Announcements

- Homework 1: Search
  - Due yesterday
  - “Show Answer”
- Project 1: Search
  - Due Friday 2/5 at 5pm.
  - Start early and ask questions. It’s longer than most!
  - Optional P1 Mini-Contest: Due Sunday 2/7 at 11:59pm.
- Homework 2: CSPs
  - About to be released! Due Monday, 2/8, at 11:59pm.

Announcements

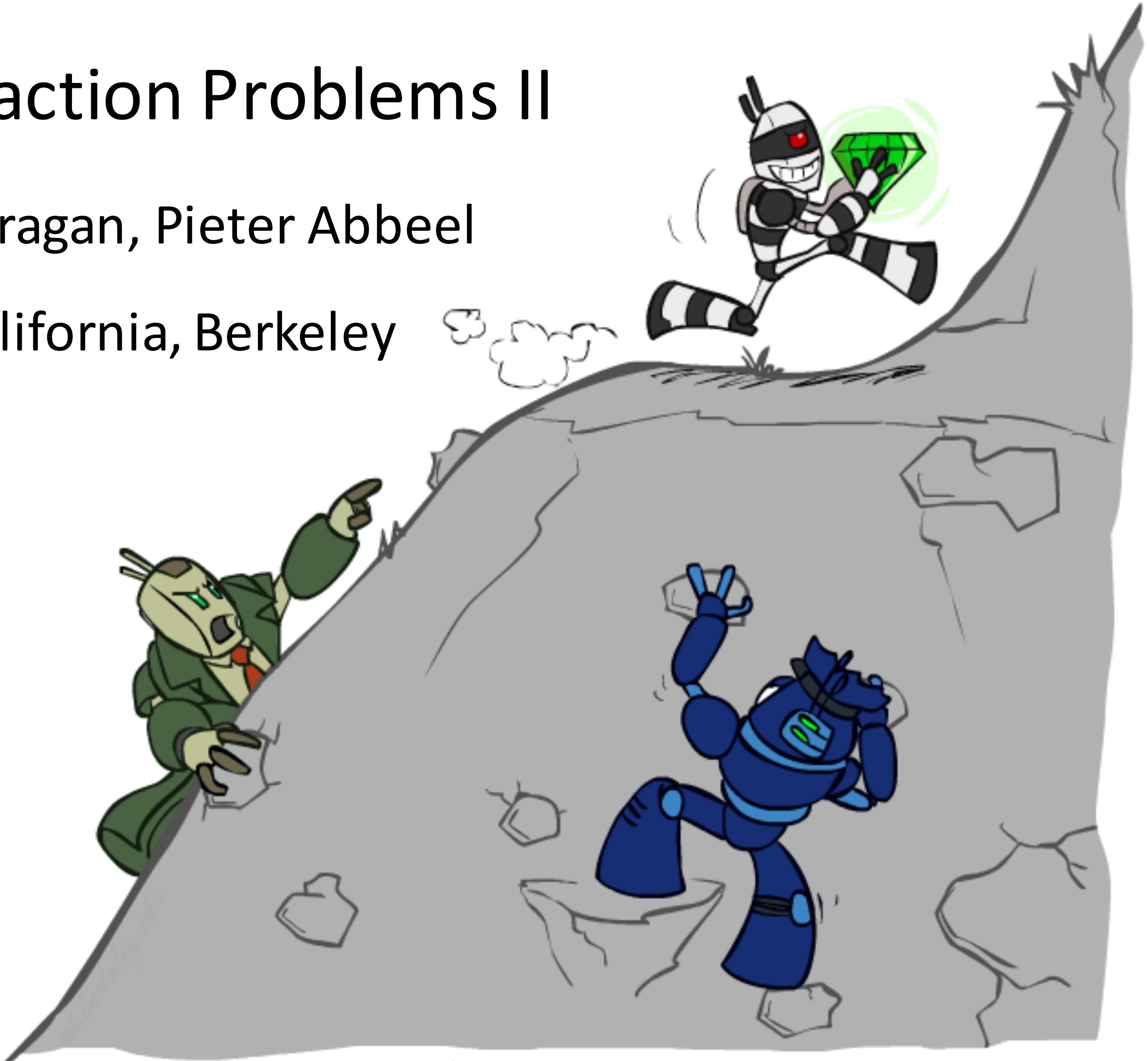
No.	Time	Location	GSI	Type	Sec 1 (Jan 25)	Sec 2 (Feb 1)
102	10-11a	102 Latimer	Christopher	dis	~6	19
104	11-12p	102 Latimer	Christopher	dis	~19	40
106	12-1p	229 Dwinelle	Davis	dis	72	~60
107	1-2p	283 Dwinelle	Davis	dis	47	~45
109	2-3p	105 Dwinelle	Karthik	dis	~40	~45
116	2:30-3:30p	310 Soda	Jacob	dis	12	11
111	3-4p	258 Dwinelle	Karthik	dis	~40	~45
117	3:30-4:30p	310 Soda	Jacob	dis	9	9
113	4-5p	205 Dwinelle	Greg	dis	35	20
118	6-7p	320 Soda	Greg	dis	8	8
101	9-10a	289 Cory	Coline	exam	~11	4
103	10-11a	105 Latimer	Tianhao	exam	~30	~12
105	11-12p	310 Hearst Min	Abhishek	exam	~40	~20
115	12-1p	310 Soda	Tianhao	exam	3	4
108	1-2p	205 Dwinelle	Abhishek	exam	~35	~32
110	2-3p	183 Dwinelle	Weicheng	exam	10	13
112	3-4p	228 Dwinelle	Weicheng	exam	8	1
114	4-5p	105 Dwinelle	Coline	exam	23	1

CS 188: Artificial Intelligence

Constraint Satisfaction Problems II

Instructors: Anca Dragan, Pieter Abbeel

University of California, Berkeley



Slides by Dan Klein, Pieter Abbeel, Anca Dragan (ai.berkeley.edu)

Today

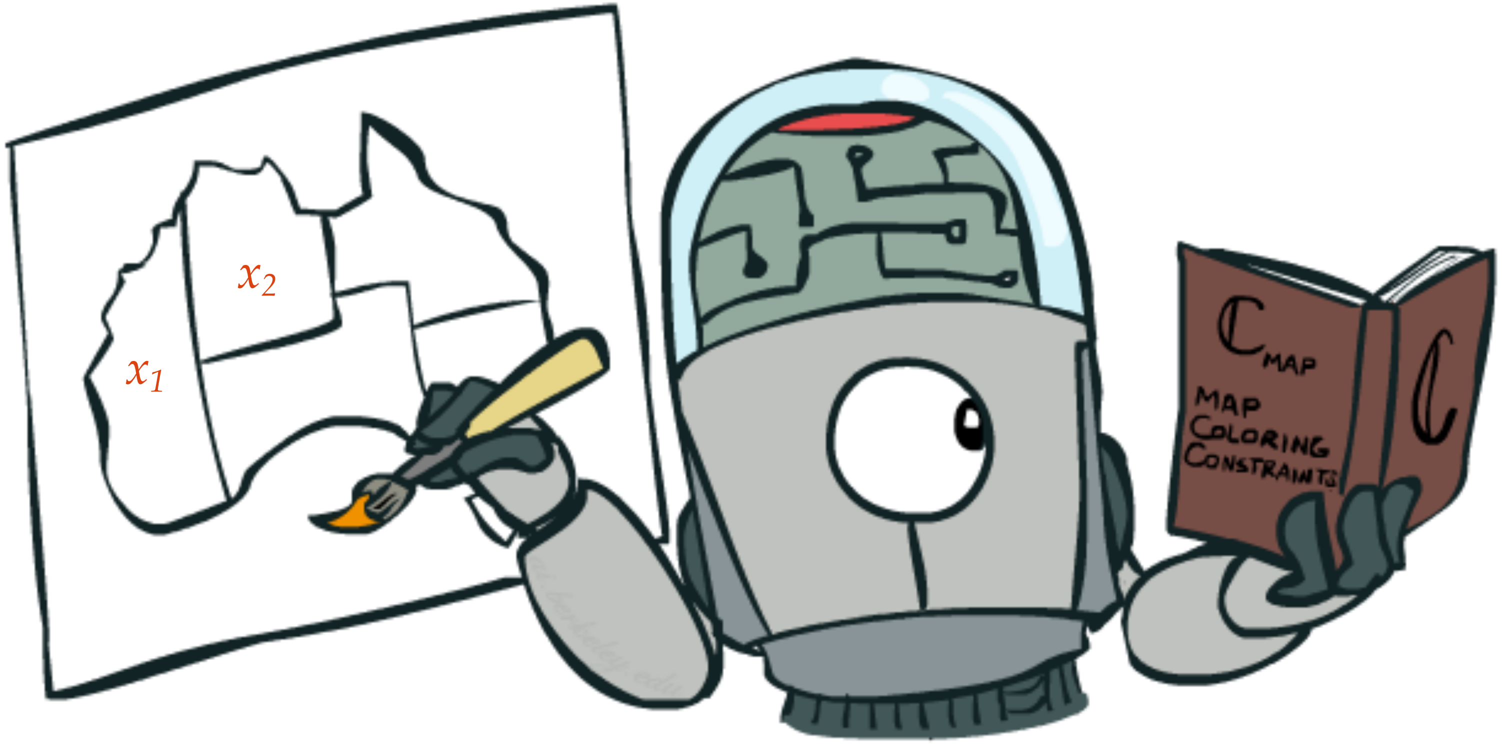
- Efficient Solution of CSPs
- Local Search

