# EE2331 Data Structures and Algorithms

Hashing

# Outline

- Hash Functions
  - Perfect Hash
  - Minimal Hash
- Collisions Resolution
  - Chaining buckets
  - Linear probing
  - Quadratic probing
  - Double hashing
- Design of Hash Function

# Indexing

- What is the index in a book?
- To help you to search the pages (that containing the keyword) quicker
- How about if the book does not have any index?
- Probably you have to search the entire book page by page, line by line and word by word (sequential search!)

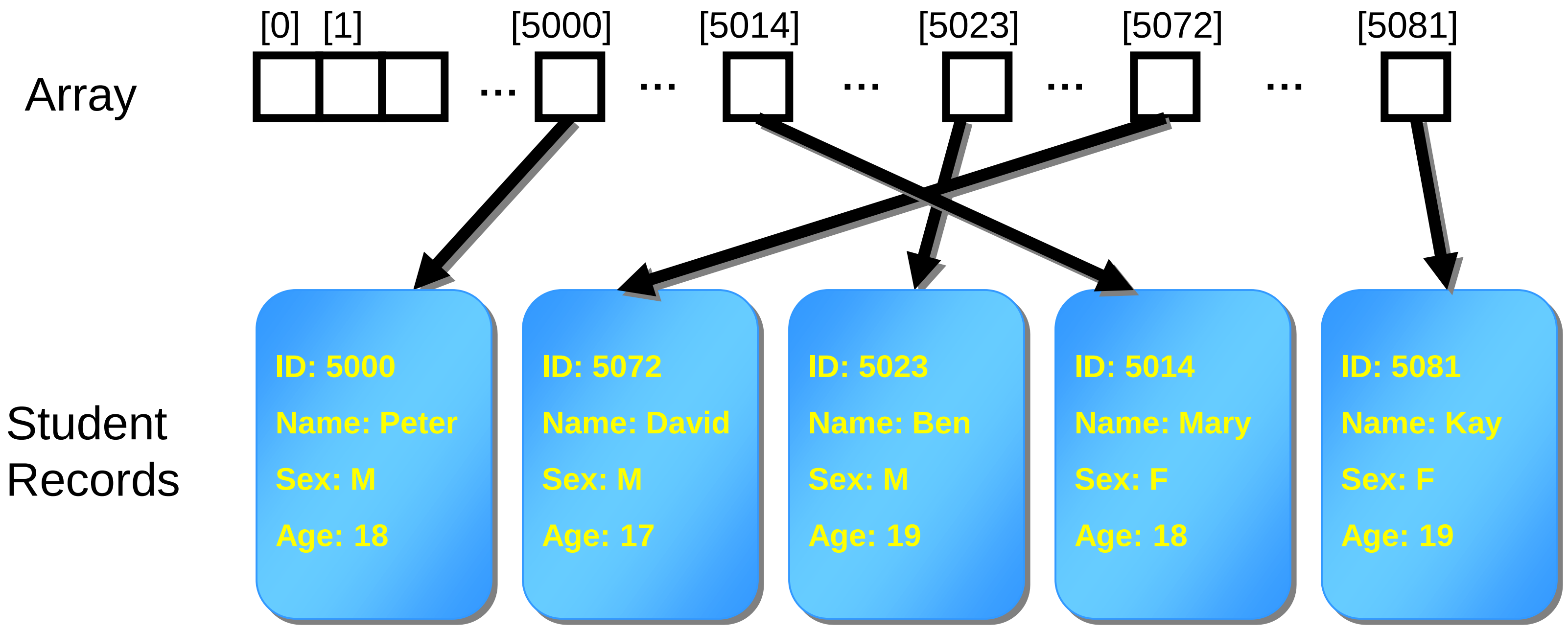
# A Practical Problem

- Given a set of data/records, how can you locate a record by the Student ID?
  - How do you sort?
    - Radix sort O(*kn*)
  - How do you search?
    - Binary search O(*log n*)

Student Records

# Indexing Data Record

- Using an array to hold pointers to the records
  - Use Student ID to index the records
  - What is the time complexity now?
  - But waste too much space…



