EE 4146 Data Engineering and Learning Systems

Lecture 10: Decision Tree

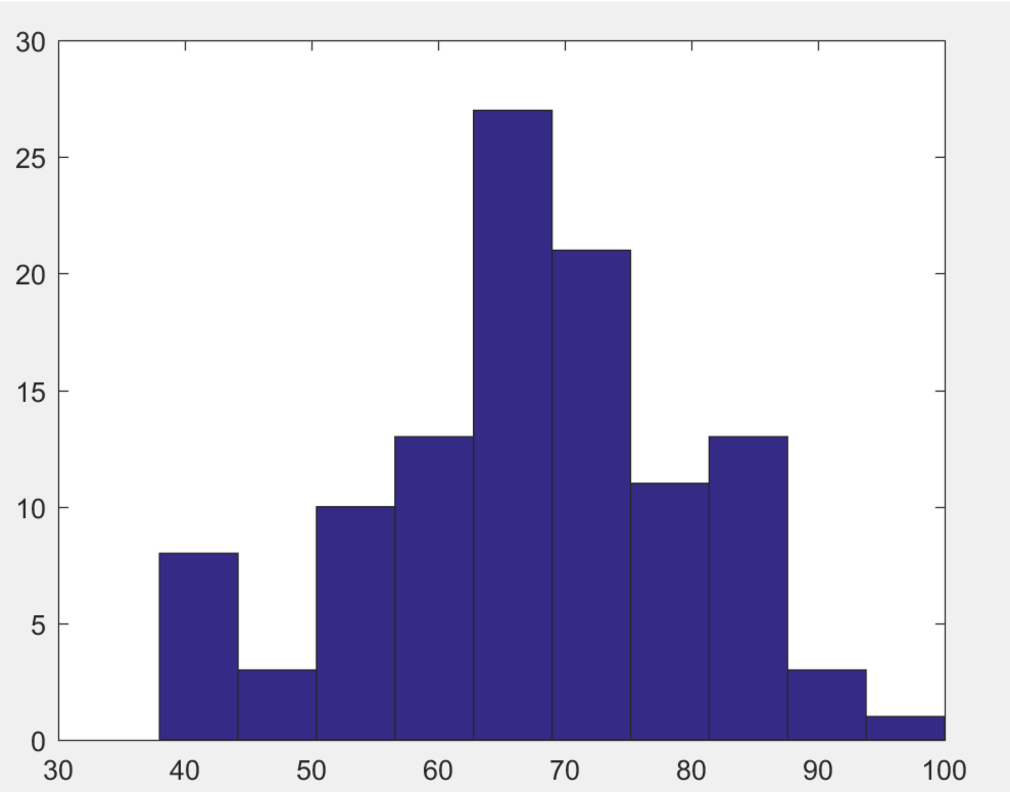
Semester A, 2021-2022

Schedules

Week	Date	Topics
1	Sep. 1	Introduction
2	Sep. 8	Data exploration
3	Sep. 15	Feature reduction and selection (HW1 out)
4	Sep. 22	Mid-Autumn Festival
5	Sep. 29	Clustering I: Kmeans based models (HW1 due in this weekend)
6	Oct. 6	Clustering II: Hierarchical/density based/fuzzing clustering
7	Oct. 13	Midterm (no tutorials this week)
8	Oct. 20	Adverse Weather
9	Oct. 27	Linear classifiers
10	Nov. 3	Classification based on decision tree (Tutorial on project) (HW2 out in Friday)
11	Nov. 10	Bayes based classifier (Tutorial on codes) (HW2 due in this weekend)
12	Nov. 17	KNN and classifier ensemble
13	Nov. 24	Deep learning based models (Quiz)
14	Make up video	Summary

Information about Midterm

- Mean=67.5273
  - You can find TA at G2325, AC1 in the open hour to get the examination paper Open hour: Monday 1:00-7:00 PM



Project

- Group information has already been uploaded to 202109EE4146->Files->Project Related->EE4146 project group information
- Please contact me, if there are some mistakes in the grouping information
- TA will illustrate the project information, related to codes, topics, submission, etc. (6:00 PM-6:50 PM)

Outline

- Basic introduction of decision tree
- Hunt’s Algorithm
- Determine how to split the records
  - How to specify the attribute test condition?
  - How to determine the best split?
- Determine when to stop splitting

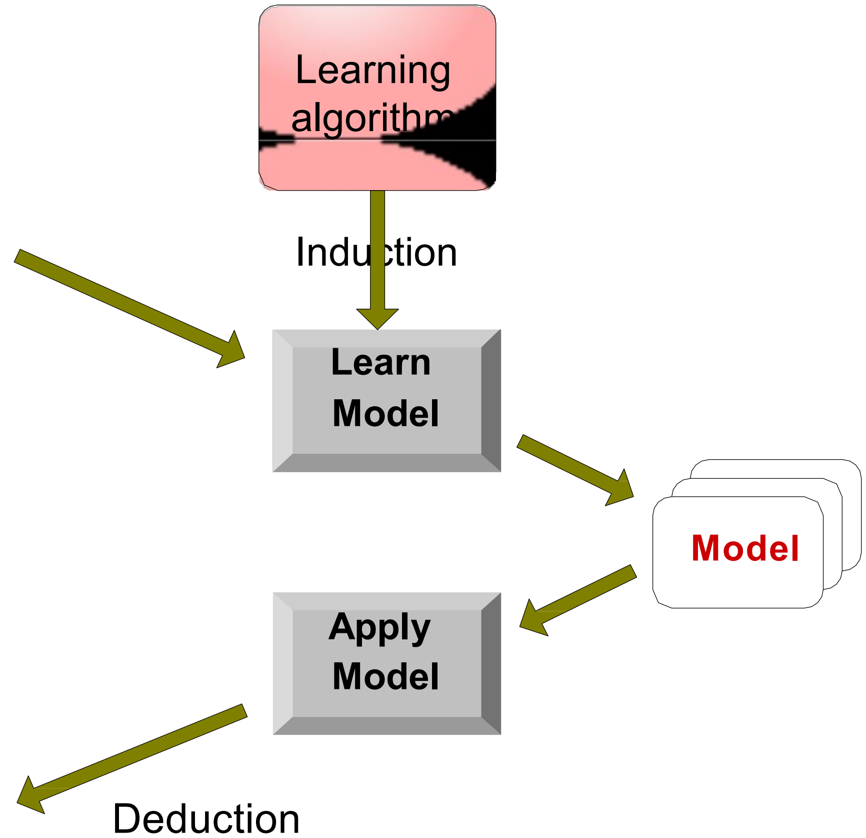
Review: Classification definition

- Given a collection of records (*training set*)
  - Each record contains a set of attributes, one of the attributes is the class.
- Find a model for class attribute as a function of the values of other attributes.
- Goal: previously unseen records should be assigned a class as accurately as possible.
  - A test set is used to determine the accuracy of the model. Usually, the given data set is divided into training and test sets, with training set used to build the model and test set used to validate it.

Illustrating Classification Task

Tid	Attrib1	Attrib2	Attrib3	Class
1	Yes	Large	125K	No
2	No	Medium	100K	No
3	No	Small	70K	No
4	Yes	Medium	120K	No
5	No	Large	95K	Yes
6	No	Medium	60K	No
7	Yes	Large	220K	No
8	No	Small	85K	Yes
9	No	Medium	75K	No
10	No	Small	90K	Yes

Training Set



Tid	Attrib1	Attrib2	Attrib3	Class
11	No	Small	55K	?
12	Yes	Medium	80K	?
13	Yes	Large	110K	?
14	No	Small	95K	?
15	No	Large	67K	?

Test Set

Examples of Classification Task

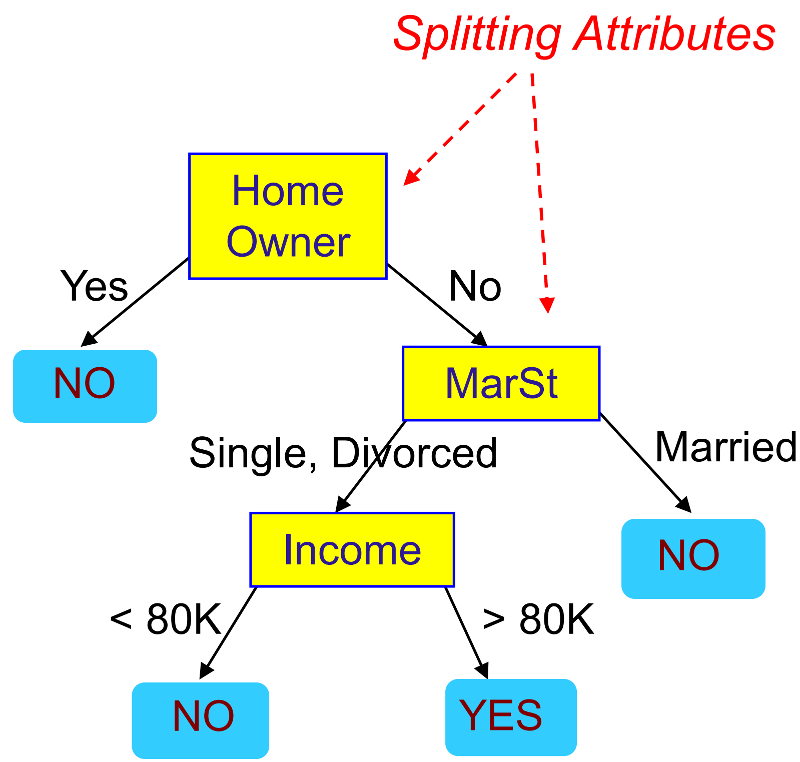
- Predicting tumor cells as benign or malignant
- Classifying credit card transactions as legitimate or fraudulent
- Classifying emails as spams or normal emails
- Categorizing news stories as finance, weather, entertainment, sports, etc

