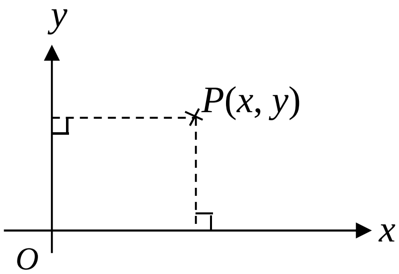
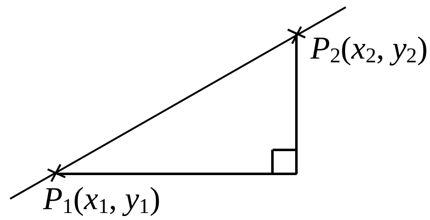
MA1200 Calculus and Basic Linear Algebra I
Chapter 1 Coordinate Geometry and Conic Sections

1 Review

In the *rectangular/Cartesian coordinates system*, we describe the location of points using coordinates.





The *distance d* between two points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ is given by $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

The midpoint M of a line segment between $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ is given by $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$.

The *slope m* of a line joining $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ is given by $m = \dfrac{y_2 - y_1}{x_2 - x_1}$.

Note: $m > 0$ for straight lines which slope upward to the right and vice versa.
m is undefined for vertical lines and $m = 0$ for horizontal lines.

Let L_1 and L_2 be two lines with slopes m_1 and m_2. Then

$$L_1 // L_2 \quad \text{if and only if} \quad m_1 = m_2\,;$$
$$L_1 \perp L_2 \quad \text{if and only if} \quad m_1 m_2 = -1\,.$$

Special forms of the equation of a straight line:

1) <u>General form of a straight line:</u>
$Ax + By + C = 0$, where A, B, C are constants such that A and B cannot both equal to zero.

2) <u>Two points form:</u>
The equation of the straight line passing through the points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is
$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}.$$

3) <u>Point-slope form:</u>
The equation of the line with slope m passing through a point $P(x_1, y_1)$ is
$$\frac{y - y_1}{x - x_1} = m\,.$$

4) <u>Slope-intercept form:</u>
The equation of straight line can also be written in the form $y = mx + c$, where m and c are the slope and the y-intercept of the straight line respectively.

Example 1

Find the equation of straight line which satisfies each of the following conditions:

(a) joining $P(3, 4)$ and $Q(1, -1)$;

(b) perpendicular to L_2: $3x - 2y + 5 = 0$ and cuts the x-axis at $(5, 0)$.

Solution:

(a) The equation of straight line L_1 is given by

$$\frac{y-4}{x-3} = \frac{4-(-1)}{3-1}$$

$$2(y-4) = 5(x-3)$$

$$5x - 2y - 7 = 0 .$$

(b) Equation of L_2 in slope-intercept form is $y = \frac{3}{2}x + \frac{5}{2}$.

Slope of L_2 is $m_2 = \frac{3}{2}$.

$\therefore$ Slope of the required line L_3 is $m_3 = -\frac{2}{3}$, since $m_2 m_3 = -1$.

The equation of straight line L_3 is given by

$$\frac{y-0}{x-5} = -\frac{2}{3}$$

$$3y = -2(x-5)$$

$$2x + 3y - 10 = 0 .$$

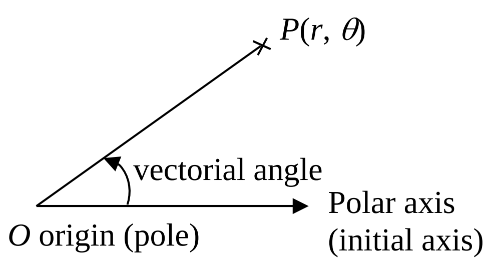
Exercise:

1) Find the coordinates of the foot of the perpendicular from $P(6, 1)$ to the straight line joining $Q(-1, 2)$ and $R(3, -4)$.

2) Find the shortest distance from the origin $O(0, 0)$ to the line joining $A(3, -4)$ to the point of intersection of the line $x - 3y + 6 = 0$ with the y-axis.

Review on Polar Coordinates

The *polar coordinates* of a point P is represented as (r, θ), where r is the distance of the point from the pole and θ is an angle formed by the polar axis and a ray from the pole through the point.



The relations between Polar and Rectangular Coordinates are:

$$x = r\cos\theta$$
$$y = r\sin\theta$$
$$x^2 + y^2 = r^2$$
$$\tan\theta = \frac{y}{x}$$

Question: Convert each of the following points from polar coordinates to rectangular coordinates.

(i) $(3, 45^\circ)$ (ii) $(7, 120^\circ)$ (iii) $(6, -60^\circ)$ (iv) $(2, 420^\circ)$

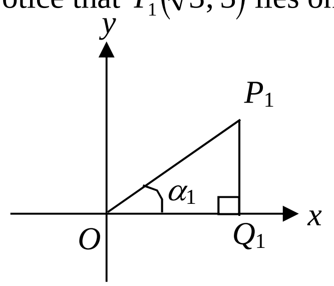
<u>Example 2</u>

Determine θ of the polar coordinates of the following points with the given rectangular coordinates. Express θ in the range $-180^\circ < \theta \le 180^\circ$.

