EE3211 Modelling Techniques

Lecture 3

Hypothesis Testing: One-sample Inference

Overview

- Null hypothesis and alternative hypothesis
- Type I error and type II error
- Hypothesis testing using one-sample inference, including:
  - calculation of t statistics, critical value and p-value for one-sided alternative
  - computation of t statistics, critical value and p-value for two-sided alternative
- Calculate the power of a test and determine the appropriate sample size

Hypothesis Testing

- Hypothesis-testing framework: null and alternative hypothesis

- Hypothesis-testing: make decisions using probabilities methods, (not rely on subjective impressions)
    - uniform and consistent decision-making criterion

- **one-sample problem**: hypotheses about a single distribution

- **two-sample problem**: compare two different distributions

Hypothesis Testing

- The **null hypothesis (*H_o*)**: hypothesis to be tested
- The **alternative hypothesis (*H_1*)**: hypothesis contradicts the null hypothesis

$$H_0: \mu = \mu_0 \quad \text{vs.} \quad H_1: \mu < \mu_0$$

- Decisions: H_0 is true or H_1 is true
- All outcomes in a hypothesis testing situation: null hypothesis

- If we decide H_0 is true→ accept H_0

  If we decide H_1 is true→ H_0 is not true (reject H_0)

Table 7.1 Four possible outcomes in hypothesis testing

Decision	Truth: H_0	Truth: H_1
Accept H_0	H_0 is true and H_0 is accepted	H_1 is true and H_0 is accepted
Reject H_0	H_0 is true and H_0 is rejected	H_1 is true and H_0 is rejected

Hypothesis Testing

		Given the Null Hypothesis Is	
		True	False
Your Decision Based On a Random Sample	Reject	α Type I Error	Correct Decision
	Do Not Reject	Correct Decision	β Type II Error

- **Probability of a type I error:**
  α : significance level of a test

- **Probability of a type II error:**
- **β: function of μ and other factors**

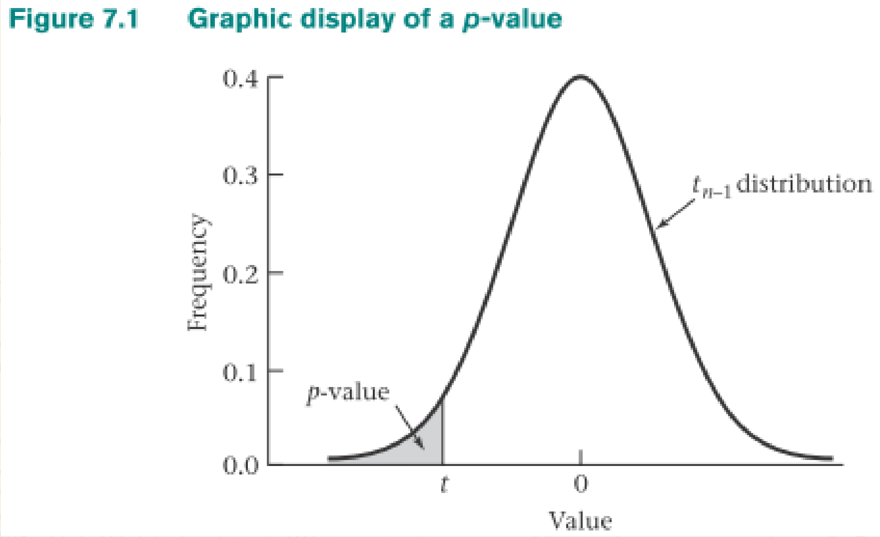
- **Power** of a test : 1 - β = 1 – probability of a type II error = Pr(rejecting $H_0|H_1$ true)

- **Objective of hypothesis testing**: use statistical tests that make α and β as small as possible

Determination of Statistical Significance for Results from Hypothesis Tests

- ***Critical-value method***: compute a test statistic and determine the outcome of a test by comparing the test statistic with a critical value determined by α (type I error)

- ***P-value:*** α level that we are not concerned between accepting or rejecting H_0



$$p = Pr(t_{n-1} \le t)$$

Determination of Statistical Significance for Results from Hypothesis Tests

Critical-value method

1) Compute test statistic t

2) Compare with critical value $t_{n-1,\alpha}$ at α level e.g. 0.05

H_0: $\mu = \mu_0$ vs. H_1: $\mu < \mu_0$

- $t < t_{n-1,0.05}$

  Reject H_0

Result: statistically significant (p<0.05)

- $t \geq t_{n-1,0.05}$

  Accept H_0

Result: not statistically significant (p≥0.05)

P-value method

-Compute exact p-value

- *If* $p < 0.05$

  Reject H_0

Result: statistically significant (p<0.05)

- if $p \geq 0.05$

  Accept H_0

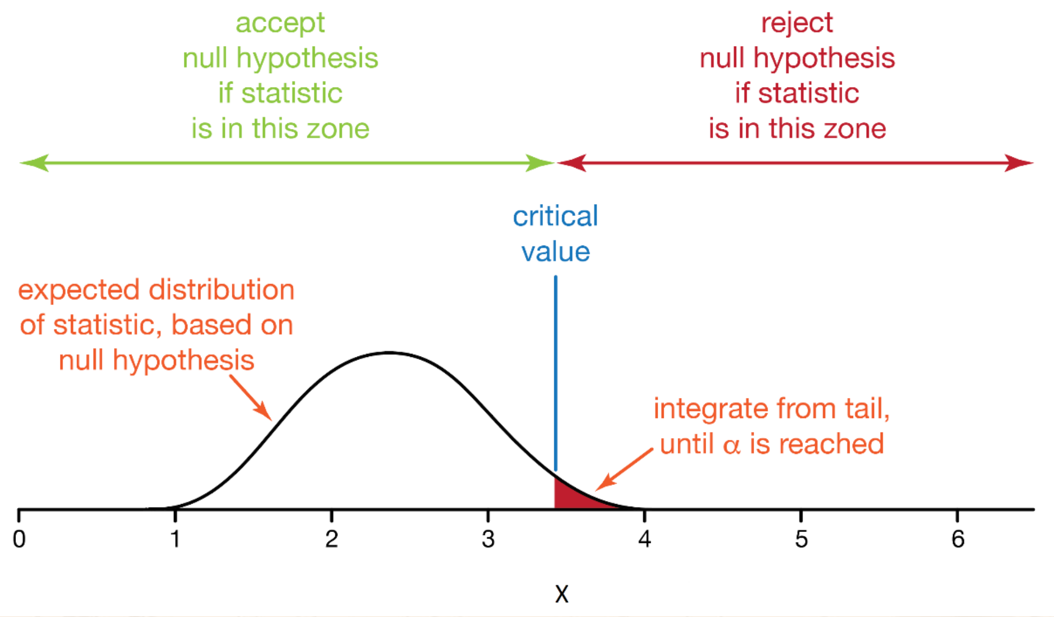
Result: not statistically significant (p≥0.05)

P-value

- ***P*-value:** probability of obtaining a test statistic as extreme as or more extreme than the actual test statistic obtained (null hypothesis is true)
  - If the null hypothesis is true → determine likelihood of getting me observed sample data
  - ”If the null hypothesis is true, are your sample data unusual?”

