Lighting and Rasterization – Creating Shadows

Intended Learning Outcomes

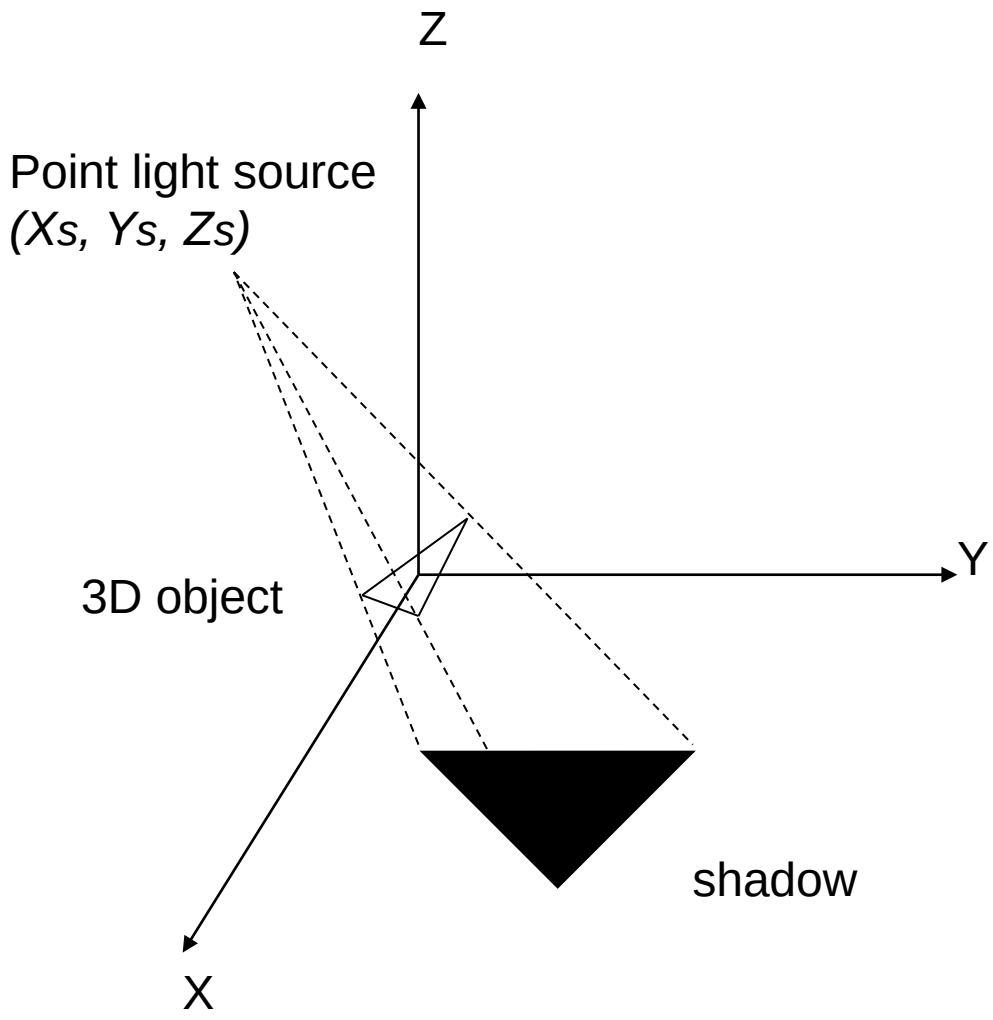
- Apply fast techniques to generate realistic shadow on the ground plane and its programming implementation
- Extend ray casting technique for general shadow creation
- Apply shadow mapping for general shadow creation

Creating Shadow on Plane

- Works only when projecting objects onto a plane and point light source

- Idea : Given a point light source s and a plane P,

1. Render the objects normally.
2. Use a coordinate system transformation that transforms s to the PRP and P to the image plane.
3. Set the object colour to the shadow colour
4. Perspective project the objects onto the image plane, creating shadows.
5. Use the inverse coordinate system transformation to transform the shadows to the normal coordinate system


