12.08.2019

PREDICATE CALCULUS

The propositional calculus does not allow us to represent many of the statements that we use in mathematics, computer science and in everyday life.

To overcome this, the propositional calculus is being generalised and this being known as predicate calculus.

Predicate ~~is part of the~~

A part of the declarative statement properties of objet or relation among objects is called predicate.

Example: Kenan is a bachelor and Errol is a bachelor. Both had the same property of being bachelor.

In propositional calculus there is ~~no~~ no symbolic representation to represent "is bachelor" ~~phrase~~. phrase since this phrase or predicate is not a statement.

In logic a predicate is used by symbolised by capital letter and the names of individual or object of small letter.

Example: The sentence x is a bachelor is represented by $P(x)$

so x is known as a predicate variable and P is known as predicate.

and $P(x)$ is also called propositional function because for each choice of x we get a proposition.

Therefore, we can define predicate as a sentence that contains a finite number of variables and becomes proposition when a specific value is given to the variables.

Domain universe or universe of discourse

The domain of the predicate variable is a set of all possible values that may be substituted in place of the variable.

Example: $P(x)$: x is a bachelor

x can be taken as set of all human beings.

Quantifier

1. universal and existantive quantifier

- quantifier are words that refer to the quantities such as some or all and indicate how frequently a statement is true. The word there exists is denoted by "$\exists$" and the phrase for all "$\forall$" and they are called universal quantifiers.

Example: consider the statement - "All human beings are mortal". To denote this statement lets consider $p(x)$: x is mortal. Then the statement "all human beings are mortal" can be represented ~~as~~ symbolically as follows:

$(\forall x \in S)\ p(x)$ OR $\forall x\ p(x)$ —— ① S: set of all human beings

This statement is called universal statement

- We notice here that $\forall x\ p(x)$ have no truth value and this statement is assigned truth value as follows:

The statement for all x $p(x)$ is true only if $p(x)$ is true only if $p(x)$ is true for all values of $x \in S$ (x to be in S)

- $\forall x\ p(x)$ is false if $p(x)$ is false for atleast one value of x.
- The value of x for which $p(x)$ is false is called example.
- There exist is an existantial quantifier

Example: consider the statement there exist x such that $x^2 \geq 5$

$p(x) = x^2 \geq 5$

that is $\exists x\ p(x)$ or $(\exists x \in \mathbb{R})\ p(x)$

The above statement is known as existantial quantifier and it has the following truth value

- there exist $\exists x\ p(x)$ is true if $p(x)$ is true for atleast one value of x and there exist $\exists x\ p(x)$ is false for each value of x in the domain.

when the quantifier used are not given any domain example : $p(x)$: $x^2 = 3$
If we take domain or universe as a set of all integer then $\exists x\ p(x)$: false

but if we take our domain as set of real number then there exist $\exists x\, P(x)$ True take x as $\sqrt{3}$

let $\mathbb{Z}$ be the set of all integers and consider the statements

i) $(\forall x \in \mathbb{Z})\ x^2 = x$

ii) $(\exists x \in \mathbb{Z})\ x^2 = x$

i) since $P(x) = x^2 = x$ is not true for all $x \in \mathbb{Z}$ hence $(\forall x \in \mathbb{Z})$ hence the statement $x^2 = x$ is false.

ii) since $P(x) : x^2 = x$ is true for $x = 1$. Therefore $\exists x\, (x \in \mathbb{Z})\ x^2 = x$ is true

Negation of quantified statement

consider the statement that "all students of this class has taken a course of discrete mathematics". This can be represented symbolically

$$\forall x\, P(x)$$

where $p(x)$: (students) x has taken course in discrete mathematics.

05.03.2019 Now its negation is given as

$$\exists x \neg (P(x))$$

The above statement is read as it is not the case that all student have taken a course in discrete mathematics.

$$\neg(\forall x\, P(x)) \equiv \exists x \neg P(x)$$

consider the existantial statement $\exists x\, P(x)$

$$\neg(\exists x\, P(x)) \equiv \forall x \neg P(x)$$

The negation of an existantial statement is logically equivalent to the universal statement (All are not).

The following table gives the list of quantified statements and their negations

	Statement	Negation
1.	$\forall x P(x)$	$\exists x [\neg P(x)]$
2.	$\exists x [\neg(P(x))]$	$\forall x P(x)$
3.	$\forall x (\neg P(x))$	$\exists x P(x)$
4.	$\exists x P(x)$	$\forall x [\neg P(x)]$

Example: $\forall$ real numbers x if $x>3$ then $x^2>9$. ~~find the~~ Find the negation of the above statement.

Let $P(x) : x>3$

$Q(x) : x^2>9$

The above statement can be expressed as

$$\forall x [P(x) \rightarrow Q(x)]$$

Therefore, $\neg[\forall x (P(x) \rightarrow Q(x))] \equiv \exists x[\neg(P(x) \rightarrow Q(x))]$

$\equiv \exists x \neg(\neg P(x) \vee Q(x))$

$\equiv \exists x [P(x) \wedge \neg Q(x)]$

That means there exist a real number x such that $x>3$ and $x^2<=9$

BOOLEAN ALGEBRA

Boolean Algebra

A non-empty set B together with two binary operation say '+' and '.' and a unary operator ''' or '—' and two distinct elements 0 and 1 is called as boolean algebra and denoted by notation $(B, +, ., ', 1, 0)$ and if the following axioms are satisfied.

1. commutativity

i) $a+b = b+a$

ii) $a.b = b.a$ $\quad \forall\ a, b \in B$

2. Associativity

i) $a+(b+c) = (a+b)+c$

ii) $a.(b.c) = (a.b).c$ $\quad \forall\ a, b, c \in B$

3. Distributivity

i) $a+(b.c) = (a+b).(a+c)$

ii) $a.(b+c) = a.b + a.c$

$\forall\ a, b, c \in B$

4. Identity laws

i) $a+0 = a$

ii) $a.1 = a$

$\forall\ a \in B$

5. complement laws

i) $a+a' = 1$

ii) $a.a' = 0$

$\forall\ a \in B$

Basic theorems of Boolean Algebra

1. Idempotence law

Prove that in a boolean algebra B

i) $a + a = a$

ii) $a . a = a$

Proof:

i) we have

$a = a + 0$ (identity law)

$= a + a.a'$ (complement law)

$= (a + a) . (a + a')$ (distributive property)

$= (a + a) . 1$ (complement law)

$= a + a$ (identity law)

ii) we have

$a = a.1$ (identity law)

$= a . (a + a')$ (complement law)

$= a.a + a.a'$ (distributive property)

$= a.a + 0$ (complement law)

$= a.a$ (identity law)

2. Boundedness law

If B is a boolean algebra then prove that

i) $a + 1 = 1$

ii) ~~a+0~~ $a.0 = 0$

Proof:

i) $a + 1 = a + (a + a')$

$= (a + a) + a'$

$= a + a'$

$= 1$

ii) $a . 0 = a . (a . a')$

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$= (a.a).a'$

$= a.a'$

$= 0$

3. Absorption law

let B be a boolean algebra then prove that

i) $a + (a.b) = a$

ii) $a.(a+b) = a$

Proof :

i) $a + (a.b) = a.1 + a.b$ (identity law)

$= a.(1+b)$ (distributive property)

$= a.1$ (Boundedness law)

$= a$ (identity law)

ii) $a.(a+b) = a.a + a.b$ (distributive property)

$= a + a.b$ (idempotence law)

$= a.1 + a.b$ (identity law)

$= a.(1+b)$ (distributive property)

$= a.1$ (Boundedness law)

$= a$ (identity law)

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4. De Morgan's laws

i) $(a+b)' = a'.b'$

ii) $(a.b)' = a' + b'$

$\forall$ $a, b \in B$ (boolean algebra)

