ASSIGNMENT 3

Q1] Solve the following 0/1 Knapsack instance

$n = 6$, $m = 165$, $(P_1, P_2, P_3, P_4, P_5, P_6) = (100, 50, 20, 10, 7, 3)$
$(w_1, w_2, w_3, w_4, w_5, w_6) = (100, 50, 20, 10, 7, 3)$

[Also write 0/1 Knapsack Algorithm]

number of element, $n = 6$
Capacity, $m = 165$
Profit $(P) = (P_1, P_2, P_3, P_4, P_5, P_6) = (100, 50, 20, 10, 7, 3)$
weight $(w) = (w_1, w_2, w_3, w_4, w_5, w_6) = (100, 50, 20, 10, 7, 3)$

State 0

$S^0 = \{(0,0)\} + (100, 100)$
$S^0_1 = \{(100, 100)\}$

State 1

$S^1 = \{(0,0)(100,100)\} + (50,50)$
$S^1_1 = \{(50,50) + (150,150)\}$

State 2

$S^2 = \{(0,0)(50,50),(100,100)(150,150)\} + (20,20)$

$S^2_1 = \{(20,20)(70,70)(120,120)(170,170)\}$

State 3

$S^3 = \{(0,0)(20,20)(50,50)(70,70)(100,100)(120,120)$
$(150,150)\ \cancel{(170,170)}\} + (10,10)$

$S^3_1 = \{(10,10)(30,30)(60,60)(80,80)(110,110)(130,130)$
$(160,160)\}$.

State 4

(20,20) (50,50)

$S^4 = \{(0,0)(10,10),(30,30),(60,60)(80,80)(110,110)(130,130)$
$(160,160)\} + (7,7)$

$S^4_1 = \{(7,7)(17,17)(37,37)(67$

State 4

$S^4 = \{(0,0)(10,10)(20,20)(30,30)(50,50)(70,70),(80,80)$
$(100,100)(110,110)(120,120)(130,130)(160,160)\} + (7,7)$

$S^4_1 = \{(7,7)(17,17)(27,27)(37,37)(57,57)(77,77)(87,87)$
$(107,107)(117,117)(127,127)(137,137)(167,167)\}$

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State 5

$S^5 = \{(0,0)\ (7,7)\ (10,10)\ (17,17)\ (20,20)\ (27,27)\ (30,30)$
$(37,37)\ (50,50)\ (57,57)\ (70,70)\ (77,77)\ (80,80)$
$(87,87)\ (100,100)\ (107,107)\ (110,110)\ (117,117)$
$(120,120)\ (127,127)\ (130,130)\ (137,137)\ (160,160)$
$\cancel{(167,167)}\} + (3,3)$

$S^5_1 = \{(3,3)\ (10,10)\ (13,13)\ (20,20)\ (23,23)\ (30,30)\ (33,33)$
$(40,40)\ (53,53)\ (60,60)\ (73,73)\ (80,80)\ (83,83)$
$(90,90)\ (103,103)\ (110,110)\ (113,113)\ (120,120)$
$(123,123)\ (130,130)\ (133,133)\ (140,140)$
$(163,163)\}$

State 6

$S^6 = \{(0,0)\ (3,3)\ (7,7)\ (10,10)\ (13,13)\ (17,17)\ (20,20)\ (23,23)$
$(27,27)\ (30,30)\ (33,33)\ (37,37)\ (40,40)\ (50,50)$
$(53,53)\ (57,57)\ (60,60)\ (70,70)\ (73,73)\ (77,77)$
$(80,80)\ (83,83)\ (87,87)\ (90,90)\ (103,103)$
$(107,107)\ (110,110)\ (113,113)\ (117,117)\ (120,120)$
$(123,123)\ (127,127)\ (130,130)\ (133,133)$
$(137,137)\ (140,140)\ (160,160)\ (163,163)\}$

Final solution :

(1,1,0,1,0,1)