Z
Point light source
(Xs, Ys, Zs)
We are the viewer
Y
3D object
X-Y is the plane
shadow
X

Coordinate transformation such that the light source becomes the origin

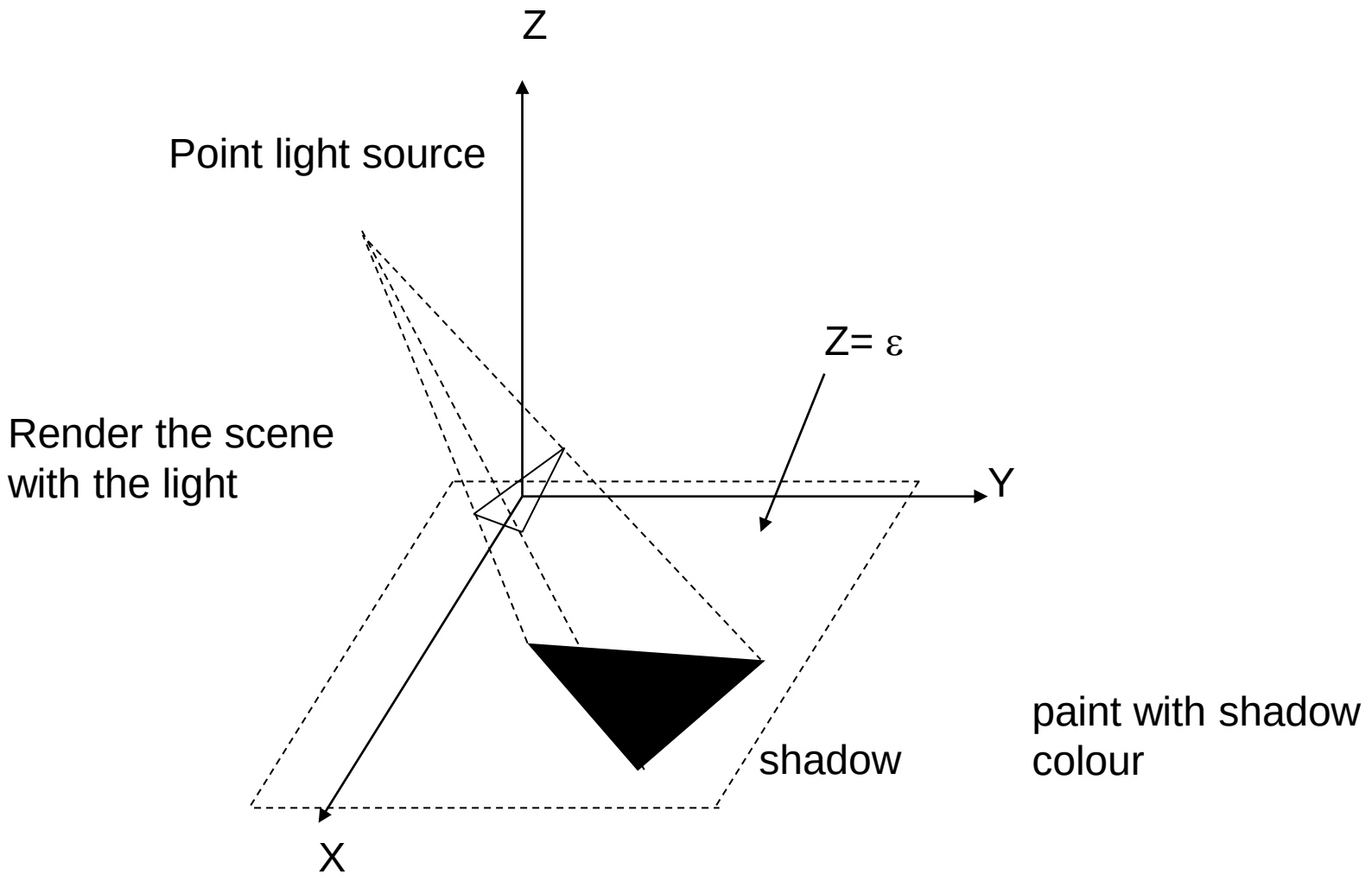
$\mathbf{M}_{s \leftarrow WC} = \mathbf{T}(-X_s, -Y_s, -Z_s)$

