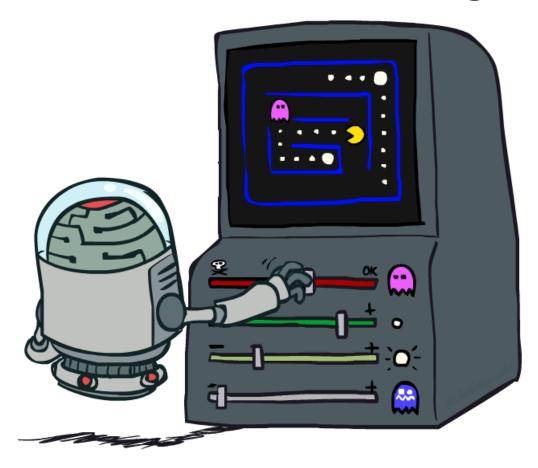
# CS 188: Artificial Intelligence

#### Reinforcement Learning II



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[These slides were created by Dan Klein, Pieter Abbeel, and Anca. http://ai.berkeley.edu.]

## Reinforcement Learning

- We still assume an MDP:
  - A set of states  $s \in S$
  - A set of actions (per state) A
  - A model T(s,a,s')
  - A reward function R(s,a,s')
- Still looking for a policy  $\pi(s)$



- New twist: don't know T or R, so must try out actions
- Big idea: Compute all averages over T using sample outcomes

#### The Story So Far: MDPs and RL

**Known MDP: Offline Solution** 

Goal

Technique

Compute V\*, Q\*,  $\pi$ \*

Value / policy iteration

Evaluate a fixed policy  $\pi$ 

Policy evaluation

Unknown MDP: Model-Based

Technique

Compute V\*, Q\*,  $\pi$ \* VI/PI on approx. MDP

Goal

Evaluate a fixed policy  $\pi$  PE on approx. MDP

Unknown MDP: Model-Free

Goal

Technique

Compute V\*, Q\*,  $\pi$ \*

Q-learning

Evaluate a fixed policy  $\pi$ 

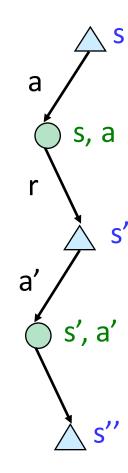
Value Learning

## Model-Free Learning

- Model-free (temporal difference) learning
  - Experience world through episodes

$$(s, a, r, s', a', r', s'', a'', r'', s'''' \dots)$$

- Update estimates each transition (s, a, r, s')
- Over time, updates will mimic Bellman updates



## Approximating Values through Samples

#### Policy Evaluation:

$$V_{k+1}^{\pi}(s) \leftarrow \sum_{s'} T(s, \pi(s), s') [R(s, \pi(s), s') + \gamma V_k^{\pi}(s')]$$



Value Iteration:

$$V_{k+1}(s) \leftarrow \max_{a} \sum_{s'} T(s, a, s') \left[ R(s, a, s') + \gamma V_k(s') \right]$$



Q-Value Iteration:

$$Q_{k+1}(s,a) \leftarrow \sum_{s'} T(s,a,s') \left[ R(s,a,s') + \gamma \max_{a'} Q_k(s',a') \right]$$



#### **Q-Learning**

We'd like to do Q-value updates to each Q-state:

$$Q_{k+1}(s, a) \leftarrow \sum_{s'} T(s, a, s') \left[ R(s, a, s') + \gamma \max_{a'} Q_k(s', a') \right]$$

- But can't compute this update without knowing T, R
- Instead, compute average as we go
  - Receive a sample transition (s,a,r,s')
  - This sample suggests

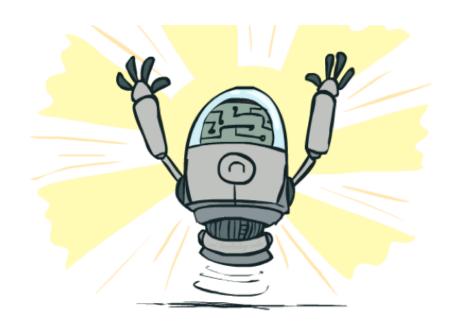
$$Q(s,a) \approx r + \gamma \max_{a'} Q(s',a')$$