Example of a Decision Tree

categorical categorical continuous class

ID	Home Owner	Marital Status	Annual Income	Defaulted Borrower
1	Yes	Single	125K	**No**
2	No	Married	100K	**No**
3	No	Single	70K	**No**
4	Yes	Married	120K	**No**
5	No	Divorced	95K	**Yes**
6	No	Married	60K	**No**
7	Yes	Divorced	220K	**No**
8	No	Single	85K	**Yes**
9	No	Married	75K	**No**
10	No	Single	90K	**Yes**

Training Data



Model: Decision Tree

Another Example of Decision Tree

categorical categorical continuous class

ID	Home Owner	Marital Status	Annual Income	Defaulted Borrower
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
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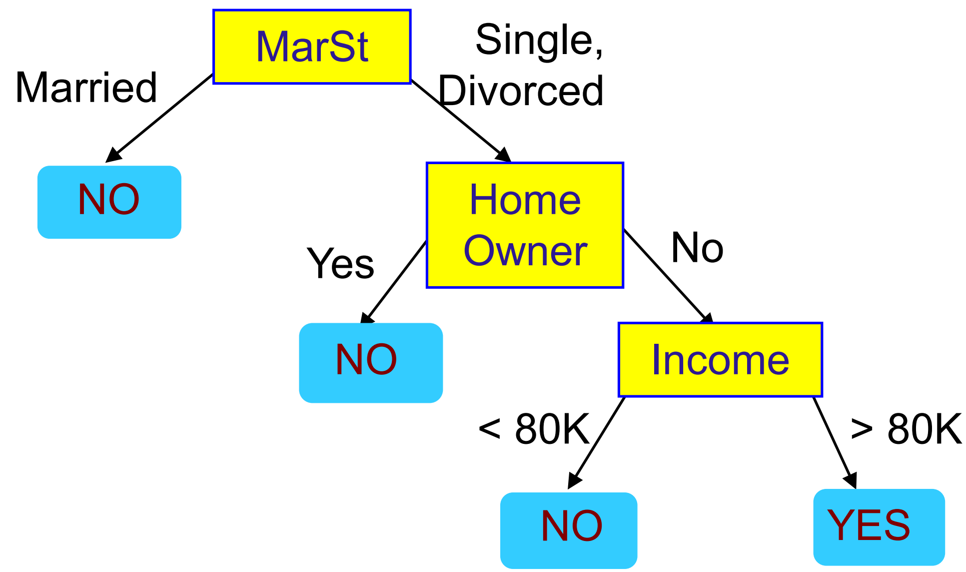
10	No	Single	90K	Yes



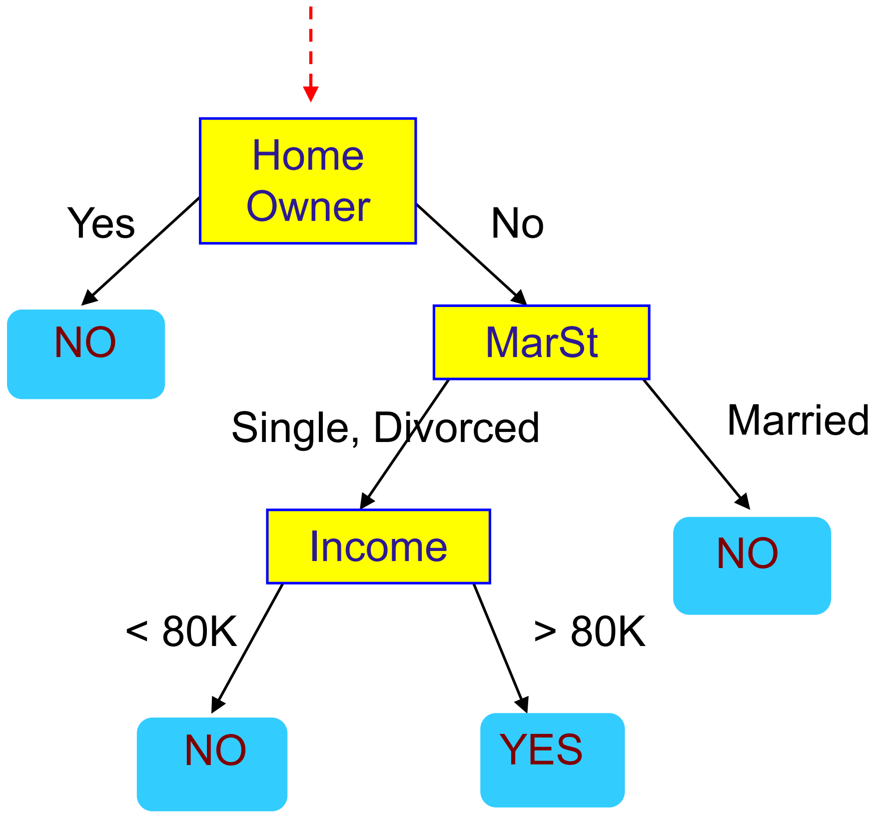
There could be more than one tree that fits the same data!

Apply Model to Test Data

Test Data

Home Owner	Marital Status	Annual Income	Defaulted Borrower
No	Married	80K	?

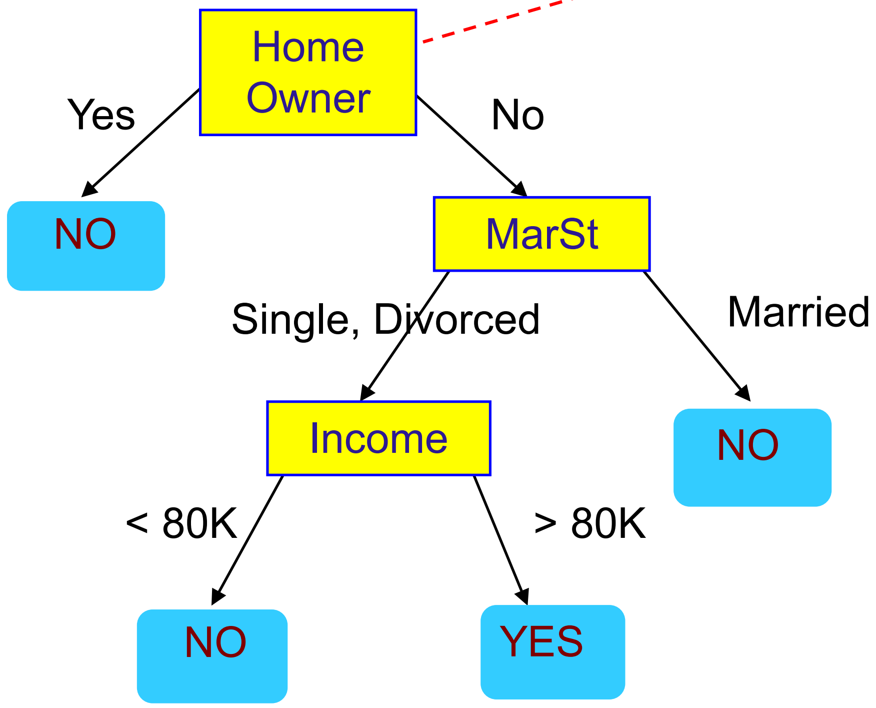
Start from the root of tree.



Apply Model to Test Data

Test Data

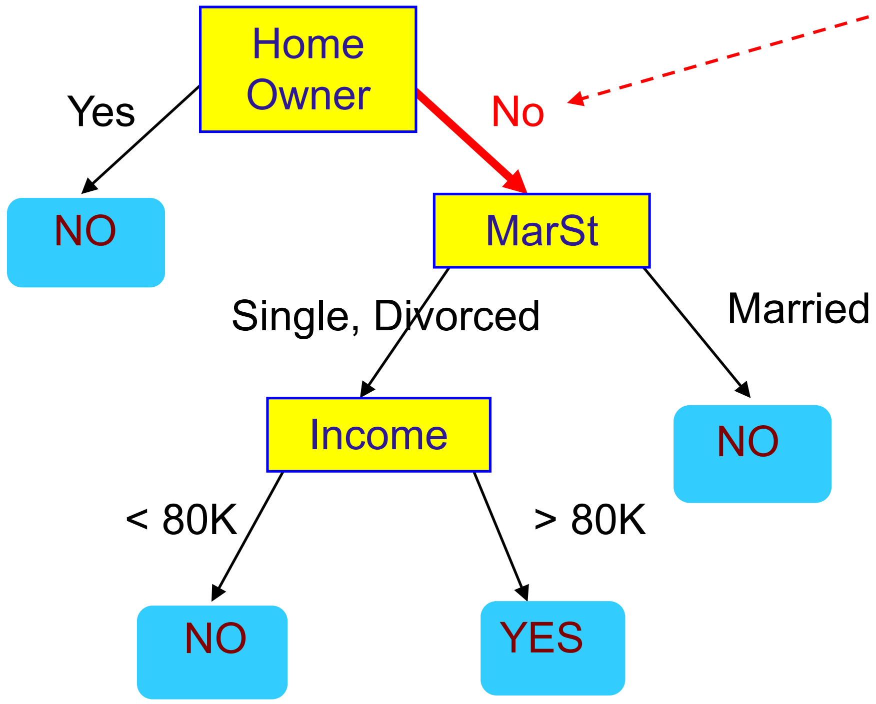
Home Owner	Marital Status	Annual Income	Defaulted Borrower
No	Married	80K	?