Perspective projection

$$\mathbf{M} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{1}{-Z_s} & 0 \end{pmatrix}$$

OpenGL code

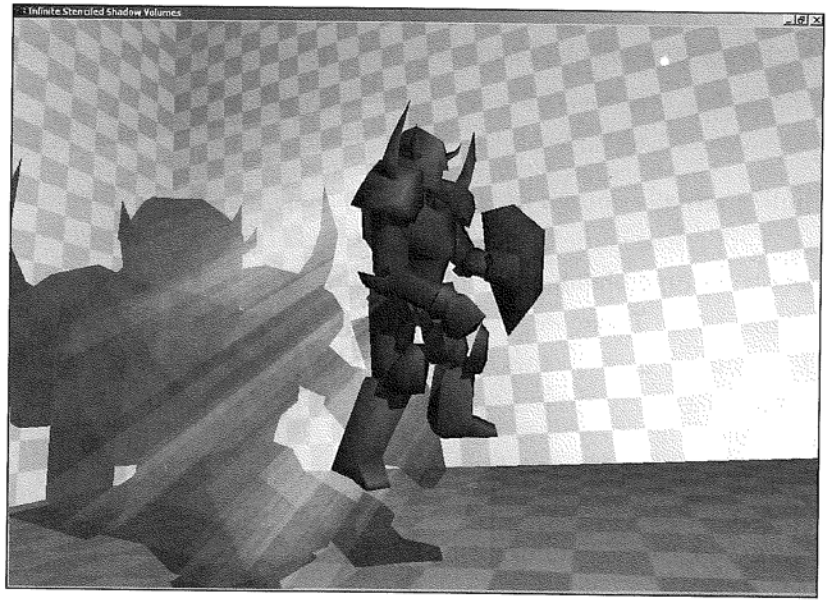
```
GLfloat  light1PosType [ ] = {Xs, Ys, Zs, 1.0};
                    :
GLfloat  M[16];                 // OpenGL is in column major format
                                //  though C is in row major format
for (i=0; i<16; i++)
     M[i]=0;
M[0]=M[5]=M[10]=1;
M[11]=-1.0/Zs;

object ( );                     // draw the objects

glPushMatrix ( );               //  save state

glMatrixMode (GL_MODELVIEW);
glTranslatef (Xs, Ys, Zs);      //  Mwc←s
glMultMatrixf (M);              //   perspective project
glTranslatef (-Xs, -Ys, -Zs);   //  Ms←wc

glColor3fv (shadowcolour);      //  set ka = kd = ks = 0 if you are using lighting model
object ( );

glPopMatrix ( );                //  restore state
```

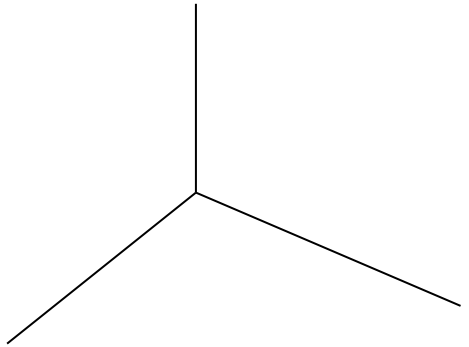


Implementation notes: in actual programming, cast the shadow on a plane $Z = \varepsilon$ after changing to the light source coordinate system, where ε is a very small number (why?)

