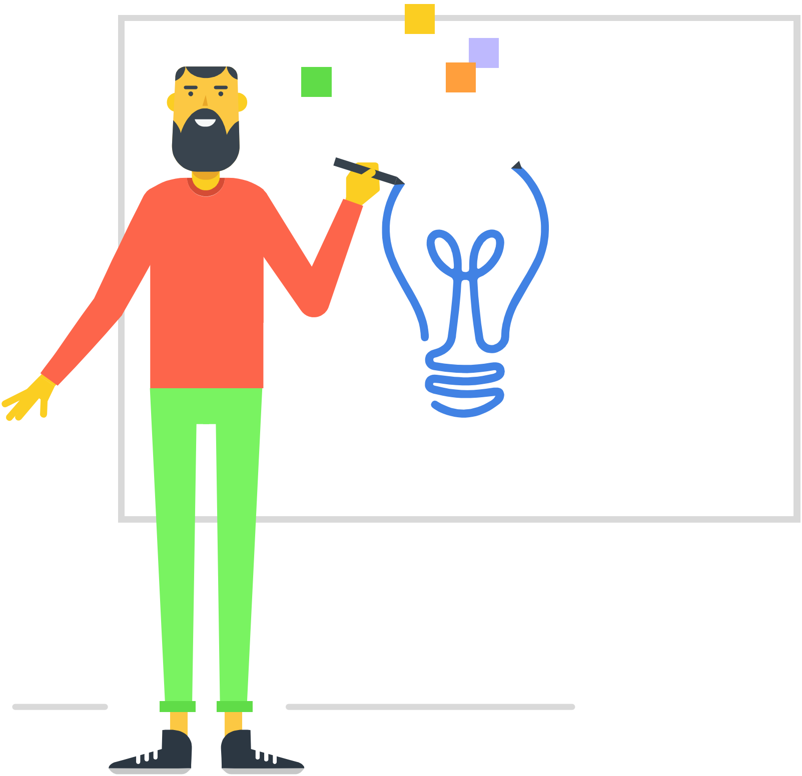
Gudrun Heinrich

*Karlsruhe Institute of Technology, Institute for Theoretical Physics, Wolfgang-Gaede-Str. 1, 76131 Karlsruhe, Germany*

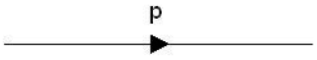
[arXiv:2009.00516[hep-ph]]

# 02-Dias vs Me

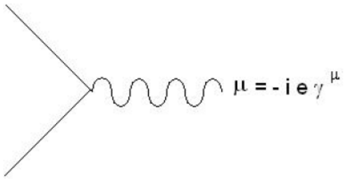
# Dias: theory

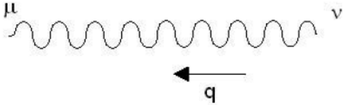


$$L = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \bar{\psi}[i\partial\!\!\!/ - m - e\gamma_\mu A^\mu]\psi$$



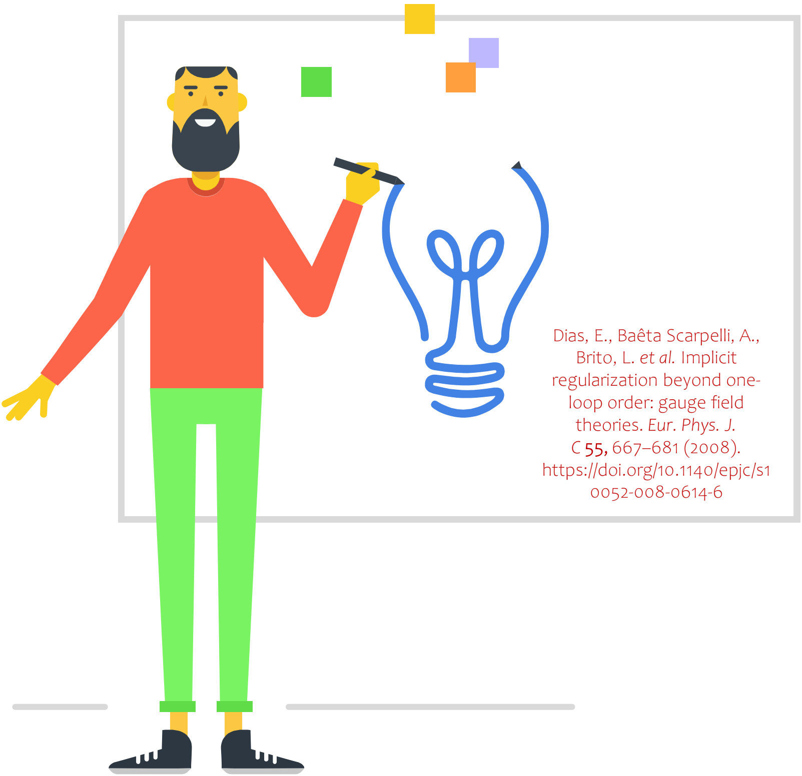

$$\frac{i}{p\!\!\!/ - m} = i\frac{(p\!\!\!/ + m)}{p^2 - m^2} \quad ;$$




$$-\frac{ig^{\mu\nu}}{q^2} \quad .$$

# Dias: diagrams and the sub-diagrams



Dias, E., Baêta Scarpelli, A., Brito, L. *et al.* Implicit regularization beyond one-loop order: gauge field theories. *Eur. Phys. J. C* **55,** 667–681 (2008). https://doi.org/10.1140/epjc/s10052-008-0614-6

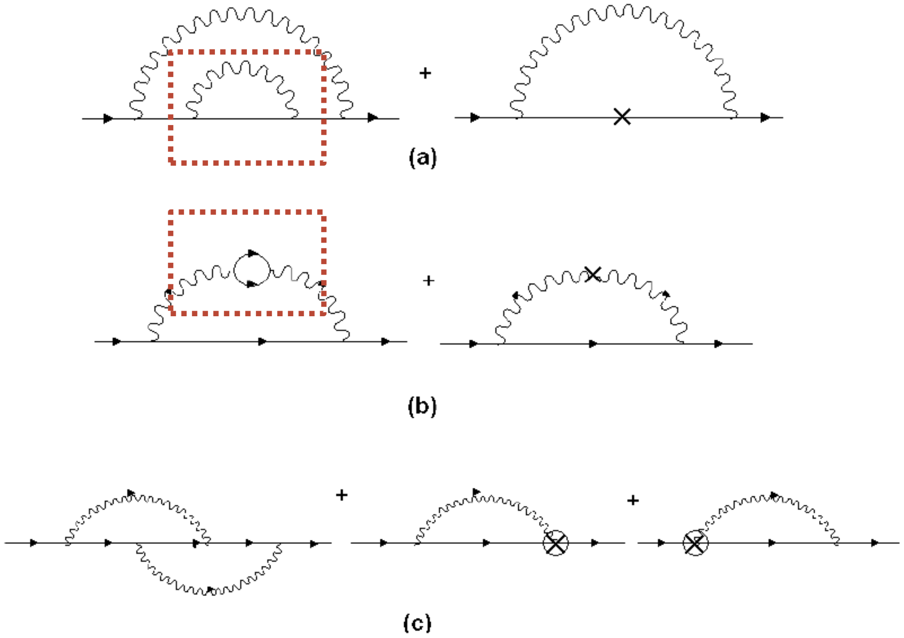
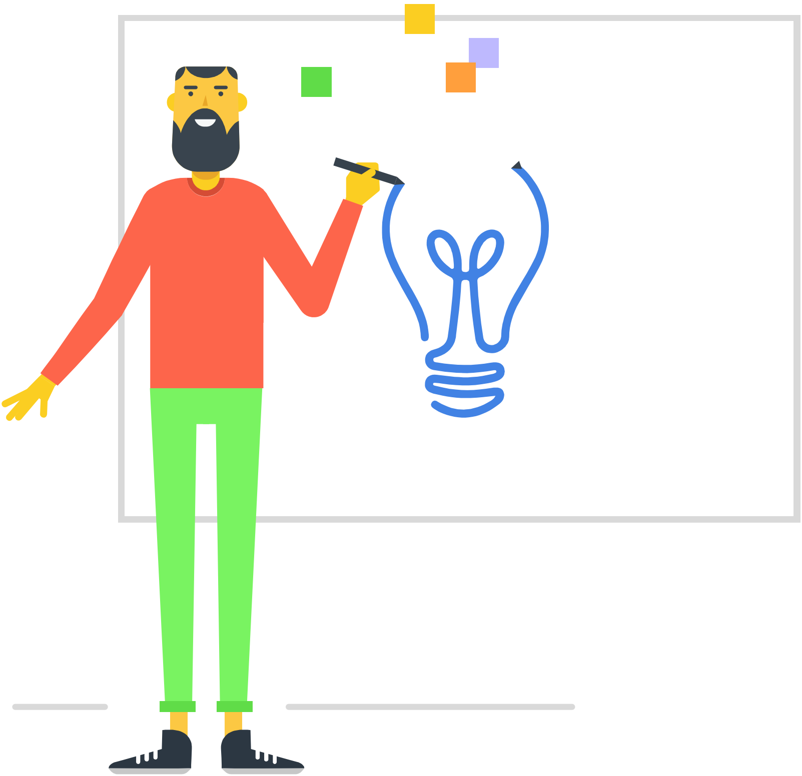
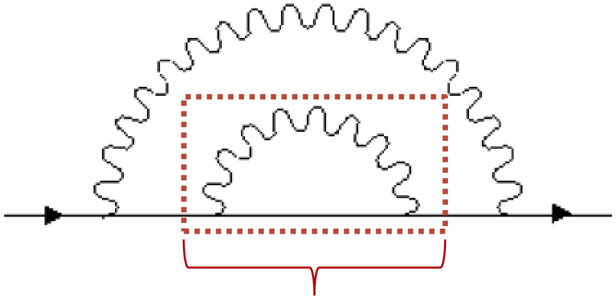