Constraint Satisfaction Problems

N variables

domain D

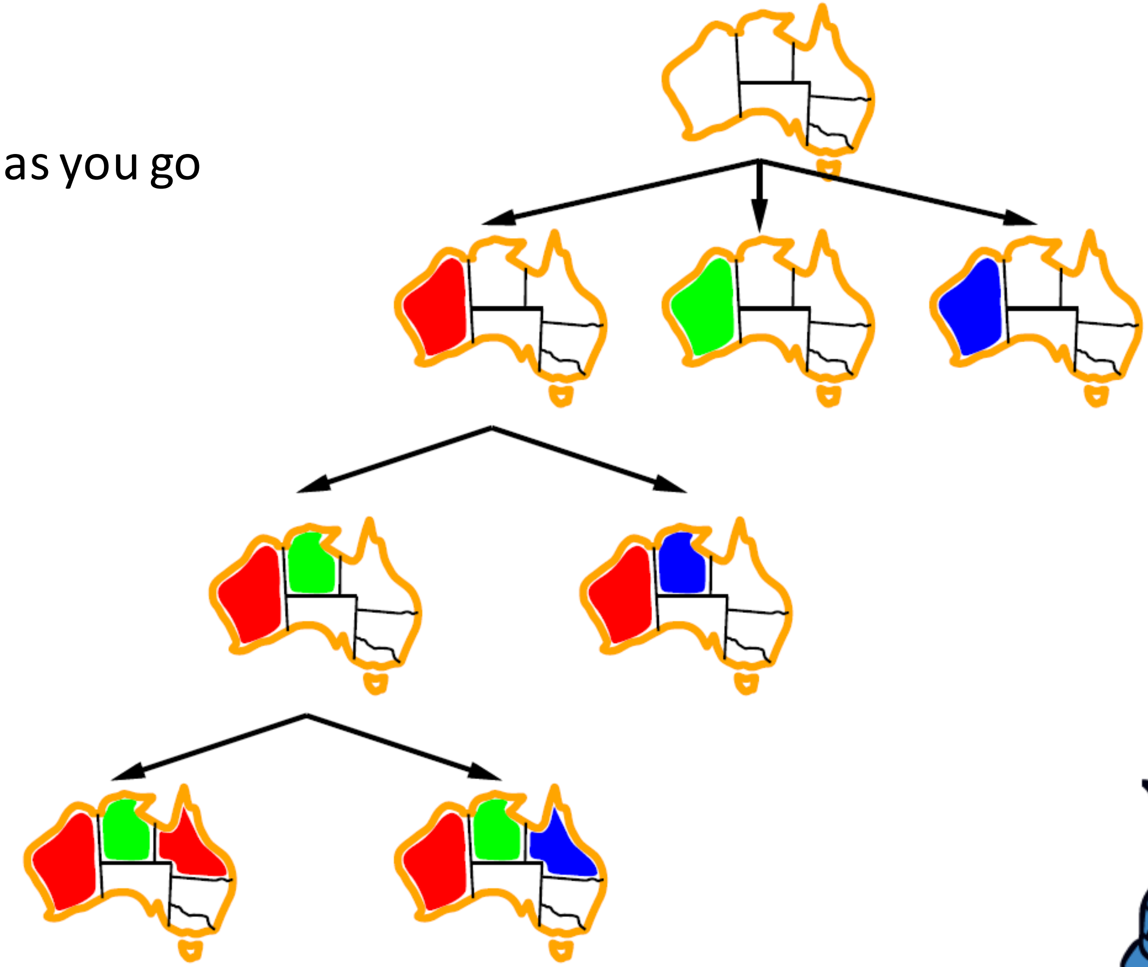
constraints

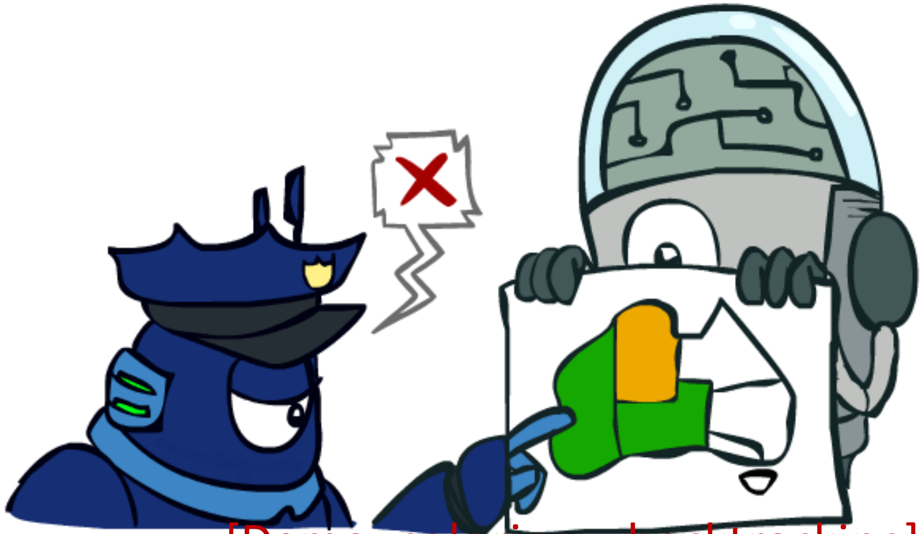


Backtracking Search

1. fix ordering

2. check constraints as you go





[Demo: coloring -- backtracking]

Improving Backtracking

- General-purpose ideas give huge gains in speed
  - … but it's all still NP-hard

- Filtering: Can we detect inevitable failure early?
- Ordering:
  - Which variable should be assigned next? (MRV)
  - In what order should its values be tried? (LCV)

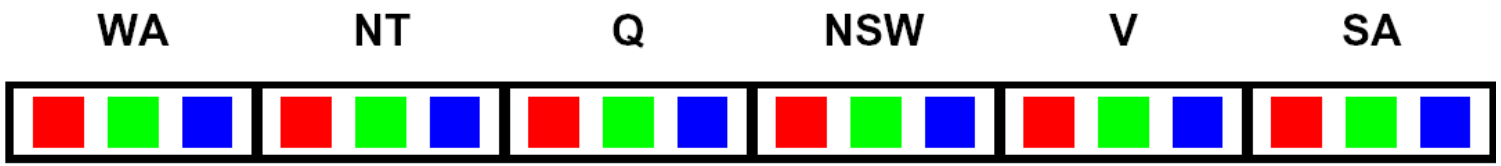
- Structure: Can we exploit the problem structure?

Filtering

Keep track of domains for unassigned variables and cross off bad options

Filtering: Forward Checking

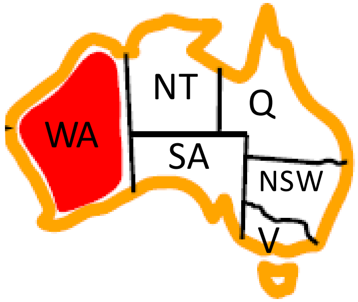
- Cross off values that violate a constraint when added to the existing assignment

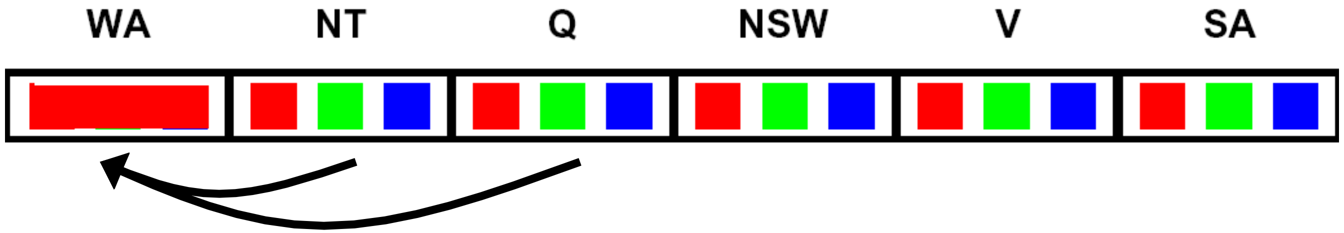


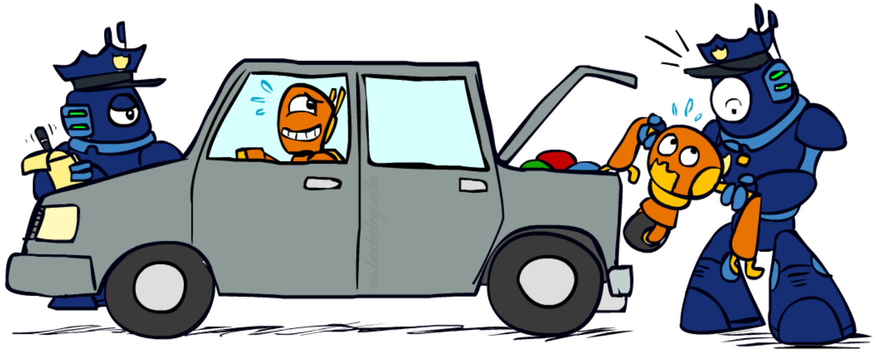
[Demo: coloring -- forward checking]

Consistency of A Single Arc

- An arc X → Y is consistent iff for *every* x in the tail there is *some* y in the head which could be assigned without violating a constraint







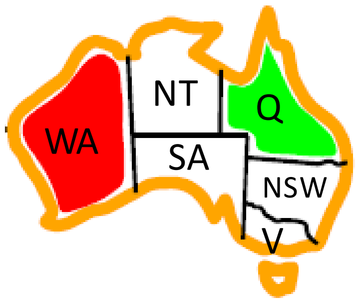
Delete from the tail!

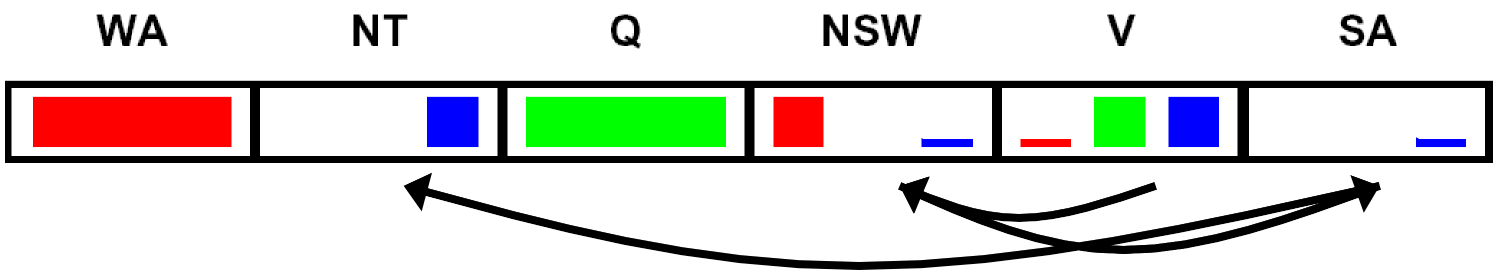
Which arcs do we make consistent in forward checking?

Enforcing consistency of arcs pointing to each new assignment

Arc Consistency of an Entire CSP

- A simple form of propagation makes sure all arcs are consistent:





- Important: If X loses a value, neighbors of X need to be rechecked!
- Arc consistency detects failure earlier than forward checking
- Can be run as a preprocessor or after each assignment
- What's the downside of enforcing arc consistency?

Remember: Delete from the tail!

[Demo: coloring -- forward checking]
[Demo: coloring -- arc consistency]

Enforcing Arc Consistency in a CSP

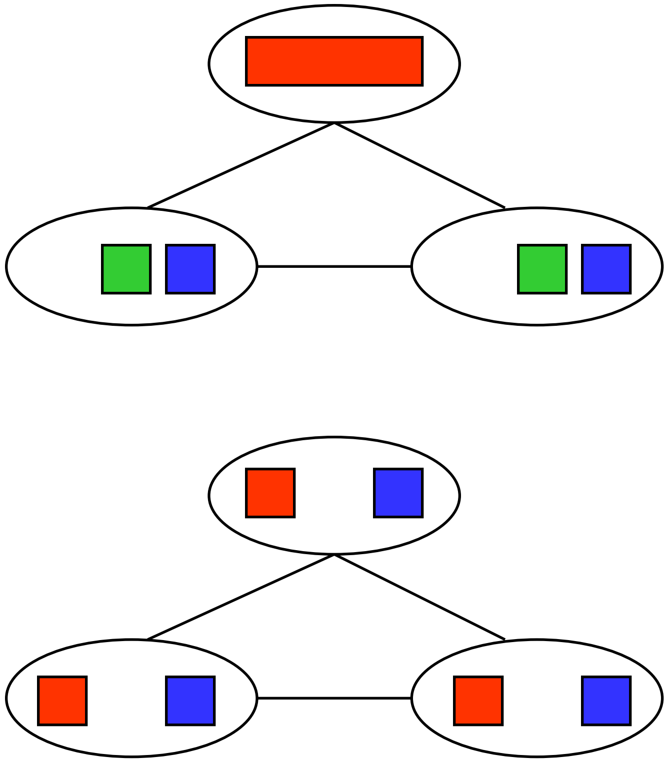
```
function AC-3( csp) returns the CSP, possibly with reduced domains
   inputs: csp, a binary CSP with variables {X1, X2, ..., Xn}
   local variables: queue, a queue of arcs, initially all the arcs in csp

   while queue is not empty do
      (Xi, Xj) ← REMOVE-FIRST(queue)
      if REMOVE-INCONSISTENT-VALUES(Xi, Xj) then
         for each Xk in NEIGHBORS[Xi] do
            add (Xk, Xi) to queue
```

```
function REMOVE-INCONSISTENT-VALUES( Xi, Xj) returns true iff succeeds
   removed ← false
   for each x in DOMAIN[Xi] do
      if no value y in DOMAIN[Xj] allows (x,y) to satisfy the constraint Xi ↔ Xj
         then delete x from DOMAIN[Xi];   removed ← true
   return removed
```

- Runtime: $O(n^2d^3)$, can be reduced to $O(n^2d^2)$
- … but detecting all possible future problems is NP-hard – why?

