

Tutorial 9: Student Name \_\_\_\_\_ Student id: \_\_\_\_\_

Question 1: Let X=aabbacab and Y=baabcbb. Find the shortest common super-sequence for X and Y. (Backtracking process is required.)

Solution:

	Y j=0	b j=1	a j=2	a	b	c	b	b
X i=0	0	1	2	3	4	5	6	7
a i=1	1	←2	↖2	↖3	←4	←5	←6	←7
a i=2	2	←3	↖3	↖3	←4	←5	←6	←7
b	3	↖3	←4	↑4	↖4	←5	↖6	↖7
b	4	↖4	←5	↑5	↖5	←6	↖6	↖7
a	5	↑5	↖5	↖6	↑6	←7	↑7	←8
c	6	↑6	↑6	←7	↑7	↖7	←8	←9
a	7	↑7	↖7	↖7	←8	↑8	←9	←10
b	8	↖8	↑8	↑8	↖8	←9	↖9	↖10

Backtracking:

b→b→a→c→a→b→b→a→a→b

Ans: baabbacabb

Question 2:

**Input:** An array  $A[1..n]$  of  $n$  integers (positive or negative).

**Task:** Use dynamic method to find a non-empty interval  $[i, j]$  such that  $A[i]+A[i+1]+\dots+A[j]$  is maximized.

Example: Given an array: -1, 2 -3, 4, 5, -1

The sum of interval  $[1,1]=-1$ ,  $[1,2]=-1+2=1$ ,  $[3, 5]=-3+4+5=6$ .

Hint: Let  $d[i]$  be the cost of the max sum of intervals ending at position  $i$ .

That is,  $d[i]=\max \{ \text{sum}[1,i], \text{sum}[2, i], \dots, \text{sum}[i,i] \}$ .

Find recursive equation and use it to design a DP algorithm.

The final solution is the subinterval with the maximal  $d$  value.

In this example,  $d[1]=\text{sum of } [1,1]=-1$ ,  $d[2]=\text{sum of } [2,2]=2$ .

$d[3]=\text{sum of } [2,3]=-1$ ,  $d[4]=\text{sum of } [4,4]=4$ .

Answer:

$$d(i) = \begin{cases} A[i] & \text{if } i = 1 \\ d[i-1] + A[i] & \text{if } d[i-1] > 0 \\ A[i] & \text{if } d[i-1] \leq 0 \end{cases}$$

Alg:

Phase 1

$d(1):=A[i]$

For  $i=2$  to  $n$  do:

If  $d(i-1)>0$ ,

$d(i)=d(i-1)+A[i]$ ,  $B[i]=1$ ,

/\* containing  $A[i]$  and optimal interval ending at  $i-1$ .

Otherwise,  $d(i)=A[i]$ ,  $B[i]=0$ ,

/\* the optimal interval ending at  $i$  contains only  $A[i]$ .

/\* B for backtracking.

Phase 2: Find  $j$  with the maximal  $d$  value. (It can also be done in phase 1)

Phase 3: Backtracking:

$i=j$ ,

while( $i>1$  &  $B(i)=1$ )

$j=j-1$ ,

The optimal interval is  $[A[i], \dots, A[j]]$ .