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# PREVIOUSLY ON...

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Regular Article - Theoretical Physics

## Two-loop renormalisation of gauge theories in $4D$ implicit regularisation and connections to dimensional methods

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**Abstract** We compute the two-loop $\beta$-function of scalar and spinorial quantum electrodynamics as well as pure Yang–Mills and quantum chromodynamics using the background field method in a fully quadridimensional setup using implicit regularization (IREG). Moreover, a thorough comparison with dimensional approaches such as conventional dimensional regularization (CDR) and dimensional reduction (DRED) is presented. Subtleties related to Lorentz algebra contractions/symmetric integrations inside divergent integrals as well as renormalisation schemes are carefully discussed within IREG where the renormalisation constants are fully defined as basic divergent integrals to arbitrary loop order. Moreover, we confirm the hypothesis that momentum routing invariance in the loops of Feynman diagrams implemented via setting well-defined surface terms to zero deliver non-abelian gauge invariant amplitudes within IREG just as it has been proven for abelian theories.

### 1 Motivations

Unravelling physics beyond the standard model (SM) has entreated theoretical predictions for particle physics precision observables beyond next-to-leading-order ($NLO$). Such predictions rely on involved Feynman diagram calculations to evaluate scattering amplitudes both in the SM and its extensions. Theoretical models beyond the SM (BSM) can be constructed, for instance, as an extension in the Higgs sector by either changing the number of scalar multiplets or considering the Higgs boson as a composite particle – the so called Composite Higgs Models [1,2]. Supersymmetric and dark matter extensions have also been considered in order to explain SM deviations from experimental results [3] in electroweak precision observables (EWPO) which are known with an accuracy at the per cent level or better [4–6]. On the other hand, precise measurements and calculations of known particles and interactions are just as important to validate, redress, or refute new models. Also, in order to evade from unphysical scale dependence at low order, higher order terms are needed to smooth out such dependence in the resulting, more accurate, predictions. For example, a full $N^3LO$ calculation for QCD corrections to gluon-fusion Higgs boson production was performed in [7] at center-of-mass energy 13 TeV. The considerably low residual theoretical uncertainty ($\approx$ 5–6%) and small sensitivity to scale variation ($\approx$ 2% ) superseded earlier results below $N^3LO$. Because experimental uncertainties are expected to drop below the accuracy of theoretical data, as expected from future experimental measurements at the future circular collider (FCC-$e^-e^+$) [8], $QCD$ theoretical uncertainties ought to be reduced at many

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## May the four be with you: Novel IR-subtraction methods to tackle NNLO calculations

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**Abstract**

NNLO

In this report, we present a discussion about different frameworks to perform precise higher-order computations for high-energy physics. These approaches implement novel strategies to deal with infrared and ultraviolet singularities in quantum field theories. A special emphasis is devoted to the local cancellation of these singularities, which can enhance the efficiency of computations and lead to discover novel mathematical properties in quantum field theories.

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arXiv:2012.02567v1

symmetry

MDPI

*Article*

## A Brief Review of Implicit Regularization and Its Connection with the BPHZ Theorem

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# Today:

**Title:**

Dimensional reduction applied to QCD at three loops

**Author:**

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JHEP 09 (2006) 053

July 2006

## Dimensional reduction applied to QCD at three loops

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ABSTRACT: Dimensional Reduction is applied to QCD in order to compute various renormalization constants in the $\overline{\mathrm{DR}}$ scheme at higher orders in perturbation theory. In particular, the $\beta$ function and the anomalous dimension of the quark masses are derived to three-loop order. Special emphasis is put on the proper treatment of the so-called $\varepsilon$-scalars and the additional couplings which have to be considered.



KEYWORDS: QCD, Supersymmetry Phenomenology.

# What they compute?

**1**

The β-function of QCD is derived within DRED to three-loop order.

**2**

The $\gamma_m$-function of QCD is derived within DRED to three-loop order.

**3**

Do a proper treatment of the ε-scalars.

# Table of contents

# 01

# Introduction

Which are the motivation of this work?

# Motivations

- Compare precision data with higher order calculations.
- Precision calculations at 3 or 4-loops.
- Such calculations currently all rely on Dimensional Regularization (DReg).

# Dimensional Regularization (DReg)

['t Hooft, Veltman 72]

Extremely successful in the Standard Model. In DReg, the number of space-time dimensions is altered:

D-dimensions

4-dimensions

$$\int_{-\infty}^{+\infty} \frac{d^4k_{[4]}}{(2\pi)^4} \to \underbrace{\mu^{4-d}}_{\text{scale}} \int_{-\infty}^{+\infty} \frac{d^dk_{[d]}}{(2\pi)^d}$$

DReg analytically continues the integral into D=4-2ε.

