GE2262 Business Statistics

Topic 6
Hypothesis Testing

Reference
Levine, D.M., Krehbiel, T.C. and Berenson, M.L., *Business Statistics: A First Course*, Pearson Education Ltd, Chapter 9

Outline

- Hypothesis Testing Procedure
- Hypothesis Test for the Population Mean
  - Critical Value Approach
  - p-Value Approach
- Potential Pitfalls and Ethical Issues

Live Chicken Supply Suspended in HK

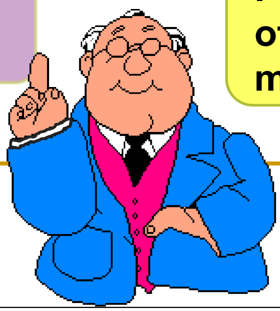
http://www.scmp.com/news/hong-kong/health-environment/article/1965394/live-chicken-supply-suspended-hong-kong-after

- SCMP, 05 June 2016: Sample taken from Yan Oi Market in Tuen Mun tests positive for bird flu virus. Live chicken supply suspended in Hong Kong
- During the year 2015, there were 1,442 poultry imported daily on average. No more than 30 poultry were tested daily for bird flu virus
- Decision on suspending live chicken supply is based on the test results of samples
  - If a sample is tested positive for the virus, then live poultry supply will be suspended for 21 days
  - If the tests for all samples are negative, no further action is required

- Do you think this checking process is reliable?
- What is the risk of making a wrong decision in either way?

What is a Hypothesis?

- The precursor to a hypothesis is a research or business problem, usually framed as a question
  - E.g., A teacher might want to know “Are the students performing well in academic?”
- The question is then converted to a testable hypothetical statement
  - A statistical hypothesis is a claim about the population parameter
    - E.g. population mean, population standard deviation, or population proportion, etc.



Hypothesis Testing Procedure

Step 1: Define hypotheses

Step 2: Collect the data and identify the rejection region(s)

Step 3: Compute test statistic

Step 4: Make statistical decision

Hypothesis Testing Procedure

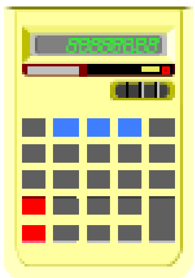
Cont'd

<u>Define Null Hypothesis</u>
Assume the population mean GPA (μ) is 3.5

<u>Identify the Population</u>

<u>Take a Random Sample</u>

<u>Compute Sample Statistic</u>



<u>Draw the Conclusion</u>
When μ = 3.5, is the sample statistic ($\bar{X}$) likely to occur? Or Is $\bar{X}$ very close to μ?
If not likely or not very close
→ REJECT Null Hypothesis

Step 1: Define Hypotheses

- The null hypothesis, H_0
  - Always about a population parameter (μ), rather than a sample statistic $(\bar{X})$
  - Always contains the "=" sign
  - Always assumed to be true at start
    - Similar to the notion of innocent unless proved guilty
  - To be tested numerically
  - The final decision is either "to reject" or "not to reject" it

Step 1: Define Hypotheses

Cont'd

- Example
  - You are in charge of a cereal-filling operation
  - You want to ensure that, on average, 368 g of cereals are in the boxes
  - Your filling machine is working properly so far
  - As a routine check, you take a random sample of 25 boxes and their average weight determined to see if it is close to 368 g
  - You null hypothesis might be

$$H_0\text{: } \mu = 368$$

Step 1: Define Hypotheses

Cont'd

- The alternative hypothesis, H_1
  - The opposite of the null hypothesis
  - Never contains the "=" sign
  - It is mutually exclusive and collectively exhaustive from the null hypothesis

- There are three different sets of hypotheses to be tested
  - Two-tail test: $H_0: \mu = \mu_0$ against $H_1: \mu \neq \mu_0$
  - Lower-tail test: $H_0: \mu \geq \mu_0$ against $H_1: \mu < \mu_0$
  - Upper-tail test: $H_0: \mu \leq \mu_0$ against $H_1: \mu > \mu_0$

Step 1: Define Hypotheses

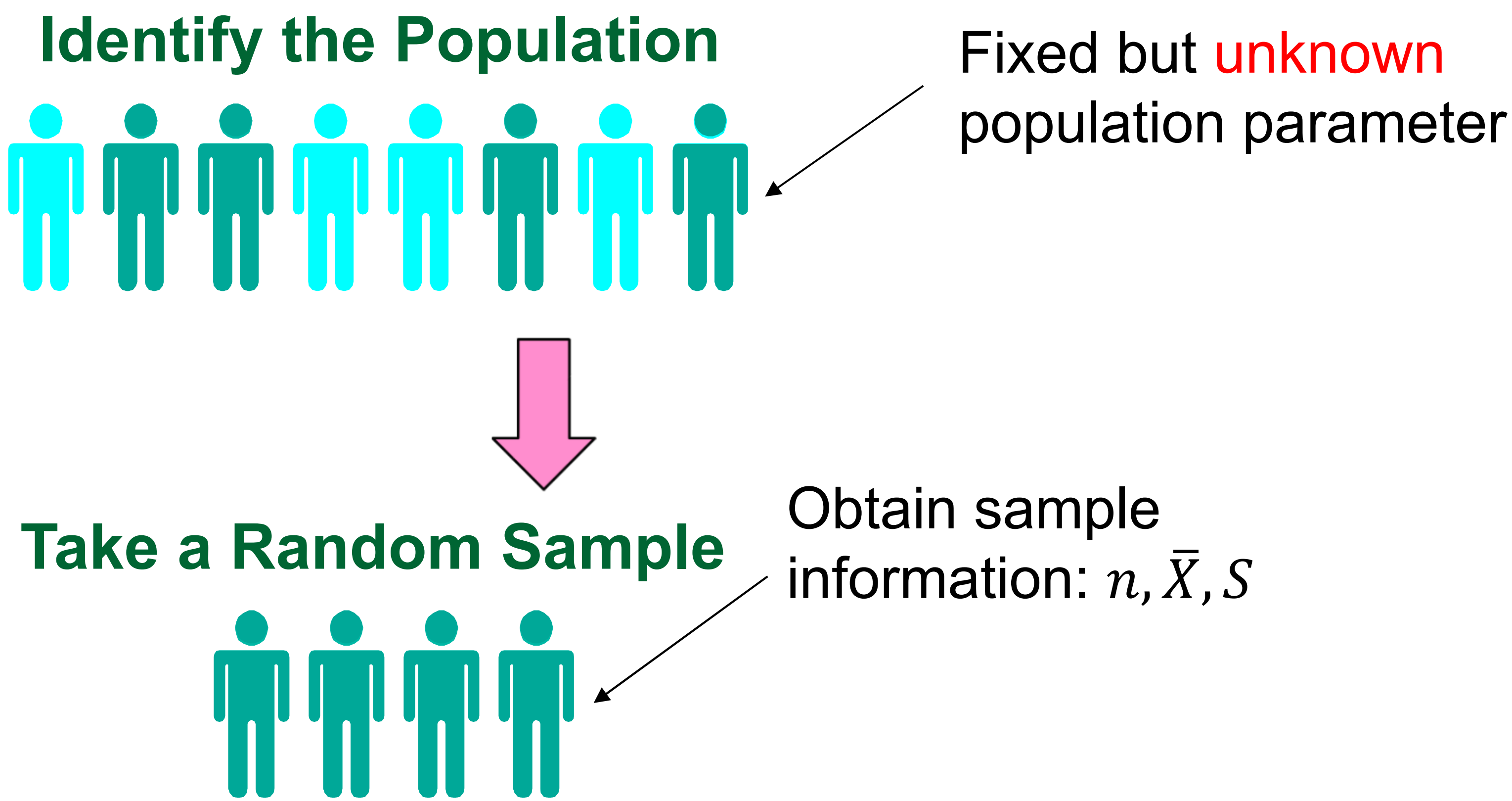
Cont'd

- Example
  - Recently, you receive complaints from customers concerning the amount of cereal being less than the specified 368 g
  - Your null and alternative hypothesis would then be