Extension to corners

- It can be used to cast shadows on corners of the room (treat it as three planes)





- However, it cannot be used to cast shadows on general non-plane objects

General Shadow creation

Limitations:

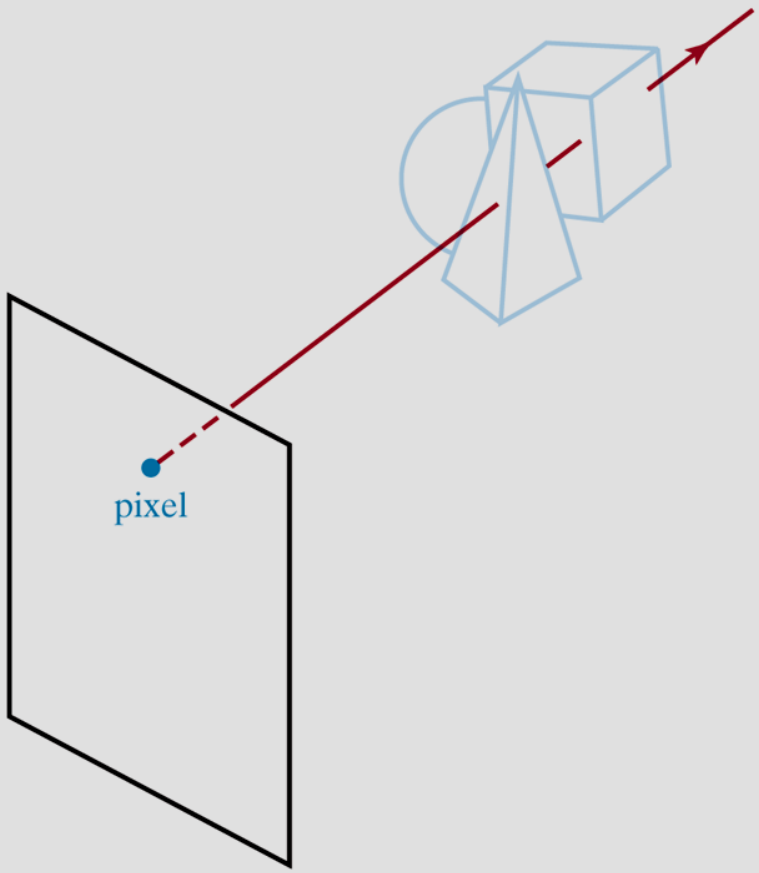
- Up to now, shadows can only be casted on planes or corners
- No shade differences for overlapping shadows
- The shadow boundary is too sharp

- Below we introduce two techniques: ray casting using shadow ray and shadow mapping, that overcome the first two limitations.
- One way to create soft shadows is radiosity, which is a sophisticated model of ambient reflection

Ray Casting

- retrace the light paths of the rays that arrive at the pixel

- for each pixel, send a ray from PRP that goes through the pixel
- find all intersections of the ray with the surfaces
- the nearest intersections is the visible part of the surface for that pixel



Ray casting

A ray along the line of sight from a pixel position through a scene.