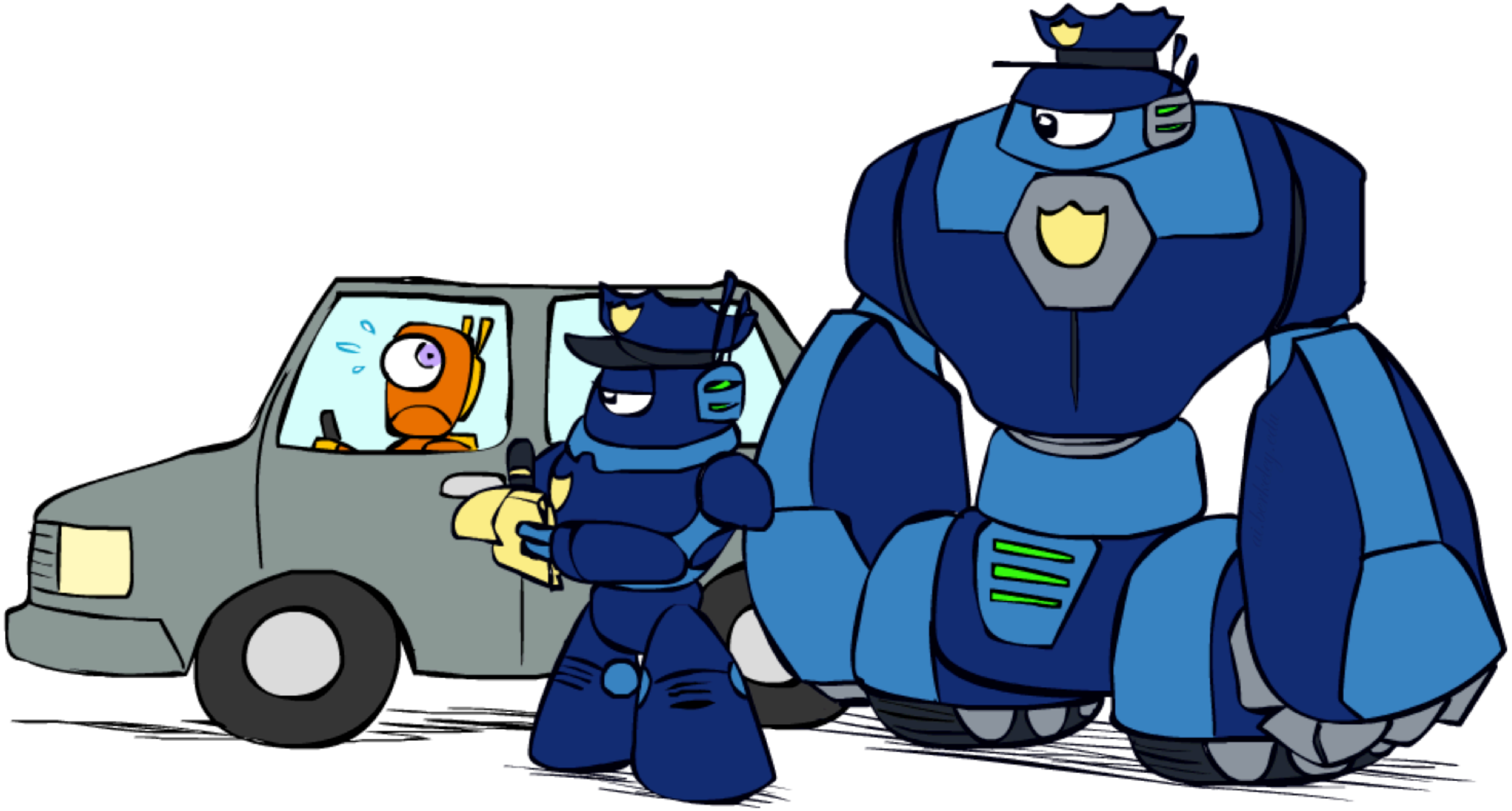
Limitations of Arc Consistency

- After enforcing arc consistency:
  - Can have one solution left
  - Can have multiple solutions left
  - Can have no solutions left (and not know it)



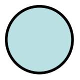
- Arc consistency still runs inside a backtracking search!

K-Consistency

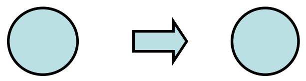


K-Consistency

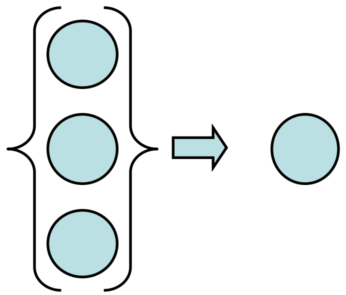
- Increasing degrees of consistency
  - 1-Consistency (Node Consistency): Each single node's domain has a value which meets that node's unary constraints



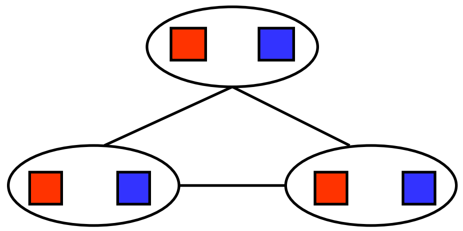
  - 2-Consistency (Arc Consistency): For each pair of nodes, any consistent assignment to one can be extended to the other



  - K-Consistency: For each k nodes, any consistent assignment to k-1 can be extended to the k^{th} node.



- Higher k more expensive to compute
- (You need to know the k=2 case: arc consistency)



Strong K-Consistency

- Strong k-consistency: also k-1, k-2, … 1 consistent
- Claim: strong n-consistency means we can solve without backtracking!
- Why?
  - Choose any assignment to any variable
  - Choose a new variable
  - By 2-consistency, there is a choice consistent with the first
  - Choose a new variable
  - By 3-consistency, there is a choice consistent with the first 2
  - …
- Lots of middle ground between arc consistency and n-consistency! (e.g. k=3, called path consistency)

Improving Backtracking

- General-purpose ideas give huge gains in speed
  - … but it’s all still NP-hard

- Filtering: Can we detect inevitable failure early?

- Ordering:
  - Which variable should be assigned next? (MRV)
  - In what order should its values be tried? (LCV)

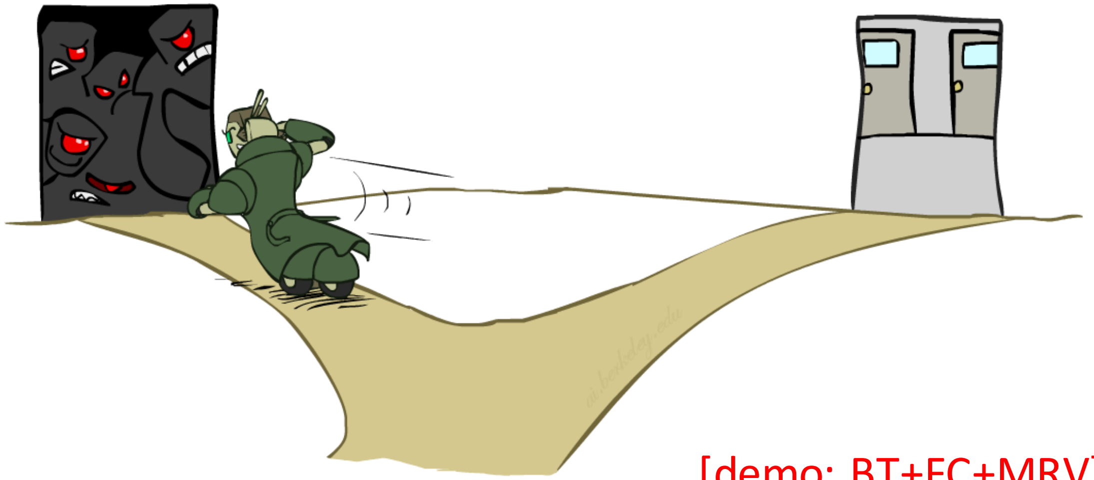
- Structure: Can we exploit the problem structure?

Ordering

Ordering: Minimum Remaining Values

- Variable Ordering: Minimum remaining values (MRV):
  - Choose the variable with the fewest legal left values in its domain

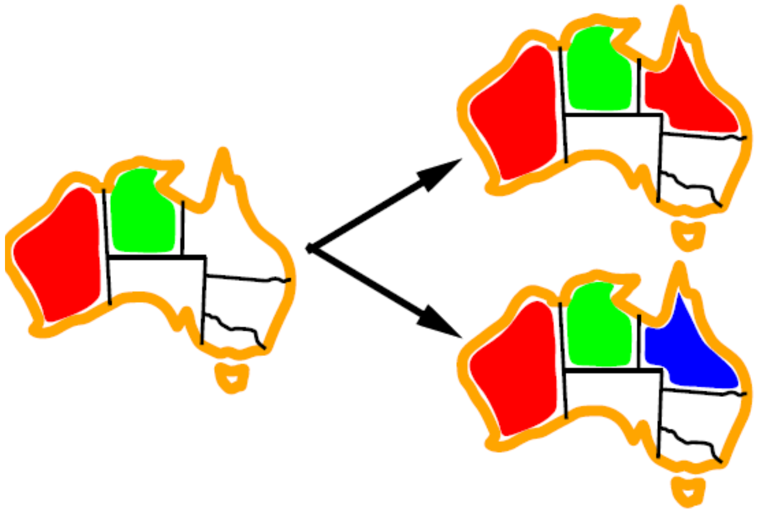
- Why min rather than max?
- Also called "most constrained variable"
- "Fail-fast" ordering



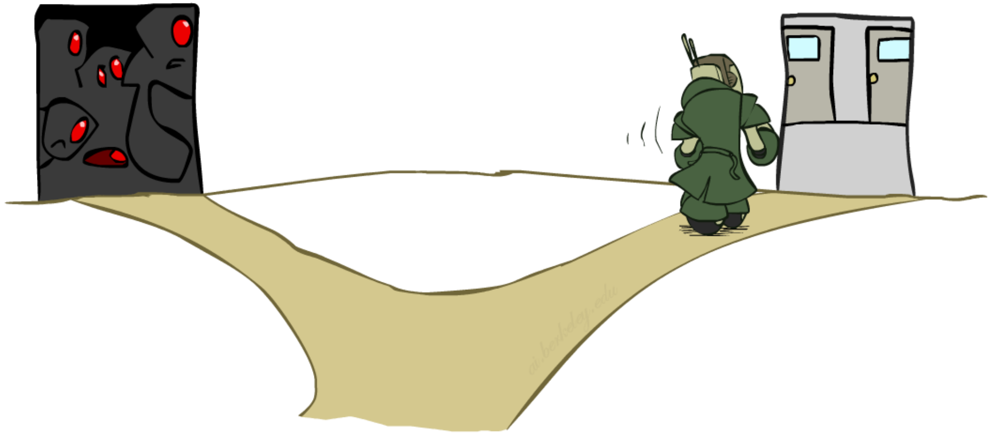
[demo: BT+FC+MRV]

Ordering: Least Constraining Value

- Value Ordering: Least Constraining Value
  - Given a choice of variable, choose the *least constraining value*
  - I.e., the one that rules out the fewest values in the remaining variables
  - Note that it may take some computation to determine this! (E.g., rerunning filtering)



- Why least rather than most?
- Combining these ordering ideas makes 1000 queens feasible



Improving Backtracking

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- Filtering: Can we detect inevitable failure early?

