

MEAM 211  
**Synthesis of Simple Planar Linkages**

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## 1 Introduction

Planar linkages are used in a wide range of applications for transforming a given input motion into a desired output motion, for transforming a specified input force or torque into a desired output force or torque, or for guiding the motion of a rigid body (or a specific point on a rigid body) through a desired path or trajectory. For example, the linkage of a casement window transforms rotary motion into the desired motion of a window sash so it can be moved from a fully-open to a fully-closed position. The damper on a door offers a resistance to linear motion that is transformed into resistance to rotary motion. The slider crank mechanism transforms the force from the high pressure gases inside a closed cylinder into a torque on a crankshaft in an internal combustion engine. The rotary motion of an input crank is converted into the rocking motion that is suitable for a lawn sprinkler.

We will consider the special class of linkage design problems associated with rigid body guidance and solve this class of problems using four bar linkages. The design problem is stated as follows.

**Problem 1.** *Given  $n$  successive positions and orientations of a rigid body, synthesize a planar four-bar linkage such that its coupler link assumes these  $n$  positions and orientations during the course of the crank motion.*

## 2 Two position synthesis

The two position synthesis problem is the following design problem.

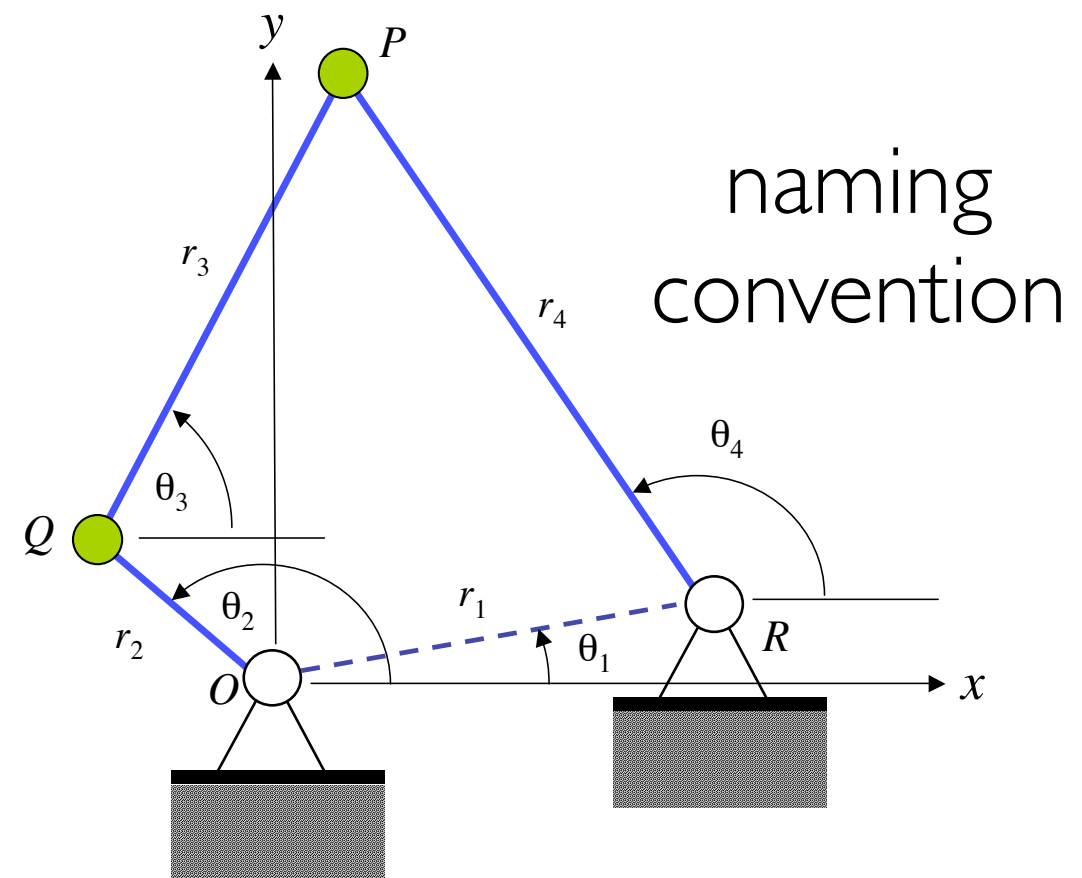
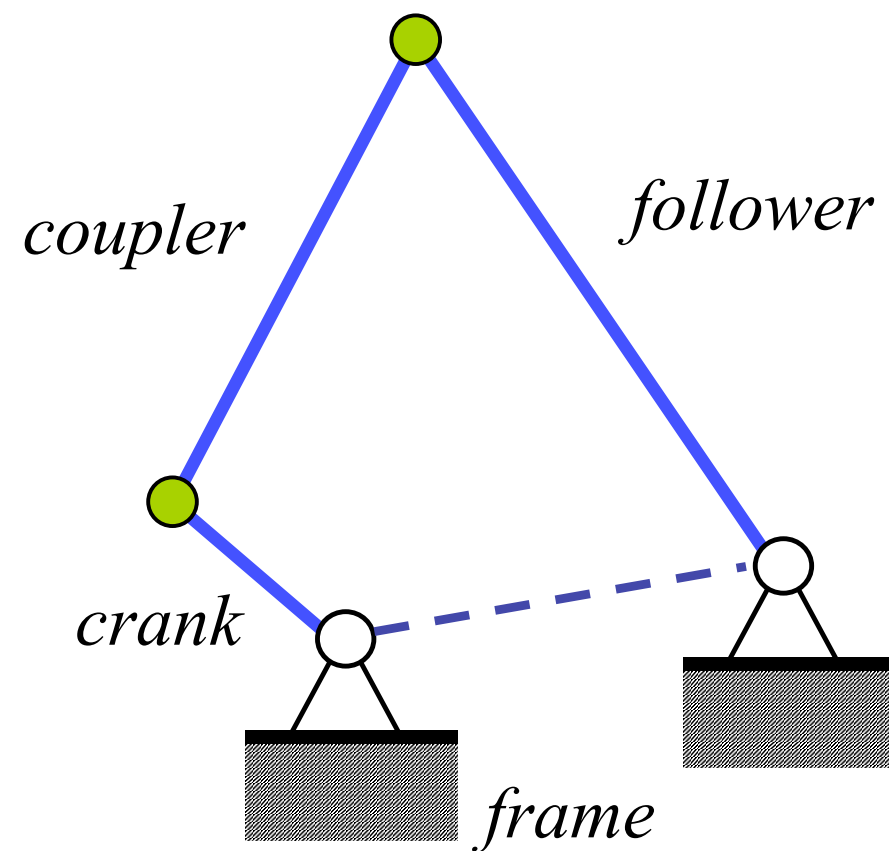
**Problem 2.** *Given two positions and orientations of a coupler link, synthesize a planar four-bar linkage such that its coupler assumes these two positions and orientations during the course of the motion of the linkage.*