(a) $P_1\left(\sqrt{3}, 3\right)$ (b) $P_2\left(-3, \sqrt{3}\right)$ (c) $P_3(-2, -2)$.

<u>Solution:</u>

(a) Notice that $P_1\left(\sqrt{3}, 3\right)$ lies on the first quadrant.

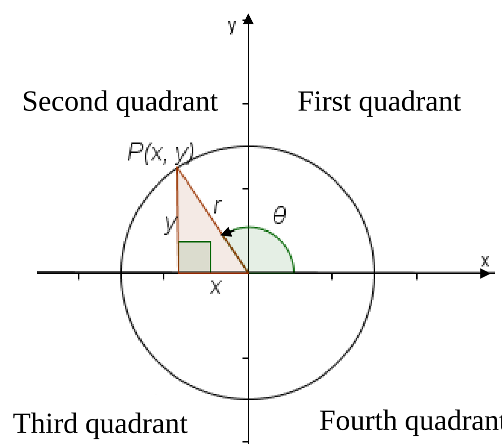


In ΔOP_1Q_1,

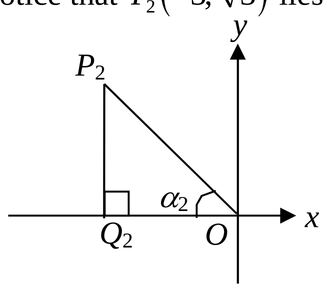
$$\tan\alpha_1 = \frac{P_1Q_1}{OQ_1} = \frac{3}{\sqrt{3}}$$

$\therefore \quad \alpha_1 = 60°$

$\therefore \quad \theta = \alpha_1 = 60°$



(b) Notice that $P_2\left(-3, \sqrt{3}\right)$ lies on the second quadrant.



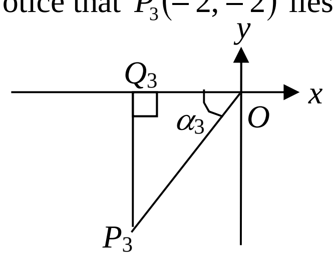
In ΔOP_2Q_2,

$$\tan\alpha = \frac{P_2Q_2}{OQ_2} = \frac{\sqrt{3}}{3}$$

$\therefore \quad \alpha_2 = 30°$

$\therefore \quad \theta = 180° - \alpha_2 = 180° - 30° = 150°$

(c) Notice that $P_3(-2, -2)$ lies on the third quadrant.



In ΔOP_3Q_3,

$$\tan\alpha = \frac{P_3Q_3}{OQ_3} = \frac{2}{2} = 1$$

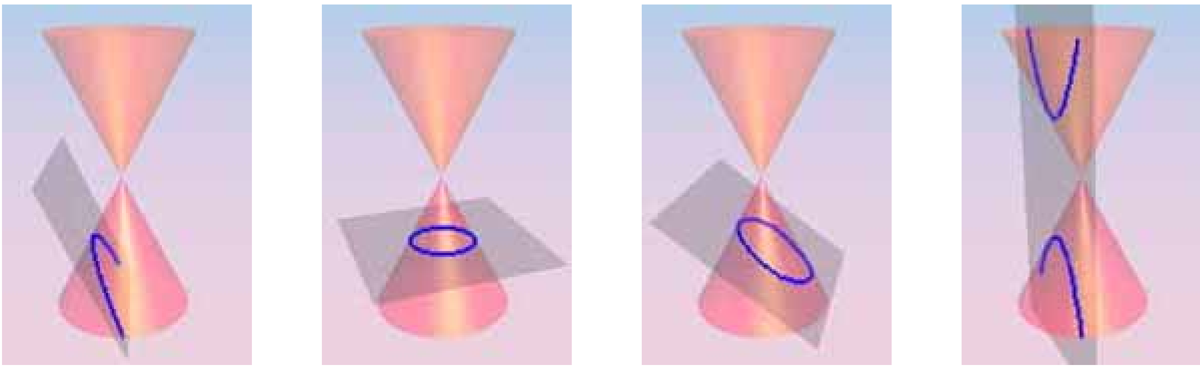
$\therefore \quad \alpha_3 = 45°$

$\therefore \quad \theta = -180° + \alpha_3 = -180° + 45° = -135°$

Question: Convert $P\left(5, -5\sqrt{3}\right)$ from rectangular to polar coordinates with $-180^\circ < \theta \le 180^\circ$.

2 Conic Sections

Mathematics is present in different aspects, for example, the movements of planets, bridge and tunnel construction, manufacture of lenses for telescopes and so on. The mathematics behind these applications involves conic sections. Conic sections are curves that result from intersecting a right circular cone with a plane. The following figure illustrates the four conic sections: the *parabola*, the *circle*, the *ellipse* and the *hyperbola*.