- But we want to average over results from (s,a) (Why?)
- So keep a running average

$$Q(s, a) \leftarrow (1 - \alpha)Q(s, a) + (\alpha) \left[ r + \gamma \max_{a'} Q(s', a') \right]$$

## **Q-Learning Properties**

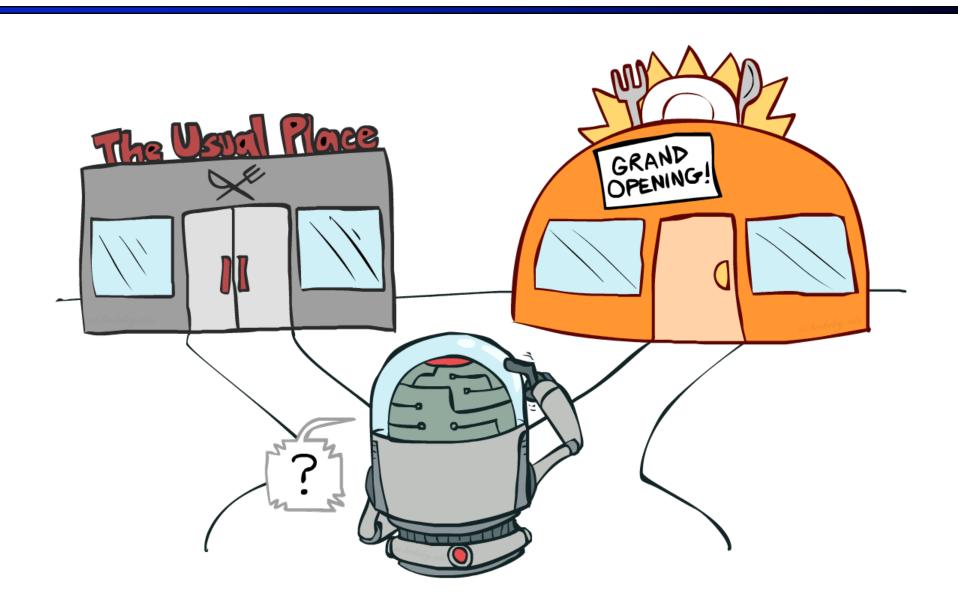
- Amazing result: Q-learning converges to optimal policy -- even if you're acting suboptimally!
- This is called off-policy learning
- Caveats:
  - You have to explore enough
  - You have to eventually make the learning rate small enough
  - ... but not decrease it too quickly
  - Basically, in the limit, it doesn't matter how you select actions (!)



## Video of Demo Q-Learning Auto Cliff Grid



# Exploration vs. Exploitation



### How to Explore?

- Several schemes for forcing exploration
  - Simplest: random actions (ε-greedy)
    - Every time step, flip a coin
    - With (small) probability ε, act randomly
    - With (large) probability 1- $\varepsilon$ , act on current policy
  - Problems with random actions?
    - You do eventually explore the space, but keep thrashing around once learning is done
    - One solution: lower ε over time
    - Another solution: exploration functions



#### Video of Demo Q-learning – Manual Exploration – Bridge Grid



#### Video of Demo Q-learning – Epsilon-Greedy – Crawler



### **Exploration Functions**

#### When to explore?

- Random actions: explore a fixed amount
- Better idea: explore areas whose badness is not (yet) established, eventually stop exploring

#### Exploration function

■ Takes a value estimate u and a visit count n, and returns an optimistic utility, e.g. f(u, n) = u + k/n

Regular Q-Update: 
$$Q(s, a) \leftarrow_{\alpha} R(s, a, s') + \gamma \max_{a'} Q(s', a')$$

Modified Q-Update: 
$$Q(s, a) \leftarrow_{\alpha} R(s, a, s') + \gamma \max_{a'} f(Q(s', a'), N(s', a'))$$

Note: this propagates the "bonus" back to states that lead to unknown states as well!