Apply Model to Test Data

Test Data

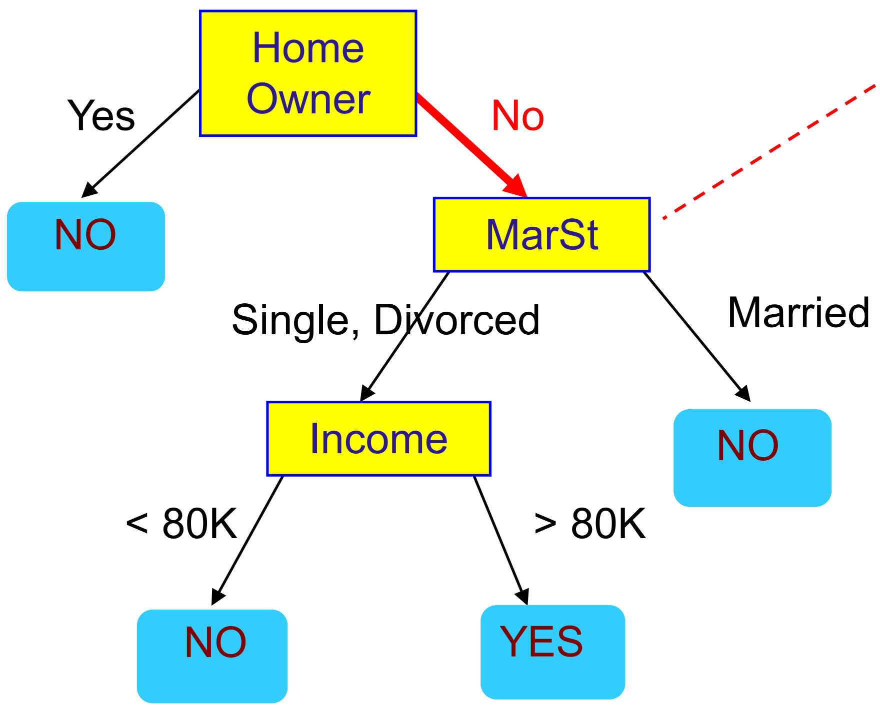
Home Owner	Marital Status	Annual Income	Defaulted Borrower
No	Married	80K	?



Apply Model to Test Data

Test Data

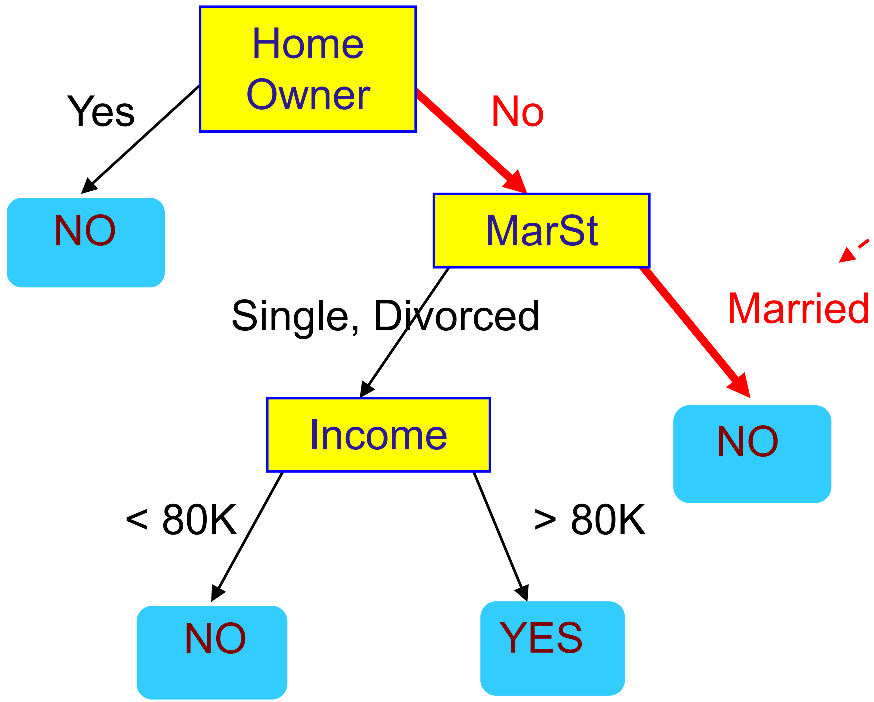
Home Owner	Marital Status	Annual Income	Defaulted Borrower
No	Married	80K	?



Apply Model to Test Data

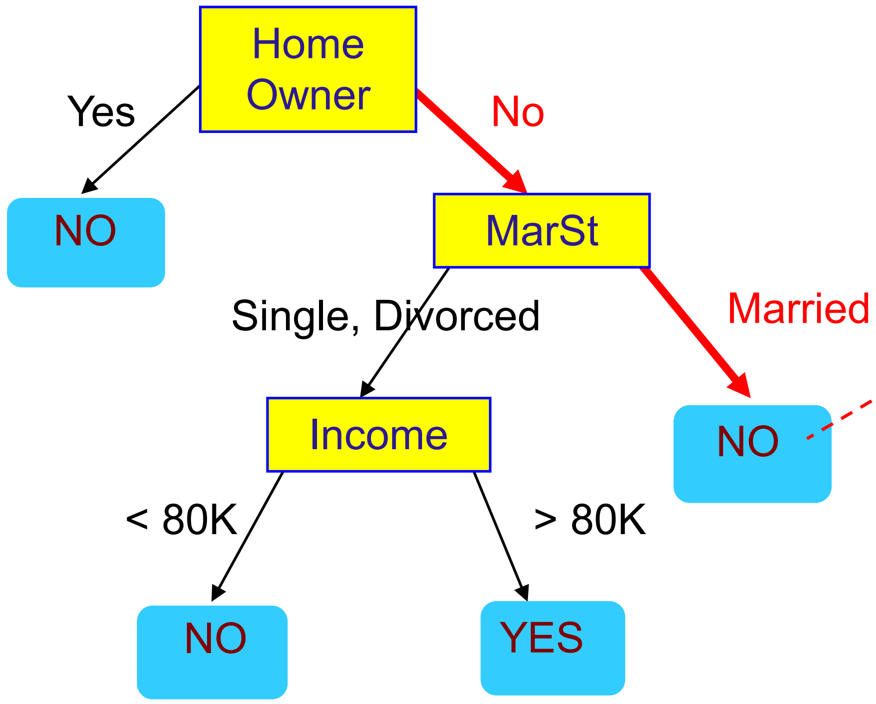
Test Data

Home Owner	Marital Status	Annual Income	Defaulted Borrower
No	Married	80K	?



How many tests do you need to reach a decision?

Apply Model to Test Data



Test Data

Home Owner	Marital Status	Annual Income	Defaulted Borrower
No	Married	80K	?

Decision Tree Classification Task

Tid	Attrib1	Attrib2	Attrib3	Class
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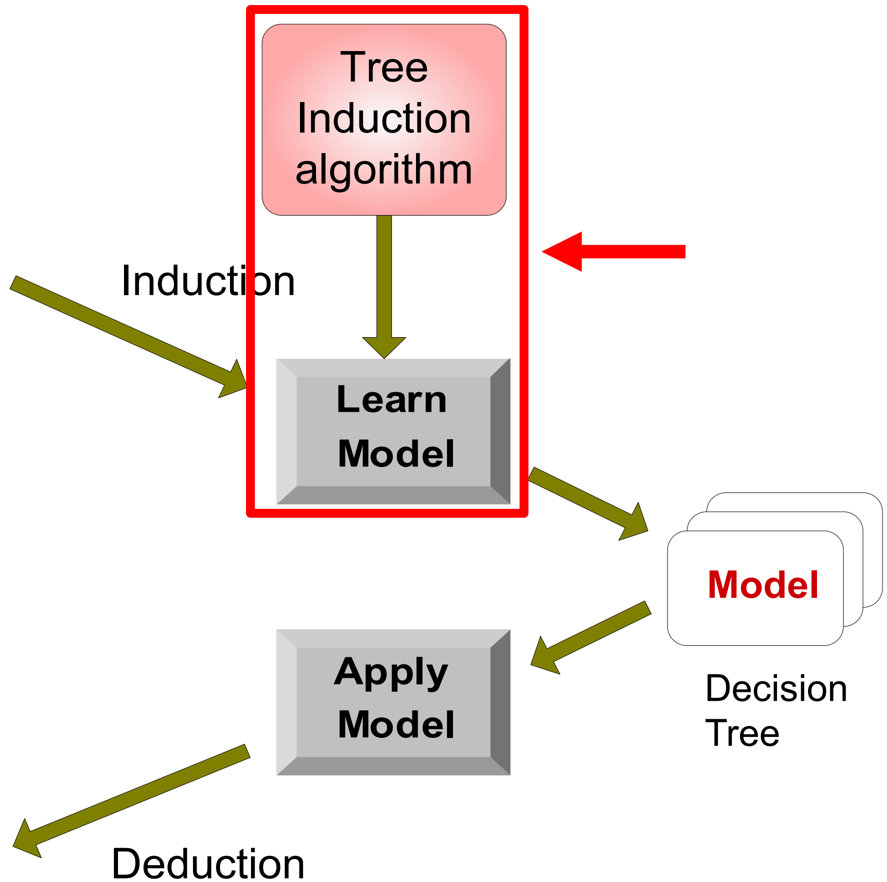
Training Set



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15	No	Large	67K	?

Test Set

Decision Tree Induction

- Many Algorithms:
  - Hunt's Algorithm (one of the earliest)
  - CART
  - ID3, C4.5
  - SLIQ,SPRINT

Outline

- Basic introduction of decision tree
- Hunt's Algorithm
- Determine how to split the records
  - How to specify the attribute test condition?
  - How to determine the best split?
- Determine when to stop splitting

General Structure of Hunt’s Algorithm

- Let D_t be the set of training records that reach a node t
- General Procedure:
  - If D_t contains records that belong the same class y_t, then t is a leaf node labeled as y_t
  - If D_t contains records that belong to more than one class, use an attribute to split the data into smaller subsets. Recursively apply the procedure to each subset.

