MA1200 Exercise for Chapter 7 Techniques of Differentiation

First Principle

1. Use the First Principle to find the derivative of the following functions:

(a) $f(x)=\dfrac{2x-3}{3x+4}$ (b) $f(x)=\sqrt{2x+1}$

Product/Quotient/Chain Rules

2. Differentiate the following functions:

(a) $y=7x^4-6x^2+x-5-\dfrac{1}{x}+\dfrac{3}{x^2}+\dfrac{2}{x^3}$ (b) $y=\sqrt[3]{3x^2}$

(c) $y=\sqrt{25-x^2}$ (d) $y=\dfrac{3-2x}{3+2x}$

(e) $y=\dfrac{x^2}{\sqrt{4-x^2}}$ (f) $y=(1+2x-5x^2)^{3000}$

(g) $f(x)=(x^2+4)^2(2x^3-1)^3$ (h) $y=\sin^3(2x)-5\cos(x^3+1)$

(i) $y=\ln(\ln(\ln x))$ (j) $y=x\ln x-x$

(k) $y=\ln(\sin x)$ (l) $y=\ln(\cos x)$

(m) $y=\ln(3xe^{-x})$ (n) $y=\cos(e^x)$

(o) $y=x^3e^x\cos x$ (p) $y=\dfrac{(2x+\sin x)e^x}{3x^2-5x}$

(q) $y=\dfrac{x^n-1}{x-1}$ (r) $y=\dfrac{x+2}{x^2-3x}$

(s) $y=\sin(2x)\cos(3x)$ (t) $y=\sin(\ln[\cos(\ln(2x)+1)]+1)$

3. Find $\dfrac{dy}{dx}$ using implicit differentiation:

(a) $x^2-y^2=1$

(b) $x^2+xy+y^2=9$

(c) $(x^2+y^2)^2=4xy$

4. Find $\dfrac{dy}{dx}$ and the equation of the tangent line and normal line to the parametric curves at the specified point:

(a) $x=2t^2+1 \quad y=3t^3+2 \quad t=1$

(b) $x=\sqrt{t} \quad y=t-\dfrac{1}{\sqrt{t}} \quad t=4$

5. Find the derivative of y:

(a) $y=(x+1)^{\cot x}$ (b) $y=(x^4+2x^2)^{e^x}$

(c) $y=(\sin 5x)^{x^2+2}+3x$

6. Differentiate the following functions n times:
   (a) $f(x)=3e^{4x}$
   (b) $h(x)=5\sin(6x-7)$
   (c) $f(x)=\dfrac{2}{2x-1}$
   (d) $g(x)=x^{11}-6x^3+2$

<u>Leibniz's rule</u>

7. Use Leibniz's rule to differentiate the following functions *n* times:
   (a) $y=x^3e^{2x}$
   (b) $y=(x^2-4x+7)\cos(2x-1)$
   (c) $y=\dfrac{x^2}{1+x}$

<u>Miscellaneous</u>

8. If $y=x\sin x$, prove that $x^2y''-2xy'+\left(2+x^2\right)y=0$.

9. If $u=\sqrt{ax^2+2bx+c}$, prove that $\dfrac{d}{dx}(xu)=\dfrac{2ax^2+3bx+c}{u}$.

*10. Find the values of *a* and *b* (in terms of *c*) such that $f'(c)$ exists, where $f(x)=\begin{cases}\dfrac{1}{|x|} & \text{if } |x|>c\\ ax+b & \text{if } |x|\le c\end{cases}$.

11. A function *y* of *x* is defined by the equation $\sin\left(x-y\right)=m\sin y$. Express *y* explicitly in terms of *x*. Hence, or otherwise, show that $\dfrac{dy}{dx}=\dfrac{1+m\cos x}{1+2m\cos x+m^2}$.

12. Let $y=\left(x+1\right)^{\cot x}$, find $\dfrac{dy}{dx}$.

13. Show that $f'\left(x\right)=0$, where $f\left(x\right)=\tan^{-1}x+\tan^{-1}\dfrac{1}{x}$.

14. Consider the parametric curve $\begin{cases}x=t^2+2\\ y=t^3\end{cases}$ where $-\infty<t<\infty$.

    Find $\dfrac{dy}{dx}$, $\dfrac{d^2y}{dx^2}$ and the equation of the tangent line to the curve at the point $\left(3,1\right)$.

15. By Leibnitz’s theorem on repeated differentiation, find the *n*th derivatives of the function $y = e^x \cos 2x$.

-End-