Proof :

i) To prove that $(a+b)' = a'.b'$ we need to show

$(a+b) + a'.b' = 1$

and $(a+b).a'b' = 0$

$(a+b) + a'.b'$

$= (a+b+a').(a+b+b')$

$= (1+b).(a+1)$

$= 1.1$

$= 1$

$(a+b).a'b'$

$= \cancel{aa'b'+} a'b'.(a+b)$

$= a'.b'.a + a'.b'.b$

$= a.a'.b' + a'.b.b'$

$= 0 + 0$

$= 0$

ii) To prove that $(a.b)' = a'+b'$ we need to show

$(a.b) + a'+b' = 1$

and $a.b.(a'+b') = 0$

$(a.b) + a'+b' \cancel{= 1}$

$= (a'+b') + a.b$

$= (a'+b'+a).(a'+b'+b)$

$= (a+a'+b').(a'+b+b')$

$= (1+b').(a'+1)$

$= 1.1$

$= 1$

$a.b.(a'+b')$

$= a.b.a' + a.b.b'$

$= a.a'.b + a.b.b'$

$= \cancel{a}\ 0.b + a.0$

$= 0+0$

$= 0$

Let B be boolean algebra then show that for each element a, b in B

1. $a+b=b \Leftrightarrow a'+b=1$

2. $a'+b=1 \Leftrightarrow a.b'=0$

Proof:

1. Assume $a+b=b$

Now $a'+b = a'+a+b$

$= 1+b$

$= 1$

Let $a'+b=1$

Now $a+b = (a+b).1$

$= (a+b).(a'+b)$

$= \cancel{(a+b).a'} + \cancel{(a+b).b}$

$= \cancel{a.a'+} \; b + (a.a')$

$= b+0$

$= b$

2. Assume $\cancel{a.b'=0}$ $a'+b=1$

Now $a.b' = (a.b').1$

$= (a.b').(a'+b)$

$= a.a'.b' + a.b.b'$

$= 0+0$

$= 0$

Let $a.b'=0$

Now $a'+b = a'+b+0$

$= a'+b + a.b'$

$= (a+a'+b).(a'+b+b')$

$= (1+b).(a'+1)$

$= 1.1$

$= 1$

Boolean function or expression

Boolean expression is defined as follows

Any boolean constant or boolean variable is a boolean expression.

If E_1 and E_2 are two boolean expressions then E_1', E_2', $E_1 + E_2$, $E_1 . E_2$ are boolean expressions.

Literal

A boolean expression that contains n distinct variable is usually referred as Boolean expression in n variables.

Each appearance of a variable or its complement in boolean expression is known as a literal.

Examples of boolean expressions

1. $a + a'.b + b'$
2. $a + b + c.c'$

Prove that

1. $ab + ab' + a'b = a + b$
2. $(a+c).b + ab' + bc' + c = a + b + c$

1. L.H.S. $= ab + ab' + a'b$

$= a(b+b') + a'b$

$= a.1 + a'b$

$= a + a'b$

$= (a+a').(a+b)$

$= 1.(a+b)$

$= a+b$

$=$ R.H.S.

2. L.H.S. $= (a+c).b + ab' + bc' + c$

$= a.b + b.c + ab' + bc' + c$

$= a(b+b') + \cancel{b.c + b.c' + c} \; b(c+c') + c$

$= a.1 + b.1 + c$

$= a + b + c$

$=$ R.H.S.

Let B be a boolean algebra.

Define a relation R on B as follows

aRb iff $\cancel{a.\bar{b} = 0}$ $a.b' = 0$

Show that R is ~~an equivalence relation on B~~ partial order

i) Reflexivity

$a.a' = 0 \quad \forall a \cancel{,b} \in B$

$\Rightarrow aRa \quad \forall a \in B$

Hence R is a ~~reflexive relation~~ ~~partial order~~ reflexive relation

~~iii)~~

ii) Anti-symmetry

Let a and b ∈ B

Let aRb and bRa

$a.b' = 0$ and $b.a' = 0$

Let $a = a + 0 \cancel{+}$

$= a + \cancel{a.b' + 0} \; b.a'$

$= \cancel{a + a.b' + b.a'} \; (a+b).(a+a')$

$= \cancel{(a+a).(a+b')} \; a+b$ —— ①

$= \cancel{a.(a+b') + b.a'}$

$= \cancel{a}$

$b = b + 0$

$= b + \cancel{b.a'} \; a.b'$

$= (a+b).(b+b')$

$= a+b$ —— ②

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from ① and ② $a = b$

R is antisymmetric

iii) aRb and bRc

$a.b' = 0$ and $b.c' = 0$

$a.c' = \cancel{a'} \cancel{a.c' + 0}$

$= \cancel{a.c' + a.b' + b.c'}$

$= \cancel{a.(b'+c') + b.c'}$

$= a.c'(b+b')$

$= a(b.c') + (a.b').c'$

$= a.0 + 0.c'$

$= 0$ R is transitive

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Literal

A boolean variable either in true form or complemented form is called a literal.

Minterms

A minterm of n variables is product of exactly n literals where each variable appears exactly once either in the true for or complemented form but not both.

For example, minterms of a and b are

i) $a.b$"

ii) $a.b'$

iii) $a'.b$

iv) $a'.b'$

Minterms of a, b and c are

i) $a'.b'.c'$

ii) $a'.b'.c$

iii) $a'.b.c'$

iv) $a'.b.c$

v) $a.b'.c'$

vi) $a.b'.c$

vii) $a.b.c'$

viii) $a.b.c$

Maxterm

A maxterm in n variable is sum of exactly n literals such that each variable appears exactly once either in true form or in the complemented form.

Example :

Maxterms in variables a and b are :

i) $a+b$ ii) $a+b'$ iii) $a'+b$ iv) $a'+b'$

Maxterms in three variables a, b and c are :

i) $a'+b'+c'$ ii) $a'+b'+c$ iii) $a'+b+c'$ iv) $a'+b+c$

v) $a+b'+c'$ vi) $a+b'+c$ vii) $a+b+c'$ viii) $a+b+c$

Principal disjunctive and principal conjunctive normal form of a boolean expression

Principal disjunctive normal form

Principal disjunctive normal form of a boolean expression is sum of the min terms.

Principal conjunctive normal form

Principal conjunctive normal form of a boolean expression is product of maxterms.

Find the principal disjunctive normal form of the following boolean expression

1. $f(x,y,z) = x + y' + z$ (Disjunctive normal form)

$= x.(y+y') + y'(x+x') + z(x+x')$

$= xy + xy' + xy' + x'y' + xz + x'z$

$= xy(z+z') + xy'(z+z') + \cancel{xy'(z+} + x'y'(z+z') + xz(y+y') + x'z(y+y')$

$= xyz + xyz' + xy'z + xy'z' + x'y'z + x'y'z' + xyz + xy'z + x'yz + x'y'z$

$= xyz + xyz' + xy'z + xy'z' + x'y'z + x'y'z' + \cancel{x'y'z} + x'yz$

(Principal disjunctive normal form)

2. i) $f(x,y,z) = xy'z + x'z$

ii) $f = (x+y).(y'z')'$

iii) $f = (x'+y)'.(x+z)' + (yz)'$

iv) $f = [x+y'+(y+z)'] + yz$

i) $f(x,y,z) = xy'z + x'z$

$= xy'z + x'z.(y+y')$

$= xy'z + x'yz + x'y'z$

ii) $f(x,y,z) = (x+y).(y'z')'$

$= (x+y).(y+z)$

$= y + xz$

$= y(x+x')(z+z') + xz(y+y')$

$= xyz + xyz' + x'yz + x'yz' + xyz + xy'z$

$= xyz + xyz' + x'yz + x'yz' + xy'z$

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iii) $f(x,y,z) = (x'+y)' \cdot (x+z)' + (yz)'$

