

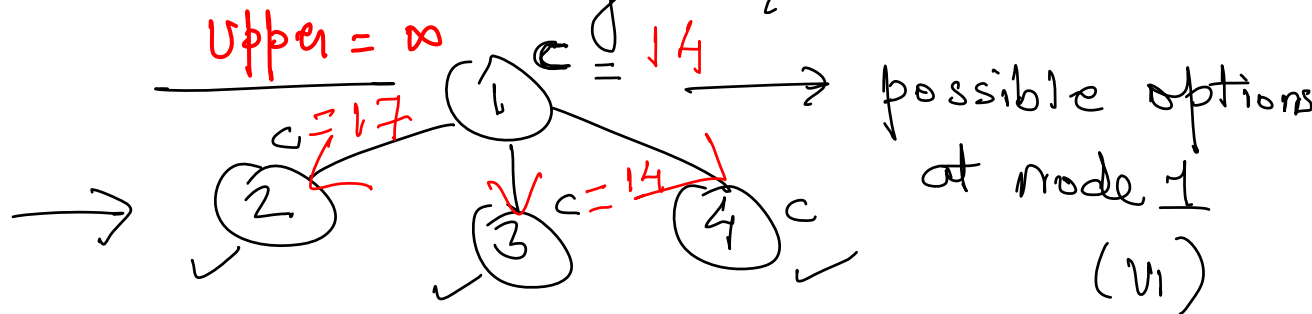
# \* TSP Using Branch & Bound

① Initial Cost Matrix  $\Rightarrow$   $R_1$   $\begin{bmatrix} \infty & 5 & 2 & 3 \end{bmatrix}$   $\begin{matrix} 2 \\ 2 \\ 2 \\ 6 \end{matrix}$   
 (A vertex Graph)

	1	2	3	4	
$R_1$	$\infty$	5	2	3	2
$R_2$	4	$\infty$	2	3	2
$R_3$	4	2	$\infty$	3	2
$R_4$	7	6	8	$\infty$	6

Find the TSP-tree using B & B

Solution



TSP Starts at node 1

Step 1 : Do Row Reduction

$R_1$  reduced 2 (min in row)  
 $R_2$  " 2 "  
 $R_3$  " 2 "  
 $R_4$  " 6 "

	$C_1$		$C_4$
$A =$	$\begin{bmatrix} \infty & 3 & 0 & 1 \\ 2 & \infty & 0 & 1 \\ 2 & 0 & \infty & 1 \\ 1 & 0 & 2 & 8 \end{bmatrix}$		
	$\downarrow$	$\downarrow$	$\downarrow$
	1	0	0

Step 2 : Do Column Reduction

$C_1$  reduced by 1  
 $C_4$  reduced by 1

~~minimize~~

$A =$

	1	2	3	4
1	$\infty$	3	0	0
2	1	$\infty$	0	0
3	1	0	$\infty$	0
4	0	0	2	$\infty$

$\downarrow$  1                       $\downarrow$  1

Node 1 matrix/  
 $\text{Cost}(1)$   
 Reduced matrix  
 at node 1  
 (to be used later)

$\therefore$  Cost of node 1 (RCL) =  $\checkmark 2 + \checkmark 2 + \checkmark 2 + \checkmark 6 + \checkmark 1 + \checkmark 1$

(Cost of Reduction / Reduced cost) = 14

\* TSP Choose to go to Vertex 2 (ie Node 2)

$\therefore$  edge  $\langle \overset{i}{1}, \overset{j}{2} \rangle$  is selected

$\checkmark$  - set row 1 & column 2 to  $\infty$

$\checkmark$  - set  $A[2,1]$  entry in matrix as  $\infty$

∴  $A =$

	1	2	3	4
1	0	∞	∞	∞
2	∞	0	0	0
3	1	∞	∞	0
4	0	∞	2	∞

$A[2,1] = \infty$   
 $R_1$  as  $\infty$   
 $C_2$  as  $\infty$   
 min

— perform row reduction & column reduction  
 (min in all rows is 0 & min in all col is 0)

∴  $RCL = 0$  at node 2

∴  $\text{cost}(2) = \text{cost}(1) + A(1,2) + RCL$   
 $= 14 + 3 + 0$   
 $\text{cost}(2) = 17$

refer from node 1 matrix  
 cost of reduction

\* TSP choose to go to vertex 3 (ie Node 3)

- edge selected is  $\langle 1, 3 \rangle$
- set row 1 & column 3 of node 1 matrix to  $\infty$
- set  $A[3,1]$  to  $\infty$