    H_0: $\mu \geq 368$

    H_1: $\mu < 368$

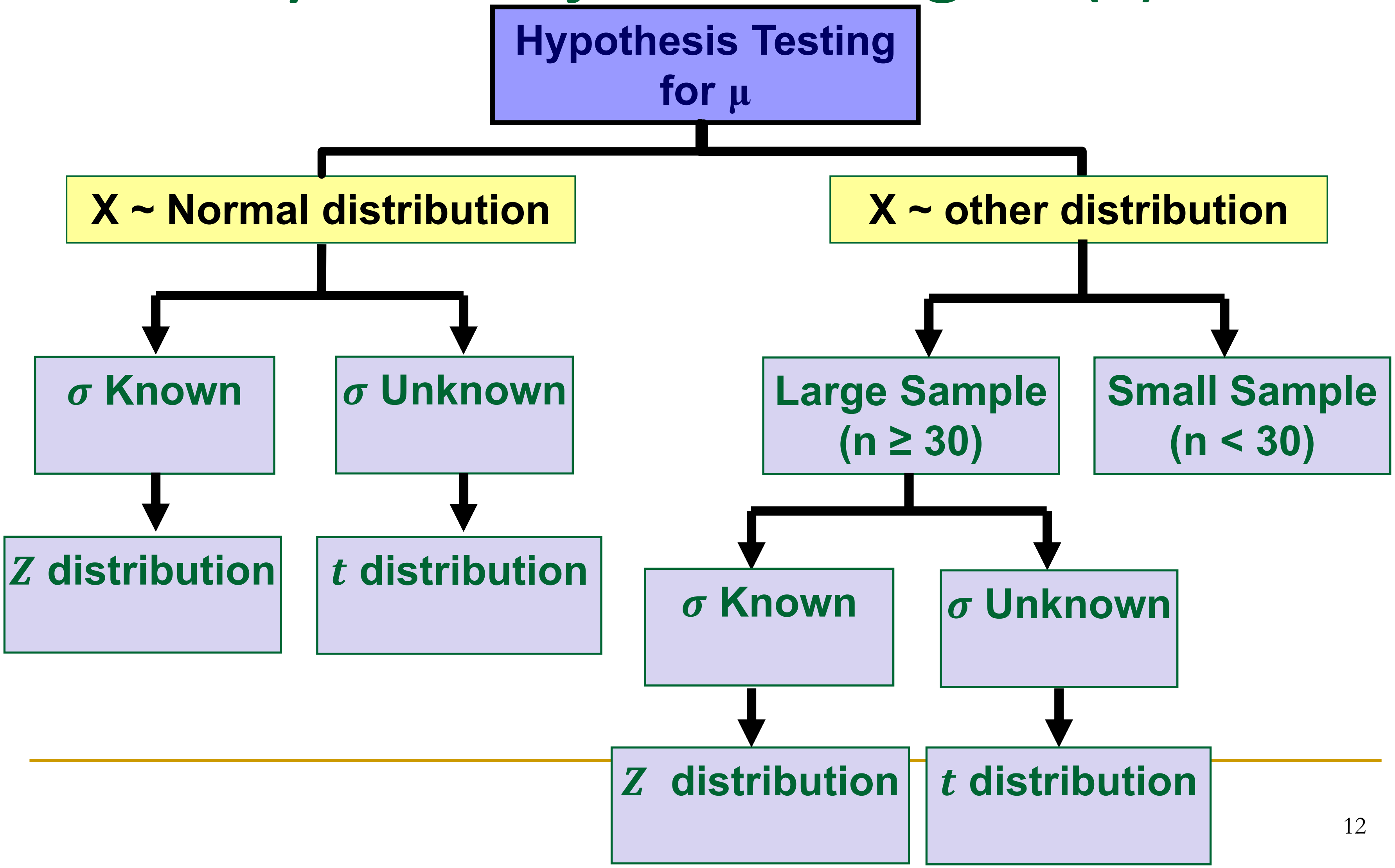
Step 2: Collect the Data and Identify the Rejection Region(s)



We will assume that the given data set is a representative sample of the population concerned

Step 2: Collect the Data and Identify the Rejection Region(s)

Cont'd



Step 2: Collect the Data and Identify the Rejection Region(s)

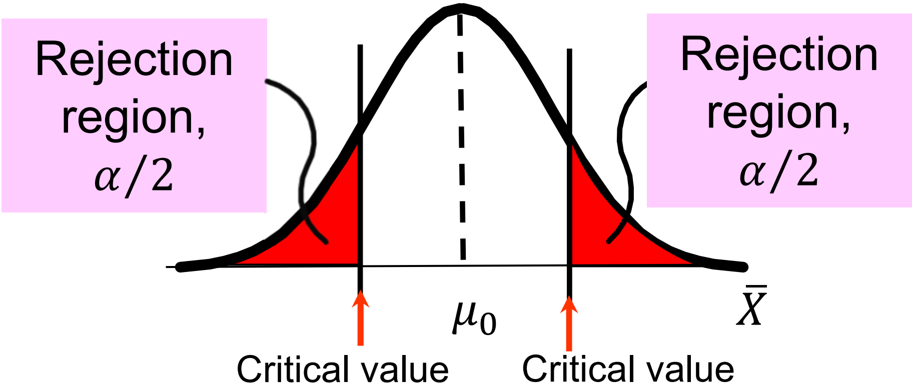
Cont'd

- The rejection region is an area containing the unlikely values of test statistic if null hypothesis is true
- The size of the rejection region is selected by the researcher at the beginning of the hypothesis test
  - Also refers to as level of significance, α
  - Typical values are 0.01, 0.05 and 0.10
  - It provides the critical value(s) of the hypothesis test
  - It controls the probability of committing Type I error
    - The acceptable risk level for rejecting the null hypothesis wrongly

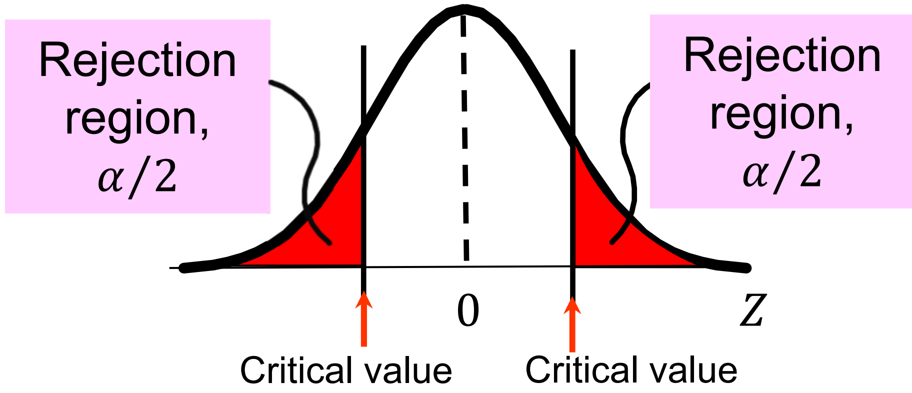
Step 2: Collect the Data and Identify the Rejection Region(s)

Cont'd

- The location of the rejection region depends on the hypotheses being tested
- For **two-tail** test: $H_0: \mu = \mu_0$ against $H_1: \mu \neq \mu_0$



