Tutorial 5

Modular Arithmetic

Question 1: Divisibility by 9

Let x be an n-digit number. Prove that

$$x \equiv a_{n-1} + a_{n-2} + \cdots + a_1 + a_0 \pmod{9},$$

where a_i is the $(i+1)$-th digit of x.

- Example 1:
  - Suppose $x = 6213$. $x \bmod 9 = 6 + 2 + 1 + 3 \bmod 9 = 3$.
- Example 2:
  - Suppose $x = 7218$. Since the digit sum $\pmod{9} = 7 + 2 + 1 + 8 \pmod{9} = 0$, x must be divisible by 9.

Question 2: Diophantie Equation

- Solve the equation

$$98x + 35y = 14,$$

where x and y are integers.

Question 3: Application

- The admission fee at a small fair is $1.5 for children and $4.00 for adult.
- On a certain day, the fair collected $5,050.
- It was known that there attended more children than adults, and that in total there were not more than 3,000 people.
- How many possible combinations of number of children and number of adults could be used to satisfy the given conditions?

Question 4: Repeat-and-Multiply

a) Use the Repeat-and-Multiply method to compute $3^{94} \bmod 17$.

b) User Fermat's Little Theorem to compute $40^{110} \bmod 37$.

Question 5: Fermat's Little Theorem

- Solve $x^{103} \equiv 4 \bmod 11$.