— set row 1 & column 4 of node 1 matrix as

— set  $A[4, 1]$  as  $\infty$

∴ ∴

$$A = \begin{matrix} & \begin{matrix} c_1 & c_2 & c_3 & c_4 \end{matrix} \\ \begin{matrix} r_1 \\ r_2 \\ r_3 \\ r_4 \end{matrix} & \begin{bmatrix} \infty & \infty & \infty & \infty \\ 1 & \infty & 0 & \infty \\ 1 & 0 & \infty & \infty \\ \infty & 0 & 2 & \infty \end{bmatrix} \end{matrix}$$

$A[4, 1] \rightarrow \infty$

$R_1 \rightarrow \infty$

$C_4 \rightarrow \infty$

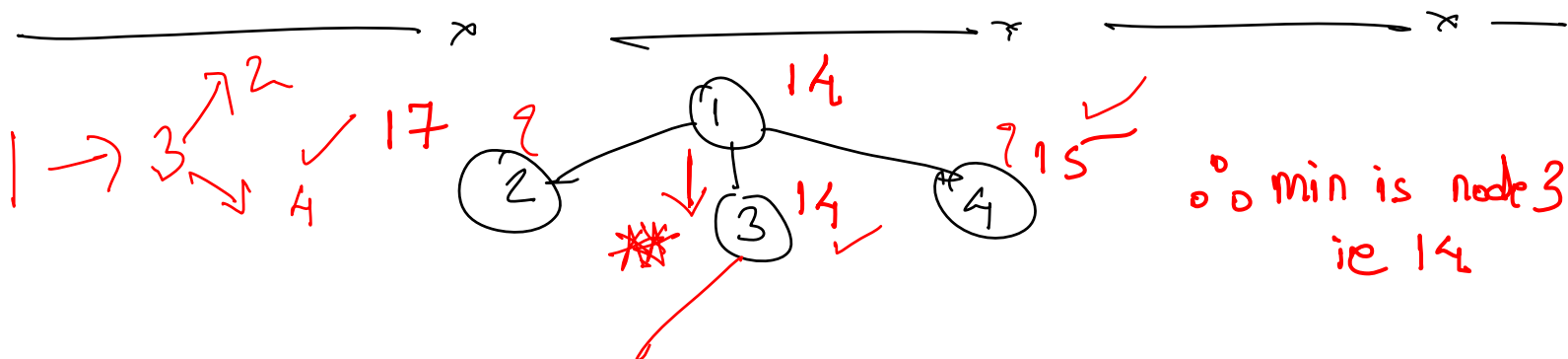
min 1

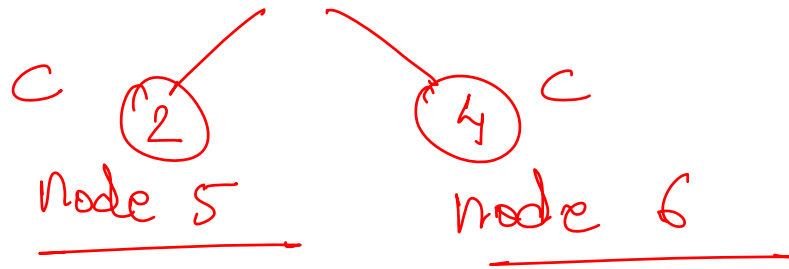
∴ ∴  $RCL = 1$

\*  $Cost(4) = cost(1) + A[1, 4] + RCL$

$= 14 + 0 + 1$

$Cost(4) = 15$





So traverse further from node 3

Note : Start with the cost matrix at node 3 as initial matrix to traverse to node 5 & node 6

$A =$

	1	2	3	4
1	0	$\infty$	$\infty$	$\infty$
2	<span style="border: 1px solid black;">1</span>	$\infty$	$\infty$	0
3	$\infty$	0	$\infty$	0
4	0	0	$\infty$	$\infty$

initial vertex 3 matrix

\* TSP choose to go to vertex 2 (ie Node 5)

1  $\rightarrow$  3  $\rightarrow$  2 ✓

$\therefore$  edge selected is  $\langle 3, 2 \rangle$  ✓

— set row 3 & col 2 of A matrix (blue color) as  $\infty$

— set  $A[2, 1]$  as  $\infty$  (becoz from vertex 2 we should not goto 1)  
 & not  $A[2, 3]$

$\therefore A = \begin{bmatrix} \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & 0 \\ \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & \infty \end{bmatrix}$

