ASSIGNMENT 4

Q1] Solve the following TSP using Branch and Bound Technique

$$A = \begin{bmatrix} \infty & 20 & 30 & 10 & 11 \\ 15 & \infty & 16 & 4 & 2 \\ 3 & 5 & \infty & 2 & 4 \\ 19 & 6 & 18 & \infty & 3 \\ 16 & 4 & 7 & 16 & \infty \end{bmatrix}$$

Step 1 Reduce the rows:

Reduce R_1 by 10

Reduce R_2 by 2

Reduce R_3 by 2

Reduce R_4 by 3

Reduce R_5 by 4

$$A = \begin{bmatrix} \infty & 10 & 20 & 0 & 1 \\ 13 & \infty & 14 & 2 & 0 \\ 1 & 3 & \infty & 0 & 2 \\ 16 & 3 & 15 & \infty & 0 \\ 12 & 0 & 3 & 12 & \infty \end{bmatrix}$$

<u>Step 2</u> Reduce the columns:

Reduce C_1 by 1

Reduce C_2 by 0

Reduce C_3 by 3

Reduce C_4 by 0

Reduce C_5 by 0

$$A = \begin{bmatrix} \infty & 10 & 17 & 0 & 1 \\ 12 & \infty & 11 & 2 & 0 \\ 0 & 3 & \infty & 0 & 2 \\ 15 & 3 & 12 & \infty & 0 \\ 11 & 0 & 0 & 12 & \infty \end{bmatrix}$$

cost of reduction $= 10+2+2+3+4+1+3$

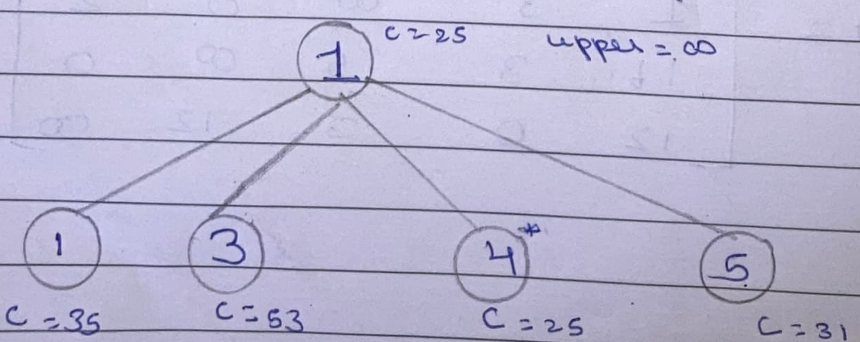
$= 14+7+4$

$= 18+7$

$= 25$

∴ RCL cost of note 1 is equal to 25

$\boxed{\therefore \text{cost}(1) = 25}$



* TSP chose to go to vertex 2 (Node 2) $(i, j) \Rightarrow (1, 2)$

$\therefore$ set row 1 and col 2 to ∞

$\therefore$ Set $A[2,2]$ to ∞

$$\therefore A = \begin{array}{c} \left[\begin{array}{ccccc} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 11 & 2 & 0 \\ 0 & \infty & \infty & 0 & 2 \\ 15 & \infty & 12 & \infty & 0 \\ 11 & \infty & 0 & 12 & \infty \end{array}\right] \begin{array}{c} \\ -0 \\ -0 \\ -0 \\ -0 \end{array} \\ \begin{array}{ccccc} 0 & & 0 & 0 & 0 \end{array} \end{array}$$

Colums and rows are already reduced

$\therefore$ RCL = 0 at node 2

$$\text{Cost}(2) = \text{cost}(1) + A[1,2] + RCL$$
$$= 25 + 10 + 0$$
$$= \underline{\underline{35}}$$

$\boxed{\therefore \text{Cost}(2) = 35}$

* TSP chose to go to node 3 (vertex 3)