$(P, W) = (163, 163)$
$(163, 163) \in S^6$ but $\notin S^5$

$\therefore x_6 = \underline{\underline{1}}$

$(163, 163) - (3, 3) = (160, 160)$

$(160, 160) \in S^5$ and $\in S^4$

$\therefore x_5 = \underline{\underline{0}}$

$(160, 160) \in S^4$ but $\notin S^3$

$\therefore x_4 = \underline{\underline{1}}$

$(160, 160) - (10, 10) = (150, 150)$

$(150, 150) \in S^3$ and $\in S^2$

$\therefore x_3 = \underline{\underline{0}}$

$(150, 150) \in S^2$ but $\notin S^1$

$\therefore x_2 = \underline{\underline{1}}$

$(150, 150) - (50, 50) = (100, 100)$

$(100, 100) \in S^1$ but $\notin S^0$

$\therefore x_1 = \underline{\underline{1}}$

$\therefore$ The final solution is $\overset{x_1, x_2, x_3, x_4, x_5, x_6}{(1, 1, 0, 1, 0, 1)}$

* Algorithm

Algorithm DKnap (p, w, x, n, m)
{
// pair[] is array of pw's
b[0] = 1; pair[1].p := pair[1].w := 0.0; // S^0
t = 1; h := 1; // start & end of S^0
b[1] := next := 2; // next free spot in pair[]
for i := 1 to n-1 do
{
// generate S^i
k := t;
u := Largest (pair, w, t, h, i, m);
for j := 1 to u do
{
// Generate S_1^{i-1} and merge
pp := pair[j].p + P[i]; ww := pair[j][w] + w[i];
// (pp, ww) is the next element in S_1^{i-1}
while ((k ≤ h) and (pair[k].w ≤ ww)) do

```
{
    pair[next].p := pair[k].p;
    pair[next].w := pair[k].w;
    next := next+1; k := k+1;
}
if ((k ≤ h) and (pair[k].w == ww)) then
{
    if pp < pair[k].p then pp := pair[k].p;
    k := k+1;
}
if pp > pair[next-1].p then
{
    pair[next] := pp; pair[next].w := ww;
    next := next + 1;
}
while ((k ≤ h) and (pair[k].p ≤ pair[next-1].p))
do k := k + 1;
}
// merge in remaining terms from S^(i-1)
while (k ≤ h) do
{
    pair[next].p := pair[k].p; pair[next].w := pair[k].w;
    next := next +1; k := k+1;
}
// initialise for S^(i-1)
t := h+1; h := next -1; b[i + 1] := next;
}
Traceback (P, w, pair, x, m, n);
}
```

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Q2] obtain OBST for the following
(a1, a2, a3, a4) = (cout, float, if, while)

$P[1:4] = \{\frac{1}{20}, \frac{1}{5}, \frac{1}{10}, \frac{1}{20}\}$
$q[0:4] = \{\frac{1}{5}, \frac{1}{10}, \frac{1}{5}, \frac{1}{20}, \frac{1}{20}\}$

Note: Consider fractional Values in calculation

[write algorithm in your terms]

	0	1	2	3	4
0	$w_{00} = 1/5$ $c_{00} = 0$ $r_{00} = 0$	$w_{11} = 1/10$ $c_{11} = 0$ $r_{11} = 0$	$w_{22} = 1/5$ $c_{22} = 0$ $r_{22} = 0$	$w_{33} = 1/20$ $c_{33} = 0$ $r_{33} = 0$	$w_{44} = 1/20$ $c_{44} = 0$ $r_{44} = 0$
1	$w_{01} = 7/20$ $c_{01} = 7/20$ $r_{01} = 1$	$w_{12} = 1/2$ $c_{12} = 1/2$ $r_{12} = 2$	$w_{23} = 7/20$ $c_{23} = 7/20$ $r_{23} = 3$	$w_{34} = 3/20$ $c_{34} = 3/20$ $r_{34} = 4$	
2	$w_{02} = 3/4$ $c_{02} = 11/10$ $r_{02} = 2$	$w_{13} = 13/20$ $c_{13} = 1$ $r_{13} = 2$	$w_{24} = 9/20$ $c_{24} = 3/5$ $r_{24} = 3$		
3	$w_{03} = 9/10$ $c_{03} = 8/5$ $r_{03} = 2$	$w_{14} = 3/4$ $c_{14} = 27/20$ $r_{14} = 2$			
4	$w_{04} = 1$ $c_{04} = 39/20$ $r_{04} = 2$				