$\downarrow$  min 0       $\downarrow$  min 0

$A[2,1] = \infty$

$C_2$  as 0

$R_3$  as 0

Since matrix is already reduced  $RCL = 0$

$\therefore \text{Cost}(5) = \text{Cost}(3) + A[3,2] + RCL$

(ie vertex 2)

$= 14 + 0 + 0$

refer from node 3 matrix

$\text{Cost}(5) = 14$

\* TSP choose to visit to vertex 4 (ie node 6)

$1 \rightarrow 3 \rightarrow 4$

$\therefore$  edge selected is  $\langle 3, 4 \rangle$

- set row 3 & column 4 of A matrix as  $\infty$  ✓

- set  $A[4, 1] = \infty$

Cost(3) matrix

$$A = \begin{bmatrix} \infty & \infty & \infty & \infty \\ \textcircled{1} & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty \\ \infty & 0 & \infty & \infty \end{bmatrix}$$

$C4 \text{ as } \infty$

min as  $\textcircled{1}$  (so subtract)

$R3 \text{ as } \infty$

$A[4, 1] = \infty$  →

$$\begin{bmatrix} \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty \\ \boxed{\infty} & 0 & \infty & \infty \end{bmatrix}$$

min 0

$$\therefore A = \begin{bmatrix} \infty & \infty & \infty & \infty \\ \textcircled{0} & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty \end{bmatrix}$$

