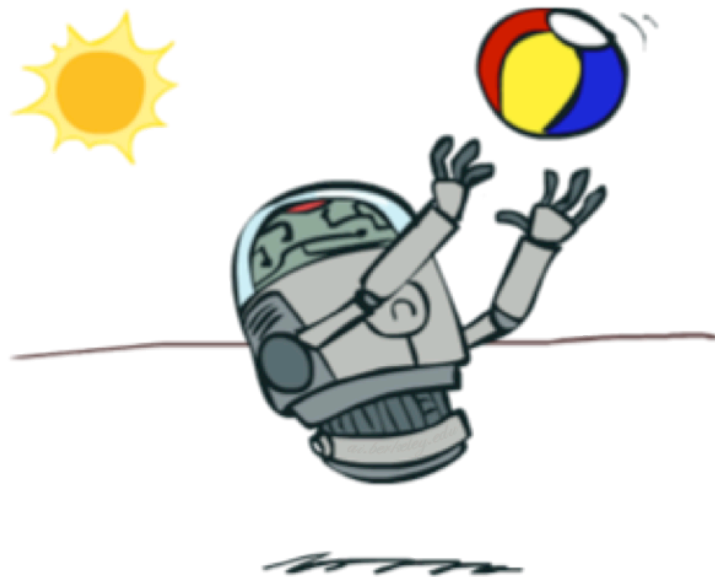# Welcome back from spring break!

Announcements

- P4 Bayes' Nets / Treasure Hunting has been released.
  - It's due Monday April 4th at 11:59pm.
- HW7 has been released.
  - It's due Tuesday April 5th, at 11:59pm. It's about twice as long as a typical homework. We encourage you to get started early!
- Regrade request window for the Midterm
  - Monday through Wednesday this week.

Probability Recap

$$P(x) = \sum_y P(x,y) \qquad\qquad P(x,y) = P(x|y)P(y)$$

$$P(X|y) = \frac{P(X,y)}{P(y)} = \frac{P(X,y)}{\sum_x P(x,y)} = \frac{P(y|X)P(X)}{\sum_x P(y|x)P(x)}$$

CS 188: Artificial Intelligence

Hidden Markov Models

Instructors: Pieter Abbeel & Anca Dragan --- University of California, Berkeley

Reasoning over Time or Space

- Often, we want to reason about a sequence of observations
    - Speech recognition
    - Robot localization
    - User attention
    - Medical monitoring

- Need to introduce time (or space) into our models

Markov Models

- Value of X at a given time is called the state

$$X_1 \rightarrow X_2 \rightarrow X_3 \rightarrow X_4 \dashrightarrow$$

$$P(X_1) \qquad P(X_t | X_{t-1})$$

$$P(X_t) = ?$$
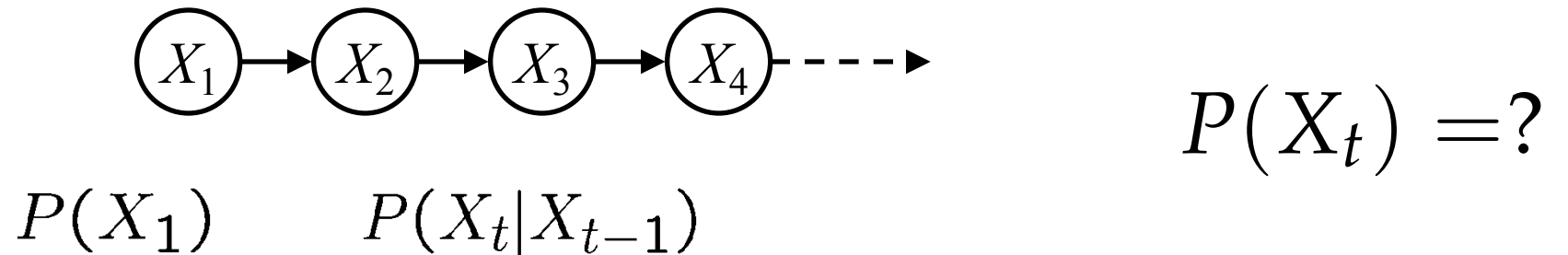
- Parameters: called transition probabilities or dynamics, specify how the state evolves over time (also, initial state probabilities)
- Stationarity assumption: transition probabilities the same at all times
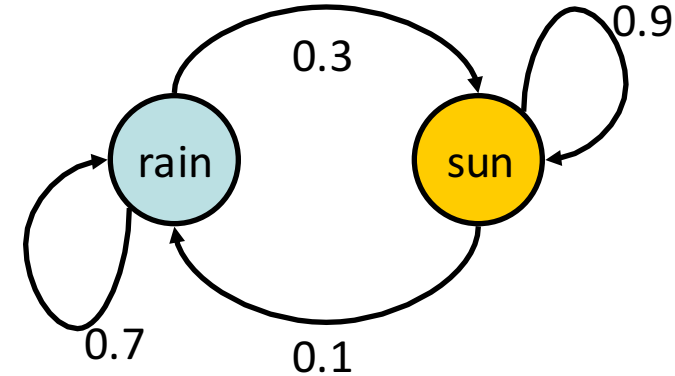
Example Markov Chain: Weather

- States: X = {rain, sun}

- Initial distribution: 1.0 sun

- CPT $P(X_t \mid X_{t-1})$:

X_{t-1}	X_t	$P(X_t\|X_{t-1})$
sun	sun	0.9
sun	rain	0.1
rain	sun	0.3
rain	rain	0.7
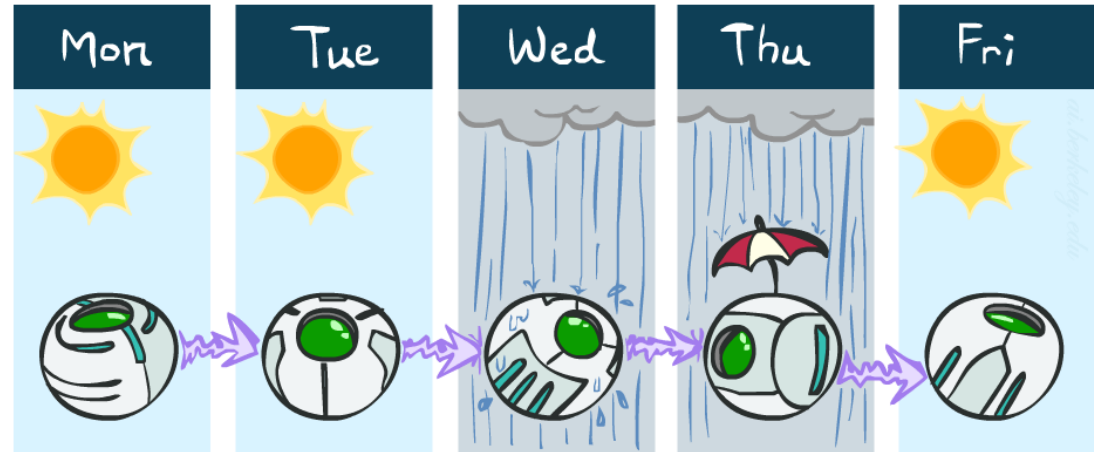
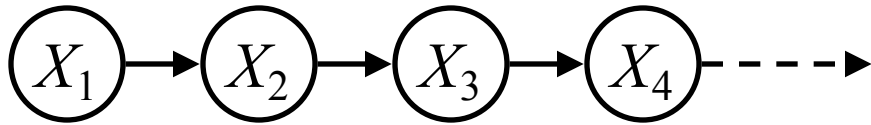# Example Markov Chain: Weather

- Initial distribution: 1.0 sun

- What is the probability distribution after one step?