1. $w_{00} = q(0) = 1/5$
$c_{00} = 0$
$r_{00} = 0$

2. $w_{11} = q(1) = 1/10$

$c_{11} = 0$

$r_{11} = 0$

3. $w_{22} = q(2) = 1/5$

$c_{22} = 0$

$r_{22} = 0$

4. $w_{33} = q(3) = 1/20$

$c_{33} = 0$

$r_{33} = 0$

5. $w_{44} = q(4) = 1/20$

$c_{44} = 0$

$r_{44} = 0$

6. $w_{01} = P(1) + q(1) + w_{00} = 1/20 + 1/10 + 1/5 = 7/20$

$c_{01} = \min\limits_{\substack{0<k\leq 1 \\ k=1}} \{c_{00} + c_{11}\} + w_{01} = (0+0) + 7/20 = 7/20$

$r_{01} = 1$

7. $w_{12} = P(2) + q(2) + w_{11} = 1/5 + 1/5 + 1/10 = 1/2$

$c_{12} = \min\limits_{\substack{1<k\leq 2 \\ k=2}} \{c_{11} + c_{22}\} + w_{12} = (0+0) + 1/2 = 1/2$

$r_{12} = 2$

8. $w_{23} = P(3) + q(3) + w_{22} = 1/10 + 1/20 + 1/5 = 7/20$

$c_{23} = \min\limits_{\substack{2<k\leq 3 \\ k=3}} \{c_{22} + c_{33}\} + w_{23} = (0+0) + 7/20 = 7/20$

$r_{23} = 3$

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9. $w_{34} = P(4) + q(4) + w_{33} = 1/20 + 1/20 + 1/20 = 3/20$

$C_{34} = \min_{\substack{3 < k \le 4 \\ k = 4}} \{C_{33} + C_{44}\} + w_{34} = (0 + 0) + 3/20 = 3/20$

$r_{34} = 4$

10. $w_{02} = P(2) + q(2) + w_{01} = 1/5 + 1/5 + 7/20 = 3/4$

$C_{02} = \min_{\substack{0 \le k \le 2 \\ k = 1, 2}} \{C_{00} + C_{12}, C_{01} + C_{22}\} + w_{02} = \min(0 + 1/2, 7/20 + 0) + 3/4$

$= \min(1/2, 7/20) + 3/4 = 7/20 + 3/4 = 11/10$

$r_{02} = 2$

11. $w_{13} = P(3) + q(3) + w_{12} = 1/10 + 1/20 + 1/2 = 13/20$

$C_{13} = \min_{\substack{1 \le k \le 3 \\ k = 2, 3}} \{C_{11} + C_{23}, C_{12} + C_{33}\} + w_{13}$

$= \min\{0 + 7/20, 1/2 + 0\} + 13/20$

$= \min(7/20, 1/2) + 13/20 = 7/20 + 13/20 = 1$

$r_{13} = 2$

12. $w_{24} = P(4) + q(4) + w_{23} = 1/20 + 1/20 + 7/20 = 9/20$

$C_{24} = \min_{\substack{2 < k \le 4 \\ k = 3, 4}} \{C_{22} + C_{34}, C_{23} + C_{44}\} + w_{24}$

$= \min\{0 + 3/20, 7/20 + 0\} + 9/20 = 3/20 + 9/20 = 3/5$

$r_{24} = 3$

13. $w_{03} = P(3) + q(3) + w_{02} = 1/10 + 1/20 + 3/4 = 9/10$

$C_{03} = \min_{\substack{0 < k \le 3 \\ k = 1, 2, 3}} \{C_{00} + C_{13}, C_{01} + C_{23}, C_{02} + C_{33}\} + w_{03}$

$= \min\{0 + 1, 7/20 + 7/20, 11/10 + 0\} + 9/10$

$= \min(1, 7/10, 11/10) + 9/10 = 7/10 + 9/10 = 8/5$

$r_{03} = 2$

14. $w_{14} = p(4) + q(4) + w_{13} = 1/20 + 1/20 + 13/20 = 3/4$

$C_{14} = \min_{1 < k \le 4} \{ C_{11} + C_{24}, C_{12} + C_{34}, C_{13} + C_{44} \} + w_{14}$