The design procedure for this problem is best described graphically. Consider the rigid body guidance problem in Figure 1. The design procedure is as follows:



**Vijay Kumar**  
**Former MEAM 211 Professor**  
**Future SEAS Dean**

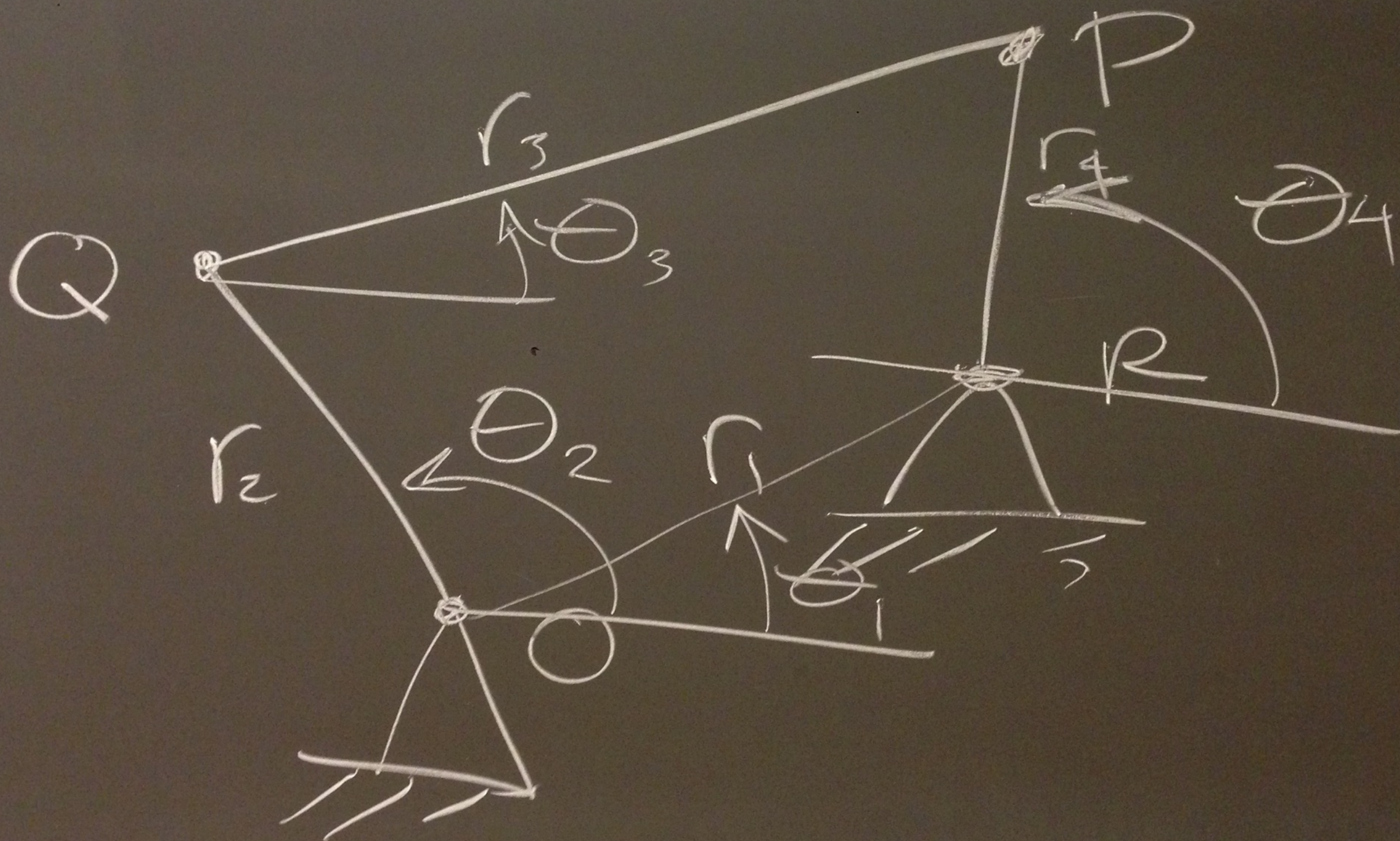
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## Two-Position Synthesis: Problem Statement

Given **two** positions and orientations of a coupler link, synthesize a planar four-bar linkage such that its coupler assumes these two positions and orientations during the course of the motion of the linkage.