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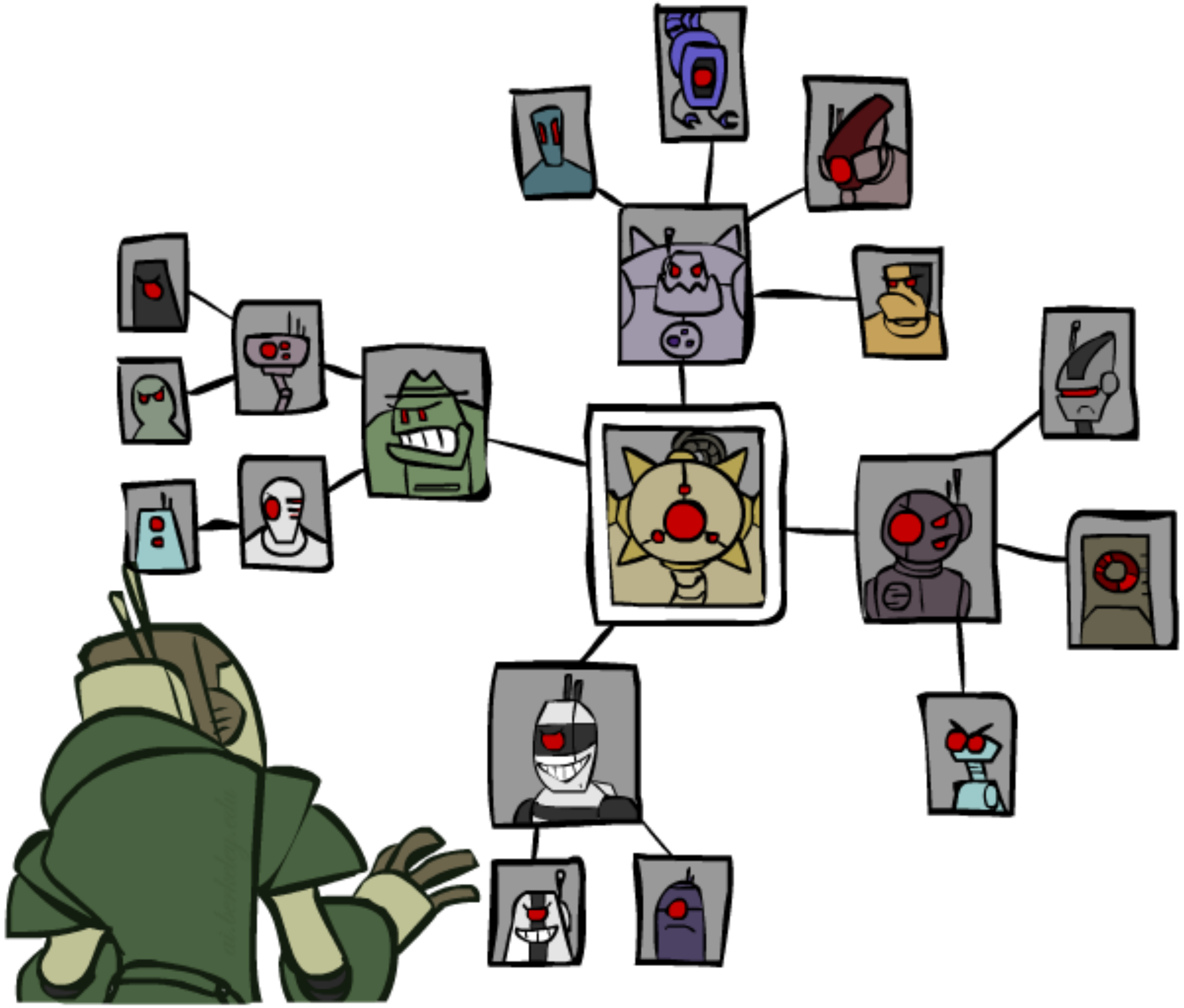
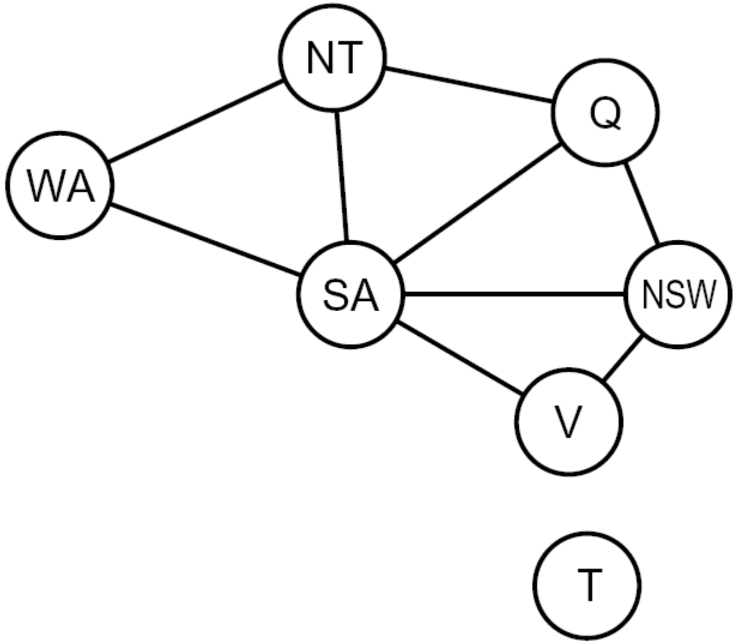
Structure

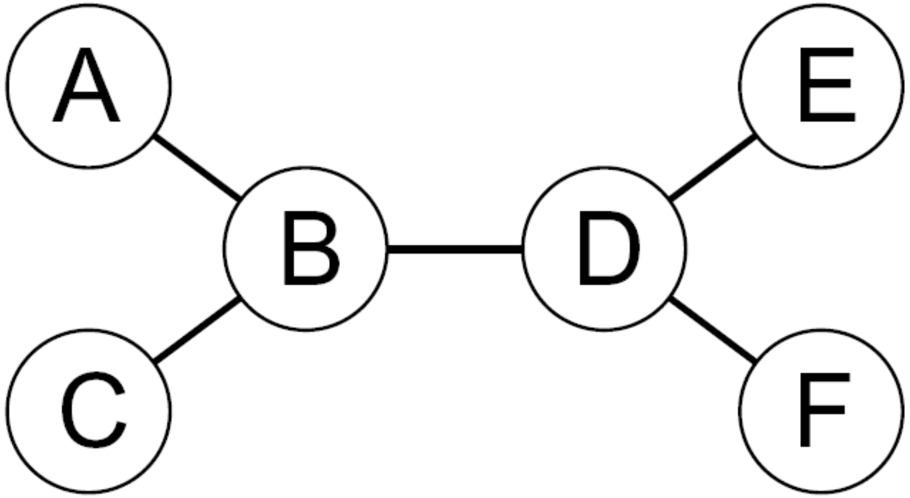


Problem Structure

- Extreme case: independent subproblems
  - Example: Tasmania and mainland do not interact
- Independent subproblems are identifiable as connected components of constraint graph
- Suppose a graph of n variables can be broken into subproblems of only c variables:
  - Worst-case solution cost is $O((n/c)(d^c))$, linear in n
  - E.g., n = 80, d = 2, c =20
  - 2^{80} = 4 billion years at 10 million nodes/sec
  - $(4)(2^{20})$ = 0.4 seconds at 10 million nodes/sec



Tree-Structured CSPs

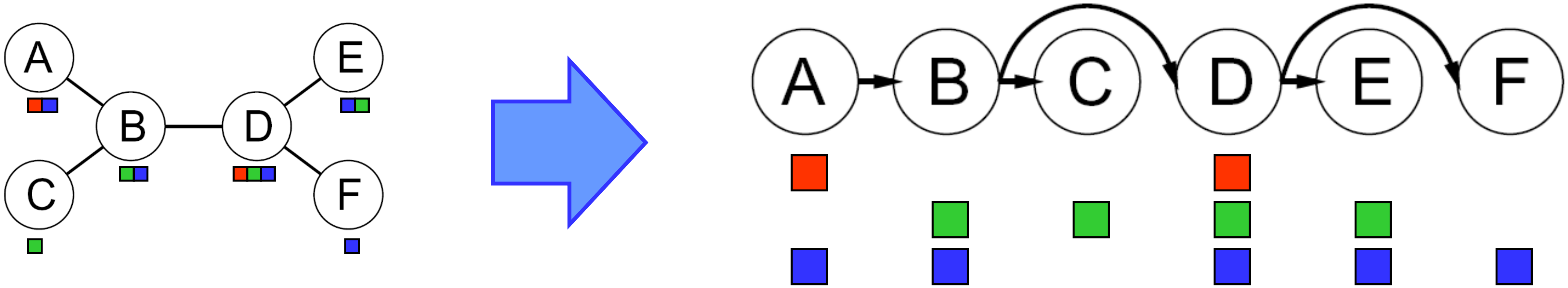


- Theorem: if the constraint graph has no loops, the CSP can be solved in $O(n\ d^2)$ time
  - Compare to general CSPs, where worst-case time is $O(d^n)$

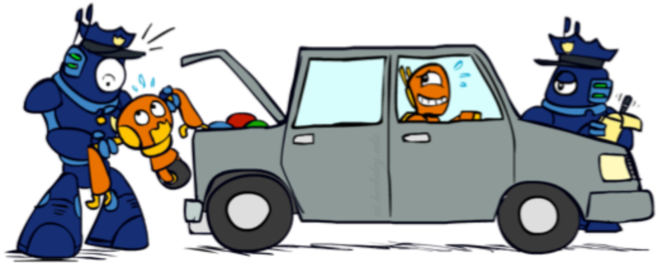
- This property also applies to probabilistic reasoning (later): an example of the relation between syntactic restrictions and the complexity of reasoning

Tree-Structured CSPs

- Algorithm for tree-structured CSPs:
  - Order: Choose a root variable, order variables so that parents precede children

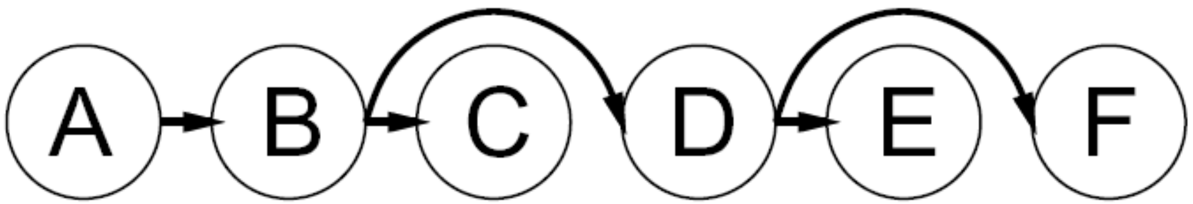


  - Remove backward: For i = n : 2, apply RemoveInconsistent(Parent(X_i),X_i)
  - Assign forward: For i = 1 : n, assign X_i consistently with Parent(X_i)
- Runtime: O(n d^2) (why?)