Parabola Circle Ellipse Hyperbola

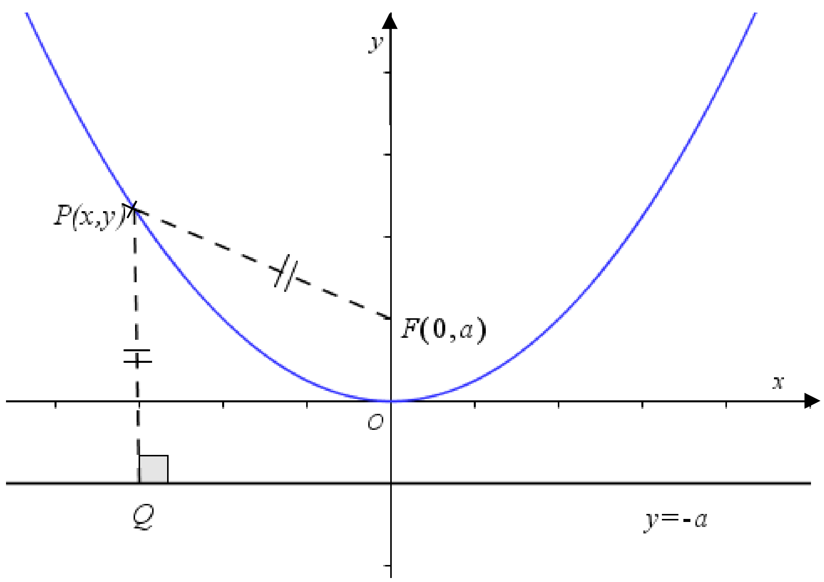
Question: What else could be obtained except the four figures shown above when a plane intersects a right circular cone?

A. Parabolae

Definition: A *parabola* is the set of all points in a plane that are equidistant from a fixed line, the *directrix*, and a fixed point, the *focus*.

The equations of a parabola with focus at the point $F(0, a)$ and the directrix $y = -a$, where $a > 0$, is

$$x^2 = 4ay .$$



Proof: The point $P(x,y)$ is on the parabola if and only if it is equidistant from the directrix $y=-a$ and the focus $F(0,\,a)$, i.e.

$$\begin{aligned}
\sqrt{(x-0)^2+(y-a)^2} &= \sqrt{(x-x)^2+(y+a)^2} \\
x^2+(y-a)^2 &= (y+a)^2 \\
x^2+y^2-2ay+a^2 &= y^2+2ay+a^2 \\
\therefore \qquad x^2 &= 4ay
\end{aligned}$$

$\square$

The parabola $x^2=4ay$ ($a>0$) has the following properties:

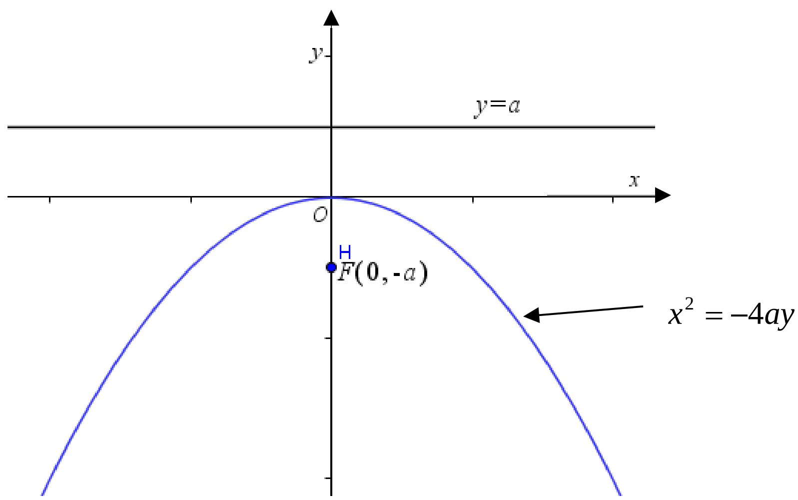
(i) It lies above the x-axis.

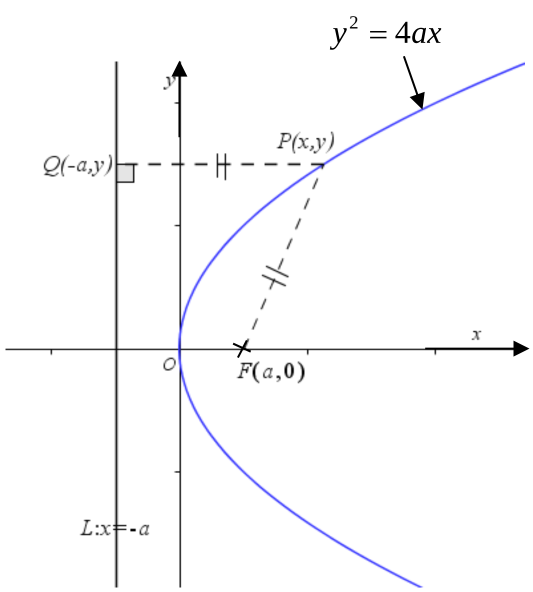
(ii) It is symmetrical about the y-axis, which is called the *axis of symmetry* of the parabola.