FIG. 4: contributions for the fermionic self energy at two loop order

# Dias: 1-loop computation with IREG





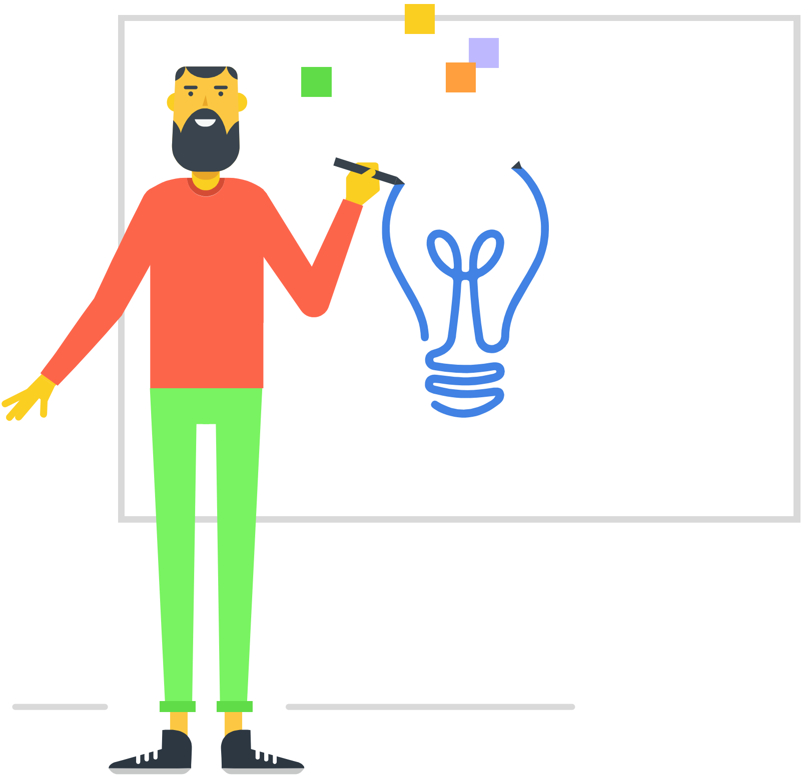
$$i\Sigma(p) = -e^2\int_k^\Lambda \frac{\gamma^\rho \not{k}\gamma_\rho}{k^2(p-k)^2}$$

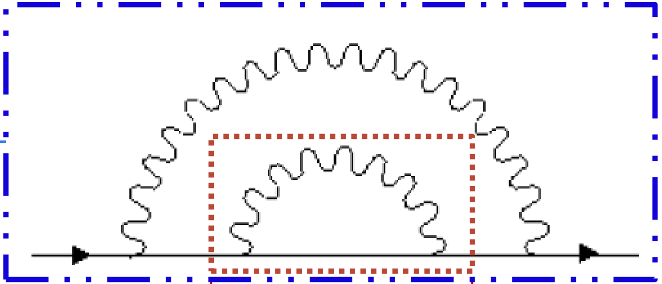
$$i\Sigma(p) = 2e^2\gamma^\alpha I_\alpha$$

$$I_\alpha = \frac{p_\alpha}{2}\left(I_{log}(\lambda^2) - b\ln\left(-\frac{p^2}{\lambda^2}\right) + 2b\right) - \frac{p^\mu}{2}\alpha_2 g_{\mu\alpha},$$

$$\boxed{i\Sigma(p) = e^2\not{p}\left\{I_{log}(\lambda^2) - b\ln\left(-\frac{p^2}{\lambda^2}\right) + 2b\right\}}$$

# Dias: 2-loop computation with IREG and counter-term



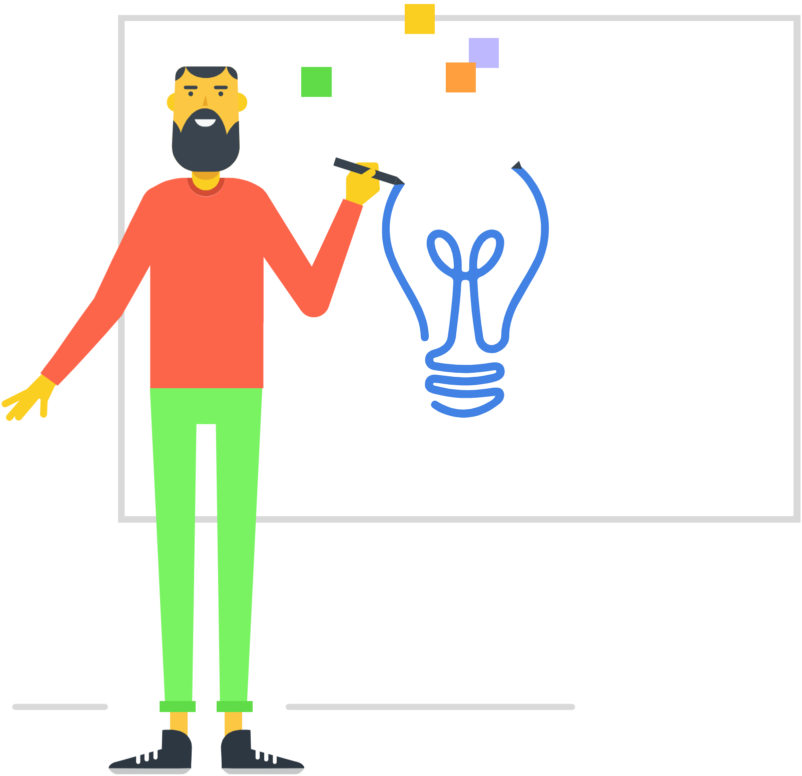


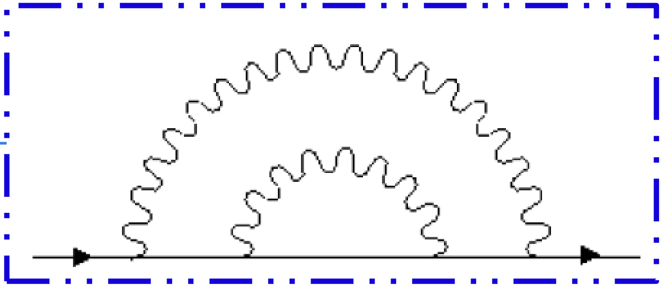
$$i\Sigma(p) = e^2 \not{p}\left\{ I_{log}(\lambda^2) - b\ln\left(-\frac{p^2}{\lambda^2}\right) + 2b\right\}$$

Now consider the nested two-loop self-energy. According to the BPHZ forest formula the subtraction of the one loop subdivergence amounts to replace the inner diagram with its finite part, namely

$$i\Sigma_1^{(2)}(p) = -ibe^4\int_k^\Lambda \frac{\gamma^\rho \not{k}\gamma_\rho}{k^2(p-k)^2}\left\{\ln\left(-\frac{k^2}{\lambda^2}\right)+2\right\}$$
$$= 2ie^4 b\gamma^\alpha\left[2I_\alpha - I_\alpha^{(2)}\right]. \qquad (20)$$

# Dias: final result for the “rainbow” diagram at 2-loop





$$i\Sigma_1^{(2)}(p) = ie^4 b\not{p}\left\{-I_{log}^{(2)}(\lambda^2) + \frac{3}{2}I_{log}(\lambda^2) + \frac{b}{2}\ln^2\left(-\frac{p^2}{\lambda^2}\right) - \frac{5}{2}b\ln\left(-\frac{p^2}{\lambda^2}\right) + \frac{5}{2}b\right\}.$$