# Hashing

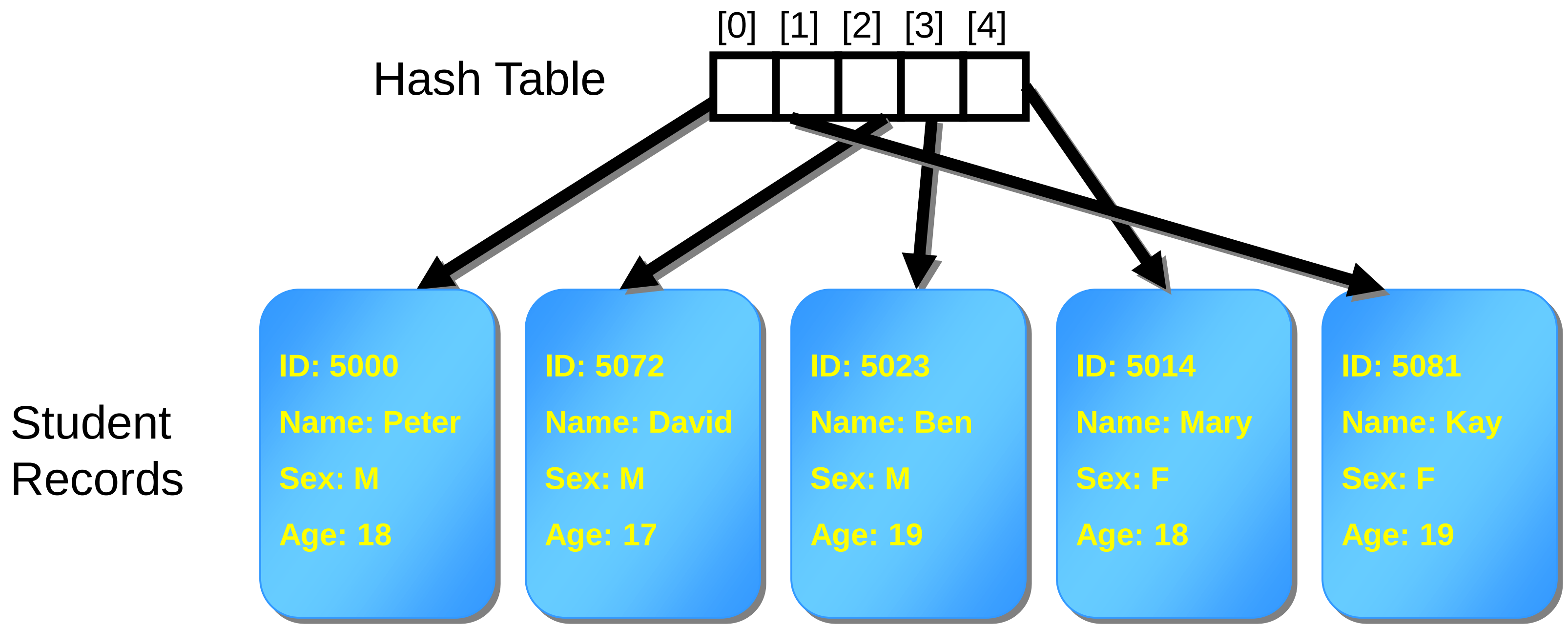
- The term “hash” means to chop and mix!
- The objectives
  - Build an index for a set of elements/records
  - To allow fast **search** (also **insert**, **delete**) operations
  - How fast? Constant time (independent of the element size!)
  - Common operations:
    - search, insert, delete and hash

# Hash Function

- A hash function is a well-defined procedure or mathematical function which converts a large, possibly variable-sized amount of data into a small datum
- The values returned by a hash function are called hash values, hash codes, hash sums, or simply hashes
- The hash value is usually a single integer that may serve as an index to an array

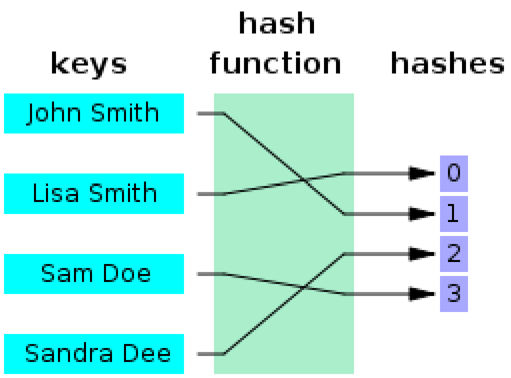
# A Simple Hash Function

- To enhance the memory utilization of the previous example, we can apply the following hash function to the key (Student ID) of the records:
  - $h(k) = k \% 5$