- **Guidance for Judging the significance of a p-value:**
  - If $0.01 \leq p < 0.05$: significant results
  - If $0.001 \leq p < 0.01$: highly significant results
  - If $p < 0.001$: very highly significant results
  - If $p > 0.05$: results are considered not statistically significant (NS)
  - If $0.05 \leq p < 0.1$: a trend toward statistical significance

One-Sample Test for the Mean of a Normal Distribution: One-Sided Alternatives



- **Acceptance region:** range of values of x that H_0 is accepted
- **Rejection region:** range of values of x that H_0 is rejected

- **One-tailed test:** values of the parameter being studied (i.e. μ) under the <u>alternative hypothesis</u> are allowed to be either > or < the values of the parameter under the <u>null hypothesis</u> (μ_0)

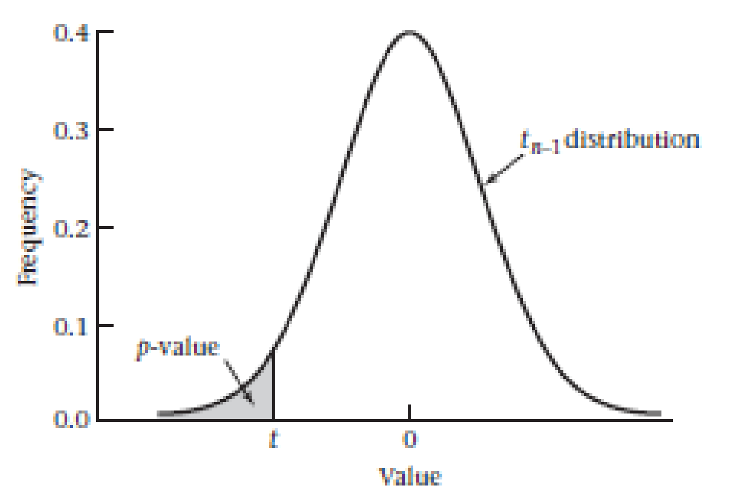
One-sample t Test for the Mean of a Normal Distribution with Unknown Variance (Alternative Mean < Null Mean)

- H_0: $\mu = \mu_0$ vs. H_1: μ **<** μ_0 * σ unknown*
  *with a significance level of α

$t = (x - \mu_0)/(s/\sqrt{n})$

- $t_{n\text{-}1,\alpha}$: **critical value**
- $t < t_{n\text{-}1,\alpha} \rightarrow$ Reject H_0
  $t \geq t_{n\text{-}1,\alpha} \rightarrow$ accept H_0

FIGURE 7.1 Graphic display of a *p*-value



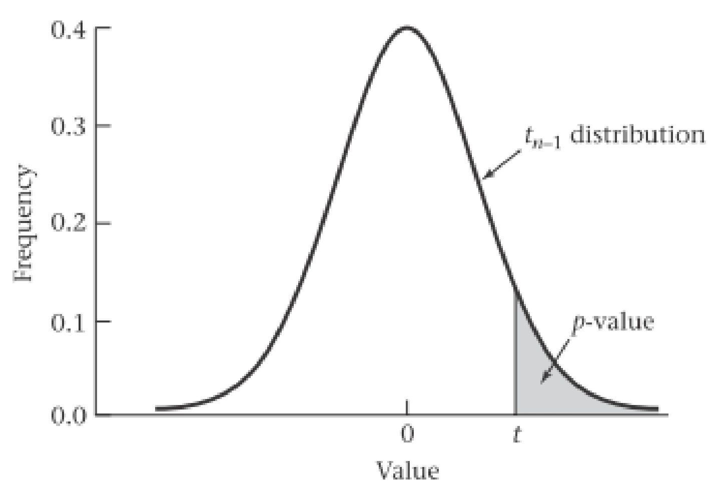
One-Sample t Test for the Mean of a Normal Distribution with Unknown Variance (Alternative Mean > Null Mean)

$H_0: \mu = \mu_0$ vs. $H_1: \mu > \mu_0$ with a significance level of α

$t = (\bar{x} - \mu_0)/(s/\sqrt{n})$

- $t > t_{n-1,1-\alpha}$ → reject H_0
- $t \leq t_{n-1,1-\alpha}$ → accept H_0
- P-value = $Pr(t > t_{n-1})$

Figure 7.2 p-value for the one-sample t test when the alternative mean (μ_1) > null mean (μ_0)



Example on One-Sample *t* Test for the Mean of a Normal Distribution with Unknown Variance: Cardiovascular Disease: Pediatrics

- A current area of research interest is the familial aggregation of cardiovascular risk factors in general and lipid levels in particular.

- Suppose the "average" cholesterol level in children is 175 mg/dL. A group of men who have died from heart disease within the past year are identified, and the cholesterol levels of their offspring are measured.

- Two hypotheses are considered:

(1) The average cholesterol level of these children is 175 mg/dL.
(2) The average cholesterol level of these children is >175 mg/dL.

- Suppose the mean cholesterol level of 10 children whose fathers died from heart disease is 200 mg/dL and the sample standard deviation is 50 mg/dL.

Q: Test the hypothesis that the mean cholesterol level is higher in this group than in the general population.

Example on One-Sample *t* Test for the Mean of a Normal Distribution with Unknown Variance: Cardiovascular Disease: Pediatrics

Solution:

- Hypothesis: $\mathrm{H}_0: \mu = 175 \quad vs. \quad \mathrm{H}_1: \mu > 175$ at α level of .05

- H_0 is rejected if: $t > t_{n-1,1-\alpha} = t_{9,.95}$

$$t = \frac{200 - 175}{50/\sqrt{10}} = \frac{25}{15.81} = 1.58$$

- *Table:* $t_{9,.95} = 1.833$
- 1.833 > 1.58 → accept *H*0 at the 5% level of significance
- use *p*-value method:
  - exact *p*-value : *p* = *Pr*(*t*9 > 1.58)
  - *t*9,.90 = 1.383 and *t*9,.95 = 1.833
  - because 1.383 < 1.58 < 1.833→ .05 < *p* < .10
  - using the pt function of R:
    exact *p*-value = *Pr*(*t*9 > 1.58) = 1 − pt (1.58,9) = .074 (*p* > .05)
- Conclusion: results are not statistically significant, and the null hypothesis is accepted
  → mean cholesterol level of these children does not differ significantly from that of an average child

TABLE 5 Percentage points of the t distribution ($t_{d,u}$)[a]

Degrees of freedom, d	u = .75	.80	.85	.90	.95	.975	.99	.995	.9995
1	1.000	1.376	1.963	3.078	6.314	12.706	31.821	63.657	636.619
2	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925	31.598
3	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	12.924
4	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	8.610
5	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	6.869
6	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	5.959
7	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	5.408
8	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	5.041
9	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	4.781
10	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	4.587
11	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106	4.437
12	0.695	0.873	1.083	1.356	1.782	2.179	2.681	3.055	4.318
13	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	4.221
14	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	4.140
15	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947	4.073
16	0.690	0.865	1.071	1.337	1.746	2.120	2.583	2.921	4.015
17	0.689	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.965
18	0.688	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.922
19	0.688	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.883
20	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.850
21	0.686	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.819
22	0.686	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.792
23	0.685	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.767
24	0.685	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.745
25	0.684	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.725
26	0.684	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.707
27	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.690
28	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.674
29	0.683	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.659
30	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.646
40	0.681	0.851	1.050	1.303	1.684	2.021	2.423	2.704	3.551
60	0.679	0.848	1.046	1.296	1.671	2.000	2.390	2.660	3.460
120	0.677	0.845	1.041	1.289	1.658	1.980	2.358	2.617	3.373
∞	0.674	0.842	1.036	1.282	1.645	1.960	2.326	2.576	3.291

[a]The uth percentile of a t distribution with d degrees of freedom.