$$P(X_2 = sun) = \sum_{x_1} P(x_1, X_2 = sun) = \sum_{x_1} P(X_2 = sun | x_1) P(x_1)$$

$$P(X_2 = \text{sun}) = \quad P(X_2 = \text{sun} | X_1 = \text{sun}) P(X_1 = \text{sun}) +$$

$$P(X_2 = \text{sun} | X_1 = \text{rain}) P(X_1 = \text{rain})$$

$$0.9 \cdot 1.0 + 0.3 \cdot 0.0 = 0.9$$

Mini-Forward Algorithm
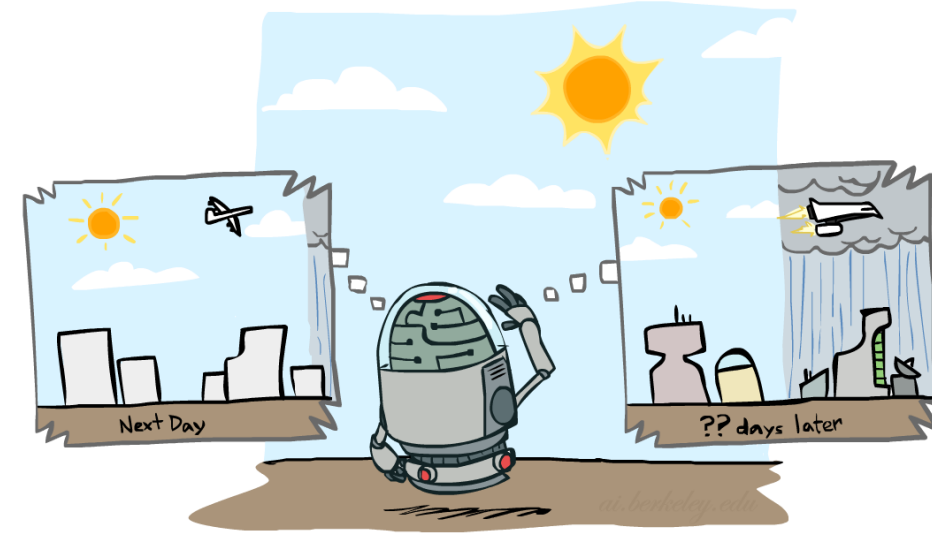
- Question: What's P(X) on some day t?

$$P(x_1) = \text{known}$$

$$P(x_t) = \sum_{x_{t-1}} P(x_{t-1}, x_t)$$

$$= \sum_{x_{t-1}} P(x_t \mid x_{t-1}) P(x_{t-1})$$

Forward simulation

Video of Demo Ghostbusters Basic Dynamics

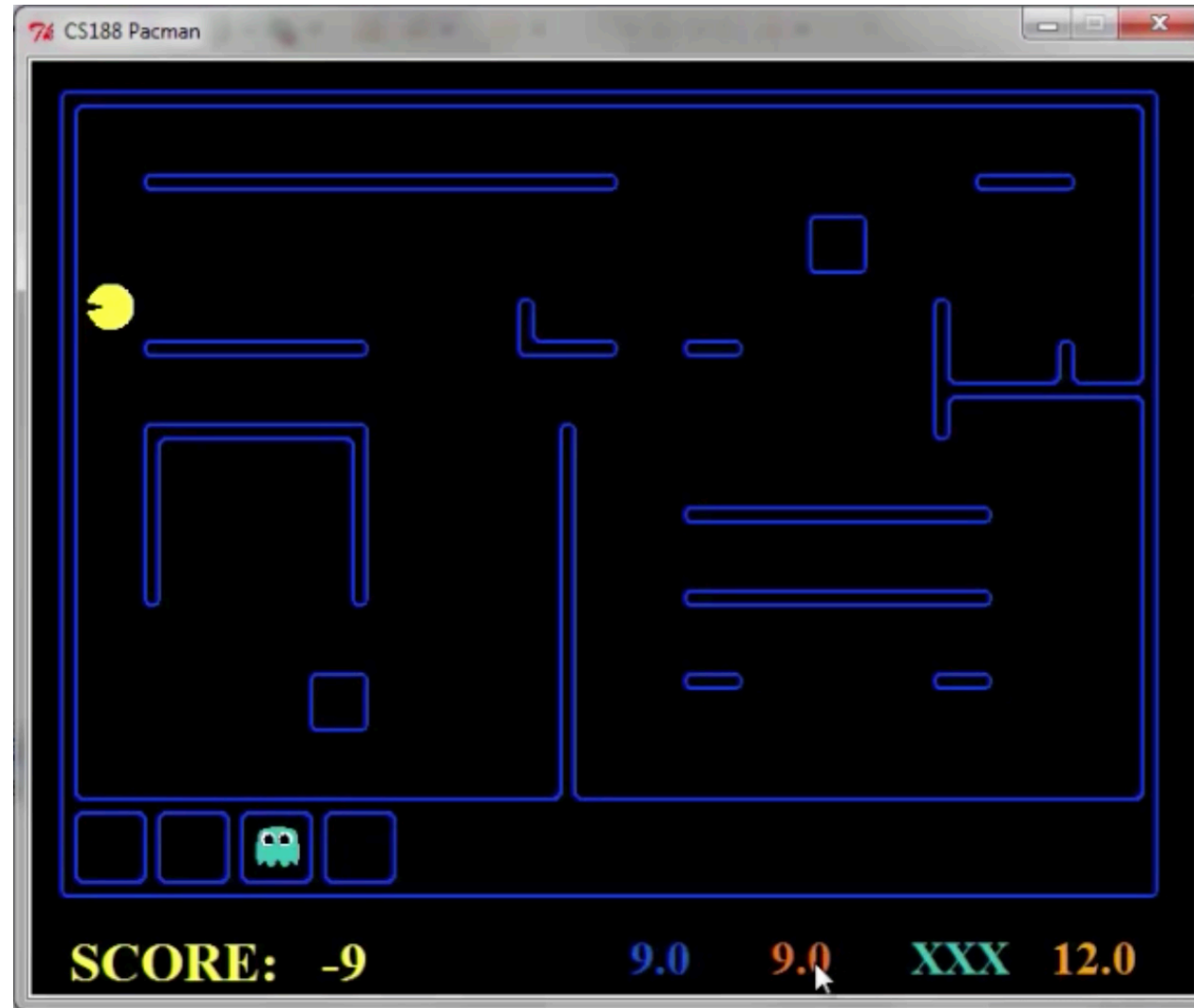# Video of Demo Ghostbusters Circular Dynamics

Video of Demo Ghostbusters Whirlpool Dynamics
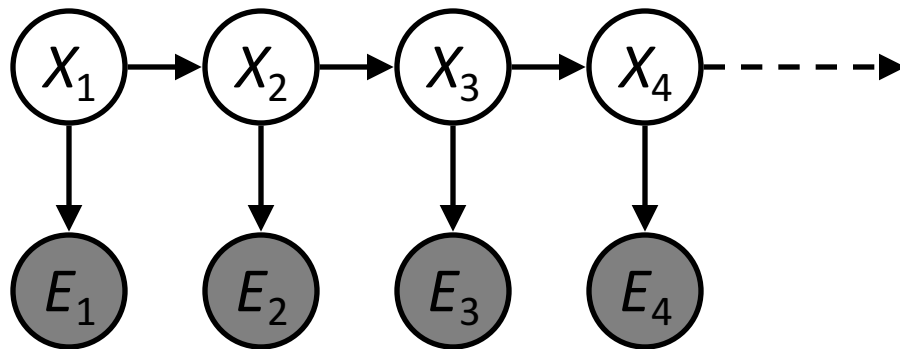
Hidden Markov Models

