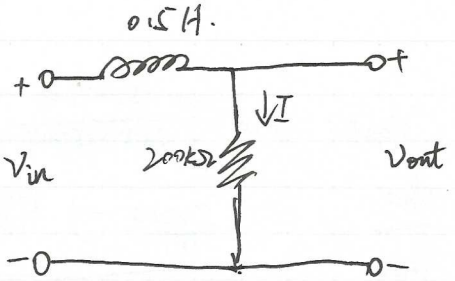
Q1

a) $Z_L = j\omega L = j\frac{\omega}{2}\,\Omega$



Input: $V_{in} = Z_{in} \cdot I$

$= I(j\frac{\omega}{2} + 200k\Omega)$.

Output: $V_{out} = Z_{out} \cdot I$

$= I \cdot 200k\Omega$.

$$\frac{V_{out}}{V_{in}} = \frac{200k\Omega}{200k\Omega + j\frac{\omega}{2}} = \frac{1}{1 + j\frac{\omega}{4\times10^6}}$$

$A = 1$, $\omega_c = 4\times10^6$ rad s^{-1}

b) When $\omega = 0$, $\frac{V_{out}}{V_{in}} = \frac{1}{1+0} = 1$ //

When $\omega = \omega_c$, $\left|\frac{V_{out}}{V_{in}}\right| = \lim_{\omega\to\omega_c} = \frac{1}{1+j} = |0.5 + j0.5| = \frac{1}{\sqrt{2}}$ //

When $\omega \to \infty$ $\frac{V_{out}}{V_{in}} = \lim_{\omega\to\infty} = \frac{1}{\infty} = 0$ //

c) When $\omega = 0$, $\angle(V_{out}/V_{in}) = 0°$.

When $\omega = \omega_c$, $V_{out}/V_{in} = 0.5 + j0.5 = \frac{1}{\sqrt{2}}\angle -45°$, $\angle(V_{out}/V_{in}) = -45°$.

When $\omega = \infty$, $\angle(V_{out}/V_{in}) = \tan^{-1}(\infty) = -90°$.

d) low pass. filter.

Q2.

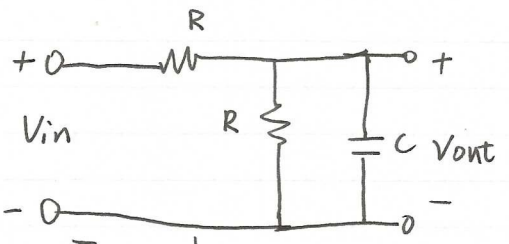
$$V_{out}/V_{in} = \frac{Z_1}{R + Z_1}$$

$$= \frac{\frac{R}{(j\omega CR+1)}}{R + \frac{R}{(j\omega CR+1)}}$$

$$= \frac{1}{j\omega CR + 1 + 1}$$

$$= \frac{\frac{1}{2}}{1 + j\omega\left(\frac{2}{CR}\right)}$$

$$= \frac{0.5}{1 + j\omega/\omega_c}$$



$Z_c = \frac{1}{j\omega C}$.

$Z_1 = \frac{R}{j\omega CR + 1}$

$A = \frac{1}{2}$, $\omega_c = \frac{2}{CR} = \frac{2}{500 \times 200\mu} = 20$ rad/s

(b) $\omega \to 0$, $|V_{out}/V_{in}| = 0.5$

$\omega \to \omega_c$, $|V_{out}/V_{in}| = \sqrt{\frac{1}{4}^2 + \frac{1}{4}^2} = \frac{1}{2\sqrt{2}}$ //

$\omega \to \infty$, $|V_0/V_i| = \frac{0.5}{\infty} = 0$ //

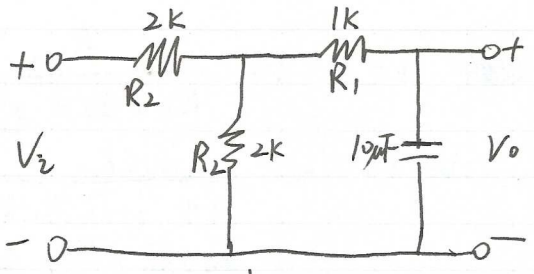
(c) $\omega \to 0$, $\angle(V_0/V_i) = 0°$

$\omega \to \omega_c$, $\angle(V_0/V_i) = -45°$

$\omega \to \infty$, $\angle(V_0/V_i) = \tan^{-1}\infty = -90°$ //

(d) low pass filter.