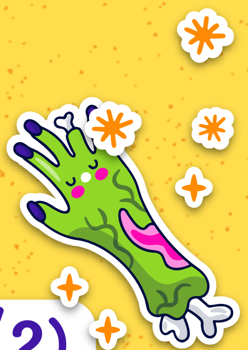
# Problem:

Dimensional regularization (DReg) breaks SUSY

# Reason

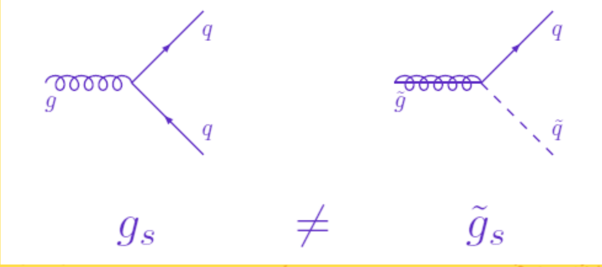
[S.P. Martin and M.T. Vaughn, Phys. Lett. B 318 331 (1993)]

$N_{spin1} = D$



$N_{spin1/2} = 2^{(D/2)}$

$$Z_g \neq \tilde{Z}_g$$



# Alternative: Dimensional Reduction (DRED)

[Siegel 79]

01

+

It keep vector fields 4-dimensional.

02

+

It compactify space-time to $D=4-2\varepsilon < 4$.

03

+

It seems consistent with SUSY so far.

# Alternative: Dimensional Reduction (DRED)

[Siegel 79]

**04**

**+**

The evaluation of the Lorentz algebra is significantly simpler.

**05**

**+**

It is a promising candidates to find links between Dreg and strictly four-dimensional approaches (IREG).

**06**

**-**

Restricted algebraic operations

02

# Framework

How exactly DRED Works?

# DRED

## Momentum-integration

D-dimensions

## Space-time

It's compactified to D=4-2ε dimensions.

## Vector-fields

They're kept 4-dimensional.

4-dimensions

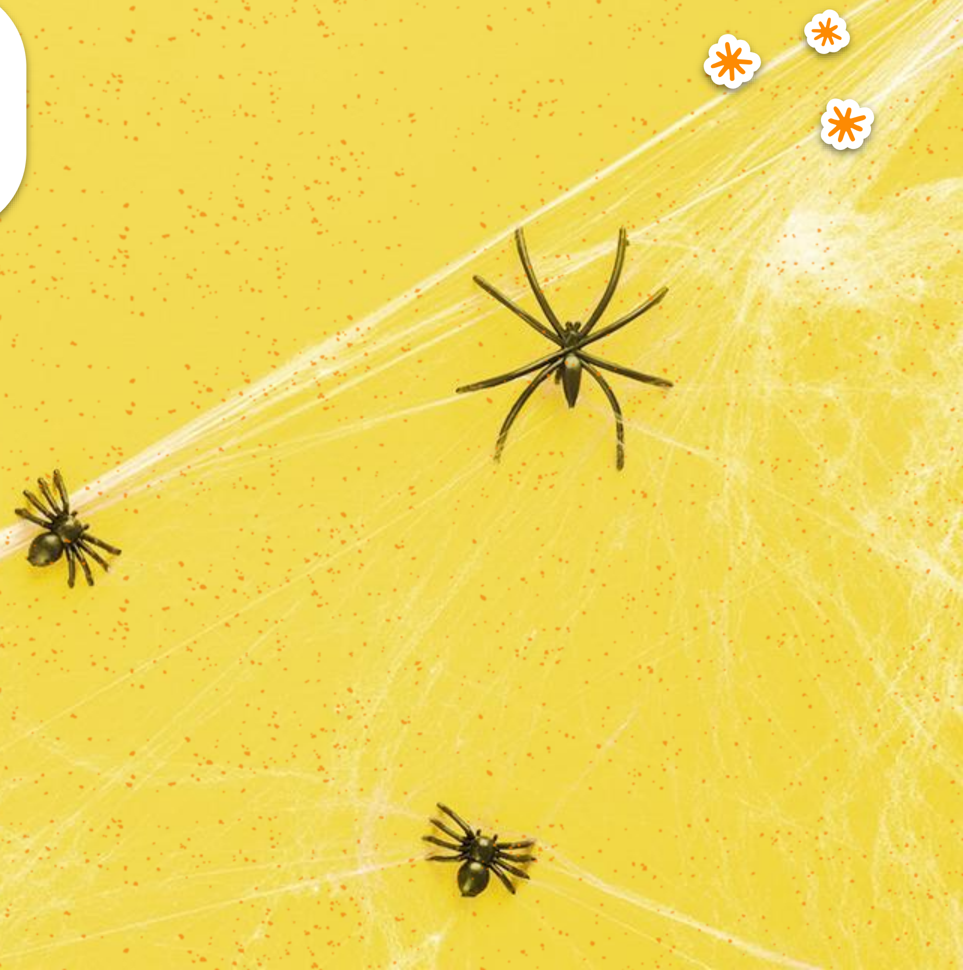
$$\frac{1}{\epsilon^n}$$

# DRED: example (electron-photon vertex)

$$\mathscr{A}_4 = \underbrace{\mathscr{A}_d}_{D\,=\,4-2\varepsilon} + \underbrace{\mathscr{A}_N}_{2\varepsilon} \Longrightarrow \mathcal{L}^{(4)} = \mathcal{L}^{(d)} + \underbrace{\mathcal{L}^{(2\epsilon)}}_{\text{New Feyman Rules}}$$

$$\bar\psi\gamma_\mu\psi A^\mu = \bar\psi\gamma_\mu\psi \underbrace{\hat A^\mu}_{D} + \bar\psi\gamma_\mu\psi \underbrace{\tilde A^\mu}_{2\varepsilon} = \bar\psi\hat\gamma_\mu\psi\hat A^\mu + \bar\psi\underbrace{\tilde\gamma_\mu}_{\text{Extra set of matrices}}\psi\tilde A^\mu$$

But they are working in QCD (not SUSY).



