

V. Statistical analysis in R (solution)

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14 March, 2022

Data

```
### Open help page for birthwt
?birthwt
```

```
### Make a tibble with the data
birthData <- as_tibble(birthwt)
birthData
```

```
## # A tibble: 189 x 10
##       low  age  lwt  race smoke  ptl   ht   ui  ftv  bwt
##   <int> <int> <int> <int> <int> <int> <int> <int> <int> <int>
## 1     0   19  182     2     0     0     0     1     0  2523
## 2     0   33  155     3     0     0     0     0     3  2551
## 3     0   20  105     1     1     0     0     0     1  2557
## 4     0   21  108     1     1     0     0     1     2  2594
## 5     0   18  107     1     1     0     0     1     0  2600
## 6     0   21  124     3     0     0     0     0     0  2622
## 7     0   22  118     1     0     0     0     0     1  2637
## 8     0   17  103     3     0     0     0     0     1  2637
## 9     0   29  123     1     1     0     0     0     1  2663
## 10    0   26  113     1     1     0     0     0     0  2665
## # ... with 179 more rows
```

```
### Make smoke into a factor
# Check if `smoke` is numerical
is.numeric(birthData$smoke)
```

```
## [1] TRUE
```

```
# Alternatively, review the summary of the whole dataset
summary(birthData)
```

```
##       low          age          lwt          race
##  Min.   :0.0000   Min.   :14.00   Min.    : 80.0   Min.    :1.000
## 1st Qu.:0.0000   1st Qu.:19.00   1st Qu.:110.0   1st Qu.:1.000
## Median :0.0000   Median :23.00   Median :121.0   Median :1.000
## Mean   :0.3122   Mean   :23.24   Mean   :129.8   Mean   :1.847
## 3rd Qu.:1.0000   3rd Qu.:26.00   3rd Qu.:140.0   3rd Qu.:3.000
## Max.   :1.0000   Max.   :45.00   Max.   :250.0   Max.   :3.000
##       smoke      ptl          ht          ui
##  Min.   :0.0000   Min.   :0.0000   Min.    :0.00000   Min.    :0.0000
## 1st Qu.:0.0000   1st Qu.:0.0000   1st Qu.:0.00000   1st Qu.:0.0000
## Median :0.0000   Median :0.0000   Median :0.00000   Median :0.0000
```

```
## Mean :0.3915 Mean :0.1958 Mean :0.06349 Mean :0.1481
## 3rd Qu.:1.0000 3rd Qu.:0.0000 3rd Qu.:0.00000 3rd Qu.:0.0000
## Max. :1.0000 Max. :3.0000 Max. :1.00000 Max. :1.0000
## ftv bwt
## Min. :0.0000 Min. : 709
## 1st Qu.:0.0000 1st Qu.:2414
## Median :0.0000 Median :2977
## Mean :0.7937 Mean :2945
## 3rd Qu.:1.0000 3rd Qu.:3487
## Max. :6.0000 Max. :4990
```

```
# Finally, make smoke into a factor
```

```
birthData <- mutate(birthData, smoke = factor(smoke))
```

```
# Check the ftv variable, make it a factor and collapse some of the levels
table(birthData$ftv)
```

```
##
## 0 1 2 3 4 6
## 100 47 30 7 4 1
```

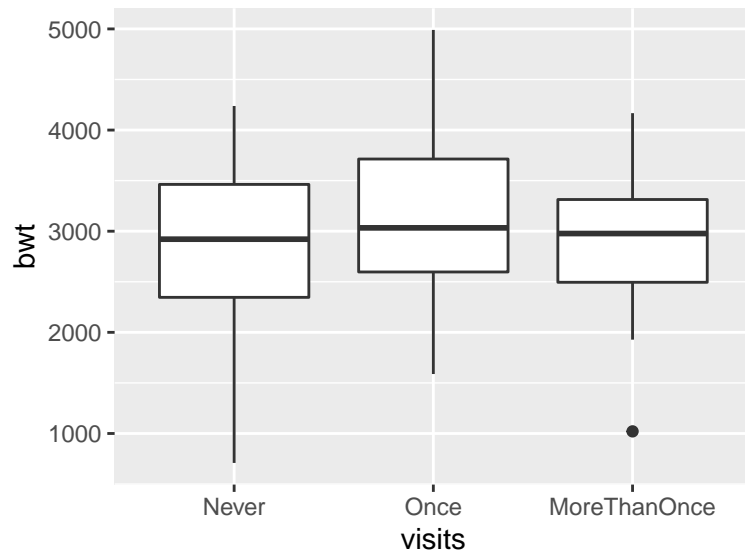
```
birthData <- mutate(birthData, ftvFac = factor(ftv))
```

```
birthData <- mutate(birthData, visits = fct_collapse(ftvFac, Never="0", Once="1", other_level="MoreThanOnce"))
summary(birthData)
```