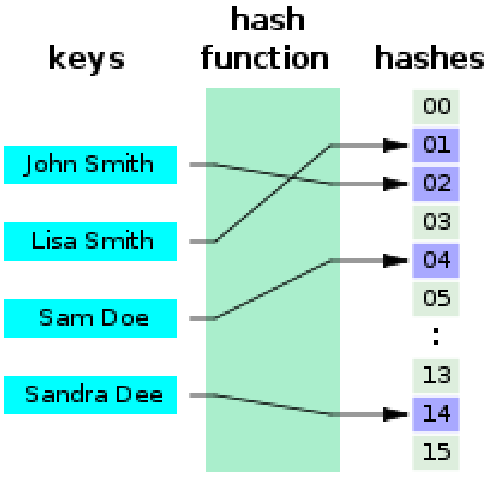


# The Hashing Approach

- By using the new hash values to index the records, we can reduce the array size to 5. A hash function maps each valid input to a different hash value is said to be **perfect**
  - With such a perfect hash function one can directly locate the desired entry in a hash table, without any additional searching
- A hash function for *n* keys is said to be **minimal** if it outputs *n* consecutive hash values




Minimal Perfect Hashing




Perfect Hashing

# Hash Collision

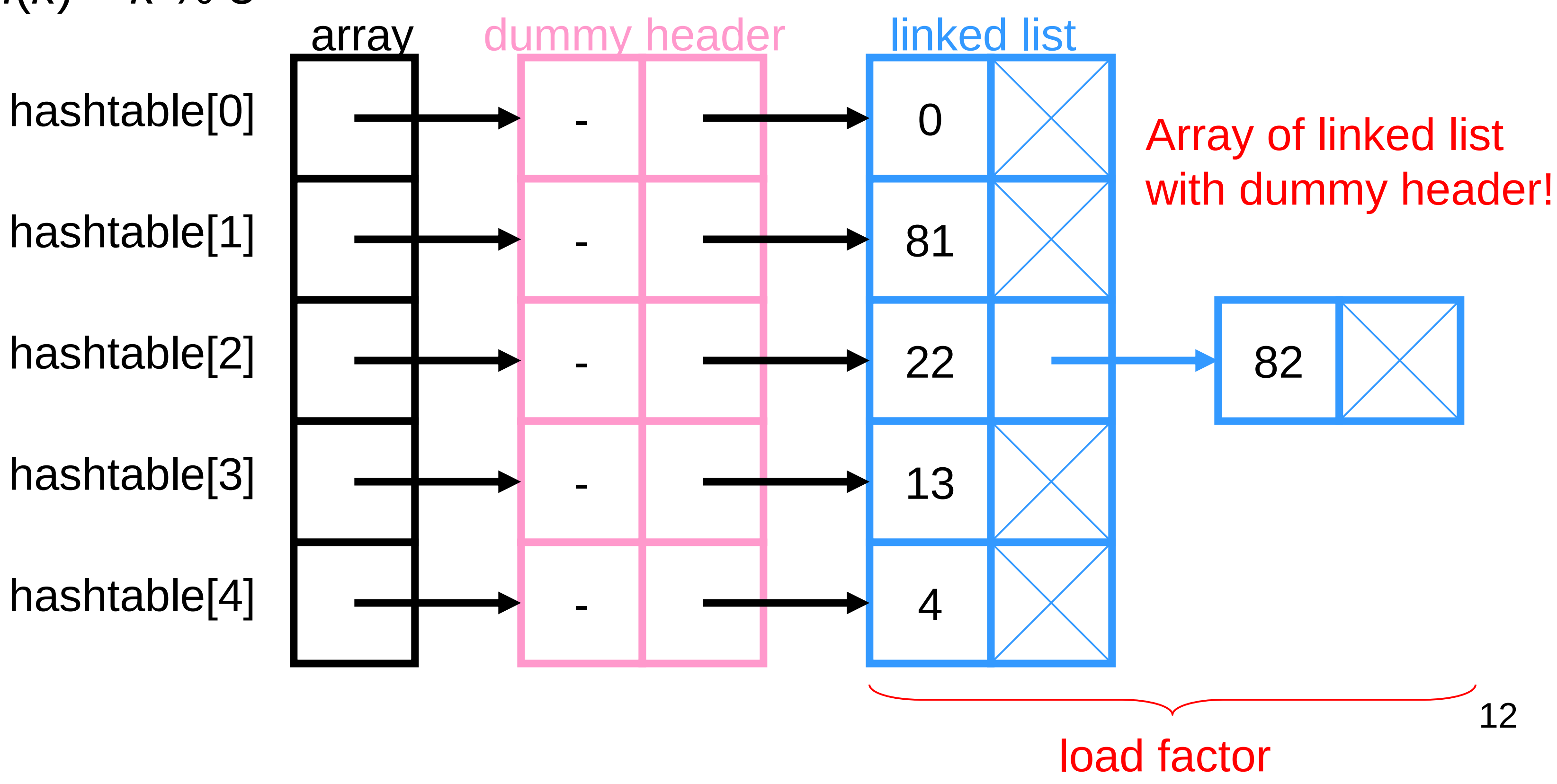
- A collision is a situation that occurs when two distinct pieces of data have the same hash value
- Collisions are unavoidable whenever members of a very large set (such as all possible person names, or all possible computer files) are mapped to a relatively short bit string
- Two types of collision resolution:
  - Closing addressing
    - Chaining buckets
  - Opening addressing
    - Linear probing, Quadratic probing, Double hashing
- Any collision in a hash table increases the average cost of lookup operations