$k = 2, 3, 4$

$= \min(0 + 3/5, 1/2 + 3/20, 1 + 0) + 3/4 = \min(3/5, 13/20, 1) + 3/4$

$= \min(3/5 + 3/4) = 27/20$

$r_{14} = 2$

15. $w_{04} = p(4) + q(4) + w_{03} = 1/20 + 1/20 + 9/10 = 1$

$C_{04} = \min_{0 < k \le 4} \{ C_{00} + C_{14}, C_{01} + C_{24}, C_{02} + C_{34}, C_{03} + C_{44} \} + w_{04}$

$k = 1, 2, 3, 4$

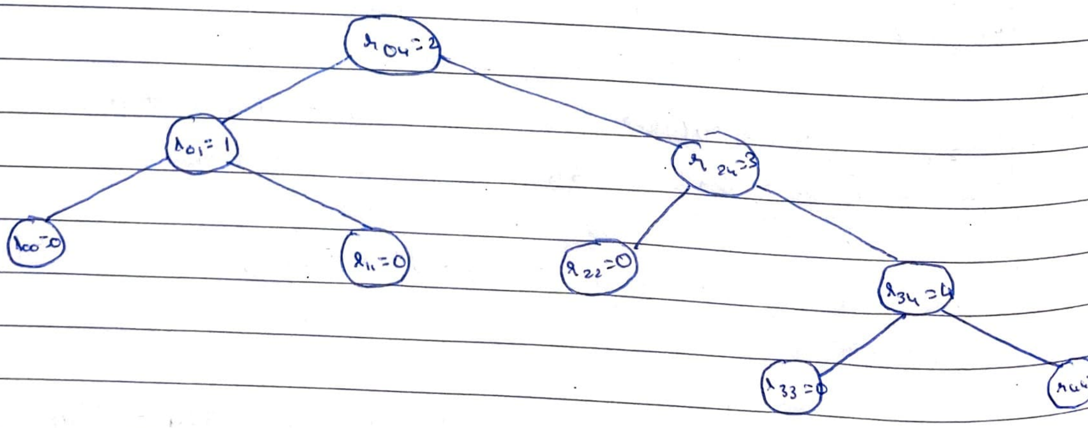
$= \min(0 + 27/20, 7/20 + 3/5, 11/10 + 3/20, 8/5 + 0) + 1$

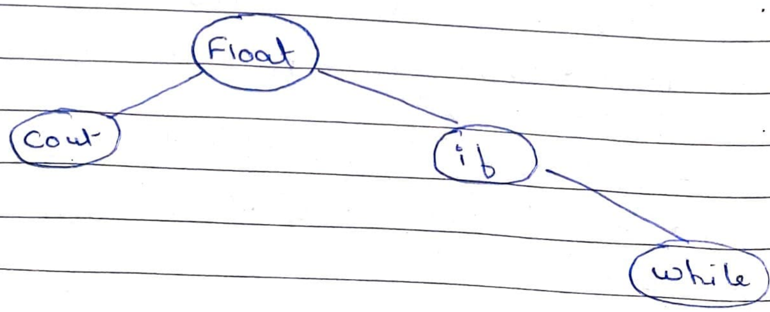
$= \min(27/20, 19/20, 5/4, 8/5) + 1$

$= 19/20 + 1 = 39/20$

$r_{04} = 2$

Root node = r[0][n-1]





* Algorithm

```
Algorithm OBST (P,q,n)
{
    for i := 0 to n-1 do
    {
        //initialize;
        w[i,i] := q[i]; r[i,i] := 0; c[i,i]:=
        //optimal trees with 1 node
        w[i,i+1] : q[i] + q[i+1] + p[i+1];
        r[i,i+1] := i+1;
        c[i,i+1] := q[i] + q[i+1] + p[i+1];
    }
    w[n,n] := q[n]; r[n,n] := 0; c[n,n] := 0.0;
    for m := 2 to n do // find optimal tree with m nodes
    for i := 0 to n-m do
    {
        j := i+m;
        w[i,j] := w[i,j-1] + p[j] + q[j];
        k = find(c,r,i,j);
        // A value of l range r[i,j-1] ≤ l
        // ≤ r[i+1,j] that minimizes c[i,l-1]+c[l,j];
        c[i,j] := w[i,j] + c[i,k-1] + c[k,j];
        r[i,j] := k;
    }
    write(c[0,n], w[0,n], r[0,n]);
}
```