Tree-Structured CSPs

- Claim 1: After backward pass, all root-to-leaf arcs are consistent
- Proof: Each X→Y was made consistent at one point and Y's domain could not have been reduced thereafter (because Y's children were processed before Y)

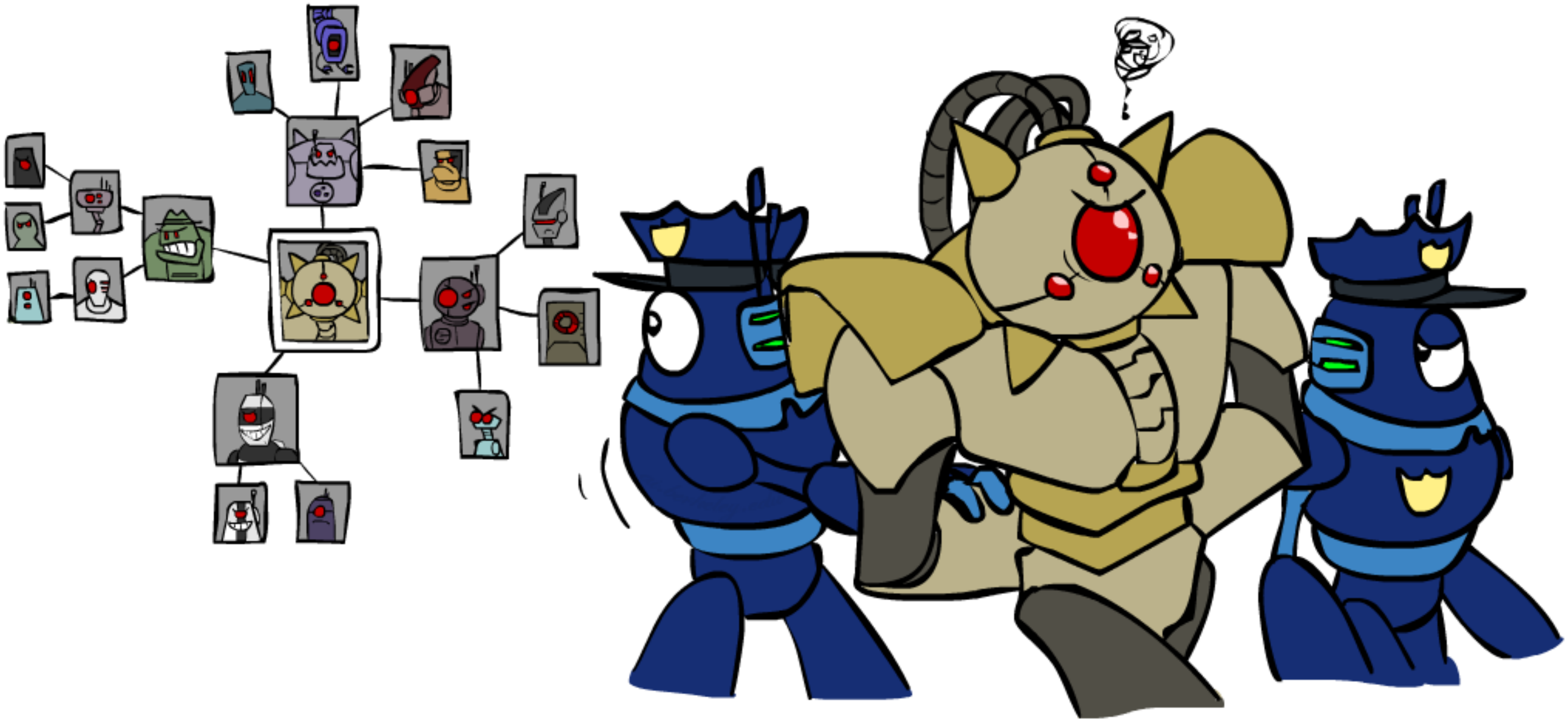


- Claim 2: If root-to-leaf arcs are consistent, forward assignment will not backtrack
- Proof: Induction on position

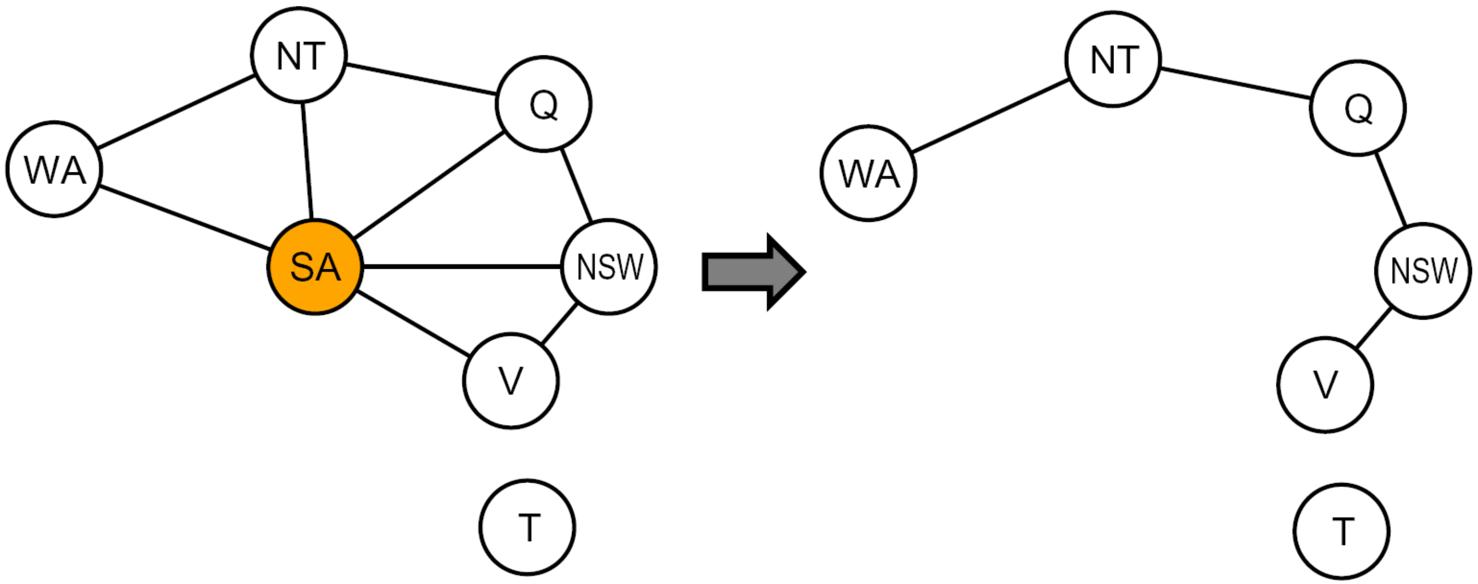
- Why doesn't this algorithm work with cycles in the constraint graph?

- Note: we'll see this basic idea again with Bayes' nets

Improving Structure



Nearly Tree-Structured CSPs



- Conditioning: instantiate a variable, prune its neighbors' domains
- Cutset conditioning: instantiate (in all ways) a set of variables such that the remaining constraint graph is a tree
- Cutset size c gives runtime $O((d^c) (n-c) d^2)$, very fast for small c

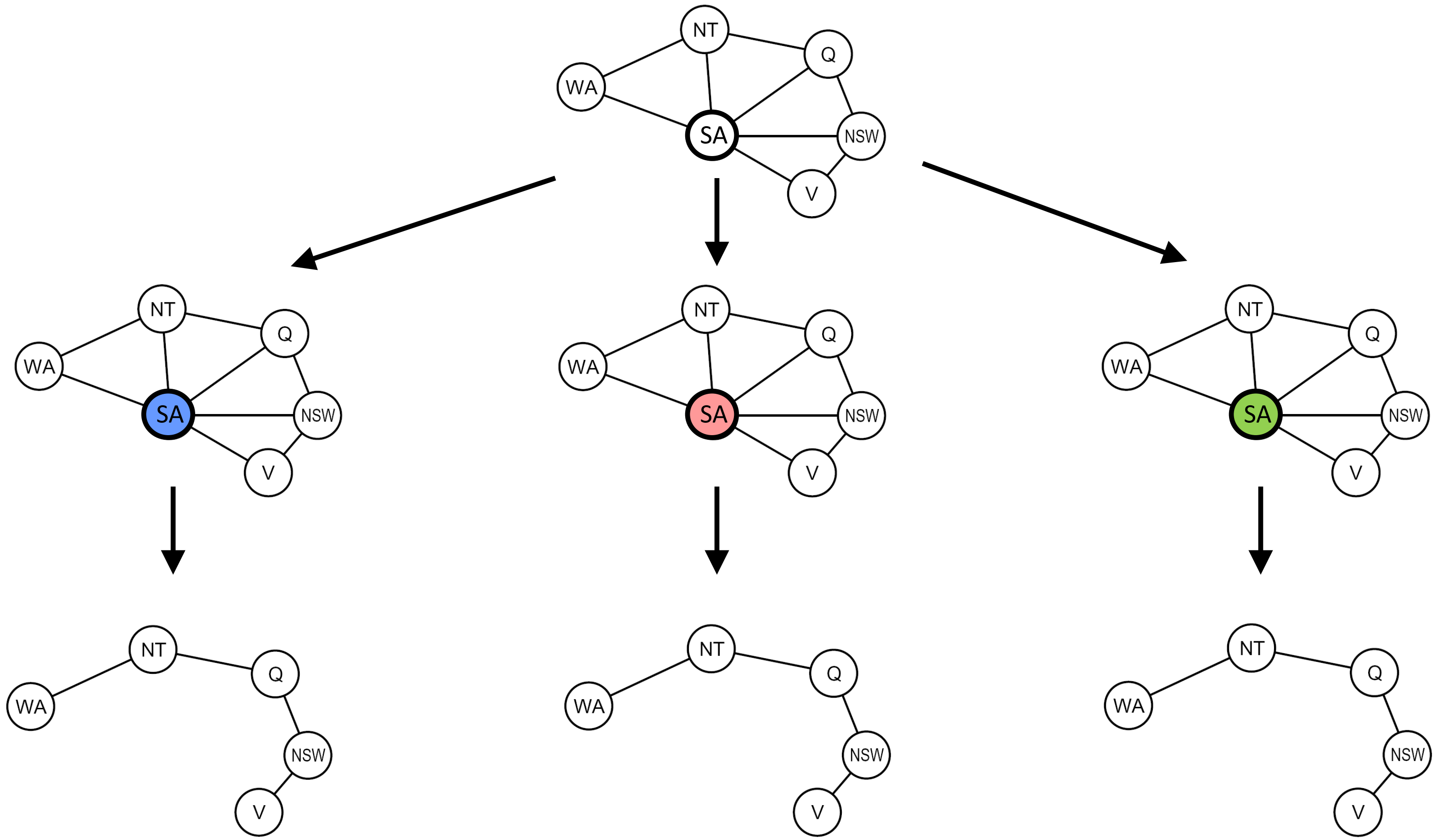
Cutset Conditioning

Choose a cutset

Instantiate the cutset
(all possible ways)

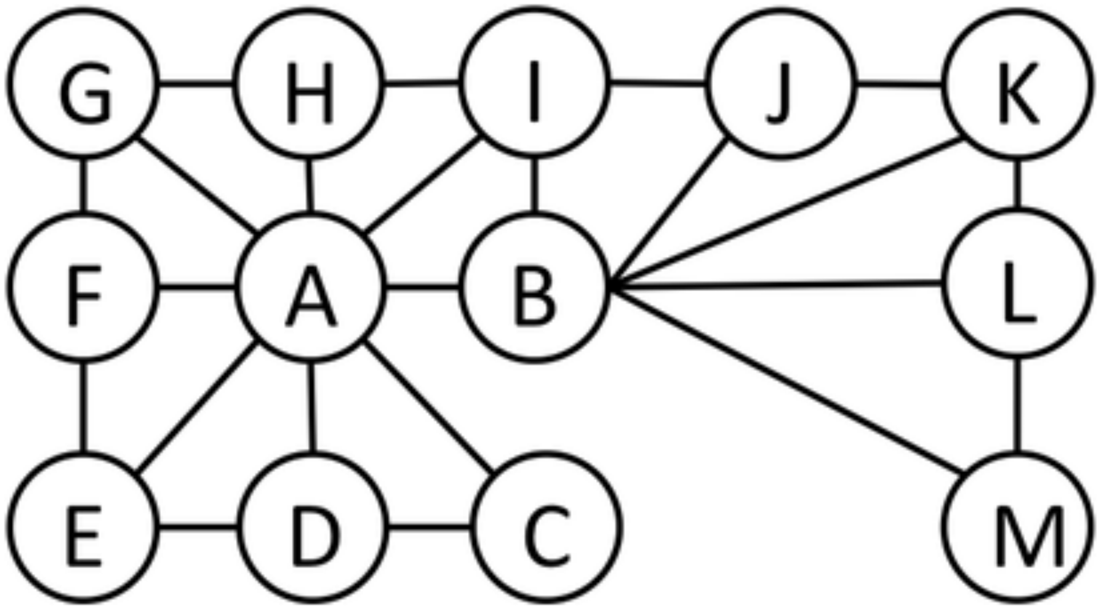
Compute residual CSP
for each assignment