$\bar{X}$ must be significantly different from μ_0 to reject H_0

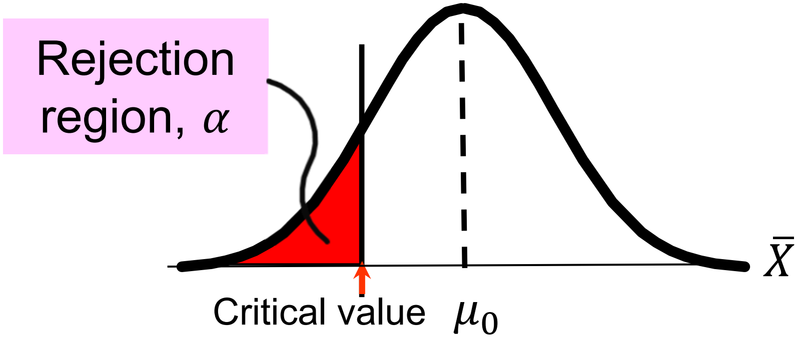


Z must be significantly different from 0 to reject H_0

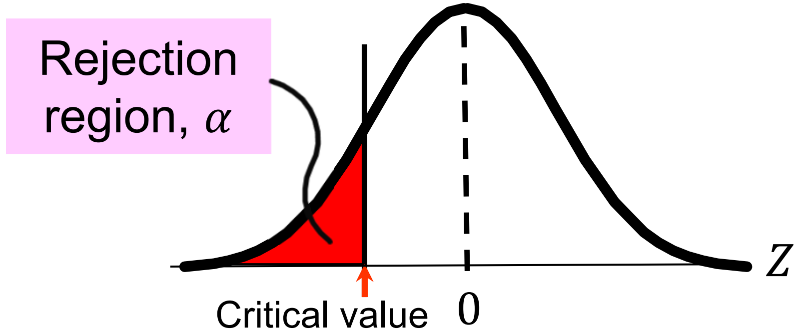
Step 2: Collect the Data and Identify the Rejection Region(s)

Cont'd

- For **lower-tail** test: $H_0: \mu \geq \mu_0$ against $H_1: \mu < \mu_0$

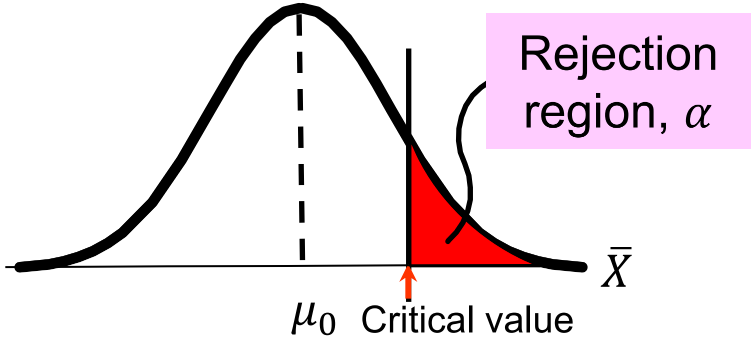


$\bar{X}$ must be significantly smaller than μ_0 to reject H_0

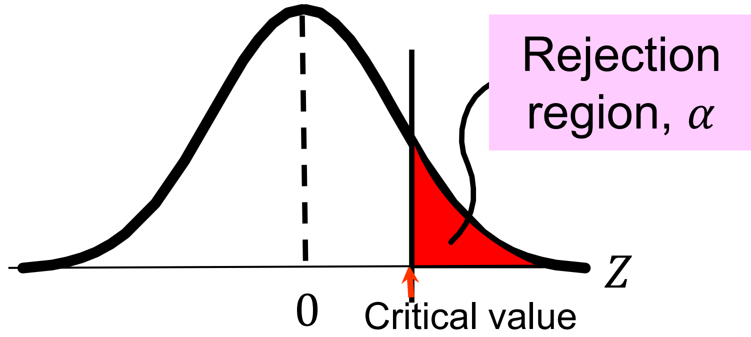


Z must be significantly smaller than 0 to reject H_0

- For **upper-tail** test: $H_0: \mu \leq \mu_0$ against $H_1: \mu > \mu_0$



$\bar{X}$ must be significantly larger than μ_0 to reject H_0



Z must be significantly larger than 0 to reject H_0

Step 3: Compute Test Statistic

- Convert sample statistic ($\bar{X}$) to test statistic (Z or t)
  - A scale free value for determining whether the sample mean is far enough from the hypothesized population mean
- Z test statistic
  - Conditions
    - Population standard deviation (σ) is known
    - Population is normally distributed ➔ $\bar{X} \sim N(\mu_{\bar{X}}, (\frac{\sigma}{\sqrt{n}})^2)$
    - If population is not normal, but with a large sample $(n \geq 30)$, by Central Limit Theorem ➔ $\bar{X} \sim N(\mu_{\bar{X}}, (\frac{\sigma}{\sqrt{n}})^2)$

$$Z = \frac{\bar{X} - \mu_{\bar{X}}}{\sigma_{\bar{X}}} = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}}$$

Step 3: Compute Test Statistic

Cont'd

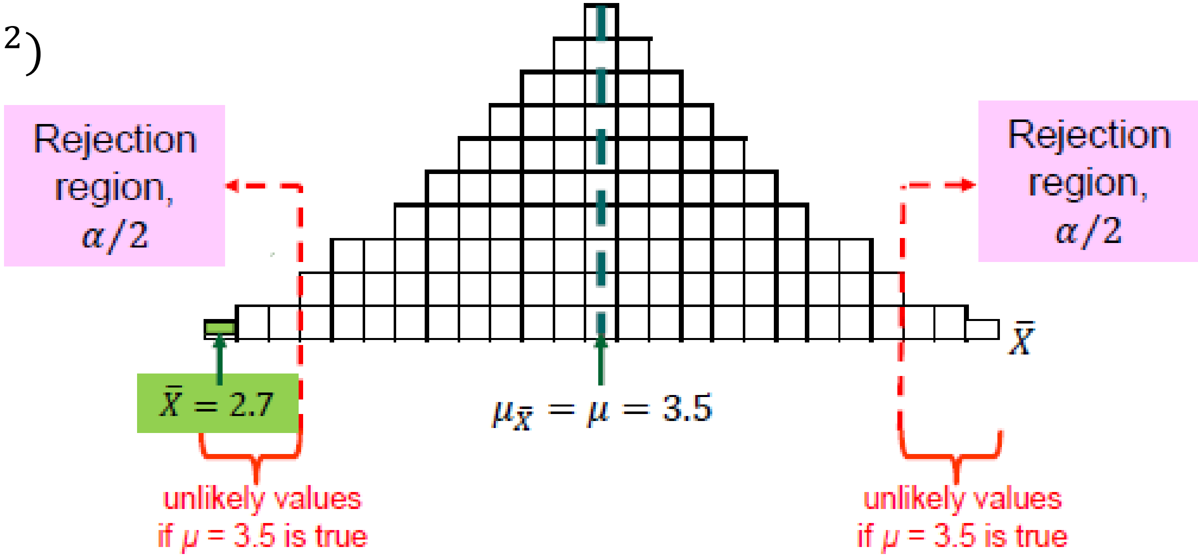
- t test statistic
  - Conditions
    - Population standard deviation (σ) is unknown
    - Population is normally distributed ➔ $\bar{X} \sim N(\mu_{\bar{X}}, (\frac{\sigma}{\sqrt{n}})^2)$
    - If population is not normal, but with a large sample $(n \geq 30)$, by Central Limit Theorem ➔ $\bar{X} \sim N(\mu_{\bar{X}}, (\frac{\sigma}{\sqrt{n}})^2)$

$$t = \frac{\bar{X} - \mu_{\bar{X}}}{\sigma_{\bar{X}}} = \frac{\bar{X} - \mu_0}{S/\sqrt{n}}$$