Source: Table 5 is taken from Table III of Fisher and Yates: "Statistical Tables for Biological, Agricultural and Medical Research," published by Longman Group Ltd., London (previously published by Oliver and Boyd Ltd., Edinburgh).

One-Sample Test for the Mean of a Normal Distribution: Two-Sided Alternatives

- **Two-tailed test:** values of the parameter being studied (μ) under H_1 are allowed to be either > or < the values of the parameter under H_0 ($\mu = \mu_0$)

- Decision rule: reject H_0 if t is either too small or too large
  - if t is either $< c_1$ or $> c_2$ for some constants c_1, c_2 → reject H_0
  - if $c_1 \le t \le c_2$ → accept H_0

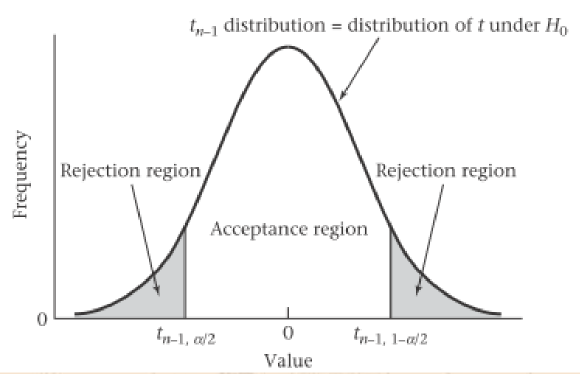
One-sample *t* Test for the Mean of a Normal Distribution with Unknown Variance (Two-Sided Alternative)

H_0: $\mu = \mu_0$ vs. H_1: $\mu \neq \mu_0$ with a significance level of α

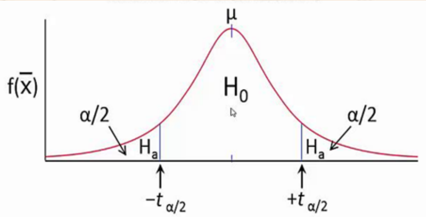
$t = (x - \mu_0)/ (s/\sqrt{n})$

- If $|t| > t_{n-1,1-\alpha/2}$ → reject H_0
- If $|t| < t_{n-1,1-\alpha/2}$ → accept H_0

Figure 7.3 One-sample *t* test for the mean of a normal distribution (two-sided alternative)



$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$



- μ: Population mean
- $\bar{x}$: Sample mean
- s: Sample standard deviation
- n: Sample size
- α: Type I error, the proportion of the time the incorrectly reject the null hypothesis
- t: The number of sample standard deviations (s) the sample mean ($\bar{x}$) is away from the population mean (μ)

Example on One-sample *t* Test for the Mean of a Normal Distribution with Unknown Variance: Cardiovascular Disease

- Suppose we want to compare fasting serum-cholesterol levels among recent Asian immigrants to the United States with typical levels found in the general U.S. population.
- Assumption: cholesterol levels in women ages 21−40 in the United States are approximately normally distributed with mean 190 mg/dL. It is unknown whether cholesterol levels among recent Asian immigrants are higher or lower than those in the general U.S. population.
- Let's assume that levels among recent female Asian immigrants are normally distributed with unknown mean μ.
- we wish to test *H*0: $\mu = \mu_0 = 190$ vs. *H*1: $\mu \neq \mu_0$. Blood tests are performed on 100 female Asian immigrants ages 21−40, and the mean level ($\bar{x}$) is 181.52 mg/dL with standard deviation = 40 mg/dL.

Q: Test the hypothesis that the mean cholesterol level of recent female Asian immigrants is different from the mean in the general U.S. population.

Example on One-sample *t* Test for the Mean of a Normal Distribution with Unknown Variance: Cardiovascular Disease

Solution:

Given that $t_{40,.975} = 2.021$, $t_{60,.975} = 2.000, t_{120,.975} = 1.980$

We compute the test statistic: $t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{181.52 - 190}{40/\sqrt{100}} = \frac{-8.48}{4} = -2.12$

- two-sided test with $\alpha = .05$: critical values are $c1 = t_{99,.025}$, $c2 = t_{99,.975}$.
- $t_{99,.975} < t_{60,.975} = 2.000 \rightarrow c2 < 2.000$
- $c1 = -c2 \rightarrow c1 > -2.000$
- $t = -2.12 < -2.000 < c1 \rightarrow$ reject H_0 at the 5% level of significance
- Conclusion: the mean cholesterol level of recent Asian immigrants is significantly different from that of the general U.S. population

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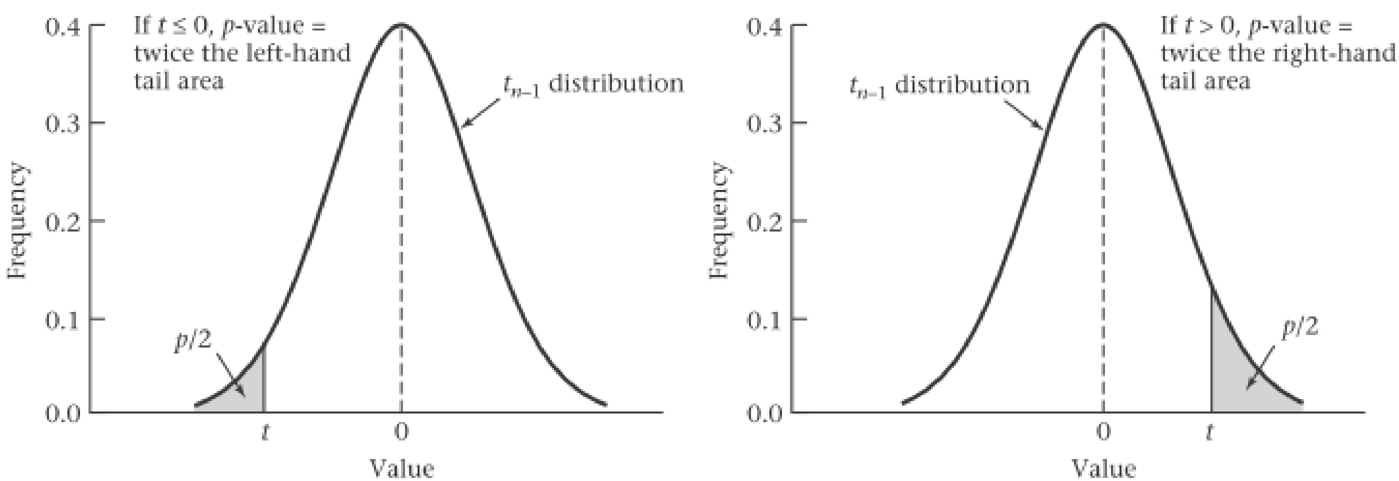
P-Value for the One-Sample t Test for the Mean of a Normal Distribution (Two-Sided Alternative)

Let $t = (\bar{x} - \mu_0)/(s/\sqrt{n})$

$$p = \begin{cases} 2 \times Pr(t_{n-1} \le t), \text{ if } t \le 0 \\ 2 \times [1 - Pr(t_{n-1} \le t)], \text{ if } t > 0 \end{cases}$$

- ***P*-value:** probability under H_0 of obtaining a test statistic as extreme as or more extreme than the observed test statistic
  - a two-sided H_1 is used
  - **absolute value** of t: measures extremeness

Figure 7.4 Illustration of the *p*-value for a one-sample *t* test for the mean of a normal distribution (two-sided alternative)



Example on P-Value calculation for the One-Sample t Test

Q: Compute the *p*-value for the hypothesis test in the previous example (cholesterol levels).