Solve the residual CSPs
(tree structured)

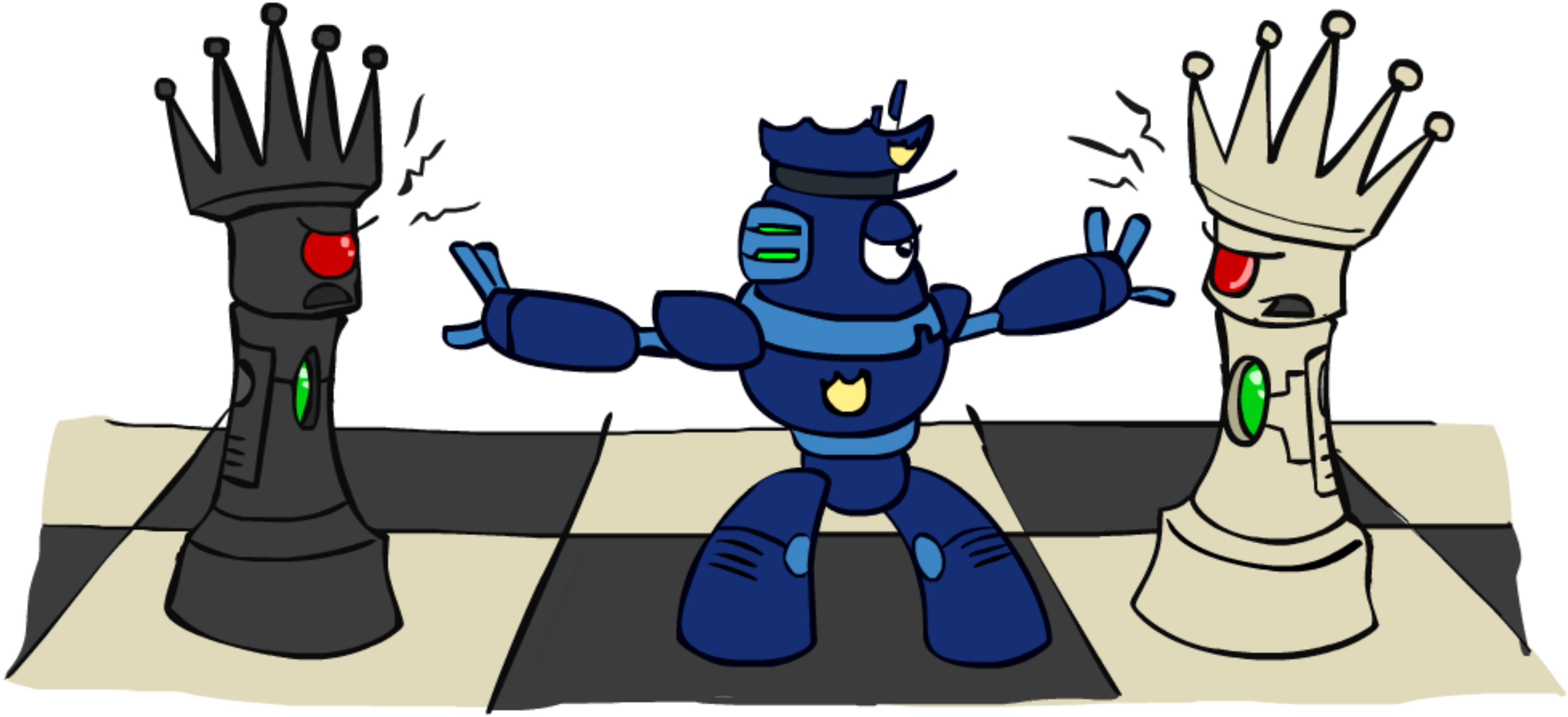


Cutset Quiz

- Find the smallest cutset for the graph below.



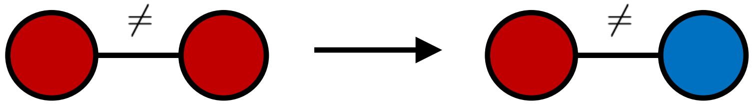
Iterative Improvement



Iterative Algorithms for CSPs

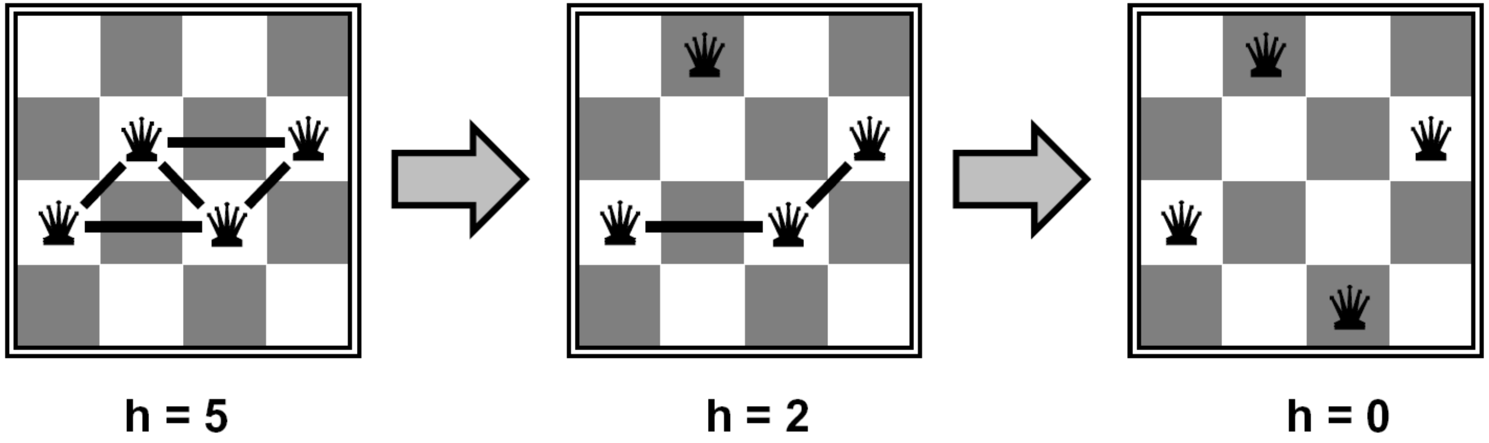
- Local search methods typically work with “complete” states, i.e., all variables assigned

- To apply to CSPs:
  - Take an assignment with unsatisfied constraints
  - Operators *reassign* variable values
  - No fringe! Live on the edge.



- Algorithm: While not solved,
  - Variable selection: randomly select any conflicted variable
  - Value selection: min-conflicts heuristic:
    - Choose a value that violates the fewest constraints
    - I.e., hill climb with h(x) = total number of violated constraints

Example: 4-Queens



- States: 4 queens in 4 columns (4^4 = 256 states)
- Operators: move queen in column
- Goal test: no attacks
- Evaluation: c(n) = number of attacks

[Demo: n-queens – iterative improvement (L5D1)]
[Demo: coloring – iterative improvement]

Video of Demo Iterative Improvement – n Queens

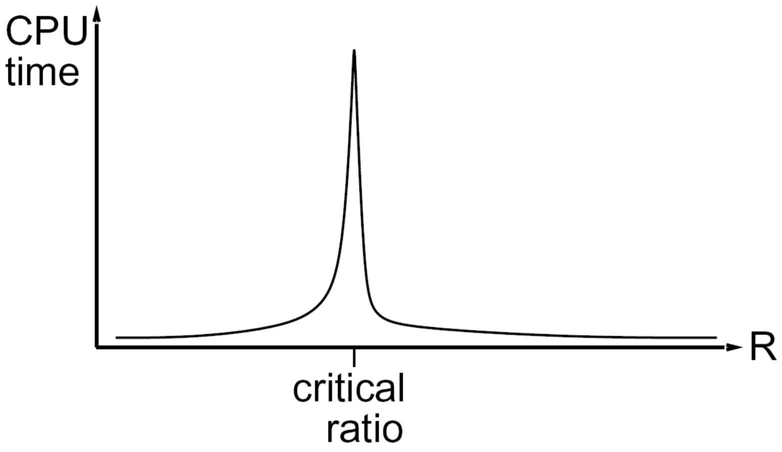
Video of Demo Iterative Improvement – Coloring

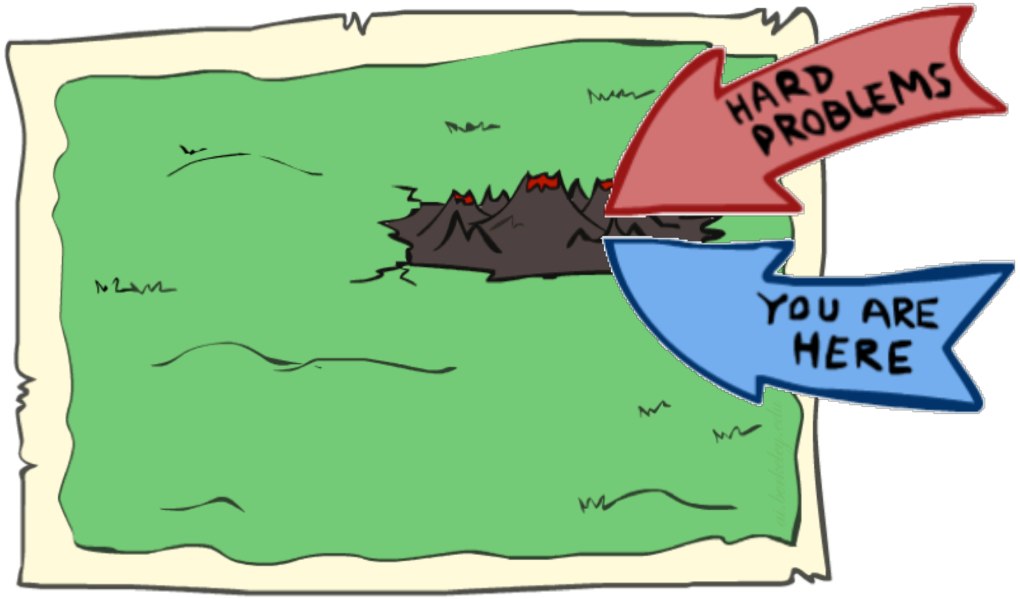
Performance of Min-Conflicts

- Given random initial state, can solve n-queens in almost constant time for arbitrary n with high probability (e.g., n = 10,000,000)!