# Non-SUSY: the evanescent coupling

$$\boxed{A_\mu^{(4)}(x) = A_\mu^{(d)}(x) + A_\mu^{(\epsilon)}(x)}$$

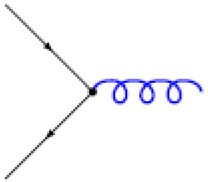
$$\Downarrow$$

$$\hat{Z} \neq \tilde{Z}$$

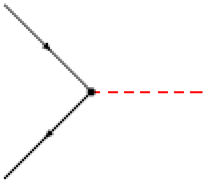
# Example: the quark-gluon vertex in DRED

$$A_\mu^{(4)}(x) = A_\mu^{(d)}(x) + A_\mu^{(\epsilon)}(x)$$

$$g_s A_\mu \bar{\psi} \gamma^\mu \psi \to \hat{g}_s \hat{A}_\mu \bar{\psi} \hat{\gamma}^\mu \psi + \tilde{g}_s \tilde{A}_\mu \bar{\psi} \tilde{\gamma}^\mu \psi$$



$\to \alpha_s$



$\to \alpha_e$

“evanescent coupling”

# Lagrange density

$$\mathcal{L}^{n} = -\tfrac{1}{4}(G_{ij}^{a})^{2} - \tfrac{1}{2}(\partial_i W_i^a)^2 + C^{*a}\partial_i D_i^{ab} C^b - \bar{\psi}_p \gamma_i D_i^{pq} \psi_q$$

$$\begin{aligned}\mathcal{L}^{E} = & -\tfrac{1}{2}(\partial_i W_\sigma^a)^2 - g f^{abc} W_i^b W_\sigma^c \partial_i W_\sigma^a \\ & -\tfrac{1}{2} g^2 f^{abc} f^{ade} W_i^b W_\sigma^c W_i^d W_\sigma^e \\ & -\tfrac{1}{4} g^2 f^{abc} f^{ade} W_\sigma^b W_{\sigma'}^c W_\sigma^d W_{\sigma'}^e + ig\bar{\psi}_p \gamma_\sigma R_{pq}^a \psi_q W_\sigma^a\end{aligned}$$

where

$$G_{\mu\nu}^a = \partial_\mu W_\nu^a - \partial_\nu W_\mu^a + g f^{abc} W_\mu^b W_\nu^c ,$$

$$D_i^{pq} = \delta^{pq}\partial_i - ig(R^a)^{pq} W_i^a$$

# Feynman rules for ε-scalars in QCD

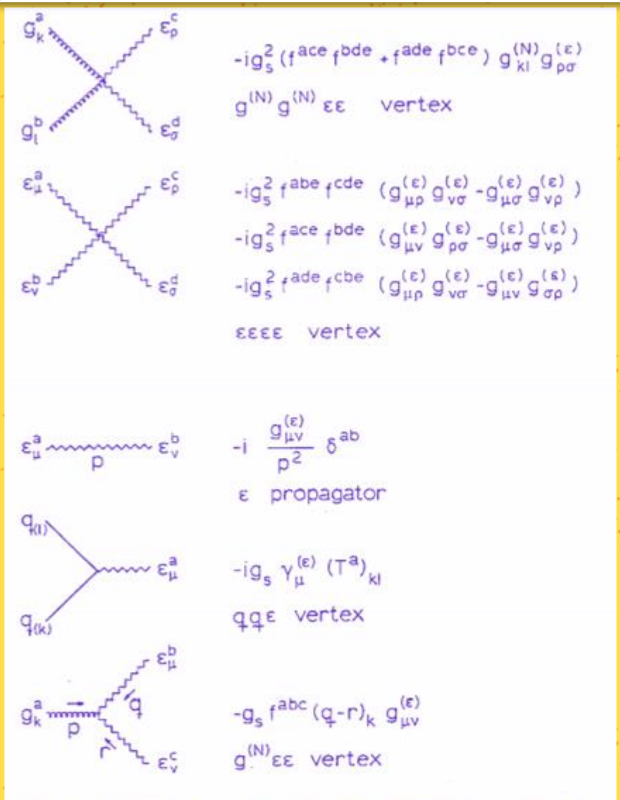
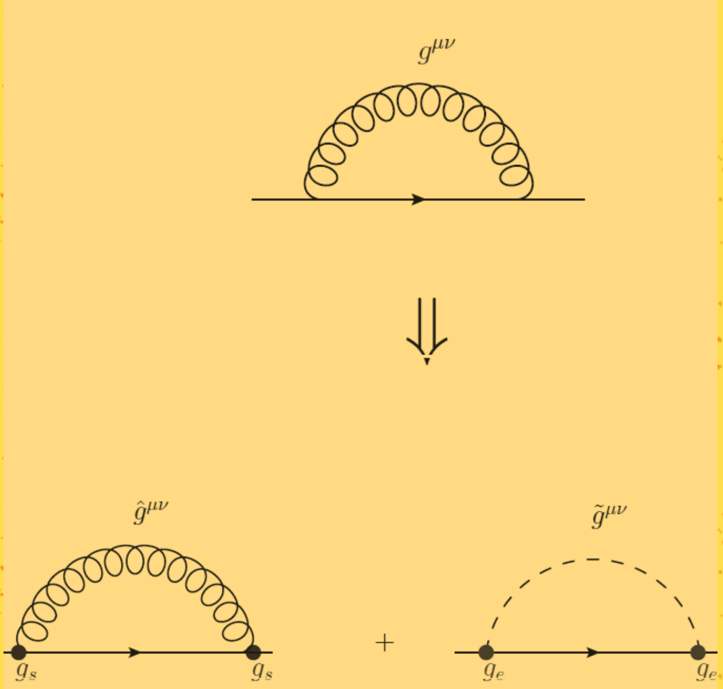


