Exercise in Lecture 5

Suppose $T(2^k) \leq T(2^{k-1})$+C for k= 1, 2,, ….
Prove that $T(2^k) = O(k)$

Solution A:

$T(2^k) \leq T(2^{k-1})$ + C
$\leq T(2^{k-2})$ + C + C
$\leq T(2^{k-3})$ + C + 2C
…
$\leq T(2^0)$ + kC
= $T(1)$ + kC

so, $T(2^k)$ = O(T(1) + kC) = O(k)

Solution B:

Let n = 2^k

$T(n) \leq T(n/2^1)$ + C
$T(n/2) \leq T(n/2^2)$ + C
$T(n/2^2) \leq T(n/2^3)$ + C
…
$T(2) \leq T(n/2^k)$ + C

Sum these inequations and simplify, and get

$T(n) \leq T(n/2^k)$ + kC
= $T(1)$ + kC

so, $T(2^k)$ = O(T(1) + kC) = O(k)