Pacman – Sonar

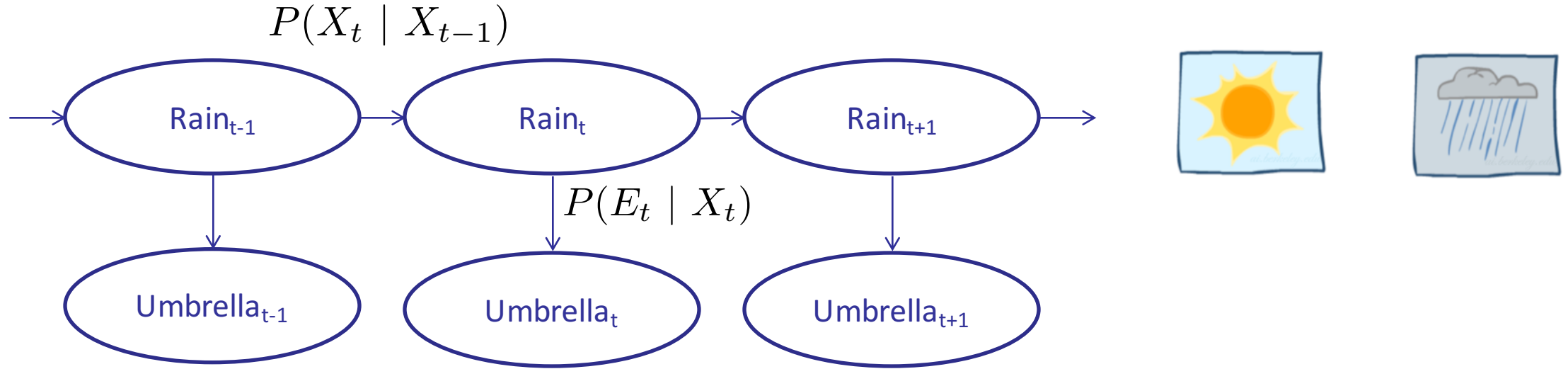# Video of Demo Pacman – Sonar (no beliefs)

Hidden Markov Models

- **Markov chains not so useful for most agents**
  - Need observations to update your beliefs

- **Hidden Markov models (HMMs)**
  - Underlying Markov chain over states X
  - You observe outputs (effects) at each time step

Example: Weather HMM

$$P(X_t \mid X_{t-1})$$
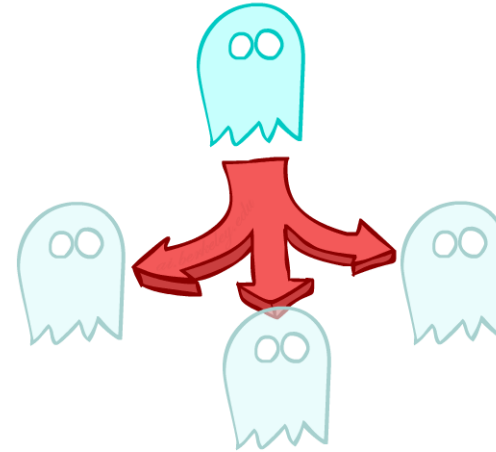

$$P(E_t \mid X_t)$$

- ■ An HMM is defined by:
  - ■ Initial distribution: $P(X_1)$
  - ■ Transitions: $P(X_t \mid X_{t-1})$
  - ■ Emissions: $P(E_t \mid X_t)$

R_{t-1}	R_t	$P(R_t|R_{t-1})$
+r	+r	0.7
+r	-r	0.3
-r	+r	0.3
-r	-r	0.7

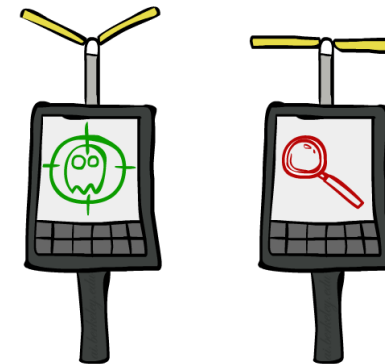| R_t	U_t	$P(U_t|R_t)$
+r	+u	0.9
+r	-u	0.1
-r	+u	0.2
-r	-u	0.8

Example: Ghostbusters HMM

- $P(X_1)$ = uniform

- $P(X|X')$ = usually move clockwise, but sometimes move in a random direction or stay in place

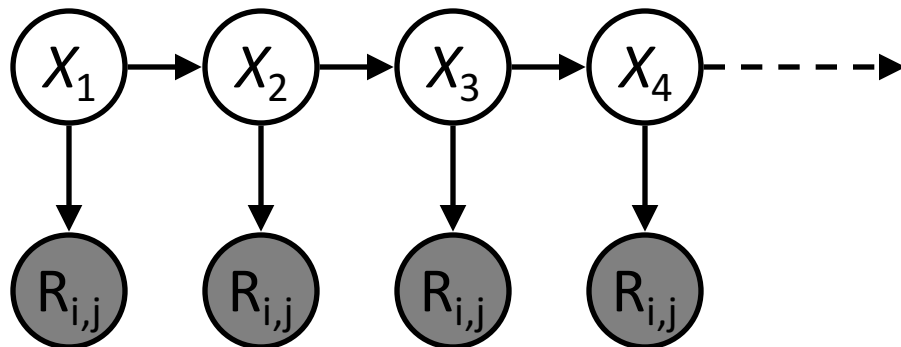- $P(R_{ij}|X)$ = same sensor model as before: red means close, green means far away.

1/9	1/9	1/9
1/9	1/9	1/9
1/9	1/9	1/9

$P(X_1)$

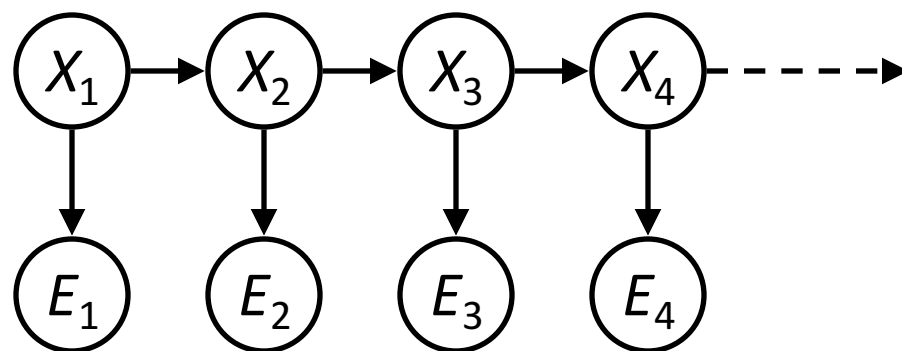| 1/6	1/6	1/2
0	1/6	0
0	0	0

$P(X|X' = <1,2>)$

Video of Demo Ghostbusters – Circular Dynamics -- HMM

Conditional Independence

- HMMs have two important independence properties:

  - Markov hidden process: future depends on past via the present

  - Current observation independent of all else given current state

- Does this mean that evidence variables are guaranteed to be independent?