$\therefore (i, j \Rightarrow (1, 3)$

$\therefore$ set row 1 and column 3 to ∞

$\therefore$ set $A[3,1]$ to ∞

$$A = \begin{array}{c} \left[\begin{array}{ccccc} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & \infty & 2 & 0 \\ \infty & 3 & \infty & 0 & 2 \\ 15 & 3 & \infty & \infty & 0 \\ 11 & 0 & \infty & 12 & \infty \end{array}\right] \begin{array}{c} \\ -0 \\ -0 \\ -0 \\ -0 \end{array} \\ \begin{array}{ccccc} 11 & 0 & & 0 & 0 \end{array} \end{array}$$

reduced matrix A = $\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 1 & \infty & \infty & 2 & 0 \\ \infty & 3 & \infty & 0 & 2 \\ 4 & 3 & \infty & \infty & 0 \\ 0 & 0 & \infty & 12 & \infty \end{bmatrix}$

RCL = 11

$$\begin{aligned} \text{Cost}(3) &= \text{cost}(1) + A[1,3] + RCL \\ &= 25 + 17 + 11 \\ &= 25 + 28 \\ &= 53 \end{aligned}$$

$\boxed{\therefore \text{Cost}(3) = 53}$

* TSP chose to go to vertex 4 (node 4)

∴ (i, j) = (1, 4)

∴ set row 1 and column 4 to ∞

∴ set A[4, 1] to ∞

$$A = \begin{array}{c} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & 11 & \infty & 0 \\ 0 & 3 & \infty & \infty & 2 \\ \infty & 3 & 12 & \infty & 0 \\ 11 & 0 & 0 & \infty & \infty \end{bmatrix} \begin{matrix} \\ -0 \\ -0 \\ -0 \\ -0 \end{matrix} \\ \begin{matrix} 0 & 0 & 0 & & 0 \end{matrix} \end{array}$$

RCL = 0

- as matrix in reduced form already

$$\begin{aligned} \text{Cost}(4) &= \text{cost}(1) + A[1,4] + RCL \\ &= 25 + 0 + 0 = 25 \end{aligned}$$

$\boxed{\therefore \text{Cost}(4) = 25}$

* TSP chose to go to vertex 5 (node 5)

∴ (i, j) ⇒ (1, 5)

∴ set row 1 and column 5 to ∞

∴ set A[5, 1] to ∞

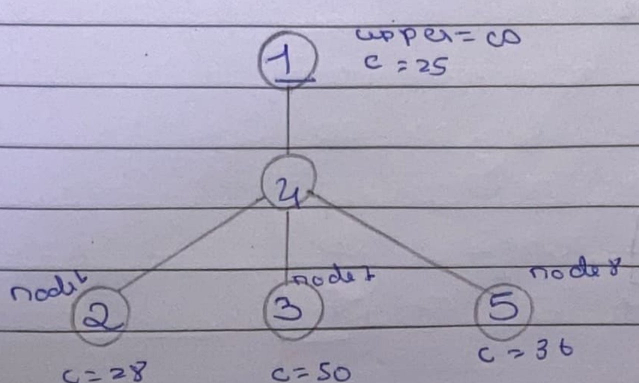
$$A = \begin{array}{c|c}\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & 11 & 2 & \infty \\ 0 & 3 & \infty & 0 & \infty \\ 15 & 3 & 12 & \infty & \infty \\ 11 & 0 & 0 & 12 & \infty \end{bmatrix} & \begin{matrix} \\ -2 \\ -0 \\ -3 \\ -0 \end{matrix} \\ \begin{matrix} 0 & \;\; 0 & \;\; 0 & \;\; 0 & \;\; \end{matrix} & \end{array}$$

reduced matrix $A = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 14 & \infty & 9 & 0 & \infty \\ 0 & 3 & \infty & 0 & \infty \\ 12 & 0 & 9 & \infty & \infty \\ \infty & 0 & 0 & 12 & \infty \end{bmatrix}$

RCL = 5

$$\begin{aligned} \text{Cost}(5) &= \text{Cost}(1) + A[1,5] + RCL \\ &= 25 + 1 + 5 \\ &= 26 + 5 \\ &= 31 \end{aligned}$$

$\boxed{\therefore \text{Cost}(5) = 31}$



$$A = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & 11 & \infty & 0 \\ 0 & 3 & \infty & \infty & 2 \\ \infty & 3 & 22 & \infty & 0 \\ 11 & 0 & 0 & \infty & \infty \end{bmatrix}$$

