MEAM 211

Synthesis of Simple Planar Linkages

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1 Introduction

Planar linkages are used in a wide range of applications for transforming a given input motion into a desired output motion, for transforming a specified input force or torque into a desired output force or torque, or for guiding the motion of a rigid body (or a specific point on a rigid body) through a desired path or trajectory. For example, the linkage of a casement window transforms rotary motion into the desired motion of a window sash so it can be moved from a fully-open to a fully-closed position. The damper on a door offers a resistance to linear motion that is transformed into resistance to rotary motion. The slider crank mechanism transforms the force from the high pressure gases inside a closed cylinder into a torque on a crankshaft in an internal combustion engine. The rotary motion of an input crank is converted into the rocking motion that is suitable for a lawn sprinkler.

We will consider the special class of linkage design problems associated with rigid body guidance and solve this class of problems using four bar linkages. The design problem is stated as follows.

Problem 1. *Given n successive positions and orientations of a rigid body, synthesize a planar four-bar linkage such that its coupler link assumes these n positions and orientations during the course of the crank motion.*

2 Two position synthesis

The two position synthesis problem is the following design problem.

Problem 2. *Given two positions and orientations of a coupler link, synthesize a planar four-bar linkage such that its coupler assumes these two positions and orientations during the course of the motion of the linkage.*

The design procedure for this problem is best described graphically. Consider the rigid body guidance problem in Figure 1. The design procedure is as follows:

1. Select suitable candidates for points Q and P on the coupler (Figure 2). Mark the locations of the corresponding points Q_1 and P_1 in Position 1 and Q_2 and P_2 in Position 2. Note that this step involves choosing two points and should be guided by practical considerations.

2. Draw the line segments Q_1Q_2 and P_1P_2. Draw the perpendicular bisector of each line segment (Figure 3). Call these perpendicular bisectors $L_{O,12}$ and $L_{R,12}$ respectively.

3. The center point or the fixed pivot, O, must lie on the line $L_{O,12}$ (why?). Similarly, the center point R, must lie on the line $L_{R,12}$. See Figure 4. Chose O and R on the lines $L_{O,12}$ and $L_{R,12}$ respectively. Once again this choice might need to be guided by practical considerations. Once these points are chosen, the four-bar linkage is uniquely determined. See Figure 5.

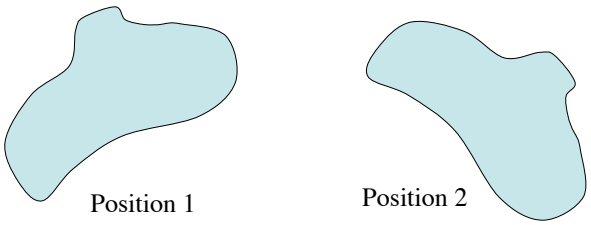

Figure 1: Rigid body guidance through two specified positions.

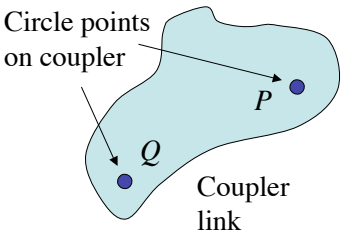

Figure 2: The choice of two circle points, P and Q, on a coupler for rigid body guidance.

3 Three position synthesis

The three position synthesis problem is the following design problem.

Problem 3. *Given three positions and orientations of a coupler link, synthesize a planar four-bar linkage such that its coupler assumes these three positions and orientations during the course of the motion of the linkage.*

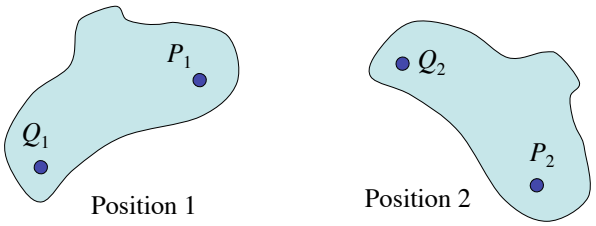

Figure 3: The locations of the circle points, P and Q, at the two specified positions.

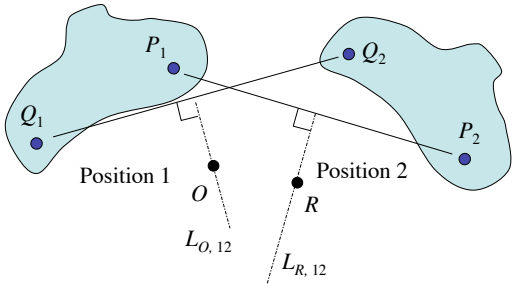

Figure 4: The center points or the fixed pivots, O and R, must lie on the perpendicular bisectors of the line segments Q_1Q_2 and P_1P_2 respectively. Thus O and R, can be chosen to lie anywhere on the lines $L_{O,12}$ and $L_{R,12}$ respectively.

Once again we will solve this problem graphically. Consider the rigid body guidance problem in Figure 6. The design procedure is as follows:

1. Select suitable candidates for points Q and P on the coupler as we did before in Figure 2. Mark the locations of the corresponding points Q_1 and P_1 in Position 1, Q_2 and P_2 in Position 2 and Q_3 and P_3 in Position 3.

