

**Question 1.1:**

$$\begin{aligned}
G' &= (V \cup \{s, t\}, E') \\
E &\subseteq E' \\
\text{for all } u \in V &\begin{cases} (s, u) \in E' \\ (u, t) \in E' \end{cases}
\end{aligned}$$

$$w_{uv} = \begin{cases} 1 & (u, v) \in E \\ \deg(v), & u = s \\ c & v = t \end{cases}$$

$$\begin{aligned}
\text{suppose } \exists S \subseteq V \text{ and } \rho(S) \geq \lambda \\
\bar{S} = V \setminus S
\end{aligned}$$

$$\begin{aligned}
\rho(S) &= \frac{\sum_{(u,v) \in E(S)} w_{uv}}{|S|} \\
\frac{\sum_{(u,v) \in E(S)} w_{uv}}{|S|} &\geq \lambda \\
\frac{2|E(S, S)|}{|S|} &\geq \lambda \\
\sum_{u \in S} \deg(u) - |E(S, \bar{S})| &\geq \lambda|S| \\
\sum_{u \in S} \deg(u) + \sum_{u \in \bar{S}} \deg(u) - \sum_{u \in \bar{S}} \deg(u) - |E(S, \bar{S})| &\geq \lambda|S| \\
\sum_{u \in \bar{S}} \deg(u) + |E(S, \bar{S})| + \lambda|S| &\leq 2|E|
\end{aligned}$$

$$\text{cut value if } S \begin{cases} = \emptyset, & 2|E| \\ \neq \emptyset, & \sum_{u \in \bar{S}} \deg(u) + |E(S, \bar{S})| + \lambda|S| \end{cases}$$

$$\begin{aligned}
WC(\{s\} \cup S, \bar{S} \cup \{t\}) &\leq \gamma \\
\frac{\sum_{(u,v) \in E', u \in \{s\} \cup S, v \in \bar{S} \cup \{t\}} w_{uv}}{|S|} &\leq \frac{\gamma}{|S|} \\
\frac{\gamma}{|S|} &\geq \rho(S) \\
\gamma &\geq \lambda|S|
\end{aligned}$$

**Question 1.2:**

$$\begin{aligned}\rho(S) &= \frac{c-1}{2} \\ \frac{\sum_{(u,v) \in E(S)} w_{uv}}{|S|} &= \frac{c-1}{2} \\ \sum_{(u,v) \in E'} w_{uv} &= \frac{|S|(c-1)}{2}\end{aligned}$$

## Question 2

$A_{ij}$											
$j \backslash i$	1	2	3	4	5	6	7	8	9	10	11
1	0	1	1	0	0	0	0	0	0	0	0
2	1	0	1	1	1	0	0	0	0	0	0
3	1	1	0	1	1	0	0	0	0	0	0
4	0	1	1	0	1	0	0	0	0	0	0
5	0	1	1	1	0	1	1	0	0	0	0
6	0	0	0	0	1	0	1	0	0	0	0
7	0	0	0	0	1	1	0	1	1	1	0
8	0	0	0	0	0	0	1	0	1	1	0
9	0	0	0	0	0	0	1	1	0	1	1
10	0	0	0	0	0	0	1	1	1	0	1
11	0	0	0	0	0	0	0	0	1	1	0
$d_1$	$d_2$	$d_3$	$d_4$	$d_5$	$d_6$	$d_7$	$d_8$	$d_9$	$d_{10}$	$d_{11}$	$ E $
= 2	= 4	= 4	= 3	= 5	= 2	= 5	= 3	= 4	= 4	= 2	= 19

$d_i d_j / 2 E $											
$j \backslash i$	1	2	3	4	5	6	7	8	9	10	11
1	2/19	4/19	4/19	3/19	5/19	2/19	5/19	3/19	4/19	4/19	2/19
2	4/19	8/19	8/19	6/19	10/19	4/19	10/19	6/19	8/19	8/19	4/19
3	4/19	8/19	8/19	6/19	10/19	4/19	10/19	6/19	8/19	8/19	4/19
4	3/19	6/19	6/19	9/38	15/38	3/19	15/38	9/38	6/19	6/19	3/19
5	5/19	10/19	10/19	15/38	25/38	5/19	25/38	15/38	10/19	10/19	5/19
6	2/19	4/19	4/19	3/19	5/19	2/19	5/19	3/19	4/19	4/19	2/19
7	5/19	10/19	10/19	15/38	25/38	5/19	25/38	15/38	10/19	10/19	5/19
8	3/19	6/19	6/19	9/38	15/38	3/19	15/38	9/38	6/19	6/19	3/19
9	4/19	8/19	8/19	6/19	10/19	4/19	10/19	6/19	8/19	8/19	4/19
10	4/19	8/19	8/19	6/19	10/19	4/19	10/19	6/19	8/19	8/19	4/19
11	2/19	4/19	4/19	3/19	5/19	2/19	5/19	3/19	4/19	4/19	2/19