Divergent Part

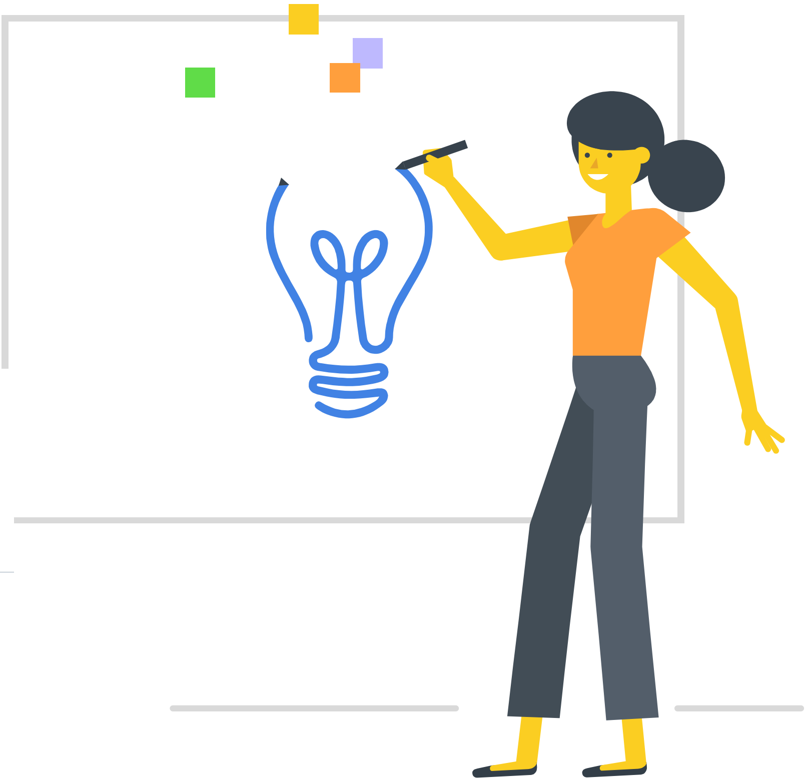
# Me: theory

## Fermion to Fermion process defined

```
ClearProcess[];
Processee = {F[2, {1}]} → {F[2, {1}]};
SetOptions[InsertFields, InsertionLevel → {Particles}, Model → FileNameJoin[{"QED", "QED"}],
asigna opciones                                          une nombre de fichero
  GenericModel → FileNameJoin[{"QED", "QED"}], ExcludeParticles → {F[2, {2 | 3}]}];
                une nombre de fichero
```

QED model created
with FeynRules

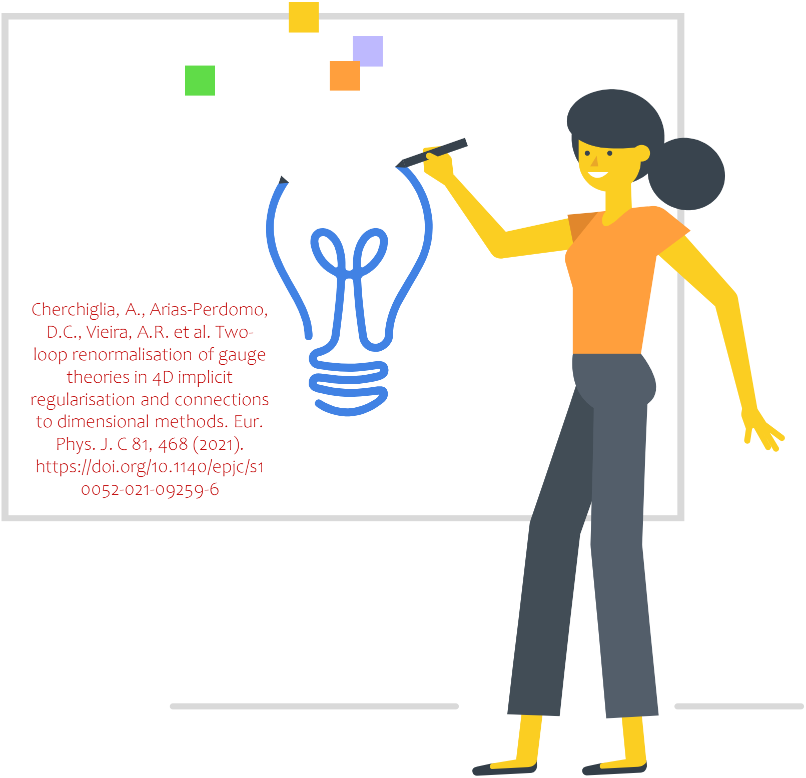
```
LQED = LQEDR + LQEDCT;
LQEDR = -1/4 FS[A, imu, inu] FS[A, imu, inu] + I lbar.Ga[imu].DC[l, imu]- Mlep[fi] lbar[s,fi].l[s,fi];
LQEDCT = FR$CT (-(ZA -1) 1/4 FS[A, imu, inu] FS[A, imu, inu]
              -1/(2GaugeXi[V[1]])(ZA/Zxi-1) del[A[imu],imu] del[A[inu],inu]
              + (Zpsi-1) I lbar.Ga[imu].del[l,imu] - (Zpsi Zm -1) Mlep[fi] lbar[s,fi].l[s,fi]
              + (Zpsi Sqrt[ZA] Ze - 1) EL lbar.Ga[imu].l A[imu]);
```



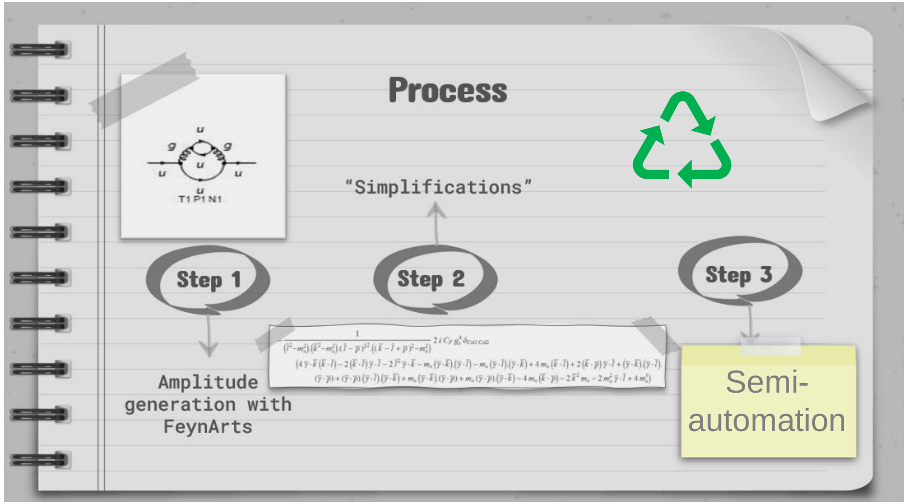
# Me: consideration

here. Moreover, in the framework of IREG, one is not allowed, in general, to evaluate a sub-diagram and join the obtained result in the full diagram. The reason can be traced back to equations similar to (3). This fact does not amount in a

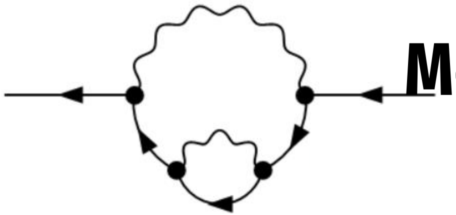
$$\left[\int_k k^{\mu_1}\cdots k^{\mu_{2m}} f(k^2)\right]^{\text{IREG}} \neq \frac{g^{\{\mu_1\mu_2}\cdots g^{\mu_{2m-1}\mu_{2m}\}}}{(2m)!}\left[\int_k k^{2m} f(k^2)\right]^{\text{IREG}}, \qquad (3)$$



# Me: algorithm




Cherchiglia, A., Arias-Perdomo, D.C., Vieira, A.R. et al. Two-loop renormalisation of gauge theories in 4D implicit regularisation and connections to dimensional methods. Eur. Phys. J. C 81, 468 (2021). https://doi.org/10.1140/epjc/s10052-021-09259-6



# Me: 2-loop computation with IREG

```
FAamplElectronR = CreateFeynAmp[therainbowdiagram, Truncated → True, GaugeRules → {},
   PreFactor → 1];
convertamplElectronR = FCFAConvert[FAamplElectronR, IncomingMomenta → {p},
   OutgoingMomenta → {p}, LoopMomenta → {l, k}, LorentzIndexNames → {μ, ν, α, β},
   UndoChiralSplittings → True, ChangeDimension → 4, List → False, SMP → True,
   DropSumOver → True, Contract → True];
```

Amplitude generation with FeynArts

```
(*We work now in Feynman gauge*)
FeynmangaugeamplElectronR = convertamplElectronR /. GaugeXi[V[1]] -> 1
```

$$\frac{i\, e^4\, \gamma^{\nu}.(\gamma\cdot(\overline{p}-\overline{l})+m_e).\gamma^{\beta}.(m_e-\gamma\cdot\overline{k}).\gamma^{\beta}.(\gamma\cdot(\overline{p}-\overline{l})+m_e).\gamma^{\nu}}{\overline{l}^2.(\overline{k}^2-m_e^2).((\overline{l}-\overline{p})^2-m_e^2)^2.(\overline{k}-\overline{l}+\overline{p})^2}$$

Choose of the Feynman Gauge

```
(*We set again the mass to zero*)
ElectronRmassless = FeynmangaugeamplElectronR //. SMP["m_e"] → 0
```