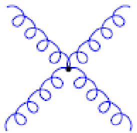

**Fig. 1.** Feynman rules for ε-scalars (all momenta flow into the vertices)

Körner, J.G., Tung, M.M. Dimensional reduction methods in QCD. *Z. Phys. C - Particles and Fields* 64, 255–265 (1994). https://doi.org/10.1007/BF01557396

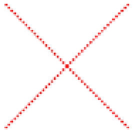
# More diagrams!



# More couplings!



$$\rightarrow \alpha_s \, f^{abe} f^{cde}$$



$$\rightarrow \lambda_1 \, f^{abe} f^{cde}, \quad \lambda_2 \, \delta^{ab} \delta^{cd}, \quad \lambda_3 \, d^{abe} d^{cde}$$

$$\eta_r = \frac{\lambda_r}{4\pi}$$

$$\alpha_s = \frac{\hat{g}_s^2}{4\pi}\,, \qquad \alpha_e = \frac{\tilde{g}_s^2}{4\pi}$$

# Renormalization constants

$$
\begin{aligned}
g_s^0 &= \mu^\epsilon Z_s g_s\,, & g_e^0 &= \mu^\epsilon Z_e g_e\,, & m^0 &= m Z_m\,,\\
1-\xi^0 &= (1-\xi)\,Z_3\,, & q^0 &= \sqrt{Z_2}\,q\,, & G_\mu^{0,a} &= \sqrt{Z_3}\,G_\mu^a\,,\\
\varepsilon_\sigma^{0,a} &= \sqrt{Z_3^\varepsilon}\,\varepsilon_\sigma^a\,, & c^{0,a} &= \sqrt{\tilde{Z}_3}\,c^a\,, & \bar{c}^{0,a} &= \sqrt{\tilde{Z}_3}\,\bar{c}^a\,,\\
\Gamma_{q\bar{q}G}^0 &= Z_1 \Gamma_{q\bar{q}G}\,, & \Gamma_{q\bar{q}\varepsilon}^0 &= Z_1^\varepsilon \Gamma_{q\bar{q}\varepsilon}\,, & \Gamma_{c\bar{c}G}^0 &= \tilde{Z}_1 \Gamma_{c\bar{c}G}\,,
\end{aligned}
$$

$$Z_s = \frac{\tilde{Z}_1}{\tilde{Z}_3\sqrt{Z_3}} = \frac{Z_1}{Z_2\sqrt{Z_3}}\,.$$

$$Z_e = \frac{Z_1^\varepsilon}{Z_2\sqrt{Z_3^\varepsilon}}$$

# 03

## β-function and the γm-function of QCD

The renormalization group functions for DRED.

# The β-function of QCD

In DReg:

$$\beta^{\overline{\mathrm{MS}}}(\alpha_s^{\overline{\mathrm{MS}}}) = \mu^2 \frac{\mathrm{d}}{\mathrm{d}\mu^2} \frac{\alpha_s^{\overline{\mathrm{MS}}}}{\pi}$$

In DRED: $\alpha_s = \frac{\hat{g}_s^2}{4\pi}\,, \qquad \alpha_e = \frac{\tilde{g}_s^2}{4\pi}$

$$\mu^2 \frac{\mathrm{d}}{\mathrm{d}\mu^2} \alpha_s = \beta_s^{\overline{\mathrm{DR}}}(\alpha_s, \alpha_e)\,, \qquad \mu^2 \frac{\mathrm{d}}{\mathrm{d}\mu^2} \alpha_e = \beta_e(\alpha_s, \alpha_e)$$

# Usual β—function relation in DReg

$$\beta^{\overline{\mathrm{MS}}}(\alpha_s^{\overline{\mathrm{MS}}}) = \mu^2 \frac{\mathrm{d}}{\mathrm{d}\mu^2} \frac{\alpha_s^{\overline{\mathrm{MS}}}}{\pi}$$

$$\frac{\alpha_{SO}^{\overline{MS}}}{\pi} = Z_s \mu^{2\varepsilon} \frac{\alpha_{SR}^{\overline{MS}}}{\pi}$$

$$\frac{1}{Z_s} \mu^2 \frac{d}{\partial \mu^2} (Z_s) = \mu^2 \frac{d}{\partial \mu^2} \ln Z_s$$

$$\mu^2 \frac{d}{\mathrm{d}\mu^2} = 2\mu^2 \cdot \frac{\mathrm{d}\alpha_{SR}^{\overline{MS}}}{\mathrm{d}\mu^2} \frac{d}{\mathrm{d}\alpha^{\overline{MS}}{}_{SR}} = 2\beta\left(\alpha_{SR}^{\overline{MS}}\right) \frac{d}{\mathrm{d}\alpha^{\overline{MS}}{}_{SR}}$$

$$\beta^{\overline{\mathrm{MS}}}(\alpha_s^{\overline{\mathrm{MS}}}) = -\epsilon \frac{\alpha_s^{\overline{\mathrm{MS}}}}{\pi} \left(1 + 2\alpha_s^{\overline{\mathrm{MS}}} \frac{\partial \ln Z_s^{\overline{\mathrm{MS}}}}{\partial \alpha_s^{\overline{\mathrm{MS}}}}\right)^{-1}$$