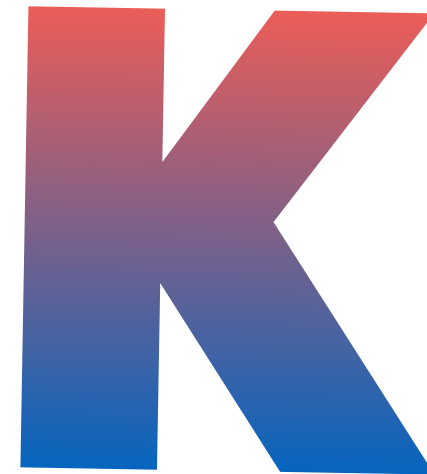






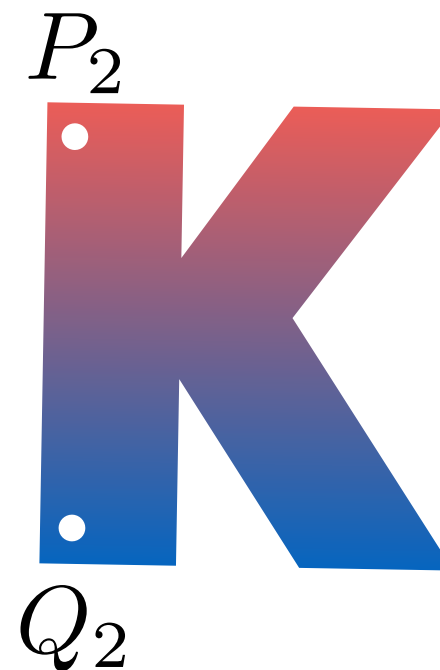
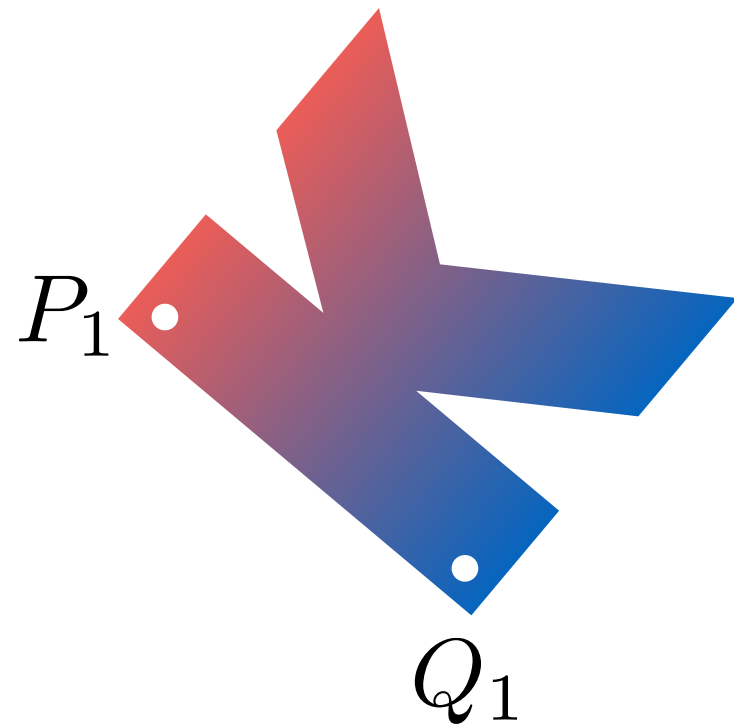
## Step 0

Choose the design of your coupler and select the two desired configurations (positions and orientations).



