

0/1 Knapsack Problem (Dynamic Programming)

① $n = 3$, $m = 6$ $(w_1, w_2, w_3) = (2, 3, 4)$
 $(p_1, p_2, p_3) = (1, 2, 5)$

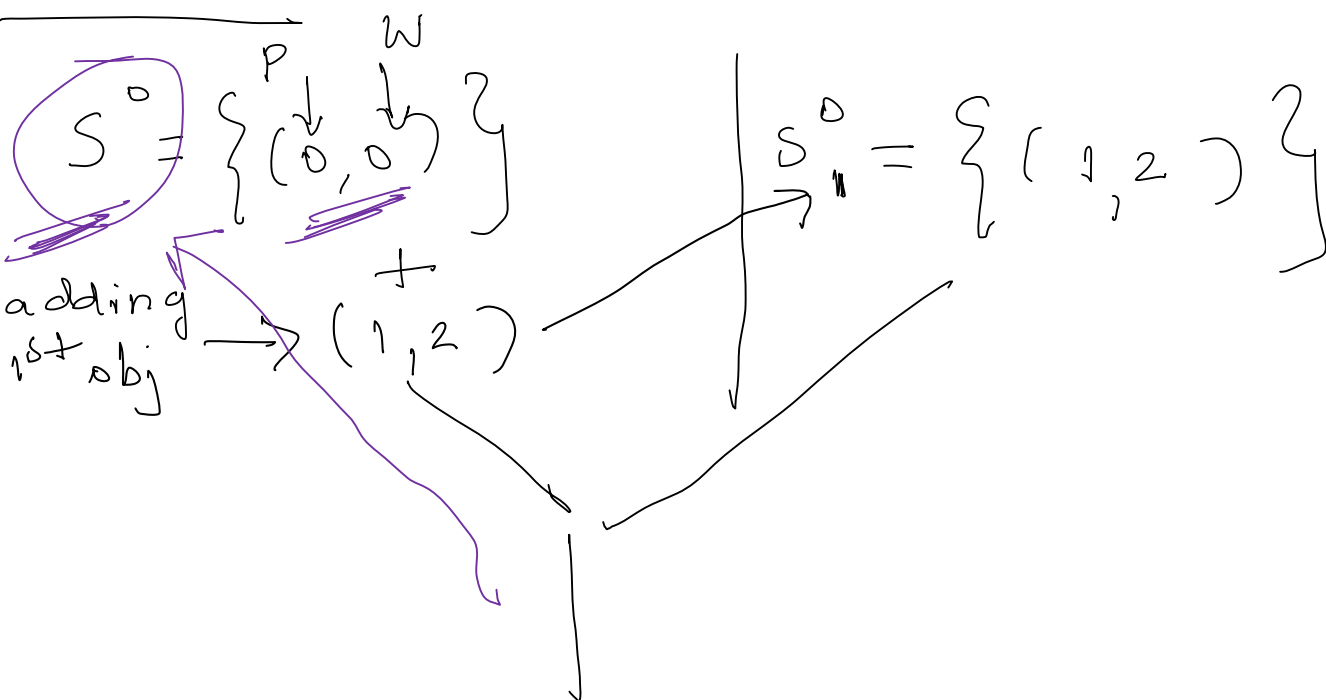
Solution: optimal solⁿ $\Rightarrow (x_1, x_2, x_3)$