# Closing Addressing

Place the key in the same slot
even when collisions has occurred

# Chaining Buckets

- Example: store 0, 4, 13, 22, 81 and 82 into the hash table using the chaining buckets
- For simplicity, let the key be the same as the element
- $h(k) = k \% 5$



# Opening Addressing

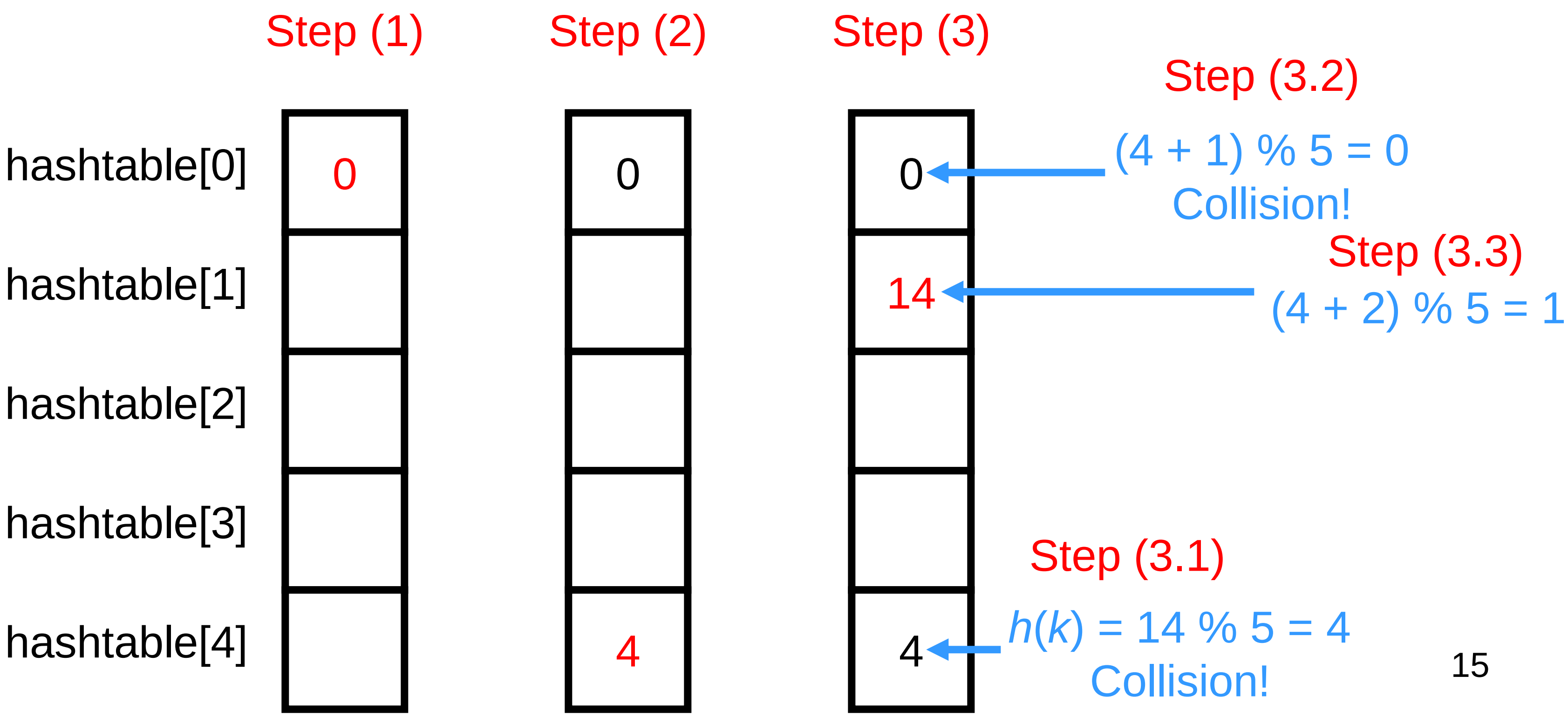
Place the key in other free slot
when collisions has occurred