[T. van Ritbergen 1997 at 4-loop]

# β—function relation in DRED

$$\beta_s^{\overline{\mathrm{DR}}}(\alpha_s^{\overline{\mathrm{DR}}},\alpha_e,\{\eta_r\}) = \mu^2\frac{\mathrm{d}}{\mathrm{d}\mu^2}\frac{\alpha_s^{\overline{\mathrm{DR}}}}{\pi}$$

$$= -\left(\epsilon\frac{\alpha_s^{\overline{\mathrm{DR}}}}{\pi} + 2\frac{\alpha_s^{\overline{\mathrm{DR}}}}{Z_s^{\overline{\mathrm{DR}}}}\frac{\partial Z_s^{\overline{\mathrm{DR}}}}{\partial\alpha_e}\beta_e + \boxed{2\frac{\alpha_s^{\overline{\mathrm{DR}}}}{Z_s^{\overline{\mathrm{DR}}}}\sum_r\frac{\partial Z_s^{\overline{\mathrm{DR}}}}{\partial\eta_r}\beta_{\eta_r}}\right)\left(1+2\frac{\alpha_s^{\overline{\mathrm{DR}}}}{Z_s^{\overline{\mathrm{DR}}}}\frac{\partial Z_s^{\overline{\mathrm{DR}}}}{\partial\alpha_s^{\overline{\mathrm{DR}}}}\right)^{-1}$$

$$\beta_e(\alpha_s^{\overline{\mathrm{DR}}},\alpha_e,\{\eta_r\}) = \mu^2\frac{\mathrm{d}}{\mathrm{d}\mu^2}\frac{\alpha_e}{\pi}$$

$$= -\left(\epsilon\frac{\alpha_e}{\pi} + 2\frac{\alpha_e}{Z_e}\frac{\partial Z_e}{\partial\alpha_s^{\overline{\mathrm{DR}}}}\beta_s^{\overline{\mathrm{DR}}} + \boxed{2\frac{\alpha_e}{Z_e}\sum_r\frac{\partial Z_e}{\partial\eta_r}\beta_{\eta_r}}\right)\left(1+2\frac{\alpha_e}{Z_e}\frac{\partial Z_e}{\partial\alpha_e}\right)^{-1},$$

$$\beta_s^{\overline{\mathrm{DR}}} \neq \beta_e$$

(In QCD)

$$\boxed{\beta_s^{\mathrm{SYM}} = \beta_e^{\mathrm{SYM}}}$$

(In SUSY)

# Results: β–function with DRED at 3-loop

Don't forget:

$$Z_e = \frac{Z_1^{\varepsilon}}{Z_2\sqrt{Z_3^{\varepsilon}}}$$

# 11000 diagrams

Generated by QGRAF and process with a C++ program call EXP. For the integrals they used MINCER.

# Results: β—function with DRED at 3-loop

Re-writting:

$$\beta_s^{\overline{\mathrm{DR}}}(\alpha_s^{\overline{\mathrm{DR}}}, \alpha_e, \{\eta_r\}) = -\epsilon \frac{\alpha_s^{\overline{\mathrm{DR}}}}{\pi} - \sum_{i,j,k,l,m} \boxed{\beta_{ijklm}^{\overline{\mathrm{DR}}}} \left(\frac{\alpha_s^{\overline{\mathrm{DR}}}}{\pi}\right)^i \left(\frac{\alpha_e}{\pi}\right)^j \left(\frac{\eta_1}{\pi}\right)^k \left(\frac{\eta_2}{\pi}\right)^l \left(\frac{\eta_3}{\pi}\right)^m ,$$

$$\beta_e(\alpha_s^{\overline{\mathrm{DR}}}, \alpha_e, \{\eta_r\}) = -\epsilon \frac{\alpha_e}{\pi} - \sum_{i,j,k,l,m} \boxed{\beta_{ijklm}^{e}} \left(\frac{\alpha_s^{\overline{\mathrm{DR}}}}{\pi}\right)^i \left(\frac{\alpha_e}{\pi}\right)^j \left(\frac{\eta_1}{\pi}\right)^k \left(\frac{\eta_2}{\pi}\right)^l \left(\frac{\eta_3}{\pi}\right)^m ,$$

