EE3210 Signals & Systems

Due on Noon, 12:00 PM, May 10, 2020

Homework #3

1. Total mark is 20 points (= 4 points per problem × 5 problems)
2. Solution will be posted on May 12 on Canvas website
3. Submission due by May 10, 2020, noon.
4. Online submission through Canvas
   - Scan or taking a photo of your anwser sheet, then upload to Canvas
   - After initial submission to Canvas, you can resubmit through email to yjchun@cityu.edu.hk
     - For revision purpose or if the submitted file is corrupted

Problem 1 (4pts)

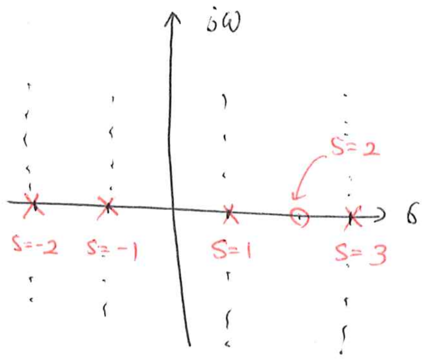
(Laplace Transform) Consider an LTI system with the following system function $H(s)$

$$H(s) = \frac{2(s-2)}{(s+2)\,(s+1)\,(s-1)\,(s-3)}$$

(2pts) a) Draw the pole-zero diagram and indicate all possible ROC that can be associaated with this diagram.

(2pts) b) For each ROC, specify whether the associated system is stable and/or causal.

a)



one zero at $s=2$

four pole at $s=-2,\ s=-1,\ s=1,\ s=3$

possible ROC
① $Re(s) > 3$
② $1 < Re(s) < 3$
③ $-1 < Re(s) < 1$
④ $-2 < Re(s) < -1$
⑤ $Re(s) < -2$

b) based on the Causality & Stability test,

Causal system is $\{Re(s) > 3\}$ and stable system is $\{-1 < Re(s) < 1\}$. Therefore

Case 1. $\{Re(s) > 3\}$: Causal, Unstable

Case 2. $\{+1 < Re(s) < 3\}$: Non-causal, Unstable

Case 3. $\{-1 < Re(s) < 1\}$: Non-causal, Stable

Case 4. $\{-2 < Re(s) < -1\}$: Non-causal, Unstable

Case 5. $\{Re(s) < -2\}$: Non-causal, Unstable

Problem 2 (4pts)

(Laplace Transform) Use the uni-lateral Laplace transform to solve the following problems.

(2pts) a) Find the system output $y(t)$ for a given input $x(t) = e^{-4t}u(t)$

$$\frac{d^2y(t)}{dt^2} + 5\frac{dy(t)}{dt} + 6y(t) = \frac{dx(t)}{dt} + x(t), \quad y(0^-) = 2, \quad y'(0^-) = 1$$

(2pts) b) Solve the following integral equation

$$y(t) = 2 + 2\int_0^t y(\tau)\, d\tau, \quad t \geq 0$$

a) Apply the Unilateral LT,

$$s^2 Y_I(s) - s\,y(0^-) - y'(0^-) + 5\left(s Y_I(s) - y(0^-)\right) + 6\, Y_I(s) \quad \text{—①}$$

$$= s X_I(s) - \cancel{x(0^-)}^{\,0} + X_I(s) \quad (x(0^-)=0 \text{ but } x(0^+)=1)$$

Since $x(t) = e^{-4t}u(t)$, $\rightarrow X(s) = \frac{1}{s+4}$

$$① \rightarrow Y_I(s) = \frac{2s^2+20s+45}{(s+2)(s+3)(s+4)} = \frac{13/2}{s+2} - \frac{3}{s+3} + \frac{-3/2}{s+4}$$

$$y(t) = \left(\frac{13}{2}e^{-2t} - 3e^{-3t} - \frac{3}{2}e^{-4t}\right)u(t)$$

b) $y(0^-) = 2$ from the integral equation. Now differentiate the eq,

$$\frac{dy}{dt} = 2y(t) \rightarrow \text{apply Uni-lateral LT}$$

$$\hookrightarrow \quad sY_I(s) - y(0^-) = 2Y_I(s) \Rightarrow Y_I(s) = \frac{2}{s-2}$$

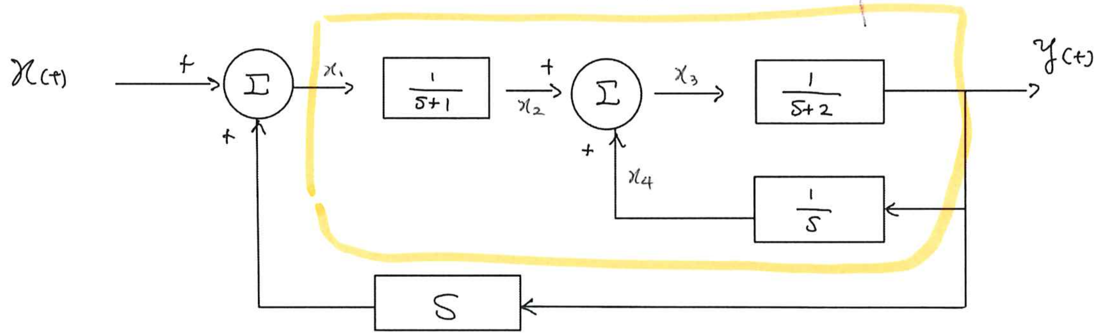
$$\Rightarrow y(t) = 2e^{at}u(t) \text{ and } a=2$$

$$= 2e^{2t}u(t)$$

Problem 3 (4pts)

$F(s)$

(Laplace Transform) Determine the overall system function $H(s)$ for the following system model



Let's divide the system into two parts. First, we will find the system function for the highlighted block, $F(s)$. Then, we will find the overall system function $H(s)$.