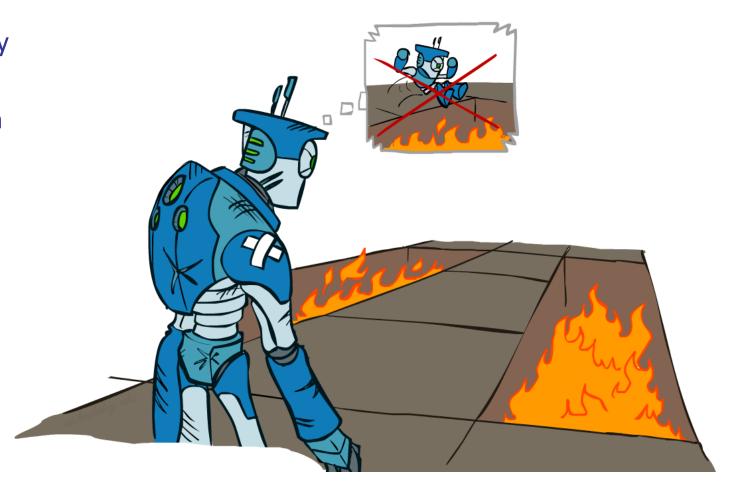
[Demo: exploration – Q-learning – crawler – exploration function (L11D4)]

#### Video of Demo Q-learning – Exploration Function – Crawler

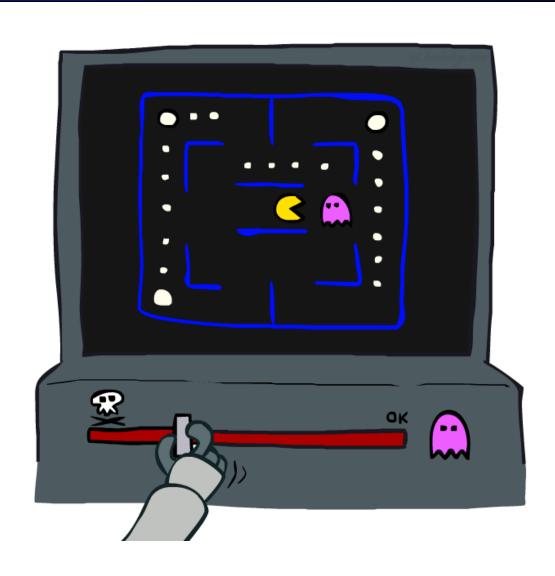


#### Regret

- Even if you learn the optimal policy, you still make mistakes along the way
- Regret is a measure of your total mistake cost: the difference between your (expected) rewards, including youthful suboptimality, and optimal (expected) rewards
- Minimizing regret goes beyond learning to be optimal – it requires optimally learning to be optimal
- Example: random exploration and exploration functions both end up optimal, but random exploration has higher regret

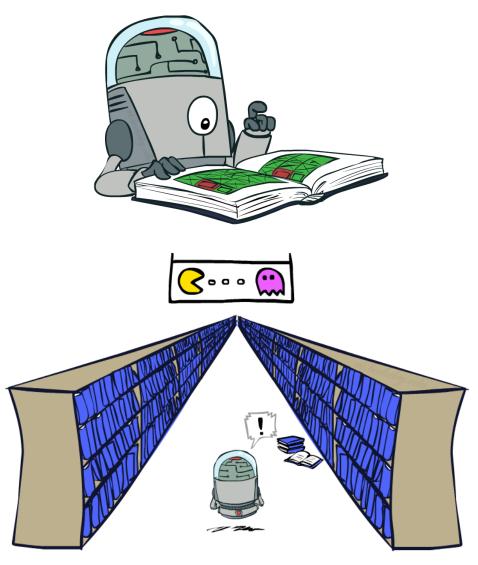


# Approximate Q-Learning



## Generalizing Across States

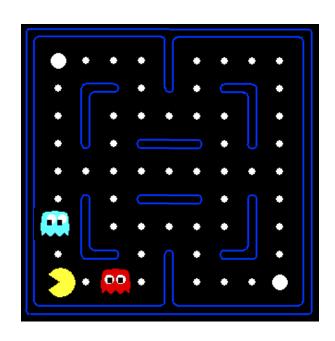
- Basic Q-Learning keeps a table of all q-values
- In realistic situations, we cannot possibly learn about every single state!
  - Too many states to visit them all in training
  - Too many states to hold the q-tables in memory
- Instead, we want to generalize:
  - Learn about some small number of training states from experience
  - Generalize that experience to new, similar situations
  - This is a fundamental idea in machine learning, and we'll see it over and over again