$B_{ij}$											
$j \backslash i$	1	2	3	4	5	6	7	8	9	10	11
1	- 2/19	15/19	15/19	- 3/19	- 5/19	- 2/19	- 5/19	- 3/19	- 4/19	- 4/19	- 2/19
2	15/19	- 8/19	11/19	13/19	9/19	- 4/19	- 10/19	- 6/19	- 8/19	- 8/19	- 4/19
3	15/19	11/19	- 8/19	13/19	9/19	- 4/19	- 10/19	- 6/19	- 8/19	- 8/19	- 4/19
4	- 3/19	13/19	13/19	- 9/38	23/38	- 3/19	- 15/38	- 9/38	- 6/19	- 6/19	- 3/19
5	- 5/19	9/19	9/19	23/38	- 25/38	14/19	13/38	- 15/38	- 10/19	- 10/19	- 5/19
6	- 2/19	- 4/19	- 4/19	- 3/19	14/19	- 2/19	14/19	- 3/19	- 4/19	- 4/19	- 2/19
7	- 5/19	- 10/19	- 10/19	- 15/38	13/38	14/19	- 25/38	23/38	9/19	9/19	- 5/19
8	- 3/19	- 6/19	- 6/19	- 9/38	- 15/38	- 3/19	23/38	- 9/38	13/19	13/19	- 3/19
9	- 4/19	- 8/19	- 8/19	- 6/19	- 10/19	- 4/19	9/19	13/19	- 8/19	11/19	15/19
10	- 4/19	- 8/19	- 8/19	- 6/19	- 10/19	- 4/19	9/19	13/19	11/19	- 8/19	15/19
11	- 2/19	- 4/19	- 4/19	- 3/19	- 5/19	- 2/19	- 5/19	- 3/19	15/19	15/19	- 2/19

### Question 3

Iteration 1	Distance to			Assign to			
	Centroid 1	Centroid 2	Centroid 3			x	y
A1	0	3.6056	8.0623	Centroid 1	Centroid 1	2	10
A2	5	4.2426	3.1623	Centroid 3	Centroid 2	6	6
A3	8.4853	5	7.2801	Centroid 2	Centroid 3	1.5	3.5
A4	3.6056	0	7.2111	Centroid 2			
A5	7.0711	3.6056	6.7082	Centroid 2			
A6	7.2111	4.1231	5.3852	Centroid 2			
A7	8.0623	7.2111	0	Centroid 3			
A8	2.2361	1.4142	7.6158	Centroid 2			

Iteration 2	Distance to			Assign to			
	Centroid 1	Centroid 2	Centroid 3			x	y
A1	0	5.6569	6.5192	Centroid 1	Centroid 1	3	9.5
A2	5	4.1231	1.5811	Centroid 3	Centroid 2	6.5	5.25
A3	8.4853	2.8284	6.5192	Centroid 2	Centroid 3	1.5	3.5
A4	3.6056	2.2361	5.7009	Centroid 2			
A5	7.0711	1.4142	5.7009	Centroid 2			
A6	7.2111	2	4.5277	Centroid 2			
A7	8.0623	6.4031	1.5811	Centroid 3			
A8	2.2361	3.6056	6.0415	Centroid 1			

Iteration 3	Distance to			Assign to			
	Centroid 1	Centroid 2	Centroid 3			x	y
A1	1.1180	6.5431	6.5192	Centroid 1	Centroid 1	3.6667	9
A2	4.6098	4.5069	1.5811	Centroid 3	Centroid 2	7	4.3333
A3	7.4330	1.9526	6.5192	Centroid 2	Centroid 3	1.5	3.5
A4	2.5	3.1325	5.7009	Centroid 1			
A5	6.0208	0.5590	5.7009	Centroid 2			
A6	6.2650	1.3463	4.5277	Centroid 2			
A7	7.7621	6.3885	1.5811	Centroid 3			
A8	1.1180	4.5069	6.0415	Centroid 1			

Iteration 4	Centroid 1	Distance to Centroid 2	Centroid 3	Assign to		x	y
A1	1.9437	7.5572	6.5192	Centroid 1	Centroid 1	3.6667	9
A2	4.3333	5.0442	1.5811	Centroid 3	Centroid 2	7	4.3333
A3	6.6165	1.0541	6.5192	Centroid 2	Centroid 3	1.5	3.5
A4	1.6667	4.1767	5.7009	Centroid 1			
A5	5.2068	0.6667	5.7009	Centroid 2			
A6	5.5176	1.0541	4.5277	Centroid 2			
A7	7.4907	6.4377	1.5811	Centroid 3			
A8	0.3333	5.5478	6.0415	Centroid 1			