$= (xy') \cdot (x'z) + y' + z'$

$= (x \cdot x' \cdot y' \cdot z) + y' + z'$

$= 0 + y' + z'$

$= y' + z'$

$= y'(x+x')(z+z') + (x+x')(y+y')z'$

$= xy'z + xy'z' + x'y'z + x'y'z' + xyz' + \cancel{xy'z'} + x'yz' + x'y'z'$

$= xy'z + xy'z' + x'y'z + x'y'z' + xyz' + x'yz'$

iv) $f(x,y,z) = [x + y' + (y+z)'] + yz$

$= x + y' + y'z' + yz$

$= x(y+y')(z+z') + y'(x+x')(z+z') + (x+x')y'z' + (x+x')yz$

$= xyz + xyz' + xy'z + xy'z' + xy'z + xy'z' + x'y'z + x'y'z' + xy'z'$

$+ x'y'z' + xyz + x'yz$

$= xyz + xyz' + xy'z + xy'z' + x'y'z + x'y'z' + x'yz$

Find the principal conjunctive normal form of the following boolean expression

i) $f(a,b,c) = ab + a'c$

ii) ~~$(x+y$~~ $f(x,y,z) = (x+y+z)(xy+xz)'$

iii) $f(x,y,z) = xy' + xz + xy$

iv) $f(x,y,z) = xyz + (x+y)(x+z)$

v) $f(x,y,z) = (xy' + xz)'$

i) $f(a,b,c) = ab + a'c$

$= (ab + a')(ab + c)$

$= (a+a') \cdot (a'+b) \cdot (a+c) \cdot (\cancel{a+} b + c)$

$= 1 \cdot (a'+b)(a+c) \cdot (b+c)$

$= (a'+b) \cdot (a+c) \cdot (b+c)$

$= (a'+b+cc').(a+c+b.b').(b+c+a.a')$

$= (a'+b+c).(a'+b+c').(a+b+c).(a+b'+c).(a+b+c).(a'+b+c)$

$= (a'+b+c).(a'+b+c').(a+b+c).(a+b'+c)$

(Principal conjunctive normal form)

~~iii~~

ii) $f(x,y,z) = (x+y+z).(xy+xz)'$

$= (x+y+z).(xy)'.(xz)'$

$= (x+y+z).(x'+y').(x'+z')$

$= (x+y+z).(x'+y'+z.z').(x'+z'+y.y')$

$= (x+y+z).(x'+y'+z).(x'+y'+z').(x'+y+z').(x'+y'+z')$

$= (x+y+z).(x'+y'+z).(x'+y'+z').(x'+y+z')$

(Principal conjunctive normal form)

iii) $f(x,y,z) = xy' + xz + xy$

$= x.(y'+y) + xz$

$= x + xz$

$= \cancel{x+x}\, x$

$= x + yy' + zz'$

$= (x+y+z).(x+y+z').(x+y'+z).(x+y'+z')$

(Principal conjunctive normal form)

iv) $f(x,y,z) = xyz + (x+y).(x+z)$

$= (xyz + x) + yz$

$= x + yz$

$= (x+y).(x+z)$

$= (x+y+zz')(x+yy'+z)$

$= (x+y+z).(x+y+z').(x+y+z).(x+y'+z)$

$= (x+y+z).(x+y+z').(x+y'+z)$

(Principal conjunctive normal form)

v) $f(x,y,z) = (xy' + xz)'$

$= (xy')' \cdot (xz)'$

$= (x' + y) \cdot (x' + z')$

$= (x' + y + zz') \cdot (x' + z' + yy')$

$= (x' + y + z) \cdot (x' + y + z') \cdot (x' + z' + y) \cdot (x' + z' + y')$

$= (x' + y + z) \cdot (x' + y + z') \cdot (x' + y' + z')$

(Principal conjunctive normal form)

11.03.2019

PRINCIPLE OF MATHEMATICAL INDUCTION

Let $P(n)$ be a statement involving natural number 'n' such that $P(n)$ is true for $n = n_0$.

If $P(n)$ is true for $n = k+1$ whenever $P(k)$ is assumed to be true then $P(n)$ is true for all natural numbers n.

1. Show that $1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6} \quad \forall n \geq 1$

Proof :

$P(n) : 1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$

1. Let $n = 1$

$1^2 = \frac{1.(1+1).(2+1)}{6} = 1$

Therefore $P(1)$ is true

2. Let us assume that $P(n)$ is true for $n = k$, $k > 1$

$\Rightarrow 1^2 + 2^2 + 3^2 + \dots + k^2 = \frac{k(k+1)(2k+1)}{6}$ ——— ⊛

3. To prove $P(k+1)$ is true that is

$1^2 + 2^2 + 3^2 + \dots + (k+1)^2 = \frac{(k+1)(k+2)(2k+3)}{6}$

Now, $1^2 + 2^2 + 3^2 + \dots + (k+1)^2$

$= 1^2 + 2^2 + 3^2 + \dots + k^2 + (k+1)^2$

$= \frac{k(k+1)(2k+1)}{6} + (k+1)^2$

$= \frac{k(k+1)(2k+1) + 6(k+1)(k+1)}{6}$

$= \frac{(k+1)\left[k(2k+1) + 6(k+1)\right]}{6}$

$$= \frac{(k+1)(2k^2+7k+6)}{6}$$

$$= \frac{(k+1)(k+2)(2k+3)}{6}$$

Therefore, $P(k+1)$ is true. Hence $P(n)$ is true for all $n \geq 1$

2. Show ~~that~~ using principle of mathematical induction that $6^{n+2} + 7^{2n+1}$ is divisible by 43 $\forall n$

$P(n)$: $6^{n+2} + 7^{2n+1}$ is divisible by 43 $\forall$ n

1. Let $n=1$, $6^3 + 7^3 = 216 + 343$

$= 559$

$= 43(13)$

2. Let us assume that $P(n)$ is true for $n=k$, $k>1$

$\Rightarrow 6^{k+2} + 7^{2k+1}$ is divisible by 43

that is $6^{k+2} + 7^{2k+1} = 43m$, $m \in \mathbb{N}$

$\Rightarrow 6^{k+2} = 43m - 7^{2k+1}$ —— ⊛

3. To show $P(k+1)$ is true

that is $6^{k+1+2} + 7^{2(k+1)+1}$ is divisible by 43.

$6^{k+1+2} + 7^{2k+2+1}$

$= 6^{k+2}.6 + 7^{2k+3}$

$= (43m - 7^{2k+1})6 + 7^{2k+1}.7^2$

$= 43(6m) - 7^{2k+1}.6 + 7^{2k+1}.7^2$

$= 43(6m) - 7^{2k+1}(6-7^2)$

$= 43(6m) + 43.7^{2k+1}$

$= 43[6m + 7^{2k+1}]$

$= 43(P)$

$\Rightarrow 6^{(k+1)+2} + 7^{2(k+1)+1}$ is divisible by 43.

$\Rightarrow P(k+1)$ is true

Hence, $P(n)$ is true for all $n \in \mathbb{N}$

3. Show that $n^2 > 2n+1$, $n \geq 3$

Proof :

$P(n)$: $n^2 > 2n+1$, $n \geq 3$

1. Let $n=3$

$9 > 2(3)+1$

$9 > 7$

$P(3)$ is true

2. Let us assume that $P(k)$ is true for $k > 3$

$\Rightarrow k^2 > 2k+1$ ——————— ⊛

3. To show $P(k+1)$ is true

that is $(k+1)^2 > 2k+3$

$(k+1)^2 = k^2 + 2k+1$

$> 2k+1 + 2k+1$ (By assumption ⊛)

$> 2k+2 + 2k$ $(2k > 1)$

$> 2k+2+1$

$> 2k+3$

Hence, $P(k+1)$ is true.

Therefore, $P(n)$ is true $\forall$ $n \geq 3$

UNIT - 03

The fundamental principles of counting

1. Sum Rule

let $E_1, E_2, \ldots, E_k$ are events or tasks such that no two can occur at the same time and if E_1 can occur n_1 ways, E_2 can occur n_2 ways, . . . , E_k can occur n_k ways then anyone can happen in $n_1 + n_2 + \ldots + n_k$ ways

Example: If there are 14 boys and 12 girls in a class. Find number of ways in which class representative can be selected.

The number of ways in which a class representative can be selected

$= 14 + 12 = 26$

E_1 - selecting a boy class representative

E_2 - selecting a girl class representative

Number of ways E_1 can happen - 14 ways

Number of ways E_2 can happen - 12 - ways

Number of ways either E_1 or E_2 can happen $= 14 + 12$

$= 26$

2. Product Rule

let $E_1, E_2, \ldots, E_k$ are k events such that E_1 can occur n_1 ways, E_2 can occur n_2 ways, . . . , E_k can occur n_k ways ~~then~~ then these k events can occur simultaneously in $n_1 . n_2 \ldots n_k$ ways

Example: 3 persons ~~occur~~ can enter in an car where there are 5 seats. In how many ways they can take up all the 3 seats.