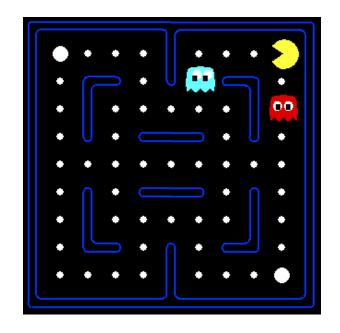


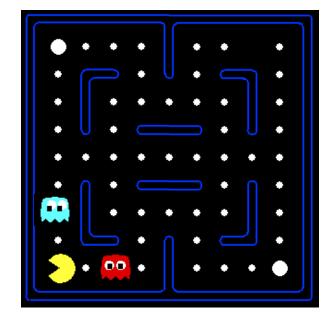
#### Example: Pacman

Let's say we discover through experience that this state is bad: In naïve q-learning, we know nothing about this state:

Or even this one!







[Demo: Q-learning – pacman – tiny – watch all (L11D5)]

[Demo: Q-learning – pacman – tiny – silent train (L11D6)]

[Demo: Q-learning – pacman – tricky – watch all (L11D7)]

#### Video of Demo Q-Learning Pacman – Tiny – Watch All



#### Video of Demo Q-Learning Pacman – Tiny – Silent Train

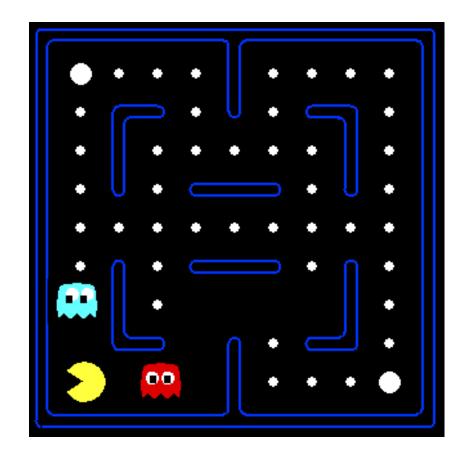


#### Video of Demo Q-Learning Pacman – Tricky – Watch All



#### Feature-Based Representations

- Solution: describe a state using a vector of features (properties)
  - Features are functions from states to real numbers (often 0/1) that capture important properties of the state
  - Example features:
    - Distance to closest ghost
    - Distance to closest dot
    - Number of ghosts
    - 1 / (dist to dot)<sup>2</sup>
    - Is Pacman in a tunnel? (0/1)
    - ..... etc.
    - Is it the exact state on this slide?
  - Can also describe a q-state (s, a) with features (e.g. action moves closer to food)



#### Linear Value Functions

Using a feature representation, we can write a q function (or value function) for any state using a few weights:

$$V(s) = w_1 f_1(s) + w_2 f_2(s) + \dots + w_n f_n(s)$$

$$Q(s,a) = w_1 f_1(s,a) + w_2 f_2(s,a) + \dots + w_n f_n(s,a)$$

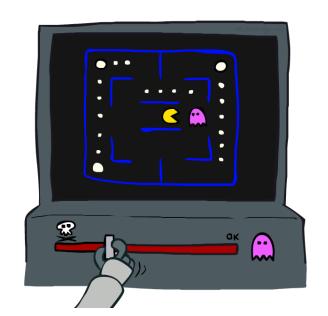
- Advantage: our experience is summed up in a few powerful numbers
- Disadvantage: states may share features but actually be very different in value!

