MA1200 CALCULUS AND BASIC LINEAR ALGEBRA

LECTURE: CG1

REVIEW ON CHAPTER 6 TO 8

Chapter 6: Limit, continuity and differentiability of functions

- Limit of a function: Left hand limit, Right hand limit

  Indeterminate form

  $$\lim_{x\to 0}\frac{\sin x}{x}=1$$

  Limit at infinity

  Sandwich Theorem

- Continuity of a function

  - $f(x)$ is **continuous** at $x=c$ if and only if $\boxed{\lim_{x\to c} f(x)=f(c)}$.

- Differentiability of a function

  - $f(x)$ is **differentiable** at $x=c$ if and only if

    $\boxed{\lim_{h\to 0}\frac{f(c+h)-f(c)}{h}}$ or $\boxed{\lim_{x\to c}\frac{f(x)-f(c)}{x-c}}$ exists.

- Differentiability of $f(x)$ at $x = c$ $\Rightarrow$ Continuity of $f(x)$ at $x = c$

$$\not\Leftarrow$$

- First Principle: $\boxed{f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$

Chapter 7: Techniques of Differentiation

- Use of table of derivatives
- Chain rule, product rule, quotient rule
- Implicit differentiation
- Inverse function theorem
- Logarithmic differentiation

- Differentiation of parametric equations
- Higher derivatives
- Leibnitz' rule:

$$(fg)^{(n)}(x) = \sum_{k=0}^{n} \binom{n}{k} f^{(k)}(x)\ g^{(n-k)}(x)$$

Chapter 8: Applications of Differentiation

- Equations of tangent and normal to the curve
- Rate of change problems
- Local extrema of functions
- Optimization problems
- L'Hôpital's rule
- Taylor series / Maclaurin series

  - Taylor series of $f(x)$ at $x = a$: $$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x-a)^n$$

  - Maclaurin series of $f(x)$: $$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$$

    (An expansion of $f(x)$ in ascending powers of x.)