

How to find the inverse in $M_{VC \leftarrow WC}$

The matrix is in a special form which makes the inverse finding easy.

$$\begin{aligned} M_{VC \leftarrow WC} &= \begin{pmatrix} X_{VC} & Y_{VC} & Z_{VC} & VRP \\ 0 & 0 & 0 & 1 \end{pmatrix}^{-1} \\ &= \left[\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} X_{VC} & Y_{VC} & Z_{VC} \\ 0 & 0 & 0 \end{pmatrix} \right]^{-1} \\ &= \begin{pmatrix} X_{VC} & 0 \\ Y_{VC} & 0 \\ Z_{VC} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}^{-1} \quad [\quad (AB)^{-1} = B^{-1}A^{-1} \quad] \end{aligned}$$

This is so since

$$\begin{pmatrix} X_{VC} & Y_{VC} & Z_{VC} \\ 0 & 0 & 0 \end{pmatrix}$$

is a rotation, and we make use of $R^{-1} = R^T$

Thus,

$$M_{VC \leftarrow WC} = \begin{pmatrix} X_{VC} & -X_{VC} \cdot VRP \\ Y_{VC} & -Y_{VC} \cdot VRP \\ Z_{VC} & -Z_{VC} \cdot VRP \\ 0 & 0 & 0 & 1 \end{pmatrix}$$