

**MA1200****Practice Exercise for Ch. 4 Trigonometric Functions and Inverse Trigonometric Functions**

1. (a) Convert the following angles to radians.

(i)  $48^\circ$

(ii)  $120^\circ$

(iii)  $315^\circ$

- (b) Convert the following angles to degree.

(i)  $\frac{\pi}{6}$  rad

(ii)  $\frac{123\pi}{180}$  rad

(iii)  $-\frac{2\pi}{5}$  rad

2. For each of the functions below, (i) plot its graph from
- $x = -\frac{\pi}{2}$
- to
- $x = 2\pi$
- ; (ii) find its domain and range; (iii) determine whether the function is periodic or not. If it is periodic, find its period.

(a)  $f(x) = 5 \sin x + 1$

(b)  $f(x) = \cos \frac{x}{2}$

(c)  $f(x) = 2 \sin\left(x - \frac{\pi}{3}\right)$

(d)  $f(x) = \tan\left(x + \frac{\pi}{2}\right)$

(e)  $f(x) = |-2 \sin x|$

(f)  $f(x) = -2|\sin x|$

3. For each of the functions below, (i) plot its graph from
- $x = -\frac{\pi}{2}$
- to
- $x = 2\pi$
- ; (ii) find its domain and range.

(a)  $f(x) = u_\pi(x) \cos x$

(b)  $f(x) = u_{\frac{\pi}{2}}(x) + \sin x$

(c)  $f(x) = \frac{x}{x} + \cos x$

Remark: (Remark:  $u_a(x) = \begin{cases} 0 & \text{if } x < a, \\ 1 & \text{if } x \geq a. \end{cases}$ )

4. Prove the following identities.

(a) 
$$\frac{\frac{1}{\cos \theta} - \cos \theta}{\frac{1}{\sin \theta} - \sin \theta} = \tan^3 \theta$$

(b) 
$$\frac{\csc^2 \theta}{1 + \tan^2 \theta} = \csc^2 \theta - 1$$

5. If
- $\cos \theta = -\frac{4}{5}$
- and
- $\theta$
- is in Quadrant III, find

(a)  $\sin \theta$

(b)  $\tan \theta$

(c)  $\csc \theta$

6. Simplify each of the following:

(a) 
$$\frac{\sin\left(\frac{\pi}{2} + \theta\right) \cos\left(\frac{3\pi}{2} - \theta\right)}{\sec(\theta - \pi)}$$

(b) 
$$\frac{\tan\left(\theta + \frac{3\pi}{2}\right) \cot\left(\frac{3\pi}{2} + \theta\right)}{\csc\left(\theta - \frac{\pi}{2}\right)}$$

7. Find the exact values of the following:

(a)  $\sin\left(\sin^{-1}\frac{2}{5}\right)$       (b)  $\sin^{-1}\left(\sin\frac{\pi}{4}\right)$       (c)  $\sin^{-1}\left(\sin\left(-\frac{2\pi}{3}\right)\right)$   
(d)  $\sin^{-1}\left(\tan\left(\frac{3\pi}{4}\right)\right)$       (e)  $\cos\left(\cos^{-1}\frac{3}{4}\right)$       (f)  $\cos^{-1}\left(\cos\frac{5\pi}{4}\right)$   
(g)  $\cos^{-1}\left(\sin\left(-\frac{\pi}{6}\right)\right)$       (h)  $\tan^{-1}(\tan\pi)$

8. Find the value of the following in surd form.

(a)  $\sin 35^\circ \cos 25^\circ + \sin 25^\circ \cos 35^\circ$       (b)  $\tan 165^\circ$

9. Prove the following identities.

(a)  $\cos(A+B)\cos(A-B) = \cos^2 A - \sin^2 B$       (b)  $\frac{\sin 2A}{\cos 2A + 1} = \tan A$   
(c)  $(\sin A - \cos A)^2 = 1 - \sin 2A$

10. Find the value of  $\frac{\sin 75^\circ - \sin 15^\circ}{\cos 75^\circ + \cos 15^\circ}$  in surd form.

11. Prove the identity  $4\cos A \cos\left(\frac{2\pi}{3} + A\right) \cos\left(\frac{2\pi}{3} - A\right) = \cos 3A$ .

12. (a) Express  $\sin(x + 45^\circ)$  in terms of  $\sin x$  and  $\cos x$ .

(b) Using (a), plot the graph of  $y = \cos x + \sin x$  for  $x = -\frac{\pi}{2}$  to  $x = 2\pi$ .

13. Solve each of the following equations.

(a)  $\cos\frac{x}{2} = \frac{\sqrt{3}}{2}$       (b)  $2\sin^2 x + \sin x - 1 = 0$