 - [No, they tend to correlated by the hidden state]

Real HMM Examples

- **Robot tracking:**
  - Observations are range readings (continuous)
  - States are positions on a map (continuous)

- **Speech recognition HMMs:**
  - Observations are acoustic signals (continuous valued)
  - States are specific positions in specific words (so, tens of thousands)

- **Machine translation HMMs:**
  - Observations are words (tens of thousands)
  - States are translation options

Filtering / Monitoring

- Filtering, or monitoring, is the task of tracking the distribution $B_t(X) = P_t(X_t \mid e_1, \ldots, e_t)$ (the belief state) over time

- We start with $B_1(X)$ in an initial setting, usually uniform

- As time passes, or we get observations, we update $B(X)$

- The Kalman filter was invented in the 60's and first implemented as a method of trajectory estimation for the Apollo program
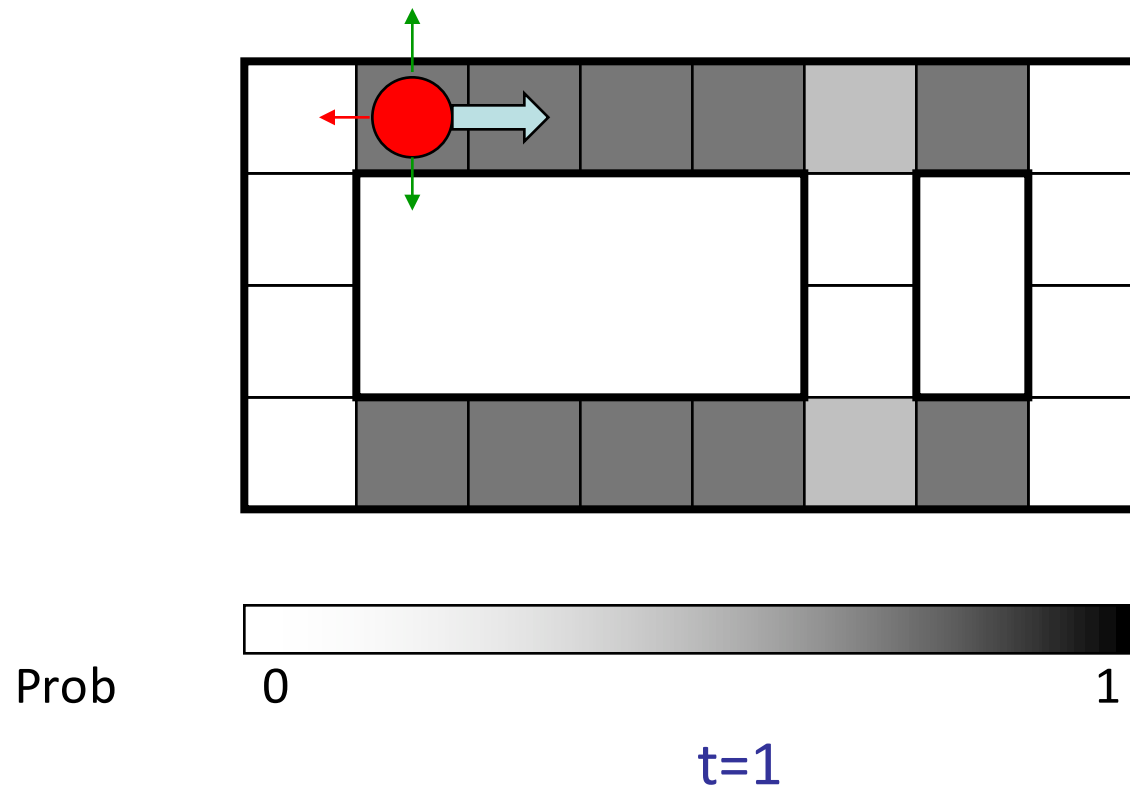
Example: Robot Localization

