Ex 4-7)

a) $x(t) = 2\cos(200\pi t) + \sin(500\pi t)$

$\underbrace{200\pi}_{\omega_1} = 2\pi f_1$, $f_1 = 100$.

$\cos(\boxed{\omega} t)$

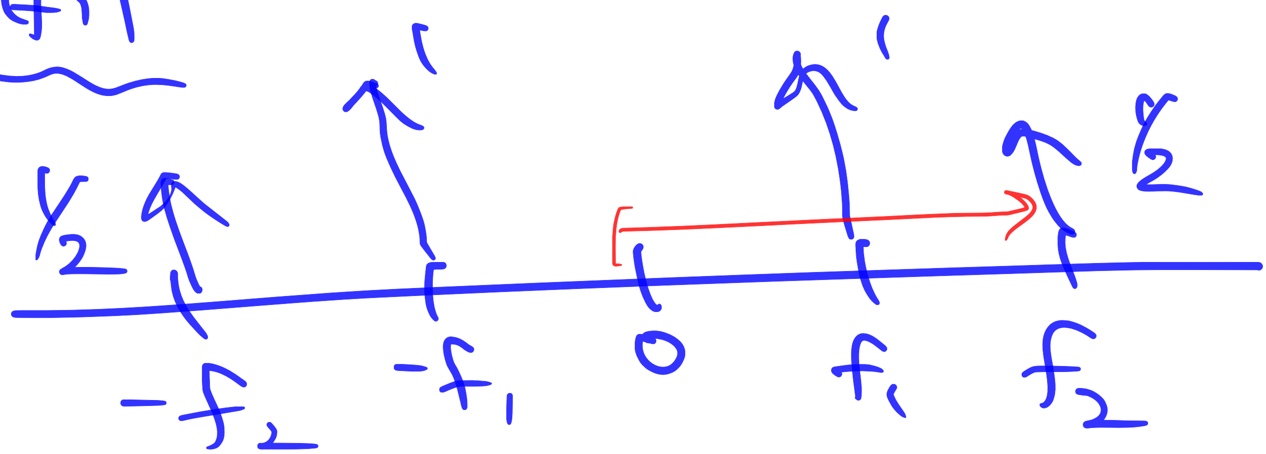
$\omega = 2\pi f$.

$\underbrace{500\pi}_{\omega_2} = 2\pi f_2$, $f_2 = 250$.

$X(f) = \left(\delta(f+f_1) + \delta(f-f_1)\right)$

$+ \frac{1}{2j}\left(\delta(f+f_2) - \delta(f-f_2)\right)$

$|X(f)|$



$f_M = f_2 = 250$

$f_S = 2f_M$

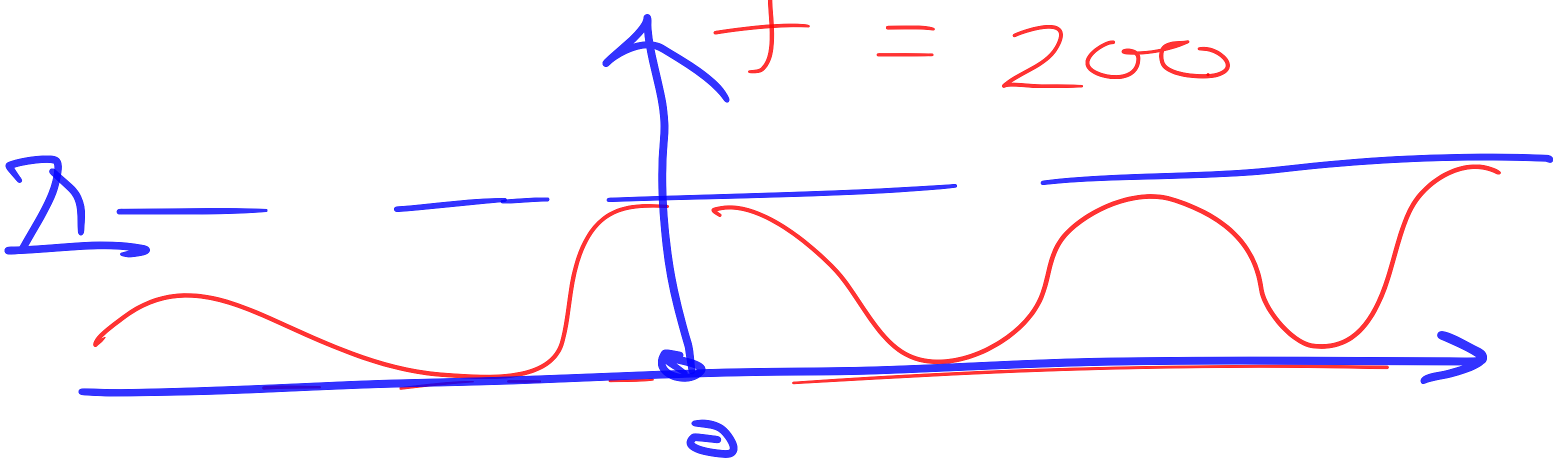
$= 500$.