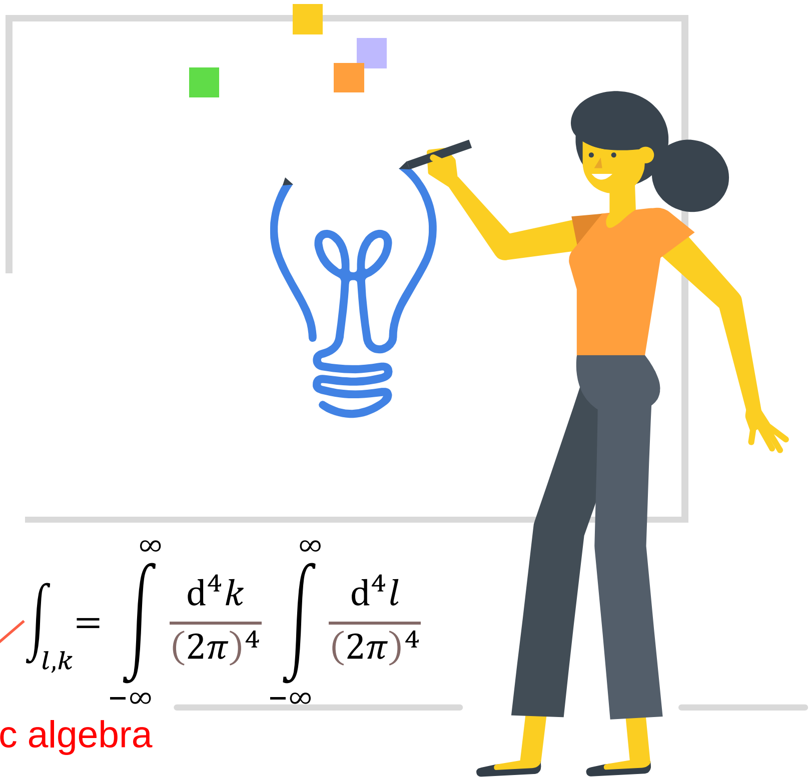
$$\frac{i\, e^4\, \gamma^{\nu}.(\gamma\cdot(\overline{p}-\overline{l})).\gamma^{\beta}.(-(\gamma\cdot\overline{k})).\gamma^{\beta}.(\gamma\cdot(\overline{p}-\overline{l})).\gamma^{\nu}}{\overline{l}^2.\overline{k}^2.(\overline{l}-\overline{p})^{22}.(\overline{k}-\overline{l}+\overline{p})^2}$$

M->0

```
(*Same procedure as above, we need to apply DiracSimplify twice to be sure we are doing all the vector
 slash vector slash products*)
AlgebraDiracElectronRAmassless = DiracSimplify[ElectronRmassless];
AlgebraDiracElectronRAmassless2 = DiracSimplify[AlgebraDiracElectronRAmassless]
```

$$-\frac{8\, i\, e^4\, (\overline{k}\cdot\overline{l})\, \gamma\cdot\overline{l}}{\overline{l}^2.\overline{k}^2.(\overline{l}-\overline{p})^{22}.(\overline{k}-\overline{l}+\overline{p})^2}+\frac{8\, i\, e^4\, (\overline{k}\cdot\overline{l})\, \gamma\cdot\overline{p}}{\overline{l}^2.\overline{k}^2.(\overline{l}-\overline{p})^{22}.(\overline{k}-\overline{l}+\overline{p})^2}+\frac{8\, i\, e^4\, (\overline{k}\cdot\overline{p})\, \gamma\cdot\overline{l}}{\overline{l}^2.\overline{k}^2.(\overline{l}-\overline{p})^{22}.(\overline{k}-\overline{l}+\overline{p})^2}-\frac{8\, i\, e^4\, (\overline{k}\cdot\overline{p})\, \gamma\cdot\overline{p}}{\overline{l}^2.\overline{k}^2.(\overline{l}-\overline{p})^{22}.(\overline{k}-\overline{l}+\overline{p})^2}-\frac{8\, i\, e^4\, \gamma\cdot\overline{k}\,(\overline{l}\cdot\overline{p})}{\overline{l}^2.\overline{k}^2.(\overline{l}-\overline{p})^{22}.(\overline{k}-\overline{l}+\overline{p})^2}+\frac{4\, i\, e^4\, \overline{l}^2\, \gamma\cdot\overline{k}}{\overline{l}^2.\overline{k}^2.(\overline{l}-\overline{p})^{22}.(\overline{k}-\overline{l}+\overline{p})^2}+\frac{4\, i\, e^4\, \overline{p}^2\, \gamma\cdot\overline{k}}{\overline{l}^2.\overline{k}^2.(\overline{l}-\overline{p})^{22}.(\overline{k}-\overline{l}+\overline{p})^2}$$

Dirac algebra

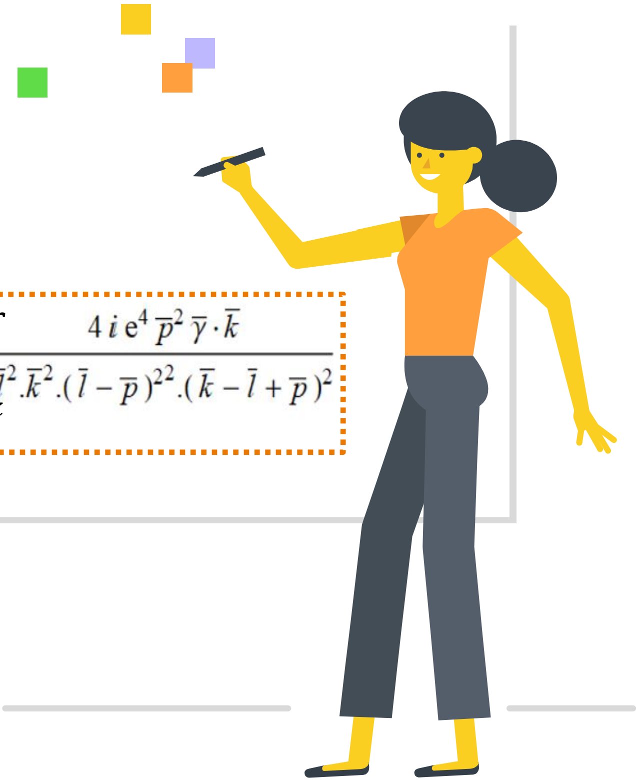


$$\int_{l,k} = \int_{-\infty}^{\infty} \frac{d^4 k}{(2\pi)^4} \int_{-\infty}^{\infty} \frac{d^4 l}{(2\pi)^4}$$

# Me: semi-automation and shift l->l+p

$$\int_{l,k}\frac{8\,i\,e^4\left(\bar{k}\cdot\bar{l}\right)\bar{\gamma}\cdot\bar{l}}{\bar{l}^2.\bar{k}^2.(\bar{l}-\bar{p})^{22}.(\bar{k}-\bar{l}+\bar{p})^2}+\int_{l,k}\frac{8\,i\,e^4\left(\bar{k}\cdot\bar{l}\right)\bar{\gamma}\cdot\bar{p}}{\bar{l}^2.\bar{k}^2.(\bar{l}-\bar{p})^{22}.(\bar{k}-\bar{l}+\bar{p})^2}+\int_{l,k}\frac{8\,i\,e^4\left(\bar{k}\cdot\bar{p}\right)\bar{\gamma}\cdot\bar{l}}{\bar{l}^2.\bar{k}^2.(\bar{l}-\bar{p})^{22}.(\bar{k}-\bar{l}+\bar{p})^2}-$$

$$\int_{l,k}\frac{8\,i\,e^4\left(\bar{k}\cdot\bar{p}\right)\bar{\gamma}\cdot\bar{p}}{\bar{l}^2.\bar{k}^2.(\bar{l}-\bar{p})^{22}.(\bar{k}-\bar{l}+\bar{p})^2}-\int_{l,k}\frac{8\,i\,e^4\,\bar{\gamma}\cdot\bar{k}\left(\bar{l}\cdot\bar{p}\right)}{\bar{l}^2.\bar{k}^2.(\bar{l}-\bar{p})^{22}.(\bar{k}-\bar{l}+\bar{p})^2}+\int_{l,k}\frac{4\,i\,e^4\,\bar{l}^2\,\bar{\gamma}\cdot\bar{k}}{\bar{l}^2.\bar{k}^2.(\bar{l}-\bar{p})^{22}.(\bar{k}-\bar{l}+\bar{p})^2}+\int_{l,k}\frac{4\,i\,e^4\,\bar{p}^2\,\bar{\gamma}\cdot\bar{k}}{\bar{l}^2.\bar{k}^2.(\bar{l}-\bar{p})^{22}.(\bar{k}-\bar{l}+\bar{p})^2}$$



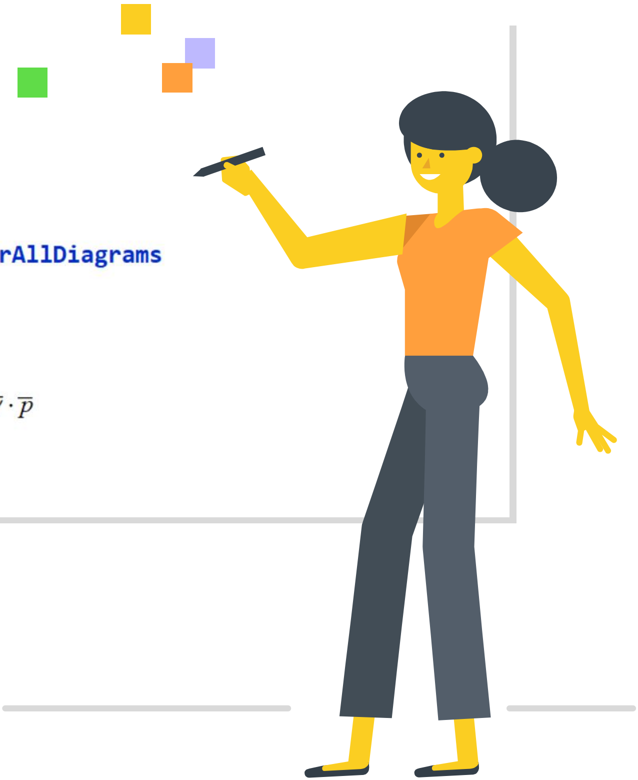
# Me: amplitude regularized with IREG (only div. part)