Example from
Michael Pfeiffer

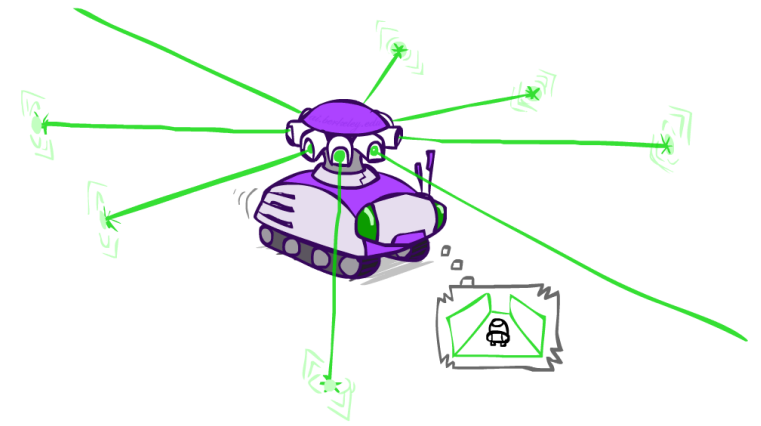Prob   0   1

t=0
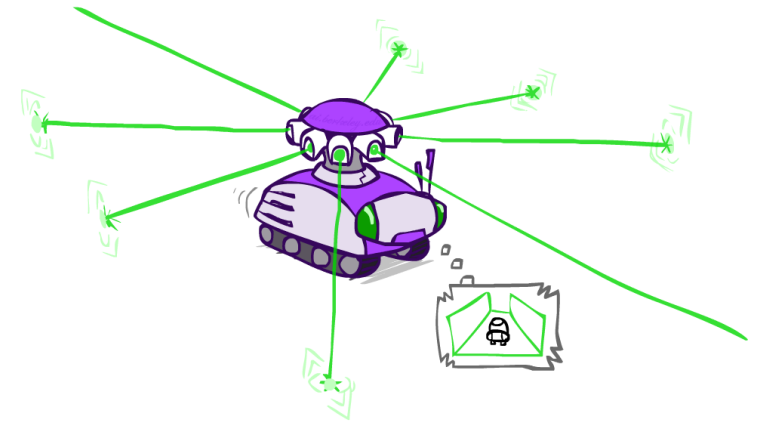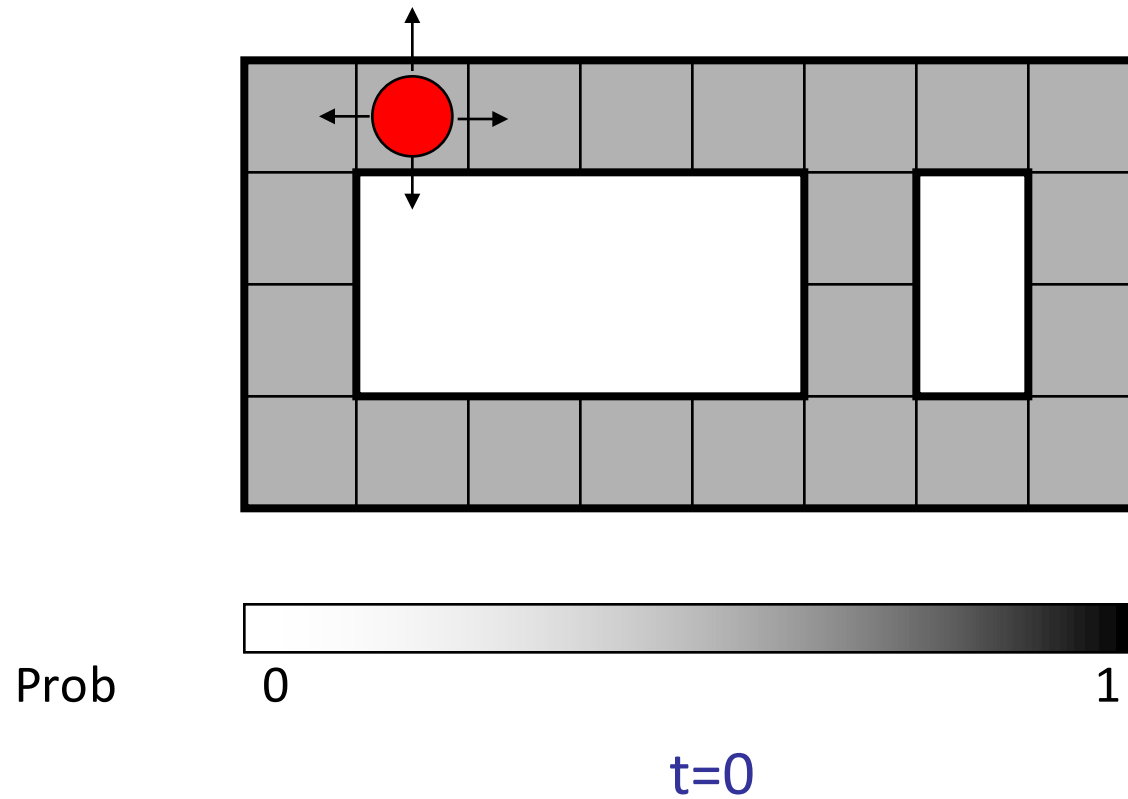
Sensor model: can read in which directions there is a wall,
never more than 1 mistake
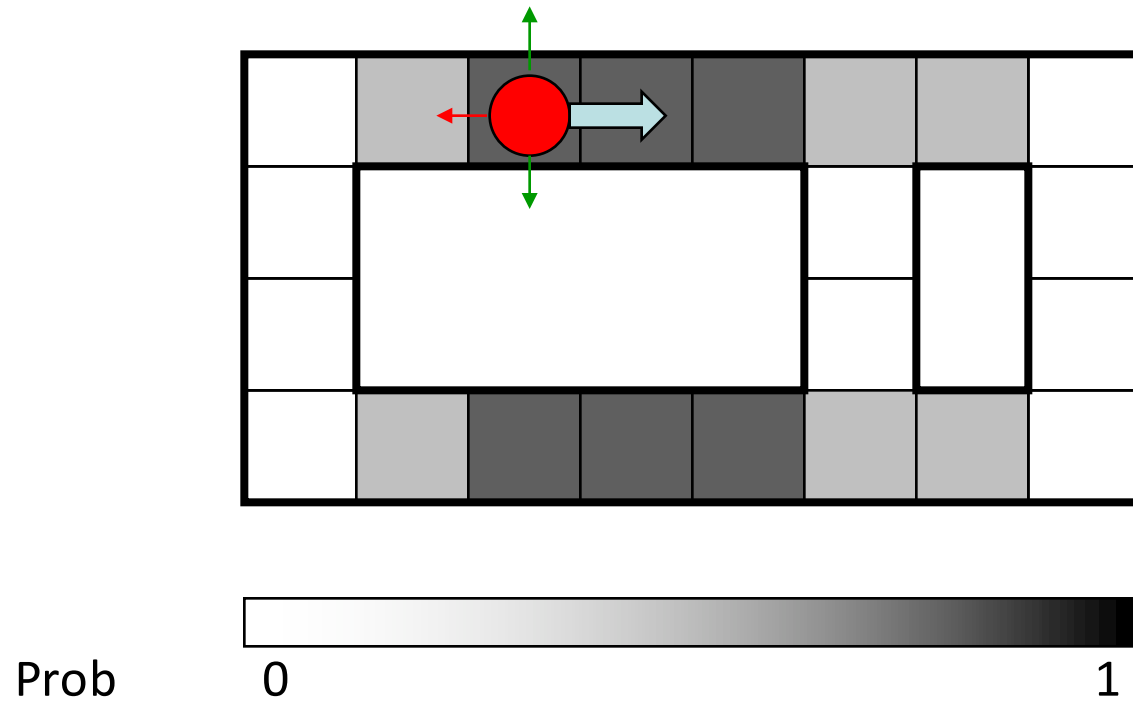
Motion model: may not execute action with small prob.
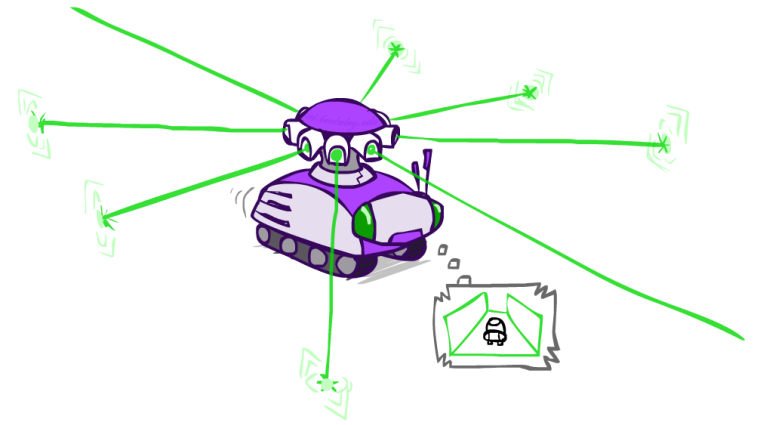
Example: Robot Localization

Prob   0   1

t=1

Lighter grey: was possible to get the reading, but less likely b/c
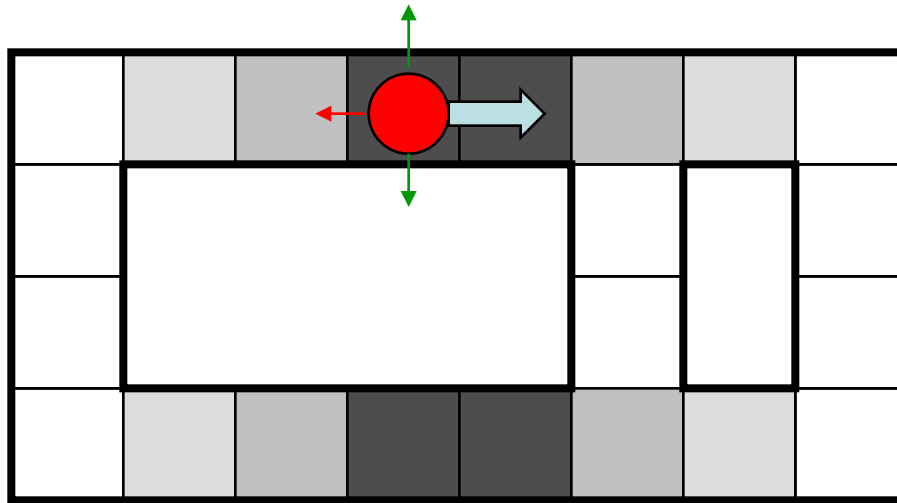required 1 mistake