# Linear Probing

- Place the key in the next free slot when collisions has occurred (i.e. sequentially search the hash table for a free location)
  - $h(k, i) = (h(k) + i) \% n$
  - where $i$ is the step size, $n$ is the table size and $h(k)$ is the original hash function
- If the slot of $h(k)$ mod $n$ has been used, try
  - $(h(k) + 1) \% n$
- If unlucky that the new slot has also been used, try
  - $(h(k) + 2) \% n$
- And so on until a free slot has been found

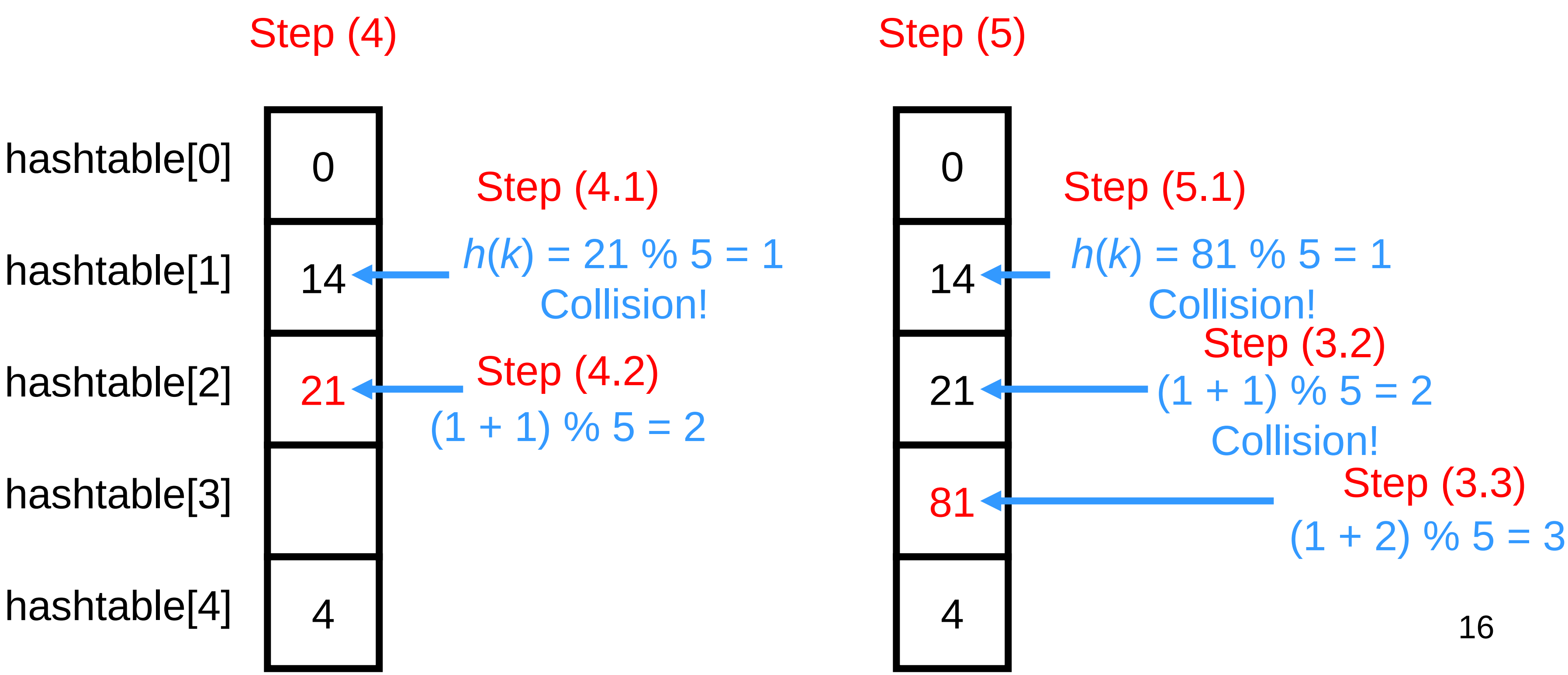
# Linear Probing

- Store 0, 4, 14, 21 and 81 into hash table using linear probing. Let $h(k) = k \% 5$



# Linear Probing

- Store 0, 4, 14, 21 and 81 into hash table using linear probing. Let $h(k) = k \% 5$

