MA1200 Practice Exercise 1 Answers

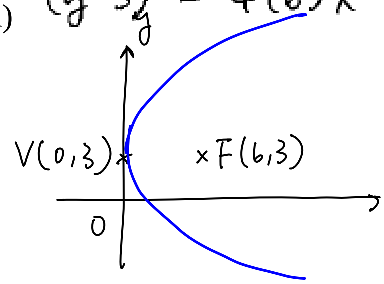
1. (a) $(x+6)^2-39$ (b) $(y-9)^2-76$ (c) $(x+3)^2-9$

   (d) $(y+\frac{9}{2})^2-\frac{77}{4}$ (e) $(x-\frac{7}{2})^2-\frac{33}{4}$ (f) $(y-\frac{21}{2})^2-\frac{441}{4}$

2. (a) $(2x+6)^2-45$ (b) $(3y-3)^2-4$ (c) $(5x+8)^2-39$

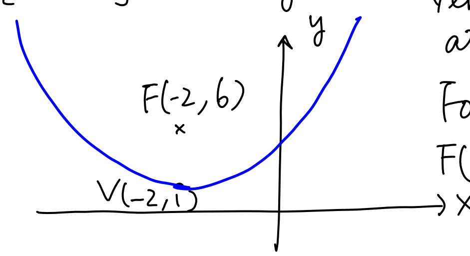
   (d) $(\sqrt{3}y+1)^2+14$ (e) $(\sqrt{2}x-3)^2-4$ (f) $(\sqrt{7}y+6)^2-15$

3. (a) $(y-3)^2=4(6)x$



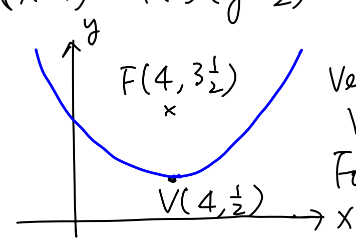
Vertex at V(0,3), Focus at F(6,3)

(b) $[x-(-2)]^2=4(5)(y-1)$



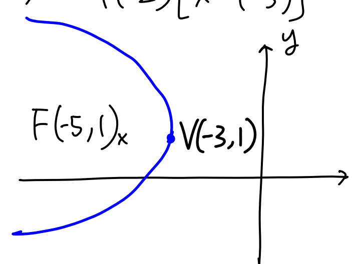
Vertex at V(-2,1) Focus at F(-2,6)

(c) $(x-4)^2=4(3)(y-\frac{1}{2})$



Vertex at V(4,½), Focus at F(4,3½)

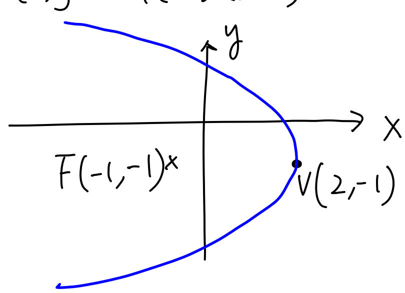
(d) $(y-1)^2=4(-2)[x-(-3)]$



Vertex at V(-3,1), Focus at F(-5,1)

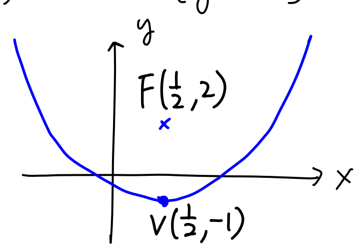
(e) $[y-(-1)]^2=4(-3)(x-2)$

Vertex at V(2,-1), Focus at F(-1,-1)



(f) $(x-\frac{1}{2})^2=4(3)[y-(-1)]$

Vertex at V(½,-1), Focus at F(½,2)



e.g.: To locate $(y-1)^2=4(2)(x-3)$, you may first think of a sketch of $y^2=4(2)x$, then $(y-1)^2=4(2)(x-3)$ is just to translate $y^2=4(2)x$ 3 units to the right and 1 unit upward.