- The same appears to be true for any randomly-generated CSP *except* in a narrow range of the ratio

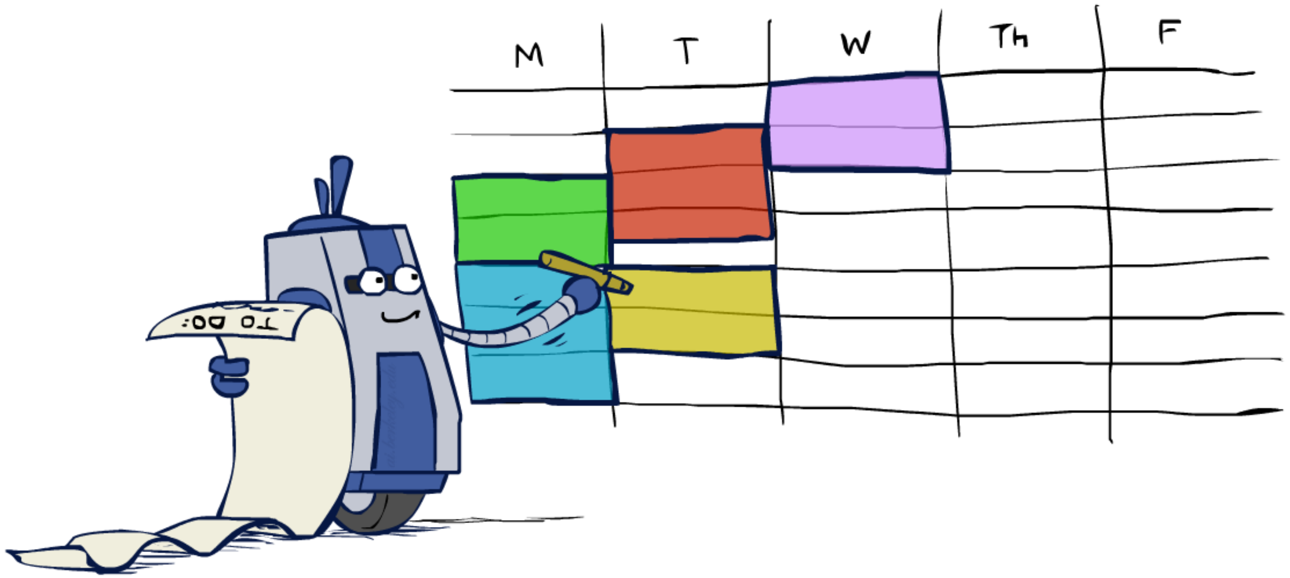
$$R = \frac{\text{number of constraints}}{\text{number of variables}}$$





Summary: CSPs

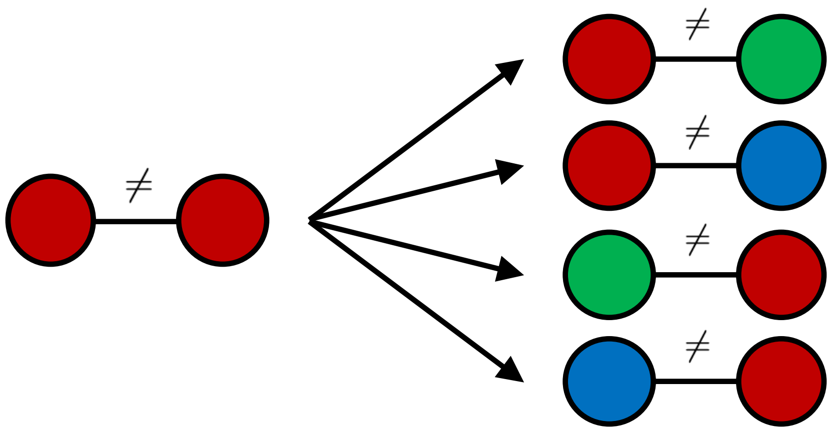
- CSPs are a special kind of search problem:
  - States are partial assignments
  - Goal test defined by constraints
- Basic solution: backtracking search
- Speed-ups:
  - Ordering
  - Filtering
  - Structure
- Iterative min-conflicts is often effective in practice



Local Search

Local Search

- Tree search keeps unexplored alternatives on the fringe (ensures completeness)
- Local search: improve a single option until you can't make it better (no fringe!)
- New successor function: local changes

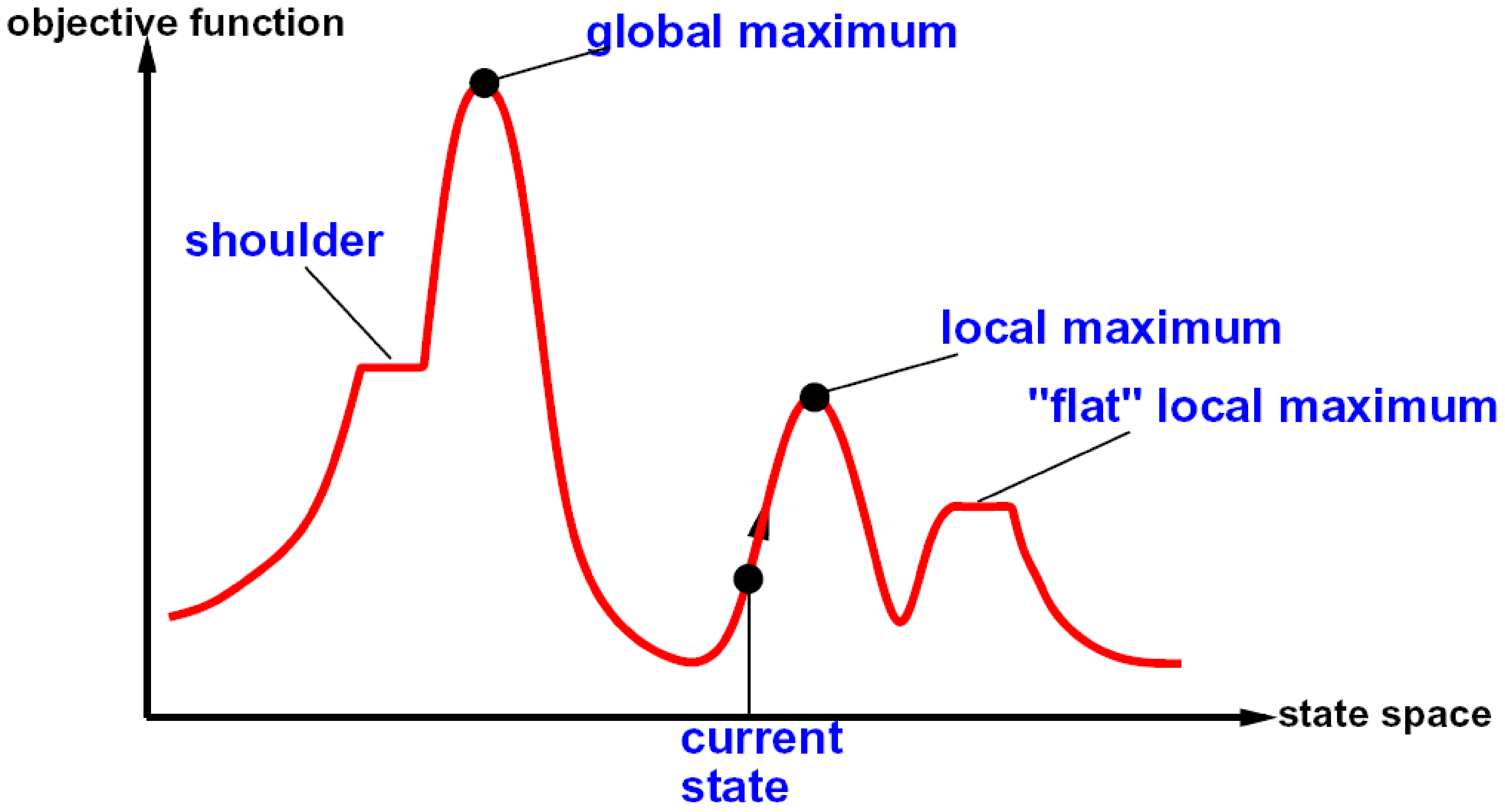


- Generally much faster and more memory efficient (but incomplete and suboptimal)

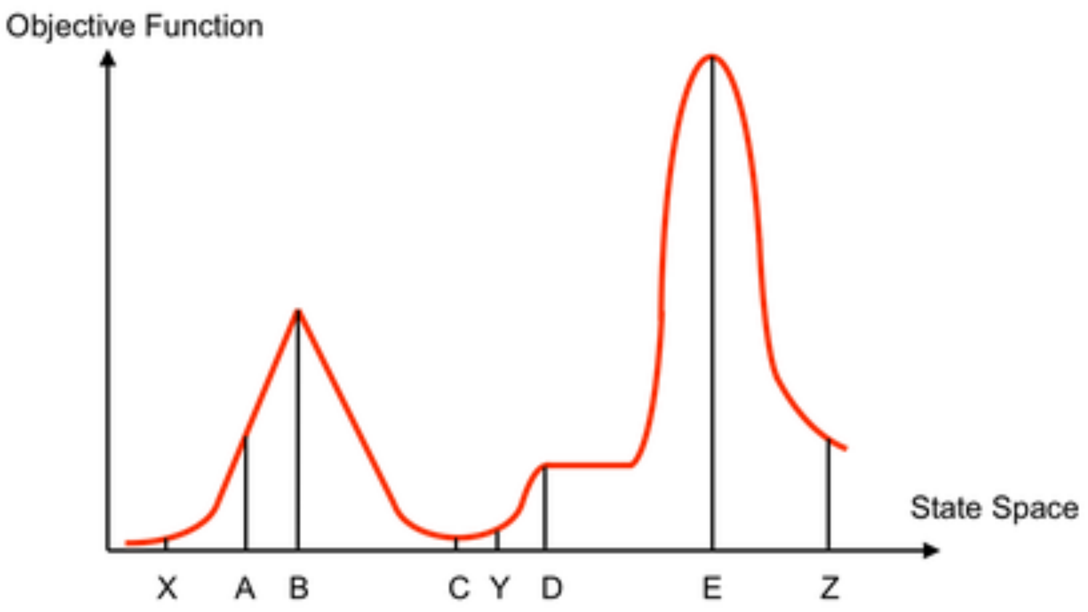
Hill Climbing

- Simple, general idea:
  - Start wherever
  - Repeat: move to the best neighboring state
  - If no neighbors better than current, quit
- What's bad about this approach?
  - Complete?
  - Optimal?
- What's good about it?

Hill Climbing Diagram



Hill Climbing Quiz



Starting from X, where do you end up ?

Starting from Y, where do you end up ?

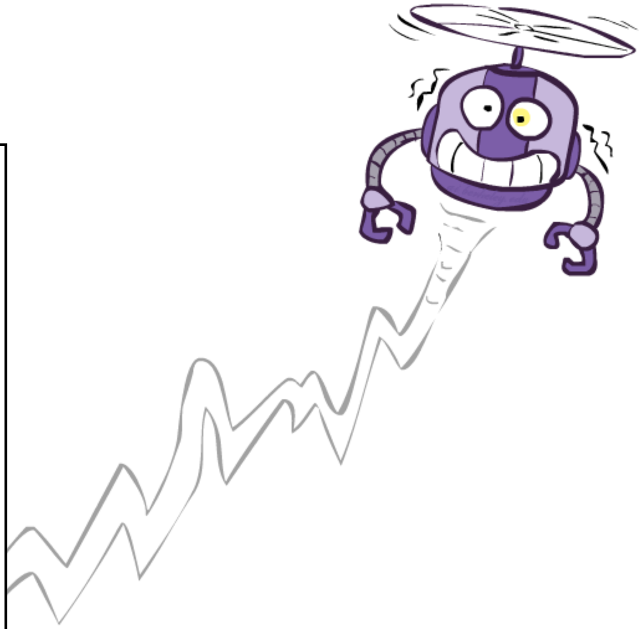
Starting from Z, where do you end up ?