Algorithm find (c, r, i, j)
{
min := ∞;
for m := r[i, j-1] to r[i+1, j] do
if (c[i, m-1] + c[m, j]) < min then
{
min := c[i, m-1] + c[m, j]; l := m;
}
return l;
}

Q. 3] Find the optimal salesperson tour for the following graph

$$\begin{bmatrix} 0 & 12 & 5 & 7 \\ 11 & 0 & 13 & 6 \\ 4 & 9 & 0 & 18 \\ 10 & 3 & 2 & 0 \end{bmatrix}$$

1. $S = \phi \quad 1 \le i \le 4 \quad g(i, \phi) c_{i1}$

$g(1, \phi) = C_{11} = 0$
$g(2, \phi) = C_{21} = 11$
$g(3, \phi) = C_{31} = 4$
$g(4, \phi) = C_{41} = 10$

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2. $s = 1$ $i = 2, 3, 4$

a) for $i = 2$

$g(2, \{3\}) = C_{23} + g(3, \phi) = 13 + 4 = 17$

$g(2, \{4\}) = C_{24} + g(4, \phi) = 16 + 10 = 16$

b) for $i = 3$

$g(3, \{2\}) = C_{32} + g(2, \phi) = 9 + 11 = 20$

$g(3, \{4\}) = C_{24} + g(4, \phi) = 18 + 10 = 28$

c) for $i = 4$

$g(4, \{2\}) = C_{42} + g(2, \phi) = 3 + 11 = 14$

$g(4, \{3\}) = C_{43} + g(3, \phi) = 2 + 4 = 6$

3. $s = 2$ $i = 2, 3, 4$ $s = \{2, 3, 4\}$

a) for $i = 2$

$g(2, \{3, 4\}) = \min\{C_{23} + g(3\{4\}), C_{24} + g(4\{3\})\}$

$= \min\{13 + 28, 6 + 6\}$

$= \min\{41, 12\} = 12$. $j = 4$

b) for $i = 3$

$g(3, \{2, 4\}) = \min\{C_{32} + g(2, \{4\}), C_{34} + g(4, \{2\})$

$= \min\{9 + 16, 18 + 14\}$

$= \min(25, 22)$ $j = 4$

$= 22$

c) for $i = 4$

$g(4, \{2,3\}) = \min\{c_{42} + g(2, \{3\}), c_{43} + g(3, \{2\})\}$

$= \min\{3 + 17, 2 + 20\}$

$= \min\{20, 22\} \quad j = 2$

$= 20$

4. $S = 3$

$g(1, \{2,3,4\}) = \min\{c_{12} + g(2, \{3,4\}), c_{13} + g(3, \{2,4\}), c_{14} + g(4, \{2,3\})\}$

$= \min\{12 + 12, 5 + 22 + 7 + 20\}$

$= \min\{24, 27, 27\}$

$= 24 \quad j = 2$

$\therefore$ min cost of the tour is : 24

Path := $1 \rightarrow 2 \rightarrow 4 \rightarrow 3 \rightarrow 1$

Tour can be constructed if we retain with each $g(i, s)$ the value of "j" that minimizes RHS of $g(i, s)$