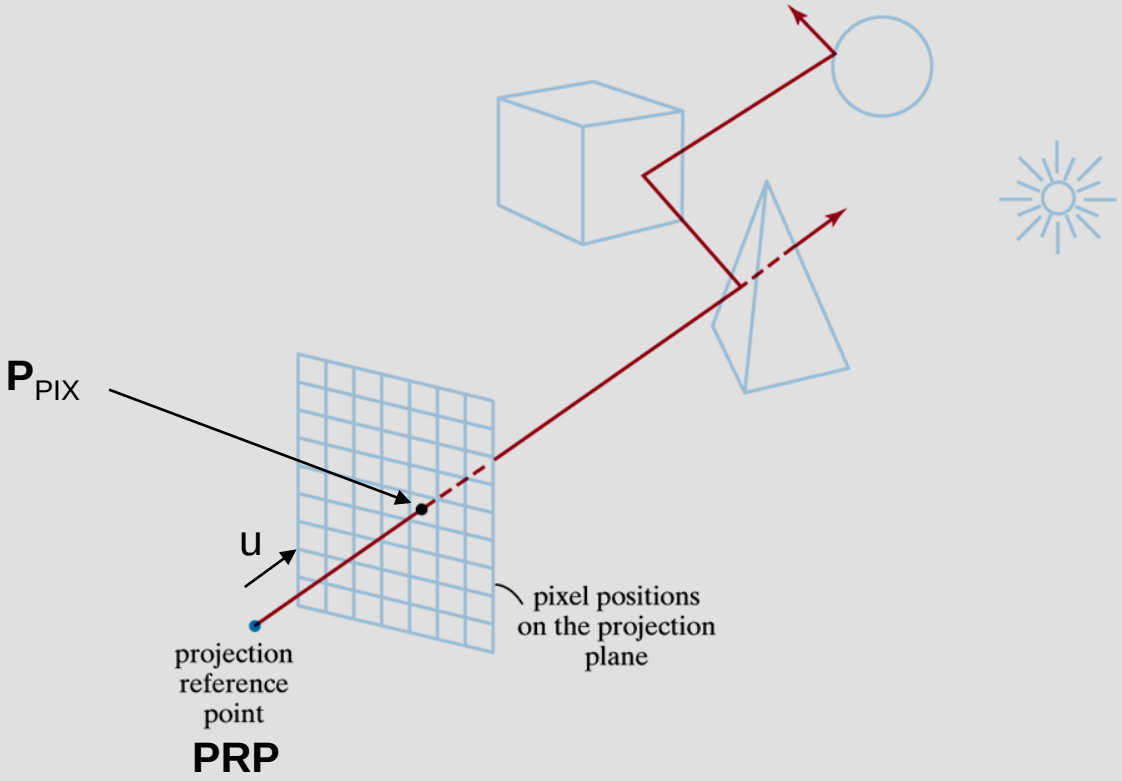
- Consider the math.

$$\boldsymbol{P} = \boldsymbol{P}_0 + s\,\boldsymbol{u} \qquad \text{(Pixel Ray Equation)}$$

$\boldsymbol{P}_0$ may either be $\boldsymbol{P}_{PRP}$ or $\boldsymbol{P}_{PIX}$

$\boldsymbol{P}_{pix}$ is the (X, Y, Z) coordinates of the pixel

$$\mathbf{u} = \frac{\mathbf{P}_{PIX} - \mathbf{P}_{PRP}}{\left|\mathbf{P}_{PIX} - \mathbf{P}_{PRP}\right|}$$ is a unit vector pointing out from **PRP**



Multiple reflection and transmission paths for a ray from the projection reference point through a pixel position and on into a scene containing several objects.

Ray – Surface Intersections

- Suppose the CG scene consists of n surfaces or polygons
- Compute the intersection point(s) of the pixel ray with each of the n surfaces/polygons
- The surface/polygons whose intersection point has the smallest s is the visible surface
- since it is the nearest