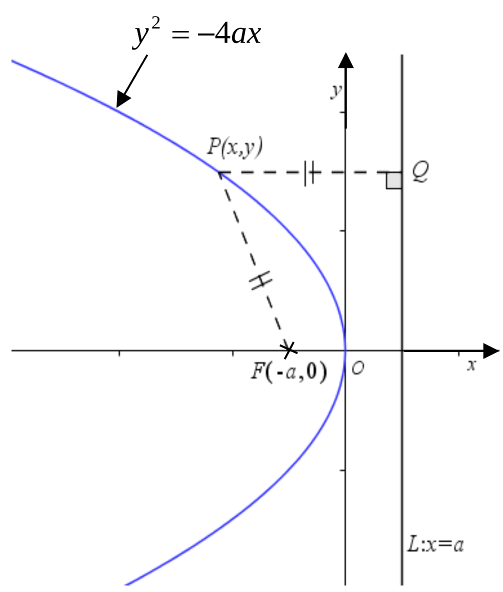
(iii) It cuts the y-axis at the origin O, which is called the vertex.

(iv) As the value of a increases, the parabola opens wider.

Depending on the location of focus and the orientation of the directrix, there are other forms of parabolae (with the vertex at the *origin* and $a>0$):







Example 3

The vertex and the axis of symmetry of a parabola are the origin and the *x*-axis respectively. If the parabola passes through the point (6, 3), find its equation.

Solution:

Since the parabola is symmetrical about the *x*-axis, we let its equation be $y^2 = 4ax$.

This parabola passes through the point (6, 3),

$\therefore \quad 3^2 = 4a(6)$,

get $a = \dfrac{3}{8}$. Hence the equation of the parabola is

$y^2 = 4\left(\dfrac{3}{8}\right)x$, i.e. $y^2 = \dfrac{3}{2}x$. □

Translations of Parabolae

The graph of a parabola can have its vertex at (h, k) rather than at the origin. Horizontal and vertical translations are accomplished by replacing x with $x-h$ and y with $y-k$ in the standard form of parabola's equation.

Equation	Vertex	Axis of Symmetry	Description
$(y-k)^2 = 4p(x-h)$	(h, k)	$y = k$	If $p > 0$, opens to the right; if $p < 0$, opens to the left.
$(x-h)^2 = 4p(y-k)$	(h, k)	$x = h$	If $p > 0$, opens upward; if $p < 0$, opens downward.

More on $y = ax^2 + bx + c$

(i) The graph of the quadratic equation $y = ax^2 + bx + c$ is a parabola which opens upward when $a > 0$ and opens downward when $a < 0$.

(ii) The parabola
- intersects the *x*-axis at 2 distinct points iff the discriminant, $b^2 - 4ac > 0$;
- touches the *x*-axis at 1 point iff $b^2 - 4ac = 0$;
- does not cut the *x*-axis iff $b^2 - 4ac < 0$.

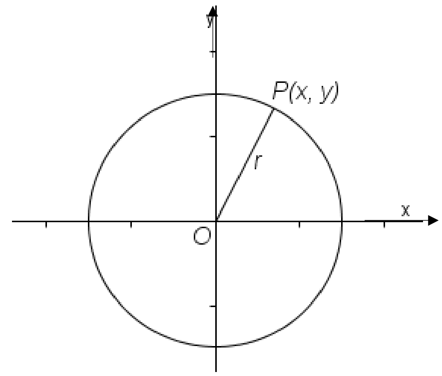
B. Circle

Definition: A *circle* is the set of all points in a plane that the distance of the point from a fixed point is a constant. The fixed point is called the *centre* and a fixed distance is called the *radius* of the circle.

The equations of a circle with centre at the origin $O(0, 0)$ and radius r is

$$x^2 + y^2 = r^2 .$$

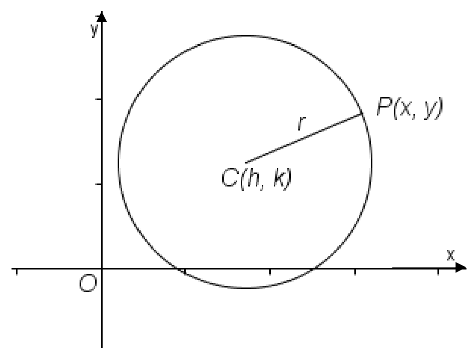
This is known as the *standard form of the equation of circle centered at the origin*.
Proof: Leave as exercise.



The equations of a circle with centre at the point $C(h, k)$ and radius r is

$$(x-h)^2 + (y-k)^2 = r^2 .$$

. Proof: Leave as exercise.



Example 4
Find the equations of the circles with the following centres and radii.
(a) Centre at $(0, 0)$, radius: 7 units (b) Centre at $(-3, 1)$, radius: $\sqrt{2}$ units

Solution:
(a) The equation of the circle is: $(x-0)^2 + (y-0)^2 = 7^2$
i.e. $x^2 + y^2 = 49$
(b) The equation of the circle is: $[x-(-3)]^2 + (y-1)^2 = (\sqrt{2})^2$
i.e. $(x+3)^2 + (y-1)^2 = 2$

Example 5
Find the centre and radius of the circle represented by each of the following equations.
(a) $(x-2)^2 + (y+5)^2 = 10$ (b) $x^2 + y^2 + 8x - 10y - 8 = 0$

Solution:
(a) Rewrite the equation in standard form:

$$(x-2)^2 + (y+5)^2 = (\sqrt{10})^2$$

$\therefore$ Centre is at $(2, -5)$, radius is $\sqrt{10}$ units.