b) $x(t) = 2\cos^2(200\pi t)$

$= 1 + \cos(\underbrace{400\pi}\, t)$

$400\pi = \omega = 2\pi f$

$f = 200$



$f_m = 200$

$f_s = 400$

Distortionless transmission.

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t) = K\,x(t-t_d)$$

$$x(t) \longleftrightarrow X(f) \qquad y(t) \longleftrightarrow Y(f) = K e^{-j2\pi f t_d} \times X(f)$$

$$H(f) = \frac{Y(f)}{X(f)} = K e^{-j2\pi f t_d}$$

$$h(t) = K\,\delta(t - t_d)$$

$$x(t) * K\,\delta(t - t_d)$$

$$= K\,x(t - t_d)$$

$\underline{H(f)}$

Differentiator.

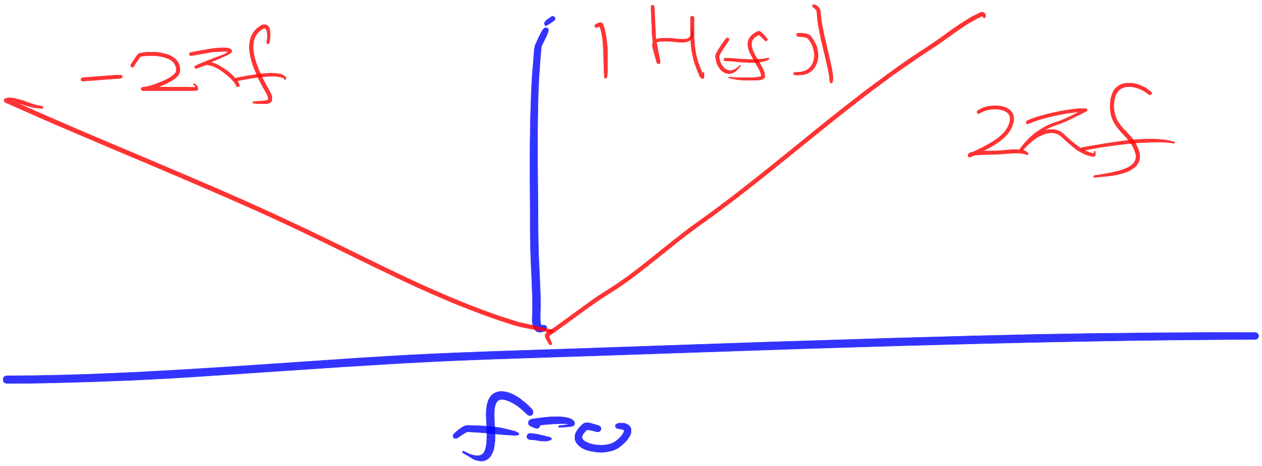
$$y(t) = \frac{dx}{dt}$$

$$\updownarrow$$

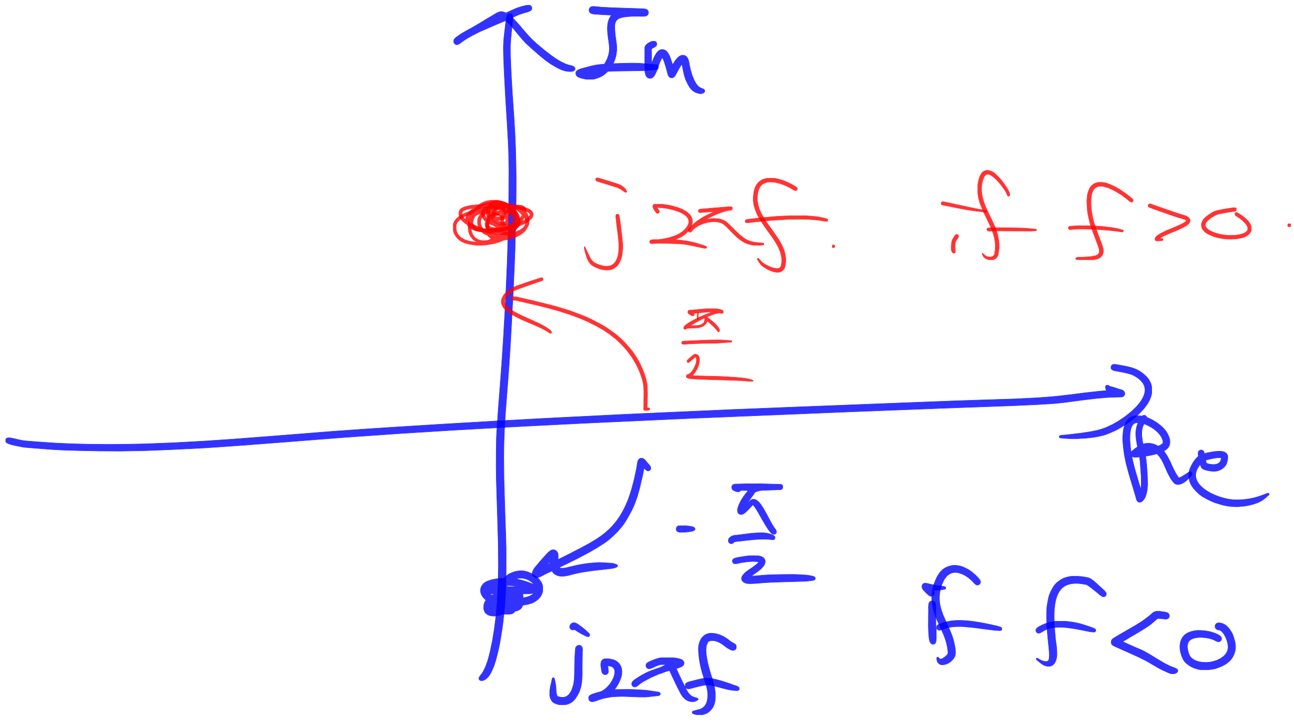
$$Y(f) = (j2\pi f)\, X(f)$$

$$\boxed{H(f) = \frac{Y(f)}{X(f)} = \boxed{j2\pi f.}}$$

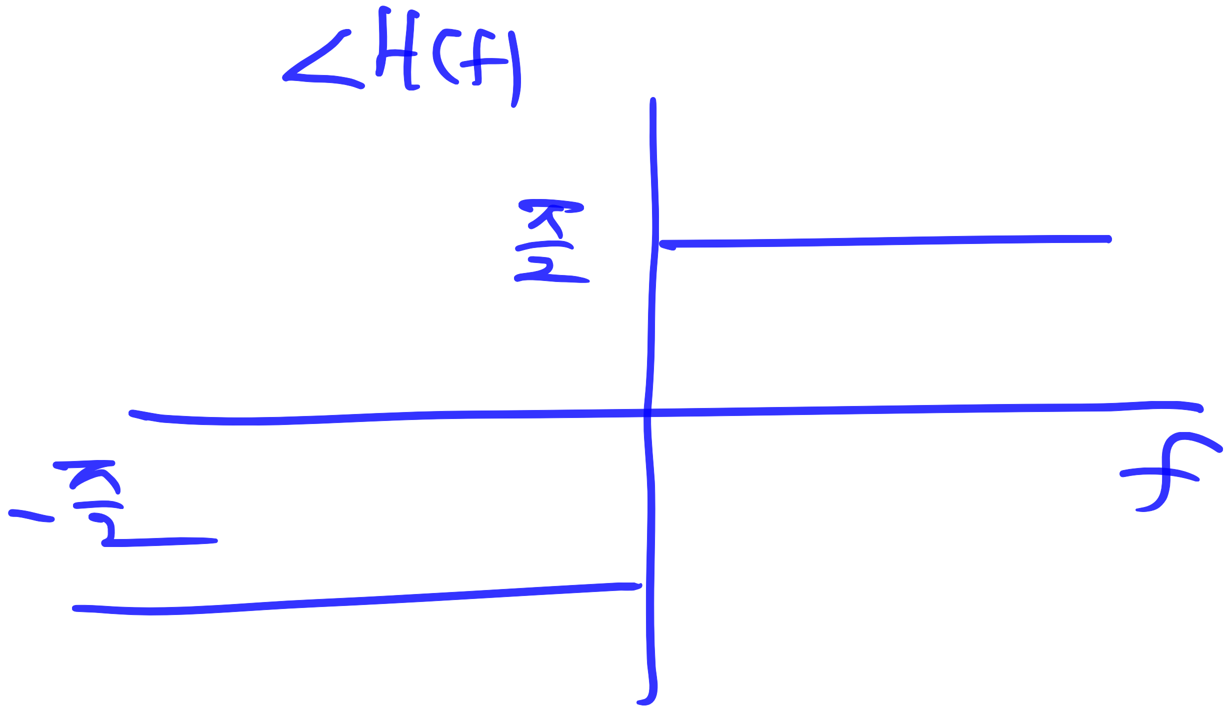
$$|H(f)| = 2\pi|f|.$$



$\angle H(f)$. if $f > 0$

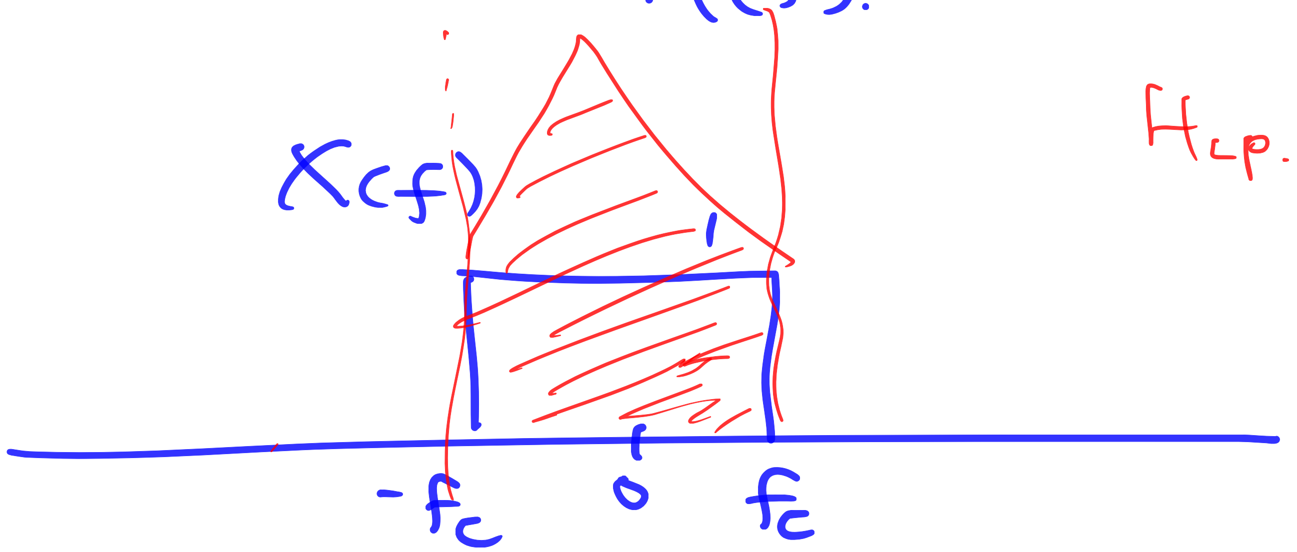