ID	Home Owner	Marital Status	Annual Income	Defaulted Borrower
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
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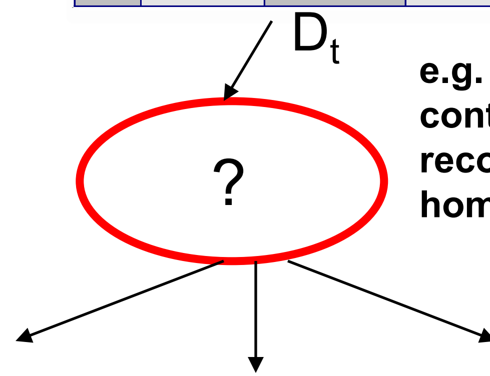

e.g. D_t contains all records “not home owner”

Hunt’s Algorithm

Defaulted = No

(7,3)

(a)

ID	Home Owner	Marital Status	Annual Income	Defaulted Borrower
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
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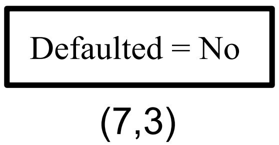
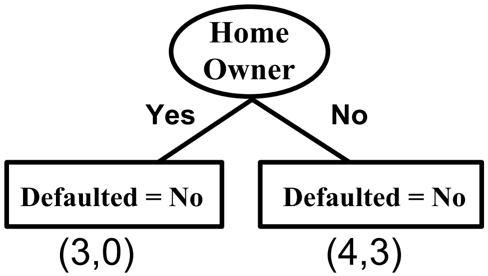
Hunt’s Algorithm



(a)



(b)

ID	Home Owner	Marital Status	Annual Income	Defaulted Borrower
1	Yes	Single	125K	No
2	No	Married	100K	No
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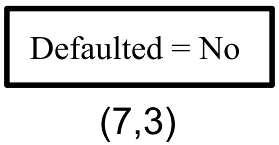
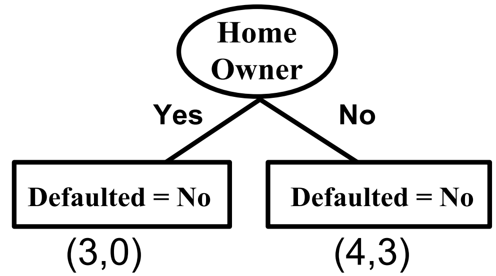
Hunt's Algorithm



(a)



(b)

ID	Home Owner	Marital Status	Annual Income	Defaulted Borrower
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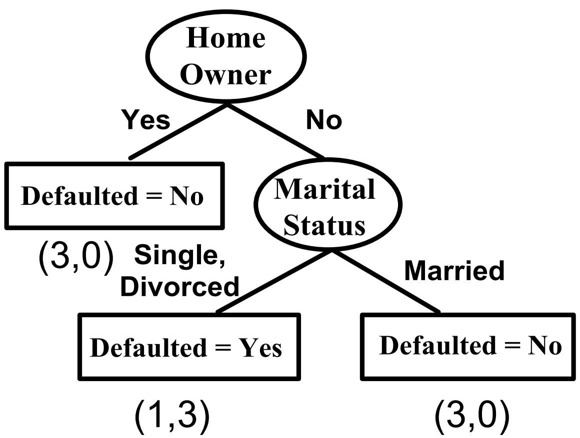

(c)

Hunt's Algorithm

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1	Yes	Single	125K	No
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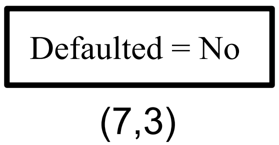
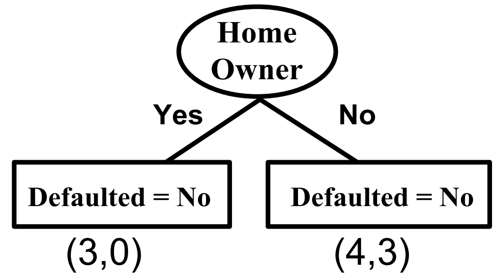
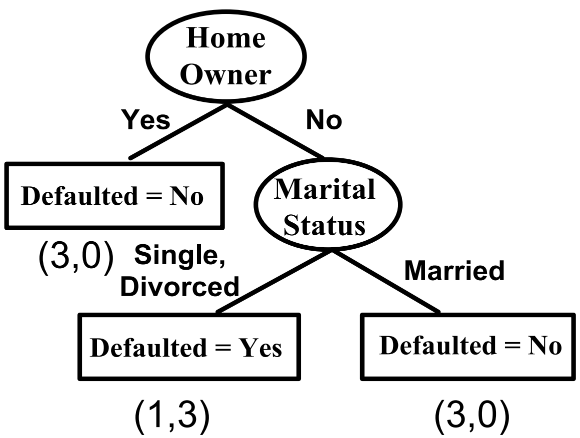


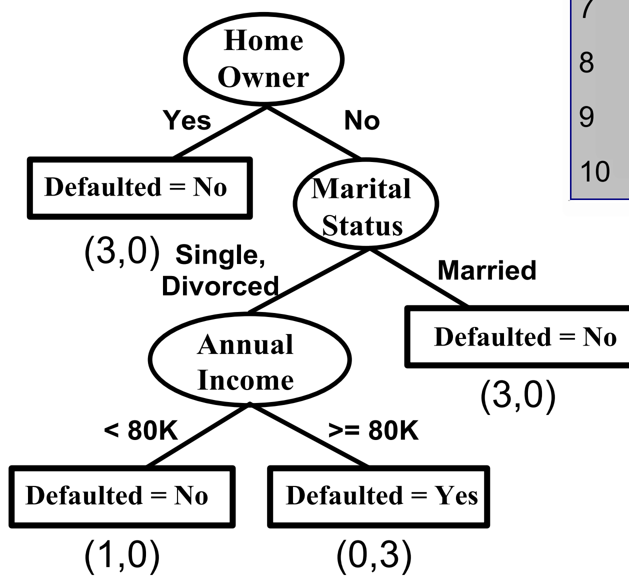
(a)



(b)



(c)



(d)

Tree Induction

- Greedy strategy
  - Split the records based on an attribute test that optimizes certain criterion.

- Issues
  - Determine how to split the records
    - How to specify the attribute test condition?
    - How to determine the best split?
  - Determine when to stop splitting

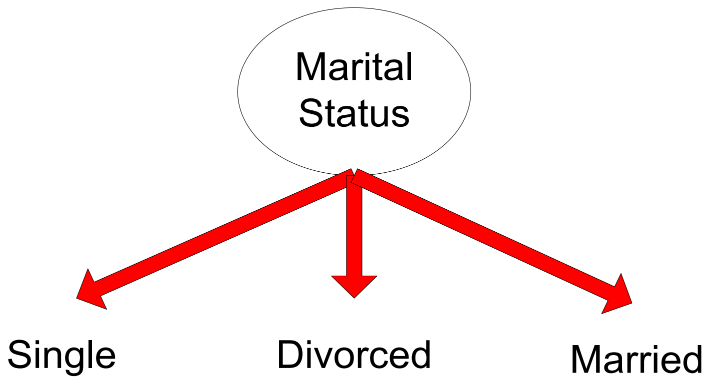
Methods for Expressing Test Conditions

- Depends on attribute types
  - Nominal
  - Ordinal
  - Continuous

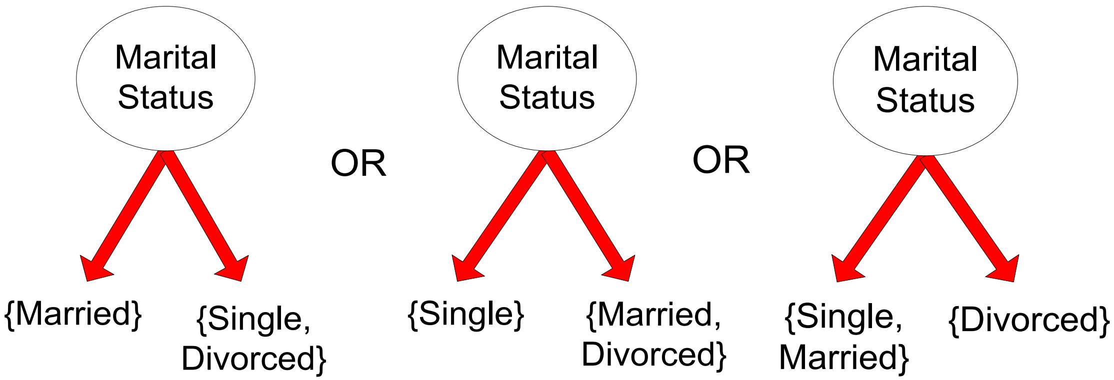
- Depends on number of ways to split
  - 2-way split
  - Multi-way split

Test Condition for Nominal Attributes

- Multi-way split:
  - Use as many partitions as distinct values.