* From vertex 4

- TSP chose to go to ~~ver~~ vertex 2 (node 6) $(i, j) \Rightarrow (4, 2)$
$\therefore$ set row 4 and column 2 and $A[2,4]$ and $A[2,1]$ to ∞

$$
\begin{array}{c c}
\begin{bmatrix}
\infty & \infty & \infty & \infty & \infty \\
\infty & \infty & 11 & \infty & 0 \\
0 & \infty & \infty & \infty & 2 \\
\infty & \infty & \infty & \infty & \infty \\
11 & \infty & 0 & \infty & \infty
\end{bmatrix} &
\begin{array}{c}
\\ -0 \\ -0 \\ \\ -0
\end{array} \\
\begin{array}{ccccc}
-0 & & -0 & & -0
\end{array} &
\end{array}
$$

matrix already reduced,
RCL = 0

$cost(2) = cost(4) + A[4,2] + RCL$
$= 25 + 3 + 0$
$= 28$

$\boxed{\therefore cost(2) = 28}$

- TSP chose to go vertex 3 (node 7) $(i, j) \Rightarrow (4, 3)$
$\therefore$ set row 4, colum 3 and $A[3,4]$ and $[3,1]$ to ∞

Reduce matrix $A =$
$$
\begin{bmatrix}
\infty & \infty & \infty & \infty & \infty \\
12 & \infty & \infty & \infty & 0 \\
\infty & 3 & \infty & \infty & 2 \\
\infty & \infty & \infty & \infty & \infty \\
11 & 0 & \infty & \infty & \infty
\end{bmatrix}
$$

RCL = 13

$cost(3) = cost(4) + RCL + A[4,3]$
$= 25 + 13 + 12$
$= 25 + 25$
$= 50$

$\boxed{\therefore cost(3) = 50}$

- TSP chose to go to vertex 5 (node 8) $(i, j) \Rightarrow (4, 5)$
  ∴ set row 4, column 5, A [5, 4] and A [5, 1] to ∞

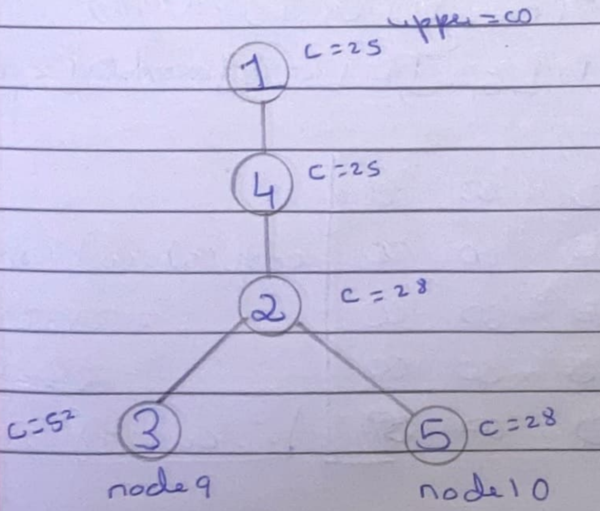
$$A = \begin{array}{c} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & 11 & \infty & \infty \\ 0 & 3 & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & 0 & 0 & \infty & \infty \end{bmatrix} \begin{array}{l} \\ -11 \\ -0 \\ \\ \\ \end{array} \\ \begin{array}{ccccc} -0 & -0 & -0 & & \end{array} \end{array}$$

$$\text{reduced matrix } A = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 1 & \infty & 0 & \infty & \infty \\ 0 & 3 & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & 0 & 0 & \infty & \infty \end{bmatrix}$$

$\underline{\underline{RCL = 11}}$

$$\text{cost}(5) = \text{cost}(4) + RCL + A[4, 5]$$
$$= 25 + 11 + 0$$
$$= \underline{\underline{36}}$$

$\boxed{\therefore \text{cost}(5) = 36}$



$$A = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 11 & \infty & 0 \\ 0 & \infty & \infty & \infty & 2 \\ \infty & \infty & \infty & \infty & \infty \\ 11 & \infty & 0 & \infty & \infty \end{bmatrix}$$

