** Approximation Algorithms

poly-time det-sol^n

- is a way of dealing with NP-completeness optimization problems (Maximization / Minimization)
- Goal of approx. algo is to come as close to optimal solution in polynomial time

* Notations Used

$C \rightarrow$ cost of Solution (ie using Approx-algo)

$C^* \rightarrow$ cost of optimal Solution

rowe

$\rho(n) \rightarrow$ approximation ratio ($n \rightarrow$ i/p size)

* Maximization problem $\Rightarrow \dfrac{C^*}{C} \leq \rho(n)$
* Minimization problem $\Rightarrow \dfrac{C}{C^*} \leq \rho(n)$

→ Always $\boxed{\rho(n) \nless 1}$ ie it may be 1 or greater than 1

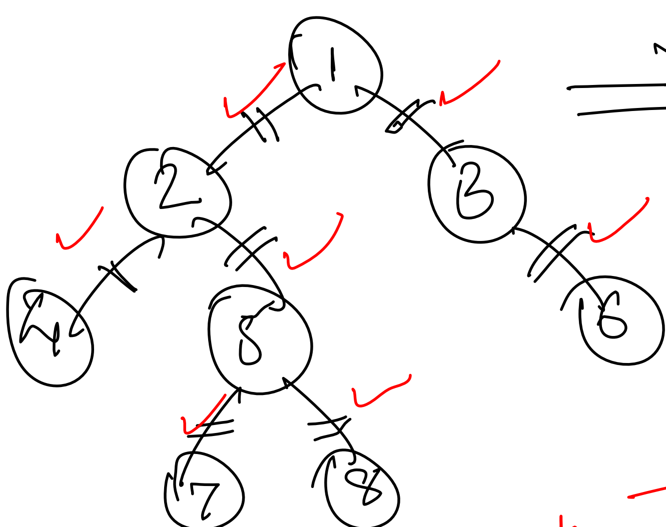
Constraint

— x — x — x —

Ex: Vertex Cover problem (NP-hard)

goal → to obtain mini of vertices needed to covers all the edges of Graph 'G'

Ex:



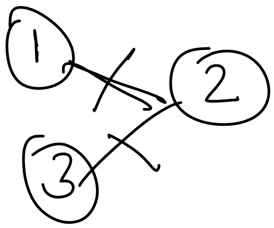
$\Longrightarrow$ $\{\underbrace{1, 3, 2, 5}_{v=4}\}$

$\{5, 6, 2, 1\} \rightarrow v=4$

* → $\{\underbrace{2, 3, 5}_{V=3}\}$

$V = 3$ → near to 0.5

(preferred ans)

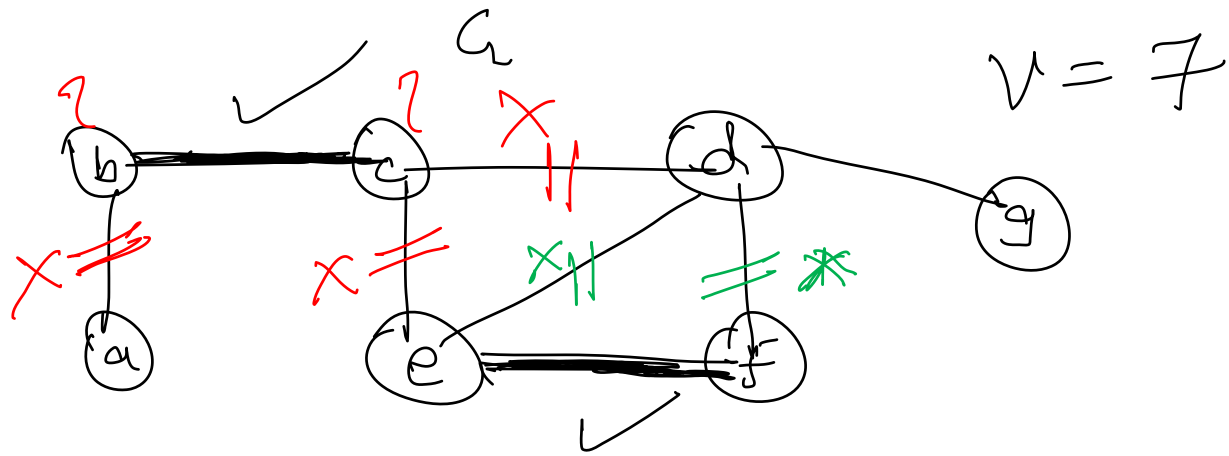
Ex:  $\xrightarrow{\text{sol}}$ $\{1, 3\}$ ✓