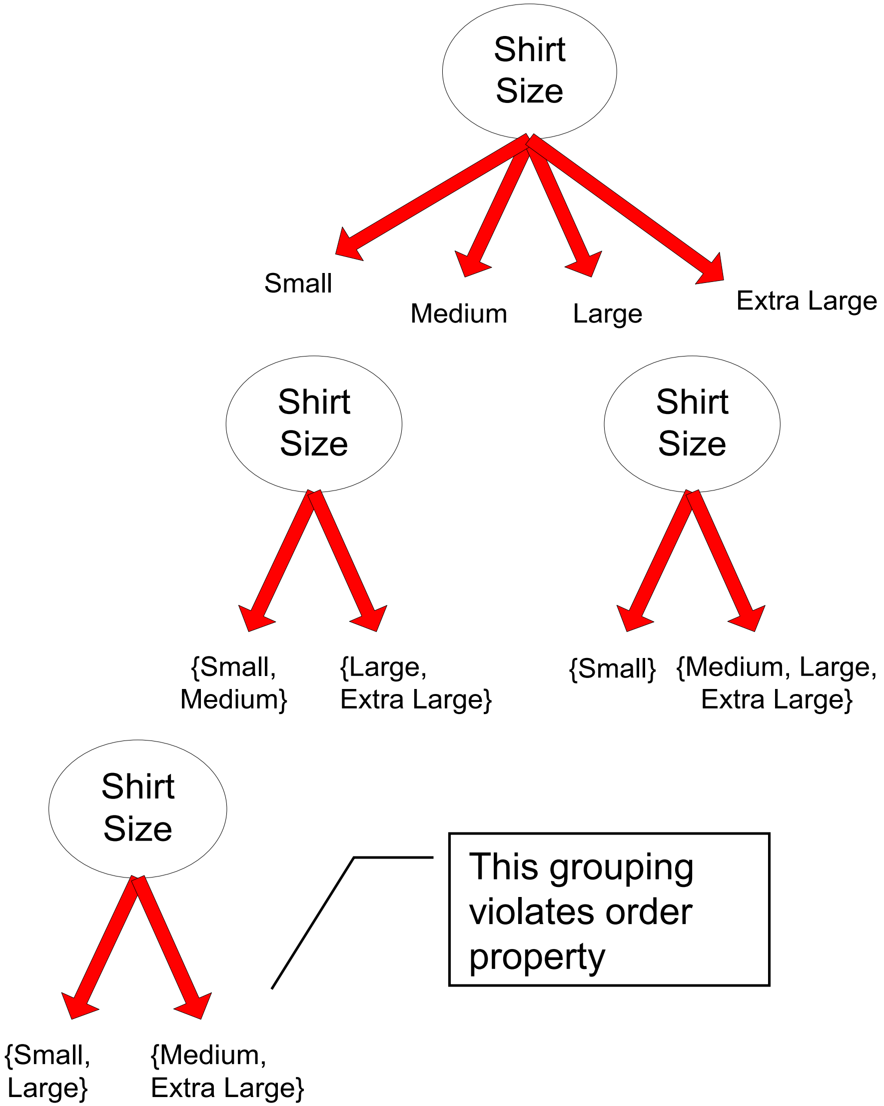
- Binary split:
  - Divides values into two subsets



Test Condition for Ordinal Attributes

- Multi-way split:
  - Use as many partitions as distinct values

- Binary split:
  - Divides values into two subsets
  - Preserve order property among attribute values

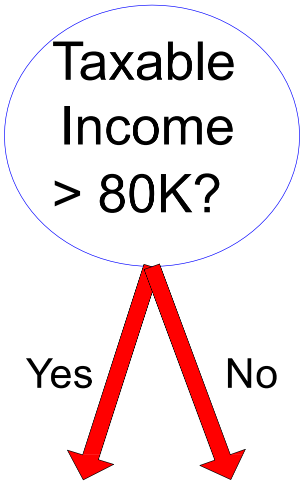


Splitting Based on Continuous Attributes

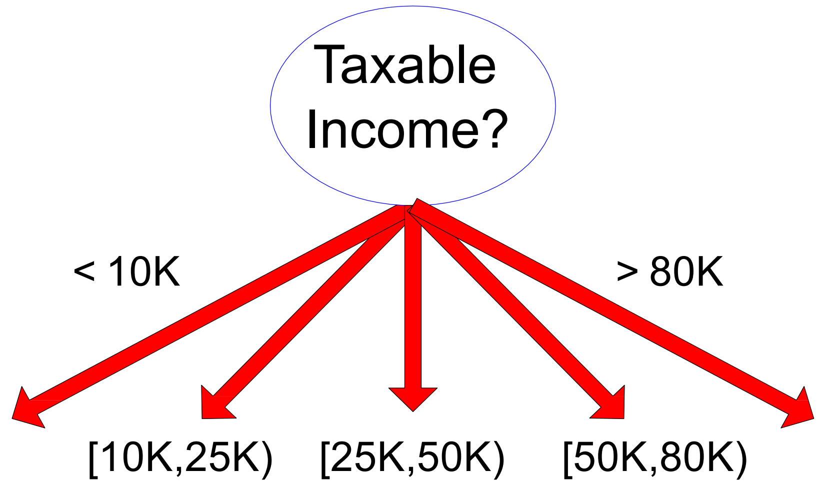
- Different ways of handling
  - Discretization to form an ordinal categorical attribute
  - Static – discretize once at the beginning
  - Dynamic – repeat at each node for discretization

  - Binary Decision: $(A < v)$ or $(A \geq v)$
  - consider all possible splits and finds the best cut
  - can be more compute intensive

Splitting Based on Continuous Attributes



(i) Binary split



(ii) Multi-way split

Tree Induction

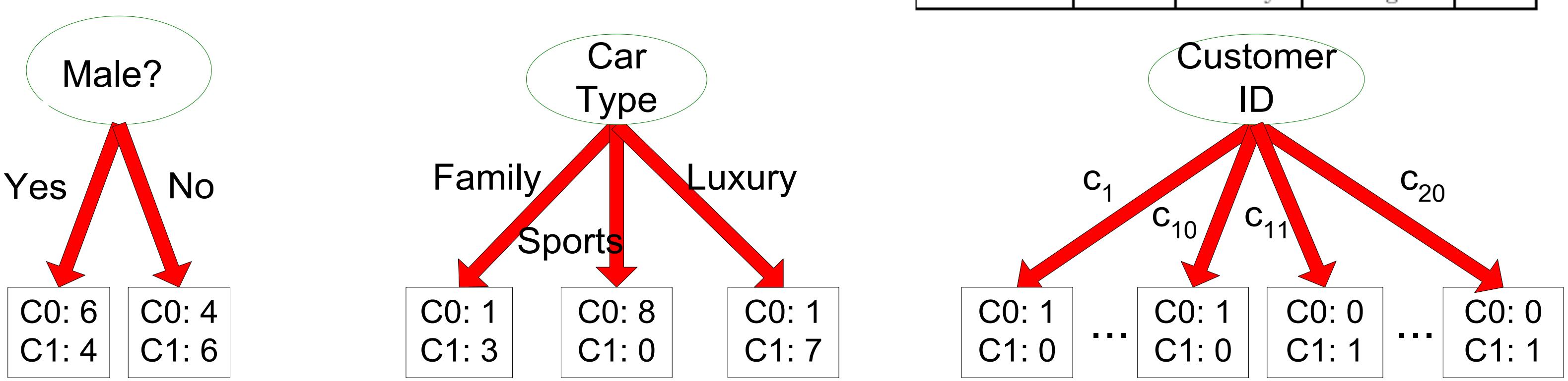
- Greedy strategy
  - Split the records based on an attribute test that optimizes certain criterion.

- Issues
  - Determine how to split the records
    - How to specify the attribute test condition?
    - How to determine the best split?
  - Determine when to stop splitting

How to determine the Best Split

Customer Id	Gender	Car Type	Shirt Size	Class
1	M	Family	Small	C0
2	M	Sports	Medium	C0
3	M	Sports	Medium	C0
4	M	Sports	Large	C0
5	M	Sports	Extra Large	C0
6	M	Sports	Extra Large	C0
7	F	Sports	Small	C0
8	F	Sports	Small	C0
9	F	Sports	Medium	C0
10	F	Luxury	Large	C0
11	M	Family	Large	C1
12	M	Family	Extra Large	C1
13	M	Family	Medium	C1
14	M	Luxury	Extra Large	C1
15	F	Luxury	Small	C1
16	F	Luxury	Small	C1
17	F	Luxury	Medium	C1
18	F	Luxury	Medium	C1
19	F	Luxury	Medium	C1
20	F	Luxury	Large	C1

Before Splitting: 10 records of class 0,
10 records of class 1



Which test condition is the best?

How to determine the Best Split

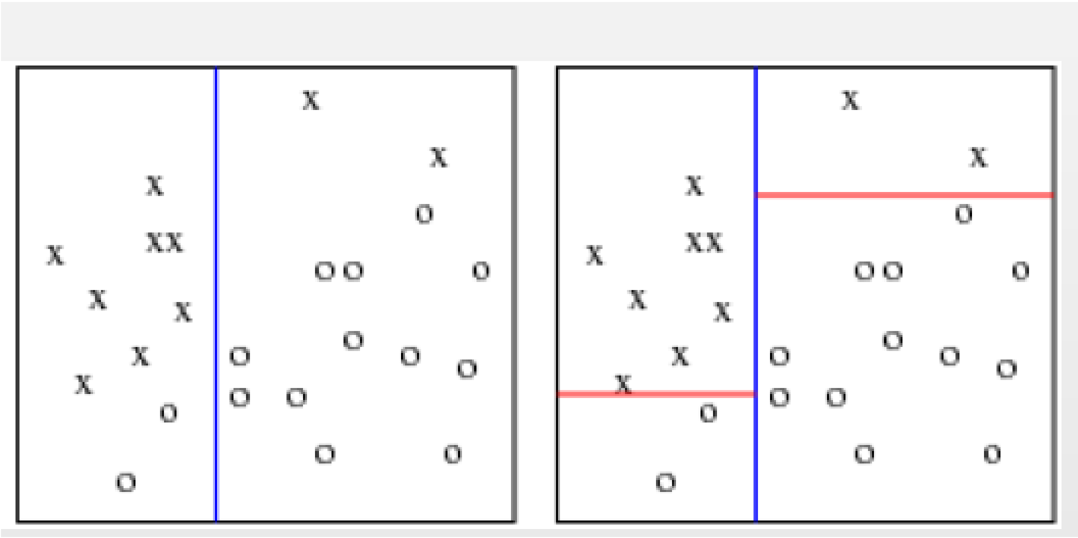
- Greedy approach:
  - Nodes with purer class distribution are preferred

- Need a measure of node impurity:

C0: 5 C1: 5	C0: 9 C1: 1
High uncertainty	Low uncertainty
High degree of impurity	Low degree of impurity

Goodness of Split

- The goodness of split is measured by an impurity function defined for each node.
- Intuitively, we want each leaf node to be “pure”, that is, one class dominates.