Example: Robot Localization

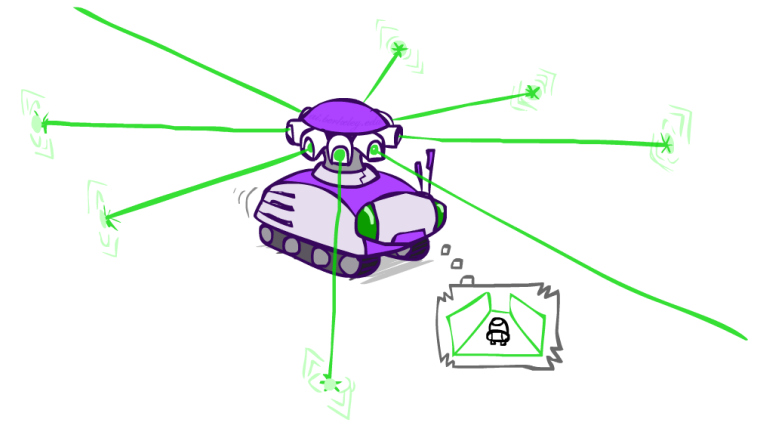Prob 0   1
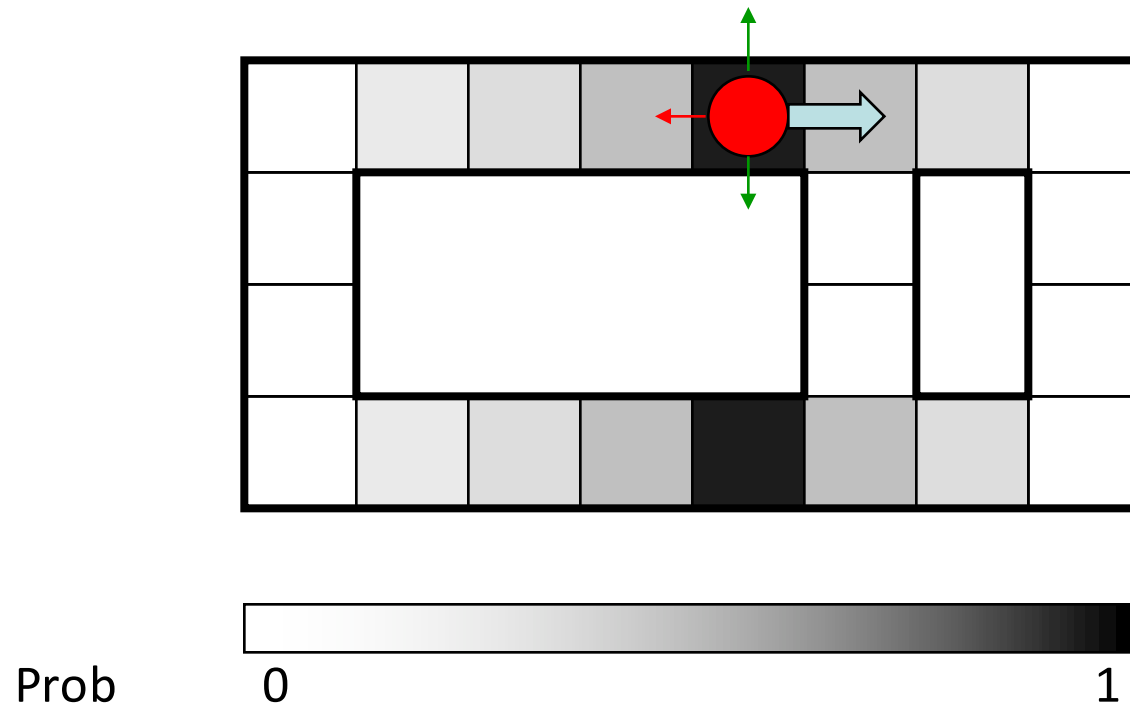
t=2