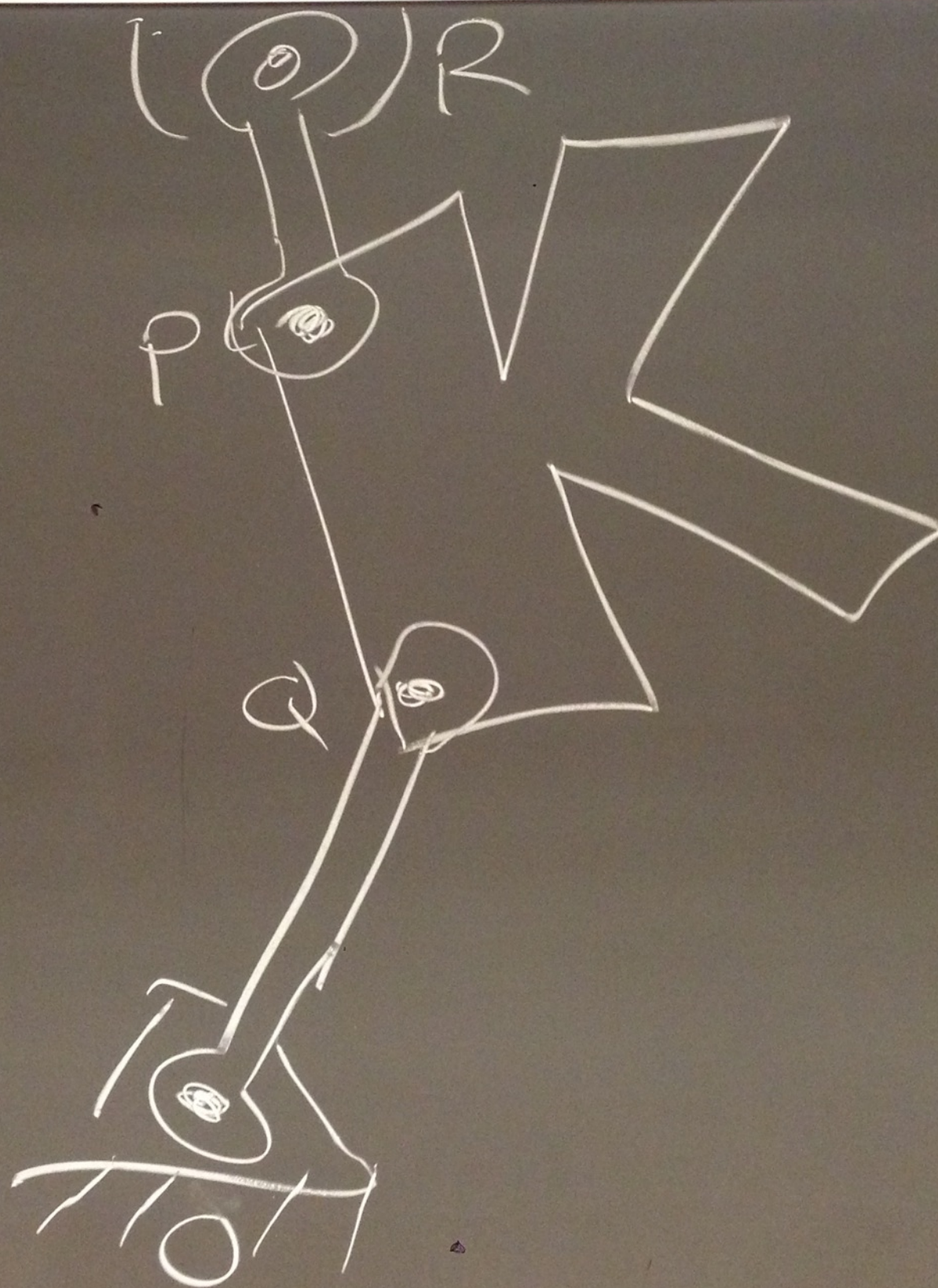
## Step I

Select suitable candidates for points Q and P on the coupler.  
This choice should be guided by practical considerations.



Mark locations of the corresponding points in both poses.