# Results: β—function with DRED at 3-loop

$$
\begin{aligned}
\beta_{20}^{\overline{\mathrm{DR}}} &= \frac{11}{12}C_A - \frac{1}{3}Tn_f\,,\\
\beta_{30}^{\overline{\mathrm{DR}}} &= \frac{17}{24}C_A^2 - \frac{5}{12}C_A T n_f - \frac{1}{4}C_F T n_f\,,\\
\beta_{40}^{\overline{\mathrm{DR}}} &= \frac{3115}{3456}C_A^3 - \frac{1439}{1728}C_A^2 T n_f - \frac{193}{576}C_A C_F T n_f\\
&\quad + \frac{1}{32}C_F^2 T n_f + \frac{79}{864}C_A T^2 n_f^2 + \frac{11}{144}C_F T^2 n_f^2\,,\\
\beta_{31}^{\overline{\mathrm{DR}}} &= -\frac{3}{16}C_F^2 T n_f\,,\\
\beta_{22}^{\overline{\mathrm{DR}}} &= -C_F T n_f\left(\frac{1}{16}C_A - \frac{1}{8}C_F - \frac{1}{16}T n_f\right)\,,\\
\beta_{02}^{e} &= -C_F - \frac{1}{2}T n_f + \frac{1}{2}C_A\,,\\
\beta_{11}^{e} &= \frac{3}{2}C_F\,,\\
\beta_{03}^{e} &= \frac{3}{8}C_A^2 - \frac{5}{4}C_A C_F + C_F^2 - \frac{3}{8}C_A T n_f + \frac{3}{4}C_F T n_f\,,\\
\beta_{21}^{e} &= -\frac{7}{64}C_A^2 + \frac{55}{48}C_A C_F + \frac{3}{16}C_F^2 + \frac{1}{8}C_A T n_f - \frac{5}{12}C_F T n_f\,,\\
\beta_{12}^{e} &= -\frac{3}{8}C_A^2 + \frac{5}{2}C_A C_F - \frac{11}{4}C_F^2 - \frac{5}{8}C_F T n_f\,,
\end{aligned}
$$

$$
\beta_{02100}^{e} = -\frac{9}{8}\,,\qquad \beta_{02010}^{e} = \frac{5}{4}\,,\qquad \beta_{02001}^{e} = \frac{3}{4}\,,\qquad \beta_{01200}^{e} = \frac{27}{64}\,,
$$

$$
\beta_{01101}^{e} = -\frac{9}{16}\,,\qquad \beta_{01020}^{e} = -\frac{15}{4}\,,\qquad \beta_{01002}^{e} = \frac{21}{32}\,,
$$

… how did they check these results?

# Results: β—function with DRED at 3-loop

$$\beta^{\overline{\mathrm{DR}}}_{20} = \frac{11}{12}C_A - \frac{1}{3}Tn_f\,,$$

$$\beta^{\overline{\mathrm{DR}}}_{30} = \frac{17}{24}C_A^2 - \frac{5}{12}C_A Tn_f - \frac{1}{4}C_F Tn_f\,,$$

$$\beta^{\overline{\mathrm{DR}}}_{40} = \frac{3115}{3456}C_A^3 - \frac{1439}{1728}C_A^2 Tn_f - \frac{193}{576}C_A C_F Tn_f + \frac{1}{32}C_F^2 Tn_f + \frac{79}{864}C_A T^2 n_f^2 + \frac{11}{144}C_F T^2 n_f^2\,,$$

$$\beta^{\overline{\mathrm{DR}}}_{31} = -\frac{3}{16}C_F^2 Tn_f\,,$$

$$\beta^{\overline{\mathrm{DR}}}_{22} = -C_F Tn_f\left(\frac{1}{16}C_A - \frac{1}{8}C_F - \frac{1}{16}Tn_f\right)\,,$$

$$\beta^{e}_{02} = -C_F - \frac{1}{2}Tn_f + \frac{1}{2}C_A\,,$$

$$\beta^{e}_{11} = \frac{3}{2}C_F\,,$$

$$\beta^{e}_{03} = \frac{3}{8}C_A^2 - \frac{5}{4}C_A C_F + C_F^2 - \frac{3}{8}C_A Tn_f + \frac{3}{4}C_F Tn_f\,,$$

$$\beta^{e}_{21} = -\frac{7}{64}C_A^2 + \frac{55}{48}C_A C_F + \frac{3}{16}C_F^2 + \frac{1}{8}C_A Tn_f - \frac{5}{12}C_F Tn_f\,,$$

$$\beta^{e}_{12} = -\frac{3}{8}C_A^2 + \frac{5}{2}C_A C_F - \frac{11}{4}C_F^2 - \frac{5}{8}C_F Tn_f\,,$$

$$\beta^{e}_{02100} = -\frac{9}{8}\,, \qquad \beta^{e}_{02010} = \frac{5}{4}\,, \qquad \beta^{e}_{02001} = \frac{3}{4}\,, \qquad \beta^{e}_{01200} = \frac{27}{64}\,,$$

$$\beta^{e}_{01101} = -\frac{9}{16}\,, \qquad \beta^{e}_{01020} = -\frac{15}{4}\,, \qquad \beta^{e}_{01002} = \frac{21}{32}\,,$$