The new centroids are:

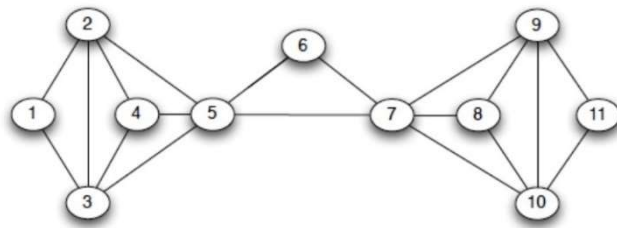
*C1: {A1, A4, A8} centre at (3.6667,9)*

*C2: {A3, A5, A6} centre at (7,4.333)*

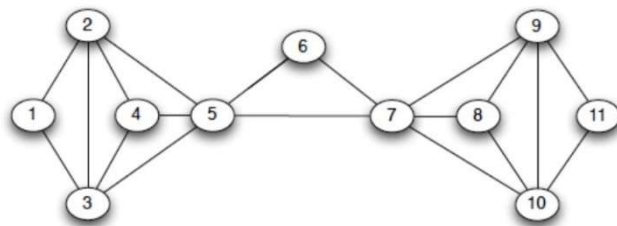
*C3: {A2, A7} centre at (1.5,3.5)*

## Question 4.1

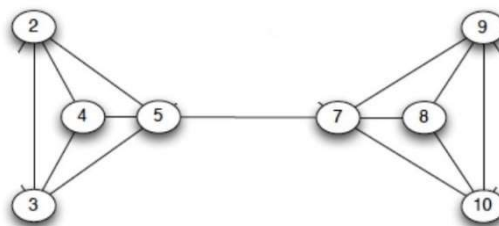
$K = 1$



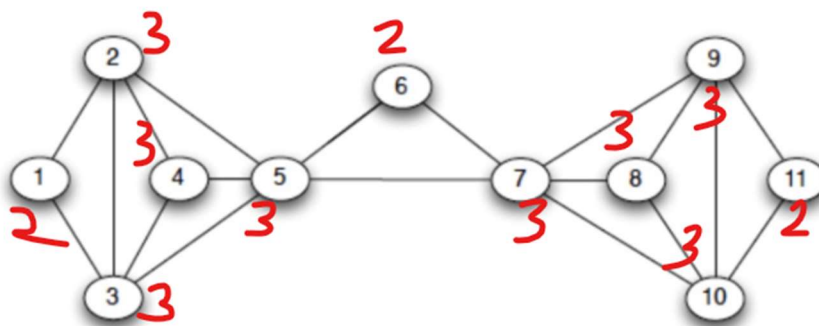
$K = 2$



$K = 3$



Core number of each node



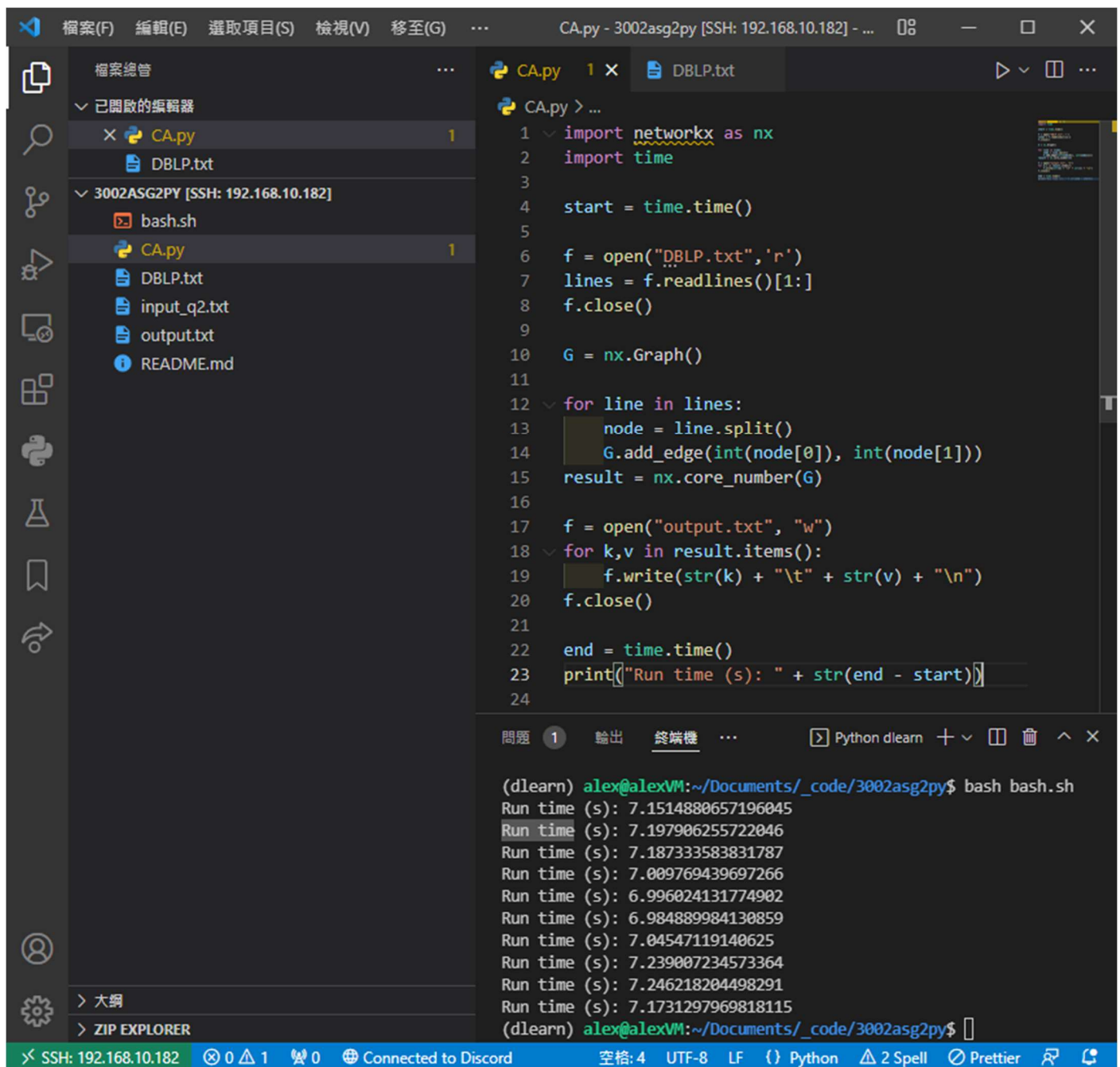
### Question 4.2:

Input: undirected graph  $G = (V, E)$

Output: array of number of core number for each node

1. Set a degree map for each node  $u$  in  $G$ , MapD, key:  $u$ , value: degree
2. Sort MapD by degree
3. Set a core number array for each node, coreD
4. Set  $G' \leftarrow G$
5. For  $k \leftarrow$  from 1 to  $n$ 
  1. If ( $G'$  empty) break from loop
  2. For all MapD [value]  $\leq k$ 
    1.  $G' \leftarrow G' \setminus u$
  3. Recalculate MapD
  4. For all MapD [value]  $\neq 0$ 
    1.  $\text{coreD}[v]++$
6. Return coreD

### Question 4.3:



The screenshot shows a VS Code editor window with a Python script named `CA.py` and its execution output in the terminal. The script reads a graph from `DBLP.txt`, calculates core numbers using `networkx`, and writes the results to `output.txt`. The terminal output shows the execution time for 10 different runs, with values ranging from approximately 6.985 to 7.246 seconds.