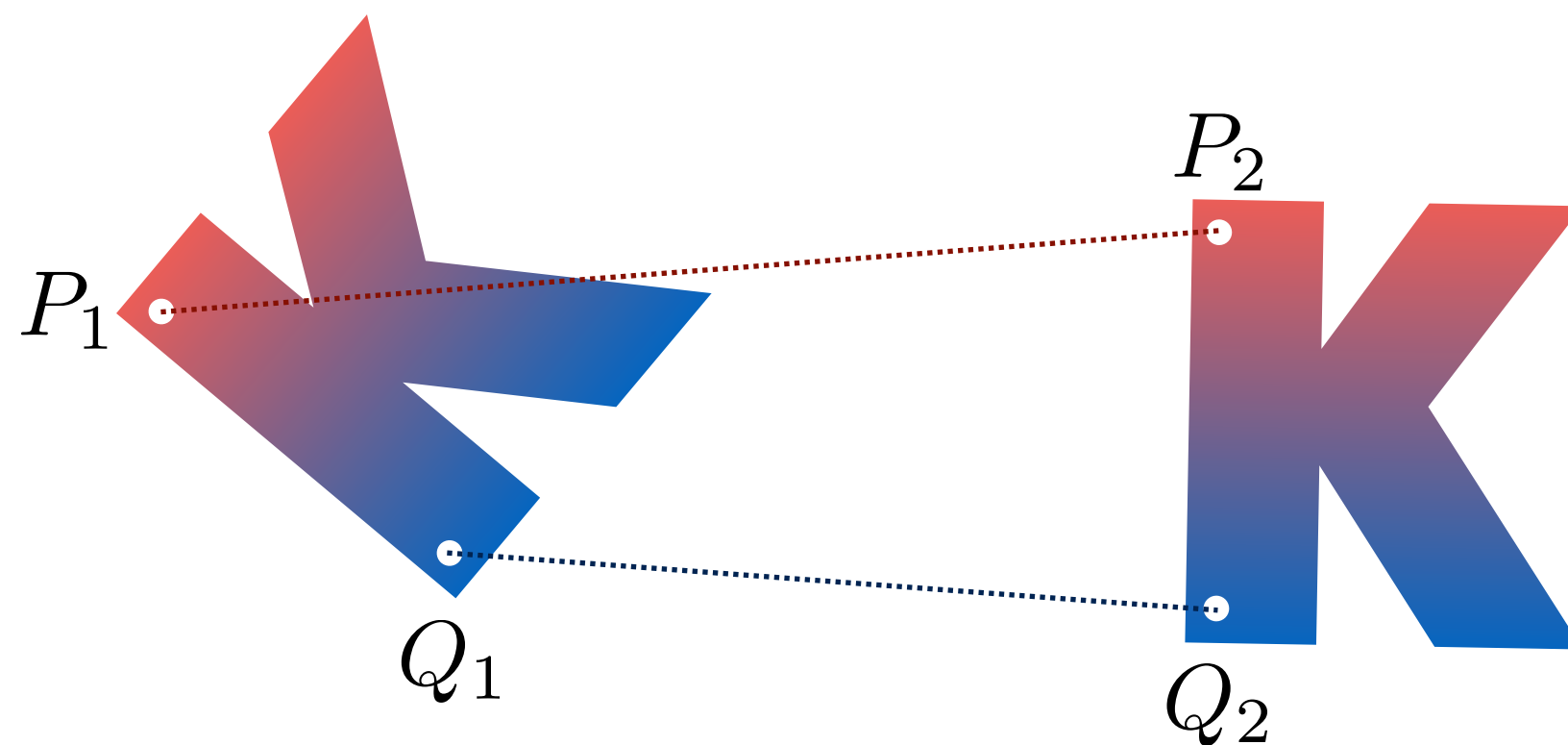






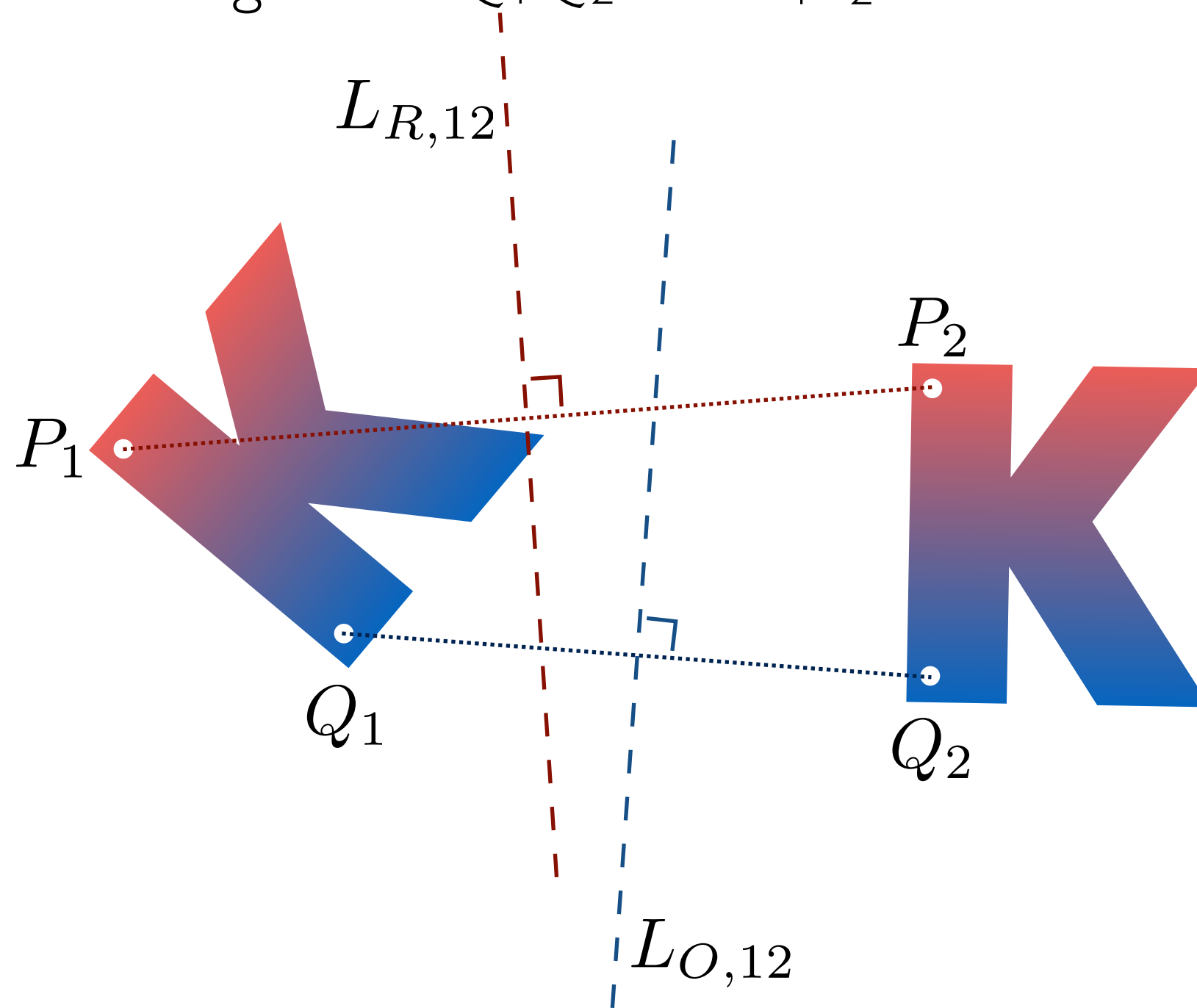
## Step 2

Draw the line segments  $Q_1Q_2$  and  $P_1P_2$ .