$g(1, \{2,3,4\}) = 24 \quad j = 2$

$g(2, \{3,4\}) = 12 \quad j = 4$

$g(4, \{3\}) = 6 \quad j = 3$

$g(3, \phi) = 4 \quad j = 1$

$\therefore$ path $= 1 \rightarrow 2 \rightarrow 4 \rightarrow 3 \rightarrow 1$

min cost = 24

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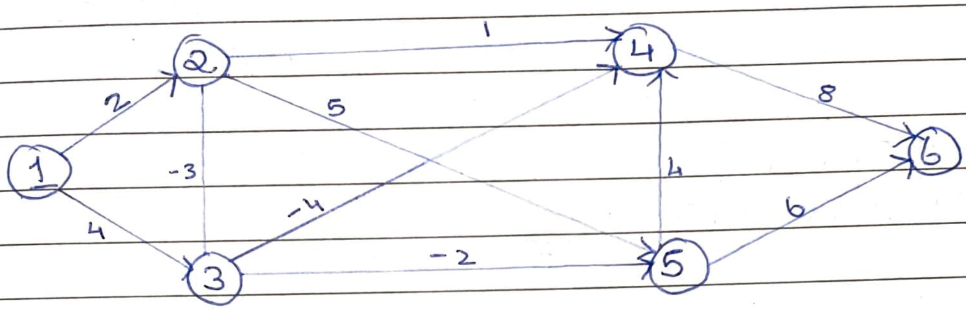
Q.4] find the shortest path from node 1 to every other node in the graph using the Bellman and ford algorithm



	$dist^K[1 \cdots 6]$					
K	1	2	3	4	5	6
1	0	2	4	∞	∞	∞
2	0	2	-1	0	2	∞
3	0	2	-1	-5	-3	8
4	0	2	-1	-5	-3	-3
5	0	2	-1	-5	-3	-3

For K = 2

* $dist^2[2] = \min\{dist^1[2], \min_{1,3,4,5,6}\{dist^1[1] + cost[1,2],$
$dist^1[3] + cost[3,2],$
$dist^1[4] + cost[4,2],$
$dist^1[5] + cost[5,2],$
$dist^1[6] + cost[6,2]\}$

$= \min\{2, \min\{2, 4+\infty, \infty+\infty, \infty+\infty, \infty+\infty\}\}$

$= \min\{2, 2\} = \underline{\underline{2}}$

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* $dist^2[3] = \min\{dist^1[3], \min_{i = 1,2,4,5,6} \{dist^1[1] + cost[1,3]$
$dist^1[2] + cost[2,3]$
$dist^1[4] + cost[4,3]$
$dist^1[5] + cost[5,3]$
$dist^1[6] + cost[6,3]\}\}$

$= \min\{4 \min\{4, 2-3, \infty+\infty, \infty+\infty, \infty+\infty\}\}$
$= \min\{4, -1\} = \underline{\underline{-1}}$

* $dist^2[4] = \min\{dist'[4], \min_{i = 1,2,3,5,6} \{dist^1[1] + cost(1,4),$
$dist^1[2] + cost(2,4)$
$dist^1[3] + cost(3,4)$
$dist^1[5] + cost(5,4)$
$dist'[6] + cost(6,4)\}$

$= \min\{\infty, \min\{\infty, 3, \infty+4, \infty+\infty\}\}$
$= \min\{\infty, 0\} = 0$ //

* $dist^2[5] = \min\{dist'[5], \min_{i = 1,2,3,4,6} \{dist^1[1] + cost(1,5),$
$dist'[2] + cost(2,5),$
$dist'[3] + cost(3,5),$
$dist'[4] + cost(4,5),$
$dist'[6] + cost(6,5)\}$

$= \min\{\infty, \min\{\infty; 7, 2, \infty+\infty, \infty+\infty\}\}$
$= \min\{\infty, 2\} = \underline{\underline{2}}$

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* $dist^2[6] = \min\{ dist^2[1] + cost(1,6),$
$\quad\quad\quad\quad\quad\quad\quad dist^1[2] + cost(2,6),$
$\quad\quad\quad\quad\quad\quad\quad dist^2[3] + cost(3,6),$
$\quad\quad\quad\quad\quad\quad\quad dist^1[4] + cost(4,6),$
$\quad\quad\quad\quad\quad\quad\quad dist^1[5] + cost(5,6), \}$

(with $i = 1,2,3,4,5$ under min)

$= \min\{\infty, \min\{\infty, 2+\infty, 4+\infty, \infty+8, \infty+6\}\}$
$= \min\{\infty, \infty\} = \underline{\underline{\infty}}$

<u>For $k = 3$</u>

* $dist^3[2] = \min\{ dist^2[2] \min_{i = 1,3,4,5,6}\{ dist^2[1] + cost(1,2)$
$\quad\quad dist^2[3] + cost(3,2)$
$\quad\quad dist^2[4] + cost(4,2)$
$\quad\quad dist^2[5] + cost(5,2)$
$\quad\quad dist^2[6] + cost(6,2)\}$