✓ min 0 ✓

↓ at node 6

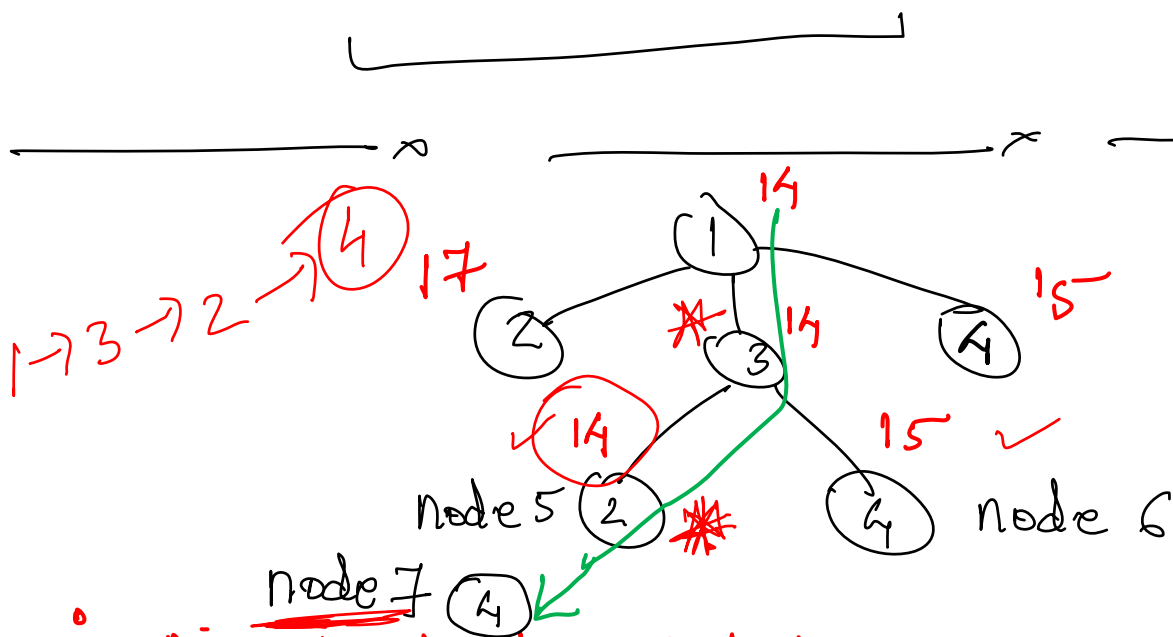
$$\therefore \boxed{RCL = 1}$$

$$\therefore \text{Cost}(6) = \text{Cost}(3) + A[3, 4] + RCL$$

(Vertex ~~4~~) =  $14 + \textcircled{0} + 1$  ✓

$$\boxed{\text{Cost}(6) = 15}$$

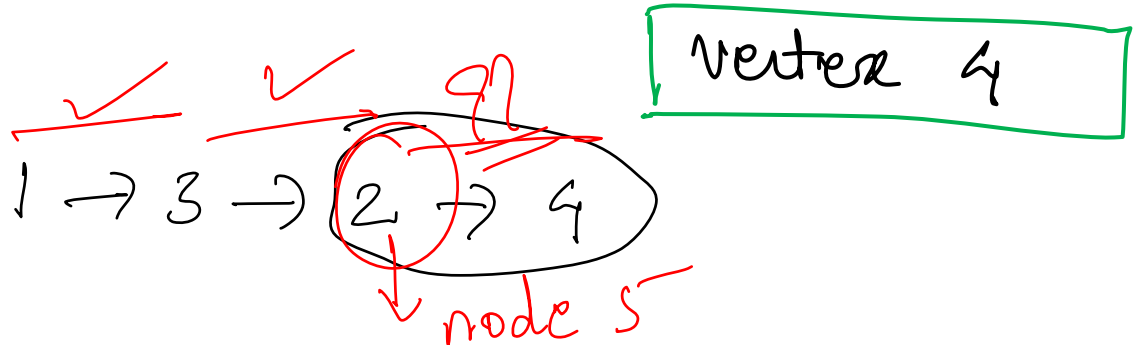




∴ min is vertex 2 path ie with cost 14

∴ vertices to explore from node 5 (ie vertex 2) is 4

— Also Consider the matrix at node 5 (ie vertex 2) as initial matrix to traverse to



$\therefore A = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} \infty & \infty & \infty & \infty \\ 1 & \infty & \infty & 0 \\ \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & \infty \end{bmatrix} \end{matrix}$

cost matrix of node 5

✓ path selected is  $\langle 2, 4 \rangle$

✓ set row 2 & col 4 of A matrix as  $\infty$

✓ set  $A[4, 1]$  as  $\infty$

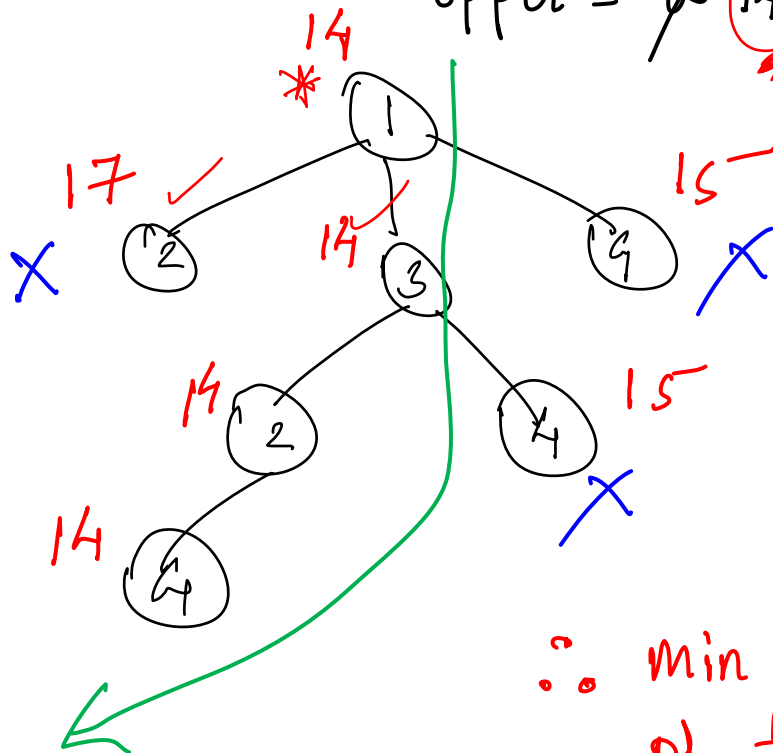
$A = \begin{bmatrix} \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty \end{bmatrix}$

$A[4, 1] \rightarrow \infty$   
 $RCL = 0$

$\therefore \text{cost}(7) = \text{cost}(5) + A[2, 4] + RCL$   
 $\text{ie (vertex 4)} = 14 + 0 + 0$   
 $\boxed{\text{cost}(7) = 14}$

∴ Tour  $\Rightarrow 1 \rightarrow 3 \rightarrow 2 \rightarrow 4 \rightarrow 1$

upper = ~~15~~ 14 (∵ all nodes with cost greater than 14 will get deleted)



∴ min cost of tour = 14

② Try for the following matrix

$$A = \begin{matrix} & \begin{matrix} \underline{1} & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} \infty & 20 & 30 & 10 & 11 \\ 15 & \infty & 16 & 4 & 2 \\ 3 & 5 & \infty & 2 & 4 \\ 19 & 6 & 18 & \infty & 3 \\ 16 & 4 & 7 & 16 & \infty \end{bmatrix} \end{matrix}$$

Soln      Min cost = 28

tour  $\Rightarrow 1 \rightarrow 4 \rightarrow 2 \rightarrow 5 \rightarrow 3 \rightarrow 1$