## Step 2

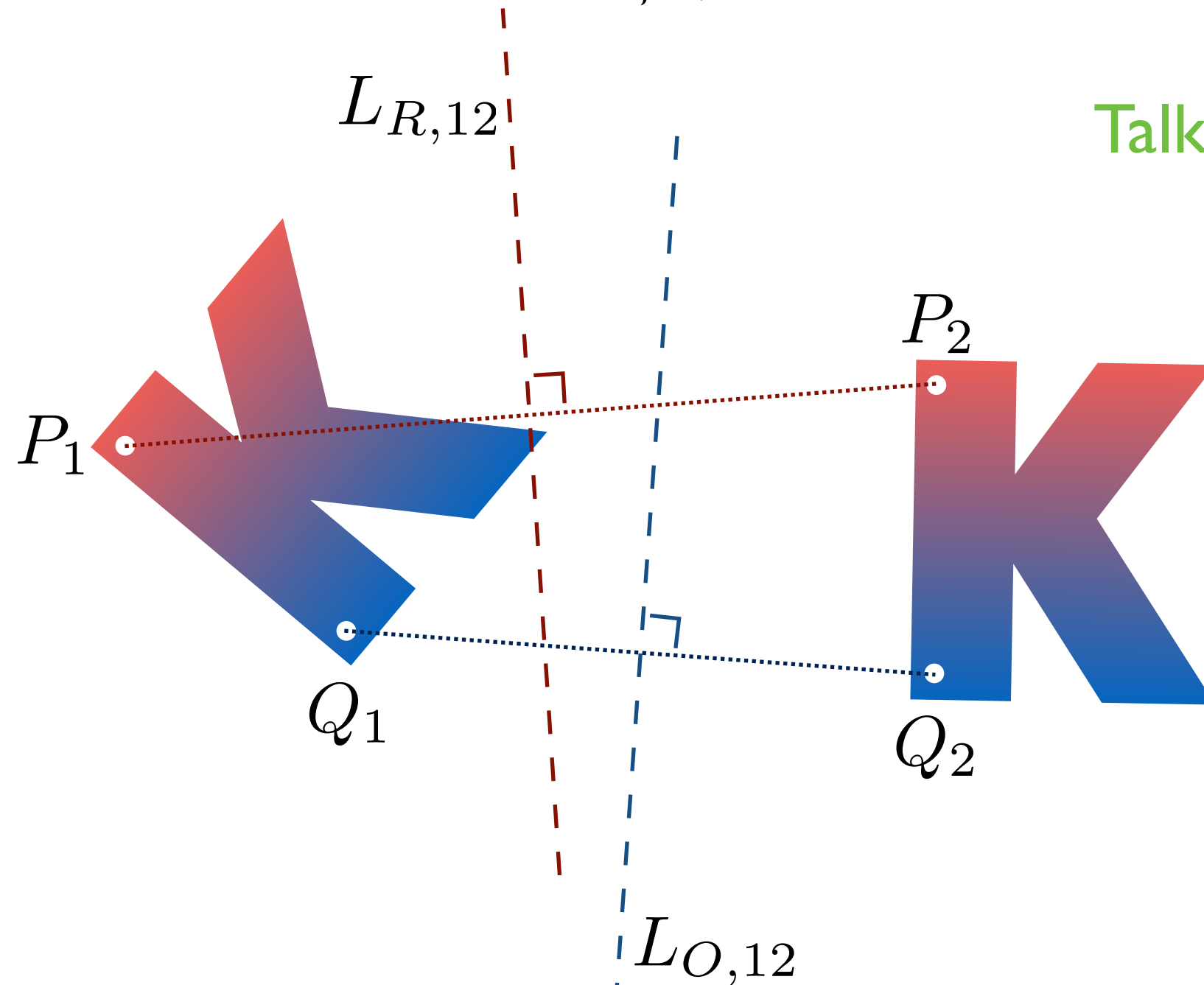
Draw the line segments  $Q_1Q_2$  and  $P_1P_2$ .



Draw the perpendicular bisector of each line segment, and call them  $L_{O,12}$  and  $L_{R,12}$  respectively.



The fixed point  $O$  must lie on the line  $L_{O,12}$ , and the fixed point  $R$  must lie on the line  $L_{R,12}$ .



Why?

