3-1
~~4~~. Q1

polar form $\Rightarrow$ Phasor Form.

$A\cos(\omega t+\theta) = A\angle\theta$

a. $V(t) = 155\cos(337t - 25°)$ V

$A = 155$, $\omega = 337$ (60Hz) $\theta = -25°$ $V(j\omega) = 155\angle -25°$ [V]

(b) $V(t) = 5\sin(1000t - 40°)$ [V] convert $\sin\theta \to \cos(\theta - 90°)$.

$= 5\cos(1000t - 40° - 90°)$

$= 5\cos(1000t - 130°)$ [V]

$V(j\omega) = 5\angle -130°$ [V]

(c) Method 1: polar form $\Rightarrow$ rectangular form $\rightarrow$ Phasor Form.

$i(t) = 10\cos(10t + 63°) + 15\cos(10t - 42°)$

$i(j\omega) = 10\angle 63° + 15\angle -42°$

$= 4.54 + j8.91 + 11.147 - j10.04$

$= 15.687 - j1.13$

$= 15.7\angle -4.12°$ [A]

$\begin{cases} A\cos\theta = 15.687 \\ A\sin\theta = -1.13 \end{cases} \Rightarrow \theta = \tan^{-1}\left(\frac{-1.13}{15.687}\right)$

Method 2: using trigonometric

$i(t) = 10\cos(10t + 63°) + 15\cos(10t - 42°)$

$= 10\cos(10t)\cos(63) - 10\sin(10t)\sin(63°)$

$+ 15\cos(10t)\cos(-42°) - 15\sin(10t)\sin(-42°)$

$= [10\cos(63) + 15\cos(-42)]\cos(10t)$

$- [10\sin(63) + 15\sin(-42)]\sin(10t)$

$= 15.687\cos(10t) - 1.13\sin(10t)$

$= \sqrt{15.687^2 + (-1.13)^2} \cdot \cos\left[10t + \tan^{-1}\left(\frac{-1.13}{15.687}\right)\right]$

$i(j\omega) = 15.7\angle -4.12°$ [A]

(d). $i(t) = 460\cos(500\pi t - 25°) - 220\sin(500\pi t + 15)$ [A]

$= 460\cos(500\pi t - 25°) - 220\cos(500\pi t - 75°)$

$= \sqrt{360^2 + 18.1^2} \angle \arctan\left(\frac{18.1}{360}\right) \Rightarrow 360.4\angle 2.88°$ [A]

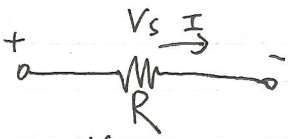
Q2

(a) $4+4j = \sqrt{4^2+4^2}\angle \arctan(\frac{4}{4}) = 5.66\angle 45°$

(b) $-3+j4 = \sqrt{(-3)^2+4^2}\angle \tan^{-1}(\frac{4}{-3}) = 5\angle -53.1°$

(c) $j+2-j4-3 = -1-j3 = \sqrt{(-1)^2+(-3)^2}\angle \tan^{-1}(\frac{-3}{-1}) = 3.16\angle 71.6°$

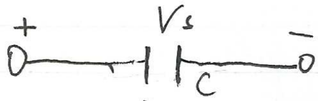
Q3



13.75

$I_s = \frac{V_s}{R} = \frac{110}{8}\cos(377t)$ [A]

Q4



$Z_C = \frac{1}{j\omega C} = \frac{1}{j2}\ \Omega$

$V_s = I\cdot Z_C = 4\cos(10^6t+25°)/j2$
$= 2\cos(10^6t+25-90) = 2\cos(10^6t-65°)$ [V]

Q5

$Z_L = j\omega L = j500\cdot 4\times10^{-3} = j2\,\Omega$

$I = \frac{V_s}{Z_L} = \frac{60}{2}\cos(500t-65°)/j = 30\cos(500t-65-90)$
$I = 30\cos(500t-155°)$ [A]

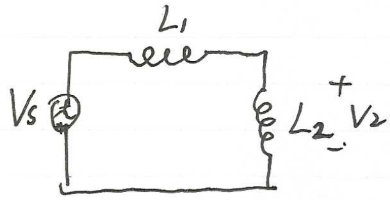
Q6 $Z = \frac{V(t)}{i(t)} = j205.88\,\Omega$. ⇒ Inductor.