$$j2\pi f$$



$$y(t) = x(t) * h(t)$$

$$Y(f) = X(f)\, H(f).$$

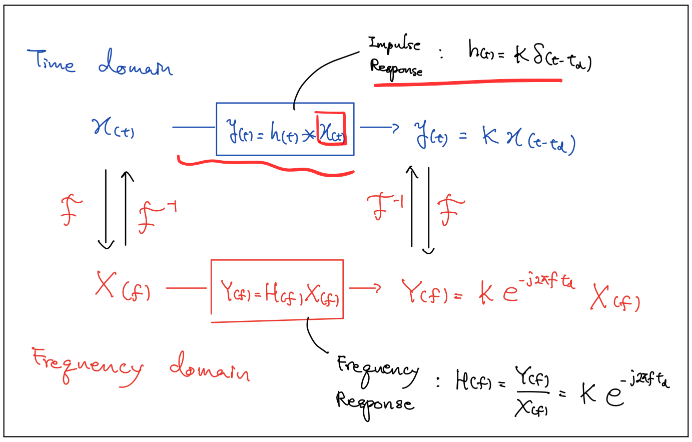


4.4 Filtering

⋇ **Distortionless Transmission:** In order to have a distortionless transmission, the output of an LTI system needs to have an identical shape to the input signal, although its amplitude may be different and it may be delayed in time.

$$\text{Input signal } x(t) \quad \Rightarrow \quad \text{Output Signal } y(t) = Kx(t - t_d),$$