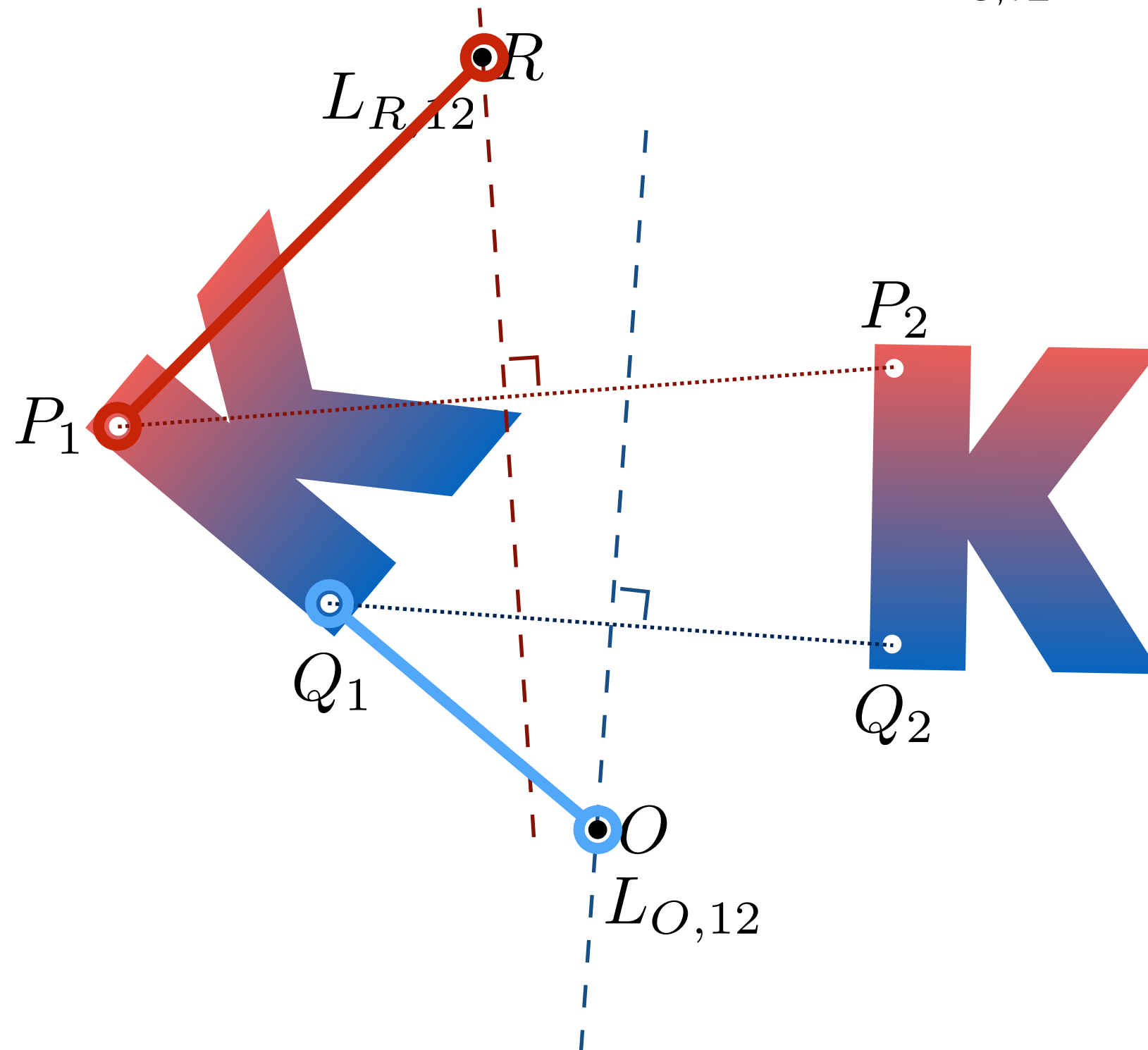
Talk to a partner.

Because the fixed end of each link must be equidistant from the two points chosen for its moving end.

Draw the perpendicular bisector of each line segment, and call them  $L_{O,12}$  and  $L_{R,12}$  respectively.

### Step 3

Choose locations for  $O$  and  $R$  on the lines  $L_{O,12}$  and  $L_{R,12}$ .