E_1 - first person enters the car

E_2 - second person enters the car

E_3 - Third person enters the car

Number of ways E_1 can happen = 5

Number of ways E_2 can happen = 4

Number of ways E_3 can happen $= 3$

Therefore, number of ways 3 persons can take up a seat $= 5 \times 4 \times 3 = 60$

In how many ways can one select 2 books from different subjects from among ~~4~~ 6 distinct computer science ~~3 di~~ books, 3 distinct mathematics books, 2 distinct chemistry books

Let E_1 denotes selection of 2 books from computer science and mathematics

Let E_2 denote selection of 2 books from mathematics and chemistry

Let E_3 denote selection of 2 books from computer science and chemistry

Number of ways E_1 can happen $= 6 \times 3$

$= 18$

Number of ways E_2 can happen $= 3 \times 2$

$= 6$

Number of ways E_3 can happen $= 6 \times 2$

$= 12$

Therefore number of ways 2 books of different subjects can be selected

= number of ways E_1 can happen

+ number

+ number

$= 18 + 6 + 12$

$= 36$

Principle of Inclusion and Exclusion

Let $A_1, A_2, \ldots, A_n$ be n finite sets then

$$\left|\bigcup_{i=1}^{n} A_i\right| = |A_1 \cup A_2 \cup \ldots \cup A_n|$$

$$= \sum_{1 \le i \le n} |A_i| - \sum_{1 \le i \le j \le n} |A_i \cap A_j| + \sum_{1 \le i \le j \le k} |A_1 \cap A_2 \cap \ldots \cap A_k|$$

$$\cancel{= \sum_{1 \le i \le j \le n} A_i \cap A_j} + \ldots + (-1)^{n+1} \left|\bigcap_{i=1}^{n} A_i\right|$$

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case i) $n=2$

$$|A_1 \cup A_2| = |A_1| + |A_2| - |A_1 \cap A_2|$$

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

case ii) $$|A_1 \cup A_2 \cup A_3| = |A_1| + |A_2| + |A_3| - |A_1 \cap A_2| - |A_2 \cap A_3| - |A_3 \cap A_1| + |A_1 \cap A_2 \cap A_3|$$

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$$

Floor and ceiling function

Let x be a real number the floor function of x is defined as

$\lfloor x \rfloor$ = Greatest integer less than or equal to x

Example : $\lfloor 2.5 \rfloor = 2$

$\lfloor -3.5 \rfloor = -4$

$\lfloor 2 \rfloor = 2$

The ceiling function of a real number x is defined as

$\lceil x \rceil$ least integer more than or equal to x

Example : $\lceil 3.5 \rceil = 4$

$\lceil -4.5 \rceil = -4$

Definition :

Let a and b are two positive integers. ~~wht~~ then the number of positive integers less than or equal to a and divisible by b is equal to $\left\lfloor \frac{a}{b} \right\rfloor$

Find the number of positive integers less than or equal to ~~2078~~ 2076 and divisible by neither 4 nor 5.

Let A denote set of positive integers less than or equal to 2076 and are divisible by 4.

let B be a set of positive integers less than or equal to 2076 and divisible by 5.

$A \cap B$ = set of integers which are positive., less than or equal to 2076 and are divisible by 4 and 5.

$$n(A) = \left\lfloor \frac{2076}{4} \right\rfloor = 519$$

$$n(B) = \left\lfloor \frac{2076}{5} \right\rfloor = 415$$

$$n(A \cap B) = \left\lfloor \frac{2076}{20} \right\rfloor = 103$$

$A \cup B$ represents set of positive integers less than or equal to 2076 and they are ~~either~~ divisible by either 4 or 5.

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$= 519 + 415 - 103$$

$$= 831$$

$A(\overline{A \cup B})$ represents set of positive integers less than or equal to 2076 and are neither divisible by 4 nor by 5.

$$n(\overline{A \cup B}) = n(U) - n(A \cup B)$$

$$= 2076 - 831$$

$$= 1245$$

Determine the integers which are positive less than or equal to 500 and are neither divisible by 2 nor 3 nor 5.

let A represents ~~int positive~~ set of positive integers less than or equal to 500 ~~are~~ and are divisible by 2

let B represent set of positive integers less than or equal to 500 and are divisible by 3

let C represent set of positive integers less than or equal to 500 and are divisible by 5

$A \cap B$ = set of positive integers less than or equal to 500 and divisible by 2 and 3

$B \cap C$ = set of positive integers less than or equal to 500 and divisible by 3 and 5

$A \cap C$ = set of positive integers less than or equal to 500 and divisible by 2 and 5

$A \cap B \cap C$ = set of positive integers less than or equal to 500 and divisible by 2 and 3 and 5

$n(A) = \cancel{20} \left\lfloor \frac{500}{2} \right\rfloor = 250$

$= \left\lfloor \frac{500}{3} \right\rfloor = 166$

$= \left\lfloor \frac{500}{5} \right\rfloor = 100$

$n(A \cap B) = \left\lfloor \frac{500}{6} \right\rfloor = 83$

$n(B \cap C) = \left\lfloor \frac{500}{15} \right\rfloor = 33$

$n(A \cap C) = \left\lfloor \frac{500}{10} \right\rfloor = 50$

$n(A \cap B \cap C) = \left\lfloor \frac{500}{30} \right\rfloor = 16$

$A \cup B \cup C$ = set of positive integers less than or equal to 500 and they are divisible by either 2 or 3 or 5

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

$= 250 + 166 + 100 - 83 - 33 - 50 + 16$

$= 532 - 166$

$= 366$

$\overline{A \cup B \cup C}$ represent set of positive integers less than or equal to 500 and are neither divisible by 2 nor 3 nor 5.

$n(\overline{A \cup B \cup C}) = n(U) - n(A \cup B \cup C)$

$= 500 - 366$

$= 134$

20.03.2019

32 people study Mathematics or physics, 14 people study physics, 30 people study physics and Mathematics. find number of people who

i) study both physics and Mathematics

ii) study only Mathematics

A - people who study Mathematics

B - people who study physics

$n(A \cup B) = 32$

$n(A \cap B) = 30$

$n(B) = 14$

we have, by principle of inclusion and exclusion

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$32 = \cancel{14 +} \; n(A) + 14 - 30$

$n(A) = 48$

Among 100 students, 32 study mathematics, 20 study physics, 45 study Biology, 15 study Mathematics and Biology, 7 study physics and Mathematics, 10 study physics and biology and 30 do not study any of the subjects. Find.

i) number of students who study all three subjects

ii) number of students who study exactly one of the three subjects

let P denote set of students who study physics

let M denote set of students who study Mathematics

let B denote set of students who study Biology

$n(P) = 20$

$n(M) = 32$

$n(B) = 45$

$n(P \cap M) = 7$

$n(M \cap B) = 15$

$n(P \cap B) = 10$

i) $n(P \cap B \cap M) = ?$

we have by principle of inclusion and exclusion

$n(P \cup M \cup B) = n(P) + n(M) + n(B) - n(P \cap M) - n(P \cap B) - n(M \cap B) + n(P \cap M \cap B)$

$100 - 30 = 20 + 32 + 45 - 7 - 10 - 15 + n(P \cap M \cap B)$

$n(P \cap M \cap B) = 70 + 32 - 97$

$= 102 - 97$

$= 5$

ii) Number of students who study ~~all~~ only maths ~~and~~ and physics

$= n(P \cap M) - n(P \cap M \cap B)$

$= 7 - 5$

$= 2$

Number of student who study only physics ~~or~~ and Biology

$= n(P \cap B) - n(P \cap M \cap B)$

$= 10 - 5$

$= 5$

Number of student who study only maths and Biology

$= n(M \cap B) - n(P \cap M \cap B)$

$= 15 - 5$

$= 10$

Number of student who study only ~~phy~~ Mathematics

$= n(M) - 2 - 10 - 5$

$= 32 - 17$

$= 15$

Number of student who study only physics

$= n(P) - 2 - 5 - 5$

$= 8$

Number of student who study only Biology

$= n(B) - 5 - 10 - 5$

$= 25$

Therefore. number of students who study exactly one of the three subjects $= \cancel{48+} 25+15+8 = 48$.