$Z_L = j\omega L = j205.88$

$L = \frac{j205.88}{j\cdot 628.3} = 327$ mH

Q7

(a)

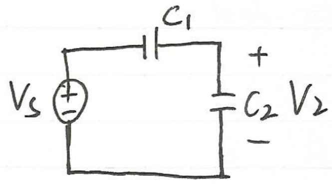


using voltage divider rule : $V_2 = V_s \cdot \frac{Z_2}{Z_1 + Z_2}$.

$Z_1 = j\omega L_1$; $Z_2 = j\omega L_2$

$$V_2 = V_s \cdot \frac{j\omega L_2}{j\omega L_1 + j\omega L_2} = V_s \cdot \frac{L_2}{L_1 + L_2} \quad //$$

(b)



$Z_1 = \frac{1}{j\omega C_1}$ $\quad Z_2 = \frac{1}{j\omega C_2}$

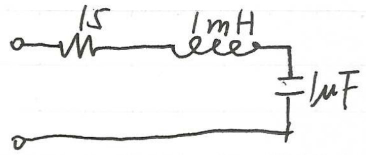
$Z_1 + Z_2 = \frac{C_1 + C_2}{j\omega(C_1 \cdot C_2)}$

$$V_2 = V_s \cdot \frac{Z_2}{Z_1 + Z_2} = \frac{V_s \cdot \frac{1}{j\omega C_2}}{\frac{C_1 + C_2}{j\omega(C_1 + C_2)}} = \frac{C_1}{C_1 + C_2} \cdot V_s$$

Q8 $\omega = 3000$. $\quad Z_L = j\omega L = j5.7\ k\Omega \quad Z_C = \frac{1}{j\omega C} = -j\,49\ k\Omega$

$$Z = R_1 + Z_L \parallel R_2 + Z_C = (3.3 + j5.7) \parallel (22 - j\,49)\,k\Omega$$
$$= (3.3 + j5.7) \cdot (22 - j49) \,/\, (25.3 - j44.3)$$
$$= \frac{72.6 - j161.7 + j125.4 + 279.3}{25.3 - j\,44.3}$$
$$= \frac{351.9 - j36.3}{25.3 - j44.3} = \frac{10511 - j14671}{25.3^2 + 44.3^2} = 4.038 - j\,5.637\ \Omega.$$

Q9. (a)



$$Z = R + j\omega L + \frac{1}{j\omega C} = 15 + j0.001\omega - j\frac{10^6}{\omega}.$$

$$= 15 + j\left(0.001\omega - \frac{10^6}{\omega}\right)$$

(b) phase $= 0$

$$j\left(0.001\omega - \frac{10^6}{\omega}\right) = 0$$

$$\omega^2 - 10^9 = 0$$

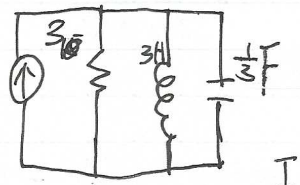
$$\omega = \sqrt{10^9} \approx 31623 \text{ rad } s^{-1}$$

Q10 Find Z_{eq} $\quad Z_L = j\omega L = j6\ \Omega \quad Z_C = \frac{1}{j\omega C} = -j1.5\ \Omega$

$$Z_{eq} = R \parallel Z_L \parallel Z_C$$

$$= \left(\frac{1}{3} + \frac{1}{j6} + \frac{1}{-j1.5}\right)^{-1}$$

$$= 0.923 - j1.38\ \Omega$$



$I = 10\cos(2t)$ A

$$V(j\omega) = I \cdot Z_{eq} = 9.23 - j13.8 = 16.64\angle -56.3^\circ$$

$$V(t) = 16.64\cos(2t - 56.3^\circ)$$