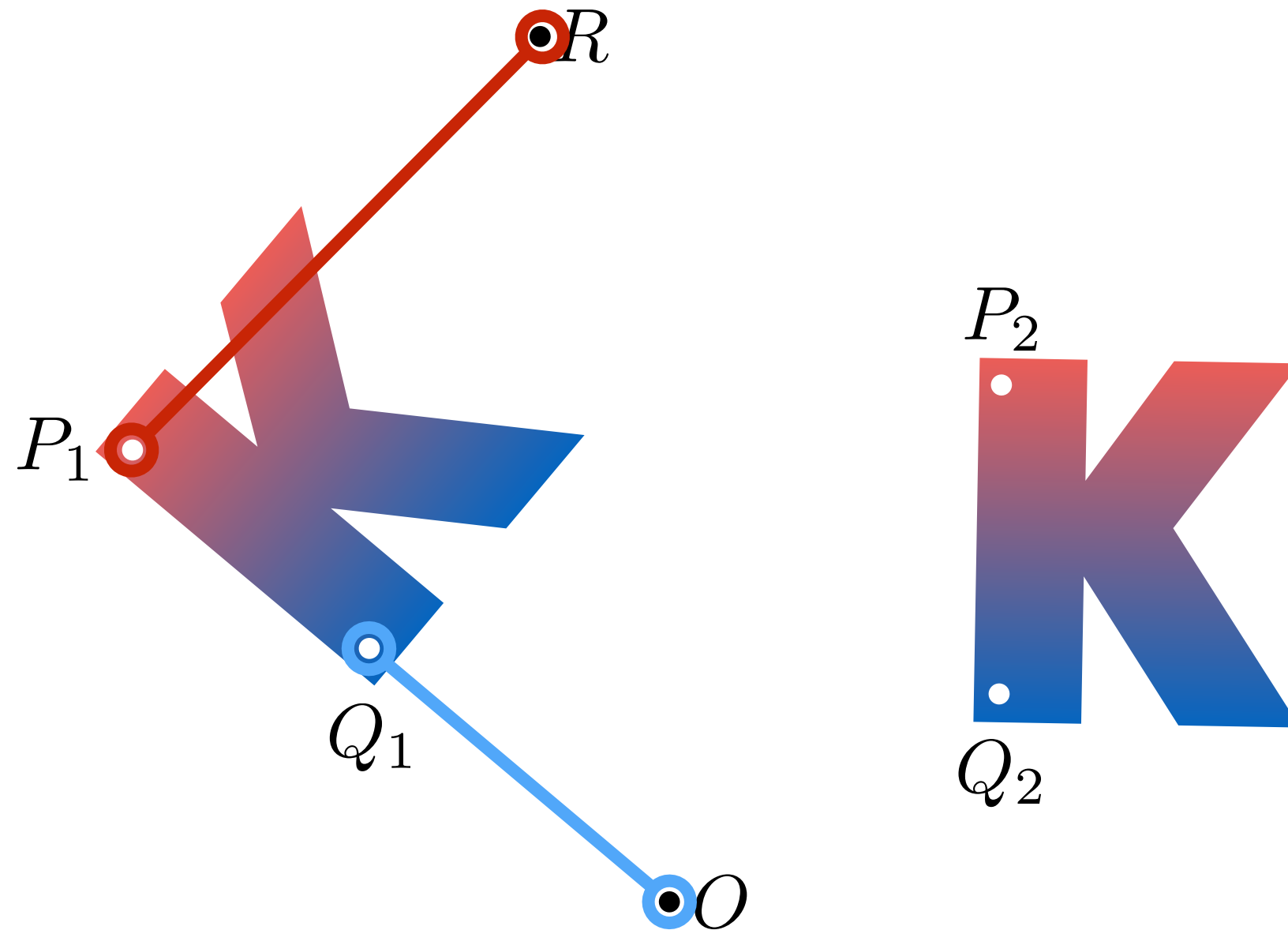


These choices should be guided by practical considerations.

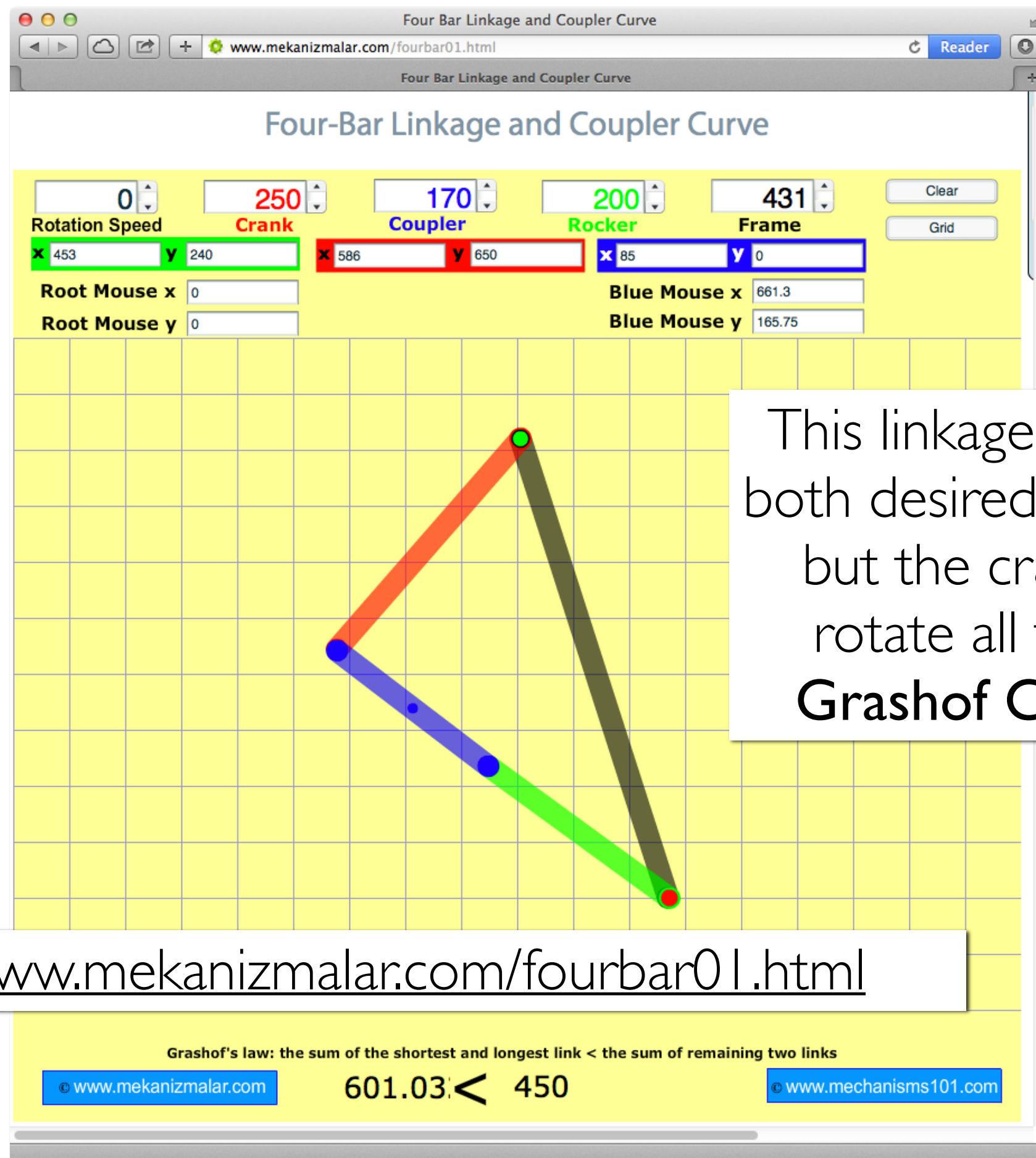


## Step 4

Make a model of your linkage and inspect its motion.



Your model can be either mechanical or computational.



This linkage can reach both desired conditions, but the crank can't rotate all the way!  
**Grashof Condition**

<http://www.mekanizmalar.com/fourbar01.html>