CSC249/449MachineVision: Homework1

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Problem 3

1.

(1) Box Filter

$$\frac{1}{9}\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix} = \frac{1}{3}\begin{bmatrix}1\\ 1\\ 1\end{bmatrix} * \frac{1}{3}\begin{bmatrix}1 & 1 & 1\end{bmatrix}$$

(2) Sobel Filter

$$\begin{bmatrix}-1 & 2 & -1\\ 0 & 0 & 0\\ 1 & 2 & 1\end{bmatrix} = \begin{bmatrix}-1\\ 0\\ 1\end{bmatrix} * \begin{bmatrix}1 & 2 & 1\end{bmatrix}$$

2.

$O(2kHW)$

Problem 4

Laplacian filter is a filter that takes image derivatives so that it is very sensitive to noise. Therefore, applying a low-pass filter such as Gaussian to smooth the image before computing the gradient is desirable. Laplacian of Gaussian operation is a two-step process which will use Gaussian filter and then applying Laplacian.

Problem 5

1.

By applying first order Taylor expansion, we get:

$$I(x+\mu, y+v, t+\omega) = I(x, y, \omega) + \ \mu\frac{\partial I}{\partial x} + v\frac{\partial I}{\partial y} + \omega\frac{\partial I}{\partial t}$$

So,

$$E(\mu, v, \omega) \approx \sum_{\mu, v, \omega \epsilon W}\left(\mu\frac{\partial I}{\partial x} + v\frac{\partial I}{\partial y} + \omega\frac{\partial I}{\partial t}\right)^2$$

$$\approx [\mu \quad v \quad \omega] \begin{bmatrix} \frac{\partial I}{\partial x}^2 & \frac{\partial I}{\partial x}\frac{\partial I}{\partial y} & \frac{\partial I}{\partial x}\frac{\partial I}{\partial t} \\ \frac{\partial I}{\partial x}\frac{\partial I}{\partial y} & \frac{\partial I}{\partial y}^2 & \frac{\partial I}{\partial y}\frac{\partial I}{\partial t} \\ \frac{\partial I}{\partial x}\frac{\partial I}{\partial t} & \frac{\partial I}{\partial y}\frac{\partial I}{\partial t} & \frac{\partial I}{\partial t}^2 \end{bmatrix} \begin{bmatrix} \mu \\ v \\ \omega \end{bmatrix}$$

The 3D structure tensor is:

$$\begin{bmatrix} \frac{\partial I}{\partial x}^2 & \frac{\partial I}{\partial x}\frac{\partial I}{\partial y} & \frac{\partial I}{\partial x}\frac{\partial I}{\partial t} \\ \frac{\partial I}{\partial x}\frac{\partial I}{\partial y} & \frac{\partial I}{\partial y}^2 & \frac{\partial I}{\partial y}\frac{\partial I}{\partial t} \\ \frac{\partial I}{\partial x}\frac{\partial I}{\partial t} & \frac{\partial I}{\partial y}\frac{\partial I}{\partial t} & \frac{\partial I}{\partial t}^2 \end{bmatrix}$$

Because it is a symmetric matrix, its eigenvalues λ_1, λ_2, λ_3 are real.

For criterion, as λ_1, λ_2, λ_3 are in decreasing order, so the λ_3 is the smallest value. According to Harris operator for 2D corner $\frac{1}{\frac{1}{\lambda_1}+\frac{1}{\lambda_2}}$, 3D corner criterion could be $\frac{1}{\frac{1}{\lambda_1}+\frac{1}{\lambda_2}+\frac{1}{\lambda_3}}$ and the λ_3 should be $\lambda_3 \gg 0$. In my opinion, the eigenvalue of time dimension actually represents the position change among frames. Therefore, when the video is static, 3D corner will become a 2D corner.

2.

To be a 3D corner, all three λ_i should be large enough which means a 3D corner should be a 2D corner in every frame of a video. In other words, 3D corner will always be detected as a corner. This also meet the criterion that the smallest λ_i should be large enough no matter the λ_i is for time dimension or other two dimensions (remember in Harris 2D corner detector, both $\lambda_1$and λ_2 are required to be large).

Specifically, a desk corner is detected at 1s in a video at position (1, 1), then it changes its position to (1, 2), but remains as a corner. The position change is denoted by λ_{time}. If it remains a corner no matter it position change, it can be considered as a 3D corner. Or, the desk corner is detected at 1s at position (1, 1), but it has disappeared later. It cannot be considered as a 3D corner.