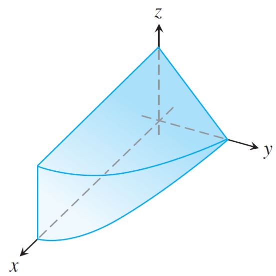
Take Home Assignment MA2001 #3

1. Let V be a solid in first octant bounded by the coordinate planes, the plane $y + z = 2$, and the cylinder $x = 4 - y^2$. Set up (No need evaluate) the iterated integrals for the triple



integral $\iiint_V f(x, y, z)\, dV$ with order $dxdydz$, $dydxdz$, $dzdxdy$, respectively.

2. (a) Solve the system

$$u = x - y, \qquad v = 2x + y$$

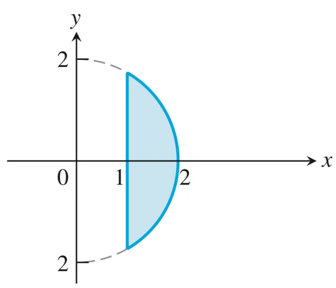
for x and y in terms of u and v. Then find the value of the Jacobian $\frac{\partial(x,y)}{\partial(u,v)}$.

(b) Find the image under the substitution $u = x - y, v = 2x + y$ of the region R in the first quadrant bounded by the lines $y = -2x+4, y = -2x+7, y = x-2$, and $y = x+1$ in the $xy-$plane. Sketch the transformed region in the $uv-$plane.

(c) Use the above substitution to evaluate the integral

$$\iint\limits_R (2x^2 - xy - y^2)\, dx\, dy.$$

3. Let R be the region illustrated below.



(a) Describe the region R in polar coordinates.

(b) Evaluate the double integral

$$\iint_R \frac{1}{\sqrt{x^2 + y^2}} dxdy.$$

4. Evaluate the integral

$$\int_{-1}^{1}\int_{0}^{\sqrt{1-y^2}}\int_{0}^{x}(x^2+y^2)\,dz\,dx\,dy$$

[Hint: Use cylindrical coordinates transformation.]

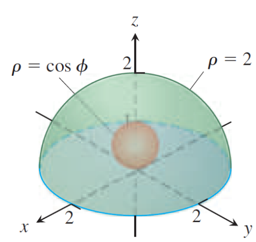
5. Set up (No need evaluate) iterated integrals for the volumes of following region.

   (a) The region between the cylinder $z = y^2$ and xy-plane that is bounded by the planes $x = 0, x = 1, y = -1, y = 1$.

   (b) The wedge cut from the cylinder $x^2 + y^2 = 1$ by the planes $z = -y$ and $z = 0$.

   (c) The region enclosed above by $z = 1 - x^2 - y^2$ and below by $z = -\sqrt{1 - x^2 - y^2}$. (Hint: use cylindrical coordinates substitution.)

   (d) The solid between the sphere $\rho = \cos\phi$ and the hemisphere $\rho = 2, z \geq 0$, where (ρ, θ, ϕ) is spherical coordinates.



6. Consider a vector field

$$\mathbf{F} = (y+z)\mathbf{i} + (z+x)\mathbf{j} + (x+y)\mathbf{k}$$

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   (a) Determine if $\mathbf{F}$ is conservative. If so, find its potential function.

   (b) Calculate the divergence of $\mathbf{F}$, i.e. div($\mathbf{F}$) at $(1, 2, 3)$.

   (c) Calculate the curl field of $\mathbf{F}$, i.e. Curl($\mathbf{F}$).

7. (a) Sketch the graphs for the straight line path $C_1 : \mathbf{r}(t) = t\mathbf{i} + t\mathbf{j} + t\mathbf{k}, \quad 0 \leq t \leq 1$ and the curve path $C_2 : \mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + \frac{2}{3}t^3\mathbf{k}, \quad 0 \leq t \leq 1$.

   (b) Evaluate the first kind line integral for the three-variable function $f(x, y, z) = xz$ along C_2.

   (c) Evaluate the second kind line integral for the vector field

$$\mathbf{F} = (y+z)\mathbf{i} + (z+x)\mathbf{j} + (x+y)\mathbf{k}$$

   along C_1 from $t = 0$ to $t = 1$.

8. Consider the surface S cut from the parabolic cylinder $z = 4 - y^2$ by the planes $x = 0, x = 2$, and $z = 0$.

   (a) Set up a proper parametric expression for the surface S.

   (b) Evaluate the first kind surface integral for the three-variable function $f(x, y, z) = xz$ over S.

   (c) Evaluate the second kind surface integral for the vector field

$$\mathbf{F} = (y + z)\mathbf{i} + (z + x)\mathbf{j} + (x + y)\mathbf{k}$$

   over S with outward orientation.