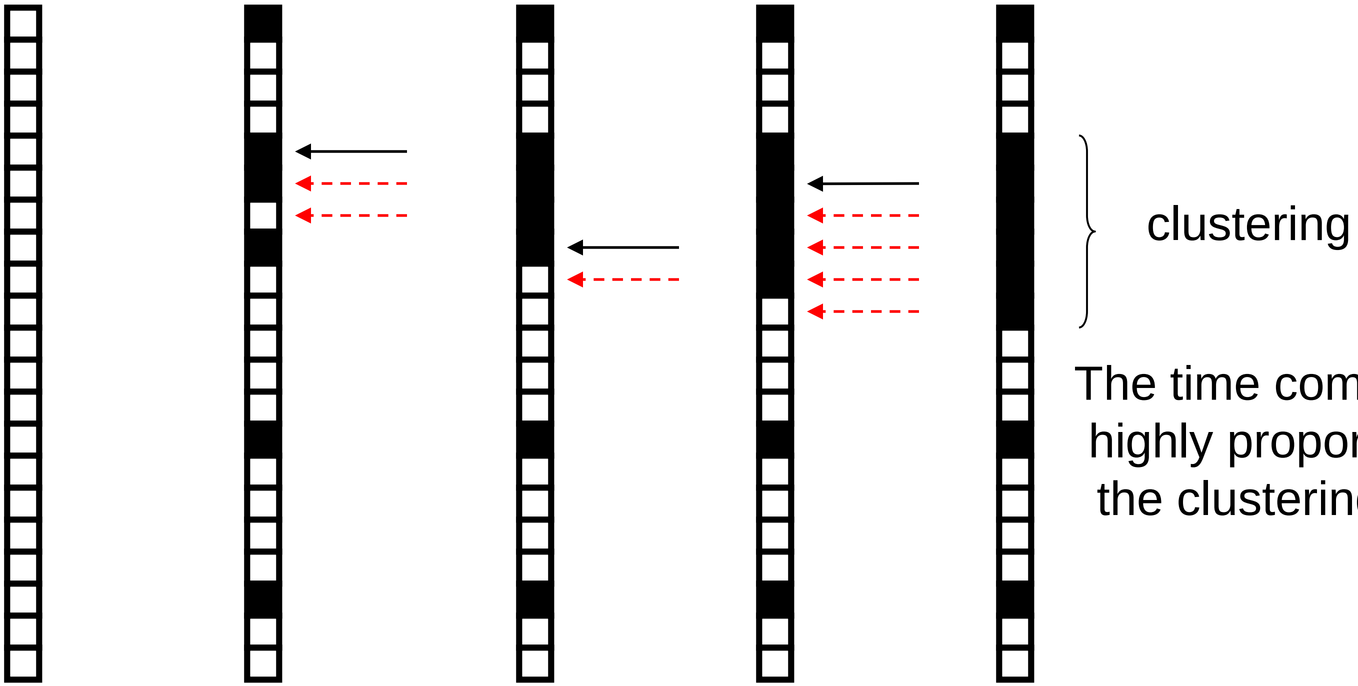


# Analysis

- The 1st element, key 0, located in its home position
- The 2nd element, key 4, also located in its home position
- The 3rd element, key 14, tried 3 positions before finding an empty slot
- The 4th element, key 21, tried 2 positions
- The last element, key 81, tried 3 positions
- The **total number of comparisons** required to search for all these 5 entities is
  - 1 + 1 + 3 + 2 + 3 = 10
- **Average number of comparisons for a successful search**
  - = 10 / 5 = 2

# Another Problem of Linear Probing

- Overflow addresses tends to group in a region of the array