(b) Rewrite the equation in standard form:

$$x^2+y^2+8x-10y-8=0$$
$$x^2+8x+y^2-10y-8=0$$
$$x^2+8x+16-16+y^2-10y+25-25-8=0$$
$$(x+4)^2-16+(y-5)^2-25-8=0$$
$$(x+4)^2+(y-5)^2=49$$

$\therefore$ Centre is at $(-4, 5)$, radius is 7 units.

Remark: The technique used in (b) is called *completing the square*. □

Example 6

Find the equation of the circle centred at the point $(-4, 3)$ and passing through the origin.

Solution:

The radius of the required circle is

$$\sqrt{(-4-0)^2+(3-0)^2}=\sqrt{16+9}=5$$

Thus, the equation of the circle is

$$(x+4)^2+(y-3)^2=25.$$ □

Example 7

Find the equation of the circle through

$P(1, 2)$, $Q(3, 4)$, $R(7, 6)$.

Solution: Method I

Since the side PQ has slope $m_1 = \dfrac{4-2}{3-1} = 1$ and its mid-point is $(2, 3)$.

The side PR has slope $m_2 = \dfrac{6-2}{7-1} = \dfrac{2}{3}$ and its mid-point is $(4, 4)$.

$\therefore$ The perpendicular bisectors of PQ and PR are

$\dfrac{y-3}{x-2} = -1$, or $y-3 = -x+2$

$$y=-x+5 \quad \cdots \quad (1)$$

and $\dfrac{y-4}{x-4}=-\dfrac{3}{2}$, or $y-4=-\dfrac{3}{2}x+6$

$$y=-\frac{3}{2}x+10 \quad \cdots \quad (2)$$

Solving the equations (1) and (2), we obtain

$$-x+5=-\frac{3}{2}x+10$$

$$\frac{1}{2}x=5\ ,\ x=10\ ,\ y=-5.$$

$\therefore$ These two perpendicular bisectors meet at $(10,-5)$, which is the centre of the circle.

The radius of the circle is given by

$r=PC=\sqrt{(10-1)^2+(-5-2)^2}=\sqrt{81+49}=\sqrt{130}$ units.

The equation of the circle is

$$(x-10)^2+(y-(-5))^2=(\sqrt{130})^2\ ,$$

Which reduces to

$$x^2+y^2-20x+10y-5=0\ .$$

<u>Method II</u>

Let $x^2+y^2+Dx+Ey+F=0$ be the equation of the circle through the points $P(1,\ 2)$, $Q(3,\ 4)$, $R(7,\ 6)$.

Then

$$\begin{cases} 1^2+2^2+D+2E+F=0 \\ 3^2+4^2+3D+4E+F=0 \\ 7^2+6^2+7D+6E+F=0 \end{cases} \Rightarrow \begin{cases} D+2E+F=-5 & \cdots\ (1) \\ 3D+4E+F=-25 & \cdots\ (2) \\ 7D+6E+F=-85 & \cdots\ (3) \end{cases}$$

$$\begin{cases} Eq(2)-eq(1): 2D+2E=-20 \\ Eq(3)-eq(1): 6D+4E=-80 \end{cases} \Rightarrow \begin{cases} D+E=-10 & \cdots\ (4) \\ \frac{3}{2}D+E=-20 & \cdots\ (5) \end{cases}$$

$Eq(5)-eq(4)$: $\frac{1}{2}D=-10$, $\therefore D=-20$

From $eqn(4)$, we have $E=10$

From $eqn(1)$, we have $F=-5+20-20=-5$.

$\therefore$ The equation of the circle is

$$x^2+y^2-20x+10y-5=0\ .$$

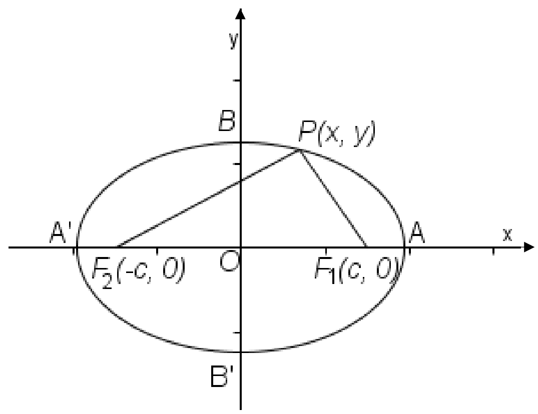
C. Ellipses

Definition: An *ellipse* is the set of all points P in a plane that the sum of whose distances from two fixed points, F_1 and F_2, is a constant. The two fixed points are called the *foci* (plural of focus). The midpoint of the segment connecting the foci is the *centre* of the ellipse.

The equations of an ellipse with foci at the points $F_1(-c, 0)$ and $F_2(c, 0)$, where $c > 0$, is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1,$$

where $a > b > 0$, $a^2 - b^2 = c^2$ and $2a$ is the sum of the distances from any point on the ellipse to the two foci. The equation above is the *standard form of the equation of an ellipse centered at the origin*.



