SE419

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S.E.(Computer) (Sem-IV) (Revised Course 2016-2017) EXAMINATION MAY/JUNE 2019
Discrete Mathematics

[Duration : Three Hours] **[Total Marks : 100]**

Instructions:
1) Attempt any five questions, at least 2 questions from Part A, at least 2 questions from Part B and at least 1 question from Part C.
2) Assume suitable data, if necessary.

Part A

Q.1 a) Let $\{A_k : k = 1,2 \ldots\}$ be a collection of subsets of some universal set U then S.T. $\left(\bigcup_{k\epsilon I} A_k\right)^{`} = \bigcap_{k\epsilon I} A'_k$ 06

b) Let Z be the set of integers and 'n' be a fixed positive integer. Let R be a relation on Z defined by: for $x, y \in Z$, xRy if and only if $x \equiv y(modn)$.
Show that R is an equivalence relation on Z. Express Z as a disjoint union of distinct equivalence classes of R. 08

c) If c divides ab and $\gcd(a, c) = 1$,prove that c divides b. Prove or Disprove : If c divides ab and $\gcd(a, c) \neq 1$, c divides b. 06

Q.2 a) If a mapping $f: A \rightarrow B$ is one to one and onto. Prove or Disprove that the inverse mapping is also one to one and onto. 06

b) Draw the Hasse Diagram for the Poset $(S, \leq)$where $S = \{2, 3, 4, 5, 6, 8, 9, 12, 18, 72\}$ and where aRb if and only if a divides b; $\forall a, b \in S$. Find the greatest and the least element (if they exist). 08

c) Prove by Mathematical Induction that $6^{n+2} + 7^{2n+1}$ is divisible by 43 for each positive integer n. 06

Q.3 a) In the Boolean Algebra $(B, +, .,)$express the Boolean function $f(x, y, z) = (x + y).(x + z) + y + z`$ in its disjunctive normal form. 06

b) If $(B, +, .,)$ is a Boolean Algebra and ; $\forall a, b \in B$, prove with proper justification that $(a.b)' = a' + b'$ 06

c) Define Tautology and contradiction. Without using truth tables prove that $(\neg p \wedge (\neg q \wedge r)) \vee (q \wedge r) \vee (p \wedge r) \equiv r$ 08

Part B

Q.4 a) There are 13 squares of side 1 positioned inside a circle of radius 2. Show that at least 2 of the squares have a common point. 06

b) A person invests Rs. 50,000 @ 71/2 % interest compound annually. How much will be the total amount at the end of 14 years. 06

c) Find the total solution of the following recurrence relation 08
$a_n + 5a_{n+1} + 4a_{n-2} = 56(3)^n, n \geq 2\ with\ a_0 = 22; a_1 = 47$

Q.5 a) Define 06
(i) Path
(ii) Eulerian Graph
(iii) Hamiltonion Graph

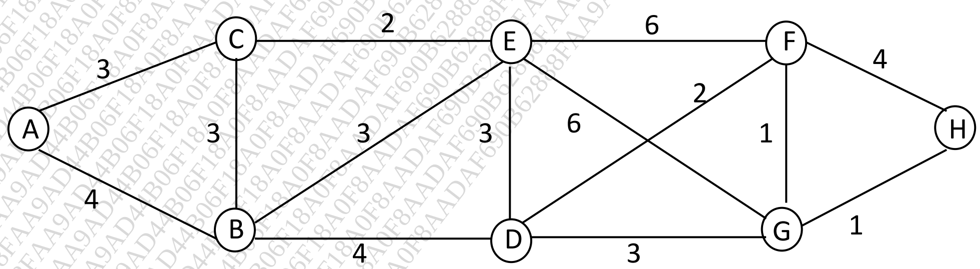
b) Consider the following Adjacency matrix 06

$$A(G_1) = \begin{pmatrix} 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 \end{pmatrix}$$

$$A(G_2) = \begin{pmatrix} 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \end{pmatrix}$$

Check whether the graph G_1 and G_2 are isomorphic. Justify.

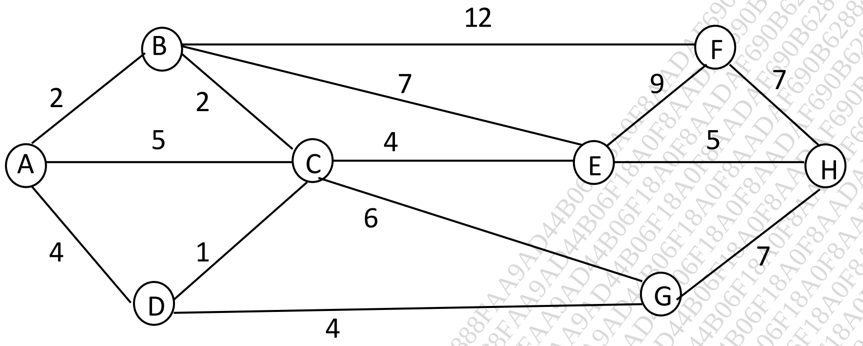
c) Using Dijkstra's algorithm find the shortest path between the vertices and h for the following weighted graphs 06



Q.6 a) 7 women and 9 men are on the faculty in the computer science department at the school. 05
(i) How many ways are there to select a committee of five members of the department if at least one woman must be on the committee?
(ii) How many ways are there to select a committee of five members of the department if at least one woman and at least one man must be on the committee?

b) Prove that a non-trivial simple graph without any isolated vertex must have at least one pair of vertices whose degrees are equal. 07

c) Using Prim's algorithm, find the minimum spanning tree from the following Graph. 08



Part C

Q.7 a) If A and B denote non empty sets then: 06
(1) Prove that $P(A) \cap P(B) = P(A \cap B)$
(2) $P(A) \cup P(B) \subseteq P(A \cup B)$. Give an example to show that $(P(A \cup B)$ need not be a subset of $P(A) \cup P(B)$

b) Use the pigeon hold principle to prove that if any five points are chosen at random within a square of length 2, then there are atleast two points whose distance apart is atmost $\sqrt{2}$ 06

c) How many positive integers not exceeding 2000 are divisible by 7 or 13? 08

Q.8 a) Find the recurrence relation for a number of n-digit binary sequence having no pair of consecutive (successive) 0's. State the initial conditions. If a_n denotes the number of different binary sequences of length n satisfying the condition that there was no consecutive zeros. Find a_5 and a_6 08

b) State and prove the Hand Shaking Lemma. 05

c) Use Mathematical Induction to prove that for all positive integers n, $2.7^n + 3.5^n - 5$ is divisible by 24 07