- Called **clustering**



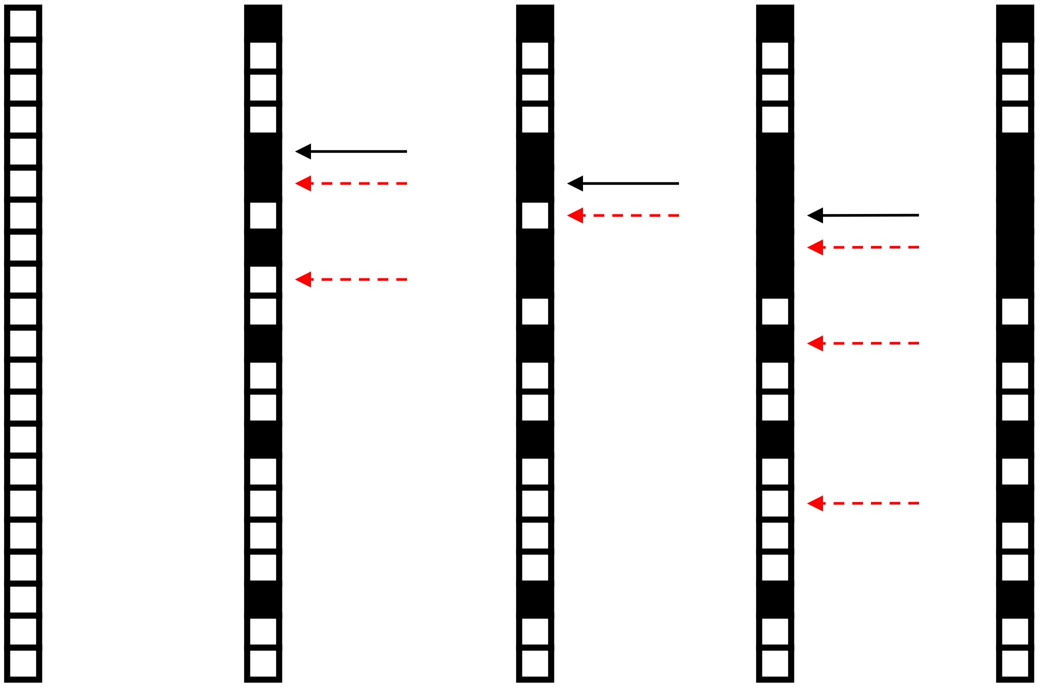

The time complexity is highly proportional to the clustering length

# Solution of Clustering

- Instead of using linear probing, try **Quadratic Probing**
  - $h(k, i) = (h(k) + i^2) \% n$
  - where $i$ is the step size, $n$ is the table size and $h(k)$ is the original hash function
- So the try sequence is
  - $h(k) \% n$
  - $(h(k) + 1^2) \% n$ Jump 1 slot
  - $(h(k) + 2^2) \% n$ Jump 3 slots more
  - $(h(k) + 3^2) \% n$ Jump 5 slots more again
  - And so on until a free slot is found
- To "jump" away from clustering