Finding the Best Split

- Compute impurity measure (P) before splitting
- Compute impurity measure (M) after splitting
  - Compute impurity measure of each child node
  - M is the weighted impurity of children
- Choose the attribute test condition that produces the highest gain

$$\text{Gain} = P - M$$

or equivalently, lowest impurity measure after splitting (M)

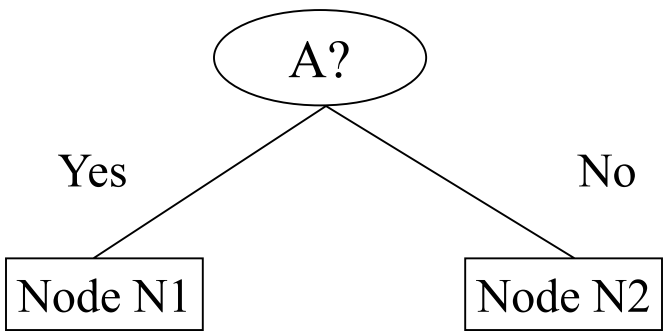
Finding the Best Split

Before Splitting:

C0	**N00**
C1	**N01**

→ P

N00: number of records with label 0 at node N0

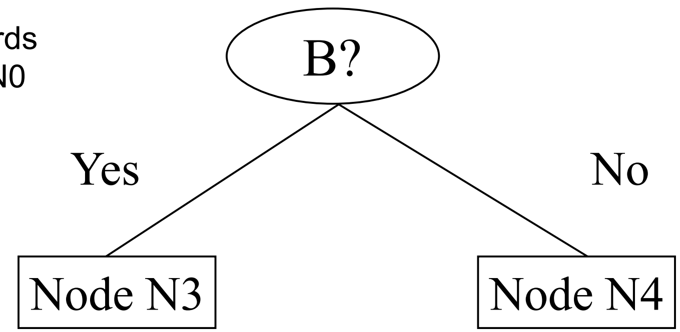


C0	**N10**
C1	**N11**

C0	**N20**
C1	**N21**

$\downarrow$ M11 $\quad$ $\downarrow$ M12

$$\underbrace{M11 \qquad M12}_{M1}$$



C0	**N30**
C1	**N31**

C0	**N40**
C1	**N41**

$\downarrow$ M21 $\quad$ $\downarrow$ M22

$$\underbrace{M21 \qquad M22}_{M2}$$

Gain = P – M1 vs P – M2

Measures of Node Impurity

- Gini Index

$$\boxed{GINI(t) = 1 - \sum_j [p(j \mid t)]^2}$$

t is a node

- Entropy

$$\boxed{Entropy(t) = -\sum_j p(j \mid t) \log p(j \mid t)}$$

- Misclassification error

Entropy quantifies uncertainty

$$\boxed{Error(t) = 1 - \max_i P(i \mid t)}$$

Measure of Impurity: GINI

- Gini Index for a given node t :

$$GINI(t) = 1 - \sum_j [p(j \mid t)]^2$$

(NOTE: *p(j | t)* is the relative frequency of class j at node t).

  - Maximum ($1 - 1/n_c$) when records are equally distributed among all classes, implying least information
  - Minimum (0.0) when all records belong to one class, implying most information

Measure of Impurity: GINI

- Gini Index for a given node t :

$$GINI(t) = 1 - \sum_j [p(j \mid t)]^2$$

(NOTE: *p(j / t)* is the relative frequency of class j at node t).

  - For 2-class problem (p, 1 – p):
  - GINI = $1 - p^2 - (1 - p)^2 = 2p(1-p)$

C1	**0**
C2	**6**
Gini=0.000	

C1	**1**
C2	**5**
Gini=0.278	

C1	**2**
C2	**4**
Gini=0.444	

C1	**3**
C2	**3**
Gini=0.500	

Computing Gini Index of a Single Node

$$GINI(t) = 1 - \sum_{j} [p(j \mid t)]^2$$

C1	**0**
C2	**6**

P(C1) = 0/6 = 0   P(C2) = 6/6 = 1

Gini = 1 – $P(C1)^2$ – $P(C2)^2$ = 1 – 0 – 1 = 0

C1	**1**
C2	**5**

P(C1) = 1/6   P(C2) = 5/6

Gini = 1 – $(1/6)^2$ – $(5/6)^2$ = 0.278

C1	**2**
C2	**4**

P(C1) = 2/6   P(C2) = 4/6

Gini = 1 – $(2/6)^2$ – $(4/6)^2$ = 0.444

Computing Gini Index for a Collection of Nodes

- When a node p is split into k partitions (children)

$$GINI_{split} = \sum_{i=1}^{k} \frac{n_i}{n} GINI(i)$$

where, n_i = number of records at child i,

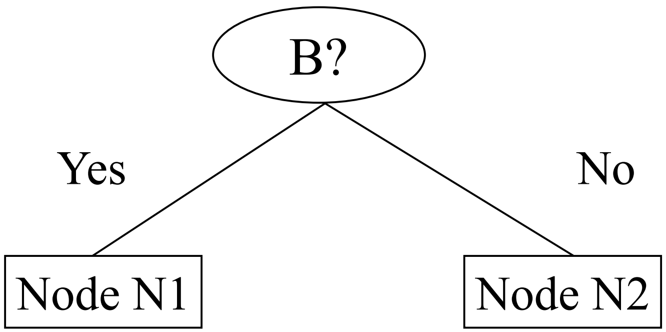
n = number of records at parent node p.

- Choose the attribute that minimizes weighted average Gini index of the children

- Gini index is used in decision tree algorithms such as CART, SLIQ, SPRINT

Binary Attributes: Computing GINI Index

- Splits into two partitions
- Effect of Weighing partitions:
  - Larger and Purer Partitions are sought for.



	Parent
C1	**7**
C2	**5**
Gini = 0.486	

Gini(P)
$= 1 - (5/12)^2 - (7/12)^2$
$= 0.486$

	N1	N2
C1	**5**	**2**
C2	**1**	**4**
Gini=0.361		

Gini(N1)
$= 1 - (5/6)^2 - (1/6)^2$
$= 0.278$

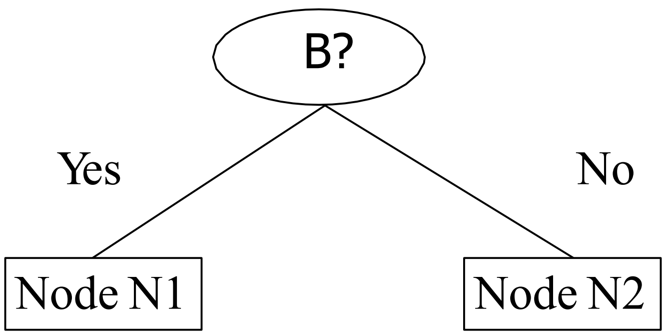
Gini(N2)
$= 1 - (2/6)^2 - (4/6)^2$
$= 0.444$

Weighted Gini of N1 N2
$= 6/12 * 0.278 +$
$6/12 * 0.444$
$= 0.361$

Gain = 0.486 – 0.361 = 0.125

Binary Attributes: Computing GINI Index

- Splits into two partitions
- Effect of Weighing partitions:
  - Larger and Purer Partitions are sought for



	Parent
C1	**6**
C2	**6**
Gini = 0.500	

Gini(P)
$= 1 – (6/12)^2 – (6/12)^2$
$= 0.5$

	N1	N2
C1	**5**	**1**
C2	**2**	**4**

Gini(N1)
$= 1 – (5/7)^2 – (2/7)^2$
$= 0.408$

Gini(N2)
$= 1 – (1/5)^2 – (4/5)^2$
$= 0.32$

Gini(Children)
$= 7/12 * 0.408 +$
$5/12 * 0.32$
$= 0.371$

Gain = 0.5 – 0.371 = 0.129

Categorical Attributes: Computing Gini Index

- For each distinct value, gather counts for each class in the dataset
- Use the count matrix to make decisions

Multi-way split

	CarType		
	Family	Sports	Luxury
C1	1	8	1
C2	3	0	7
Gini	0.1625		

Two-way split
(find best partition of values)

Case 1	CarType	
	{Sports, Luxury}	{Family}
C1	9	1
C2	7	3
Gini	0.468	

Case 2	CarType	
	{Sports}	{Family, Luxury}
C1	8	2
C2	0	10
Gini	0.167	

16/20*(1-(7/16)^2-(9/16)^2)+4/20*(1-(1/4)^2-(3/4)^2)

8/20*(1-(0/8)^2-(8/8)^2)+12/20*(1-(2/12)^2-(10/12)^2)

Which of these is the best?

Continuous Attributes: Computing Gini Index

- Use Binary Decisions based on one value
- Each splitting value has a count matrix associated with it
  - Class counts in each of the partitions, A < v and A ≥ v
- Simple method to choose best v
  - For each v, scan the database to gather count matrix and compute its Gini index
  - Computationally Inefficient! Repetition of work.

ID	Home Owner	Marital Status	Annual Income	Defaulted
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes

Annual Income ?