Answer $\rightarrow (x_1, x_2, x_3) = (1, 0, 1)$

$\checkmark P = 6$, $\checkmark W = 6$

1	1	0	- 3/5
1	0	1	- 6/6
0	1	1	- 7/7
1	1	1	- 8/9

State 0: tuple $\Rightarrow (P, w)$



merge

State 1

$$S^1 = \{(0,0), (1,2)\}$$

1st obj (a_1) — 1

$$S^1_1 = \{(2,3), (3,5)\}$$

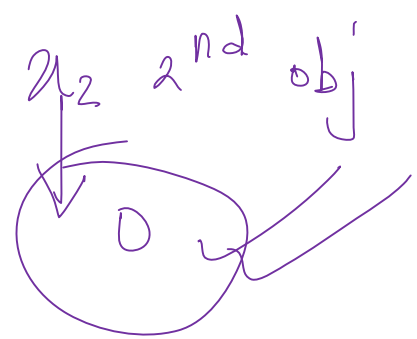
adding
2nd obj

$$\rightarrow (2,3)$$

State 2

merged

$$S^2 = \{(0,0), (1,2), (2,3), (3,5)\}$$

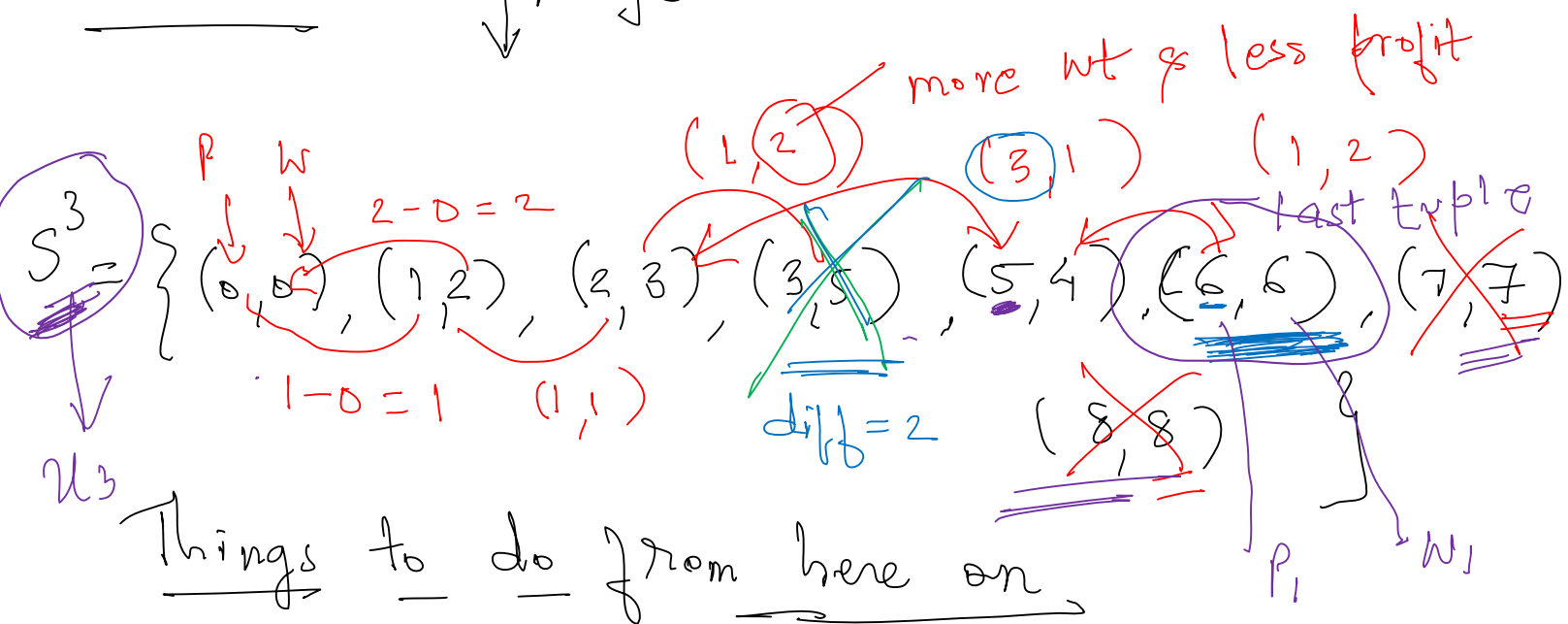


adding
3rd obj

$$\rightarrow (5,4)$$

$$S^2_1 = \{(5,4), (6,6), (7,7), (8,9)\}$$

State 3 $\xrightarrow{\text{merge}}$ u_3 object



① Discard the tuples that has $\boxed{\text{wt} > m}$

② Apply the purging rule (ie discard the tuple giving more wt & less profit)

ie (p_j, w_j) & (p_k, w_k) then
 if $p_j \leq p_k$ & $w_j \geq w_k$ (p_j, w_j) is discarded & decide accordingly