with $(n-1)$ degrees of freedom

Step 4: Make Statistical Decision

$\bar{X} \sim N(\mu_{\bar{X}}, (\frac{\sigma}{\sqrt{n}})^2)$

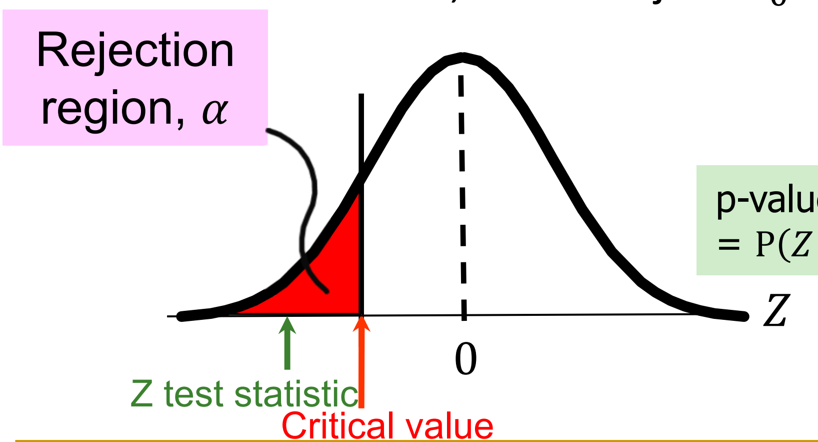


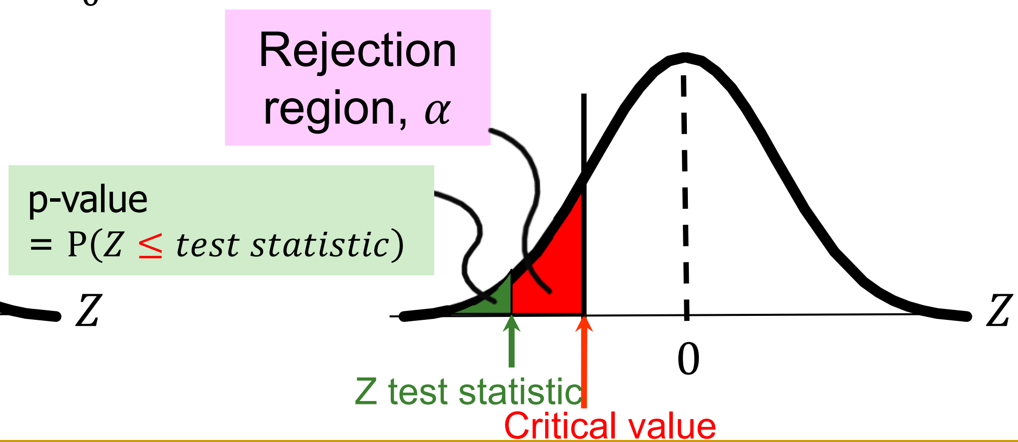
- Critical value approach
  - Based on the level of significance (α), obtain critical value(s) from the Z or t table
  - Set up the decision rule to identify where is (are) the rejection region(s)
  - Check if the Z or t test statistic falls in the rejection region or not
    - If yes, then reject H_0
    - Otherwise, do not reject H_0

Step 4: Make Statistical Decision

Cont'd

- p-value approach
  - Convert the Z or t test statistic to p-value
    - The p-value is the probability of obtaining a test statistic as extreme or more extreme ($\leq$ or $\geq$) than the observed sample statistic given H_0 is true
  - Compare the p-value with the level of significance (α)
    - If p-value $< \alpha$, then reject H_0
    - Otherwise, do not reject H_0





Reject H_0 if Z test statistic < Critical value

Reject H_0 if p-value < α

Step 4: Make Statistical Decision

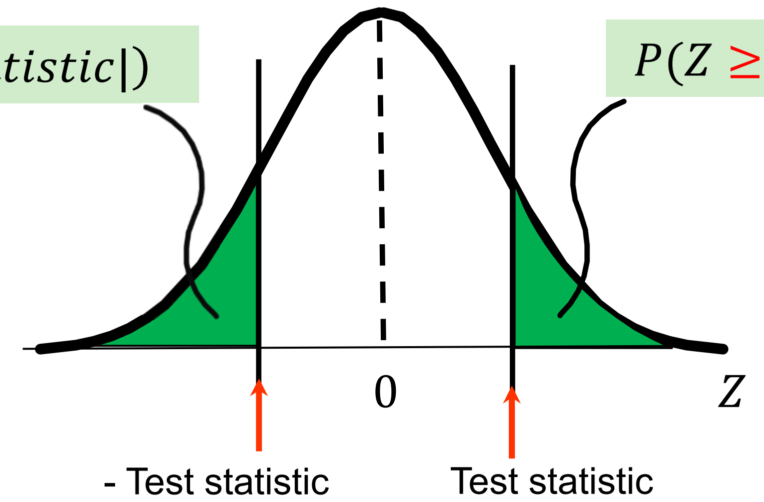
Cont'd

- For **two-tail** test: $H_0: \mu = \mu_0$ against $H_1: \mu \neq \mu_0$

p-value = $P(Z \leq -|test\ statistic|) + P(Z \geq |test\ statistic|)$

$P(Z \leq -|test\ statistic|)$

$P(Z \geq |test\ statistic|)$



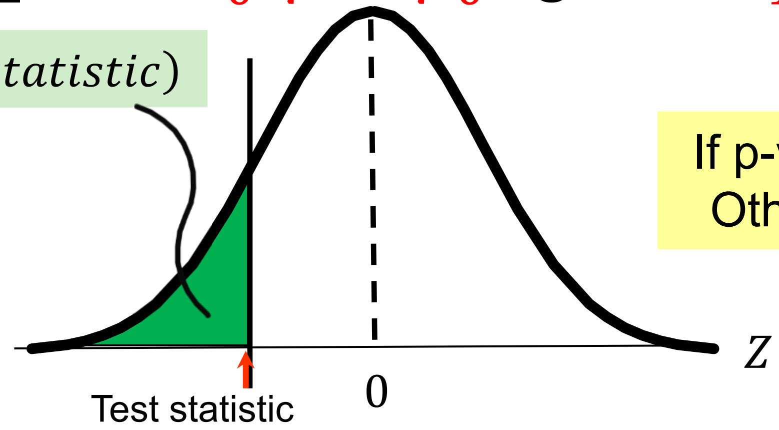
If p-value $< \alpha$, then reject H_0
Otherwise, do not reject H_0

Step 4: Make Statistical Decision

Cont'd

- For **lower-tail** test: $H_0: \mu \geq \mu_0$ against $H_1: \mu < \mu_0$

p-value = $\mathrm{P}(Z \leq \mathit{test\ statistic})$

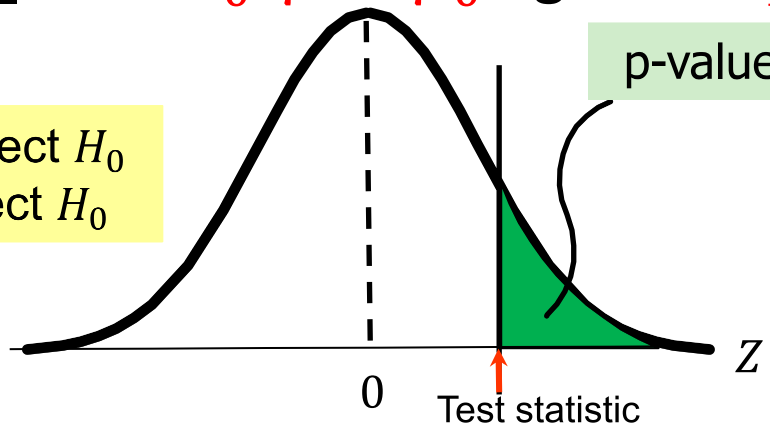


If p-value $< \alpha$, then reject H_0
Otherwise, do not reject H_0

- For **upper-tail** test: $H_0: \mu \leq \mu_0$ against $H_1: \mu > \mu_0$

p-value = $P(Z \geq \mathit{test\ statistic})$

If p-value $< \alpha$, then reject H_0
Otherwise, do not reject H_0



Step 4: Make Statistical Decision

Cont'd

- In statistical hypothesis testing, we make the decision based on only one sample, we do not have the information to claim that the null hypothesis is true or false with 100% certainty
- Whether the null hypothesis is rejected or not rejected, we always facing a risk of making a wrong decision
- We never prove any one of the two hypotheses is true or false, we simply reject or do not reject the null hypothesis with a risk