## Rainbow Diagram

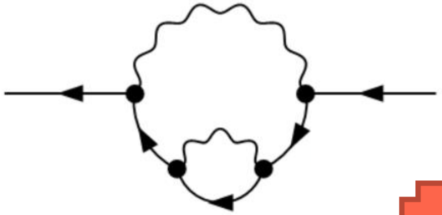
```
RegulatedAmplitudeRainbowDiagram = AlgebraDiracElectronRAmassless2 //. SubRuleforAllDiagrams
```

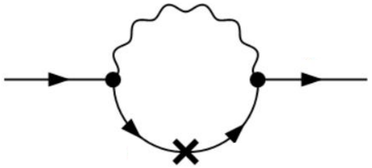
$$8\,i\,e^4\left(-\frac{1}{2}\,b\,\text{Ilog2}\lambda2\,\overline{\gamma}\cdot\overline{p}-\frac{1}{2}\boxed{b\,\text{Ilog}\lambda2\,\log\left(-\frac{p^2}{\lambda^2}\right)}\overline{\gamma}\cdot\overline{p}+\frac{5}{4}\,b\,\text{Ilog}\lambda2\,\overline{\gamma}\cdot\overline{p}+\frac{1}{2}\,\text{Ilog}\lambda2^2\,\overline{\gamma}\cdot\overline{p}\right)-$$
$$8\,i\,e^4\left(-\frac{3}{8}\,b\,\text{Ilog2}\lambda2\,\overline{\gamma}\cdot\overline{p}-\frac{3}{8}\,b\,\text{Ilog}\lambda2\log\left(-\frac{p^2}{\lambda^2}\right)\overline{\gamma}\cdot\overline{p}+b\,\text{Ilog}\lambda2\,\overline{\gamma}\cdot\overline{p}+\frac{3}{8}\,\text{Ilog}\lambda2^2\,\overline{\gamma}\cdot\overline{p}\right)+2\,i\,b\,e^4\,\text{Ilog}\lambda2\,\overline{\gamma}\cdot\overline{p}$$

No local-terms



# Me: renormalization





$$\frac{2\,e^2\,\bar{l}^2\,Z_\psi\,\bar{\gamma}\cdot\bar{l}}{(\bar{l}^2)^2.(\bar{l}+\bar{p})^2}-\frac{2\,e^2\,\bar{l}^2\,\bar{\gamma}\cdot\bar{l}}{(\bar{l}^2)^2.(\bar{l}+\bar{p})^2}$$

```
(*The final rainbow C.T*)
RainbowCounterTermFinal = AlgebraDiracmasslessCTRainbow2 //. SubRuleforRainbowCT
```

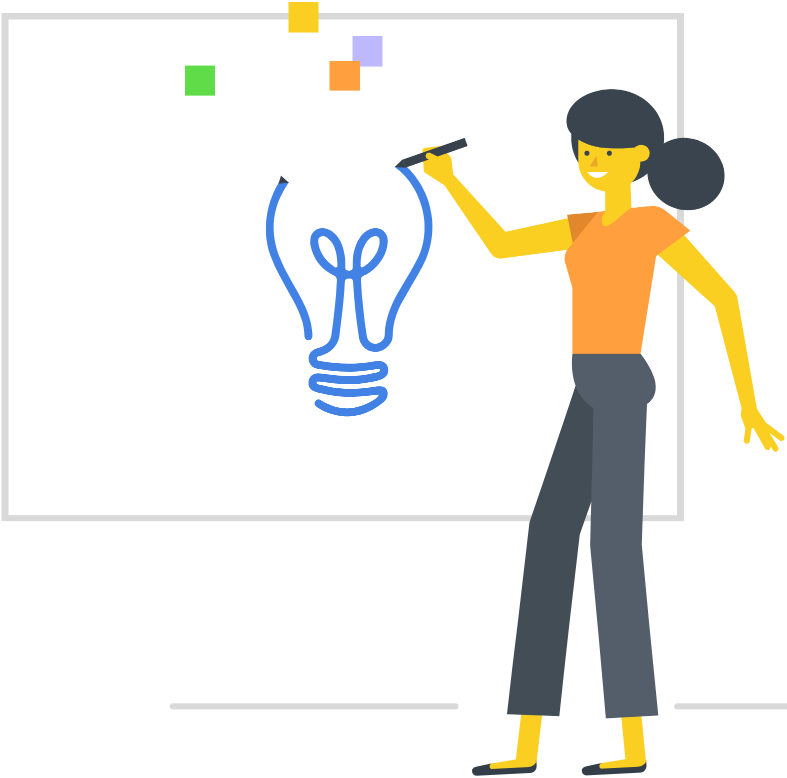
$$e^2\,\bar{\gamma}\cdot\bar{p}\left(b\left(-\log\left(-\frac{p^2}{\lambda^2}\right)\right)+2\,b+\mathrm{Ilog}\lambda 2\right)-e^2\,Z_\psi\,\bar{\gamma}\cdot\bar{p}\left(b\left(-\log\left(-\frac{p^2}{\lambda^2}\right)\right)+2\,b+\mathrm{Ilog}\lambda 2\right)$$

```
(*The renormalization constant of the field for IREG*)
Z2 = 1 + SMP["e"]^2 * SMP["d_psi"]
```

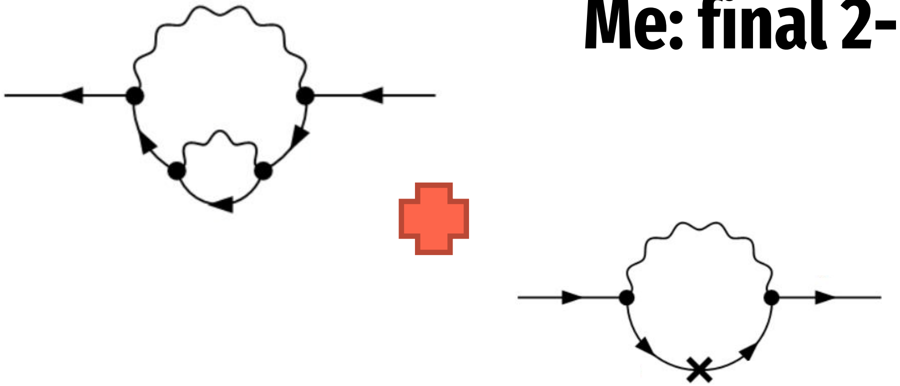
$$e^2\,\delta_\psi+1$$

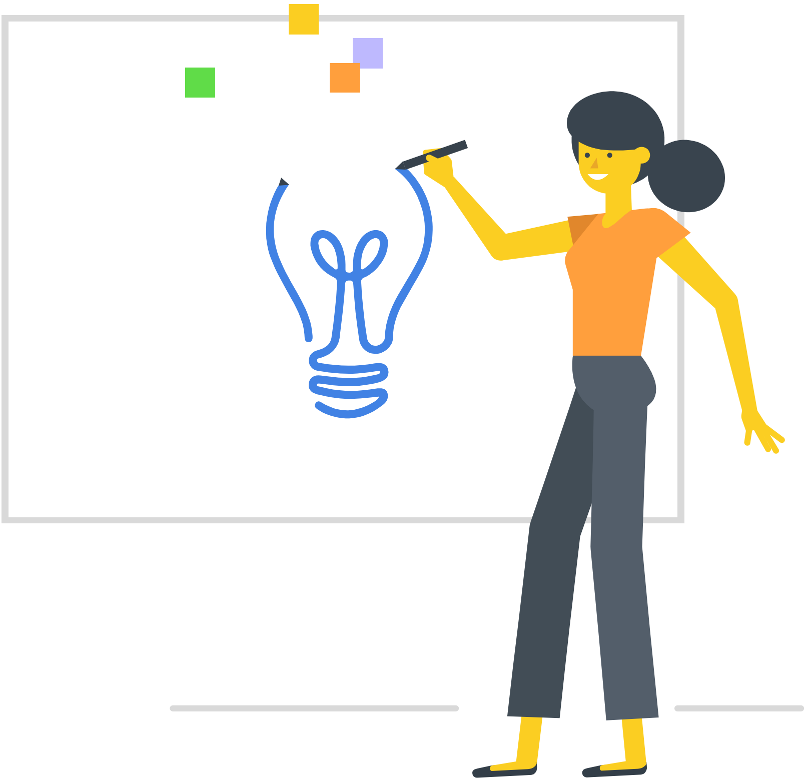
```
NewRuleCTRainbow = {SMP["Z_psi"] → Z2};
RainbowCounterTermFinal2 = RainbowCounterTermFinal //. NewRuleCTRainbow
```

$$e^2\,\bar{\gamma}\cdot\bar{p}\left(b\left(-\log\left(-\frac{p^2}{\lambda^2}\right)\right)+2\,b+\mathrm{Ilog}\lambda 2\right)-e^2\,(e^2\,\delta_\psi+1)\,\bar{\gamma}\cdot\bar{p}\left(b\left(-\log\left(-\frac{p^2}{\lambda^2}\right)\right)+2\,b+\mathrm{Ilog}\lambda 2\right)$$