③ Consider the objects from u_n to include into bag (ie the last tuple)

④ If the last tuple from S^i belongs to S^{i-1} then the entry is Zero

otherwise the entry is One ✓

{ i.e. we doing the choice of objects to be considered }

Note :- if last tuple is not there in S^{i-1} , then find from which tuple it got generated

Final solⁿ

vector $\Rightarrow (x_1, x_2, x_3)$

(1 , 0 , 1)

Since ↙

(1,2) was
not there
in S^0

↓

(1,2)
belonging
in S^1

↓

(6,6) not in
 S^2

② $n=3$, $m=4$ $(w_1, w_2, w_3) = (1, 2, 2)$
 $(p_1, p_2, p_3) = (18, 16, 16)$

Soln $\Rightarrow (x_1, x_2, x_3)$

both are feasible

	x_1	x_2	x_3	P	w
$\rightarrow \checkmark$	1	0	0	18	3
$\rightarrow \checkmark$	0	1	1	32	4
	1	1	1	X	

\therefore Consider

the Combination

which gives max-profit

$(34, 3)$, $(32, 4)$
 \downarrow
 final tuple

$$S^0 = \{ (0,0) \} + (18,1)$$

$$S_1^0 = \{ (18,1) \}$$

$$S^1 = \{ (0,0), (18,1) \} + (16,2)$$

\downarrow
 $\pi_1 \rightarrow 1$

$$S_1^1 = \{ (16,2), (34,3) \}$$

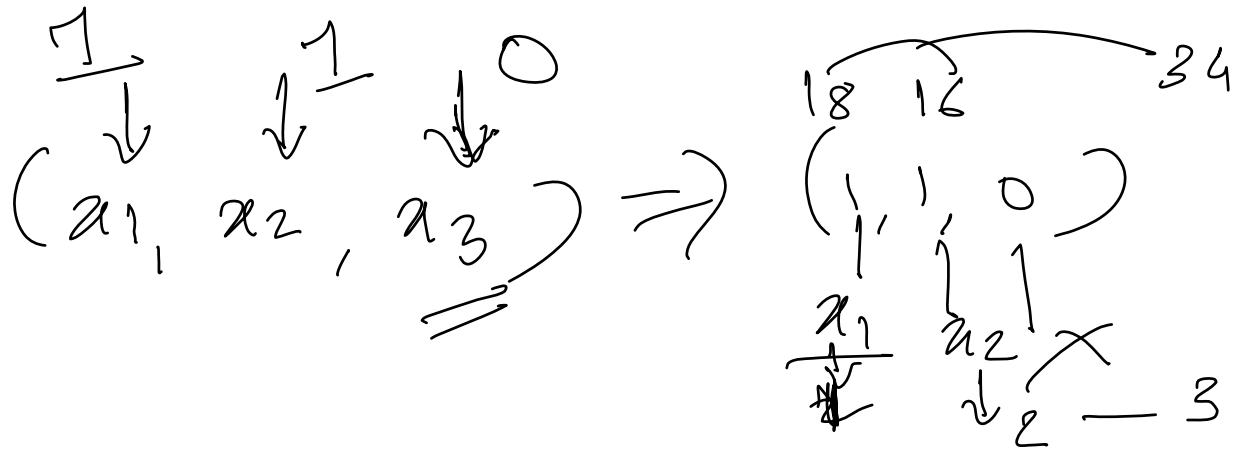
$$S^2 = \{ (0,0), (18,1), (16,2), (34,3) \} + (16,2)$$

\downarrow
 $\pi_2 \rightarrow 1$

$$S_1^2 = \{ (16,2), (34,3), (32,4), (50,5) \}$$

$$S^3 = \{ (0,0), (18,1), (16,2), (34,3), (16,2), (34,3), (32,4), (50,5), (34,3) \}$$

\downarrow
 π_3



Purging rule $\Rightarrow (p_j, w_j) \& (p_k, w_k)$

if $(p_j \leq p_k) \& (w_j \geq w_k)$

then (p_j, w_j) gets discarded