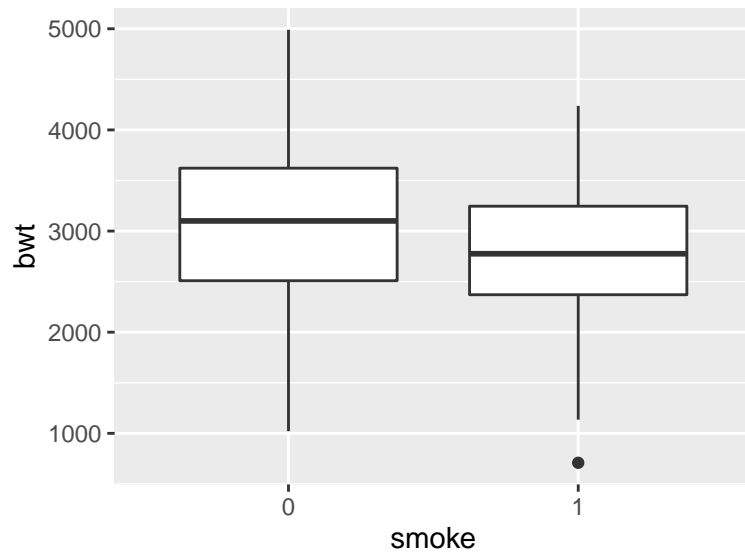
```
## low age lwt race smoke
## Min. :0.0000 Min. :14.00 Min. : 80.0 Min. :1.000 0:115
## 1st Qu.:0.0000 1st Qu.:19.00 1st Qu.:110.0 1st Qu.:1.000 1: 74
## Median :0.0000 Median :23.00 Median :121.0 Median :1.000
## Mean :0.3122 Mean :23.24 Mean :129.8 Mean :1.847
## 3rd Qu.:1.0000 3rd Qu.:26.00 3rd Qu.:140.0 3rd Qu.:3.000
## Max. :1.0000 Max. :45.00 Max. :250.0 Max. :3.000
## ptl ht ui ftv
## Min. :0.0000 Min. :0.00000 Min. :0.00000 Min. :0.0000
## 1st Qu.:0.0000 1st Qu.:0.00000 1st Qu.:0.00000 1st Qu.:0.0000
## Median :0.0000 Median :0.00000 Median :0.00000 Median :0.0000
## Mean :0.1958 Mean :0.06349 Mean :0.1481 Mean :0.7937
## 3rd Qu.:0.0000 3rd Qu.:0.00000 3rd Qu.:0.00000 3rd Qu.:1.0000
## Max. :3.0000 Max. :1.00000 Max. :1.00000 Max. :6.0000
## bwt ftvFac visits
## Min. : 709 0:100 Never :100
## 1st Qu.:2414 1: 47 Once : 47
## Median :2977 2: 30 MoreThanOnce: 42
## Mean :2945 3: 7
## 3rd Qu.:3487 4: 4
## Max. :4990 6: 1
```

```
### Parallell boxplots
```

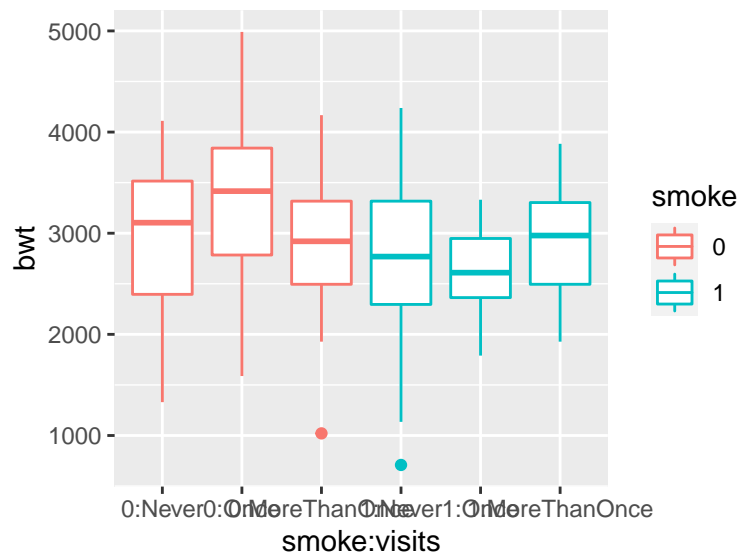
```
ggplot(birthData, aes(x=visits, y=bwt)) + geom_boxplot()
```