Let's compare with SUSY Yang Mills!
For that, they set:

$$C_F = C_A = T\,, \qquad n_f = 1/2$$

So, for SUSY Yang Mills at 3-loop:

$$\beta_s^{\mathrm{SYM}} = \beta_e^{\mathrm{SYM}}$$

# Is $\overline{DRED}$ a viable renormalization scheme?

$\overline{MS}$

$\overline{DRED}$

$\alpha_s \neq \alpha_e$

# Is $\overline{DRED}$ a viable renormalization scheme?

The value of $\alpha_s$ in a physical scheme is independent of regularization:

$$\alpha_s^{\mathrm{ph}} = \left(z_s^{\mathrm{ph,X}}\right)^2 \alpha_s^{\mathrm{X}}\,, \qquad z_s^{\mathrm{ph,X}} = Z_s^{\mathrm{X}}/Z_s^{\mathrm{ph,X}}\,, \qquad \mathrm{X} \in \{\overline{\mathrm{MS}}, \overline{\mathrm{DR}}\}$$

$$\Rightarrow \alpha_s^{\overline{\mathrm{DR}}} = \left(\frac{Z_s^{\mathrm{ph},\overline{\mathrm{DR}}}\, Z_s^{\overline{\mathrm{MS}}}}{Z_s^{\mathrm{ph},\overline{\mathrm{MS}}}\, Z_s^{\overline{\mathrm{DR}}}}\right)^2 \alpha_s^{\overline{\mathrm{MS}}}\,,$$

$$\alpha_s^{\overline{\mathrm{DR}}} = \alpha_s^{\overline{\mathrm{MS}}}\left[1 + \frac{\alpha_s^{\overline{\mathrm{MS}}}}{\pi}\frac{C_A}{12} + \left(\frac{\alpha_s^{\overline{\mathrm{MS}}}}{\pi}\right)^2 \frac{11}{72}C_A^2 - \frac{\alpha_s^{\overline{\mathrm{MS}}}}{\pi}\frac{\alpha_e}{\pi}\frac{1}{8}C_F T n_f + \cdots\right]$$

# Another way to check previous results:

$$\alpha_s^{\overline{\mathrm{DR}}} = \alpha_s^{\overline{\mathrm{MS}}}\left[1 + \frac{\alpha_s^{\overline{\mathrm{MS}}}}{\pi}\frac{C_A}{12} + \left(\frac{\alpha_s^{\overline{\mathrm{MS}}}}{\pi}\right)^2 \frac{11}{72}C_A^2 - \frac{\alpha_s^{\overline{\mathrm{MS}}}}{\pi}\frac{\alpha_e}{\pi}\frac{1}{8}C_F T n_f + \cdots\right]$$

$$\begin{aligned}\beta_s^{\overline{\mathrm{DR}}}(\alpha_s^{\overline{\mathrm{DR}}}, \alpha_e, \{\eta_r\}) &= \mu^2 \frac{\mathrm{d}}{\mathrm{d}\mu^2}\frac{\alpha_s^{\overline{\mathrm{DR}}}}{\pi} \\ &= \beta_s^{\overline{\mathrm{MS}}}(\alpha_s^{\overline{\mathrm{MS}}})\frac{\partial \alpha_s^{\overline{\mathrm{DR}}}}{\partial \alpha_s^{\overline{\mathrm{MS}}}} + \beta_e(\alpha_s^{\overline{\mathrm{DR}}}, \alpha_e, \{\eta_r\})\frac{\partial \alpha_s^{\overline{\mathrm{DR}}}}{\partial \alpha_e} + \cdots\end{aligned}$$

Same coefficients as before!

# The γm-function of QCD

In DReg:

$$\gamma_m^{\overline{\mathrm{MS}}}(\alpha_s^{\overline{\mathrm{MS}}}) = \frac{\mu^2}{m^{\overline{\mathrm{MS}}}}\frac{\mathrm{d}}{\mathrm{d}\mu^2}m^{\overline{\mathrm{MS}}} = -\pi\beta^{\overline{\mathrm{MS}}}\frac{\partial \ln Z_m^{\overline{\mathrm{MS}}}}{\partial \alpha_s^{\overline{\mathrm{MS}}}}$$

[K.G. Chetyrkin 1997 at 4-loop]

In DRED: $\alpha_s = \frac{\hat{g}_s^2}{4\pi}\,, \qquad \alpha_e = \frac{\tilde{g}_s^2}{4\pi}$

$$\begin{aligned}\gamma_m^{\overline{\mathrm{DR}}}(\alpha_s^{\overline{\mathrm{DR}}}, \alpha_e, \{\eta_r\}) &= \frac{\mu^2}{m^{\overline{\mathrm{DR}}}}\frac{\mathrm{d}}{\mathrm{d}\mu^2}m^{\overline{\mathrm{DR}}} \\ &= -\pi\beta_s^{\overline{\mathrm{DR}}}\frac{\partial \ln Z_m^{\overline{\mathrm{DR}}}}{\partial \alpha_s^{\overline{\mathrm{DR}}}} - \pi\beta_e\frac{\partial \ln Z_m^{\overline{\mathrm{DR}}}}{\partial \alpha_e} - \pi\sum_r \beta_{\eta_r}\frac{\partial \ln Z_m^{\overline{\mathrm{DR}}}}{\partial \eta_r}\,.\end{aligned}$$