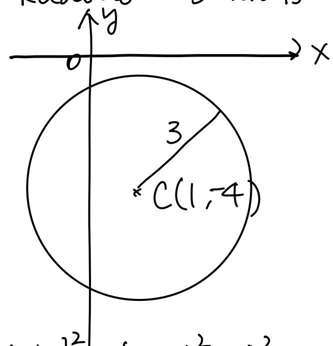
4. (a) $(-1-\sqrt{6},0)$ and $(-1+\sqrt{6},0)$ (b) $(-\frac{3}{2},0)$

   (c) The parabola does not cut the x-axis.

5. (a) $(x-1)^2+[y-(-4)]^2=3^2$

$\therefore$ Centre at $C(1,-4)$

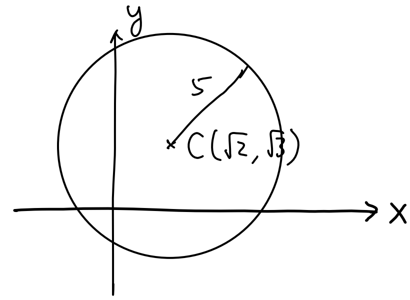
Radius $=3$ units



(b) $(x-\sqrt{2})^2+(y-\sqrt{3})^2=5^2$

$\therefore$ Centre at $C(\sqrt{2},\sqrt{3})$,

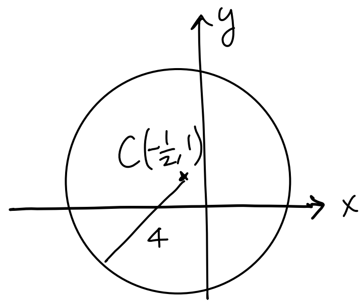
Radius $=5$ units



(c) $[x-(-\frac{1}{2})]^2+(y-1)^2=4^2$

$\therefore$ Centre at $C(-\frac{1}{2},1)$

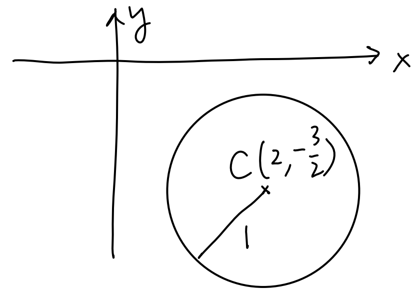
radius $=4$ units



(d) $(x-2)^2+[y-(-\frac{3}{2})]^2=1^2$

$\therefore$ Centre at $C(2,-\frac{3}{2})$,

radius $=1$ unit

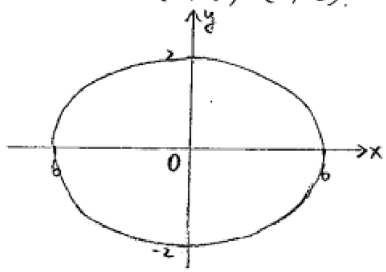


6. (a) $(-\frac{3}{2}, \frac{3\sqrt{3}}{2})$ (b) $(\frac{7\sqrt{3}}{2}, -\frac{7}{2})$ (c) $(-3\sqrt{2}, -3\sqrt{2})$ (d) $(0,2)$

7. (a) $(4\sqrt{3}, 30^\circ)$ (b) $(2, 120^\circ)$ (c) $(8, -150^\circ)$ (d) $(4, -30^\circ)$

8. (a) $\frac{x^2}{6^2} + \frac{y^2}{2^2} = 1$

Vertices at $(6,0)$, $(-6,0)$, $(0,2)$, $(0,-2)$.

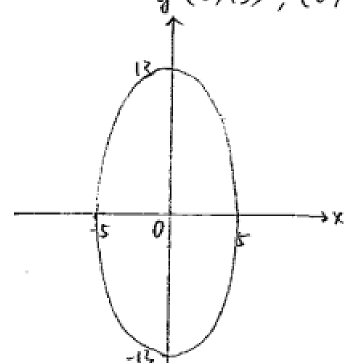


Two foci are at $(4\sqrt{2},0)$ and $(-4\sqrt{2},0)$

Centre is at (0, 0).

(b) $\frac{x^2}{5^2} + \frac{y^2}{13^2} = 1$

Vertices at $(5,0)$, $(-5,0)$, $(0,13)$, $(0,-13)$.