$\{2\} \rightarrow$ prefered soln

Algo Approx_cover_Prob (G)

1. $C \leftarrow \emptyset$
2. $E' \leftarrow E[G]$
3. while $E' \neq \emptyset$
4. do let (u, v) be an arbitary edge in E'
5. $C \leftarrow C \underset{}{\overset{\text{union}}{\cup}} \{u, v\}$
6. Remove from E' every edge incident on u or v
7. Return C

Ex:



Sol^n

$C = \emptyset$

$E' = \{ (a,b), (b,c), (c,e), \ldots \}$

(I) Arbitarly / randomly select any edge from G

(for eg edge selected is (b, c))

$C = C \cup \{b, c\}$

$C = \{b, c\}$

(After removing all the edges incident on b or c we left with only 4 edges

(II) Assume next edge selected is (e, f)

$$C = C \cup \{e, f\}$$
$$= \{(b, c)\} \cup \{(e, f)\}$$
$$C = \{(b, c, e, f)\}$$

(After removing the edges incident on e or f we are left with only one edge ie (d, g))

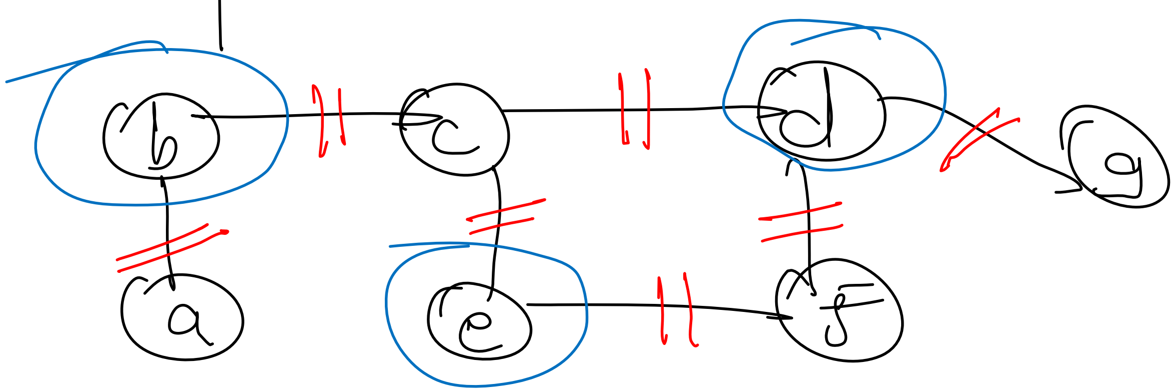
(III) Edge selected will be (d, g)

$$C = \{(b, c, e, f)\} \cup \{(d, g)\}$$
$$C = \{b, c, d, e, f, g\}$$

min $\rightarrow$ 6 vertices to cover all edges of G

By random observation

Actual optimal sol^n



$V = \{b, d, e\} \longrightarrow V = \{3\}$

needed to cover all edges of G

* Minimization $\quad \frac{C}{C^*} \le \rho(n) \qquad \boxed{\rho \ge 1}$

$$\frac{6}{3} \Rightarrow 2 \le \rho(n)$$