Example: Robot Localization

Prob   0   1
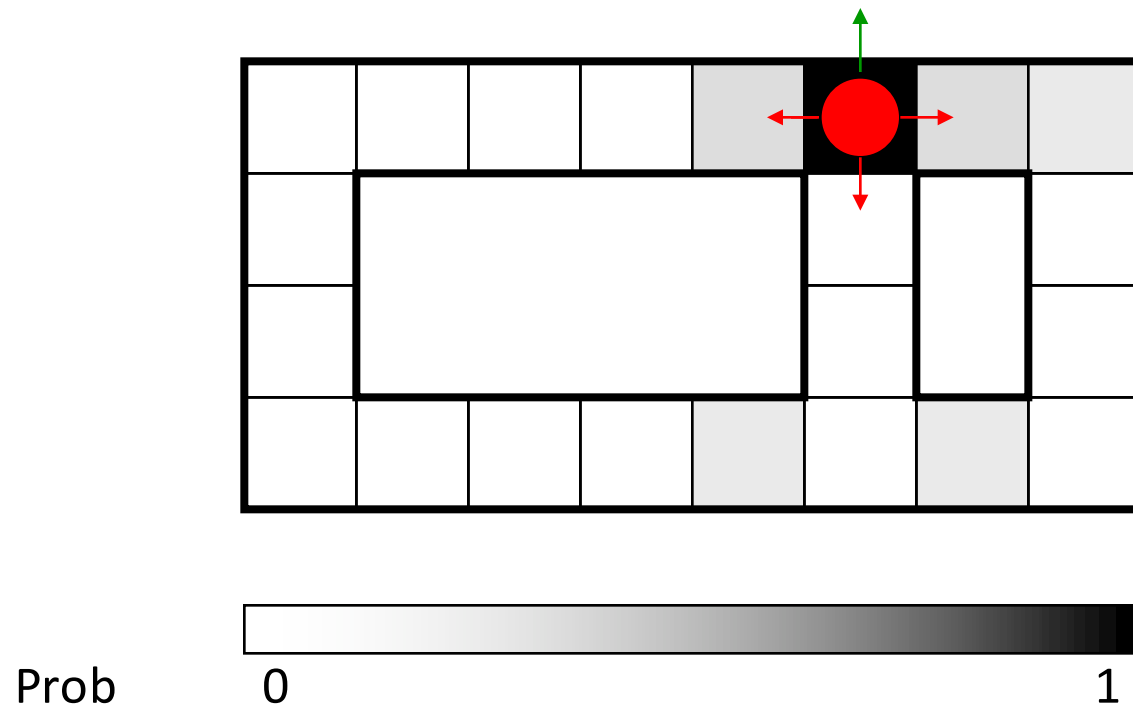
t=3

Example: Robot Localization

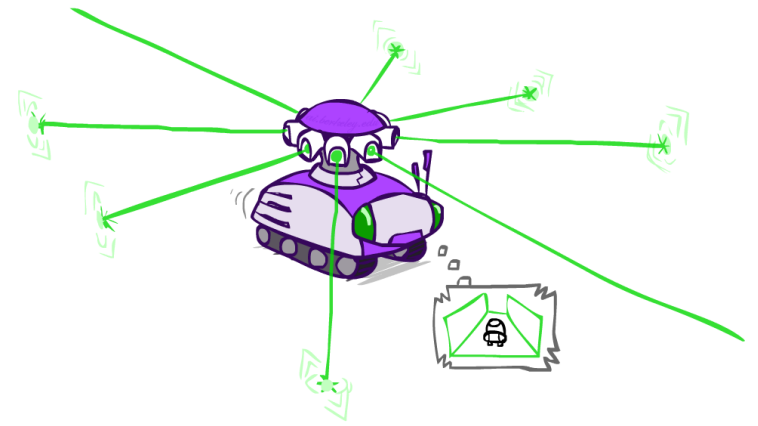Prob   0   1

t=4

Example: Robot Localization

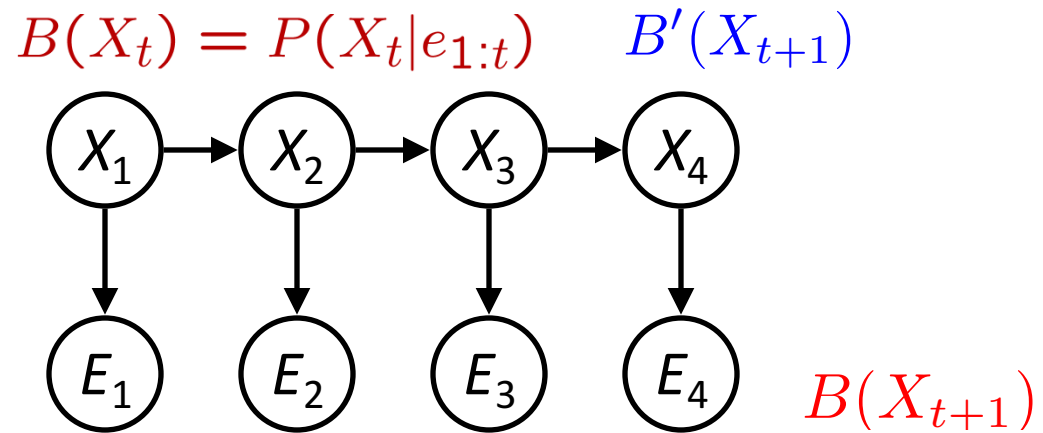

Prob   0   1
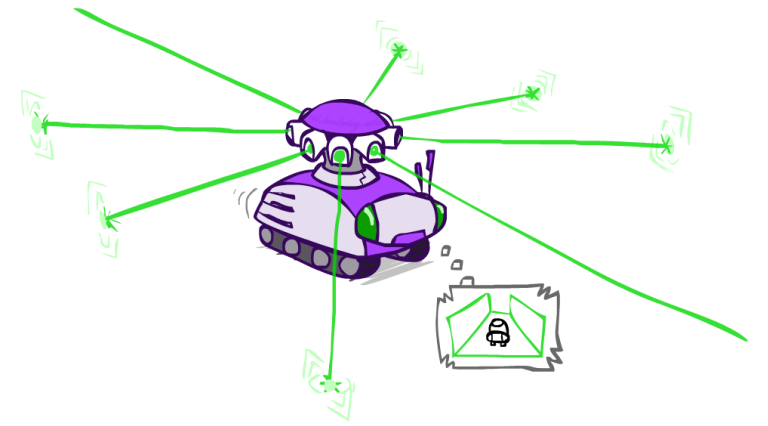
t=5

Inference: Find State Given Evidence

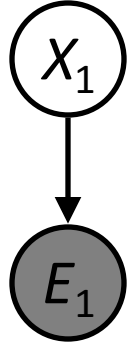- We are given evidence at each time and want to know

$$B_t(X) = P(X_t | e_{1:t})$$

- Idea: start with $P(X_1)$ and derive B_t in terms of B_{t-1}
    - equivalently, derive B_{t+1} in terms of B_t

Two Steps: Passage of Time + Observation

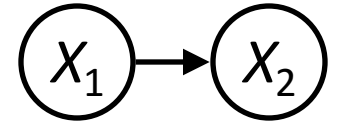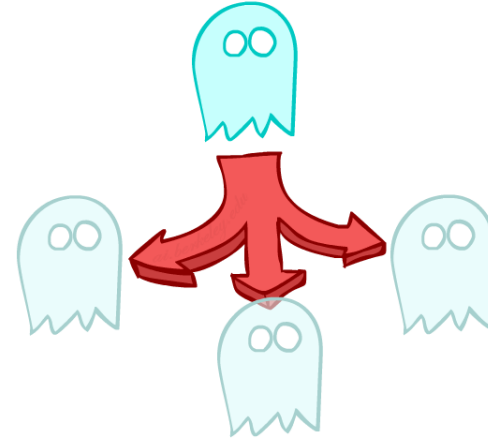$$B(X_t) = P(X_t|e_{1:t}) \qquad B'(X_{t+1})$$

$$B(X_{t+1})$$

Inference: Base Cases

$$P(X_1|e_1)$$

$$P(X_1|e_1) = \frac{P(X_1,e_1)}{\sum_{x_1} P(x_1,e_1)}$$

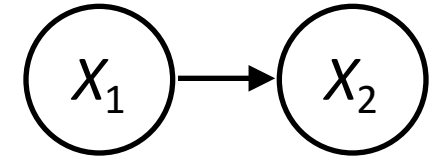$$P(X_1|e_1) = \frac{P(X_1|e_1)P(X_1)}{\sum_{x_1} P(x_1|e_1)P(x_1)}$$

$$P(X_2)$$

$$P(X_2) = \sum_{x_1} P(x_1, X_2)$$

$$P(X_2) = \sum_{x_1} P(X_2|x_1)P(x_1)$$

Passage of Time

- Assume we have current belief P(X | evidence to date)

$$B(X_t) = P(X_t|e_{1:t})$$

- Then, after one time step passes:

$$P(X_{t+1}|e_{1:t}) = \sum_{x_t} P(X_{t+1}, x_t|e_{1:t})$$

$$= \sum_{x_t} P(X_{t+1}|x_t, e_{1:t})P(x_t|e_{1:t})$$

- Or compactly:

$$= \sum_{x_t} P(X_{t+1}|x_t)P(x_t|e_{1:t})$$

$$B'(X_{t+1}) = \sum_{x_t} P(X'|x_t)B(x_t)$$

- Basic idea: beliefs get "pushed" through the transitions
  - With the "B" notation, we have to be careful about what time step t the belief is about, and what evidence it includes

Example: Passage of Time

- As time passes, uncertainty "accumulates"

(Transition model: ghosts usually go clockwise)

<0.01	<0.01	<0.01	<0.01	<0.01	<0.01
<0.01	<0.01	<0.01	<0.01	<0.01	<0.01
<0.01	<0.01	1.00	<0.01	<0.01	<0.01
<0.01	<0.01	<0.01	<0.01	<0.01	<0.01

T = 1

<0.01	<0.01	<0.01	<0.01	<0.01	<0.01
<0.01	<0.01	0.06	<0.01	<0.01	<0.01
<0.01	0.76	0.06	0.06	<0.01	<0.01
<0.01	<0.01	0.06	<0.01	<0.01	<0.01

T = 2

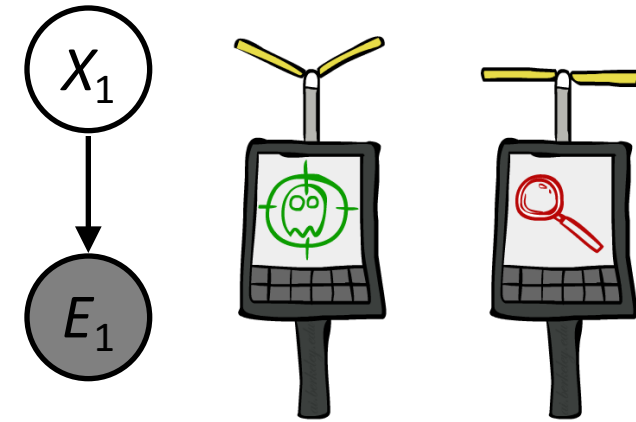| 0.05	0.01	0.05	<0.01	<0.01	<0.01
0.02	0.14	0.11	0.35	<0.01	<0.01
0.07	0.03	0.05	<0.01	0.03	<0.01
0.03	0.03	<0.01	<0.01	<0.01	<0.01

T = 5
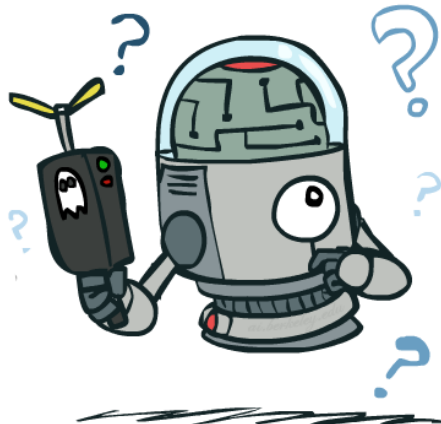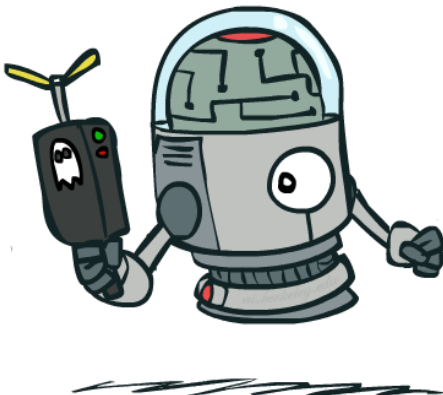
Observation