$= \min\{2, \min\{2, -1+\infty, \infty, 2+\infty, \infty+\infty\}\}$
$= \min\{2,2\} = \underline{\underline{2}}$

* $dist^3[3] = \min\{-1, \min_{i = 1,2,4,5,6}\{4, -1, 0+\infty; 7+\infty, \infty+\infty\}\}$
$= \min\{-1, -1\} = \underline{\underline{-1}}$

* $dist^3[4] = \min\{0, \min_{i = 1,2,3,5,6}\{\infty, 3, -5, 11, \infty\}\}$
$= \min\{0, -5\} = \underline{\underline{-5}}$

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* $dist^3[5] = \min\{7, \min_{1,2,3,4,6}\{\infty, 7, -3, \infty, \infty\}\}$

$= \min\{7, -3\} = -3$

* $dist^3[6] = \min\{\infty, \min_{1,2,3,4,5}\{\infty \cancel{2} + \infty, -1+\infty, 8, 13\}\}$

$= \min\{\infty, 8\} = 8$

For K = 4

* $dist^4[2] = \min\{dist^3[2], \min_{1,3,4,5,6}\{2, 6, -5+\infty, -3+\infty, 8+\infty\}\}$

$= \min\{2, 2\} = 2$

* $dist^4[3] = \min\{-1, \min_{1,2,4,5,6}\{3, 4, -5+\infty, -3+\infty, 8+\infty\}\}$

$= \min\{-1, 3\} = -1$

* $dist^4[4] = \min\{-5, \min_{1,2,3,5,6}\{\infty, 3, -5, 1, 8+\infty\}\}$

$= \min\{-5, -5\} = -5$

* $dist^4[5] = \min\{-3, \min_{1,2,3,4,6}\{\infty, 7, -3, -5+\infty, 8+\infty\}\}$

$= \min\{-3, -3\} = -3$

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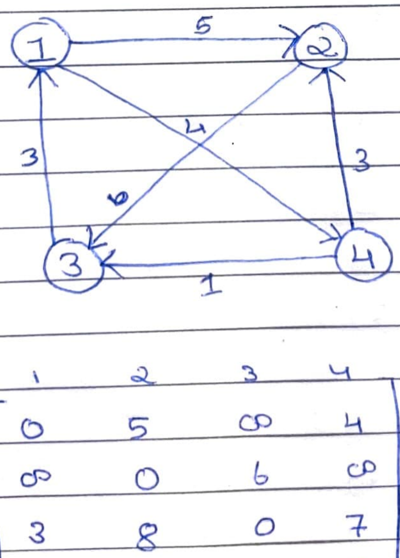
* $dist^4[6] = \min\{8, \min_{1,2,3,4,5}\{\infty, 2+\infty, -1+\infty, 3, -3\}\}$

$= \min\{8, -3\} = -3$

Q5] Find all pair shortest path for the directed graph $G = (V, E, W)$ where $V = \{1, 2, 3, 4\}$
$E = \{<1,2>, <1,4>, <4,3>, <4,2>, <2,3>, <3,1>\}$ &
corresponding weights on edges are $W = \{5, 4, 1, 3, 6, 3\}$

[write the Algorithm as well]



$$A^0 = \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{bmatrix} 0 & 5 & \infty & 4 \\ \infty & 0 & 6 & \infty \\ 3 & \infty & 0 & \infty \\ \infty & 3 & 1 & 0 \end{bmatrix}$$

* $$A^1 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 5 & \infty & 4 \\ 2 & \infty & 0 & 6 & \infty \\ 3 & 3 & 8 & 0 & 7 \\ 4 & \infty & 3 & 1 & 0 \end{array}$$

$A^0(2,3) \quad A^0(2,1) + A^0(1,3$

$6 < \infty + \infty$

$A^0(2,4) \quad A^0(2,1) + A^0(1,4)$

$\infty < \infty + 4$

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$A^0(3,2)$ $\quad A^0(3,1) + A^0(1,2)$