Step 4: Make Statistical Decision

Cont'd

Decision	The Truth	
	H_0 True	H_0 False
Do not reject H_0	Level of Confidence $(1-\alpha)$	Type II Error (β)
Reject H_0	Type I Error (α)	Power of the Test $(1-\beta)$

Step 4: Make Statistical Decision

Cont'd

- Type I Error
  - Reject a true null hypothesis
  - Probability of Type I error is denoted α
    - $\alpha = P(Reject\ H_0 | H_0\ true)$
    - Also called level of significance
      - Set by researcher
  - $(1 - \alpha)$ is called level of confidence
- Type II Error
  - Fails to reject a false null hypothesis
  - Probability of Type II error is denoted β
    - $\beta = P(Do\ not\ reject\ H_0 |\ H_0\ false)$
  - $(1 - \beta)$ is called power of the test

Step 4: Make Statistical Decision

Cont'd

- Naturally, we would like both type of errors to be as small as possible
- While the Type I error is often pre-specified before the test (e.g. $\alpha = 0.05$), we cannot do much about the Type II error as the value of β depends on the true value of the parameter to be tested, which is often unknown to us if the null hypothesis is rejected
- Ways to reduce the probability of making a Type II error
  - By increasing α. This is preferred if the cost of committing Type II error is higher than that of Type I error
  - By increasing the sample size for the test. This is preferred if there are sufficient resources to do so

Z Test for the Population Mean (σ Known)

- Conditions
  - Population standard deviation (σ) is known
  - Population is normally distributed ➔ $\bar{X} \sim N(\mu_{\bar{X}}, (\frac{\sigma}{\sqrt{n}})^2)$
  - If population is not normal, but with a large sample $(n \geq 30)$, by Central Limit Theorem ➔ $\bar{X} \sim N(\mu_{\bar{X}}, (\frac{\sigma}{\sqrt{n}})^2)$

- Obtain critical value(s) from the Z-table

- Test statistic, $Z = \frac{\bar{X} - \mu_{\bar{X}}}{\sigma_{\bar{X}}} = \frac{\bar{X} - \mu_0}{\sigma / \sqrt{n}}$

Z Test for the Population Mean (σ Known) – Example

- A random sample of 25 boxes of cereals gave a mean 364.5 g
- The company has specified the population distribution is Normal and the standard deviation to be 15 g
- Test at the 5% level of significance and see if the average weight is close to 368 g

Z Test for the Population Mean (σ Known) – Example

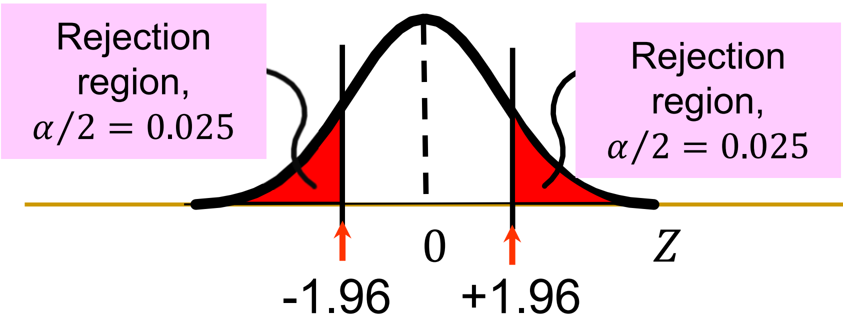
Cont'd

$H_0: \mu = 368$
$H_1: \mu \neq 368$

At $\alpha = 0.05$
$n = 25$

Critical Value = ± 1.96

Reject H_0 if $Z < -1.96$ or $Z > +1.96$



$$Z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} = \frac{364.5 - 368}{15/\sqrt{25}}$$
$$= -1.17$$

At $\alpha = 0.05$, do not reject H_0

There is no evidence that the true mean weight is not 368 g

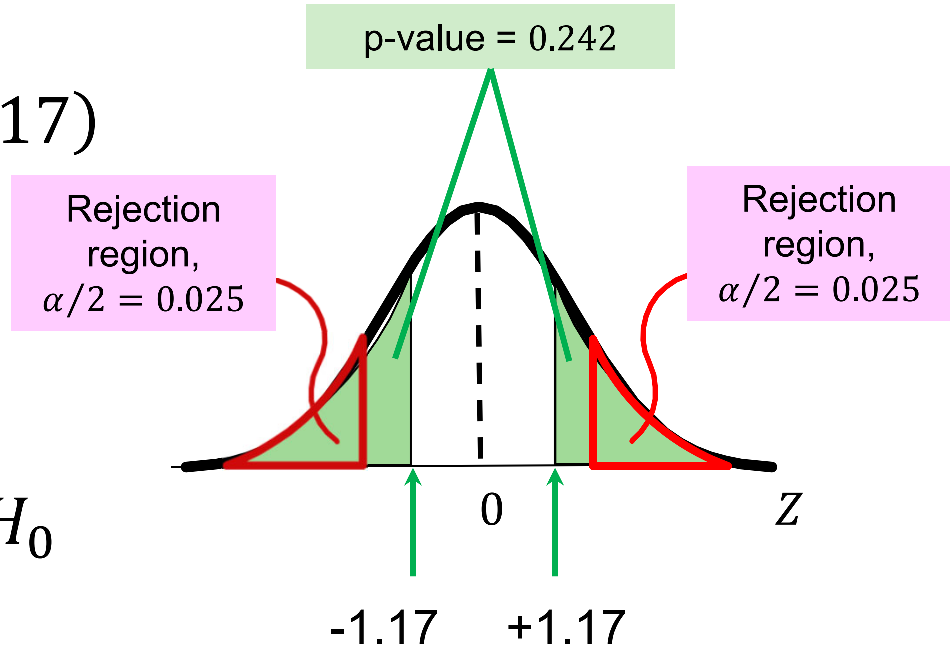
Z Test for the Population Mean (σ Known) – Example

Cont'd

$H_0: \mu = 368$
$H_1: \mu \neq 368$

$$Z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} = \frac{364.5 - 368}{15/\sqrt{25}} = -1.17$$

p-value
$= P(Z \leq -1.17) + P(Z \geq 1.17)$
$= 2 \times P(Z \leq -1.17)$
$= 2 \times 0.1210$
$= 0.242$



As p-value > α, do not reject H_0

There is no evidence that the true mean weight is not 368 g

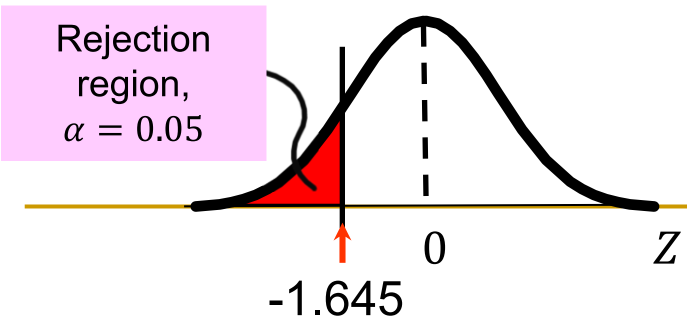
Z Test for the Population Mean (σ Known) – Exercise

Cont'd

- How would you revise the analysis if you need to deal with the customers' concerning about the amount of cereal being less than the specified 368 g?
- Noted that
  - The company has specified the population distribution is Normal
  - The population standard deviation is 15 g
  - Test at the 5% level of significance