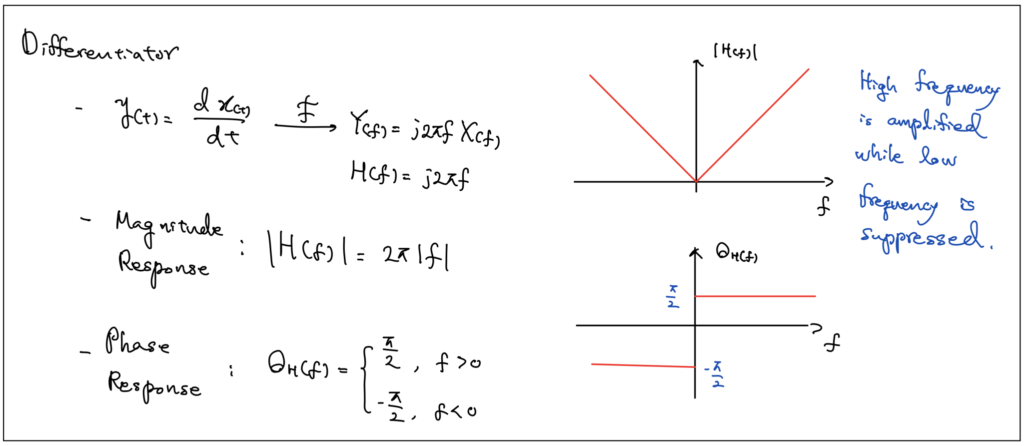
where t_d is the time delay, K is a gain constant. By using the FT, we can obtain the Frequency response as follows



Hence, the systems with distortionless transmission should have Frequency response $H(f) = Ke^{-j2\pi f t_d}$, constant magnitude response $|H(f)| = K$, and linear phase response $\theta_H(f) = -j2\pi f t_d$ over the entire frequency range.