Proof:

The point $P(x, y)$ is on the ellipse if and only if the sum of distances from $F_1(-c, 0)$ and $F_2(c, 0)$ is $2a$,

i.e. $$\sqrt{(x+c)^2 + y^2} + \sqrt{(x-c)^2 + y^2} = 2a$$

Simplifying, get

$$(a^2 - c^2)x^2 + a^2y^2 = a^2(a^2 - c^2)$$

Observed that $F_1F_2 = 2c < PF_1 + PF_2$, so that $c < a$ and $a^2 - c^2 > 0$.

For convenience, let $b^2 = a^2 - c^2$. We have

$$b^2x^2 + a^2y^2 = a^2b^2$$

$$\therefore \quad \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

The ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ($a > b > 0$) has the following properties:

(i) It is symmetrical about the x-axis, the y-axis and the origin.

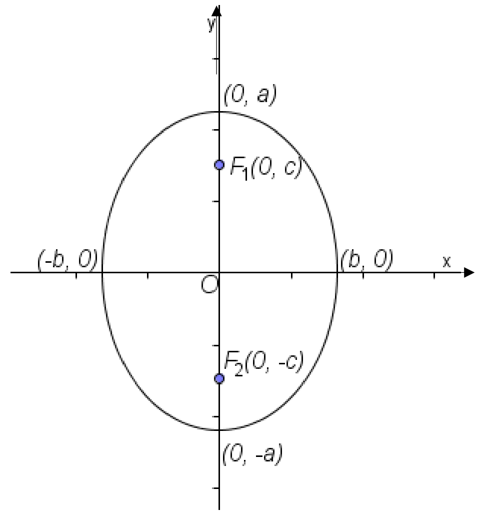
(ii) Its foci are $F_1(-c, 0)$ and $F_2(c, 0)$, and can be found by the equation $c^2 = a^2 - b^2$.

(iii) It cuts the x-axis at $A'(-a, 0)$ and $A(a, 0)$, and the y-axis at $B'(0, -b)$ and $B(0, b)$.
These four points are called the vertices of the ellipse.
The line segments $A'A$ and $B'B$ intersect at the origin O, which is the *centre* of the ellipse.
The length of the line segment $A'A$ is greater than that of the line segment $B'B$.

Depending on the location of foci, there is another form of ellipse (with the centre also at the *origin*):

$$\frac{x^2}{b^2}+\frac{y^2}{a^2}=1 \ ,$$

where $a>b>0$, $a^2-b^2=c^2$.



Example 8

Find the coordinates of the vertices and the foci of each ellipse, and sketch its graph.

(a) $4x^2+9y^2=36$ (b) $8x^2+y^2=8$

Solution:

(a) Rewrite the equation of the ellipse as $\dfrac{x^2}{3^2}+\dfrac{y^2}{2^2}=1$,

We have $a=3$ and $b=2$.

Note that the equation represents an ellipse with the centre at the origin.

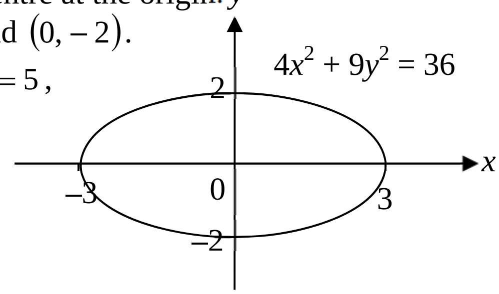
The vertices of the ellipse are $(3,0)$, $(-3,0)$, $(0,2)$ and $(0,-2)$.

Moreover, $c^2=a^2-b^2=3^2-2^2=5$,

i.e. $c=\sqrt{5}$ (c is positive.)

The foci of this ellipse are $F'\left(-\sqrt{5},0\right)$ and $F\left(\sqrt{5},0\right)$.

The sketch of the ellipse is shown on right.



(b) Rewrite the equation of the ellipse as $x^2+\dfrac{y^2}{\left(2\sqrt{2}\right)^2}=1$,

so that $a=2\sqrt{2}$ and $b=1$.

Note that the equation represents an ellipse with the centre at the origin.

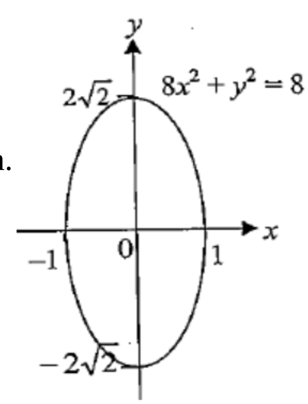
The vertices of the ellipse are $(1,0)$, $(-1,0)$, $\left(0,2\sqrt{2}\right)$ and $\left(0,-2\sqrt{2}\right)$.

Moreover, $c^2=a^2-b^2=\left(2\sqrt{2}\right)^2-1=7$,

i.e. $c=\sqrt{7}$ (c is positive.)

The foci of this ellipse are $\left(0,\sqrt{7}\right)$ and $\left(0,-\sqrt{7}\right)$.

The sketch of the ellipse is shown on right.



□

Example 9

Find the equation of the ellipse whose centre is the origin and the ellipse passes through the points $\left(2\sqrt{2}, 0\right)$ and $\left(-2, \sqrt{3}\right)$.