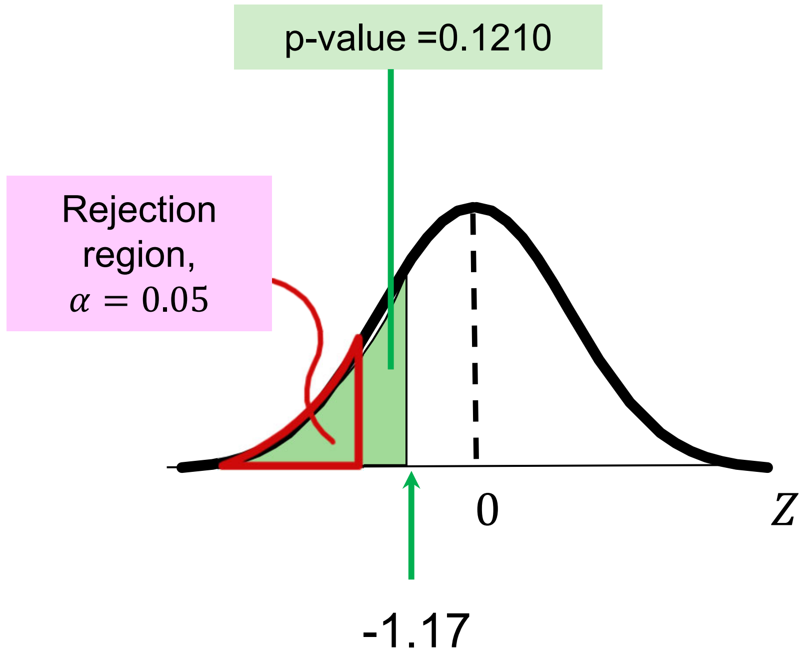
Z Test for the Population Mean (σ Known) – Exercise

Cont'd



Z Test for the Population Mean (σ Known) – Exercise

Cont'd

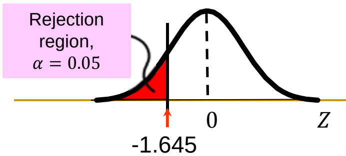


Z Test for the Population Mean (σ Known) – Exercise

Cont'd

H_0: $\mu \geq 368$
H_1: $\mu < 368$

At $\alpha = 0.05$
$n = 25$
Critical Value = -1.645
Reject H_0 if $Z < -1.645$



$$Z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} = \frac{364.5 - 368}{15/\sqrt{25}}$$
$$= -1.17$$

At $\alpha = 0.05$, do not reject H_0

There is no evidence that the true mean weight is less than 368 g

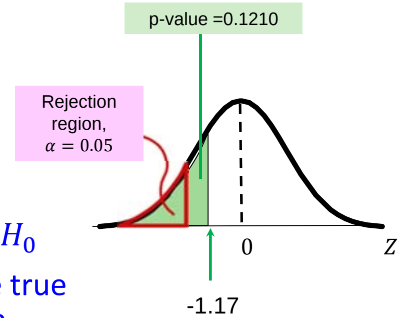
Z Test for the Population Mean (σ Known) – Exercise

Cont'd

H_0: $\mu \geq 368$
H_1: $\mu < 368$

$$Z = \frac{\bar{X}-\mu_0}{\sigma/\sqrt{n}} = \frac{364.5-368}{15/\sqrt{25}} = -1.17$$

p-value
$= P(Z \leq -1.17)$
$= 0.1210$



As p-value > α, do not reject H_0

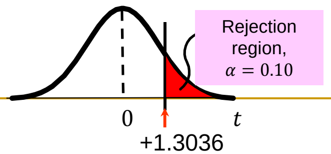
There is no evidence that the true mean weight is less than 368 g

t Test for the Population Mean (σ Unknown) – Exercise

Cont'd

H_0: $\mu \leq 1$
H_1: $\mu > 1$

At $\alpha = 0.10$
$n = 40$ $\quad df = 39$
Critical Value = $+1.3036$
Reject H_0 if $t > +1.3036$



$$t = \frac{\bar{X} - \mu_0}{S/\sqrt{n}} = \frac{1.03 - 1}{0.08/\sqrt{40}}$$
$$= 2.37$$

At $\alpha = 0.10$, reject H_0

There is evidence that the true mean amount is more than 1 L

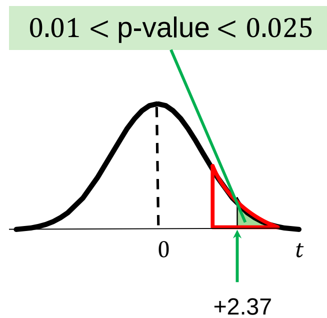
t Test for the Population Mean (σ Unknown) – Exercise

Cont'd

H_0: $\mu \leq 1$
H_1: $\mu > 1$

$$t = \frac{\bar{X}-\mu_0}{S/\sqrt{n}} = \frac{1.03-1}{0.08/\sqrt{40}} = 2.37$$

p-value
$= P(t \geq 2.37)$
$= (0.01, 0.025)$



As p-value < α, H_0 is rejected

There is evidence that the true mean amount is more than 1 L

Using Excel "T.DIST" function, the p-value is found to be 0.0114

t Test for the Population Mean (σ Unknown)

- Conditions
  - Population standard deviation (σ) is unknown
  - Population is normally distributed ➔ $\bar{X} \sim N(\mu_{\bar{X}}, (\frac{\sigma}{\sqrt{n}})^2)$
  - If population is not normal, but with a large sample $(n \geq 30)$, by Central Limit Theorem ➔ $\bar{X} \sim N(\mu_{\bar{X}}, (\frac{\sigma}{\sqrt{n}})^2)$
- Obtain critical value(s) from the t-table with $(n-1)$ degrees of freedom
- Test statistic, $t = \frac{\bar{X} - \mu_{\bar{X}}}{\sigma_{\bar{X}}} = \frac{\bar{X} - \mu_0}{S/\sqrt{n}}$

t Test for the Population Mean (σ Unknown) – Example

- In addition to cereals, the company newly set up the filling machine for milk
- Each bottle should contain 1 L of milk
- A random sample of 40 bottles are selected, giving an average 1.03 L and standard deviation 0.08 L
- At 10% level of significance, test to see is the filling machine is working properly

t Test for the Population Mean (σ Unknown) – Example

Cont'd

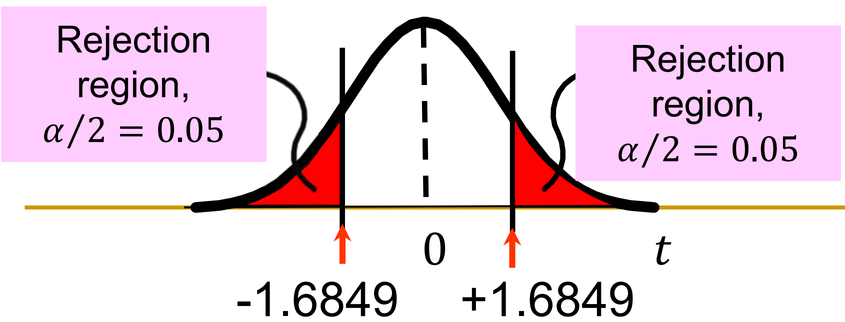
$H_0: \mu = 1$
$H_1: \mu \neq 1$

At $\alpha = 0.10$

$n = 40 \qquad df = 39$

Critical Value = ± 1.6849

Reject H_0 if $t < -1.6849$ or $t > +1.6849$



$$t = \frac{\bar{X} - \mu_0}{S/\sqrt{n}} = \frac{1.03 - 1}{0.08/\sqrt{40}} = 2.37$$