Among the first 1000 positive integers determine

i) integers which are not divisible by 5 or 7 or 9

ii) integers that are divisible ~~but not~~ by 5 but not by 7 or 9

let A denote set of ~~first~~ integers which are positive and less than or equal to 1000 and divisible by 5

let B denote set of integers which are positive and less than or equal to 1000 and divisible by 7

let C denote set of integers which are positive and less than or equal to 1000 and divisible by 9

~~A∩B denotes set of positive integers which are~~

$$n(A) = \left\lfloor \frac{1000}{5} \right\rfloor = 200$$

$$n(B) = \left\lfloor \frac{1000}{7} \right\rfloor = 142$$

$$n(C) = \left\lfloor \frac{1000}{9} \right\rfloor = 111$$

$$n(A\cap B) = \left\lfloor \frac{1000}{35} \right\rfloor = 28$$

$$n(B\cap C) = \left\lfloor \frac{1000}{63} \right\rfloor = 15$$

$$n(A\cap C) = \left\lfloor \frac{1000}{45} \right\rfloor = 22$$

$$n(A\cap B\cap C) = \left\lfloor \frac{1000}{315} \right\rfloor = 3$$

$$n(A\cup B\cup C) = n(A) + n(B) + n(C) - n(A\cap B) - n(B\cap C) - n(A\cap C) + n(A\cap B\cap C)$$

$$= 200 + 142 + 111 - 28 - 15 - 22 + 3$$

$= 391$

$n(\overline{A \cup B \cup C}) = 1000 - 391$

$= 609$

ii) number of integers divisible only by 5 and 7 $= \cancel{342} - 3 = \cancel{339}$ 25 (28)

number of integers divisible only by 5 and 9 $= 22 - 3 = 19$

number of integers divisible only by 5 ~~and not by 7 or 9~~ $= 200 - 25 - 19 - 3$

$= 153$

32 students like Mathematics ~~and P, 13~~ or Physics, ~~13~~ 30 like Mathematics 14 like Physics. Find number of student

i) who like both

ii) who like only Mathematics

iii) who like only Physics

M - set of student who like Mathematics

P - set of students who like Physics

$n(M \cup P) = 32$

$n(M) = \cancel{13}\ 30$

$n(P) = 14$

i) $n(M \cup P) = n(M) + n(P) - n(M \cap P)$

$32 = 30 + 14 - n(M \cap P)$

$n(M \cap P) = \cancel{5}\ 12$

ii) number of student who like only mathematics $= \cancel{13 - 5 = 8}\ 30 - 12 = 18$

iii) number of student who like only physics $= \cancel{14 - 5 = 9}\ 14 - 12 = 2$

22.08.2019

Pigeon - hole principle

It states that if there are n pigeons and they are assigned to m pigeon holes where n is more than m then atleast one pigeon hole contains two or more than two pigeons.

Generalised Pigeon hole principle

It states if n objects are to be placed in k boxes where n is greater than k then there is atleast one box containing atleast $\left\lceil \frac{n}{k} \right\rceil$ objects

Show that if any 5 numbers from 1 to 8 are choosen, then two of them will add up to 9.

$A_1 = \{1, 8\}$, $A_2 = \{2, 7\}$, $A_3 = \{3, 6\}$, $A_4 = \{4, 5\}$

Now each of the 5 numbers chosen must belong to the above 4 sets. Since, there are only 4 sets and we have to choose 5 numbers therefore, by pigeon hole principle, two of the chosen numbers belong to the same set. Hence, these 2 numbers will add up to 9.

Suppose 30 objects are to be numbered from 1 to 30 ~~are~~ and are ~~to be~~ placed in a large box. Show that if 18 objects are drawn randomly, there must be a pair among them whose sum of the numbers appearing on them is 35.

Let $A = \{1, 2, 3, 4, \ldots, 30\}$

$A_1 = \{17, 18\}$, $A_2 = \{16, 19\}$, $A_3 = \{15, 20\}$, $A_4 = \{14, 21\}$, $A_5 = \{13, 22\}$, $A_6 = \{12, 23\}$, $A_7 = \{11, 24\}$, $A_8 = \{10, 25\}$, $A_9 = \{9, 26\}$, $A_{10} = \{8, 27\}$, $A_{11} = \{7, 28\}$, $A_{12} = \{6, 29\}$, $A_{13} = \{5, 30\}$, $A_{14} = \{\cancel{4}\,1\}$, $A_{15} = \{2\}$, $A_{16} = \{3\}$, $A_{17} = \{4\}$

Since 18 objects are drawn, 4 of them may belong to in each of $\{A_{14}\}$, A_{15}, A_{16}, A_{17}. For the remaining 14 objects there are only 13 sets. Therefore, by pigeon hole principle two of them will lie in the same set. Hence two numbers will add up to 35.

show that if any 11 numbers are chosen from set 1 to 20 then one of them will be a multiple of other.

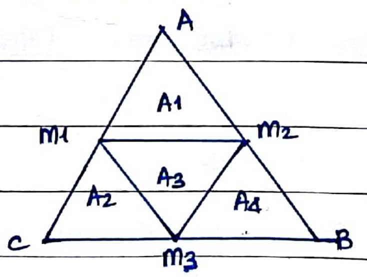
$A_1 = \{1, 2\}$, $A_2 = \{3, 6\}$, $A_3 = \{4, 8\}$, $A_4 = \{5, 10\}$, $A_5 = \{6, 12\}$, $A_6 = \{7, 14\}$, $A_7 = \{8, 16\}$, $A_8 = \{9, 18\}$, $A_9 = \{10, 20\}$

In a group of ~~30 c~~ 13 children show that there are atleast 2 children who are born in the same month.

The 13 children can be considered as pigeons and 12 months of the year are considered to be pigeon holes. Therefore, by pigeon-hole principle 2 of the children are born in the same month.

suppose there are 5 points lying on or inside an equilateral triangle of side 4 cms. show that atleast 2 of them are not more than 2 cms apart.

let ABC be an equilateral triangle with each side equal to 4 cms.
let m_1, m_2, m_3 are the mid point of the sides AC, AB and BC respectively.
Join m_1, m_2 ; m_2, m_3 and m_1, m_3.



let A_1 be the subtriangle bounded by m_1, A, m_2
let A_2 be the subtriangle bounded by c, m_1, m_3

let A_3 be subtriangle bounded by ~~c, m_1, m_3~~ m_1, m_2, m_3

let A_4 be subtriangle bounded by m_3, m_2, B

Now, we know that distance between 2 points on or inside these subtriangles will be less than or equal to 2.

Now these 4 subtriangles A_1, A_2, A_3, A_4 can be considered as pigeon holes.

Now by pigeon hole principle if we choose any 5 points inside or on equilateral triangle ABC then atleast 2 of them will lie in either A_1, A_2, A_3 or A_4. Therefore, 2 of them will be ~~more than 2cms~~ not more than 2cms apart.

Show that if any 8 positive integers are chosen, 2 of them will have same remainders when divided by 7.

when any 8 positive integers are divided by 7 each of them will have some remainder.

Since there are 8 integers and only 7 distinct remainders because 7 can generate only 7 distinct remainders.

So, by pigeon hole principle 2 of the integers must have same remainders when they are divided by 7.

If 51 numbers are to be taken from integers 1 to 100 (both inclusive), then prove that two of them will be successive.

25.03.2019

RECURRENCE RELATION

Linear Recurrence Relation

A linear recurrence relation of order k with constant coefficients has the standard form given as

$$c_0 a_n + c_1 a_{n-1} + c_2 a_{n-2} + \dots + c_k a_{n-k} = f(n)$$

where $c_0, c_1, \dots, c_k$ are constant and $f(n)$ is a function of n where n is a whole number.

The above equation can also be represented as below

$$c_0 a_{n+k} + c_1 a_{n+k-1} + c_2 a_{n+k-2} + \dots + c_k a_n = f(n)$$

Example i) $a_{n+2} + 2a_{n+1} + 3a_n = n+1$

ii) $a_n + a_{n-1} + 2a_{n-2} = n^2$

Note: If f(n) is equal to zero then the recurrence relation is said to be homogeneous otherwise it is said to be non homogeneous.