Solution:

Let the equation of the ellipse be $\dfrac{x^2}{p}+\dfrac{y^2}{q}=1$.

Since the ellipse passes through the point $\left(2\sqrt{2}, 0\right)$, we have $\dfrac{\left(2\sqrt{2}\right)^2}{p}+\dfrac{0^2}{q}=1$, get $p=\left(2\sqrt{2}\right)^2=8$.

As it also passes through the point $\left(-2, \sqrt{3}\right)$, we have $\dfrac{(-2)^2}{\left(2\sqrt{2}\right)^2}+\dfrac{\left(\sqrt{3}\right)^2}{q}=1$, i.e. $q=6$.

Hence the equation of the ellipse is $\dfrac{x^2}{8}+\dfrac{y^2}{6}=1$.

If the ellipse $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$ is translated h units to the right and k units upward so that its new centre is at $C(h, k)$, then the equations of the new ellipse becomes

$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1.$$

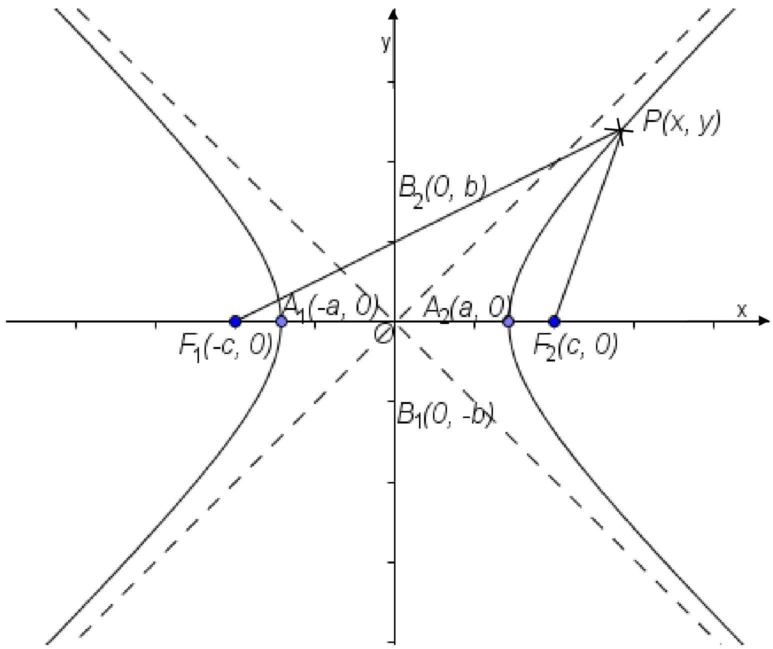
D. Hyperbolae

Definition: A *hyperbola* is the set of all points in a plane that the difference of whose distances from two fixed points, called the *foci*, is constant.

The equation of a hyperbola with foci at the points $F_1(-c, 0)$ and $F_2(c, 0)$, where $c > 0$, is

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1,$$

where $a, b > 0$, $a^2 + b^2 = c^2$ and $2a$ is the absolute difference of the distances from any point on the hyperbola to the two foci.
The equation above is called the standard form of the *equation of a hyperbola centered at the origin*.



Proof:
The point $P(x, y)$ is on the hyperbola if and only if the absolute difference of whose distances from $F_1(-c, 0)$ and $F_2(c, 0)$ is $2a$,

i.e. $$\left|\sqrt{(x+c)^2 + y^2} - \sqrt{(x-c)^2 + y^2}\right| = 2a$$

Simplifying, get

$$(c^2 - a^2)x^2 - a^2y^2 = a^2(c^2 - a^2)$$

Observed that $F_1F_2 = 2c > 2a$, so that $c^2 - a^2 > 0$.
For convenience, let $b^2 = c^2 - a^2$. We have

$$b^2x^2 - a^2y^2 = a^2b^2$$

$$\therefore \quad \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

The hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ ($a, b > 0$) has the following properties:

(i) It is symmetrical about the *x*-axis, the *y*-axis and the origin.

(ii) Its foci are $F_1(-c, 0)$ and $F_2(c, 0)$, and can be found by the equation $c^2 = a^2 + b^2$. The mid-point of these two foci is the origin *O*, which is the *centre* of the hyperbola.

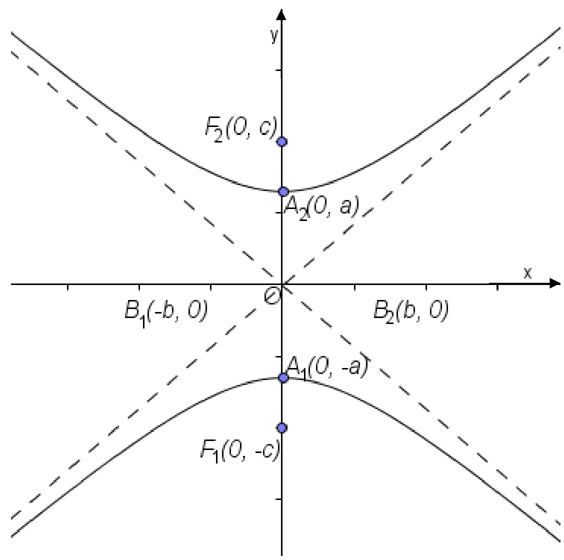
(iii) It cuts the *x*-axis at $A_1(-a, 0)$ and $A_2(a, 0)$, which are called the *vertices* of the hyperbola.