2. Draw the line segments Q_1Q_2 and Q_2Q_3. Draw the perpendicular bisector of each line segment (Figure 7). Call these perpendicular bisectors $L_{O,12}$ and $L_{O,23}$ respectively. The center point or the fixed pivot, O, must lie on the intersection of lines $L_{O,12}$ and $L_{O,23}$.

3. Draw the line segments P_1P_2 and P_2P_3. Draw the perpendicular bisector of each line segment (Figure 7). Call these perpendicular bisectors $L_{R,12}$ and $L_{R,23}$ respectively. The center point or the fixed pivot, R, must lie on the intersection of lines $L_{R,12}$ and $L_{R,23}$.

4. Once the fixed pivots O and R are determined, the four-bar linkage is uniquely determined. See Figure 8.

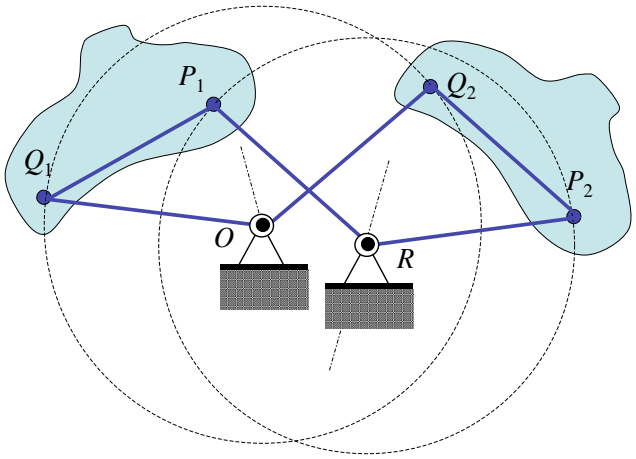

Figure 5: Once the location of O and R on the lines $L_{O,12}$ and $L_{R,12}$ is chosen, the four-bar linkage is uniquely determined.

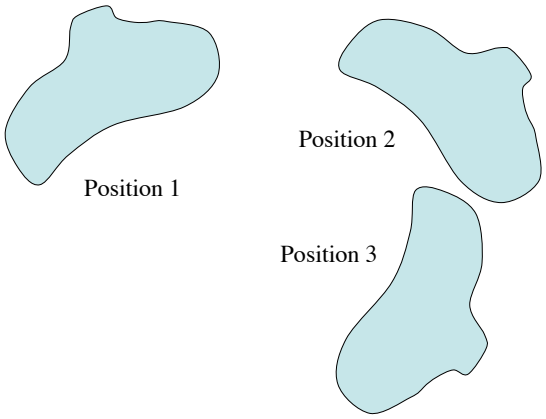

Figure 6: Rigid body guidance through three specified positions.

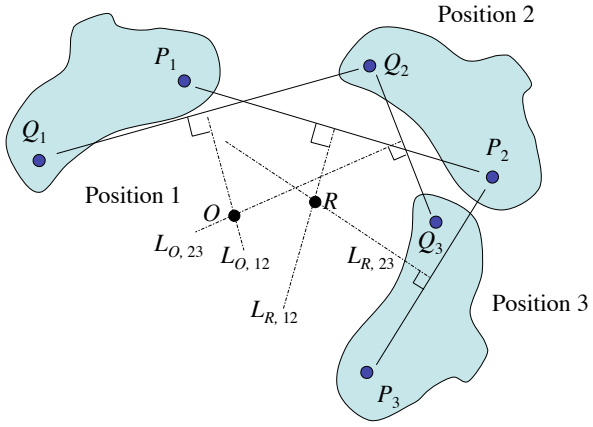

Figure 7: The fixed pivot O must lie on the perpendicular bisectors of the line segments Q_1Q_2 and Q_2Q_3. Similarly the fixed pivot R must lie on the perpendicular bisectors of the line segments P_1P_2 P_2P_3. Thus the intersection of $L_{O,12}$ and $L_{O,23}$ determines the location of O and the intersection of $L_{R,12}$ and $L_{R,23}$ determines the location of R.

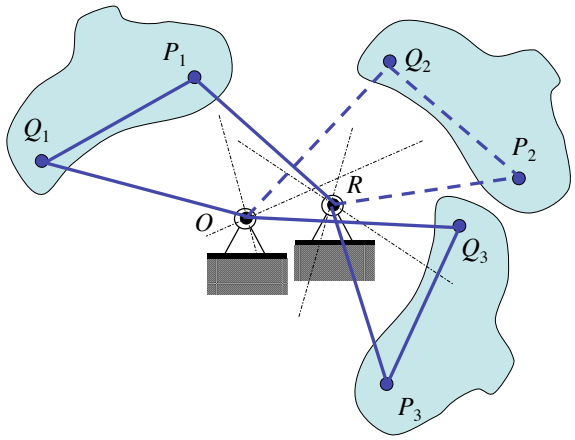

Figure 8: The choice of P and Q leads to the unique determination of O and R and the four-bar linkage $OQPR$.