* From vertex 2

- TSP chose to go to vertex 3 (node 9) $(i, j) = (2, 3)$
  ∴ set row 2, colum = 3 and $A[3,2]$, $A[3,4]$, $A[3,1]$ to ∞

$$A = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & 2 \\ \infty & \infty & \infty & \infty & \infty \\ 11 & \infty & \infty & \infty & \infty \end{bmatrix} \begin{matrix} \\ \\ -2 \\ \\ -11 \end{matrix}$$

RCL = 13

$$\text{Cost}(3) = \text{Cost}(2) + \text{RCL} + A[2,3]$$
$$= 28 + 13 + 11$$
$$= 52$$

$\boxed{\therefore \text{Cost}(3) = 52}$

Reduce matrix

$$A = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & 0 \\ \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & \infty & \infty \end{bmatrix}$$

$\boxed{\text{Cost}(3) = 52}$

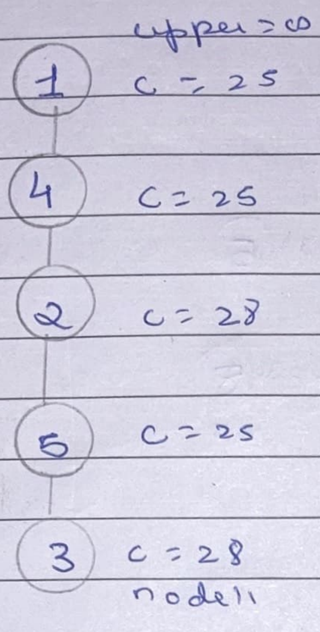
- TSP chose to go to vertex 5 (node 10) $(i, j) = (2, 5)$
  ∴ set row 2 and colum 5, $A[5,1]$, $A[5,2]$ and $A[5,4]$ to ∞

$$A = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 0 & \infty & \infty \end{bmatrix}$$

matrix is already reduced, RCL = 0

$$\text{cost}(5) = \text{cost}(2) + \text{RCL} + A[2,5]$$
$$= 28 + 0 + 0$$
$$= 28$$

$\boxed{\therefore \text{Cost}(5) = 28}$



From vertex 5, TSP chose vertex 3 (node 11), $(i, j) = (5,3)$

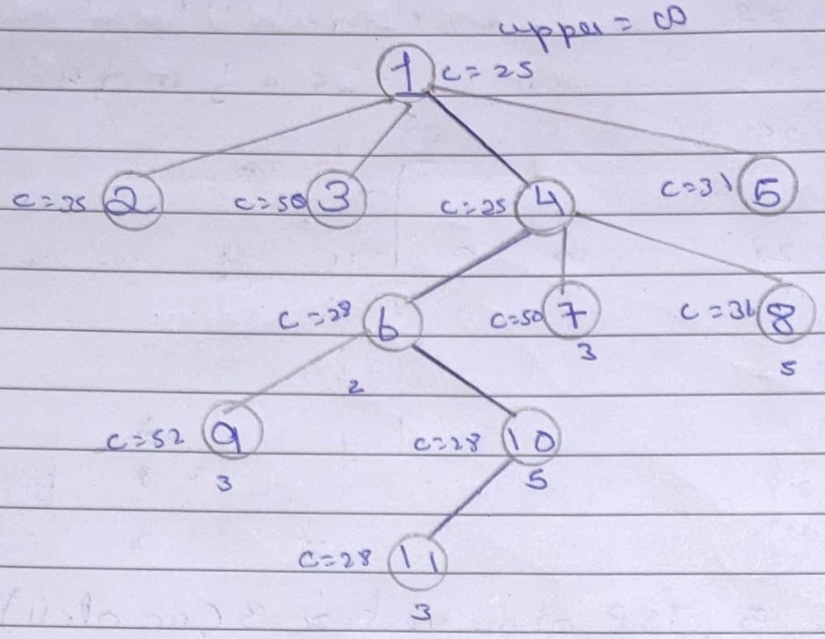
$\therefore$ set rows 5, column 3, $A[3,1]$, $A[3,2]$, $A[3,4]$, $A[3,5]$ to ∞

$$A = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \end{bmatrix}$$

RCL = 0

$$\text{cost}(3) = \text{cost}(5) + RCL + A[5,3]$$
$$= 28 + 0 + 0$$
$$= 28$$

$\boxed{\therefore \text{cost}(3) = 28}$



∴ This tour is : 1 → 4 → 2 → 5 → 3 → 1
and the minimum cost is 28

Q2] Find the longest common subsequence of the following string using dynamic programming