(iv) As *x* and *y* get larger, the two branches of the graph approach a pair of intersecting straight lines $y = \frac{b}{a}x$ and $y = -\frac{b}{a}x$. These are called the *asymptotes* of the hyperbola.

Depending on the location of foci, there is another form of hyperbola (with the centre also at the *origin*):

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1,$$

$a, b > 0$, $a^2 + b^2 = c^2$



Example 10

Arrange $9x^2 - 4y^2 = 144$ into the standard form of hyperbola.

Solution:

$$9x^2 - 4y^2 = 144$$

$$\frac{x^2}{16} - \frac{y^2}{36} = 1$$

$$\frac{x^2}{4^2} - \frac{y^2}{6^2} = 1$$

If the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is translated *h* units to the right and *k* units upward so that its new centre is at $C(h, k)$, then the equations of the new hyperbola becomes

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1.$$

When the equation of a conic section is expressed in the form $Ax^2 + Cy^2 + Dx + Ey + F = 0$, one can identify the conic section by completing the squares so that the equation is expressed into the corresponding standard form.

Example 11

Classify the type of conic section described by each of the following equations using completing the squares.

(a) $4x^2 - 16x + 25y^2 - 84 = 0$ (b) $4x^2 + 8x + 4y^2 - 24y + 15 = 0$

Solution:

(a) $4x^2 - 16x + 25y^2 - 84 = 0$

$4(x^2 - 4x) + 25y^2 = 84$

$4(x^2 - 4x + 4) + 25y^2 = 100$

$4(x-2)^2 + 25y^2 = 100$

$\dfrac{(x-2)^2}{5^2} + \dfrac{y^2}{2^2} = 1$ $\therefore$ The equation represents an ellipse.

(b) $4x^2 + 8x + 4y^2 - 24y + 15 = 0$

$4(x^2 + 2x) + 4(y^2 - 6y) = -15$

$4(x^2 + 2x + 1) + 4(y^2 - 6y + 9) = 25$

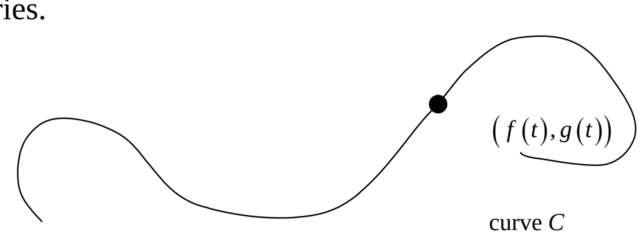
$4(x+1)^2 + 4(y-3)^2 = 25$

$(x+1)^2 + (y-3)^2 = \dfrac{25}{4}$ $\therefore$ The equation represents a circle.

3 Parametric Equations of Conic Sections

Consider the pair of equations $\begin{cases} x = f(t) \\ y = g(t) \end{cases}$.

To each real value of *t*, there corresponds to a point (*x*, *y*) in the Cartesian plane. The point (*x*, *y*) moves and traces a curve as *t* varies.



The pair of equations $\begin{cases} x = f(t) \\ y = g(t) \end{cases}$ is called parametric equations and the independent variable *t* is called a parameter.

The following table lists the equations of conic sections in rectangular coordinate form and in parametric form.

Type of Conics	Equation in Rectangular Coordinate Form	Equation in Parametric Form
Parabola Type 1	$x^2 = 4ay$ (or $y = \frac{x^2}{4a}$)	$\begin{cases} x = 2at \\ y = at^2 \end{cases}, -\infty < t < \infty$
Parabola Type 2	$y^2 = 4ax$	$\begin{cases} x = at^2 \\ y = 2at \end{cases}, -\infty < t < \infty$
Circle, centred at (0,0) and radius r	$x^2 + y^2 = r^2$	$\begin{cases} x = r\cos t \\ y = r\sin t \end{cases}, 0 \le t \le 2\pi$
Ellipse "Fat"	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1; (a > b)$	$\begin{cases} x = a\cos t \\ y = b\sin t \end{cases}; (a > b)$, $0 \le t \le 2\pi$
Ellipse "Thin"	$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1; (a > b)$	$\begin{cases} x = b\cos t \\ y = a\sin t \end{cases}; (a > b)$, $0 \le t \le 2\pi$
Hyperbola (East-West Openings)	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	$\begin{cases} x = a\sec t \\ y = b\tan t \end{cases}, -\frac{\pi}{2} < t < \frac{\pi}{2}$
Hyperbola (North-South Openings)	$\frac{x^2}{b^2} - \frac{y^2}{a^2} = 1$	$\begin{cases} x = b\sec t \\ y = a\tan t \end{cases}, -\frac{\pi}{2} < t < \frac{\pi}{2}$
Rectangular Hyperbola	$xy = c^2$; c is a constant	$\begin{cases} x = t \\ y = \frac{c^2}{t} \end{cases}, -\infty < t < \infty, t \ne 0$