Solution:

- Because $t = -2.12$: *p*-value for the test is twice the left-hand tail area, or

$$p = 2 \times \Pr(t_{99} < -2.12) = 2 \times pt\,(-2.12{,}99) = .037$$

(using pt function of R)

- Conclusion: results are statistically significant with a *p*-value of .037

When is a one-sided test more appropriate than a two-sided test?

- One-sided test is easier to reject H_0: sample mean falls in the expected direction from μ_0

- One-sided test is better: only alternatives on one side of the null mean are of interest or possible
  - more power (easier to reject H_0 based on a finite sample if H_1 is true)

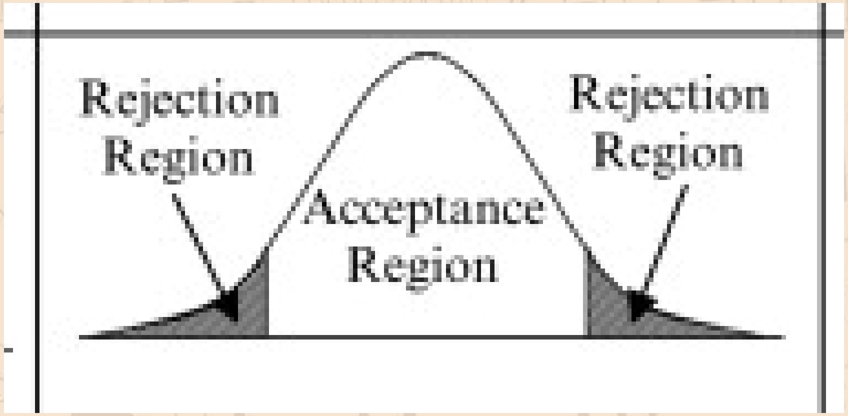
- Decision about using one-side or two-sided test should be made <u>before the data analysis</u> (or <u>before data collection</u>)
  - not to bias conclusions based on results of hypothesis testing
  - Do not change from a two-sided to a one-sided test after looking at the data

One-Sample z Test for the Mean of a Normal Distribution with Known Variance (Two-Sided Alternative)

H_0: $\mu = \mu_0$ vs. H_1: $\mu \neq \mu_0$ with a significance level of α

- standard deviation σ is known
  $z = (\bar{x} - \mu_0)/(\sigma/\sqrt{n})$

- If $z < z_{\alpha/2}$ or $z > z_{1-\alpha/2}$ → reject H_0
- If $z_{\alpha/2} \leq z \leq z_{1-\alpha/2}$ → accept H_0



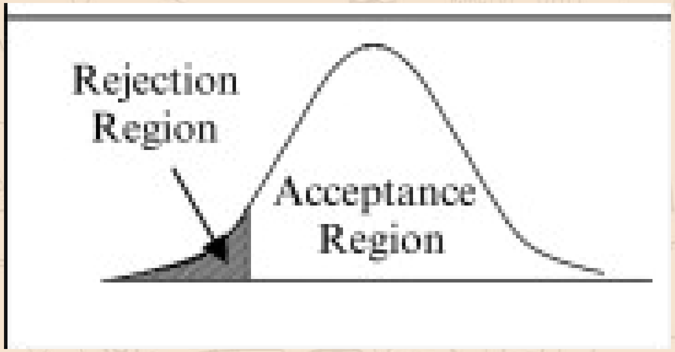
- Two-sided p-value calculation:
  $p = 2\Phi(z)$ if $z \leq 0$
  $p = 2[1 - \Phi(z)]$ if $z > 0$

One-Sample z Test for the Mean of a Normal Distribution with Known Variance (One-Sided Alternative)

H_0: $\mu = \mu_0$ vs. H_1: $\mu < \mu_0$ with a significance level of α

- standard deviation σ is known

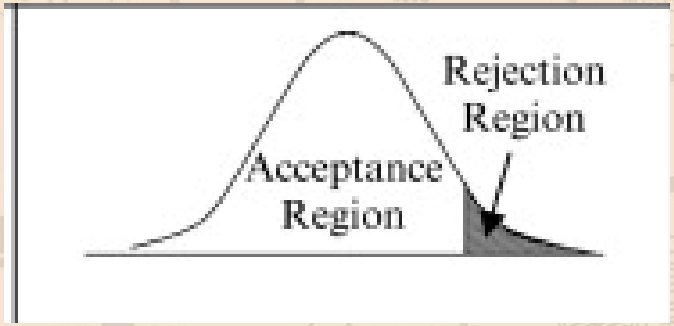
  $z = (x - \mu_0)/(\sigma/\sqrt{n})$
- If $z < z_\alpha$ → reject H_0
- if $z \geq z_\alpha$, → accept H_0 is accepted
- *p*-value: $p = \Phi(z)$



H_0: $\mu = \mu_0$ vs. H_1: $\mu > \mu_0$ with a significance level of α

- standard deviation σ is known

  $z = (x - \mu_0)/(\sigma/\sqrt{n})$
- If $z > z_{1-\alpha}$ → reject H_0
- if $z \leq z_{1-\alpha}$ → accept H_0
- *p*-value: $p = 1 - \Phi(z)$.



Example on One-Sample z Test for the Mean of a Normal Distribution with Known Variance

Q: Consider the cholesterol data in the cholesterol example. Assume that the standard deviation is known to be 40 and the sample size is 200 instead of 100. Assess the significance of the results.

Solution:

The test statistic : $$z = \frac{181.52 - 190}{40/\sqrt{200}} = \frac{-8.48}{2.828} = -3.00$$

- Critical-value method with α = 0.05
  - critical values are −1.96 and 1.96
- $z = -3.00 < -1.96$ -> reject $H0$ at a 5% level of significance
- two-sided *p*-value : $2 \times \Phi(-3.00) = .003$

TABLE 3 The normal distribution

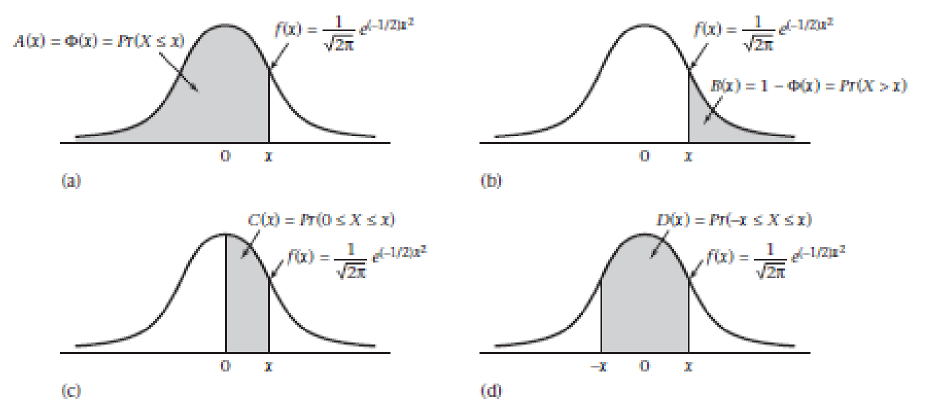


TABLE 3 The normal distribution (continued)