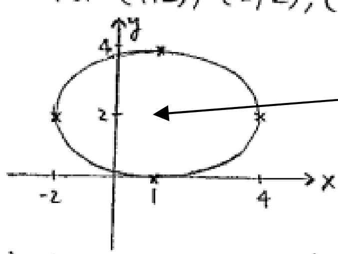
Two Foci are at $(0,12)$ and $(0,-12)$

Centre is at (0, 0).

(c) $\frac{(x-1)^2}{3^2} + \frac{(y-2)^2}{2^2} = 1$

Vertices at $(1+3, 2+0)$, $(1-3, 2+0)$, $(1+0, 2+2)$, $(1+0, 2-2)$,

i.e. $(4,2)$, $(-2,2)$, $(1,4)$, $(1,0)$



Centre is at (1, 2)

$\therefore$ The two Foci are at $(\sqrt{5}+1, 2)$ and $(-\sqrt{5}+1, 2)$

8 (d).

$$25x^2 + y^2 - 150x + 2y + 201 = 0$$

$$25x^2 - 150x + y^2 + 2y = -201$$

$$25(x^2 - 6x) + y^2 + 2y = -201$$

$$25(x^2 - 6x + 9) + y^2 + 2y + 1 = -201 + 225 + 1$$

$$25(x-3)^2 + (y+1)^2 = 25$$

$$\frac{(x-3)^2}{1^2} + \frac{[y-(-1)]^2}{5^2} = 1$$

$\therefore$ $a = 5$, $b = 1$

$c = \sqrt{5^2 - 1^2} = 2\sqrt{6}$

Note: For the equation $\frac{x^2}{1^2} + \frac{y^2}{5^2} = 1$, It will represent an ellipse with Vertices at $(1,0)$, $(-1,0)$, $(0,5)$, $(0,-5)$ Foci at $(0, 2\sqrt{6})$, $(0, -2\sqrt{6})$.

Note: The ellipse we have:

$$\frac{(x-3)^2}{1^2} + \frac{[y-(-1)]^2}{5^2} = 1 \quad \text{---}(*)$$

is just to translate $\frac{x^2}{1^2} + \frac{y^2}{5^2} = 1$ for 3 units to the right and (-1) unit upward (i.e. 1 unit downward)

$\therefore$ Vertices for $(*)$ are:

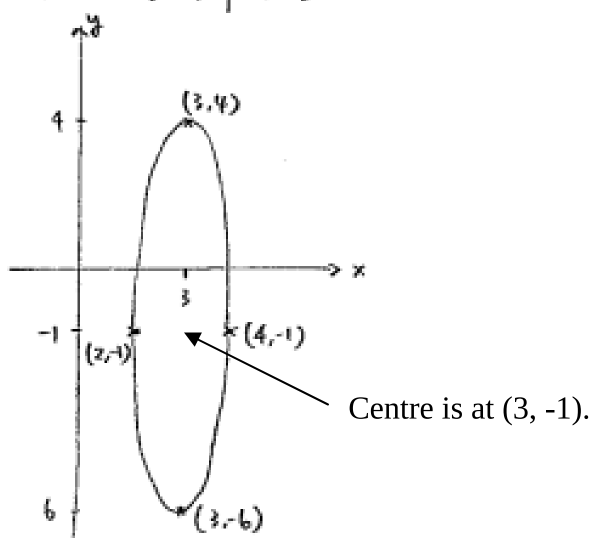
$(1+3, 0-1)$, $(-1+3, 0-1)$,
$(0+3, 5-1)$, $(0+3, -5-1)$,

i.e. $(4,-1)$, $(2,-1)$,
$(3, 4)$, $(3, -6)$.

Foci for $(*)$ are: $(0+3, 2\sqrt{6}-1)$
$(0+3, -2\sqrt{6}-1)$

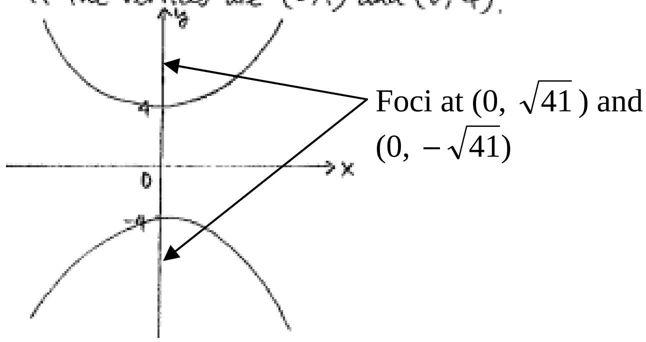
i.e. $(3, 2\sqrt{6}-1)$, $(3, -2\sqrt{6}-1)$.