At $\alpha = 0.10$, reject H_0

There is evidence that the true mean amount is not 1 L

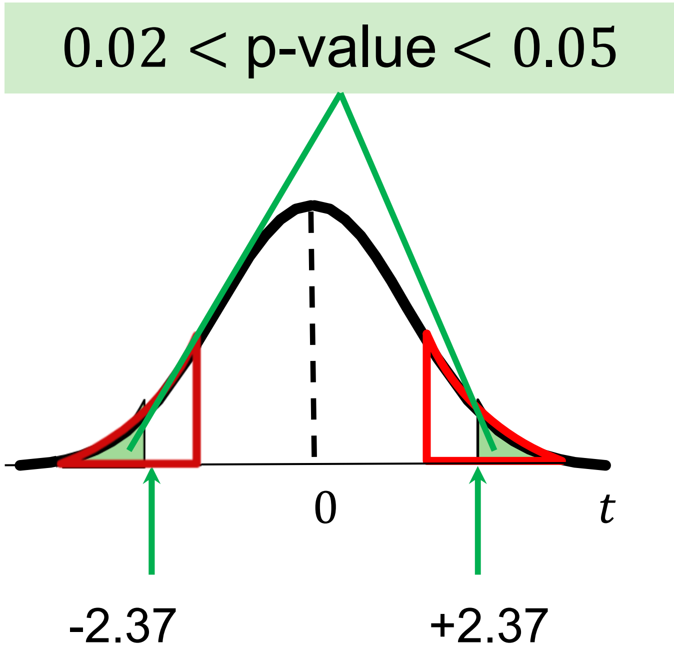
t Test for the Population Mean (σ Unknown) – Example

Cont'd

$H_0: \mu = 1$
$H_1: \mu \neq 1$

$$t = \frac{\bar{X} - \mu_0}{S/\sqrt{n}} = \frac{1.03 - 1}{0.08/\sqrt{40}} = 2.37$$

p-value
$= P(t \leq -2.37) + P(t \geq 2.37)$
$= 2 \times P(t \geq 2.37)$
$= 2 \times (0.01, 0.025)$
$= (0.02, 0.05)$



As p-value < α, H_0 is rejected

There is evidence that the true mean amount is not 1 L

Using Excel "T.DIST" function, the p-value is found to be 0.0228

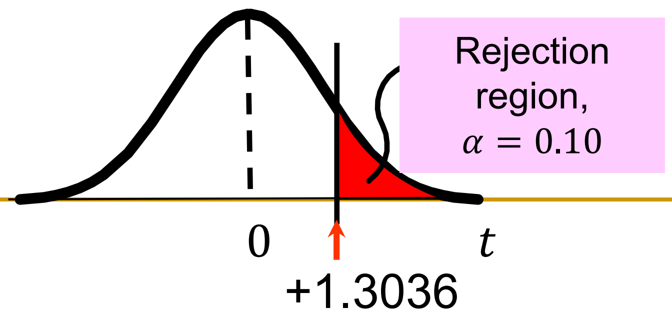
t Test for the Population Mean (σ Unknown) – Exercise

Cont'd

- In the last example, we found that the mean amount of milk is not 1 L
- Now, test to see if the mean amount is more than 1 L at 10% level of significance

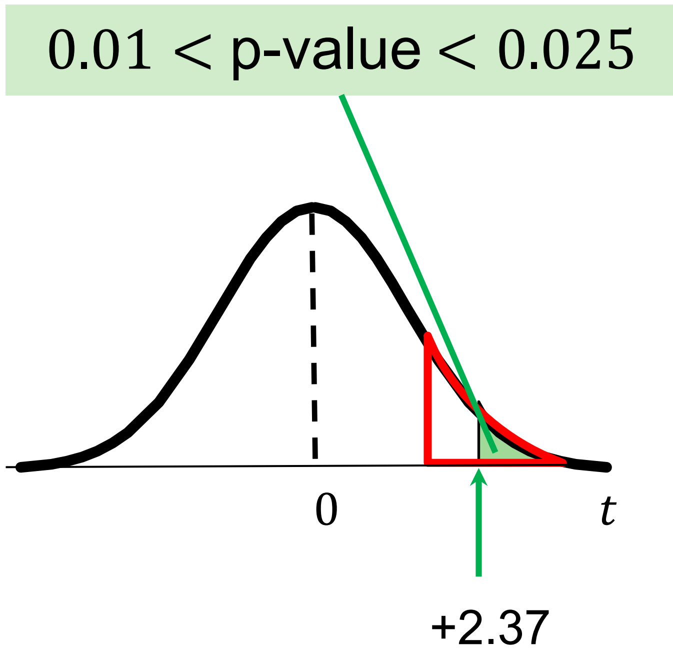
t Test for the Population Mean (σ Unknown) – Exercise

Cont'd



t Test for the Population Mean (σ Unknown) – Exercise

Cont'd



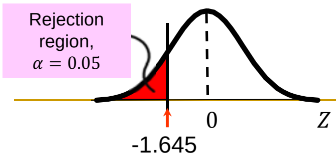
Using Excel "T.DIST" function, the p-value is found to be 0.0114

Z Test for the Population Mean (σ Known) – Exercise

Cont'd

H_0: $\mu \geq 368$
H_1: $\mu < 368$

At $\alpha = 0.05$
$n = 25$
Critical Value = -1.645
Reject H_0 if $Z < -1.645$



$$Z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} = \frac{364.5 - 368}{15/\sqrt{25}}$$
$$= -1.17$$

At $\alpha = 0.05$, do not reject H_0

There is no evidence that the true mean weight is less than 368 g

Z Test for the Population Mean (σ Known) – Exercise

Cont'd

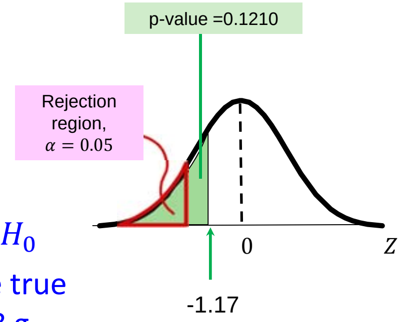
H_0: $\mu \geq 368$
H_1: $\mu < 368$

$$Z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} = \frac{364.5 - 368}{15/\sqrt{25}} = -1.17$$

p-value
$= P(Z \leq -1.17)$
$= 0.1210$

As p-value > α, do not reject H_0

There is no evidence that the true mean weight is less than 368 g

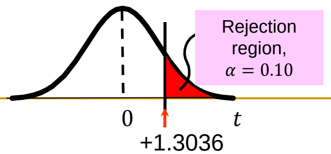


t Test for the Population Mean (σ Unknown) – Exercise

Cont'd

H_0: $\mu \leq 1$
H_1: $\mu > 1$

At $\alpha = 0.10$
$n = 40$ $\quad df = 39$
Critical Value = $+1.3036$
Reject H_0 if $t > +1.3036$



$$t = \frac{\bar{X} - \mu_0}{S/\sqrt{n}} = \frac{1.03 - 1}{0.08/\sqrt{40}}$$
$$= 2.37$$

At $\alpha = 0.10$, reject H_0

There is evidence that the true mean amount is more than 1 L

t Test for the Population Mean (σ Unknown) – Exercise

Cont'd

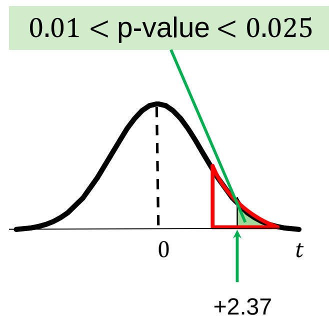
H_0: $\mu \leq 1$
H_1: $\mu > 1$

$$t = \frac{\bar{X} - \mu_0}{S/\sqrt{n}} = \frac{1.03 - 1}{0.08/\sqrt{40}} = 2.37$$

p-value
$= P(t \geq 2.37)$
$= (0.01, 0.025)$

As p-value < α, H_0 is rejected

There is evidence that the true mean amount is more than 1 L



Using Excel "T.DIST" function, the p-value is found to be 0.0114

Hypothesis Test – More Exercise

- Besides direct selling to the consumers, the milk is used to make processed cheese
- It is known that excess water will change the freezing point of the milk
- The freezing point of natural milk is distributed with a mean of -0.545 °C
- 14 randomly selected bottles of milk shows a mean -0.550 °C and standard deviation 0.016 °C

- At 5% level of significance, is the milk containing excess water?