Ray – Sphere Intersection

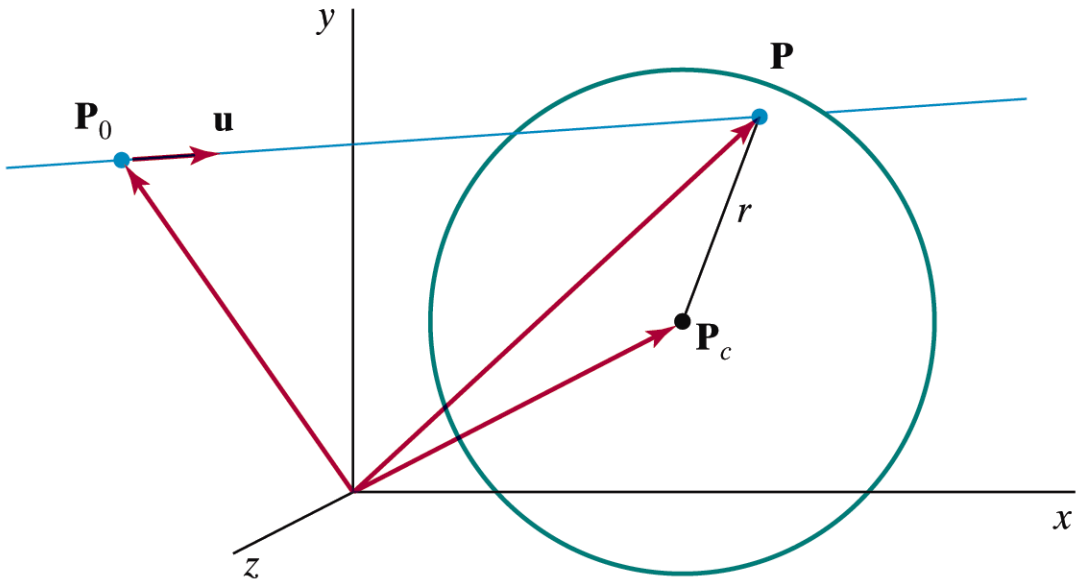
- Sphere is the simplest surface with analytical equation

$$(X - X_C)^2 + (Y - Y_C)^2 + (Z - Z_C)^2 = r^2$$

$\mathbf{P}_C(X_C, Y_C, Z_C)$ Center

r Radius

$$\left|\mathbf{P} - \mathbf{P}_c\right|^2 - r^2 = 0$$ Vector Equation



A ray intersecting a sphere with radius r and center position $\mathbf{P}_c$.

Ray-Sphere Intersections (2)

- Sub. $\mathbf{P} = \mathbf{P}_0 + s\mathbf{u}$ gives a quadratic equation
- Solution:

$$s = \mathbf{u} \cdot \Delta\mathbf{P} \pm \sqrt{r^2 - \left|\Delta\mathbf{P} - (\mathbf{u} \cdot \Delta\mathbf{P})\mathbf{u}\right|^2} \qquad \Delta\mathbf{P} = \mathbf{P}_C - \mathbf{P}_0$$

- If discriminant < 0, does not intersect
- Otherwise choose the intersection with the smaller s

- Solution is more difficult and time consuming for more complicated surfaces

Shadowing

- The ray casting method above can be used to determine the visible surface
- For each visible surface point **P**, the question is how to determine whether **P** is in shadow
- Given a set of light sources, **P** can be in shadow to a subset of light sources and illuminated by the rest
- If in shadow with respect to source $\mathbf{S_o}$, then the light intensity due to $\mathbf{S_o}$ is set to zero

Shadow ray

- To test whether **P** is in shadow w.r.t. a point light source $\mathbf{S_o}$:
- Send a pixel ray from **P** to $\mathbf{S_o}$
- If the pixel ray intersects ANY surface /polygon on its way, **P** is in shadow w.r.t. $\mathbf{S_o}$
- The pixel ray is called “shadow ray”

Shadow Mapping

- Idea: A point is in shadow iff it is not visible to the light source (a visibility determination problem)
- Change the coordinate system such that the light position is the PRP. We call this the lighting coordinate system
- Perform a perspective projection. Keep the depth buffer. The depth buffer holds, for each pixel, the nearest distance to the light source
- For each 3D point to be rendered, change to the lighting coordinate system.
- Project the point. Compare its depth to the value in the depth buffer.
- The point is in shadow if it is not the same value as that in the depth buffer.