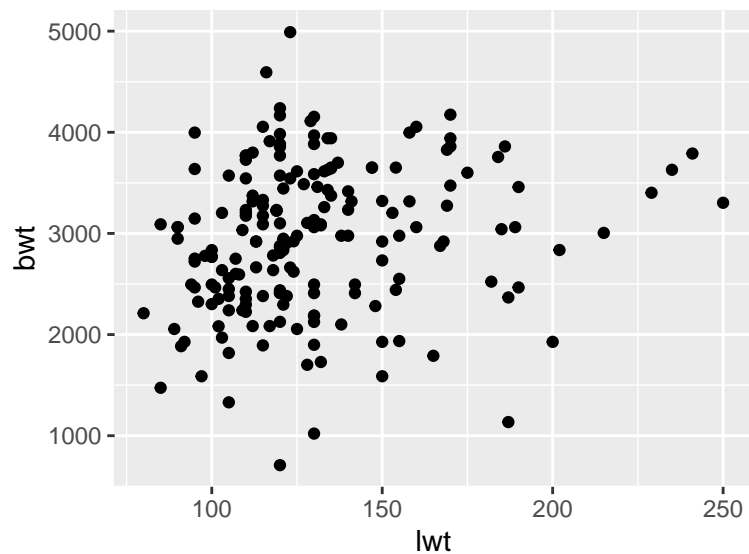
```
ggplot(birthData, aes(x=smoke, y=bwt)) + geom_boxplot()
```



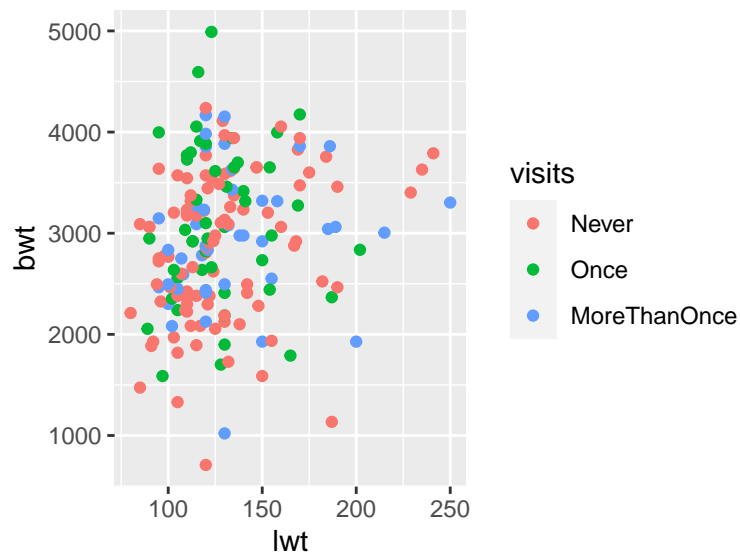
```
# Groupwise boxplots
ggplot(birthData, aes(x=smoke:visits, y=bwt, col=smoke)) + geom_boxplot()
```



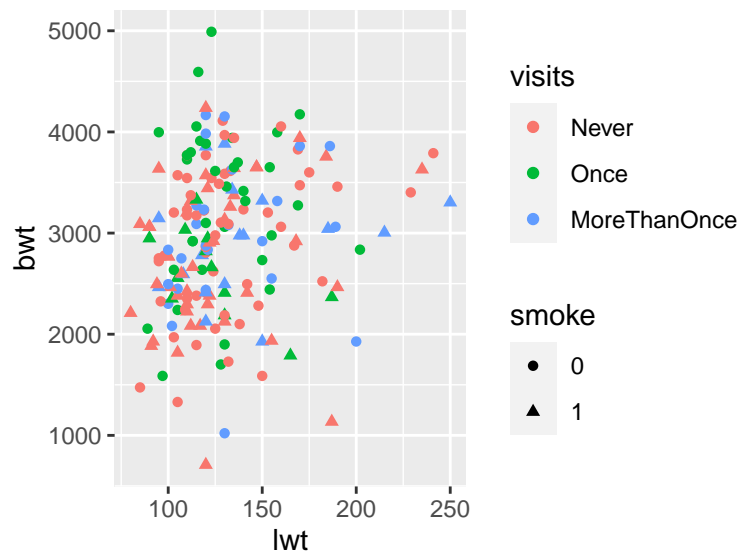
```
### Scatterplot
ggplot(birthData, aes(x=lwt, y=bwt)) + geom_point()
```



```
### Scatter plot with points coloured after visits, and point types after smoke status
ggplot(birthData, aes(x=lwt, y=bwt, col=visits)) + geom_point()
```



```
ggplot(birthData, aes(x=lwt, y=bwt, col=visits, pch=smoke)) + geom_point()
```