# Results: γm-function with DRED at 3-loop

Re-writting:

$$\gamma_m^{\overline{\mathrm{DR}}}(\alpha_s^{\overline{\mathrm{DR}}}, \alpha_e, \{\eta_r\}) = -\sum_{i,j,k,l,m} \gamma_{ijklm}^{\overline{\mathrm{DR}}} \left(\frac{\alpha_s^{\overline{\mathrm{DR}}}}{\pi}\right)^i \left(\frac{\alpha_e}{\pi}\right)^j \left(\frac{\eta_1}{\pi}\right)^k \left(\frac{\eta_2}{\pi}\right)^l \left(\frac{\eta_3}{\pi}\right)^m$$

# Results: γm-function with DRED at 3-loop

$$\gamma_{10}^{\overline{\mathrm{DR}}} = \frac{3}{4}C_F\,,$$

$$\gamma_{20}^{\overline{\mathrm{DR}}} = \frac{3}{32}C_F^2 + \frac{91}{96}C_AC_F - \frac{5}{24}C_FTn_f\,,$$

$$\gamma_{11}^{\overline{\mathrm{DR}}} = -\frac{3}{8}C_F^2\,,$$

$$\gamma_{02}^{\overline{\mathrm{DR}}} = \frac{1}{4}C_F^2 - \frac{1}{8}C_AC_F + \frac{1}{8}C_FTn_f\,,$$

$$\gamma_{30}^{\overline{\mathrm{DR}}} = \frac{129}{128}C_F^3 - \frac{133}{256}C_F^2C_A + \frac{10255}{6912}C_FC_A^2 + \frac{-23+24\zeta_3}{32}C_F^2Tn_f - \left(\frac{281}{864} + \frac{3}{4}\zeta_3\right)C_AC_FTn_f - \frac{35}{432}C_FT^2n_f^2\,,$$

$$\gamma_{21}^{\overline{\mathrm{DR}}} = -\frac{27}{64}C_F^3 - \frac{21}{32}C_F^2C_A - \frac{15}{256}C_FC_A^2 + \frac{9}{32}C_F^2Tn_f\,,$$

$$\gamma_{12}^{\overline{\mathrm{DR}}} = \frac{9}{8}C_F^3 - \frac{21}{32}C_F^2C_A + \frac{3}{64}C_FC_A^2 + \frac{3}{64}C_AC_FTn_f + \frac{3}{8}C_F^2Tn_f\,,$$

$$\gamma_{03}^{\overline{\mathrm{DR}}} = -\frac{3}{8}C_F^3 + \frac{3}{8}C_F^2C_A - \frac{3}{32}C_FC_A^2 + \frac{1}{8}C_AC_FTn_f - \frac{5}{16}C_F^2Tn_f - \frac{1}{32}C_FT^2n_f^2$$

$$\gamma_{02100}^{\overline{\mathrm{DR}}} = \frac{3}{8}\,,\qquad \gamma_{02010}^{\overline{\mathrm{DR}}} = -\frac{5}{12}\,,\qquad \gamma_{02001}^{\overline{\mathrm{DR}}} = -\frac{1}{4}\,,\qquad \gamma_{01200}^{\overline{\mathrm{DR}}} = -\frac{9}{64}\,,$$

$$\gamma_{01020}^{\overline{\mathrm{DR}}} = \frac{5}{4}\,,\qquad \gamma_{01101}^{\overline{\mathrm{DR}}} = \frac{3}{16}\,,\qquad \gamma_{01002}^{\overline{\mathrm{DR}}} = -\frac{7}{32}\,.$$

# How to check previous results:

$$m^{\overline{\mathrm{DR}}} = m^{\overline{\mathrm{MS}}}\left[1 - \frac{\alpha_e}{\pi}\frac{1}{4}C_F + \left(\frac{\alpha_s^{\overline{\mathrm{MS}}}}{\pi}\right)^2 \frac{11}{192}C_A C_F - \frac{\alpha_s^{\overline{\mathrm{MS}}}}{\pi}\frac{\alpha_e}{\pi}\left(\frac{1}{4}C_F^2 + \frac{3}{32}C_A C_F\right) + \left(\frac{\alpha_e}{\pi}\right)^2\left(\frac{3}{32}C_F^2 + \frac{1}{32}C_F T n_f\right) + \cdots\right],$$

$$\gamma_m^{\overline{\mathrm{DR}}}(\alpha_s^{\overline{\mathrm{DR}}}, \alpha_e, \{\eta_r\}) = \gamma_m^{\overline{\mathrm{MS}}}\frac{\partial \ln m^{\overline{\mathrm{DR}}}}{\partial \ln m^{\overline{\mathrm{MS}}}} + \frac{\pi \beta_s^{\overline{\mathrm{MS}}}}{m^{\overline{\mathrm{DR}}}}\frac{\partial m^{\overline{\mathrm{DR}}}}{\partial \alpha_s^{\overline{\mathrm{MS}}}} + \frac{\pi \beta_e}{m^{\overline{\mathrm{DR}}}}\frac{\partial m^{\overline{\mathrm{DR}}}}{\partial \alpha_e} + \cdots$$

Same coefficients as before!

04

# Conclusions

Final Outlook of this work

# Conclusions

- DRED poses an attractive alternative to DReg.
- Nevertheless, DRED in non-SUSY theory becomes messy.
- $\overline{\text{DRED}}$ and the $\overline{\text{MS}}$ renormalization scheme are related by an analytic redefinition of the couplings and masses.
- **Side result:** consistency check of DRED and SUSY.

# Thanks!

Do you have any questions?

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