$\infty > 3+5 = 8$

$A^0(3,4)$ $\quad A^0(3,1) + A^0(1,4)$

$\infty > 3+4 = 7$

$A^0(4,2)$ $\quad A^0(4,1) + A^0(1,2)$

$3 < \infty + 5$

$A^0(4,3)$ $\quad A^0(4,1) + A^0(1,3)$

$1 < \infty + \infty$

$$* \quad A^2 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 5 & 11 & 4 \\ 2 & \infty & 0 & 6 & \infty \\ 3 & 3 & 8 & 0 & 7 \\ 4 & \infty & 3 & 1 & 0 \end{array}$$

$A^1(1,3)$ $\quad A^1(1,2) + A^1(2,3)$

$\infty > 5+6$

$A^1(1,4)$ $\quad A^1(1,2) + A^1(2,4)$

$4 < 5+\infty$

$A^1(3,1)$ $\quad A^1(3,2) + A^1(2,1)$

$3 < 8+\infty$

$A^1(3,4)$ $\quad A^1(3,2) + A^1(2,4)$

$7 < 8+\infty$

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$A^1(4,1)$ $A^1(4,2) + A^1(2,1)$
$\infty < 3 + \infty$

$A^1(4,3)$ $A^1(4,2) + A^1(2,3)$
$1 < 3 + 6$

$$A^3 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 5 & 11 & 4 \\ 2 & 9 & 0 & 6 & 13 \\ 3 & 3 & 8 & 0 & 7 \\ 4 & 4 & 3 & 1 & 0 \end{array}$$

$A^2(1,2)$ $A^2(1,3) + A^2(3,2)$
$5 < 11 + 8$

$A^2(1,4)$ $A^2(1,3) + A^2(3,4)$
$4 < 11 + 7$

$A^2(2,1)$ $A^2(2,3) + A^2(3,1)$
$\infty > 6 + 3$

$A^2(2,4)$ $A^2(2,3) + A^2(3,4)$
$\infty > 6 + 7$

$A^2(4,1)$ $A^2(4,3) + A^2(3,1)$
$\infty > 1 + 3$

$A^2(4,2)$ $A^2(4,3) + A^2(3,2)$
$3 < 1 + 8$

$$* \quad A^4 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 5 & 5 & 4 \\ 2 & 9 & 0 & 6 & 13 \\ 3 & 3 & 8 & 0 & 7 \\ 4 & 4 & 3 & 1 & 0 \end{array}$$

$A^3(1,2) \qquad A^3(1,4) + A^3(4,2)$

$5 < 4+3$

$A^3(1,3) \qquad A^3(1,4) + A^3(4,3)$

$11 < 14 + 1$

$A^3(2,1) \qquad A^3(2,4) + A^3(4,1)$

$9 < 13+4$

$A^3(3,1) \qquad A^3(3,4) + A^3(4,1)$

$3 < 7+4$

$A^3(2,3) \qquad A^3(2,4) + A^3(4,3)$

$6 < 13 + 1$

$A^3(3,2) \qquad A^3(3,4) + A^3(4,2)$

$8 < 7+3$

$\therefore$ length of shortest paths are given in the following directed graph

$$A^1 = \begin{bmatrix} 0 & 5 & \infty & 4 \\ \infty & 0 & 6 & \infty \\ 3 & 8 & 0 & 7 \\ \infty & 3 & 1 & 0 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 0 & 5 & 11 & 4 \\ \infty & 0 & 6 & \infty \\ 3 & 8 & 0 & 7 \\ \infty & 3 & 1 & 0 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 0 & 5 & 11 & 4 \\ 9 & 0 & 6 & 13 \\ 3 & 8 & 0 & 7 \\ 4 & 3 & 1 & 0 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} 0 & 5 & 5 & 4 \\ 9 & 0 & 6 & 13 \\ 3 & 8 & 0 & 7 \\ 4 & 3 & 1 & 0 \end{bmatrix}$$

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* Algorithm

Algorithm Allpaths (cost, A, n)

```
for i := 1 to n do
for j := 1 to n do
A [i, j] := cost[i, j]; //copy cost into A
for k := 1 to n do
for i := 1 to n do
for j := 1 to n do
A [i, j] := min [A[i, j], A [i, k] + A [k, j]);
}
```

// cost[1:n, 1:n] is cost adjancency matrix of graph with n vertices
// A [i, j] is the cost of the shortest path from vertex i to j
// cost [i, i] = 0.0, for 1 ≤ i ≤ n