x	A[a]	B[b]	C[c]	D[d]
2.96	.9985	.0015	.4985	.9969
2.97	.9985	.0015	.4985	.9970
2.98	.9986	.0014	.4986	.9971
2.99	.9986	.0014	.4986	.9972
3.00	.9987	.0013	.4987	.9973
3.01	.9987	.0013	.4987	.9974
3.02	.9987	.0013	.4987	.9975
3.03	.9988	.0012	.4988	.9976
3.04	.9988	.0012	.4988	.9976
3.05	.9989	.0011	.4989	.9977
3.06	.9989	.0011	.4989	.9978
3.07	.9989	.0011	.4989	.9979
3.08	.9990	.0010	.4990	.9979
3.09	.9990	.0010	.4990	.9980
3.10	.9990	.0010	.4990	.9981
3.11	.9991	.0009	.4991	.9981
3.12	.9991	.0009	.4991	.9982
3.13	.9991	.0009	.4991	.9983
3.14	.9992	.0008	.4992	.9983
3.15	.9992	.0008	.4992	.9984
3.16	.9992	.0008	.4992	.9984
3.17	.9992	.0008	.4992	.9985
3.18	.9993	.0007	.4993	.9985
3.19	.9993	.0007	.4993	.9986
3.20	.9993	.0007	.4993	.9986
3.21	.9993	.0007	.4993	.9987
3.22	.9994	.0006	.4994	.9987
3.23	.9994	.0006	.4994	.9988
3.24	.9994	.0006	.4994	.9988
3.25	.9994	.0006	.4994	.9988
3.26	.9994	.0006	.4994	.9989
3.27	.9995	.0005	.4995	.9989
3.28	.9995	.0005	.4995	.9990
3.29	.9995	.0005	.4995	.9990
3.30	.9995	.0005	.4995	.9990
3.31	.9995	.0005	.4995	.9991
3.32	.9995	.0005	.4995	.9991
3.33	.9996	.0004	.4996	.9991
3.34	.9996	.0004	.4996	.9992
3.35	.9996	.0004	.4996	.9992
3.36	.9996	.0004	.4996	.9992
3.37	.9996	.0004	.4996	.9992
3.38	.9996	.0004	.4996	.9993
3.39	.9997	.0003	.4997	.9993
3.40	.9997	.0003	.4997	.9993
3.42	.9997	.0003	.4997	.9994
3.43	.9997	.0003	.4997	.9994
3.45	.9997	.0003	.4997	.9994
3.46	.9997	.0003	.4997	.9995
3.47	.9997	.0003	.4997	.9995
3.48	.9997	.0003	.4997	.9995

[a] $A(x) = \Phi(x) = Pr(X \le x)$, where X is a standard normal distribution.
[b] $B(x) = 1 - \Phi(x) = Pr(X > x)$, where X is a standard normal distribution.
[c] $C(x) = Pr(0 \le X \le x)$, where X is a standard normal distribution.
[d] $D(x) = Pr(-x \le X \le x)$, where X is a standard normal distribution.

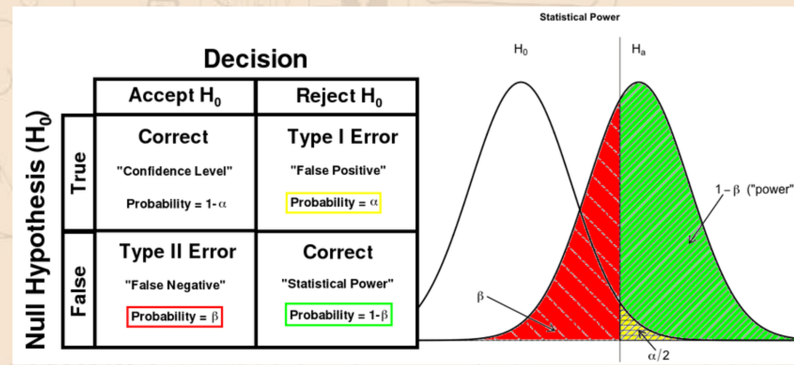
The Power of a Test

- how likely a statistically significant difference will be distinguished based on a finite sample size n

- probability that statistical test correctly rejects the null hypothesis

- Likelihood of a true positive result

- Probability of avoiding a Type II error



The power of the one-sided test (one-sample z test):

$$\Phi(z_\alpha + |\mu_0 - \mu_1|\sqrt{n}/\sigma) = \Phi(-z_{1-\alpha} + |\mu_0 - \mu_1|\sqrt{n}/\sigma)$$

The power of the two-sided test (one-sample z test):

$$\Phi\left[-z_{1-\alpha/2} + \frac{(\mu_0 - \mu_1)\sqrt{n}}{\sigma}\right] + \Phi\left[-z_{1-\alpha/2} + \frac{(\mu_1 - \mu_0)\sqrt{n}}{\sigma}\right]$$

Approx. by: $\Phi(-z_{1-\alpha/2} + |\mu_0 - \mu_1|\sqrt{n}/\sigma)$

Example on Power Calculation: Cardiovascular Disease, Pediatrics

Q: Using a 5% level of significance and a sample of size 10, compute the power of the test for the cholesterol data example, with an alternative mean of 190 mg/dL, a null mean of 175 mg/dL, and a standard deviation (σ) of 50 mg/dL.

Solution:

We have $\mu 0 = 175$, $\mu 1 = 190$, $\alpha = .05$, $\sigma = 50$, $n = 10$

$$Power = \Phi\left[-1.645 + \frac{(190-175)\sqrt{10}}{50}\right] = \Phi\left(-1.645 + \frac{15\sqrt{10}}{50}\right)$$
$$= \Phi(-0.696) = 1 - \Phi(0.696) = 1 - 0.757 = 0.243$$

- Chance of finding a significant difference in this case is only 24%
- it is not surprising that a significant difference was not found the previous example because the sample size was too small

TABLE 3 The normal distribution

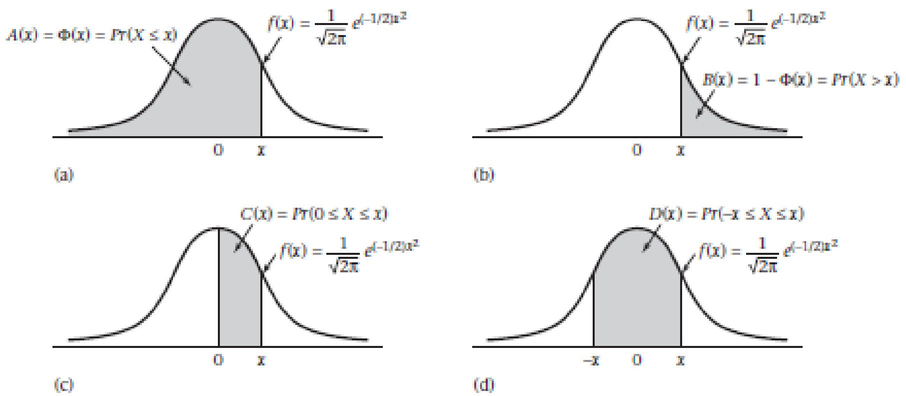


TABLE 3 The normal distribution (*continued*)