Regression

```
### Simple linear regression
reg1 <- lm(bwt ~ lwt, data=birthData)
summary(reg1)

##
## Call:
## lm(formula = bwt ~ lwt, data = birthData)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2192.12  -497.97   -3.84    508.32   2075.60
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
##
```

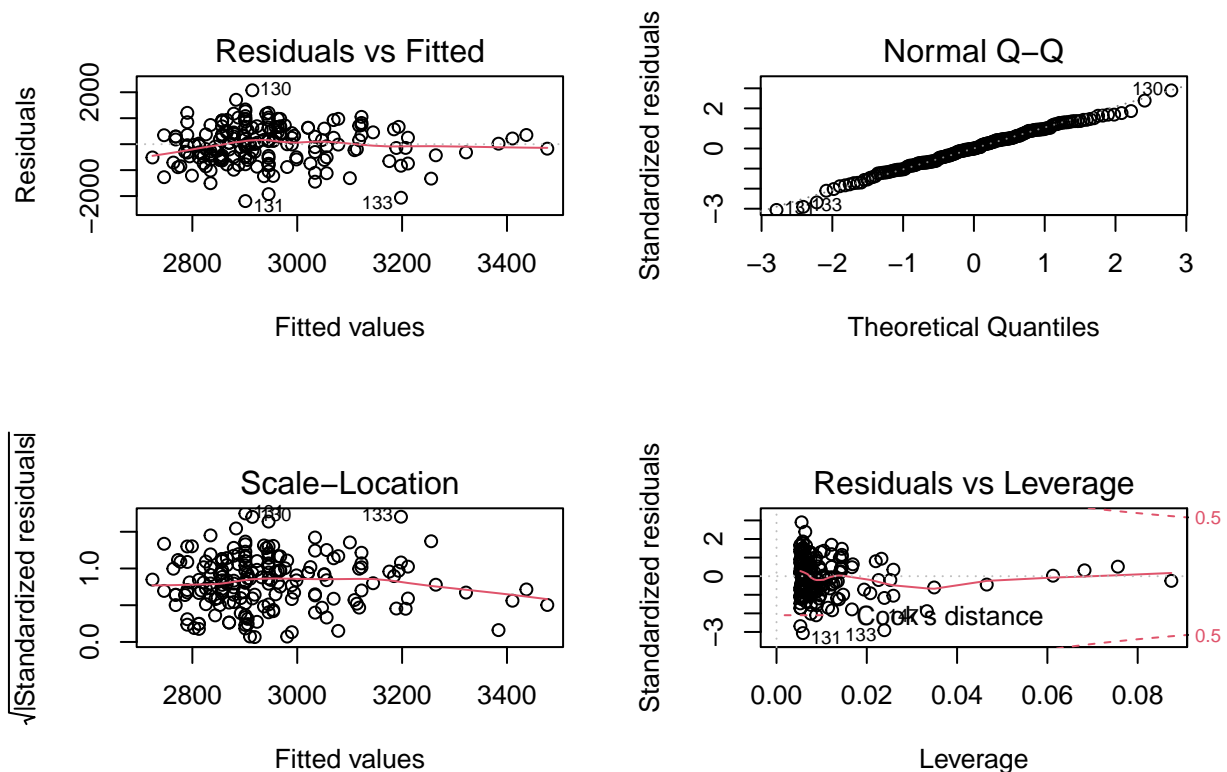
```
## (Intercept) 2369.624    228.493    10.371    <2e-16 ***
## lwt          4.429      1.713     2.585     0.0105 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 718.4 on 187 degrees of freedom
## Multiple R-squared:  0.0345, Adjusted R-squared:  0.02933
## F-statistic: 6.681 on 1 and 187 DF,  p-value: 0.0105
```

```
confint(reg1)
```

```
##              2.5 %      97.5 %
## (Intercept) 1918.867879 2820.37916
## lwt          1.048845   7.80937
```

```
### Model validation
```

```
par(mfrow=c(2,2))
plot(reg1)
```



```
### Include age and ftv as covariate
reg2 <- lm(bwt ~ lwt + age + ftv, data=birthData)
summary(reg2)
```

```
##
## Call:
## lm(formula = bwt ~ lwt + age + ftv, data = birthData)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2218.60  -488.73   11.56   516.63  1907.42
##
```

```
## Coefficients:
##           Estimate Std. Error t value Pr(>|t|)
## (Intercept) 2223.506    301.567   7.373 5.4e-12 ***
## lwt         4.121      1.758   2.344 0.0201 *
## age         7.486     10.285   0.728 0.4676
## ftv        15.364     51.114   0.301 0.7641
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 720.9 on 185 degrees of freedom
## Multiple R-squared:  0.03831,    Adjusted R-squared:  0.02271
## F-statistic: 2.457 on 3 and 185 DF,  p-value: 0.06446
```

```
# Just to view the coefficients
```

```
summary(reg2)$coefficients
```

```