# Me: final 2-loop renormalized amplitude





## Rainbow Diagram

```
(*Re-writting the counterterm*)
RainbowCounterTermFinal4 = RainbowCounterTermFinal3 //. RuleCTField
```

$$i\,e^4\,\text{Ilog}\lambda2\;\overline{\gamma}\cdot\overline{p}\left(b\log\left(-\frac{p^2}{\lambda^2}\right)-2\,b-\text{Ilog}\lambda2\right)$$

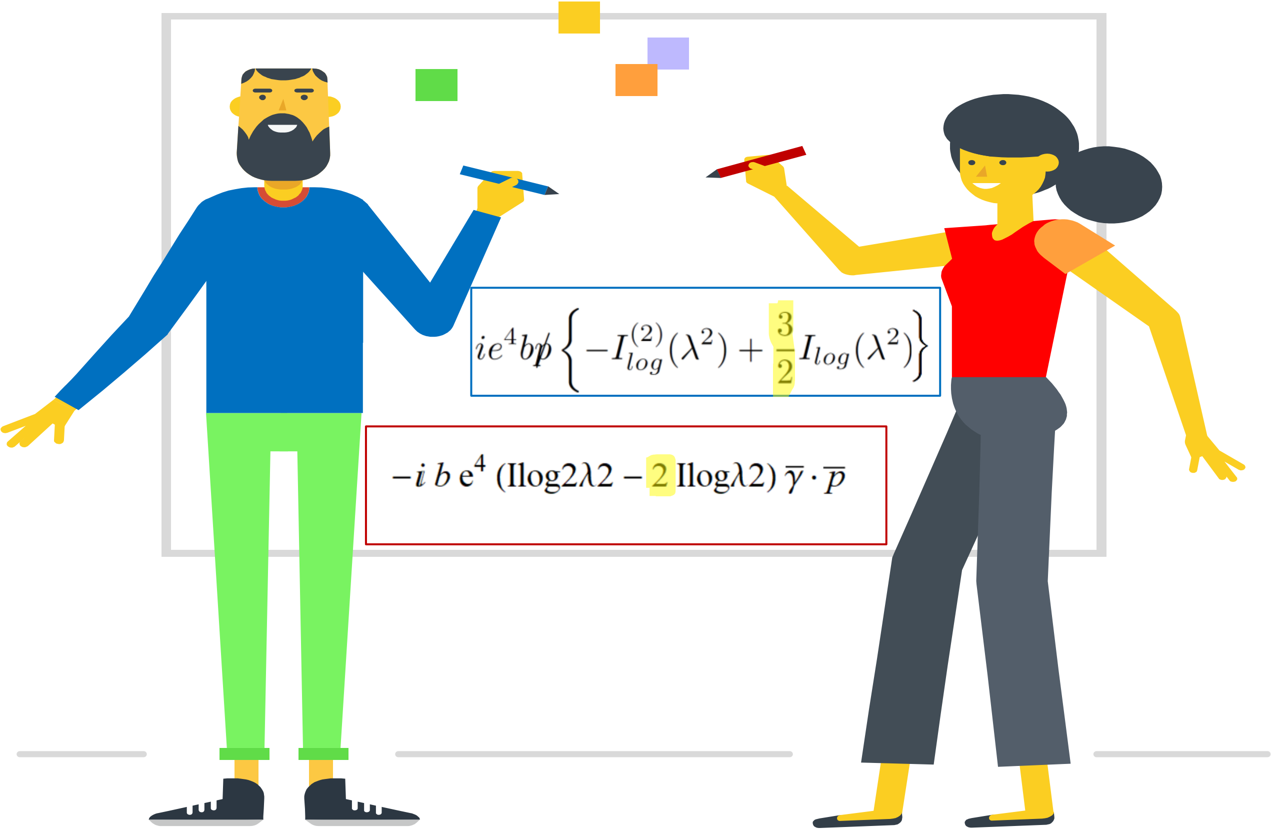
```
(*Regulated Amplitude + Counter-Term*)
ChaveRainbow = RegulatedRainbowAmplitudeIREG + RainbowCounterTermFinal4
```

$$i\,e^4\,\text{Ilog}\lambda2\;\overline{\gamma}\cdot\overline{p}\left(b\log\left(-\frac{p^2}{\lambda^2}\right)-2\,b-\text{Ilog}\lambda2\right)-i\,e^4\;\overline{\gamma}\cdot\overline{p}\left(b\,(\text{Ilog}2\lambda2-4\,\text{Ilog}\lambda2)+b\,\text{Ilog}\lambda2\,\log\left(-\frac{p^2}{\lambda^2}\right)-\text{Ilog}\lambda2^2\right)$$

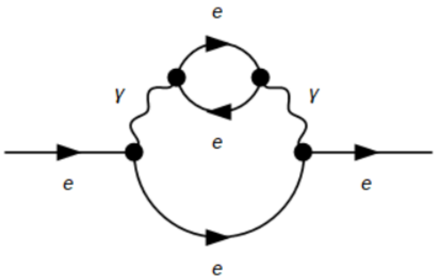
```
ChaveRainbow2 = Simplify[ChaveRainbow]
                        simplifica
```

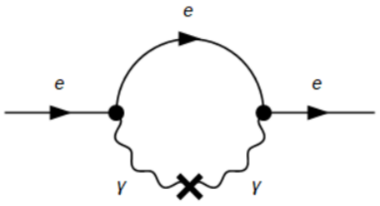
$$-i\,b\,e^4\,(\text{Ilog}2\lambda2-2\,\text{Ilog}\lambda2)\;\overline{\gamma}\cdot\overline{p}$$

# The difference...



# Other issues




### Electron-Bubble Diagram

```
(*Re-writting the counterterm*)
ElectronBubbleCounterTermFinal5 = ElectronBubbleCounterTermFinal4 //. RuleCTCoupling
```

$$\frac{4}{3}\, i\, e^4\, \mathrm{Ilog}\lambda 2\; \overline{\gamma}\cdot\overline{p}\left(b \log\left(-\frac{p^2}{\lambda^2}\right) - 2\, b - \mathrm{Ilog}\lambda 2\right)$$

```
(*Regulated Amplitude + Counter-Term*)
ChaveBubble = RegulatedElectronBubbleAmplitudeIREG + ElectronBubbleCounterTermFinal5
```

$$\frac{4}{3}\, i\, e^4\, \mathrm{Ilog}\lambda 2\; \overline{\gamma}\cdot\overline{p}\left(b \log\left(-\frac{p^2}{\lambda^2}\right) - 2\, b - \mathrm{Ilog}\lambda 2\right) - \frac{1}{3}\, i\, e^4\, \overline{\gamma}\cdot\overline{p}\left(9\, b\, \mathrm{Ilog2}\lambda 2 + 9\, b\, \mathrm{Ilog}\lambda 2 \log\left(-\frac{p^2}{\lambda^2}\right) - 35\, b\, \mathrm{Ilog}\lambda 2 - 9\, \mathrm{Ilog}\lambda 2^2\right)$$