### Approximate Q-Learning

$$Q(s,a) = w_1 f_1(s,a) + w_2 f_2(s,a) + \dots + w_n f_n(s,a)$$

Q-learning with linear Q-functions:

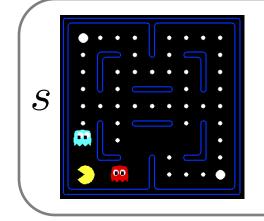
$$\begin{aligned} & \text{transition } = (s, a, r, s') \\ & \text{difference} = \left[r + \gamma \max_{a'} Q(s', a')\right] - Q(s, a) \\ & Q(s, a) \leftarrow Q(s, a) + \alpha \text{ [difference]} \end{aligned} \quad & \text{Exact Q's} \\ & w_i \leftarrow w_i + \alpha \text{ [difference]} f_i(s, a) \quad & \text{Approximate Q's} \end{aligned}$$



- Intuitive interpretation:
  - Adjust weights of active features
  - E.g., if something unexpectedly bad happens, blame the features that were on: disprefer all states with that state's features
- Formal justification: online least squares

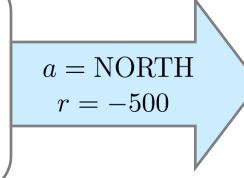
### Example: Q-Pacman

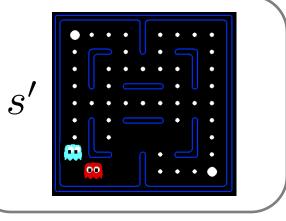
$$Q(s, a) = 4.0 f_{DOT}(s, a) - 1.0 f_{GST}(s, a)$$



$$f_{DOT}(s, NORTH) = 0.5$$

$$f_{GST}(s, NORTH) = 1.0$$





$$Q(s',\cdot)=0$$

$$Q(s, NORTH) = +1$$
  
 $r + \gamma \max_{a'} Q(s', a') = -500 + 0$ 

difference 
$$= -501$$

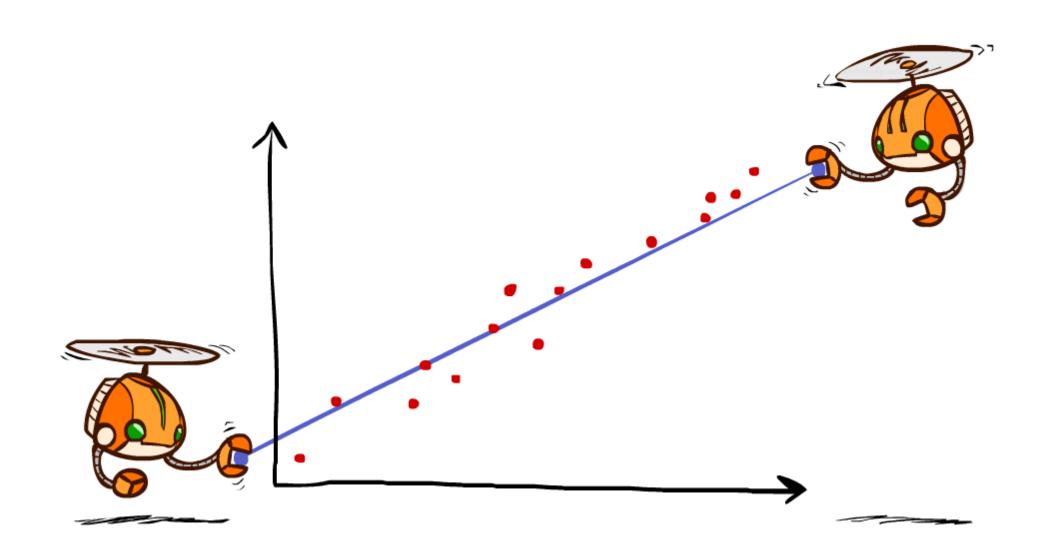
$$w_{DOT} \leftarrow 4.0 + \alpha [-501] 0.5$$
  
 $w_{GST} \leftarrow -1.0 + \alpha [-501] 1.0$ 

$$Q(s, a) = 3.0 f_{DOT}(s, a) - 3.0 f_{GST}(s, a)$$

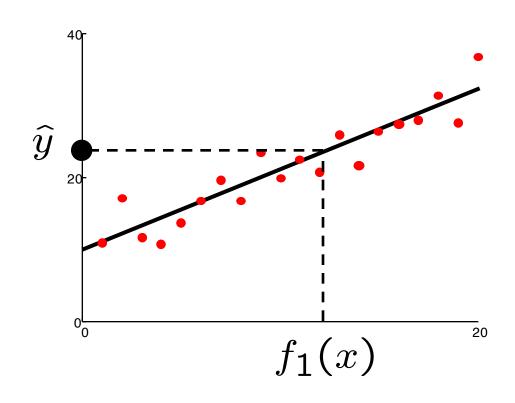
# Video of Demo Approximate Q-Learning -- Pacman