##           Estimate Std. Error  t value    Pr(>|t|)
## (Intercept) 2223.505618 301.567140 7.3731694 5.402210e-12
## lwt         4.120744    1.757787 2.3442798 2.012498e-02
## age         7.485748   10.285178 0.7278189 4.676446e-01
## ftv        15.363944   51.113921 0.3005824 7.640705e-01
```

```
### Prediction
```

```
newData <- data.frame(lwt=100, age=25, ftv=0)
newData
```

```
##   lwt age ftv
## 1 100 25  0
```

```
predict(reg2, newData)
```

```
##           1
## 2822.724
```

```
predict(reg2, newData, interval="prediction")
```

```
##           fit          lwr          upr
## 1 2822.724 1390.148 4255.299
```

```
### Use only data from mothers with weight below 160
```

```
reg3 <- lm(bwt ~ lwt + age + ftv, data=filter(birthData, lwt<160))
summary(reg3)
```

```
##
## Call:
## lm(formula = bwt ~ lwt + age + ftv, data = filter(birthData,
##           lwt < 160))
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2233.9  -487.0    25.7   476.2  1852.5
##
## Coefficients:
##           Estimate Std. Error t value Pr(>|t|)
## (Intercept) 1824.063    444.508   4.104 6.52e-05 ***
## lwt         7.182      3.342   2.149 0.0332 *
## age         9.179     11.624   0.790 0.4310
## ftv        16.986     60.012   0.283 0.7775
```

```
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 716.3 on 157 degrees of freedom
## Multiple R-squared:  0.03956,    Adjusted R-squared:  0.0212
## F-statistic: 2.155 on 3 and 157 DF,  p-value: 0.09549

# Just to view the coefficients
summary(reg3)$coefficients

##              Estimate Std. Error  t value    Pr(>|t|)
## (Intercept) 1824.063262 444.508301  4.1035528 0.0000651941
## lwt          7.182184   3.341787  2.1492047 0.0331503757
## age          9.178577  11.624301  0.7896024 0.4309508695
## ftv         16.986307   60.011783  0.2830495 0.7775117320
```