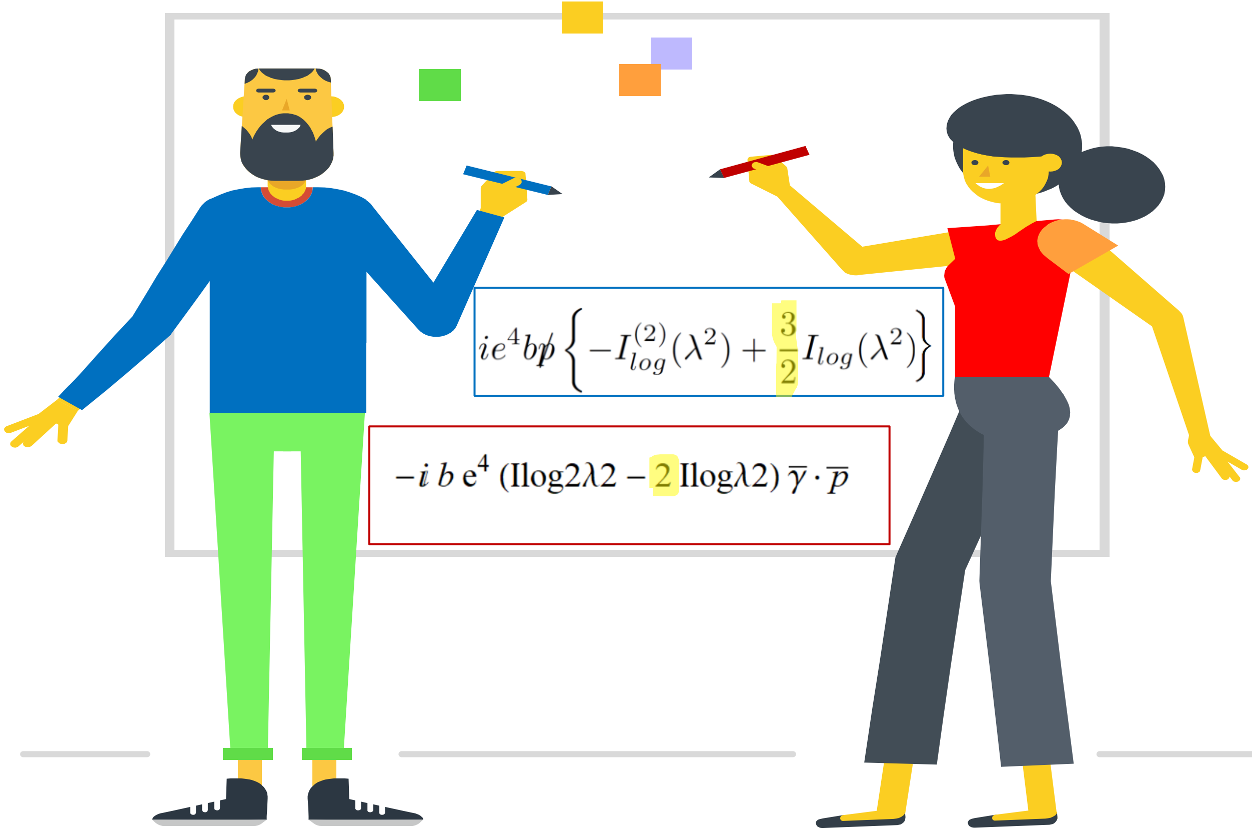
```
ChaveBubble2 = Simplify[ChaveBubble]
```

simplifica

$$-\frac{1}{3}\, i\, e^4\, \overline{\gamma}\cdot\overline{p}\left(9\, b\, (\mathrm{Ilog2}\lambda 2 - 3\, \mathrm{Ilog}\lambda 2) \boxed{+ 5\, b\, \mathrm{Ilog}\lambda 2 \log\left(-\frac{p^2}{\lambda^2}\right) - 5\, \mathrm{Ilog}\lambda 2^2}\right)$$

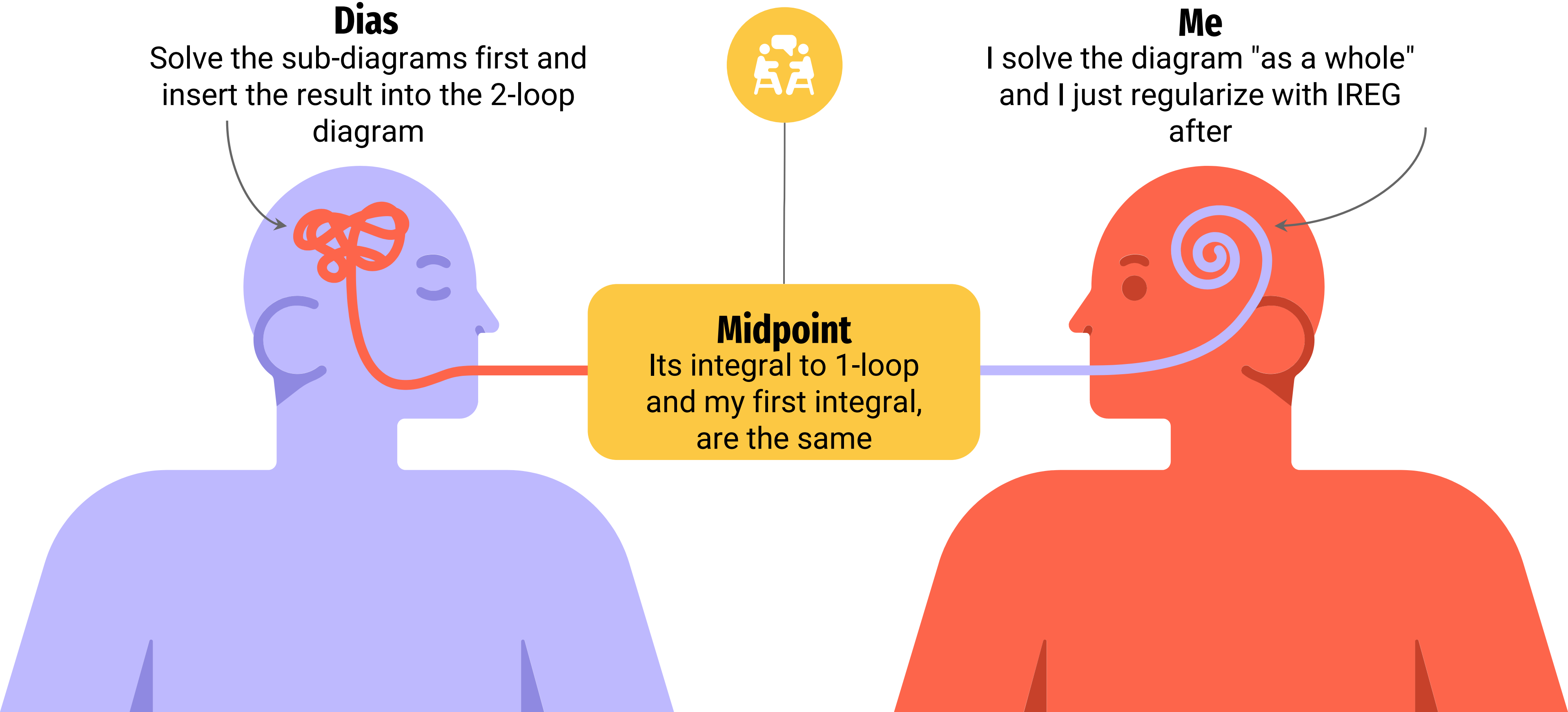
No local-terms still don't vanish…

# The difference...



# 03-Brainstorming

# Where does this difference come from?



# Shift l->l+p before Dirac algebra: Does it help to compare better?

## Problem

How do Dias' integrals differ from mine? Is there a l^2 term generation with his method or mine?

## Possible olution

Do the shift before the Dirac Algebra and compare

# Amplitude doing l->l+p before and after Dirac's algebra

~~Before~~ shift after

```
ElectronB2massless =
 FeynmangaugeamplElectronB2 //. SMP["m_e"] → 0
```

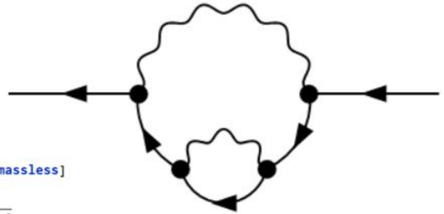
$$\frac{i\,e^4\,\bar{\gamma}^{\nu}.(\bar{\gamma}\cdot(\bar{p}-\bar{l})).\bar{\gamma}^{\beta}.(-(\bar{\gamma}\cdot\bar{k})).\bar{\gamma}^{\beta}.(\bar{\gamma}\cdot(\bar{p}-\bar{l})).\bar{\gamma}^{\nu}}{\bar{l}^2.\bar{k}^2.(\bar{l}-\bar{p})^{22}.(\bar{k}-\bar{l}+\bar{p})^2}$$

```
AlgebraDiracElectronB2Amassless =
 DiracSimplify[ElectronB2massless]
```

$$-\frac{8\,i\,e^4\,(\bar{k}\cdot\bar{l})\,\bar{\gamma}\cdot\bar{l}}{\bar{l}^2.\bar{k}^2.(\bar{l}-\bar{p})^{22}.(\bar{k}-\bar{l}+\bar{p})^2}+\frac{8\,i\,e^4\,(\bar{k}\cdot\bar{l})\,\bar{\gamma}\cdot\bar{p}}{\bar{l}^2.\bar{k}^2.(\bar{l}-\bar{p})^{22}.(\bar{k}-\bar{l}+\bar{p})^2}+\frac{8\,i\,e^4\,(\bar{k}\cdot\bar{p})\,\bar{\gamma}\cdot\bar{l}}{\bar{l}^2.\bar{k}^2.(\bar{l}-\bar{p})^{22}.(\bar{k}-\bar{l}+\bar{p})^2}-\frac{8\,i\,e^4\,(\bar{k}\cdot\bar{p})\,\bar{\gamma}\cdot\bar{p}}{\bar{l}^2.\bar{k}^2.(\bar{l}-\bar{p})^{22}.(\bar{k}-\bar{l}+\bar{p})^2}-\frac{8\,i\,e^4\,\bar{\gamma}\cdot\bar{k}\,(\bar{l}\cdot\bar{p})}{\bar{l}^2.\bar{k}^2.(\bar{l}-\bar{p})^{22}.(\bar{k}-\bar{l}+\bar{p})^2}+\frac{4\,i\,e^4\,\bar{l}^2\,\bar{\gamma}\cdot\bar{k}}{\bar{l}^2.\bar{k}^2.(\bar{l}-\bar{p})^{22}.(\bar{k}-\bar{l}+\bar{p})^2}+\frac{4\,i\,e^4\,\bar{p}^2\,\bar{\gamma}\cdot\bar{k}}{\bar{l}^2.\bar{k}^2.(\bar{l}-\bar{p})^{22}.(\bar{k}-\bar{l}+\bar{p})^2}$$