Formation of recurrence relation or recurrence relation model

1. Find a recurrence relation for a_n = the amount due after n years if P is principal borrowed from a bank and 'r' is the interest rate per year.

$a_n \to$ amount due after n years

$a_{n-1} \to$ amount due after n-1 years

$a_n = a_{n-1} + r(a_{n-1})$

$a_n = (1+r)\, a_{n-1}$

$a_0 = P$

2. Find a recurrence relation for a_n where a_n is the number of different ways to distribute either a 1\$ bill, a 2\$ bill, a 5\$ bill or a 10\$ bill on successive day until a total of n \$ bill has been distributed.

Let a_n denote number of ways n $ can be distributed.

suppose on the first day we distribute a 1$ bill then we are to distribute (n-1) $ bill on the successive days which can be done by a_{n-1} ways.

On the other hand, if we distribute a $2 bill on the first day then (n-2) $ can be distributed in a_{n-2} ways and so on.

Therefore, by sum rule, number of ways to distribute n $

$$= a_{n-1} + a_{n-2} + a_{n-5} + a_{n-10}$$

$$a_n = a_{n-1} + a_{n-2} + a_{n-5} + a_{n-10}$$

3. suppose that a school principal decides to give a prize each day. Further suppose that the principal has 3 different kinds of prizes worth 1$ each and 5 different kinds of prizes worth 4 $ each. Find a recurrence relation for a_n = number of different ways to distribute prizes worth n $

a_n = number of different ways to distribute prizes worth n $

If on the first day a prize of $1 is given away then there are (n-1) $ worth of prizes are to be given ~~to~~ later.

There are 3 ways to select a prize of worth $1. Therefore, there are $3a_{n-1}$ ways to distribute (n-1) $ worth of prizes.

On the other hand suppose on the first day a 4 $ prize is given away then there are (n-4) $ worth of prizes are to be given on successive days.

There are 5 ways to select a prize of worth $4. Therefore, number of ways to distributed (n-4) $ worth prizes is equal to $5a_{n-4}$.

Therefore, number of different ways to ~~different~~ distribute prizes of worth n $

$$= 3a_{n-1} + 5a_{n-4}$$

$$a_n = 3a_{n-1} + 5a_{n-4}$$

4. Find recurrence relation for number of ways to climb n steps if one can climb at a time either 1 step, 2 steps or 3 steps. Also find the initial conditions.

let a_n denote number of ways to climb n steps.

Now assume that only 1 step is climbed at first.

Therefore n-1 steps can be climbed at a_{n-1} ways

If 2 steps are climbed at first then n-2 steps can be climbed at a_{n-2} ways

Suppose 3 steps are climbed at first then n-3 steps can be climbed at a_{n-3} ways.

Therefore, number of ways to climb n steps = $a_{n-1} + a_{n-2} + a_{n-3}$

Therefore,

$$a_n = a_{n-1} + a_{n-2} + a_{n-3}$$

Therefore, $a_1 = 1$

$a_2 = 2$

$a_3 = 4$

5. Find the ~~number~~ recurrence relation and give initial conditions for the number of bit strings of length n that do not contain pattern 11

let c_n denote the number of bit strings of length n that do not contain ~~let c_n denote~~ pattern 11.

Solution of linear Recurrence Relation

consider the linear recurrence relation

$$c_0 a_n + c_1 a_{n-1} + c_2 a_{n-2} + \dots + c_k a_{n-k} = f(n)$$

Now a sequence a_n which satisfies the above recurrence relation is called its solution.

Method of characteristic roots to solve a linear recurrence relation

In this method the solution is obtained as the sum of two parts - the homogeneous solution which satisfies the recurrence relation when the right hand side is equal to zero. and the particular solution which satisfies the recurrence relation when the right hand side is not equal to zero.

The homogeneous solution of the recurrence relation is denoted as $a_n^{(h)}$ and the particular solution is denoted by $a_n^{(p)}$

Therefore, the general solution of the given recurrence relation

$$a_n = a_n^{(h)} + a_n^{(p)}$$

26.03.2019

To find homogeneous solution

consider a linear recurrence relation

$$c_0 a_{n+k} + c_1 a_{n+k-1} + c_2 a_{n+k-2} + \dots + c_k a_n = f(n) \text{ ——— } (1)$$

To find homogeneous solution we find the characteristic equation of the given recurrence relation given as

$$c_0 r^k + c_1 r^{k-1} + c_2 r^{k-2} + \dots + c_{k-1} r + c_k = 0$$

let $m_1, m_2, \dots, m_k$ are the k roots of the above characteristic equation.

case i) let us assume that all the k root are real and distinct then the homogeneous solution can be expressed as

$$a_n^{(h)} = b_1 m_1^n + b_2 m_2^n + \dots + b_k m_k^n$$

where $b_1, b_2, \dots, b_k$ are some arbitrary constants

case ii) let us assume that $m_1 = m_2 \neq m_3 \neq \dots \neq m_k$

then the homogeneous solution can be expressed as

$$a_n^{(h)} = (b_1 + n b_2) m_1^n + b_3 m_3^n + \dots + b_k m_k^n$$

To find particular solution

The particular solution depends on the function f(n) and to find the particular solution we consider a trial solution containing a number of unknowns which are to be determined by substituting the solution in the recurrence relation.

Further, the trial solution which can be used for different f(n) is given as below

f(n)	Trial solution
1. b^n, b is not a root of the characteristic equation	Ab^n
2. b^n, b is a root of multiplicity 's' of the characteristic equation	$Ab^n n^s$
3. p(n) - A polynomial of degree m in variable n	$A_0 + A_1 n + A_2 n^2 + \dots + A_m n^m$
4. $c^n p(n)$, c is not a root of the characteristic equation	$c^n [A_0 + A_1 n + \dots + A_m n^m]$
5. $c^n p(n)$, where c is a root of characteristic equation of multiplicity s	$c^n n^s [A_0 + A_1 n + \dots + A_m n^m]$

1. Solve the following linear recurrence relation

$a_{n+2} - 5a_{n+1} + 6a_n = 2$ or $a_n - 5a_{n-1} + 6a_{n-2} = 2$

with initial condition $a_0 = 1$, $a_1 = -1$

The given recurrence relation is

$$a_{n+2} - 5a_{n+1} + 6a_n = 2 \quad \text{——— (I)}$$

To find homogeneous solution

The characteristic equation is $r^2 - 5r + 6 = 0$

$m_1 = 2$, $m_2 = 3$

The homogeneous solution is given as

$$a_n^{(h)} = b_1 2^n + b_2 3^n$$

b_1 and b_2 are some arbitrary constants

To find particular solution

$f(n) = 2$ is a constant polynomial

Therefore, the trial solution $a_n^{(P)} = A_0$ ——— (i)

$$a_{n+1}^{(P)} = A_0 \quad \text{——— (ii)}$$

$$a_{n+2}^{(P)} = A_0 \quad \text{———— (iii)}$$

substitute (i), (ii) and (iii) in (I) we get

$$A_0 - 5A_0 + 6A_0 = 2$$

$$A_0 = 1$$

$$a_n^{(P)} = 1$$

Therefore, general solution $a_n = a_n^{(h)} + a_n^{(P)}$

$$a_n = b_1 2^n + b_2 3^n + 1$$

$a_0 = 1, \quad a_1 = -1$

$$a_0 = b_1 + b_2 + 1$$

$$1 = b_1 + b_2 + 1$$

$$b_1 + b_2 = 0 \quad \text{———— (iv)}$$

$$a_1 = 2b_1 + 3b_2 + 1$$

$$-1 = 2b_1 + 3b_1 + 1$$

$$2b_1 + 3b_1 = -2 \quad \text{———— (v)}$$

solving (iv) and (v) we get

$$b_1 = 2, \quad b_2 = -2$$

$$a_n = 2 \cdot 2^n - 2 \cdot 3^n + 1$$

$$a_n = 2^{n+1} - 2 \cdot 3^n + 1$$

2. solve the recurrence relation

$$a_{n+2} - a_{n+1} - 2a_n = n^2$$

$$a_{n+2} - a_{n+1} - 2a_n = n^2 \quad \text{———— (I)}$$

To find homogeneous solution

characteristic equation $r^2 - r - 2 = 0$

$m_1 = 2, \quad m_2 = -1$

Homogeneous solution is given as

$$a_n^{(h)} = b_1 2^n + b_2 (-1)^n$$

where b_1, ~~b_2~~ and b_2 are some arbitrary constants.