Simulated Annealing

- Idea: Escape local maxima by allowing downhill moves
  - But make them rarer as time goes on

```
function SIMULATED-ANNEALING( problem, schedule) returns a solution state
   inputs: problem, a problem
           schedule, a mapping from time to "temperature"
   local variables: current, a node
                    next, a node
                    T, a "temperature" controlling prob. of downward steps

   current ← MAKE-NODE(INITIAL-STATE[problem])
   for t ← 1 to ∞ do
       T ← schedule[t]
       if T = 0 then return current
       next ← a randomly selected successor of current
       ΔE ← VALUE[next] – VALUE[current]
       if ΔE > 0 then current ← next
       else current ← next only with probability e^(Δ E/T)
```

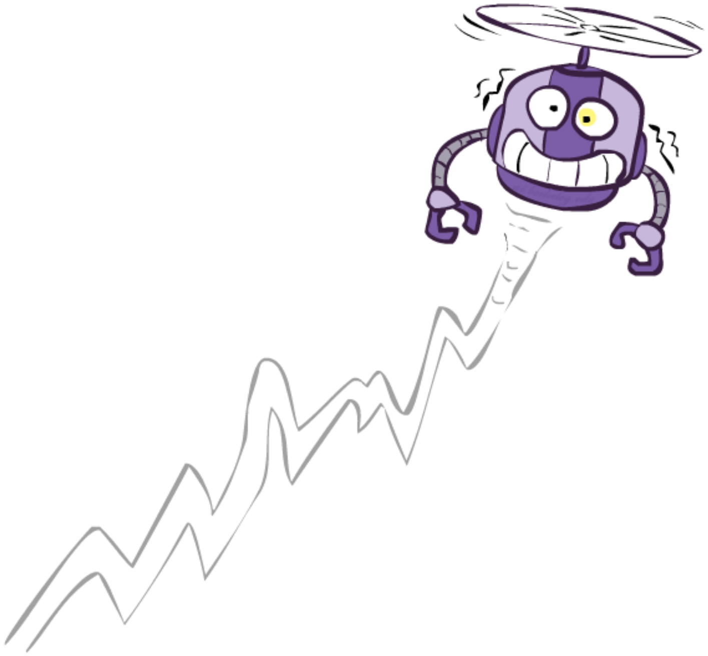


Simulated Annealing

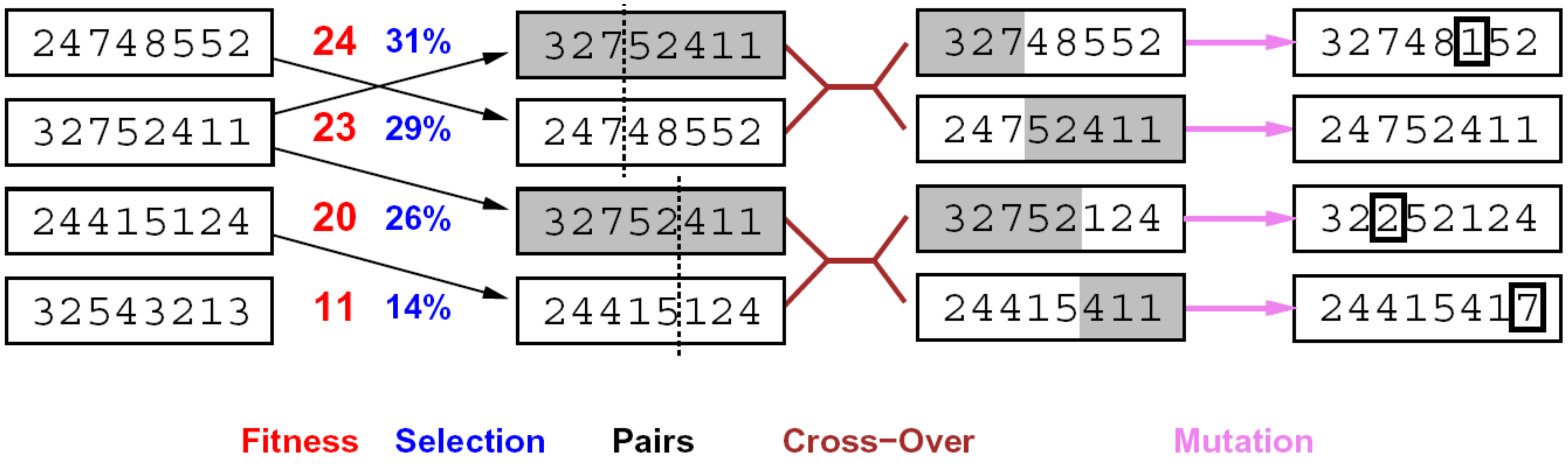
- Theoretical guarantee:
  - Stationary distribution: $p(x) \propto e^{\frac{E(x)}{kT}}$
  - If T decreased slowly enough, will converge to optimal state!

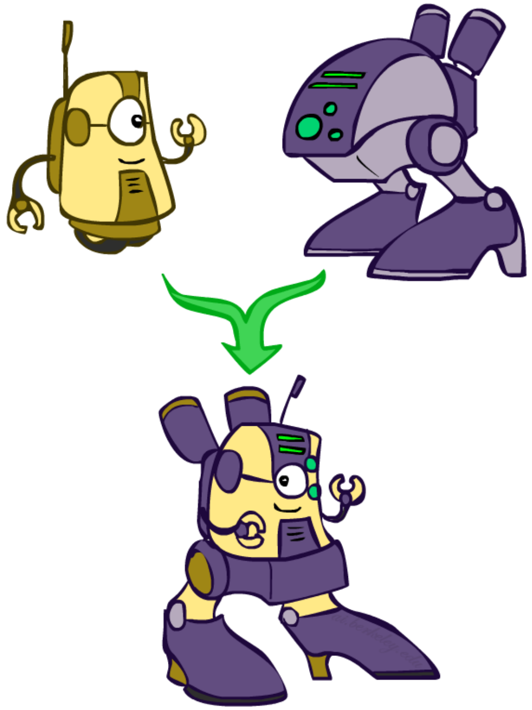
- Is this an interesting guarantee?

- Sounds like magic, but reality is reality:
  - The more downhill steps you need to escape a local optimum, the less likely you are to ever make them all in a row
  - People think hard about *ridge operators* which let you jump around the space in better ways



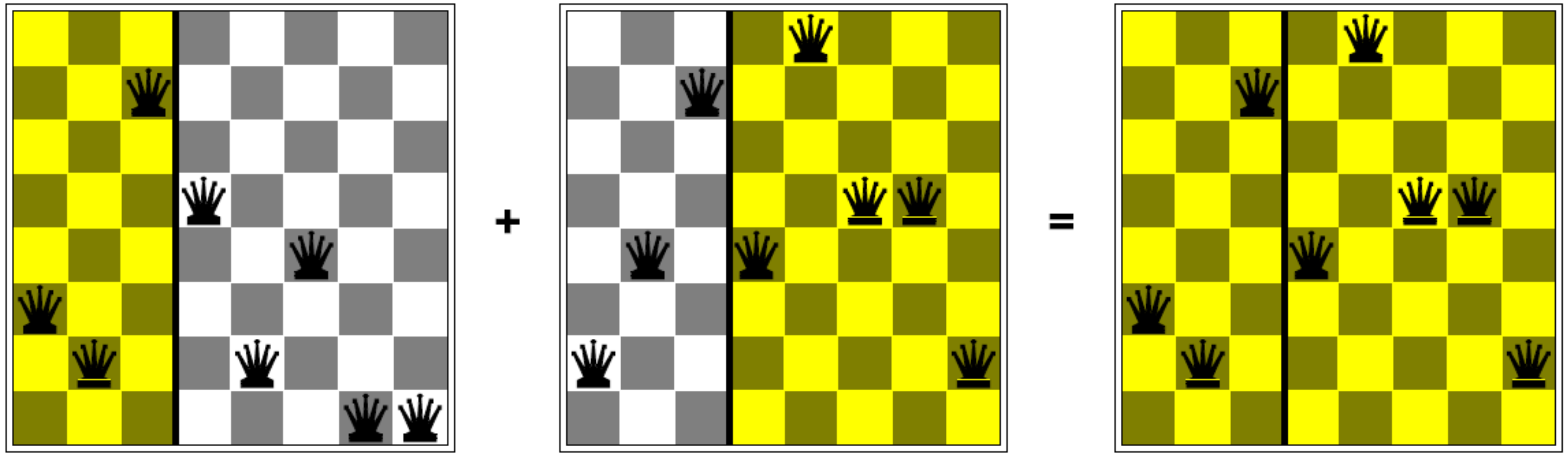
Genetic Algorithms





- Genetic algorithms use a natural selection metaphor
  - Keep best N hypotheses at each step (selection) based on a fitness function
  - Also have pairwise crossover operators, with optional mutation to give variety
- Possibly the most misunderstood, misapplied (and even maligned) technique around

Example: N-Queens

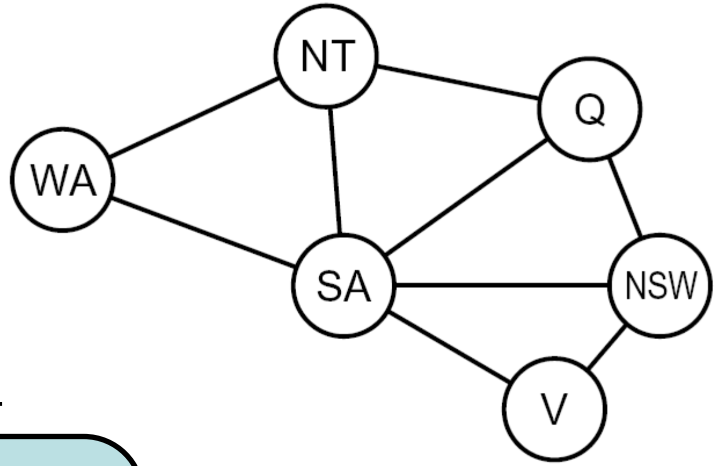


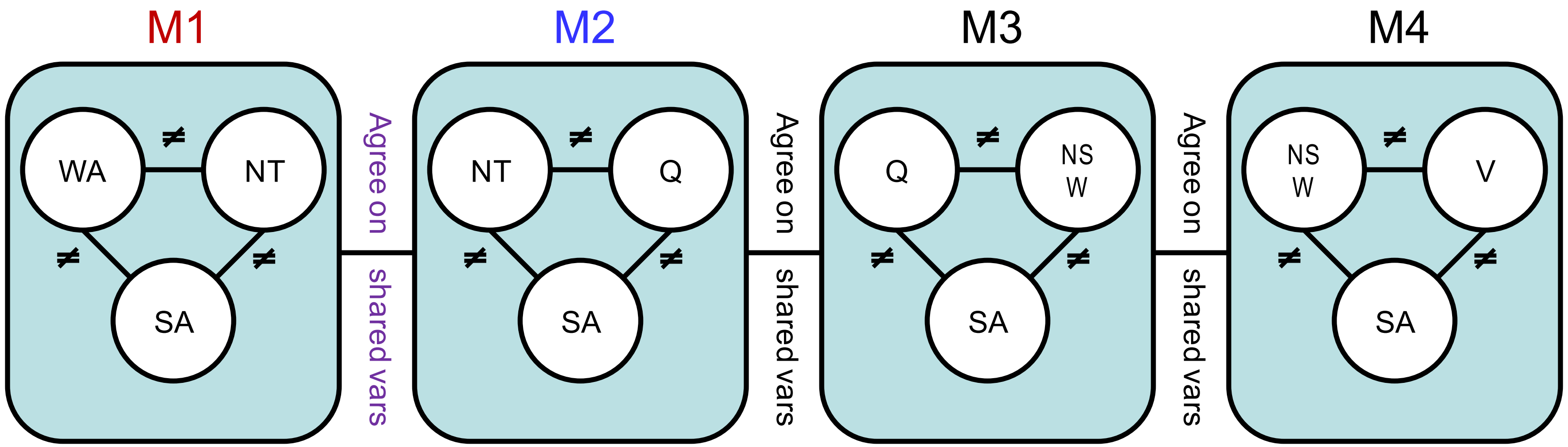
- Why does crossover make sense here?
- When wouldn’t it make sense?
- What would mutation be?
- What would a good fitness function be?

Next Time: Adversarial Search!

Tree Decomposition*

- Idea: create a tree-structured graph of mega-variables
- Each mega-variable encodes part of the original CSP
- Subproblems overlap to ensure consistent solutions





{(WA=r,SA=g,NT=b),
(WA=b,SA=r,NT=g),
…}

{(NT=r,SA=g,Q=b),
(NT=b,SA=g,Q=r),
…}

Agree: (M1,M2) ∈
{((WA=g,SA=g,NT=g), (NT=g,SA=g,Q=g)), …}