For $F(s)$, $\Rightarrow$ $X_3(s) = X_2(s) + X_4(s)$ where $X_2(s) = X_1(s) \cdot \frac{1}{s+1}$,

$X_4(s) = Y(s)\,\frac{1}{s}$, $Y(s) = X_3(s)\,\frac{1}{s+2}$.

We can multiply $\frac{1}{s+2}$ to the equation to obtain

$$Y(s) = \frac{1}{s+2}\left[\frac{X_1(s)}{s+1} + \frac{Y(s)}{s}\right] \Rightarrow F(s) = \frac{Y(s)}{X_1(s)} = \frac{s}{s^3+3s^2+s-1}$$

(2 pts)

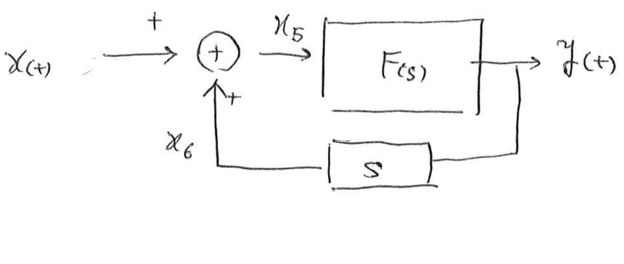
Next, for $H(s)$, use the simplified diagram plotted ~~bt~~ below

$\Rightarrow X_5(s) = X(s) + X_6(s)$ where $Y(s) = F(s)\,X_5(s)$ and $X_6(s) = Y(s)\cdot s$. Multiply $F(s)$ to the equation to get

$$Y(s) = F(s)\left(X(s) + Y(s)\cdot s\right)$$

$$\Rightarrow H(s) = \frac{Y(s)}{X(s)} = \frac{F(s)}{1 - F(s)\cdot s} = \frac{s}{s^3+2s^2+s-1}$$

(2pts)



Problem 4 (4pts)

(Z-Transform) Find the inverse Z-Transform of the given $X(z)$

(2pts) a)

$$X(z) = \frac{3}{Z-3}, \quad |Z| > 3$$

(2pts) b)

$$X(z) = \frac{1}{(1-4Z^{-1})^2}, \quad |Z| > 4$$

a) $\mathcal{Z}^{-1}(X(z)) = 3 \cdot 3^{n-1} u[n-1] = 3^n u[n-1]$

b) $X(z) = 4^{-1} z \left[\frac{4z^{-1}}{(1-4z^{-1})^2} \right]$

$\Rightarrow \mathcal{Z}^{-1}(X(z)) = 4^{-1} \left((n+1) 4^{n+1} u[n+1] \right)$

$= (n+1) 4^n u[n+1]$

Problem 5 (4pts) Give full score to every student who attempt Q5.

(Z-Transform) Determine whether the LTI system is causal ~~and/or stable~~ for the given system function $H(z)$
(just check causality)

Assume Stable LTI System

(2pts) a)

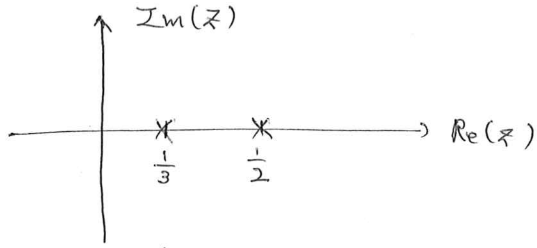
$$H(z) = \frac{1 - \frac{4}{3}Z^{-1} + \frac{1}{2}Z^{-2}}{Z^{-1}\left(1 - \frac{1}{2}Z^{-1}\right)\left(1 - \frac{1}{3}Z^{-1}\right)}$$

(2pts) b)

$$H(z) = \frac{Z - \frac{1}{2}}{Z^2 + \frac{1}{2}Z - \frac{3}{16}}$$

a)

$$H(z) = \frac{z\left(z^2 - \frac{4}{3}z + \frac{1}{2}\right)}{\left(z - \frac{1}{2}\right)\left(z - \frac{1}{3}\right)}$$

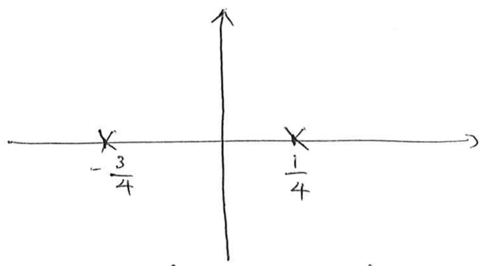


three pole at $z = \frac{1}{3},\ z = \frac{1}{2},\ z = \infty$

$$ROC = \left\{ \frac{1}{2} < |z| < \infty \right\}$$

$\Rightarrow$ Non-causal

b) $$H(z) = \frac{z - \frac{1}{2}}{\left(z + \frac{3}{4}\right)\left(z - \frac{1}{4}\right)}$$



two pole at $z = \frac{1}{4},\ z = -\frac{3}{4}$

$$ROC = \left\{ |z| > \frac{3}{4} \right\}$$

$\Rightarrow$ Causal