	≤ 80	> 80
Defaulted Yes	0	3
Defaulted No	3	4

Continuous Attributes: Computing Gini Index...

- For efficient computation: for each attribute,
  - Sort the attribute on values
  - Linearly scan these values, each time updating the count matrix and computing gini index
  - Choose the split position that has the least gini index

	Cheat	No	No	No	Yes	Yes	Yes	No	No	No	No
		Annual Income									
Sorted Values →		60	70	75	85	90	95	100	120	125	220

Continuous Attributes: Computing Gini Index...

- For efficient computation: for each attribute,
  - Sort the attribute on values
  - Linearly scan these values, each time updating the count matrix and computing gini index
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	Cheat	No	No	No	Yes	Yes	Yes	No	No	No	No
		Annual Income									
Sorted Values →		60	70	75	85	90	95	100	120	125	220

Split Positions →	55		65		72		80		87		92		97		110		122		172		230	
	<=	>	<=	>	<=	>	<=	>	<=	>	<=	>	<=	>	<=	>	<=	>	<=	>	<=	>

Continuous Attributes: Computing Gini Index...

- For efficient computation: for each attribute,
  - Sort the attribute on values
  - Linearly scan these values, each time updating the count matrix and computing gini index
  - Choose the split position that has the least gini index

Cheat	No	No	No	Yes	Yes	Yes	No	No	No	No
	Annual Income									
Sorted Values	60	70	75	85	90	95	100	120	125	220

Split Positions	55		65		72		80		87		92		97		110		122		172		230	
	<=	>	<=	>	<=	>	<=	>	<=	>	<=	>	<=	>	<=	>	<=	>	<=	>	<=	>
Yes							0	3														
No							3	4														
Gini							0.343															

Gini(N1) = $1 - (0/3)^2 - (3/3)^2 = 0$
Gini(N2) = $1 - (3/7)^2 - (4/7)^2 = 0.4898$

Gini(Children) = 3/10*0+ 7/10 * 0.4898
= 0.343

Continuous Attributes: Computing Gini Index...

- For efficient computation: for each attribute,
  - Sort the attribute on values
  - Linearly scan these values, each time updating the count matrix and computing gini index
  - Choose the split position that has the least gini index

Cheat	No	No	No	Yes	Yes	Yes	No	No	No	No
	Annual Income									
Sorted Values	60	70	75	85	90	95	100	120	125	220

Split Positions	55		65		72		80		87		92		97		110		122		172		230	
	<=	>	<=	>	<=	>	<=	>	<=	>	<=	>	<=	>	<=	>	<=	>	<=	>	<=	>
Yes							0	3	1	2												
No							3	4	3	4												
Gini							0.343		0.417													

Continuous Attributes: Computing Gini Index...

- For efficient computation: for each attribute,
  - Sort the attribute on values
  - Linearly scan these values, each time updating the count matrix and computing gini index
  - Choose the split position that has the least gini index

	Cheat	No	No	No	Yes	Yes	Yes	No	No	No	No
		Annual Income									
Sorted Values →		60	70	75	85	90	95	100	120	125	220

Split Positions →	55		65		72		80		87		92		97		110		122		172		230	
	<=	>	<=	>	<=	>	<=	>	<=	>	<=	>	<=	>	<=	>	<=	>	<=	>	<=	>
Yes	0	3	0	3	0	3	0	3	1	2	2	1	3	0	3	0	3	0	3	0	3	0
No	0	7	1	6	2	5	3	4	3	4	3	4	3	4	4	3	5	2	6	1	7	0
Gini	0.420		0.400		0.375		0.343		0.417		0.400		***0.300***		0.343		0.375		0.400		0.420	

Measure of Impurity: Entropy

- Entropy at a given node t:

$$\boxed{Entropy(t) = -\sum_j p(j\,|\,t)\log p(j\,|\,t)}$$

- (NOTE: p(j|t) is the relative frequency of class j at node t).

- Measures purity of a node
  - Maximum (log n_c) when records are equally distributed among all classes implying least information
  - Minimum (0.0) when all records belong to one class, implying most information
  - Entropy based computations are quite similar to the GINI index computations

Computing Entropy of a Single Node

$$Entropy(t) = -\sum_j p(j \mid t) \log_2 p(j \mid t)$$

C1	**0**
C2	**6**

P(C1) = 0/6 = 0   P(C2) = 6/6 = 1

Entropy = – 0 log 0 – 1 log 1 = – 0 – 0 = 0

C1	**1**
C2	**5**

P(C1) = 1/6   P(C2) = 5/6

Entropy = – (1/6) $\log_2$ (1/6) – (5/6) $\log_2$ (1/6) = 0.65

C1	**2**
C2	**4**

P(C1) = 2/6   P(C2) = 4/6

Entropy = – (2/6) $\log_2$ (2/6) – (4/6) $\log_2$ (4/6) = 0.92

Computing Information Gain After Splitting

- Information Gain:

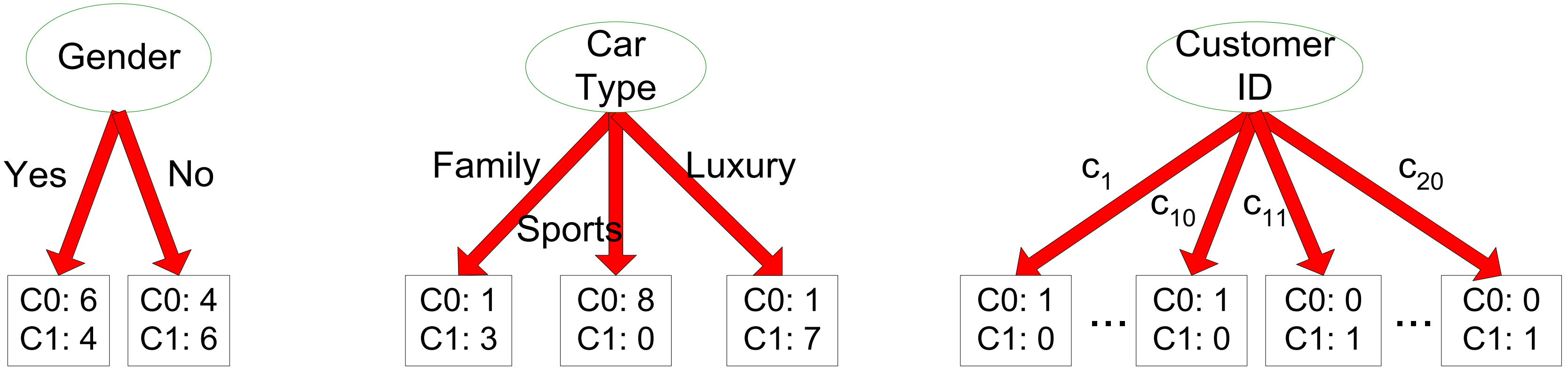
$$GAIN_{split} = Entropy(p) - \left(\sum_{i=1}^{k} \frac{n_i}{n} Entropy(i) \right)$$

Parent Node, p is split into k partitions; n_i is number of records in partition i

- Measures reduction in entropy achieved because of the split. Choose the split that achieves most reduction (maximizes GAIN)
- Used in the ID3 and C4.5 decision tree algorithms

Problem with large number of partitions

- Node impurity measures tend to prefer splits that result in large number of partitions, each being small but pure



- Customer ID has highest information gain because entropy for all the children is zero

Gain Ratio

- Gain Ratio:

$$GainRATIO_{split} = \frac{GAIN_{Split}}{SplitINFO} \qquad SplitINFO = -\sum_{i=1}^{k} \frac{n_i}{n} \log \frac{n_i}{n}$$

Parent Node, p is split into k partitions, n_i is the number of records in partition i

- Adjusts Information Gain by the entropy of the partitioning (SplitINFO).
  - Higher entropy partitioning (large number of small partitions) is penalized!
- Used in C4.5 algorithm
- Designed to overcome the disadvantage of Information Gain

Gain Ratio

- Gain Ratio

$$GainRATIO_{split} = \frac{GAIN_{Split}}{SplitINFO} \qquad SplitINFO = -\sum_{i=1}^{k} \frac{n_i}{n} \log \frac{n_i}{n}$$

Parent Node, p is split into k partitions n_i is the number of records in partition i

Case 1

	CarType		
	Family	Sports	Luxury
C1	1	8	1
C2	3	0	7
Gini	0.1625		

SplitINFO = 1.52

-[(4/20)*log2(4/20)+(8/20)*log2(8/20)+(8/20)*log2(8/20)]

Case 2

	CarType	
	{Sports, Luxury}	{Family}
C1	9	1
C2	7	3
Gini	0.4688	

SplitINFO = 0.72

-[(16/20)*log2(16/20)+(4/20)*log2(4/20)]

Case 3

	CarType	
	{Sports}	{Family, Luxury}
C1	8	2
C2	0	10
Gini	0.1667	

SplitINFO = 0.97

-[(8/20)*log2(8/20)+(12/20)*log2(12/20)]

Measure of Impurity: Classification Error

- Classification error at a node t :

$$\boxed{Error(t) = 1 - \max_i P(i \mid t)}$$

- Measures misclassification error made by a node.
  - Maximum ($1 - 1/n_c$) when records are equally distributed among all classes, implying least information
  - Minimum (0) when all records belong to one class, implying most information