$\therefore$ The sketch is as follows:



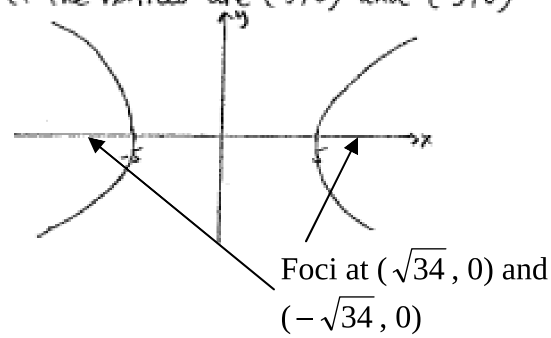
9. (a) $\dfrac{y^2}{4^2} - \dfrac{x^2}{5^2} = 1$

∴ The vertices are (0,4) and (0,-4).



(b) $\dfrac{x^2}{5^2} - \dfrac{y^2}{3^2} = 1$

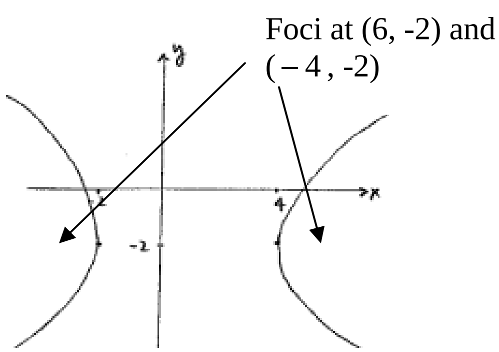
∴ The vertices are (5,0) and (-5,0)



(c) $\dfrac{(x-1)^2}{3^2} - \dfrac{(y-(-2))^2}{4^2} = 1$

∴ The vertices of (*) are at (3+1, 0-2) (-3+1, 0-2),

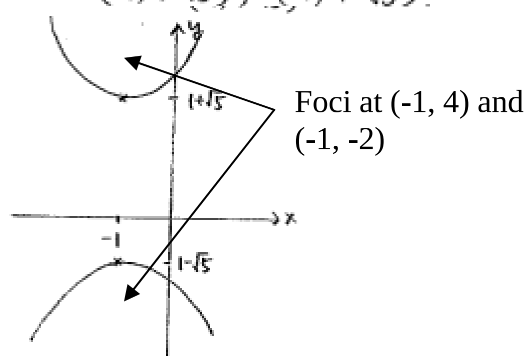
i.e. (4,-2) and (-2,-2).



(d) $\dfrac{(y-1)^2}{\sqrt{5}^2} - \dfrac{(x-(-1))^2}{2^2} = 1$

∴ Vertices of (*) are at $(0-1, \sqrt{5}+1)$, $(0-1, -\sqrt{5}+1)$

i.e. $(-1, 1+\sqrt{5})$, $(-1, 1-\sqrt{5})$.



10. (a) $\left(x-\dfrac{1}{2}\right)^2 + \left(y-\left(-\dfrac{3}{2}\right)\right)^2 = 2^2$ It represents a circle.

(b) $(y-3)^2 = 4(-1)\left(x-\left(-\dfrac{1}{2}\right)\right)$ It represents a parabola.

(c) $\dfrac{x^2}{1^2} + \dfrac{(y-1)^2}{2^2} = 1$ It represents a hyperbola.

(d) $\dfrac{(x-1)^2}{2^2} + \dfrac{(y-(-3))^2}{1^2} = 1$ It represents an ellipse.

11. (a) $\frac{y'^2}{1} - \frac{x'^2}{3} = 1$ (b) $\frac{x'^2}{4} - \frac{y'^2}{9} = 1$

12. (a) Hyperbola (b) Circle
(c) Hyperbola (d) An ellipse or a circle

13. (a) No intersection (b) $(0,-2)$ and $(1,0)$ (c) $(0,1)$ and $(0,-1)$