CITY UNIVERSITY OF HONG KONG

Module code & title : MA1201 Calculus and Basic Linear Algebra II
Session : Semester A, 2019–2020
Time allowed : Three hours

This paper has five pages (including this page).

Instruction to candidates:

1. This paper consists 6 questions.

2. Show all working.

3. Attempt ALL questions.

4. Start each question on a new page.

This is a **closed-book** examination.

Materials, aid & instruments which students are permitted to use during the examination:

Non-programmable Calculator

Materials/aids other than those stated above are not permitted. Candidates will be subject to disciplinary action if any unauthorized aerials or aids are found to them.

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1. (35 points) Evaluate the following integrals.

(a)(5 points) $\int_1^2 (2x+3)^{1/3} dx$

(b)(5 points) $\int \sin(2x)\cos(5x)dx$

(c)(7 points) $\int x^2 \tan^{-1} x dx$

(d)(7 points) $\int \frac{1}{(x^2+4)^{3/2}} dx$

(e)(11 points) $\int \frac{9x-7}{(x+2)(x^2-4x+13)} dx$

2. (a)(8 points) Let $f(x) = \begin{cases} 2e^x & x \geq 0 \\ x+2 & x < 0. \end{cases}$ Find the area of the region bounded by $x = -3$, $x = 1$, x-axis and the graph of $y = f(x)$.

(b)(7 points) Compute the arc length of the curve: $x = t - \sin t, y = 1 - \cos t$, $0 \leq t \leq \pi$.

3. (15 points, 5 points each)Let $A(3,-2,1)$, $B(1,-3,2)$ and $C(2,-1,-3)$ be three points on a plane Π.

(a) Find the angle $\angle BAC$

(b) Determine a unit vector perpendicular to the plane Π.

(c) Evaluate the shortest distance from a point $D(-4,-1,2)$ to the plane Π.

4. (a)(5 points) Simplify $\left(\frac{1-i}{1+i}\right)^{2019}$ into the polar form with principle arguments

(b)(10 points) Solve $z^4 + 1 = -\sqrt{3}i$ and express the answer in Euler form with principle arguments.

5.(5 points) $A = \begin{pmatrix} 3 & 1 & -2 \\ -3 & 3 & 3 \\ 0 & 2 & 2 \end{pmatrix}$. Compute the determinant of A and then $|A^T A^{-3}|$

6.(15 points) Given a system of linear equations as follows.

$$\begin{aligned} x - 2y + 3z - 4w &= 1 \\ -2x + 3y - 4z + 10w &= 2 \\ x - y + 2z - 3w &= 3 \end{aligned}$$

(a) (11 points) Solve the above linear system by Gaussian elimination and express the solution in vector norm.

(b) (4 points) Write down the corresponding homogeneous system and determine a non-trival solution without resolving it.

Useful Elementary Integrals

Constant and powers

1. $\int k\,dx = kx + C.$

2. $\int x^n\,dx = \begin{cases} \dfrac{x^{n+1}}{n+1} + C, & n \neq -1 \\ \ln|x| + C, & n = -1 \end{cases}$

Exponentials

3. $\int \mathrm{e}^x\,dx = \mathrm{e}^x + C.$

4. $\int a^x\,dx = \dfrac{a^x}{\ln a} + C,\ a \neq 1,\ a > 0.$

Trigonometric functions

5. $\int \sin x\,dx = -\cos x + C.$

6. $\int \cos x\,dx = \sin x + C.$

7. $\int \sec^2 x\,dx = \tan x + C.$

8. $\int \csc^2 x\,dx = -\cot x + C.$

9. $\int \sec x \tan x\,dx = \sec x + C.$

10. $\int \csc x \cot x\,dx = -\csc x + C.$

11. $\int \tan x\,dx = \ln|\sec x| + C.$

12. $\int \cot x\,dx = \ln|\sin x| + C.$

13. $\int \sec x\,dx = \ln|\sec x + \tan x| + C.$

14. $\int \csc x\,dx = \ln|\csc x - \cot x| + C.$

Algebraic functions

15. $\int \dfrac{1}{1+x^2}\,dx = \tan^{-1} x + C.$

16. $\int \dfrac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1} x + C.$

Hyperbolic functions

17. $\int \sinh x\,dx = \cosh x + C.$

18. $\int \cosh x\,dx = \sinh x + C.$

Useful Trigonometric Identities

Pythagorean identities

1. $\sin^2\theta + \cos^2\theta = 1.$

2. $1 + \tan^2\theta = \sec^2\theta.$

3. $1 + \cot^2\theta = \csc^2\theta.$

Double-angle formulas

4. $\sin 2\theta = 2\sin\theta\cos\theta.$

5. $\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta.$

Half-angle formulas

6. $\sin^2\theta = \dfrac{1}{2}(1 - \cos 2\theta).$

7. $\cos^2\theta = \dfrac{1}{2}(1 + \cos 2\theta).$

Compound-angle formulas

8. $\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B.$

9. $\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B.$

10. $\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A\tan B}.$

Sum-to-product formulas

11. $\sin A + \sin B = 2\sin\dfrac{A+B}{2}\cos\dfrac{A-B}{2}.$

12. $\sin A - \sin B = 2\cos\dfrac{A+B}{2}\sin\dfrac{A-B}{2}.$

13. $\cos A + \cos B = 2\cos\dfrac{A+B}{2}\cos\dfrac{A-B}{2}.$

14. $\cos A - \cos B = -2\sin\dfrac{A+B}{2}\sin\dfrac{A-B}{2}.$

Product-to-sum formulas

15. $\sin A\cos B = \dfrac{1}{2}\left[\sin(A+B) + \sin(A-B)\right].$

16. $\cos A\sin B = \dfrac{1}{2}\left[\sin(A+B) - \sin(A-B)\right].$

17. $\cos A\cos B = \dfrac{1}{2}\left[\cos(A+B) + \cos(A-B)\right].$

18. $\sin A\sin B = -\dfrac{1}{2}\left[\cos(A+B) - \cos(A-B)\right].$

Euler's formulas

19. $\mathrm{e}^{\pm\mathrm{i}\theta} = \cos\theta \pm \mathrm{i}\sin\theta.$

20. $\mathrm{e}^{\mathrm{i}\theta} + \mathrm{e}^{-\mathrm{i}\theta} = 2\cos\theta,\ \cos\theta = \dfrac{1}{2}(\mathrm{e}^{\mathrm{i}\theta} + \mathrm{e}^{-\mathrm{i}\theta}).$

21. $\mathrm{e}^{\mathrm{i}\theta} - \mathrm{e}^{-\mathrm{i}\theta} = 2\mathrm{i}\sin\theta,\ \sin\theta = \dfrac{1}{2\mathrm{i}}(\mathrm{e}^{\mathrm{i}\theta} - \mathrm{e}^{-\mathrm{i}\theta}).$

Remark. Formulas of the form $A \pm B = C \pm D$ contain two separate formulas

$$A + B = C + D, \qquad \text{and} \qquad A - B = C - D.$$

Likewise, formulas of the form $A \pm B = C \mp D$ contain two separate formulas

$$A + B = C - D, \qquad \text{and} \qquad A - B = C + D.$$

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