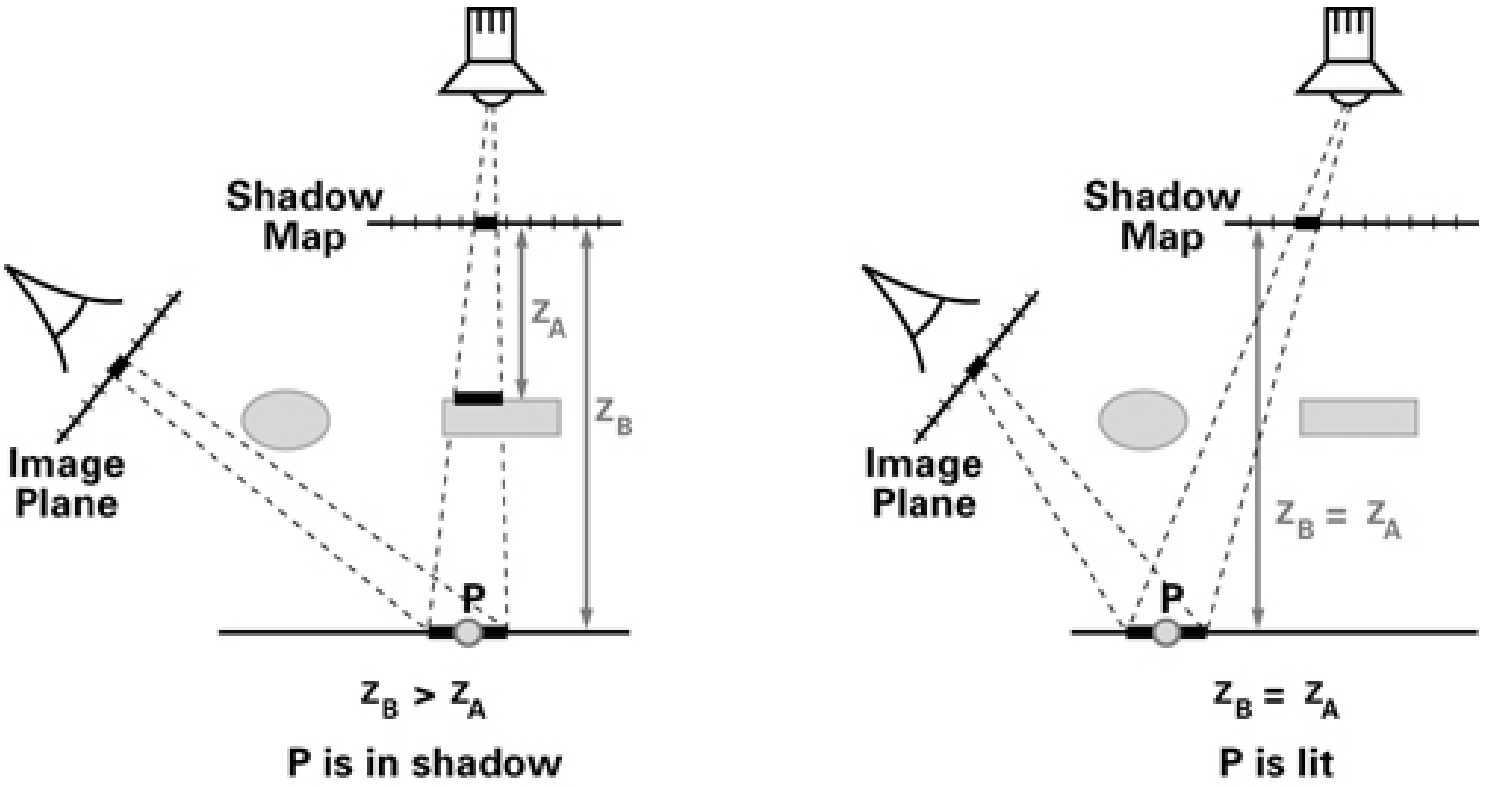


Figure from
http://http.developer.nvidia.com/CgTutorial/cg_tutorial_chapter09.html