```
CA.py > ...
1 import networkx as nx
2 import time
3
4 start = time.time()
5
6 f = open("DBLP.txt", 'r')
7 lines = f.readlines()[1:]
8 f.close()
9
10 G = nx.Graph()
11
12 for line in lines:
13     node = line.split()
14     G.add_edge(int(node[0]), int(node[1]))
15 result = nx.core_number(G)
16
17 f = open("output.txt", "w")
18 for k,v in result.items():
19     f.write(str(k) + "\t" + str(v) + "\n")
20 f.close()
21
22 end = time.time()
23 print("Run time (s): " + str(end - start))
24
```

問題 1 輸出 終端機 ... Python dlearn + - 1 0 1 0 Connected to Discord 空格: 4 UTF-8 LF {} Python 2 Spell Prettier

```
(dlearn) alex@alexVM:~/Documents/_code/3002asg2py$ bash bash.sh
Run time (s): 7.1514880657196045
Run time (s): 7.197906255722046
Run time (s): 7.187333583831787
Run time (s): 7.009769439697266
Run time (s): 6.996024131774902
Run time (s): 6.984889984130859
Run time (s): 7.04547119140625
Run time (s): 7.239007234573364
Run time (s): 7.246218204498291
Run time (s): 7.1731297969818115
(dlearn) alex@alexVM:~/Documents/_code/3002asg2py$
```

Running time in second = (6.985, 7.246)