# Problem of Quadratic Probing

- Quadratic probing eliminate primary clustering
- But produce secondary clustering



secondary clustering

Less serious than primary clustering

# To Avoid Clustering

- **Double hashing**: design 2 independent hash functions h1() and h2()
  - $h(k, i) = (h1(k) + i * h2(k)) \% n$
  - where $i$ is the step size and $n$ is the table size
- So the try sequence is
  - $h1(k) \% n$
  - $(h1(k) + h2(k)) \% n$
  - $(h1(k) + 2 * h2(k)) \% n$
  - $(h1(k) + 3 * h2(k)) \% n$
  - And so on until a free slot has been found
- The jump interval is decided using a second, independent hash function. So values mapping to the same location have different jump sequences
- This minimizes repeated collisions and the effects of clustering
- The trade off: cost more time to compute new hash value

# Design of Hash Function

Division Method
Mid-Square
Folding Method
Radix Transform

# Design of Hash Function

- An ideal hash function should have the following properties
  - Low Cost
    - Easy and fast to compute
  - Variable Range
    - Able to transform words, symbols into numbers
  - Uniformity
    - Distributes the keys evenly
    - Minimize the chance of collisions

# 1) Division Method

- Easy to implement and fast to compute
  - **Division**: key % tablesize
  - Use **prime number** as the hash table size to reduce collisions
- How about the key is not an integer?
  - Transform into integer

```
int hash(char key[SIZE]) {
    int i = SIZE, k = 0;
    while (i-- > 0)
        k += key[i];                //sum the ASCII values
        //or k ^= key[i];           //XOR the ASCII values
    return k % tablesize;           //return the hash value
}
```

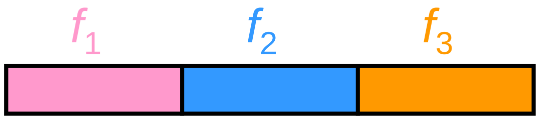
# 2) Mid-Square

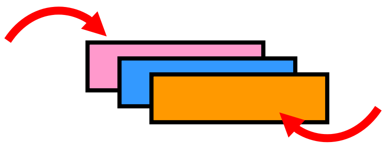
- Step 1. Transform to integer
- Step 2. Square the number
- Step 3. Select some digits from the middle
- E.g. Put these keys into a table of size 5

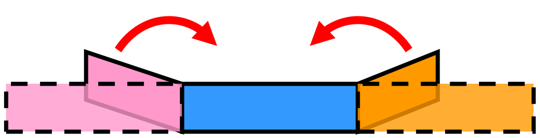
| key | key$^2$ | hash(key) |
|---|---|---|
| 281 | 78961 | 96 mod 5 = 1 |
| 99 | 9801 | 80 mod 5 = 0 |
| 123 | 15129 | 12 mod 5 = 2 |

# 3) Folding Method

- Step 1. Split the key into several parts
- Step 2. Sum the folded the key
- E.g. key = 245769908
  - Shift-folding
    - Define $f_1$ = 245, $f_2$ = 769, $f_3$ = 908
    - $h(k) = (f_1 + f_2 + f_3) \bmod size$
  - Boundary-folding
    - Define $f_1$ = 542, $f_2$ = 769, $f_3$ = 809
    - $h(k) = (f_1 + f_2 + f_3) \bmod size$





# 4) Radix Transformation

- Generate the hash value by transforming the key using new radix
- e.g. key = $358345_{(10)}$
  - Define $k_2 = 358345_{(9)}$
  - $h(k) = k_2 \bmod \text{size}$
  - $k_2 = 358345_{(9)} = 216068_{(10)}$
  - $h(k) = 216068_{(10)} \bmod \text{size}$

# Applications

- Hash functions are mostly used to speed up table lookup or data comparison tasks such as finding items in a database, detecting duplicated or similar records in a large file
- Determine if there are any duplicated numbers from the following sequence of numbers:
  - {52, 61, 18, 70, 39, 48, 28, 57, 61, 39, 43}
  - 61 and 39 repeated twice
- Can you suggest an algorithm to find the duplicated numbers?