x	A[a]	B[b]	C[c]	D[d]
1.56	.9406	.0594	.4406	.8812
1.57	.9418	.0582	.4418	.8836
1.58	.9429	.0571	.4429	.8859
1.59	.9441	.0559	.4441	.8882
1.60	.9452	.0548	.4452	.8904
1.61	.9463	.0537	.4463	.8926
1.62	.9474	.0526	.4474	.8948
1.63	.9484	.0516	.4484	.8969
1.64	.9495	.0505	.4495	.8990
1.65	.9505	.0495	.4505	.9011
1.66	.9515	.0485	.4515	.9031
1.67	.9525	.0475	.4525	.9051
1.68	.9535	.0465	.4535	.9070
1.69	.9545	.0455	.4545	.9090
1.70	.9554	.0446	.4554	.9109
1.71	.9564	.0436	.4564	.9127
1.72	.9573	.0427	.4573	.9146
1.73	.9582	.0418	.4582	.9164
1.74	.9591	.0409	.4591	.9181
1.75	.9599	.0401	.4599	.9199
1.76	.9608	.0392	.4608	.9216
1.77	.9616	.0384	.4616	.9233
1.78	.9625	.0375	.4625	.9249
1.79	.9633	.0367	.4633	.9265
1.80	.9641	.0359	.4641	.9281
1.81	.9649	.0351	.4649	.9297
1.82	.9656	.0344	.4656	.9312
1.83	.9664	.0336	.4664	.9327
1.84	.9671	.0329	.4671	.9342
1.85	.9678	.0322	.4678	.9357
1.86	.9686	.0314	.4686	.9371
1.87	.9693	.0307	.4693	.9385
1.88	.9699	.0301	.4699	.9399
1.89	.9706	.0294	.4706	.9412
1.90	.9713	.0287	.4713	.9426
1.91	.9719	.0281	.4719	.9439
1.92	.9726	.0274	.4726	.9451
1.93	.9732	.0268	.4732	.9464
1.94	.9738	.0262	.4738	.9476
1.95	.9744	.0256	.4744	.9488
1.96	.9750	.0250	.4750	.9500
1.97	.9756	.0244	.4756	.9512
1.98	.9761	.0239	.4761	.9523
1.99	.9767	.0233	.4767	.9534
2.00	.9772	.0228	.4772	.9545
2.01	.9778	.0222	.4778	.9556
2.02	.9783	.0217	.4783	.9566
2.03	.9788	.0212	.4788	.9576
2.04	.9793	.0207	.4793	.9586
2.05	.9798	.0202	.4798	.9596
2.06	.9803	.0197	.4803	.9606
2.07	.9808	.0192	.4808	.9615
2.08	.9812	.0188	.4812	.9625
2.09	.9817	.0183	.4817	.9634
2.10	.9821	.0179	.4821	.9643
2.11	.9826	.0174	.4826	.9651
2.12	.9830	.0170	.4830	.9660
2.13	.9834	.0166	.4834	.9668
2.14	.9838	.0162	.4838	.9676
2.15	.9842	.0158	.4842	.9684
2.16	.9846	.0154	.4846	.9692
2.17	.9850	.0150	.4850	.9700
2.18	.9854	.0146	.4854	.9707
2.19	.9857	.0143	.4857	.9715
2.20	.9861	.0139	.4861	.9722
2.21	.9864	.0136	.4864	.9729
2.22	.9868	.0132	.4868	.9736
2.23	.9871	.0129	.4871	.9743
2.24	.9875	.0125	.4875	.9749
2.25	.9878	.0122	.4878	.9756
2.26	.9881	.0119	.4881	.9762
2.27	.9884	.0116	.4884	.9768
2.28	.9887	.0113	.4887	.9774
2.29	.9890	.0110	.4890	.9780
2.30	.9893	.0107	.4893	.9786
2.31	.9896	.0104	.4896	.9791
2.32	.9898	.0102	.4898	.9797
2.33	.9901	.0099	.4901	.9802
2.34	.9904	.0096	.4904	.9807
2.35	.9906	.0094	.4906	.9812
2.36	.9909	.0091	.4909	.9817
2.37	.9911	.0089	.4911	.9822
2.38	.9913	.0087	.4913	.9827

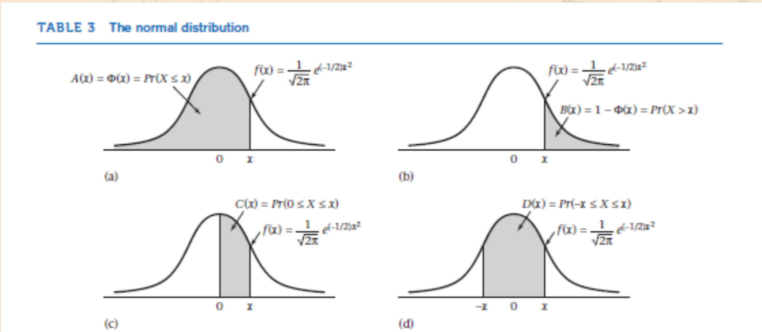
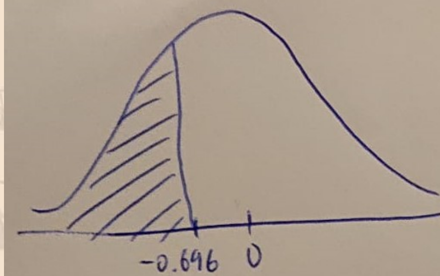
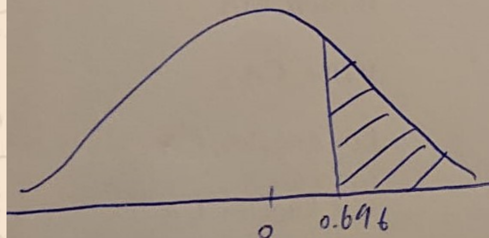
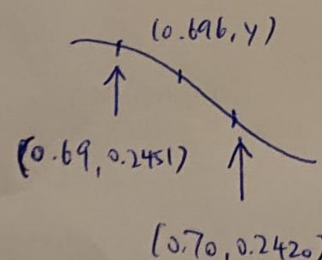


TABLE 3 The normal distribution *(continued)*

x	A^a	B^b	C^c	D^d
0,64	,7389	,2611	,2389	,4778
0,65	,7422	,2578	,2422	,4843
0,66	,7454	,2546	,2454	,4907
0,67	,7486	,2514	,2486	,4971
0,68	,7517	,2483	,2517	,5035
0,69	,7549	$\boxed{,2451}$	,2549	,5098
0,70	,7580	$\boxed{,2420}$	,2580	,5161
0,71	,7611	,2389	,2611	,5223
0,72	,7642	,2358	,2642	,5285
0,73	,7673	,2327	,2673	,5346
0,74	,7703	,2297	,2703	,5407
0,75	,7734	,2266	,2734	,5467
0,76	,7764	,2236	,2764	,5527
0,77	,7793	,2207	,2793	,5587
0,78	,7823	,2177	,2823	,5646
0,79	,7852	,2148	,2852	,5705
0,80	,7881	,2119	,2881	,5763
0,81	,7910	,2090	,2910	,5821
0,82	,7939	,2061	,2939	,5878
0,83	,7967	,2033	,2967	,5935
0,84	,7995	,2005	,2995	,5991
0,85	,8023	,1977	,3023	,6047
0,86	,8051	,1949	,3051	,6102
0,87	,8078	,1922	,3078	,6157
0,88	,8106	,1894	,3106	,6211
0,89	,8133	,1867	,3133	,6265
0,90	,8159	,1841	,3159	,6319
0,91	,8186	,1814	,3186	,6372
0,92	,8212	,1788	,3212	,6424