ANOVA

```
### Oneway ANOVA against smoke
oneway1 <- lm(bwt ~ smoke, data=birthData)
summary(oneway1)

##
## Call:
## lm(formula = bwt ~ smoke, data = birthData)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2062.9  -475.9    34.3   545.1  1934.3
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  3055.70      66.93  45.653 < 2e-16 ***
## smoke1       -283.78     106.97  -2.653  0.00867 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 717.8 on 187 degrees of freedom
## Multiple R-squared:  0.03627,    Adjusted R-squared:  0.03112
## F-statistic: 7.038 on 1 and 187 DF,  p-value: 0.008667

summary(oneway1)$coefficients

##              Estimate Std. Error  t value    Pr(>|t|)
## (Intercept) 3055.6957    66.93326 45.652875 2.463035e-103
## smoke1      -283.7767   106.96877 -2.652893  8.666726e-03

emmeans(oneway1,~smoke)

##  smoke emmean   SE  df lower.CL upper.CL
##    0     3056 66.9 187    2924     3188
##    1     2772 83.4 187    2607     2937
##
## Confidence level used: 0.95

pairs(emmeans(oneway1,~smoke))
```



```
## contrast estimate SE df t.ratio p.value
## 0 - 1          284 107 187    2.653  0.0087

### Oneway ANOVA against visits
oneway2 <- lm(bwt ~ visits, data=birthData)
summary(oneway2)

##
## Call:
## lm(formula = bwt ~ visits, data = birthData)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2156.14  -484.88    26.12   578.86  1882.00
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)    2865.14      72.62  39.456 <2e-16 ***
## visitsOnce      242.86     128.42   1.891  0.0602 .
## visitsMoreThanOnce  85.74     133.52   0.642  0.5216
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 726.2 on 186 degrees of freedom
## Multiple R-squared:  0.01888,    Adjusted R-squared:  0.008335
## F-statistic:  1.79 on 2 and 186 DF,  p-value: 0.1698

emmeans(oneway2,~visits)

## visits      emmean    SE df lower.CL upper.CL
## Never        2865    72.6 186    2722    3008
## Once         3108   105.9 186    2899    3317
## MoreThanOnce  2951   112.1 186    2730    3172
##
## Confidence level used: 0.95

pairs(emmeans(oneway2,~visits))

## contrast      estimate SE df t.ratio p.value
## Never - Once    -242.9 128 186  -1.891  0.1441
## Never - MoreThanOnce -85.7 134 186  -0.642  0.7970
## Once - MoreThanOnce  157.1 154 186   1.019  0.5658
##
## P value adjustment: tukey method for comparing a family of 3 estimates

drop1(oneway2,test="F")

## Single term deletions
##
## Model:
## bwt ~ visits
##      Df Sum of Sq    RSS    AIC F value Pr(>F)
## <none>          98081730 2493.2
## visits  2    1887925 99969656 2492.8  1.7901 0.1698

### Twoway ANOVA without interaction
twoway1 <- lm(bwt ~ visits + smoke, data=birthData)
```

```
summary(twoway1)
```

```
##
## Call:
## lm(formula = bwt ~ visits + smoke, data = birthData)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2034.02  -504.02   34.98   536.36  1816.31
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)    2980.92     86.74  34.367  <2e-16 ***
## visitsOnce      192.77    128.60   1.499   0.1356
## visitsMoreThanOnce  74.10    131.98   0.561   0.5752
## smoke1        -257.28    108.37  -2.374   0.0186 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 717.3 on 185 degrees of freedom
## Multiple R-squared:  0.04789,    Adjusted R-squared:  0.03245
## F-statistic: 3.102 on 3 and 185 DF,  p-value: 0.02795
```

```
summary(twoway1)$coefficients
```

```
##              Estimate Std. Error    t value    Pr(>|t|)
## (Intercept)    2980.91672    86.73734  34.3671676 3.021284e-82
## visitsOnce      192.77220   128.59583   1.4990548 1.355639e-01
## visitsMoreThanOnce  74.10202   131.98092   0.5614601 5.751634e-01
## smoke1        -257.28159   108.37493  -2.3739954 1.862096e-02
```

```
### Tway ANOVA with interaction, test for interaction in two ways
```

```
twoway2 <- lm(bwt ~ visits * smoke, data=birthData)
```

```
summary(twoway2)
```

```
##
## Call:
## lm(formula = bwt ~ visits * smoke, data = birthData)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2045.82  -460.29   50.71   505.18  1713.77
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)    2955.40     96.22  30.714  <2e-16 ***
## visitsOnce      320.83    154.30   2.079   0.039 *
## visitsMoreThanOnce  12.20    172.13   0.071   0.944
## smoke1        -200.58    143.44  -1.398   0.164
## visitsOnce:smoke1  -458.32    278.50  -1.646   0.102
## visitsMoreThanOnce:smoke1  159.27    266.27   0.598   0.550
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 713.6 on 183 degrees of freedom
```

```
## Multiple R-squared:  0.06783,    Adjusted R-squared:  0.04236
## F-statistic: 2.663 on 5 and 183 DF,  p-value: 0.02378
```

```
anova(twoway2, twoway1)
```

```
## Analysis of Variance Table
##
## Model 1: bwt ~ visits * smoke
## Model 2: bwt ~ visits + smoke
##   Res.Df      RSS Df Sum of Sq    F Pr(>F)
## 1     183 93189162
## 2     185 95182096 -2  -1992934 1.9568 0.1443
```

```
drop1(twoway2, test="F")
```

```
## Single term deletions
##
## Model:
## bwt ~ visits * smoke
##           Df Sum of Sq    RSS    AIC F value Pr(>F)
## <none>                93189162 2489.5
## visits:smoke    2    1992934 95182096 2489.5  1.9568 0.1443
```

```
# Compute the expected birth weight of infants for smokers and non-smokers, respectively, on average over
emmeans(twoway2, ~smoke)
```

```
## NOTE: Results may be misleading due to involvement in interactions
```

```
##   smoke emmean   SE  df lower.CL upper.CL
## 0        3066 70.1 183    2928    3205
## 1        2766 96.4 183    2576    2956
##
```

```
## Results are averaged over the levels of: visits
## Confidence level used: 0.95
```