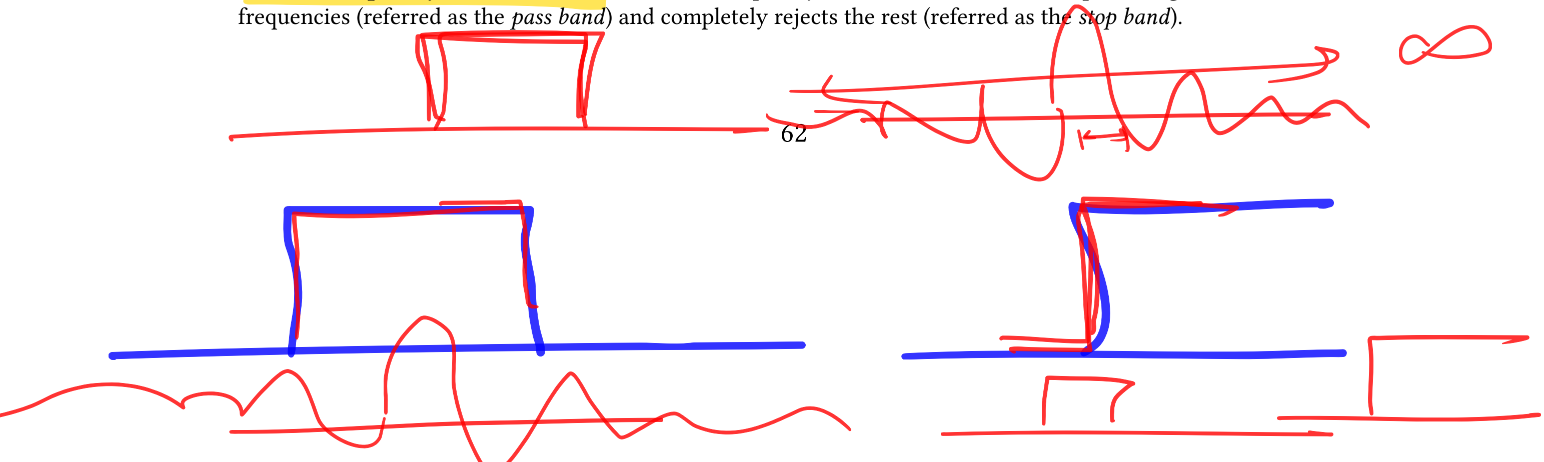
⋇ **Filtering** is a process that changes the amplitude (or phase) of some frequency components of an input signal.

1. Frequency-Shaping Filter
   - amplify some frequency components while suppress some other frequency components
   - e.g. differntiator, equalizer in a Hi-Fi system



2. Frequency-Selective Filter
   - select some bands of frequencies and reject others.
   - e.g. low-pass filter, high-pass filter, band-pass filter, band-stop filter