- Assume we have current belief P(X | previous evidence):

$$B'(X_{t+1}) = P(X_{t+1}|e_{1:t})$$

- Then, after evidence comes in:

$$P(X_{t+1}|e_{1:t+1}) = P(X_{t+1}, e_{t+1}|e_{1:t})/P(e_{t+1}|e_{1:t})$$

$$\propto_{X_{t+1}} P(X_{t+1}, e_{t+1}|e_{1:t})$$

$$= P(e_{t+1}|e_{1:t}, X_{t+1})P(X_{t+1}|e_{1:t})$$

$$= P(e_{t+1}|X_{t+1})P(X_{t+1}|e_{1:t})$$

- Or, compactly:

$$B(X_{t+1}) \propto_{X_{t+1}} P(e_{t+1}|X_{t+1})B'(X_{t+1})$$

- Basic idea: beliefs "reweighted" by likelihood of evidence

- Unlike passage of time, we have to renormalize

Example: Observation

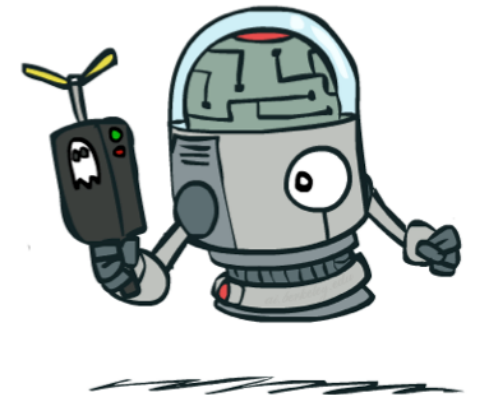- As we get observations, beliefs get reweighted, uncertainty "decreases"

Before observation

After observation

$$B(X) \propto P(e|X)B'(X)$$

Example: Weather HMM

B(+r) = 0.5
B(-r) = 0.5

B'(+r) = 0.5
B'(-r) = 0.5

B(+r) = 0.818
B(-r) = 0.182

B'(+r) = 0.627
B'(-r) = 0.373

B(+r) = 0.883
B(-r) = 0.117
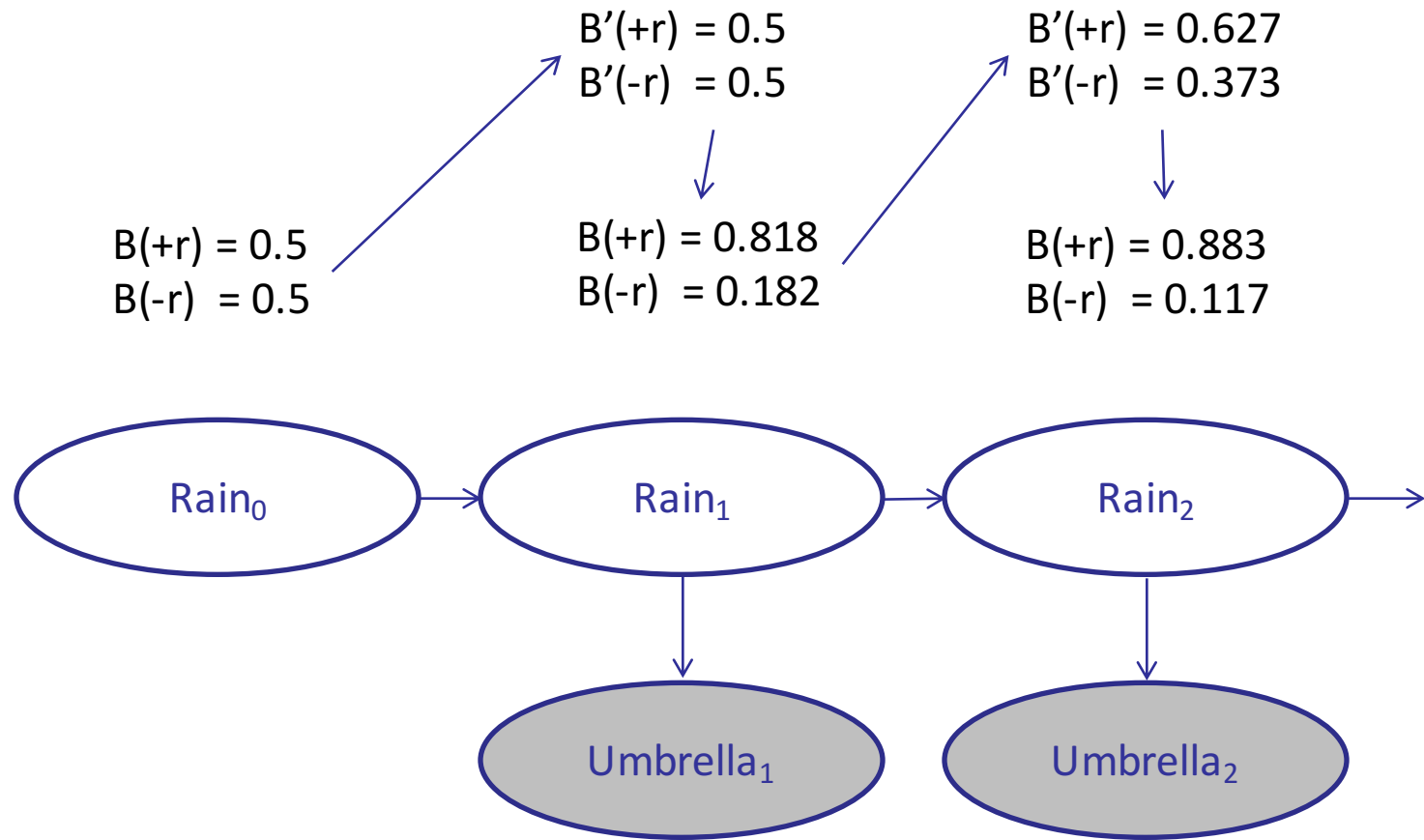
R_t	R_{t+1}	$P(R_{t+1}|R_t)$
+r	+r	0.7
+r	-r	0.3
-r	+r	0.3
-r	-r	0.7

R_t	U_t	$P(U_t|R_t)$
+r	+u	0.9
+r	-u	0.1
-r	+u	0.2
-r	-u	0.8

Online Belief Updates
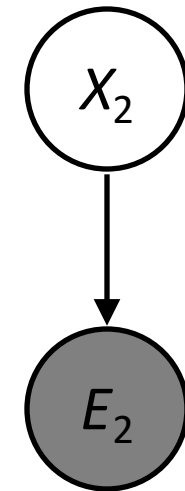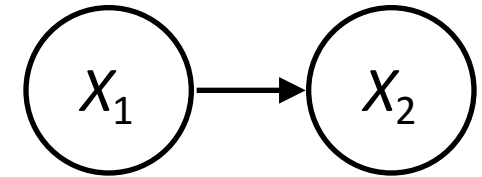
- Every time step, we start with current P(X | evidence)

- We update for time:

$$P(x_t|e_{1:t-1}) = \sum_{x_{t-1}} P(x_{t-1}|e_{1:t-1}) \cdot P(x_t|x_{t-1})$$

- We update for evidence:

$$P(x_t|e_{1:t}) \propto_X P(x_t|e_{1:t-1}) \cdot P(e_t|x_t)$$

- The forward algorithm does both at once (and doesn't normalize)

The Forward Algorithm

- We are given evidence at each time and want to know

$$B_t(X) = P(X_t|e_{1:t})$$

- We can derive the following updates

$$P(x_t|e_{1:t}) \propto_X P(x_t, e_{1:t})$$

We can normalize as we go if we want to have P(x|e) at each time step, or just once at the end...

$$= \sum_{x_{t-1}} P(x_{t-1}, x_t, e_{1:t})$$

$$= \sum_{x_{t-1}} P(x_{t-1}, e_{1:t-1})P(x_t|x_{t-1})P(e_t|x_t)$$

$$= P(e_t|x_t) \sum_{x_{t-1}} P(x_t|x_{t-1})P(x_{t-1}, e_{1:t-1})$$

Pacman – Sonar (P4)

Video of Demo Pacman – Sonar (with beliefs)

Next Time: Particle Filtering and Applications of HMMs