Models with numerical as well as categorical predictors

```
### Model with linear (lwt,bwt) association.
### Intercept differ between smokers and non-smokers, one common slope.
model1 <- lm(bwt ~ lwt + smoke, data=birthData)
summary(model1)
```

```
##
## Call:
## lm(formula = bwt ~ lwt + smoke, data = birthData)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2030.90  -445.69    29.16   521.76  1967.76
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  2501.125    230.836  10.835  <2e-16 ***
## lwt           4.237      1.690    2.507  0.0130 *
## smoke1       -272.081    105.591  -2.577  0.0107 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
##
## Residual standard error: 707.8 on 186 degrees of freedom
## Multiple R-squared:  0.06777,    Adjusted R-squared:  0.05775
## F-statistic: 6.761 on 2 and 186 DF,  p-value: 0.001464

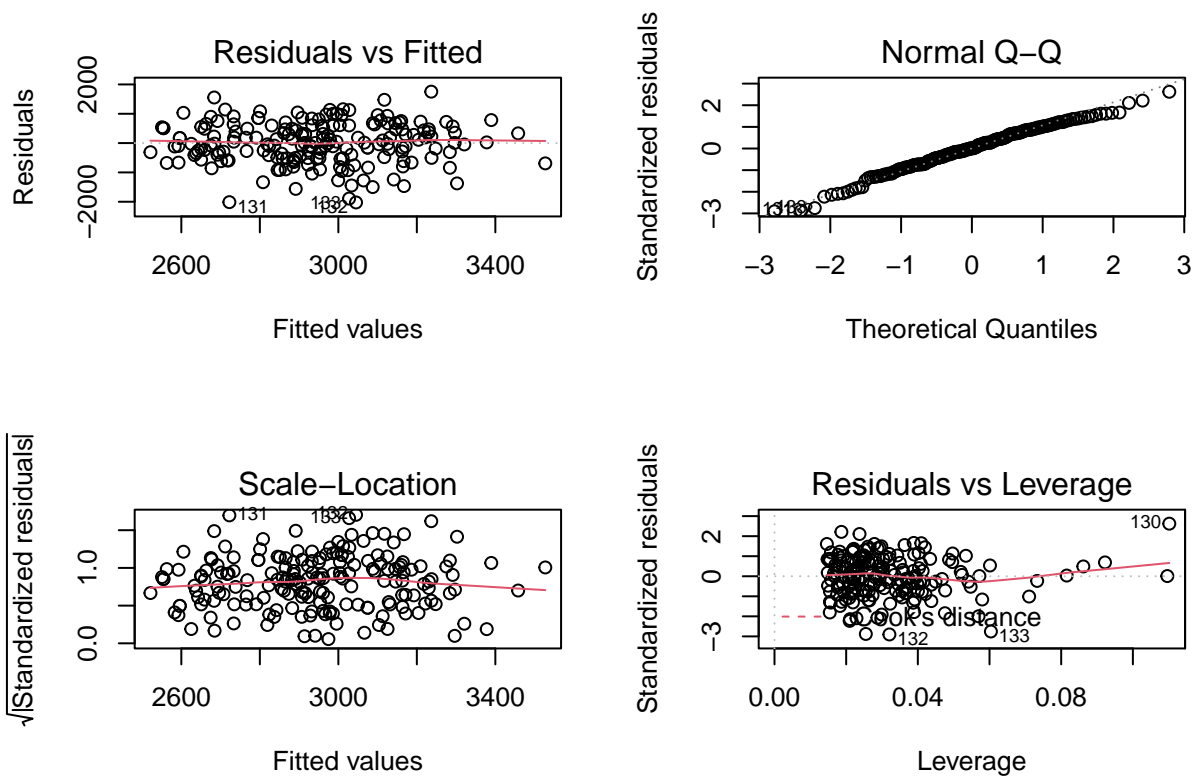
### Model with linear (lwt,bwt) association. Intercept and slope both differ between visit groups
model2 <- lm(bwt ~ lwt * smoke, data=birthData)
summary(model2)

##
## Call:
## lm(formula = bwt ~ lwt * smoke, data = birthData)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2038.80  -454.76   28.36   530.84  1976.84
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  2350.578    312.733   7.516 2.35e-12 ***
## lwt           5.387      2.335   2.307  0.0222 *
## smoke1       41.384     451.187   0.092  0.9270
## lwt:smoke1    -2.422      3.388  -0.715  0.4757
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 708.8 on 185 degrees of freedom
## Multiple R-squared:  0.07034,    Adjusted R-squared:  0.05527
## F-statistic: 4.666 on 3 and 185 DF,  p-value: 0.003621

### Test if slopes differ between visit groups
anova(model2, model1)

## Analysis of Variance Table
##
## Model 1: bwt ~ lwt * smoke
## Model 2: bwt ~ lwt + smoke
##   Res.Df    RSS Df Sum of Sq    F Pr(>F)
## 1     185 92937722
## 2     186 93194298 -1    -256576 0.5107 0.4757

### Model with many effects (no interactions)
model3 <- lm(bwt ~ lwt + smoke + age + visits, data=birthData)
par(mfrow=c(2,2))
plot(model3)
```



```
summary(model3)
```

```
##
## Call:
## lm(formula = bwt ~ lwt + smoke + age + visits, data = birthData)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2024.13  -491.65    6.56   507.56  1753.25
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   2346.442    301.420   7.785   5e-13 ***
## lwt             4.167      1.727   2.413  0.0168 *
## smoke1        -244.402    107.187  -2.280  0.0238 *
## age             4.300     10.252   0.419  0.6754
## visitsOnce     184.314    130.194   1.416  0.1586
## visitsMoreThanOnce 32.318    133.410   0.242  0.8089
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 708.6 on 183 degrees of freedom
## Multiple R-squared:  0.08076,    Adjusted R-squared:  0.05564
## F-statistic: 3.215 on 5 and 183 DF,  p-value: 0.008306
```