To find particular solution $a_n^{(p)}$

$f(n) = n^2$ (A polynomial of degree 2)

Therefore, trial solution $a_n^{(p)} = A_0 + A_1 n + A_2 n^2$ ——— (i)

$a_{n+1}^{(p)} = A_0 + A_1(n+1) + A_2(n+1)^2$ ——— (ii)

$a_{n+2}^{(p)} = A_0 + A_1(n+2) + A_2(n+2)^2$ ——— (iii)

Substituting (i), (ii) and (iii) in ① we get

$A_0 + A_1(n+2) + A_2(n+2)^2 - [A_0 + A_1(n+1) + A_2(n+1)^2] - 2[A_0 + A_1 n + A_2 n^2] = n^2$

$(A_2 - A_2 - 2A_2)n^2 + (A_1 + 4A_2 - A_1 - 2A_2 - 2A_1)n + A_0 + 2A_1 + 4A_2 - A_0 - A_1 - A_2 - 2A_0 = n^2$

$-2A_2 n^2 + (2A_2 - 2A_1)n - 2A_0 + A_1 + 3A_2 = n^2$

Equating the coefficient on both sides, we get

$-2A_2 = 1$ ——— (iv)

$2A_2 - 2A_1 = 0 \Rightarrow A_1 = A_2$ ——— (v)

$-2A_0 + A_1 + 3A_2 = 0$ ——— (vi)

From (iv) $A_2 = -\frac{1}{2}$

From (v) $A_1 = -\frac{1}{2}$

Substituting A_1, A_2 in (vi)

$-2A_0 - \frac{1}{2} - \frac{3}{2} = 0$

$-2A_0 = 2$

$A_0 = -1$

The particular solution $a_n^{(p)} = -1 - \frac{1}{2}n - \frac{1}{2}n^2$

$a_n = a_n^{(h)} + a_n^{(p)}$

$= b_1 2^n + b_2(-1)^n - \frac{1}{2}n - \frac{1}{2}n^2 - 1$

27.03.2019

Solve the following recurrence relations

1. $a_{n+2} - 4a_{n+1} + 4a_n = 2^n$

2. $a_n - 5a_{n-1} + 6a_{n-2} = 1$

with $a_0 = 0, \ a_1 = 1$

3. $a_{n+2} + 2a_{n+1} - 15a_n = 6n + 10$

with $a_0 = 1, \ a_1 = -\frac{1}{2}$

4. $a_n - 4a_{n-1} + 3a_{n-2} = 2^n$

with $a_0 = 1, \ a_1 = 0$

1. To find homogeneous solution

$$a_{n+2} - 4a_{n+1} + 4a_n = 2^n \quad \text{——— (1)}$$

The characteristic equation $r^2 - 4r + 4 = 0$

$m_1 = 2, \quad m_2 = 2$

$a_n^{(h)} = (b_1 + b_2 n)\, 2^n$

~~To find P~~

To find particular solution $a_n^{(p)}$

$f(n) = 2^n$

Since 2 is the root of characteristic equation with multiplicity 2.

the trial solution $a_n^{(p)} = n^2 A_0 2^n$ ——— (i)

$$a_{n+1}^{(p)} = (n+1)^2 A_0 2^{n+1} \quad \text{——— (ii)}$$

$$a_{n+2}^{(p)} = (n+2)^2 A_0 2^{n+2} \quad \text{——— (iii)}$$

$A_0 (n+2)^2 2^{n+2} - 4A_0 (n+1)^2 2^{n+1} + 4A_0 n^2 2^n = 2^n$

$4A_0 (n^2 + 4n + 4) 2^n - 8A_0 (n^2 + 2n + 1) 2^n + 4A_0 n^2 2^n = 2^n$

comparing coefficients of 2^n on both sides

$16A_0 - 8A_0 = 1$

$A_0 = \frac{1}{8}$

$a_n^{(p)} = \frac{1}{8} n^2 2^n$

Therefore, general solution $a_n = a_n^{(h)} + a_n^{(p)}$

$a_n = (b_1 + b_2 n) 2^n + \frac{1}{2} 2^n n^2$

2. $a_n - 5a_{n-1} + 6a_{n-2} = 1$ ——— (1)

$a_0 = 0, \quad a_1 = 1$ ——— (2)

To find homogeneous solution

The characteristic equation is given as $r^2 - \cancel{5a}\, 5r + 6 = 0$

$m_1 = 2, \quad m_2 = 3$

$a_n^{(h)} = b_1 2^n + b_2 3^n$

where b_1 and b_2 are some arbitrary constants

To find particular solution

$f(n) = 1$ which is a constant polynomial

Therefore trial solution $a_n^{(p)} = A_0$ ——— (i)

$a_{n-1}^{(p)} = A_0$ ——— (ii)

$a_{n-2}^{(p)} = A_0$ ——— (iii)

substituting (i), (ii) and (iii) in equation (1) we get

$A_0 - 5A_0 + 6A_0 = 1$

$A_0 = \frac{1}{2}$

$a_n^{(p)} = \frac{1}{2}$

$a_n = a_n^{(h)} + a_n^{(p)}$

$= b_1 2^n + b_2 3^n + \frac{1}{2}$ ——— (3)

$a_0 = b_1 + b_2 + \frac{1}{2}$

$0 = b_1 + b_2 + \frac{1}{2}$

$b_1 + b_2 = -\frac{1}{2}$ ——— (iv)

$a_1 = 2b_1 + 3b_2 + \frac{1}{2}$

$2b_1 + 3b_2 = 1 - \frac{1}{2} \Rightarrow 2b_1 + 3b_2 = \frac{1}{2}$ ——— (v)

~~substituting~~ solving (iv) and (v)

$b_1 = -2, \quad b_2 = \frac{3}{2}$

$a_n = -2 \cdot 2^n + \frac{3}{2} \cdot 3^n + \frac{1}{2}$

$= -2^{n+1} + \frac{1}{2} 3^{n+1} + \frac{1}{2}$

3. $a_{n+2} + 2a_{n+1} - 15a_n = 6n + 10$ ——— (1)

$a_0 = 1, \quad a_1 = -\frac{1}{2}$ ——— (2)

To find homogeneous solution

The characteristic equation given as $r^2 + 2r - 15 = 0$

$m_1 = 3, \quad m_2 = -5$

$a_n^{(h)} = b_1 3^n + b_2 (-5)^n$

To find particular solution $a_n^{(p)}$

$f(n) = 6n + 10$

$a_n^{(p)} = A_0 + A_1 n$ ——— (i)

$a_{n+1}^{(p)} = A_0 + A_1(n+1)$ ——— (ii)

$a_{n+2}^{(p)} = A_0 + A_1(n+2)$ ——— (iii)

substituting (i), (ii) and (iii) in equation (1)

$A_0 + A_1(n+2) + 2A_0 + 2A_1(n+1) - 15A_0 - 15A_1 n = 6n + 10$

$(A_1 + 2A_1 - 15A_1) n + A_0 + 2A_1 + 2A_0 + 2A_1 - 15A_0 = 6n + 10$

comparing the coefficients

$\cancel{A_1} \; -12A_1 = 6$

$A_1 = -\frac{1}{2}$

$-12A_0 + 4A_1 = 10$

$-12A_0 - 2 = 10$

$A_0 = -1$

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$a_n^{(p)} = -1 - \frac{1}{2}n$

$a_n = a_n^{(h)} + a_n^{(p)}$

$= b_1 3^n + b_2 (-5)^n - 1 - \frac{1}{2}n$

$a_0 = b_1 + b_2 - 1 - \cancel{0}$

$1 = b_1 + b_2 - 1$

$b_1 + b_2 = 2$ ——— (iv)