S 1: RINGING

S 2: ENGINEERING

		E	N	G	I	N	E	E	R	I	N	G
	0	1	2	3	4	5	6	7	8	9	10	11
0	0	0	0	0	0	0	0	0	0	0	0	0
R 1	0	0	0	0	0	0	0	0	1	1	1	1
I 2	0	0	0	0	+1	1	1	1	1	+2	2	2
N 3	0	0	+1	1	1	+2	2	2	2	2	+3	3
G 4	0	0	1	+2	2	2	2	2	2	2	3	+4
I 5	0	0	1	2	+3	3	3	3	3	+3	3	4
N 6	0	0	1	2	3	+4	4	4	4	4	+4	4
G 7	0	0	1	2	4	4	4	4	4	4	4	+5

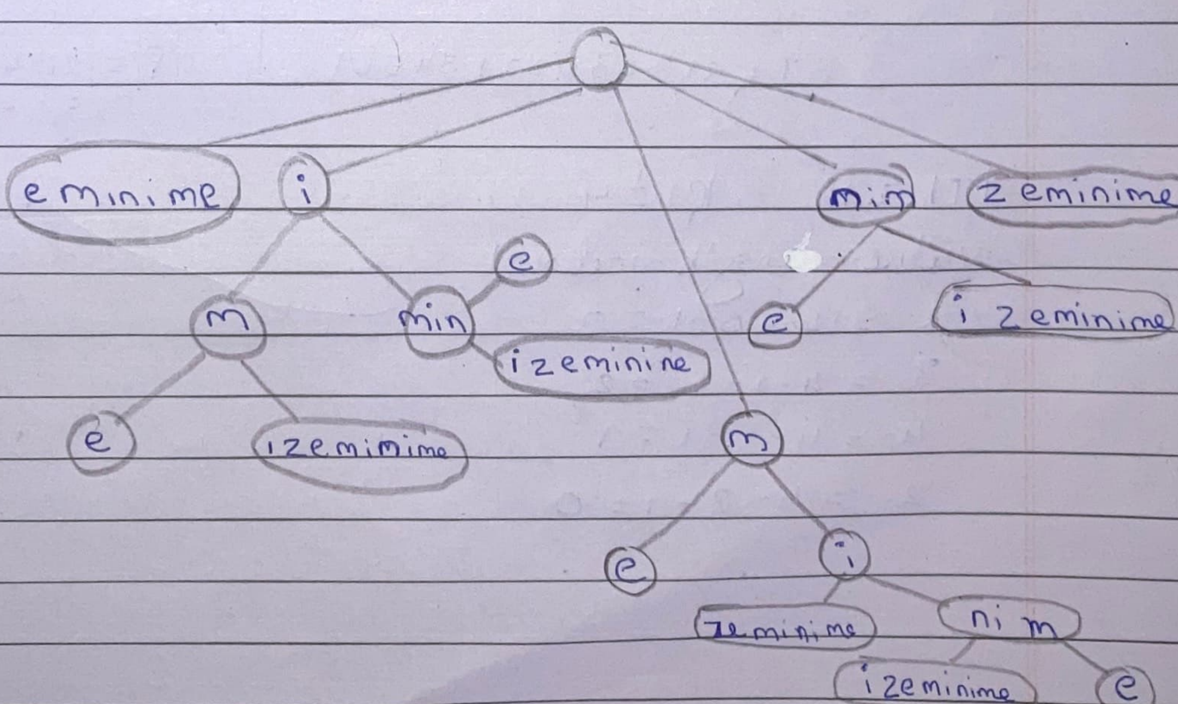
N G I N G

LCS = "NGING"

Q3] Draw a suffix tree and compact representation of the suffix for the string " Minimize minime "