Logistic regression

```
### Logistic regression with many predictors (no interactions)
logreg1 <- glm(low ~ lwt + smoke + age + visits, data=birthData, family="binomial")
```

```
summary(logreg1)
```

```
##
## Call:
## glm(formula = low ~ lwt + smoke + age + visits, family = "binomial",
##      data = birthData)
##
## Deviance Residuals:
##      Min       1Q   Median       3Q      Max
## -1.3128  -0.8665  -0.6732   1.2390   2.0197
##
## Coefficients:
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept)      1.370311   1.016623   1.348  0.1777
## lwt             -0.012276   0.006138  -2.000  0.0455 *
## smoke1           0.619077   0.330428   1.874  0.0610 .
## age             -0.031764   0.033933  -0.936  0.3492
## visitsOnce      -0.413043   0.424527  -0.973  0.3306
## visitsMoreThanOnce -0.148285  0.420965  -0.352  0.7247
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##      Null deviance: 234.67  on 188  degrees of freedom
## Residual deviance: 221.91  on 183  degrees of freedom
## AIC: 233.91
##
## Number of Fisher Scoring iterations: 4
```

Linear mixed models

```
### Make artificial center variable
set.seed(123)
center <- sample(rep(1:19, each=10)[1:189])
birthData <- mutate(birthData, center=factor(center))

### Remember to install lme4 before this can run
# install.packages("lme4")
library(lme4)

## Loading required package: Matrix
##
## Attaching package: 'Matrix'
## The following objects are masked from 'package:tidyr':
##
##      expand, pack, unpack
### Linear mixed model with random effect of center
lmm1 <- lmer(bwt ~ lwt + smoke + age + visits + (1|center), data=birthData)

## boundary (singular) fit: see ?isSingular
```

```
summary(lmm1)
```

```
## Linear mixed model fit by REML ['lmerMod']
## Formula: bwt ~ lwt + smoke + age + visits + (1 | center)
## Data: birthData
##
## REML criterion at convergence: 2958
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -2.85637 -0.69380  0.00926  0.71625  2.47412
##
## Random effects:
## Groups Name Variance Std.Dev.
## center (Intercept) 9.668e-12 3.109e-06
## Residual 5.022e+05 7.086e+02
## Number of obs: 189, groups: center, 19
##
## Fixed effects:
##              Estimate Std. Error t value
## (Intercept)  2346.442    301.420   7.785
## lwt          4.167      1.727    2.413
## smoke1      -244.402    107.187  -2.280
## age          4.300     10.252    0.419
## visitsOnce   184.314    130.194    1.416
## visitsMoreThanOnce 32.318    133.410    0.242
##
## Correlation of Fixed Effects:
##              (Intr) lwt    smoke1 age    vstsOn
## lwt          -0.613
## smoke1       -0.196  0.046
## age          -0.620 -0.173  0.003
## visitsOnce   -0.026  0.048  0.160 -0.218
## vstsMrThnOn  0.055 -0.058  0.031 -0.190  0.335
## optimizer (nloptwrap) convergence code: 0 (OK)
## boundary (singular) fit: see ?isSingular
```