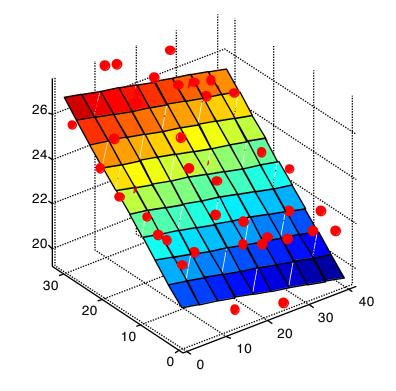


# Q-Learning and Least Squares



### Linear Approximation: Regression\*





Prediction:

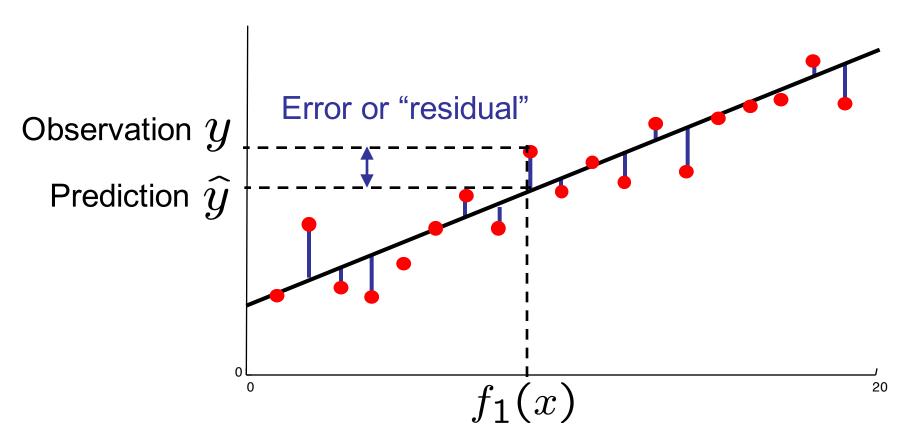
$$\hat{y} = w_0 + w_1 f_1(x)$$

Prediction:

$$\hat{y}_i = w_0 + w_1 f_1(x) + w_2 f_2(x)$$

## Optimization: Least Squares\*

total error = 
$$\sum_{i} (y_i - \hat{y_i})^2 = \sum_{i} \left(y_i - \sum_{k} w_k f_k(x_i)\right)^2$$



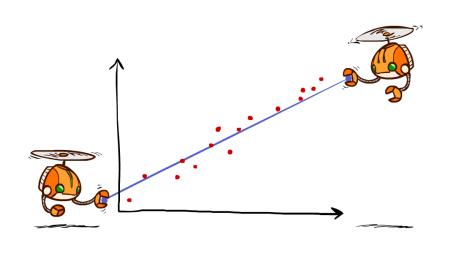
## Minimizing Error\*

Imagine we had only one point x, with features f(x), target value y, and weights w:

$$\operatorname{error}(w) = \frac{1}{2} \left( y - \sum_{k} w_{k} f_{k}(x) \right)^{2}$$

$$\frac{\partial \operatorname{error}(w)}{\partial w_{m}} = -\left( y - \sum_{k} w_{k} f_{k}(x) \right) f_{m}(x)$$

