S.E. (Computer Engineering) (Semester – IV) (RC 2016 – 17) Examination, November/December 2018
DISCRETE MATHEMATICS

Duration : 3 Hours   Max. Marks : 100

Instructions : 1) *Attempt **any five** questions, **any two** questions **each** from Part – **A** and Part – **B** and **one** from Part – **C**.*
2) *Assume suitable data, **if** necessary.*
3) *Figures to the **right** indicate full marks.*

PART – A

Answer **any two** questions from the following : **(2×20=40)**

1. a) Let A, B and C be any three non empty sets. Show that
$A \cap (B \oplus C) = (A \cap B) \oplus (A \cap C)$. **4**

b) Without using Truth tables, prove that
$(\sim p \to (\sim p \to (\sim p \wedge q))) \equiv p \vee q$. **5**

c) Let $f : R \to R$, determine if the function $f(x) = \begin{cases} x+4; & x<1 \\ x^2+4; & x \geq 1 \end{cases}$ is bijective. If it is bijective, find its inverse. **6**

d) A relation R is defined on the set of integers as xRy if (x + y) is even. Show that R is an equivalence relation on Z having two equivalence classes. **5**

2. a) Draw the Hasse diagram representing the partial ordering R on the set A = {1, 2, 3, 4, 6, 8, 12} given by aRb if 'a divides b'. Find the maximal elements and minimal elements of the above POSET. Also find the upper bounds, lower bounds, supremum and infimum of the subset B = {2, 6, 8, 12}. **6**

b) Without actually carrying out multiplication, find the remainder when the integer $[9 \times 85 \times 89\ (67)^2 \times 539 \times (1269)^3]$ is divided by 16. **6**

c) Use mathematical induction to prove that for all positive integers n, **6**
$$\frac{1^2}{1.3}+\frac{2^2}{3.5}+\frac{3^3}{5.7}+\ldots+\frac{n^2}{(2n-1)(2n+1)}=\frac{n(n+1)}{2(2n+1)}.$$

d) Give an example of a function which is one-one but not onto. Justify your answer. **2**

3. a) Using Euclidean Algorithm, find the greatest common divisor of 785 and 1210 and express in the form 785x + 1210y where x and y are integers. **6**

b) Determine the validity of the following argument. **6**

"My father praises me only if I can be proud of myself. Either I do well in sports or I can't be proud of myself. If I study hard, then I can't do well in sports. Therefore, if father praises me, then I do not study hard."

c) Without using truth tables, obtain the principal disjunctive normal form of $\sim (p \to (q \wedge r))$. **6**

d) Let B be a Boolean Algebra. Without using truth tables, prove that **2**

$a + (a \cdot b) = a \;\; \forall\, a, b \in B$

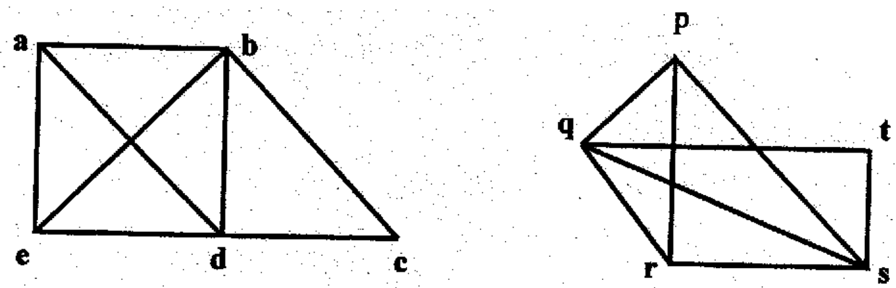
PART – B

Answer **any two** questions from the following : **(2×20=40)**

4. a) In how many ways can 10 oranges be distributed among 4 children so that each child gets at least one orange ? **5**

b) Define graph isomorphism. **5**

Check whether the following graphs are Isomorphic or not.



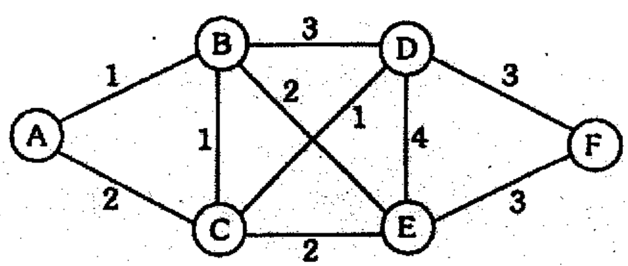
c) Find the recurrence relation for the number of ways of climbing n steps if a person can climb one or two or three steps of a time. Also give the initial conditions. **5**

d) Find the coefficient of x^3y^2 in the expansion of $(3x + y)^5$. **5**

5. a) Solve the recurrence relation $a_n - 2a_{n-1} + a_{n-2} = 7$ with $a_0 = 1$ and $a_1 = 2$. **8**

b) Show that graph K_5 is non planar. Also find its chromatic number. **4**

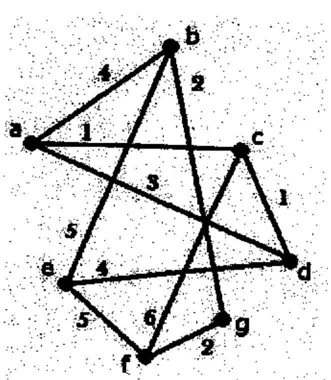
c) Apply Dijkstra's algorithm to find the shortest path between A and F in the following weighted graph. **6**



d) Give an example of an undirected graph with degree sequence 1, 3, 3, 4, 5, 6. **2**

6. a) Show that a tree with n vertices has n – 1 edges. **6**

b) Using Prim's Algorithm, find the minimum spanning tree for the weighted graph given below. **5**



c) Find the number of positive integers not exceeding 400 which are **7**

i) divisible by 5 or 3 or 7.

ii) divisible by 3 not by 5 nor by 7.

d) Give an example of a graph which is bipartite but not complete bipartite. Justify your answer. **2**

PART – C

Answer **any one** question from the following : **(1×20=20)**

7. a) Let $f : A \to B$ and $g : B \to C$ be two bijective functions. Then prove that $g \circ f : A \to C$ is also bijective. **6**

   b) Show that 103 is a prime number. **4**

   c) Using Venn diagram, show that $A - (B \cup C) = (A - B) \cap (A - C)$. **5**

   d) Define functionally complete set of connectives. **5**

   Show that the NAND operator $\{\uparrow\}$ is functionally complete.

8. a) State pigeonhole principle.

   If 5 points are randomly chosen in a square of side 2 units, using pigeonhole principle, show that atleast two of them are no more than $\sqrt{2}$ units apart. **6**

   b) Explain briefly Konigsberg's Bridge problem and draw a graph representing the problem. **5**

   c) Prove the Pascal's identity ${}^{n}C_{r} + {}^{n}C_{r-1} = {}^{n+1}C_{r}$. **5**

   d) A tree has two vertices of degree 2, one vertex of degree 3 and there vertices of degree 4. How many vertices of degree 1 does it have ? **4**