$a_1 = 3b_1 - 5b_2 - 1 - \frac{1}{2}$

$\frac{3}{2} - \frac{1}{2} = 3b_1 - 5b_2$

$3b_1 - 5b_2 = 1$ ——— (v)

solving (iv) and (v) we get

$b_1 = \frac{11}{8}$

$b_2 = \frac{5}{8}$

$a_n = \frac{11}{8} 3^n + \frac{5}{8} (-5)^n - 1 - \frac{1}{2}n$

4. $a_n - 4a_{n-1} + 3a_{n-2} = 2^n$ ——— ①

$a_0 = 1, \quad a_1 = 0$ ——— ②

To find homogeneous solution

The characteristic equation given as $r^2 - 4r + 3 = 0$

$m_1 = 1, \quad m_2 = 3$

$a_n^{(h)} = b_1 (1)^n + b_2 3^n$

To find particular solution $a_n^{(p)}$

$f(n) = 2^n$

$a_n^{(p)} = A_0 2^n$ ——— (i)

$$a_{n-1}^{(p)} = A_0 2^{n-1} \text{ ——— (II)}$$

$$a_{n-2}^{(p)} = A_0 2^{n-2} \text{ ——— (III)}$$

substituting (I), (II) and (III) in equation (I)

$$A_0 2^n - 4A_0 2^{n-1} + 3A_0 2^{n-2} = 2^n$$

$$A_0 2^n - 2A_0 2^n + \frac{3}{4} A_0 2^n = 2^n$$

comparing the coefficient of 2^n

$$A_0 - 2A_0 + \frac{3}{4} A_0 = 1$$

$$-\frac{1}{4} A_0 = 1$$

$$A_0 = -4$$

$$a_n^{(p)} = -4 \cdot 2^n$$

$$= -2^{n+2}$$

$$a_n = a_n^{(h)} + a_n^{(p)}$$

$$= b_1 + b_2 3^n - 2^{n+2}$$

$$a_0 = b_1 + b_2 - 4$$

$$1 = b_1 + b_2 - 4$$

$$b_1 + b_2 = 5 \text{ ——— (IV)}$$

$$a_1 = b_1 + 3b_2 - 8$$

$$0 = b_1 + 3b_2 - 8$$

$$b_1 + 3b_2 = 8 \text{ ——— (V)}$$

solving (IV) and (V) we get

$$b_1 = \frac{7}{2}$$

$$b_2 = \frac{3}{2}$$

$$a_n = \frac{7}{2} + \frac{3}{2} \cdot 3^n - 2^{n+2}$$

$$= \frac{7}{2} + \frac{3^{n+1}}{2} - 2^{n+2}$$

.3.2019

UNIT 4

GRAPH THEORY

Simple Graph

A simple graph $G = (V, E)$

V is called set of vertices and E is a set containing two element subsets of distinct elements of V and it is called set of edges.

Multigraph

A multigraph G consists of set of vertices and set of edges $G = (V, E)$ and

$f : E - \{ \{u, v\}, u \neq v , u, v \in V \}$

Two edges in a multigraph are ~~set of~~ said to be parallel edges $f(e_1) = f(e_2)$

Pseudograph

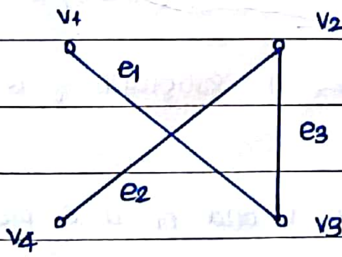
A graph $G = (V, E)$ is said to be pseudo if it consists set of vertices and set of edges and $f : E - \{ \{u, v\}, u, v \in V \}$

An edge is said to be loop if $f(e) = \{u, u\}$

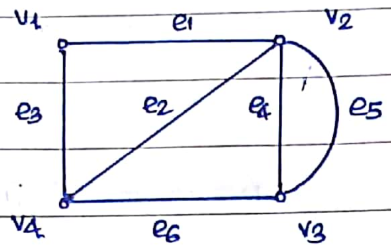
Representation of a graph on a plane

A graph $G = (V, E)$ can be represented on a plane by denoting its vertices as dots or small circles on a plane and edges by straight lines or curves joining two vertices.

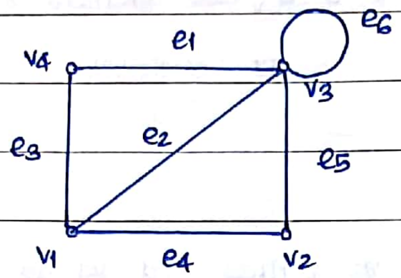
Example :



simple graph



Multigraph



Pseudograph

Finite and infinite Graphs

A graph $G = (V, E)$ is said to be finite graph if it contains finite number of vertices otherwise it is said to be an inf

Size and order of a graph

A graph $G = (V, E)$ is said to be of order p and size q if it has p number of vertizes and q number of edges.

Subgraph

Let $G = (V, E)$ be a graph.

A graph $H = (V_1, E_1)$ is said to be a subgraph of G if V_1 is a subset of V and E_1 is a subset of E.

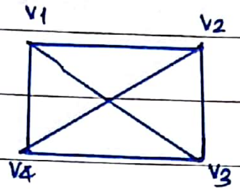
Note: If V_1 is a proper subset of V and E_1 is a proper subset of E then H is called a proper subgraph of G.

Spanning subgraph

Let $G = (V, E)$ be a graph, a graph ~~H is equal to~~ $H = (V_1, E_1)$ is said to be a spanning subgraph of G if and only if $V_1 = V$

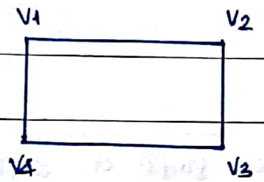
Example:

Let G :



Then the subgraph H given below is a spanning subgraph of G

H :

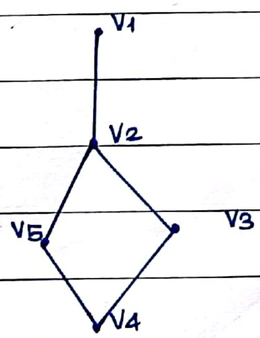


Induced subgraph

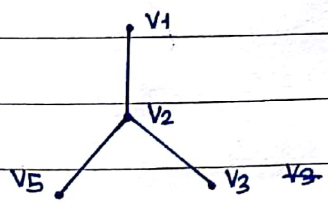
A subgraph $H = (V_1, E_1)$ of a graph $G = (V, E)$ is called induced subgraph of G if whenever $(u, v) \in V(H)$ and $uv \in E(G)$ then $uv \in E(H)$

For example consider the graph G

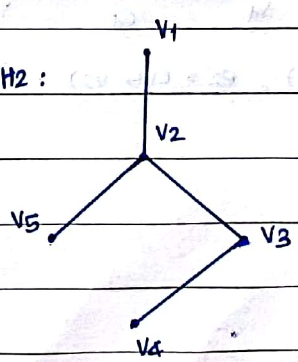
G :



H_1:



H2 :



H_1 is an induced subgraph.

H_2 is not an induced subgraph because $v_2, v_4 \in V(H_2)$ and $v_2 v_4 \in E(G)$ but $v_2 v_4 \notin$ ~~E(G)~~ $E(H_2)$. Hence H_2 is a subgraph of G but not an induced subgraph.

Induced subgraph

Let G be a graph $G = (V, E)$

Let S be a non-empty subset of V then the induced subgraph generated by set S is denoted by notation $G[S]$

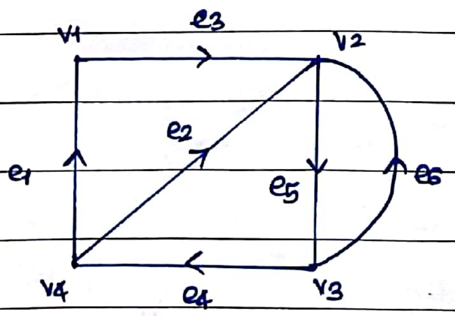
Directed Graph or Digraph

A graph $G = (V, E)$ is said to be directed graph or digraph if each edge of G is associated with an ordered pair (u, v) where $u, v \in V(G)$

In other words we say that edge e is directed from vertex u to v that is if $e = (u, v)$ then

- u is called the initial vertex of e
- v is called terminal or final vertex of e
- and ~~we say~~ u is said to be adjacent to v ~~and~~
- and v is said to be adjacent to u

For example



$e_1 = (v_4, v_1)$, $e_2 = (v_1, v_2)$