* Traveling Salesperson Problem (TSP)

- Let $G = (V, E)$ be a directed graph with edge cost "c_{ij}"
- $\boxed{c_{ij} > 0}$ $\forall$ i & j & $\boxed{c_{ij} = \infty}$ — if $<i, j> \in E$
- A tour of 'G' is a $\overbrace{\text{directed simple cycle}}^{\text{tour}}$ that starts at vertex 1 in G, visits every vertex of G & ends at vertex 1
- The cost of the tour is the cost of the edges on the tour
- TSP is to find a tour of minimum cost that starts & ends at vertex 1
- Every tour consists of an edge $<1, k>$ for

Some $k \in V - \{1\}$ & a path from vertex k to vertex 1, which goes through each vertex in $\boxed{V - \{1, k\}}$ exactly once

$\{1,2,3,4\} \Rightarrow \{2,3,4\}$

- The length of the optimal salesperson tour is given by

$$g(1, \underbrace{V - \{i\}}_{S}) = \min_{2 \le k \le n} \left\{ c_{1k} + g(k, V - \{1, k\}) \right\} \qquad 1 \le i \le n$$

<u>Generalizing</u>

to be remembered

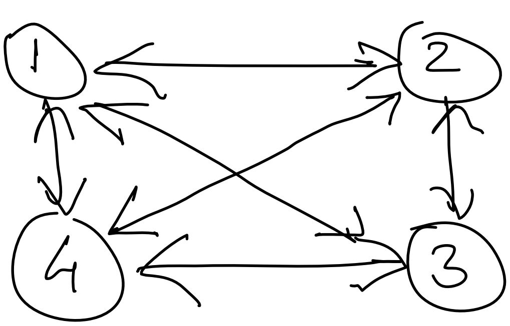
$$\boxed{\begin{array}{c} g(i, S^*) = \min\limits_{j \in S} \left\{ c_{ij} + g(j, S^* - \{j\}) \right\} \\ i \ne 1, \quad 1 \notin S, \quad i \notin S, \quad 1 \le i \le n \end{array}}$$

$S \rightarrow$ is the set of vertices to be visited

* Initially $\boxed{g(i, \phi) = c_{i1} \quad 1 \le i \le n}$

- Let $g(i, S)$ be the length of a shortest path starting at vertex "i" going through all vertices in S & terminating at vertex 1

<u>Problem Solving</u>



$$
\begin{array}{c|cccc}
 & 1 & 2 & 3 & 4 \\
\hline
1 & 0 & 10 & 15 & 20 \\
2 & 5 & 0 & 9 & 10 \\
3 & 6 & 13 & 0 & 12 \\
4 & 8 & 8 & 9 & 0
\end{array}
$$

$\underline{\underline{Sol^n}}$ I $S = \phi$

II $S = 1 \longrightarrow i = 2, 3, 4$

III $S = 2 \longrightarrow i = 2, 3, 4$

IV $S = 3 \longrightarrow i = 1$

$$g(1, \overbrace{\{2, 3, 4\}}^{S=3}) = \min_{j \in S} \begin{cases} c_{12} + g(2, \{3, 4\}) & j = 2 \\ c_{13} + g(3, \{2, 4\}) \\ c_{14} + g(4, \{2, 3\}) \end{cases}$$

final condition = $\min\{10 + 25, 15 + 25, 20 + 23\}$

funn to obtain = $\boxed{35} \longrightarrow$ mini cost of tour

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i $c_{i1} \longrightarrow c_{ij}$

I $S = \phi$

$1 \leq i \leq n$

$1 \leq i \leq 4$

$g(1, \phi) = c_{11} = 0$

$g(2, \phi) = c_{21} = 5$

$g(3, \phi) = c_{31} = 6$

$g(4, \phi) = c_{41} = 8$

II $S = 1 \quad (i = 2, 3, 4), \quad S = \{\cancel{2}, 3, 4\}$

(a) for $\boxed{i = 2}$

$$g(2, \overbrace{\{3\}}^{S}) = \min\{C_{23} + g(3, \overset{\{3\}}{S} - \{3\})$$

$$= \min\{C_{23} + g(3, \emptyset)\}$$

$$= 9 + 6 = 15$$

$$g(2, \overbrace{\{4\}}^{S}) = 18$$

(b) for $\boxed{i = 3}$

$$g(3, \overbrace{\{2\}}^{S}) = \min\{C_{32} + g(2, S - \{2\})$$

$$\min\{C_{32} + g(2, \{2\} - \{2\})$$

$$\min\{13 + g(2, \emptyset)\} = 18$$

$$g(3, \overbrace{\{4\}}^{S}) = 20$$

(c) for $\boxed{i = 4}$

$$g(4, \overbrace{\{2\}}^{S}) = 13$$

$$g(4, \overset{S}{\{3\}}) = 15 \rightarrow \boxed{j = 3}$$

$$= \min\{C_{43} + g(3, \emptyset)\} = 15$$

III $S = 2$, $i = 2, 3, 4$, $S = \{2, 3, \cancel{4}\}$

(a) for $\boxed{i = 2}$

$$g(2, \overset{S}{\{3, 4\}}) = \min\begin{cases} C_{23} + g(3, \{3,4\} - \{3\}) \\ C_{24} + g(4, \{3,4\} - \{4\}) \end{cases}$$

$$= \min\{C_{23} + g(3, \{4\}),\ C_{24} + g(4, \{3\})\}$$

$j = 4$

$$= \min\{9 + 20, 10 + 15\} = 25$$

(b) for $\boxed{i = 3}$

$$g(3, \overset{S}{\{2, 4\}}) = \min\begin{cases} C_{32} + g(2, \{4\}) \\ C_{34} + g(4, \{2\}) \end{cases}$$

$$= \min\{13 + 18, 12 + 13\} = 25$$

(c) for $\boxed{i = 4}$

$$g(4, \overbrace{\{2,3\}}^{S}) = \min \begin{Bmatrix} C_{42} + g(2, \{3\}), \\ C_{43} + g(3, \{2\}) \end{Bmatrix}$$

$$= \min\{8+15, 9+18\} = 23$$

* Tour can be constructed if we retain with each $g(i, s)$, the value of $\boxed{\text{"}j\text{"}}$ that minimizes the R.H.S of $g(i,s)$ funn

$S = 3$

$g(1, \{2,3,4\}) = 2$ $\quad 1 \to 2$

$g(2, \overbrace{\{3,4\}}^{S=2}) = 4$ $\quad 1 \to 2 \to 4$

$g(4, \{3\}) = 3$ $\quad 1 \to 2 \to 4 \to 3$

$g(3, \cancel{\emptyset}) = \underline{1}$

$\boxed{1 \to 2 \to 4 \to 3 \to \underline{1}}$

optimal tour

of mini cost = 35