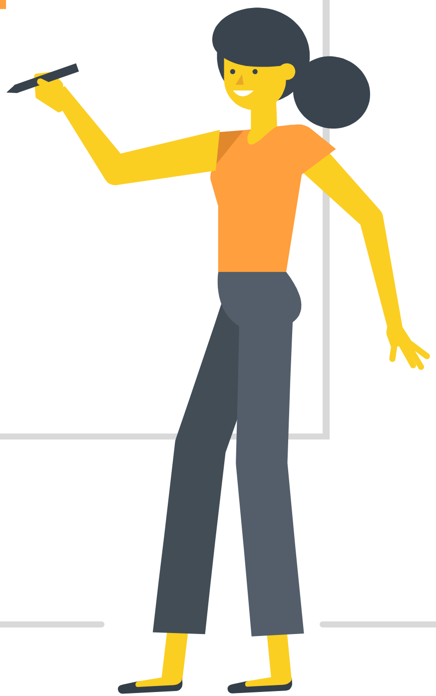
~~After~~ shift before

$$\frac{i\,e^4\,\bar{\gamma}^{\nu}.(-(\bar{\gamma}\cdot\bar{l})).\bar{\gamma}^{\beta}.(-(\bar{\gamma}\cdot\bar{k})).\bar{\gamma}^{\beta}.(-(\bar{\gamma}\cdot\bar{l})).\bar{\gamma}^{\nu}}{(\bar{l}+\bar{p})^2.\bar{k}^2.(\bar{l}^2)^2.(\bar{k}-\bar{l})^2}$$



```
AlgebraDiracElectronC2Amassless =
 DiracSimplify[AlgebraDiracElectronB2Amassless]
```

$$\frac{4\,i\,e^4\,\bar{l}^2\,\bar{\gamma}\cdot\bar{k}}{(\bar{l}+\bar{p})^2.\bar{k}^2.(\bar{l}^2)^2.(\bar{k}-\bar{l})^2}-\frac{8\,i\,e^4\,(\bar{k}\cdot\bar{l})\,\bar{\gamma}\cdot\bar{l}}{(\bar{l}+\bar{p})^2.\bar{k}^2.(\bar{l}^2)^2.(\bar{k}-\bar{l})^2}$$



# Amplitude doing l->l+p before and after Dirac's algebra

~~Before~~

Doing shift After D,A

```
RegulatedRainbowAmplitudeIREG = Simplify[RegulatedAmplitudeRainbowDiagram]
```

simplifica

$$-i\, e^4\, \bar{\gamma}\cdot\bar{p}\left(b\,(\text{Ilog}2\lambda 2 - 4\,\text{Ilog}\lambda 2) + b\,\text{Ilog}\lambda 2\,\log\left(-\frac{p^2}{\lambda^2}\right) - \text{Ilog}\lambda 2^2\right)$$

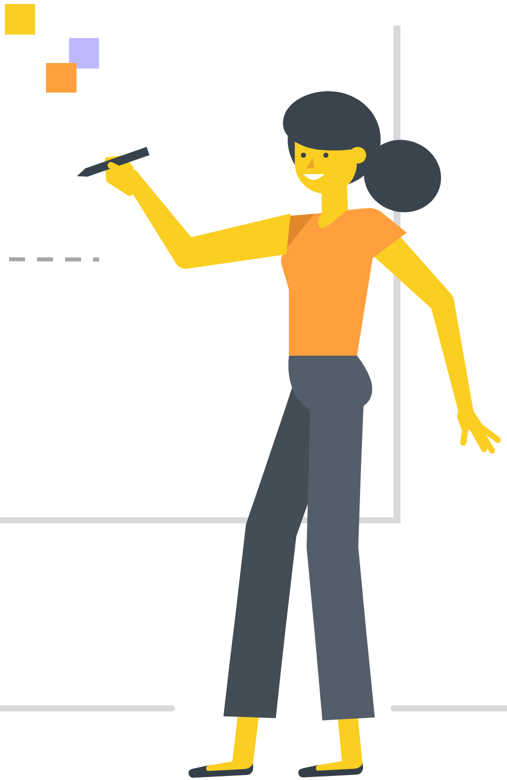
~~After~~

shift before

```
RegulatedRainbowAmplitudeIREG = Simplify[RegulatedAmplitude]
```

simplifica

$$\frac{1}{2}\, i\, e^4\, \bar{\gamma}\cdot\bar{p}\left(8\, b\,\text{Ilog}2\lambda 2 + 8\, b\,\text{Ilog}\lambda 2\,\log\left(-\frac{p^2}{\lambda^2}\right) - 23\, b\,\text{Ilog}\lambda 2 - 8\,\text{Ilog}\lambda 2^2\right)$$



# About the shift and Ward Identities (Dias)

$$\mathcal{M}(\mathrm{k}) = \epsilon_{\mu} M^{\mu}(\mathrm{k})$$

superfície. Outra identidade de Ward, que estabelece a relação entre funções de vértice e propagadores é estudada na QED espinorial a 2 loops. De posse do contratermo necessário para a renormalização do tensor de polarização do vácuo, calculamos a função $\beta$ a dois loops, comparando o valor encontrado com aquele obtido em outras referências.

A divisão do trabalho é a seguinte: no capítulo 2, apresentamos a verificação da prova diagramática das identidades de Ward, constatando que para sua validade, é necessário um procedimento de regularização que admita a possibilidade de fazermos shifts nos momentos de integração, e quando isso não é possível devido ao grau de divergência da amplitude, é necessária a introdução de termos de superfície para compensar eventuais shifts. No capí-

# About the shift and Ward Identities (Dias)

De uma forma geral, cancelamentos similares acontecem entre termos originados de outros pares de inserções adjacentes, por exemplo, nas posições $n$ e $n+1$. Quando somarmos sobre *todos* os n pontos de inserção, teremos como resultado

$$
\begin{aligned}
\Sigma = -e^{n+1} \int \frac{d^4p_1}{(2\pi)^4} tr[ & \left(\frac{\imath}{\not{p}_n - m}\right)\gamma^{\lambda_n}\left(\frac{\imath}{\not{p}_{n-1} - m}\right)\gamma^{\lambda_{n-1}} \dots \left(\frac{\imath}{\not{p}_1 - m}\right)\gamma^{\lambda_1} \\
+ \quad e^{n+1} \int \frac{d^4p_1}{(2\pi)^4} tr[ & \left(\frac{\imath}{\not{p}_n + \not{k} - m}\right)\gamma^{\lambda_n}\left(\frac{\imath}{\not{p}_{n-1} + \not{k} - m}\right)\gamma^{\lambda_{n-1}} \dots \left(\frac{\imath}{\not{p}_1 + \not{k} - m}\right)\gamma^{\lambda_1}]
\end{aligned}
\tag{2.18}
$$

Desta forma, o que podemos perceber é que um *shift* na variável de integração de $p_1$ para $p_1 + k$ no segundo termo faz com que a soma dos dois termos restantes da soma sobre todas as inserções se anule. Assim, a amplitude em que um fóton é inserido ao longo de um loop fechado é nula, quando somamos sobre todas as possíveis contribuições para tal processo (soma sobre todos os possíveis pontos de inserção).

# About the shift and Ward Identities (Dias)

De fato, no contexto da regularização implícita sabemos que a invariância por roteamento nas amplitudes de Feynman conduz a um conjunto de relações conhecidas como *condições de consistência*, que são relações necessárias para que uma amplitude de probabilidade tenha seu valor independente do rótulo adotado para os momentos nas linhas internas do diagrama. Tais relações sempre podem ser escritas em função de termos de superfície, como veremos. Qualquer seja o esquema de regularização adotado, podemos remover termos violadores de simetrias por meio de *contratermos restauradores de simetria*. Isso, na prática, é automaticamente implementado se ajustamos *todos* os termos de superfície para zero logo no início do cálculo perturbativo. É necessário ter cuidado em situações onde quebras de simetria quânticas ocorrem. Nesse caso, os termos de superfície devem ser considerados como parâmetros finitos arbitrários, que serão fixados com base em critérios físicos. Isso ocorre porque anomalias são normalmente relacionadas à dependência com a rotulação dos momentos em gráficos de Feynman (59).

A remoção dos termos de superfície pode ser feita através da introdução de *contratermos restauradores de simetria*, introduzidos na lagrangeana para restaurar alguma simetria quebrada pelo esquema de regularização empregado. Verificaremos, a partir de então, que a eliminação dos termos de

# Other ideas

| | | |
|---|---|---|
| | 01 | Or c.t are the same? |
| | 02 | Where I should insert the c.t? |

# Discussion

# Thank you for your attention and help