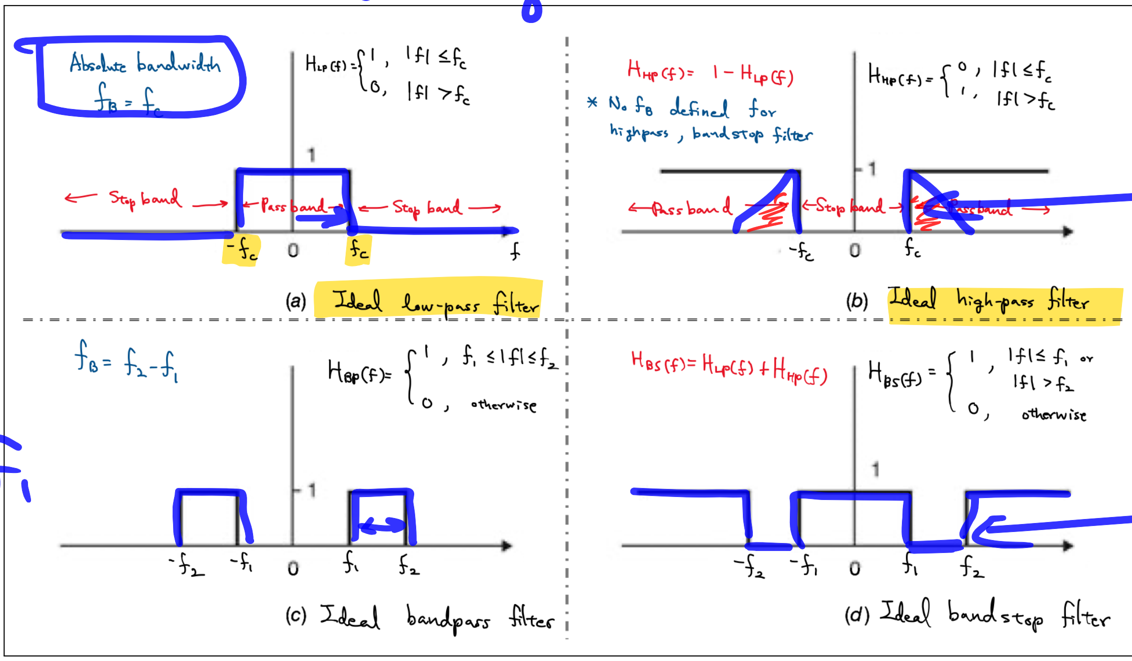
⋇ **Ideal Frequency Selective Filter:** An *ideal* frequency selective filter is one that passes signals at one set of frequencies (referred as the *pass band*) and completely rejects the rest (referred as the *stop band*).



f_c : Cut-off freq

$f_B = f_c$

$f_B = f_2 - f_1$



a) **Ideal Low-Pass Filter**

$$H(f) = \begin{cases} 1, & |f| \le f_c \\ 0, & |f| > f_c \end{cases}$$

where w_c is the *cutoff frequency*.

b) **Ideal High-Pass Filter**

$$H(f) = \begin{cases} 0, & |f| \le f_c \\ 1, & |f| > f_c \end{cases}$$

c) **Ideal Bandpass Filter**

$$H(f) = \begin{cases} 1, & f_1 < |f| < f_2 \\ 0, & \text{otherwise} \end{cases}$$

d) **Ideal Bandstop Filter**

$$H(f) = \begin{cases} 0, & f_1 < |f| < f_2 \\ 1, & \text{otherwise} \end{cases}$$

※ **Bandwidth:** There are many different definitions of filter bandwidth.

1. Absolute Bandwidth f_B
   - absolute BW (Bandwidth) of an ideal low-pass filter: $f_B = f_c$
   - absolute BW of an ideal bandpass filter: $f_B = f_2 - f_1$
   - absolute BS of an ideal high-pass or bandstop filter are not defined.
2. 3-dB Bandwidth (*Half-Power Bandwidth*) $f_{3\text{ dB}}$
   - $f_{3\text{ dB}}$ is the frequency where the peak magnitude spectrum $|H(0)|$ drops to $|H(0)|/\sqrt{2}$. Since the power is proportional to the square of $|H(f)|$, *i.e.*, $P \propto |H(f)|$, then the 3-dB bandwidth represent the frequency where the peak power reduces to the half.

$$10\log_{10}\left(\frac{P(f_{3\text{ dB}})}{P(f_{\max})}\right)\ dB = 10\log_{10}\left(\frac{|H(f_{3\text{ dB}})|^2}{|H(0)|^2}\right)\ dB \xRightarrow{|H(f_{3\text{ dB}})| = \frac{|H(0)|}{\sqrt{2}}} 10\log_{10}(1/2)\ dB$$
$$= -10\log_{10}(2)\ dB \simeq -3dB$$