0/1 Knapsack Problem (Dynamic Programming)

① $n = 3$, $m = 6$ $(w_1, w_2, w_3) = (2, 3, 4)$

$(P_1, P_2, P_3) = (1, 2, 5)$

Solution: optimal soln => (x_1, x_2, x_3)

$$\begin{array}{cccc} x_1 & x_2 & x_3 & \\ 1 & 1 & 0 & - 3 \mid 5 \\ 1 & 0 & 1 & - 6 \mid 6 \\ \cancel{0} & \cancel{1} & \cancel{1} & \cancel{- 7 \mid 7} \\ \cancel{1} & \cancel{1} & \cancel{1} & \cancel{- 8 \mid 9} \end{array}$$

Answer → $(1, 0, 1)$

$P = 6$, $W = 6$

State 0: tuple => (P, W)

$S^0 = \{(0, 0)\}$ (P, W)

adding 1st obj → + $(1, 2)$

$S^0_1 = \{(1, 2)\}$

State 1

merge

$S^1 = \{(0,0),(1,2)\} \Rightarrow S^1_1 = \{(2,3),(3,5)\}$

1st obj (x_1) — 1

adding 2nd obj $\rightarrow$ + $(2,3)$

p w

State 2

merged

$S^2 = \{(0,0),(1,2),(2,3),(3,5)\}$

p_1 w_1

x_2 2nd obj

0

adding 3rd obj $\rightarrow$ + $(5,4)$

$S^2_1 = \{(5,4),(6,6),(7,7),(8,9)\}$

State 3 — merge — x_3 object

more wt & less profit

$(1, 2)$ $(3, 1)$ $(1, 2)$

P W $2-0=2$ last tuple

$S^3 = \{(0,0), (1,2), (2,3), \cancel{(3,5)}, (5,4), (6,6), \cancel{(7,7)}$ $\cancel{(8,8)}\}$

$1-0=1$ $(1,1)$ diff $=2$

x_3 P_1 W_1

Things to do from here on

1. Discard the tuples that has $\boxed{wt > m}$

2. Apply the purging rule (ie discard the tuple giving more wt & less profit)
   ie (P_j, W_j) & (P_k, W_k), if $P_j \le P_k$ & $W_j \ge W_k$ then (P_j, W_j) is discarded & decide accordingly S^3

3. Consider the objects from x_n to include into bag (ie the last tuple)

(1) If the last tuple from S^i belongs to S^{i-1} then the entry is $\boxed{\text{zero}}$

otherwise the entry is $\boxed{\text{One}}$ ✓

{ ie we doing the choice of objects to be considered }

Note :- if last tuple is not there in S^{i-1}, then find from which tuple it got ~~generated~~

Final Soln vector $\Rightarrow (x_1, x_2, x_3)$

$(1, 0, 1)$

Since (1,2) was not there in S^0

(1,2) belonging in S^1

(6,6) not in S^2

(2) $n = 3$, $m = 4$ $(w_1, w_2, w_3) = (1, 2, 2)$

$(P_1, P_2, P_3) = (18, 16, 16)$

Sol^n ⇒ (x_1, x_2, x_3)

x_1	x_2	x_3		P	W
1	1	0	→	34	3
0	1	1	→	32	4
1	1	1	→	×	

both are feasible

∴ Consider the Combination which gives max. profit

(34, 3) , (32, 4)

final tuple

$S^0 = \{(0,0)\}$

$+ (18,1)$

$S^0_1 = \{(18,1)\}$

(P_1, W_1)

$S^1 = \{(0,0), (18,1)\}$

$x_1 \to 1$ $+ (16,2)$

$S^1_1 = \{(16,2), (34,3)\}$

$(P_1 \quad W_1)$

$S^2 = \{(0,0), (18,1), (16,2), (34,3)\}$

$x_2 \to 1$ $+ (16,2)$

$S^2_1 = \{(16,2), (34,3), (32,4), (50,5)\}$

(P_1, W_1)

$S^3 = \{(0,0), (18,1), (16,2), (34,3), (16,2), (34,3), (32,4), \cancel{(50,5)}\}$

x_3

1 1 0

$$(x_1, x_2, x_3) \Rightarrow (1, 1, 0)$$

18 16 34

x_1 x_2

1 2 3

Purging rule $\Rightarrow$ (p_j, w_j) & (p_k, w_k)

if $(p_j \le p_k)$ & $(w_j \ge w_k)$

then (p_j, w_j) gets discarded