MA1200 TAKE HOME PROBLEM SET 3

The following is the third take-home assignment of MA1200, which counts 3 points of total 100 of your final score of the course.

Please submit it via canvas in a pdf file (you can handwrite the answers and take photos by your phone, then make it into a pdf file, see for example, https://www.wikihow.com/Convert-JPG-to-PDF) for how to combine jpg files to a pdf; you can also do it by note-taking apps on an iPad or an Surface)

Q1. Differentiate with respect to x

$$(0.3point)(a) \qquad \tan^{-1}(\sinh x)$$

$$(0.3point)(b) \qquad 3^{\sqrt{x}}$$

$$(0.4point)(c) \qquad \frac{\sin^2(x)e^{3x}}{(2x)^{1/2}\tan^{1/2}(3x)}$$

Q2. (0.5 point) Show from first principles that $\dfrac{d}{dx}(\cos x) = -\sin x$.

Q4. (0.5 point) $x = \cos t + \ln(\tan(t/2))$ and $y = \sin t$, compute $\dfrac{d^2y}{dx^2}$

Q5. Let $f(x) = \sin(2\ln(1+x))$

(a) (0.3 point) Show that $(1+x)^2 f''(x) + (1+x)f'(x) + 4f(x) = 0$.

(b) (0.4 point) Let n be a positive integer, show that

$$(1+x)^2 f^{(n+2)}(x) + (2n+1)(1+x)f^{(n+1)}(x) + (n^2+4)f^{(n)}(x) = 0.$$

Hint, Leibnitz' rule: $(uv)^{(n)} = \sum_{r=0}^{n} C_r^n u^{(r)} v^{(n-r)}$, $C_r^n = \frac{n!}{(n-r)!r!}$

(c) (0.3 point) Find $f^{(n)}(0)$ for $n = 0, 1, 2, 3, 4, 5, 6$.

The assignment is due on 23:59 of Nov 22, Sunday.

You will lose 1 point for each day of late submission. All submissions after the midnight of Nov 25 will be marked as 0.

Date: November 12, 2020.