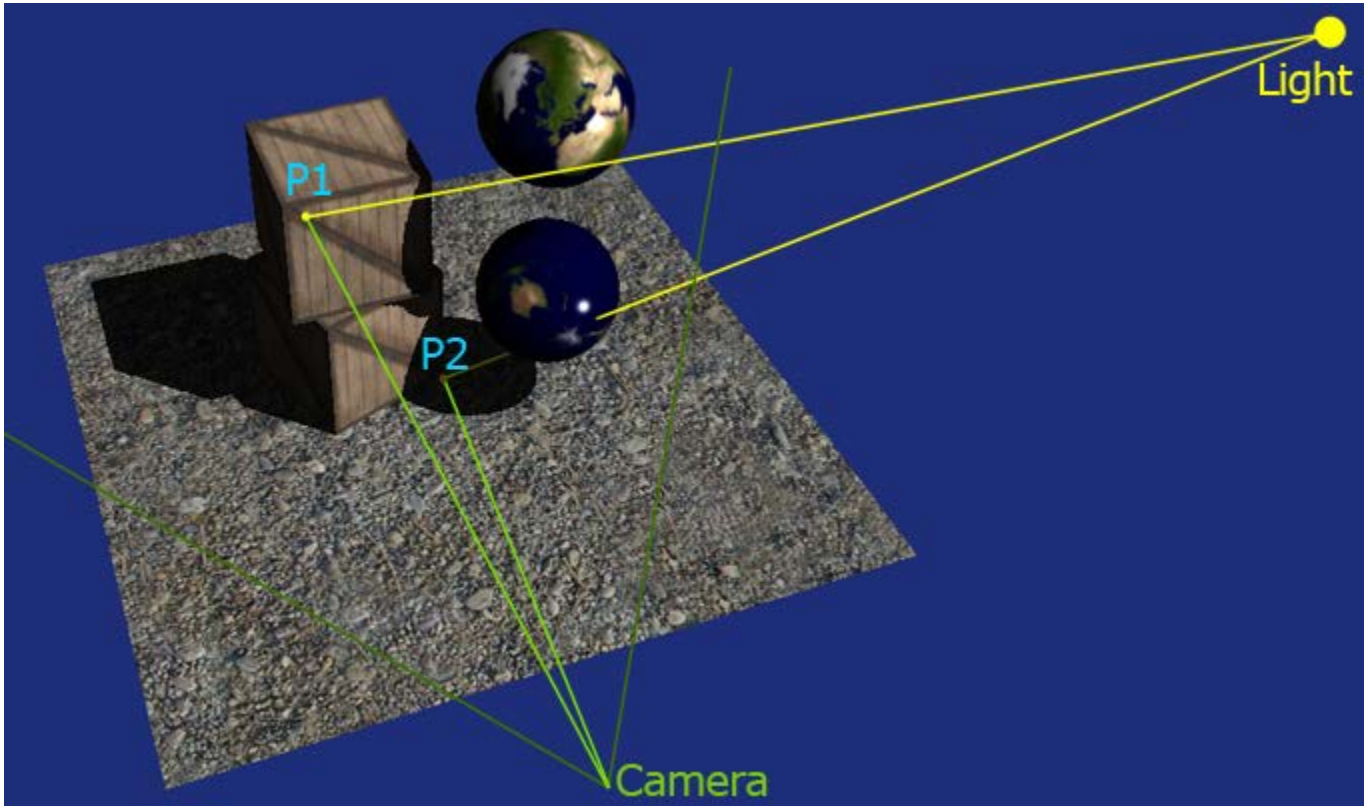


Figure from
http://www.codinglabs.net/tutorial_opengl_deferred_rendering_shadow_mapping.aspx

References

- Text: Ch. 16-10 for ray casting method
- Creating shadow on plane
  - E. Angel, Interactive Computer Graphics, A Top-Down Approach Using OpenGL, 3rd Ed., 2003, pp. 261-264.