Hypothesis Test – More Exercise

Cont'd

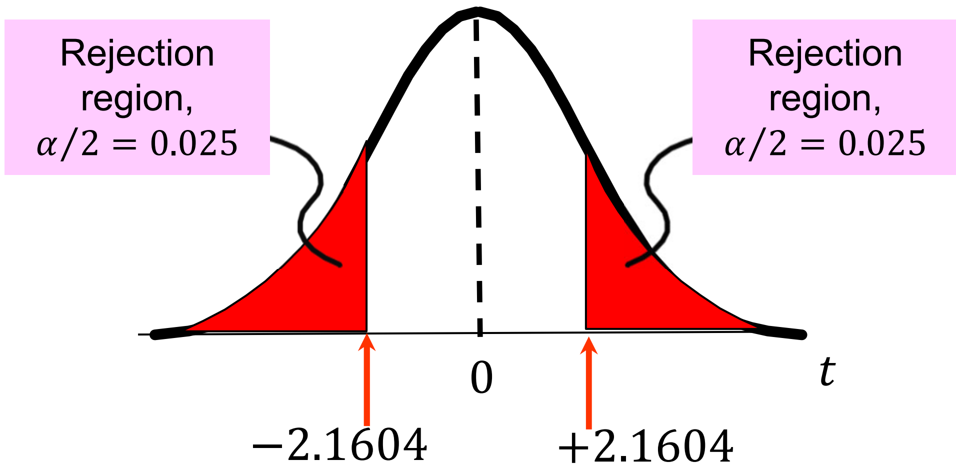
- Step 1: Define hypotheses

- Step 2: Collect data and identify rejection region(s)
  - Population distribution:
  - Sample size:
  - Any assumption needed?
    - What is the assumption?
    - Why?
  - σ:
  - Distribution to be used:

Hypothesis Test – More Exercise

Cont'd

- Step 2: Collect data and identify rejection region(s)
  - Significance level:
  - Degrees of freedom:
  - Critical value(s):
  - Decision rule:



Hypothesis Test – More Exercise

Cont'd

- Step 3: Compute test statistic
  - Test statistic =
  - p-value =

- Step 4: Make statistical decision
  - Decision:
  - Conclusion:

Hypothesis Test – More Exercise

Cont'd

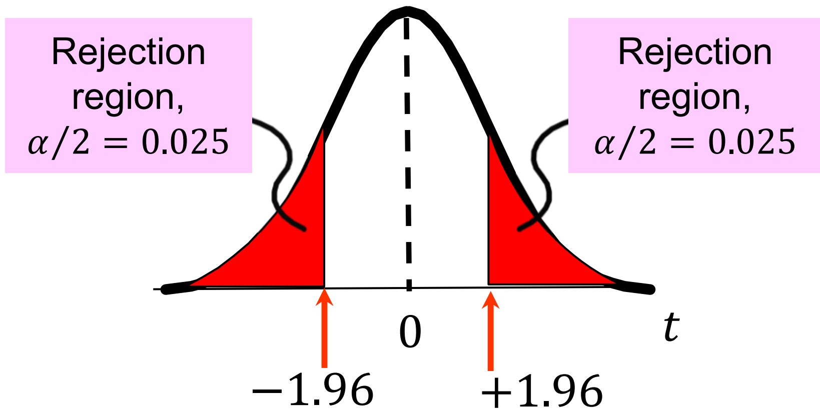
- What would happened if the sample size is 144 rather than 14?
  - Assumed the sample mean and standard deviation remain unchanged

- Step 1: Define hypotheses

Hypothesis Test – More Exercise

Cont'd

- Step 2: Collect data and identify rejection region(s)
  - Population distribution:
  - Sample size:
  - Any assumption needed?
    - What is the assumption?
    - Why?
  - σ:
  - Distribution to use:
  - Significance level:
  - Degrees of freedom:
  - Critical value(s):
  - Decision rule:



Hypothesis Test – More Exercise

Cont'd

- Step 3: Compute test statistic
  - Test statistic =
  - p-value
- Step 4: Make statistical decision
  - Decision:
  - Conclusion:

Hypothesis Test – More Exercise

Cont'd

- Step 1: Define hypotheses

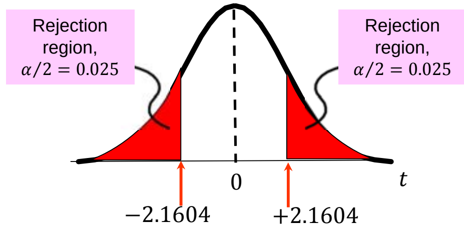
  $H_0: \mu = -0.545$

  $H_1: \mu \neq -0.545$
- Step 2: Collect data and identify rejection region(s)
  - Population distribution: Unknown
  - Sample size: 14
  - Any assumption needed? Yes
    - What is the assumption? Assume Normal population
    - Why? The sample size is too small to apply Central Limit Theorem
  - σ: unknown
  - Distribution to be used: t

Hypothesis Test – More Exercise

Cont'd

- Step 2: Collect data and identify rejection region(s)
  - Significance level: 0.05
  - Degrees of freedom: 13
  - Critical value(s): ± 2.1604
  - Decision rule: Reject H_0 if t < -2.1604 or t > +2.1604



Hypothesis Test – More Exercise

Cont'd

- Step 3: Compute test statistic
  - Test statistic = $t = \frac{\bar{X}-\mu_0}{S/\sqrt{n}} = \frac{-0.550-(-0.545)}{0.016/\sqrt{14}} = -1.17$
  - p-value = (0.20, 0.50)
- Step 4: Make statistical decision
  - Decision: At α = 0.05, do not reject H_0
  - Conclusion: There is insufficient evidence that the mean freezing point of the milk is not -0.545 °C

Hypothesis Test – More Exercise

Cont'd

- What would happened if the sample size is 144 rather than 14?
  - Assumed the sample mean and standard deviation remain unchanged
- Step 1: Define hypotheses

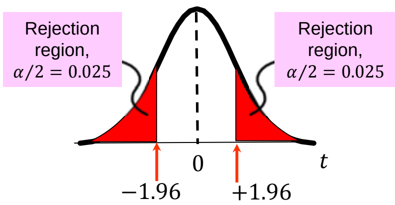
  $H_0: \mu = -0.545$

  $H_1: \mu \neq -0.545$

Hypothesis Test – More Exercise

Cont'd

- Step 2: Collect data and identify rejection region(s)
  - Population distribution: Unknown
  - Sample size: 144
  - Any assumption needed? No
    - What is the assumption? NA
    - Why? The sample size is large enough to apply Central Limit Theorem
  - σ: unknown
  - Distribution to use: t
  - Significance level: 0.05
  - Degrees of freedom: 143 $\approx \infty$
  - Critical value(s): ± 1.96
  - Decision rule: Reject H_0 if t < -1.96 or t > +1.96



Hypothesis Test – More Exercise

Cont'd

- Step 3: Compute test statistic
  - Test statistic = $t = \frac{\bar{X}-\mu_0}{S/\sqrt{n}} = \frac{-0.550-(-0.545)}{0.016/\sqrt{144}} = -3.75$
  - p-value < 0.01
- Step 4: Make statistical decision
  - Decision: At α = 0.05, reject H_0
  - Conclusion: There is sufficient evidence that the mean freezing point of the milk is not -0.545 °C

Potential Pitfalls and Ethical Issues

- What is the goal of the study? How can you translate the goal into a null hypothesis and an alternative hypothesis?
- Is the hypothesis test a two-tail test or one-tail test?
- Can you select a random sample from the underlying population of interest?
- At what level of significance should you conduct the hypothesis test?
- What conclusions and interpretations can you reach from the results of the hypothesis test?

Potential Pitfalls and Ethical Issues

Cont'd

- Some of the areas where ethical issues can arise include
  - The use of human subjects in experiments
  - The data collection method
  - The type of test (two-tail or one-tail test)
  - The choice of level of significance
  - The cleansing and discarding of data
  - The failure to report pertinent findings