Answers to Quiz 2

Qn 1

$\binom{9}{2}(0.4)^2(0.6)^7(0.4) = 0.064497254$

Qn 2

$$\frac{\binom{2}{2}\binom{8}{n-2}}{\binom{10}{n}} > 0.5$$

$$\frac{8!\,n!\,(10-n)!}{(8-n+2)!\,(n-2)!\,(10)!} = \frac{n(n-1)}{(10)(9)} > 0.5$$

The minimum n is 8

Qn 3

a) $r = 0.470588$

b) They are positively correlated

c) The two random variables are statistically linear related

Qn 4

a) $n = 5$

$\bar{x} = 5.6$
$s = 1.516575$ (sample standard deviation is used.)

$\alpha = 0.1$ d.f. $= 4$

$t_{0.05,4} = 2.132$

The confidence interval is

$$\left(\bar{x} - t_{\frac{\alpha}{2},n-1}\frac{s}{\sqrt{n}}, \ \bar{x} + t_{\frac{\alpha}{2},n-1}\frac{s}{\sqrt{n}}\right) = (5.6 - 1.445992668, 5.6 + 1.445992668)$$

$$= (4.15400733, 7.045992668)$$

b) Each of the random variable is normal and independent.

c) Take more samples.

Qn 5

a) Sign test (with normal approximation)

Patient	before	after	Sign
1	120	120	
2	130	120	-
3	135	120	-
4	140	145	+
5	155	145	-
6	140	125	-
7	165	160	-
8	115	120	+
9	130	120	-
10	140	120	-

The data of the first patient is discounted.

$T = 7 \quad n = 9$

$$Z = \frac{T - n/2}{0.5\sqrt{n}} = \frac{7 - 4.5}{0.5\sqrt{9}} = 1.666666667 \approx 1.67$$

p-value $= 2\,(1 - 0.9525) = 0.095$

b)

Patient	before	after	abs diff	Sign	Rank
1	120	120			
2	130	120	10	-	5
3	135	120	15	-	7.5
4	140	145	5	+	2
5	155	145	10	-	5
6	140	125	15	-	7.5
7	165	160	5	-	2
8	115	120	5	+	2
9	130	120	10	-	5
10	140	120	20	-	9

$T = 5 + 7.5_5 + 7.5 + 2 + 5 + 9 = 41 \qquad\qquad n = 9$

$E(T) = \frac{n(n+1)}{4} = 22.5$

$$Var(T) = \frac{n(n+1)(2n+1)}{24} = 71.25$$

$$Z = \frac{T - E(T)}{\sqrt{V(T)}} = 2.191690848 \approx 2.19$$

p-value $= 2(1 - 0.9857) = 0.0286$

Qn 6

Paired test should be used.

Patient	before	after	difference
1	120	120	0
2	130	120	-10
3	135	120	-15
4	140	145	5
5	155	145	-10
6	140	125	-15
7	165	160	-5
8	115	120	5
9	130	120	-10
10	140	120	-20

$\bar{x} = -7.5$ $n = 10$
$s = 8.579692$

$$T = \frac{\bar{X}}{s/\sqrt{n}} = -2.76432795 \approx -2.76$$

$t_{0.025,9} = 2.262$

$|T| > t_{0.025,9}$

The hypothesis will be rejected.

Assumptions:

The two distributions are both normal with unknown variance (not necessarily equal to each other), and they are independent of each other. The two variables are generated in pairs.

Qn 7

p-value $= P\{Bin(9,0.5) \geq 7\} = \left[\binom{9}{7} + \binom{9}{8} + \binom{9}{9}\right](0.5)^9 = 0.08983475 < 0.1$

The hypothesis is rejected.