Answers to Quiz 1

Qn 1

By Markov's inequality,

$$P\{X \geq a\} \leq \frac{E[X]}{a}$$

The upper bound is $80/a$, where a is the last 3 digit of the SID.

Qn 2

$P(E_1) = 1/6$

Case 1: If the difference is 0,

$P(E_2) = P\{(1,1), (2,2), \dots, (6,6)\} = 1/6$

$P(E_1 E_2) = P\{(6,6)\} = 1/36 = P(E_1)P(E_2)$

Independent

Case 2: If the difference is 1,

$P(E_2) = P\{(1,2), (2,3), \dots, (5,6), (6, 5), (5, 4), \dots, (2,1)\} = 10/36$

$P(E_1 E_2) = P\{(6,5)\} = 1/36 \neq P(E_1)P(E_2)$

Not independent

Qn 3

$$\frac{\binom{5}{2}\binom{1}{1}\binom{43}{2}}{\binom{49}{5}} = 4.736474208 \times 10^{-3}$$

Qn 4

$P(winning\ in\ one\ trial) = P\{(1,2), (2,3), \dots, (5,6), (6,5), (5,4), \dots, (2,1)\} = 10/36$

$P(does\ not\ win\ in\ n\ trials) = (1 - \frac{10}{36})^n < 0.5$

$n > 2.129992218 \quad \Longrightarrow \quad n = 3$

Qn 5

i)

Let y be the last two digits in the SID.

Let X denote the number of different numbers

$$X = X_1 + \cdots + X_6$$

where

$$X_i = \begin{cases} 1 & if\ at\ least\ one\ 'i'appear \\ 0 & otherwise \end{cases}$$

$$E[X_i] = (1)P(at\ least\ one\ 'i') + (0)P(otherwise) = P(at\ least\ one\ \ 'i') = 1 - \left(\frac{5}{6}\right)^y$$

$$E[X] = E[X_1] + \cdots + E[X_6] = 6\left(1 - \left(\frac{5}{6}\right)^y\right)$$

ii)

By coupon collector theorem,

$$1 + \frac{1}{(\frac{5}{6})} + \cdots + \frac{1}{\left(\frac{1}{6}\right)} = 1 + \frac{6}{5} + \frac{6}{4} + \frac{6}{3} + \frac{6}{2} + \frac{6}{1} = 14.7$$

Qn 6

i)

E event that the person carries the virus
F event that the person tests positive

$$P(E|F) = \frac{P(F|E)P(E)}{P(F|E)P(E) + P(F|E^c)P(E^c)}$$

$$P(E) = \frac{32}{1.78 \times 10^6}$$

$P(F|E) = 1 - 0.2 = 0.8$
$P(F|E^c) = 0.03125$

$$P(E|F) = \frac{(0.8)P(E)}{(0.8)P(E) + (0.03125)P(E^c)} = 4.60021276 \times 10^{-4}$$

ii)

$P(F^c|E) = 0.2$

$P(F) = (0.8)P(E) + (0.03125)P(E^c)$

$$P(E|F^c) = \frac{P(F^c|E)P(E)}{P(F^c)} = \frac{P(F^c|E)P(E)}{1 - P(F)} = 3.711542619 \times 10^{-6}$$

Qn 7

From the weak law of large numbers slide

$$P\left\{\left|\frac{X_1 + \cdots + X_n}{n} - \mu\right| > \epsilon\right\} \leq \frac{\sigma^2}{n\epsilon^2}$$

$$0.01 = \frac{\sigma^2}{n(0.1^2)}$$

If standard deviation $\sigma = 2,\ n = 40000$

If $\sigma = 4,\ n = 160000$