Computing Error of a Single Node

$$Error(t) = 1 - \max_i P(i \mid t)$$

C1	**0**
C2	**6**

P(C1) = 0/6 = 0   P(C2) = 6/6 = 1

Error = 1 – max (0, 1) = 1 – 1 = 0

C1	**1**
C2	**5**

P(C1) = 1/6   P(C2) = 5/6

Error = 1 – max (1/6, 5/6) = 1 – 5/6 = 1/6

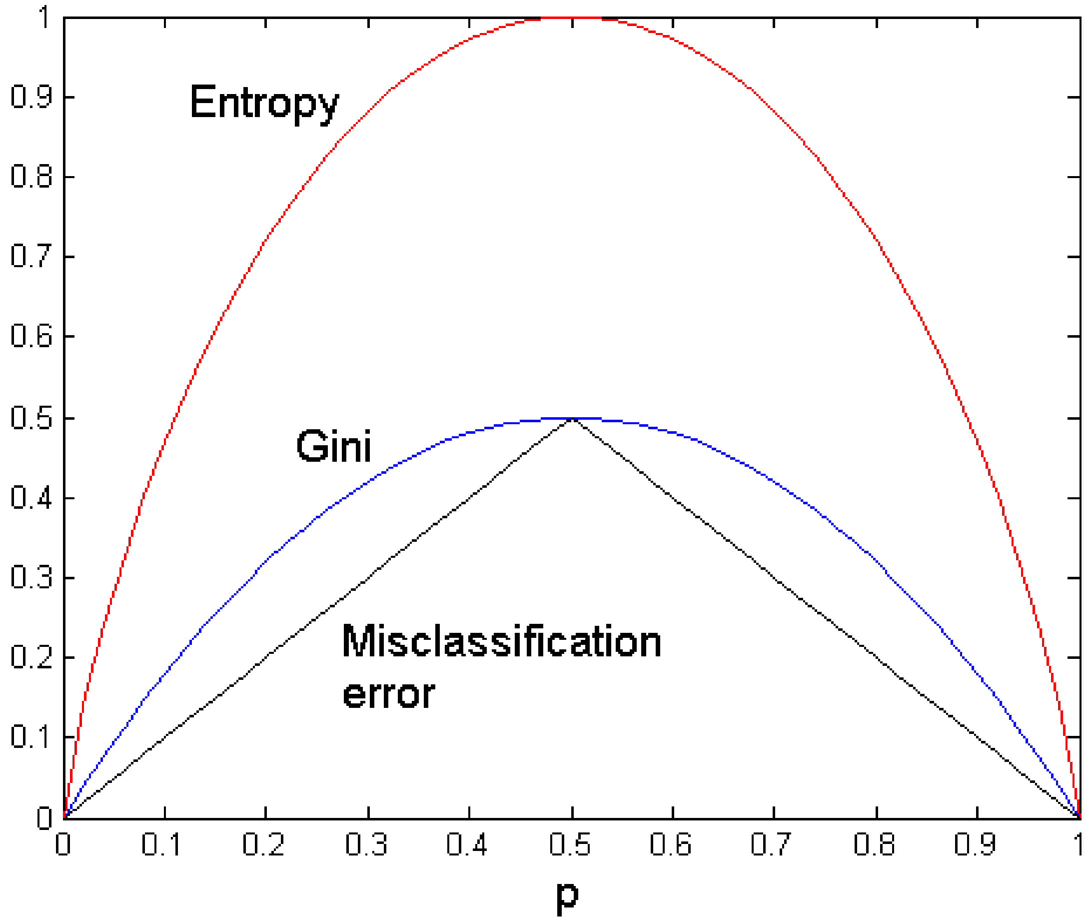
C1	**2**
C2	**4**

P(C1) = 2/6   P(C2) = 4/6

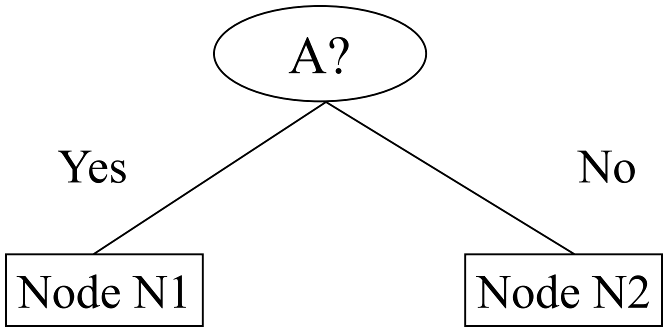
Error = 1 – max (2/6, 4/6) = 1 – 4/6 = 1/3

Comparison among Impurity Measures

For a 2-class problem:



Misclassification Error vs Gini Index



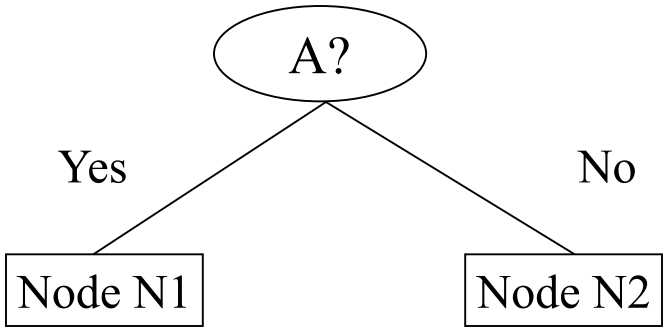
	Parent
C1	**7**
C2	**3**
Gini = 0.42	

	N1	**N2**
C1	**3**	**4**
C2	**0**	**3**
Gini=0.342		

Error for children
N1: 1-max (3/3, 0/3) = 0
N2: 1-max(4/7, 3/7) = 3/7
Weighted sum:
3/10 * 0 + 7/10 * 3/7 = 0.3

Error for parent nodes
1-max(7/10,3/10)=0.3

Misclassification Error vs Gini Index



	Parent
C1	**7**
C2	**3**
Gini = 0.42	

	N1	N2
C1	**3**	**4**
C2	**0**	**3**
Gini=0.342		

Gini(N1)
$= 1 – (3/3)^2 – (0/3)^2$
$= 0$

Gini(N2)
$= 1 – (4/7)^2 – (3/7)^2$
$= 0.489$

Gini(Children)
$= 3/10 * 0$
$+ 7/10 * 0.489$
$= 0.342$

Gini improves but error remains the same!!

Outline

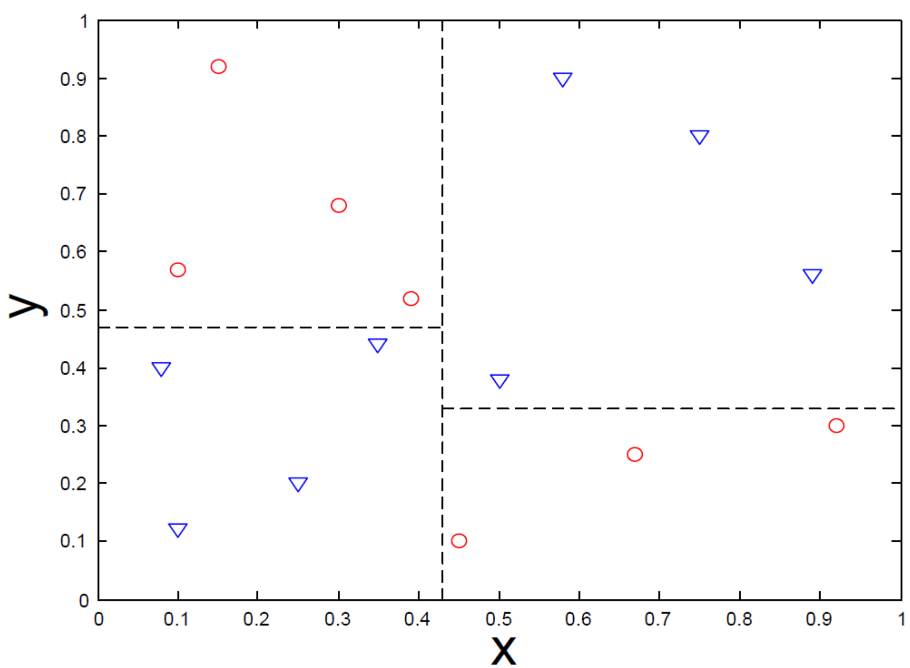
- Basic introduction of decision tree
- Hunt’s Algorithm
- Determine how to split the records
  - How to specify the attribute test condition?
  - How to determine the best split?
- Determine when to stop splitting

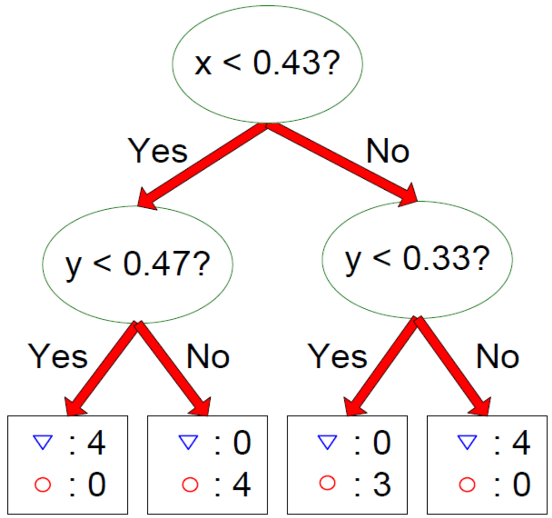
Determine when to stop splitting

- Stop expanding a node when all the records belong to the same class
- Stop expanding a node when all the records have similar attribute values

Decision Boundary

- Border line between two neighboring regions of different classes is known as decision boundary
- Decision boundary is parallel to axes because test condition involves a single attribute at-a-time





Decision Tree Based Classification

- Advantages:
  - Inexpensive to construct
  - Extremely fast at classifying unknown records
  - Easy to interpret for small-sized trees
  - Robust to noise
  - Can easily handle redundant or irrelevant attributes (unless the attributes are interacting)
- Disadvantages:
  - Space of possible decision trees is exponentially large. Greedy approaches are often unable to find the best tree.
  - Does not take into account interactions between attributes
  - Each decision boundary involves only a single attribute