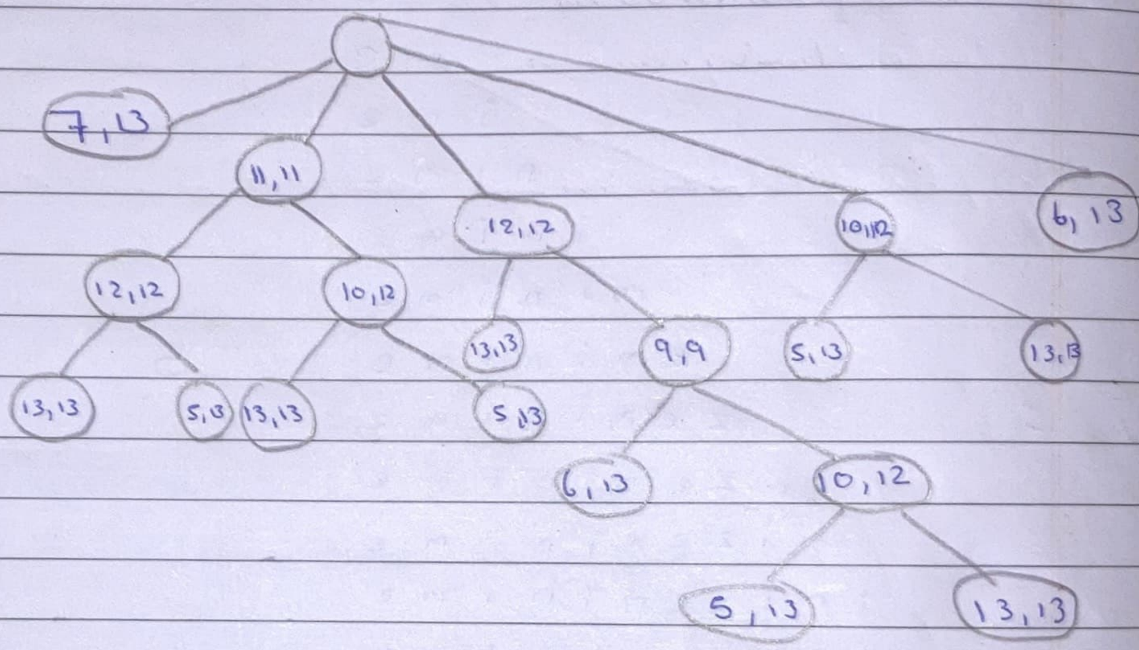
e
me
ime
nime
inime
minime
eminime
zeminime
izeminime
mizeminime
imizeminime
nimizeminime
inimizeminime
minimizeminime

5



M I N I M I Z E M I N I M E

0 1 2 3 4 5 6 7 8 9 10 11 12 13



Q4] Implement the Boyer Moore algorithm on the given text and patern

T = 21323422134561 P = 2342

0 1 2 3 - index

|T| = 14 |P| = 4

value = length - index - 1

2 = 4 - 0 - 1 = 3

3 = 4 - 1 - 1 = 2

4 = 4 - 2 - 1 = 1

2 = 4 - 3 - 1 = 0

2	3	4	*
0	2	1	4

i	2	1	3	2	3	4	2	2	1	3	4	5	6	1

j	2	3	4	2

→ mismatch at i = 2, j = 2

2	3	4	2

→ mismatch at i = 5 and j = 3

2	3	4	2

→ found at index 3 (Pattern)

∴ pattern is found in the text at position 4 (index 3)

Q5) Explain NP hard and NP complete problem, also state the cooks theorem

- NP stands for non-deterministic polynomial time an NP problem is a yes/no problem such that:

1. There is a polynomial time proof for every instance of problem with a "yes" answer, that is "yes" equivalently

2. There exists a polynomial time algorithm that has a ~~zero~~ non zero probability of answering "yes" if answer is "no"

- we are not sure there is no polynomial time solution but once you provide solution, this solution can be verified in polynomial time

* NP Hard

- A problem is said to be a NP hard problem if it is atleast as hard as hardest problem in NP, NP complete problems are also NP Hard, However, vice versa is not possible at all time

- Not all NP hard problems are NP as a NP hard problem may not have a polynomial time checking algorithm that makes those NP hard problems harder to solve

* NP Complete

P

- Refers to polynomial time we still need to find a solution but it can be verified in polynomial time. A problem x is NP - complete it

1) x is in NP and
2) For any problem y in NP, there is a "reduction" from y to x: a polynomial time algorithm that transforms any instance of y into an instance of x such that answer to y - instance is "yes" if and only if answer to x - instance is "yes"

- If x is NP complete and a deterministic polynomial as time algorithm exists that can solve all instances of x correctly then any problem in NP can be solved in deterministic polynomical time.