Q3 $V_{out} = V_i \cdot \frac{Z_{eq}}{R_2 + Z_{eq}} \cdot \frac{Z_C}{Z_1}$



$Z_C = \frac{1}{j\omega C} = -j100\ k\Omega$

$Z_1 = R_1 + \frac{1}{j\omega C} = \frac{1+j\omega CR_1}{j\omega C}$

$Z_{eq} = \frac{R_2 \cdot Z_1}{R_2 + Z_1}$

$= \frac{R_2}{\frac{R_2}{Z_1} + 1}$

$= \frac{R_2}{\frac{j\omega CR_2}{1+j\omega CR_1} + 1}$

$= \frac{R_2}{\frac{j\omega CR_2 + 1 + j\omega CR_1}{1+j\omega CR_1}}$

$= \frac{R_2}{\frac{j\omega C(R_1+R_2)+1}{j\omega CR_1 + 1}}$

$\frac{V_{out}}{V_i} = \frac{1}{1+R_2/Z_{eq}+1} \cdot \frac{\frac{1}{j\omega C}}{1+j\omega CR_1}$

$= \frac{1}{\frac{j\omega C(R_1+R_2)+1}{j\omega CR_1+1} + 1} \cdot \frac{1}{1+j\omega CR_1}$

$= \frac{1}{2 + j\omega C(2R_1+R_2)}$

$= \frac{\frac{1}{2}}{1 + j\omega\left[\frac{2}{C(2R_1+R_2)}\right]}$

$A = 0.5,\ \omega_c = \frac{2}{10\mu \times (4000)} = 50\ rad/s.$

(b)

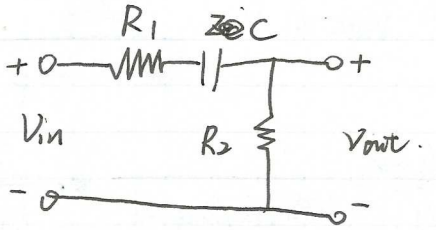
$\omega \to 0$	$\lvert \frac{V_o}{V_i} \rvert = 0.5$
$\omega \to \omega_c$	$\lvert V_o/V_i \rvert = \frac{1}{2\sqrt{2}}$ //
$\omega \to \infty$	$\lvert V_o/V_i \rvert = 0$ //

(c)

$\omega \to 0$,	$\angle \varphi = 0°$
$\omega \to \omega_c$,	$\varphi = -45°$
$\omega \to \infty$	$\varphi = -90°$

d) low pass.

Q4 $V_o = V_i \dfrac{R_2}{R_1 + R_2 + \frac{1}{j\omega C}}$



$$V_o/V_i = \frac{\frac{R_2}{R_1+R_2}}{1 - j\frac{1}{\omega \cdot C(R_1+R_2)}}$$

$$= \frac{A}{1 - j\frac{\omega_c}{\omega}}$$

$$A = \frac{R_2}{R_1+R_2}, \quad \omega_c = \frac{1}{C(R_1+R_2)}$$

b) $\omega \to 0$, $\dfrac{V_o}{V_i} = \dfrac{1}{\infty} = 0$.

$\omega \to \omega_c$, $\left|\dfrac{V_o}{V_i}\right| = \dfrac{R_2}{R_1+R_2} \cdot \dfrac{1}{\sqrt{2}}$.

$\omega \to \infty$, $\left|\dfrac{V_o}{V_i}\right| = \dfrac{A}{1+0} = \dfrac{R_1}{R_1+R_2}$ //

c) $\omega \to 0$, $\varphi = \tan^{-1}(\frac{1}{0}) = 90°$

$\omega \to \omega_c$, $\varphi = 45°$

$\omega \to \infty$, $\varphi = \tan^{-1}(\frac{1}{\infty}) = 0°$.

d) high-pass filter.