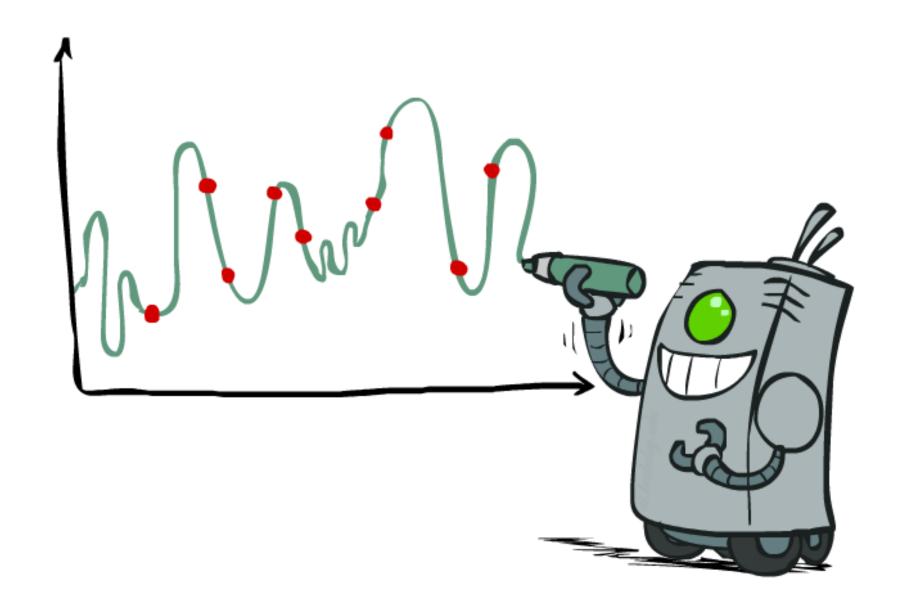
$$w_{m} \leftarrow w_{m} + \alpha \left( y - \sum_{k} w_{k} f_{k}(x) \right) f_{m}(x)$$

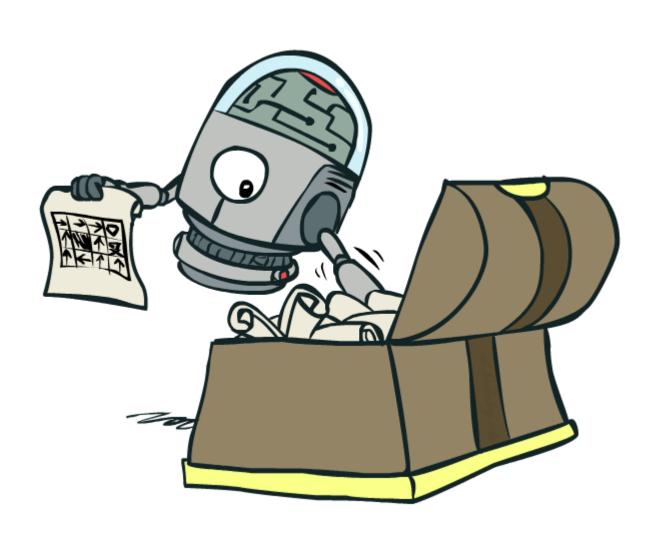


Approximate q update explained:

$$w_m \leftarrow w_m + \alpha \left[ r + \gamma \max_a Q(s', a') - Q(s, a) \right] f_m(s, a)$$
"target" "prediction"

# Overfitting: Why Limiting Capacity Can Help\*





- Problem: often the feature-based policies that work well (win games, maximize utilities) aren't the ones that approximate V / Q best
  - E.g. your value functions from project 2 were probably horrible estimates of future rewards, but they still produced good decisions
  - Q-learning's priority: get Q-values close (modeling)
  - Action selection priority: get ordering of Q-values right (prediction)
  - We'll see this distinction between modeling and prediction again later in the course
- Solution: learn policies that maximize rewards, not the values that predict them
- Policy search: start with an ok solution (e.g. Q-learning) then fine-tune by hill climbing on feature weights

- Simplest policy search:
  - Start with an initial linear value function or Q-function
  - Nudge each feature weight up and down and see if your policy is better than before
- Problems:
  - How do we tell the policy got better?
  - Need to run many sample episodes!
  - If there are a lot of features, this can be impractical
- Better methods exploit lookahead structure, sample wisely, change multiple parameters...



[Andrew Ng] [Video: HELICOPTER]

#### Conclusion

- We're done with Part I: Search and Planning!
- We've seen how AI methods can solve problems in:
  - Search
  - Constraint Satisfaction Problems
  - Games
  - Markov Decision Problems
  - Reinforcement Learning
- Next up: Part II: Uncertainty and Learning!