* Cook's theorem

- It states that satisfiability is in P, i.e. only if P = NP, Hence, if P = NP then satisfiability is in P.

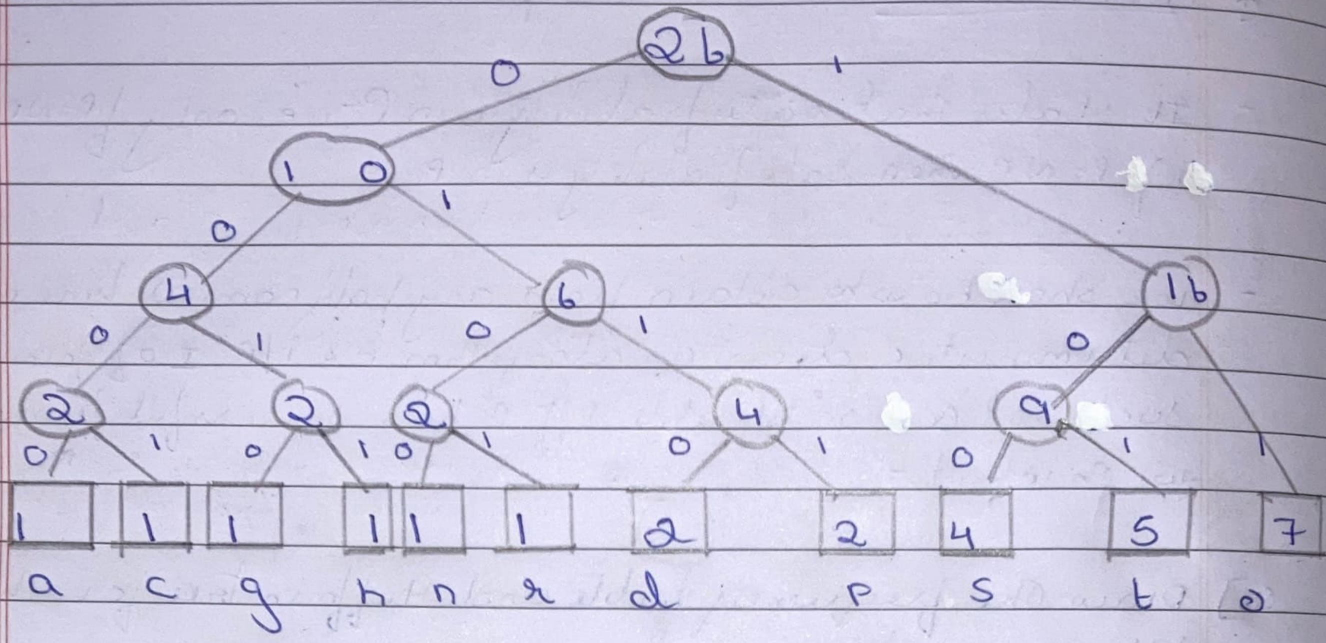
- we show how to obtain from any polynomial time non-deterministic decision algorithm A & i/p I of formula Q(A,I) such that Q is satisfiable iff A has successful termination with input I

Q6] Draw the frequency table and Huffman tree for the following string.
"Dogs do not spot hot spots or cat"

message length = 26
message size = 26 × 8 = 208 bits

Char	Frequency count	code	size
a	1	0000	1 × 4 = 4
c	1	0001	1 × 4 = 4
d	2	0110	2 × 4 = 8
g	1	0010	1 × 4 = 4
h	1	0011	1 × 4 = 4
n	1	0100	1 × 4 = 4
o	7	11	7 × 2 = 11
p	2	0111	2 × 4 = 8
r	1	0101	1 × 4 = 4
s	4	100	4 × 3 = 12
t	5	101	5 × 3 = 15
	26	40 bits	81 bits

ASCII = 11 × 8 = 88 bits



Message size = 81 bits

Table size = 88 + 40 bits

= 128 bits

Total message size = 209 bits