$0.2451 > y > 0.2420$

$$\frac{0.2451-0.2420}{0.69-0.70} = \frac{0.2451-y}{0.69-0.696}$$

$$\frac{0.0031}{-0.01} = \frac{0.2451-y}{-0.006}$$

$$-0.0000186 = -0.002451 + 0.01y$$

$$y = 0.243$$

Example on Power Calculation: Obstetrics

Suppose we want to test the hypothesis that mothers with low socioeconomic status (SES) deliver babies whose birthweights are lower than "normal." To test this hypothesis, a list is obtained of birthweights from 100 consecutive, full-term, live-born deliveries from the maternity ward of a hospital in a low-SES area. The mean birthweight (*x*) is found to be 115 oz with a sample standard deviation (*s*) of 24 oz. Suppose we know from nationwide surveys based on millions of deliveries that the mean birthweight in the United States is 120 oz. Compute the power of the test for the birthweight data with an alternative mean of 115 oz and α = .05, assuming the true standard deviation = 24 oz.

Q: Assuming a sample size of 10 rather than 100, compute the power for the birthweight data with an alternative mean of 115 oz and α = .05.

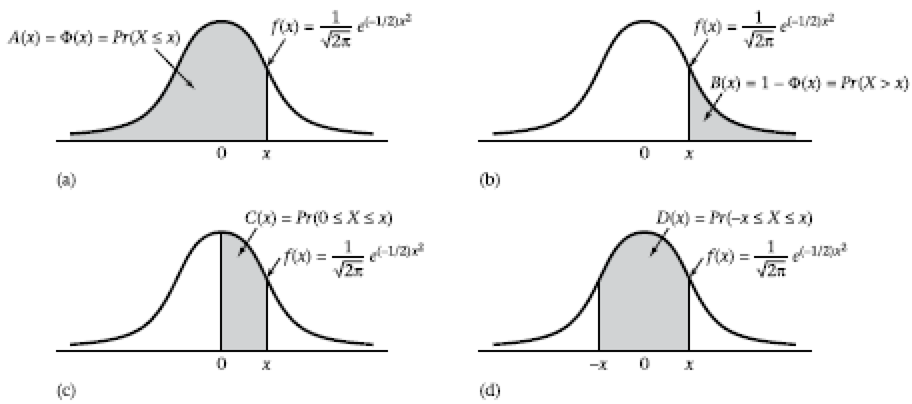
Solution:

We have $\mu 0 = 120\, oz, \mu 1 = 115\, oz, \alpha = .05, \sigma = 24, n = 100.$

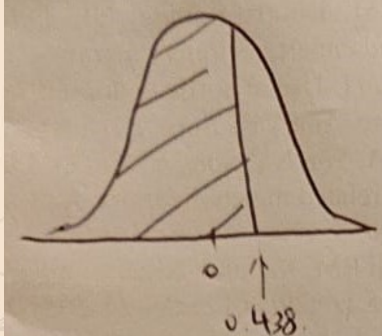
$$Power = \Phi\left[z_{.05} + \frac{(120-115)\sqrt{100}}{24}\right] = \Phi\left[-1.645 + \frac{5(10)}{24}\right]$$
$$= \Phi(0.428) = 0.669$$

- There is about a 67% chance of detecting a significant difference using a 5% significance level with this sample size

TABLE 3 The normal distribution



x	A[a]	B[b]	C[c]	D[d]	x	A	B	C	D
0,0	,5000	,5000	,0	,0	0,32	,6255	,3745	,1255	,2510
0,01	,5040	,4960	,0040	,0080	0,33	,6293	,3707	,1293	,2586
0,02	,5080	,4920	,0080	,0160	0,34	,6331	,3669	,1331	,2661
0,03	,5120	,4880	,0120	,0239	0,35	,6368	,3632	,1368	,2737
0,04	,5160	,4840	,0160	,0319	0,36	,6406	,3594	,1406	,2812
0,05	,5199	,4801	,0199	,0399	0,37	,6443	,3557	,1443	,2886
0,06	,5239	,4761	,0239	,0478	0,38	,6480	,3520	,1480	,2961
0,07	,5279	,4721	,0279	,0558	0,39	,6517	,3483	,1517	,3035
0,08	,5319	,4681	,0319	,0638	0,40	,6554	,3446	,1554	,3108
0,09	,5359	,4641	,0359	,0717	0,41	,6591	,3409	,1591	,3182
0,10	,5398	,4602	,0398	,0797	0,42	,6628	,3372	,1628	,3255
0,11	,5438	,4562	,0438	,0876	0,43	,6664	,3336	,1664	,3328
0,12	,5478	,4522	,0478	,0955	0,44	,6700	,3300	,1700	,3401
0,13	,5517	,4483	,0517	,1034	0,45	,6736	,3264	,1736	,3473
0,14	,5557	,4443	,0557	,1113	0,46	,6772	,3228	,1772	,3545
0,15	,5596	,4404	,0596	,1192	0,47	,6808	,3192	,1808	,3616
0,16	,5636	,4364	,0636	,1271	0,48	,6844	,3156	,1844	,3688
0,17	,5675	,4325	,0675	,1350	0,49	,6879	,3121	,1879	,3759
0,18	,5714	,4286	,0714	,1428	0,50	,6915	,3085	,1915	,3829
0,19	,5753	,4247	,0753	,1507	0,51	,6950	,3050	,1950	,3899
0,20	,5793	,4207	,0793	,1585	0,52	,6985	,3015	,1985	,3969
0,21	,5832	,4168	,0832	,1663	0,53	,7019	,2981	,2019	,4039
0,22	,5871	,4129	,0871	,1741	0,54	,7054	,2946	,2054	,4108
0,23	,5910	,4090	,0910	,1819	0,55	,7088	,2912	,2088	,4177
0,24	,5948	,4052	,0948	,1897	0,56	,7123	,2877	,2123	,4245
0,25	,5987	,4013	,0987	,1974	0,57	,7157	,2843	,2157	,4313
0,26	,6026	,3974	,1026	,2051	0,58	,7190	,2810	,2190	,4381
0,27	,6064	,3936	,1064	,2128	0,59	,7224	,2776	,2224	,4448
0,28	,6103	,3897	,1103	,2205	0,60	,7257	,2743	,2257	,4515
0,29	,6141	,3859	,1141	,2282	0,61	,7291	,2709	,2291	,4581
0,30	,6179	,3821	,1179	,2358	0,62	,7324	,2676	,2324	,4647
0,31	,6217	,3783	,1217	,2434	0,63	,7357	,2643	,2357	,4713



X	$\Phi(x)$
0.44	0.6700
0.438	? (y)
0.43	0.6664

$$\frac{0.67-0.6664}{0.44-0.43} = \frac{0.67-y}{0.44-0.438}$$

$$\frac{0.0036}{0.01} = \frac{0.67-y}{0.002}$$

$$0.0000072 = 0.0067 - .01y$$

$$y = 0.669$$

Example on Power Calculation: Obstetrics

$\mu 0 = 170\, oz, \mu 1 = 190\,, \alpha = .05, \sigma = 50, n = 10$:

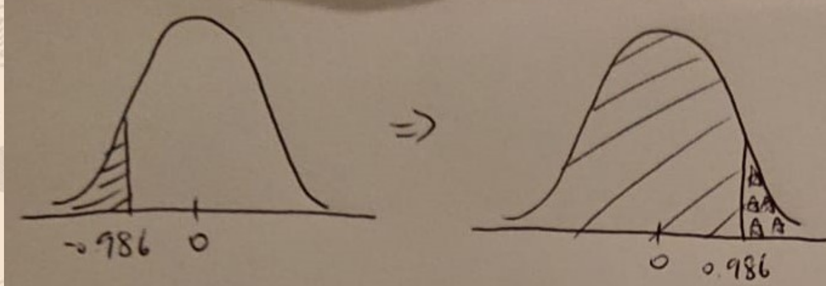
$$Power = \Phi\left[z_{.05} + \frac{(120-115)\sqrt{10}}{24}\right] = \Phi\left[-1.645 + \frac{5\sqrt{10}}{24}\right] = \Phi(-0.986)$$
$$= 1 - 0.838 = 0.162$$

- there is only a 16% chance of finding a significant difference with a sample size of 10
- whereas there was a 67% chance with a sample size of 100

→ if 10 infants were sampled, we would have virtually no chance of finding a significant difference and would almost surely report a false-negative result

TABLE 3 The normal distribution *(continued)*

x	A^a	B^b	C^c	D^d
0.64	.7389	.2611	.2389	.4778
0.65	.7422	.2578	.2422	.4843
0.66	.7454	.2546	.2454	.4907
0.67	.7486	.2514	.2486	.4971
0.68	.7517	.2483	.2517	.5035
0.69	.7549	.2451	.2549	.5098
0.70	.7580	.2420	.2580	.5161
0.71	.7611	.2389	.2611	.5223
0.72	.7642	.2358	.2642	.5285
0.73	.7673	.2327	.2673	.5346
0.74	.7703	.2297	.2703	.5407
0.75	.7734	.2266	.2734	.5467
0.76	.7764	.2236	.2764	.5527
0.77	.7793	.2207	.2793	.5587
0.78	.7823	.2177	.2823	.5646
0.79	.7852	.2148	.2852	.5705
0.80	.7881	.2119	.2881	.5763
0.81	.7910	.2090	.2910	.5821
0.82	.7939	.2061	.2939	.5878
0.83	.7967	.2033	.2967	.5935
0.84	.7995	.2005	.2995	.5991
0.85	.8023	.1977	.3023	.6047
0.86	.8051	.1949	.3051	.6102
0.87	.8078	.1922	.3078	.6157
0.88	.8106	.1894	.3106	.6211
0.89	.8133	.1867	.3133	.6265
0.90	.8159	.1841	.3159	.6319
0.91	.8186	.1814	.3186	.6372
0.92	.8212	.1788	.3212	.6424
0.93	.8238	.1762	.3238	.6476
0.94	.8264	.1736	.3264	.6528
0.95	.8289	.1711	.3289	.6579
0.96	.8315	.1685	.3315	.6629
0.97	.8340	.1660	.3340	.6680
0.98	.8365	.1635	.3365	.6729
0.99	.8389	.1611	.3389	.6778
1.00	.8413	.1587	.3413	.6827
1.01	.8438	.1562	.3438	.6875
1.02	.8461	.1539	.3461	.6923
1.03	.8485	.1515	.3485	.6970
1.04	.8508	.1492	.3508	.7017
1.05	.8531	.1469	.3531	.7063
1.06	.8554	.1446	.3554	.7109
1.07	.8577	.1423	.3577	.7154
1.08	.8599	.1401	.3599	.7199
1.09	.8621	.1379	.3621	.7243
1.10	.8643	.1357	.3643	.7287
1.11	.8665	.1335	.3665	.7330
1.12	.8686	.1314	.3686	.7373
1.13	.8708	.1292	.3708	.7415
1.14	.8729	.1271	.3729	.7457
1.15	.8749	.1251	.3749	.7499
1.16	.8770	.1230	.3770	.7540
1.17	.8790	.1210	.3790	.7580
1.18	.8810	.1190	.3810	.7620
1.19	.8830	.1170	.3830	.7660
1.20	.8849	.1151	.3849	.7699
1.21	.8869	.1131	.3869	.7737
1.22	.8888	.1112	.3888	.7775



$$\Phi(-0.986) = 1 - \Phi(0.986)$$

X	$\Phi(X)$
0.98	0.8365
0.986	?(y)
0.99	0.8389

$$\frac{0.99 - 0.98}{0.8389 - 0.8365} = \frac{0.99 - 0.986}{0.8389 - y}$$

$$\frac{0.01}{0.0024} = \frac{0.004}{0.8389 - y}$$

$$0.0000096 = 0.008389 - 0.01y$$

$$y = 0.838$$

Factors affecting the power

E.g. 2-sided one-sample test: $\Phi(-z_{1-\alpha/2} + |\mu_0 - \mu_1|\sqrt{n}/\sigma)$

- If the significance level is made smaller (α decreases)→ z_α increases → power decreases
- If the alternative mean is shifted farther away from the null mean ($|\mu_0 - \mu_1|$ increases) → power increases
- If σ of the distribution of individual observations increases (σ increases) → power decreases
- If the sample size increases (n increases) →power increases

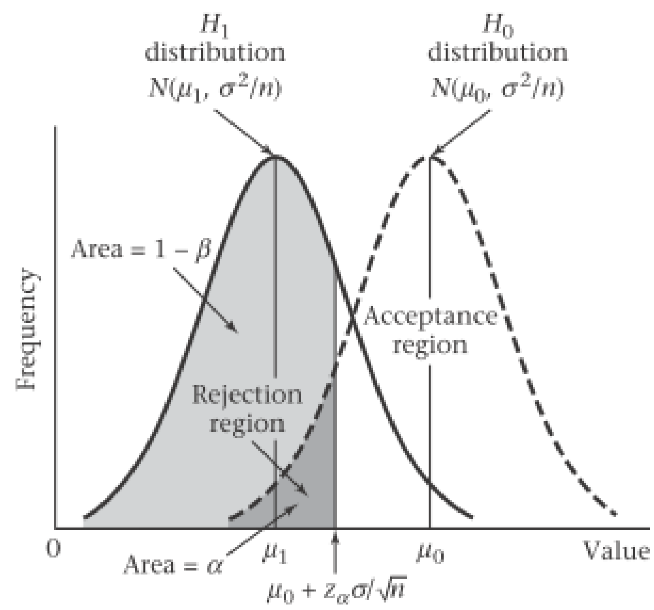
Sample-Size Determination: One-Sided Alternatives

H_0: $\mu = \mu_0$ vs. H_1: $\mu = \mu_1$, data ~ $N(\mu, \sigma^2)$

- What is the same size for a one-sided test with significance level α and probability of detecting a significant difference = 1- β?

$$n = \frac{\sigma^2 \left(z_{1-\beta} + z_{1-\alpha}\right)^2}{\left(\mu_0 - \mu_1\right)^2}$$

Figure 7.9 Requirements for appropriate sample size



Factors Affecting the Sample Size (n)

$$n = \frac{\sigma^2 \left(z_{1-\beta} + z_{1-\alpha}\right)^2}{\left(\mu_0 - \mu_1\right)^2}$$

- σ^2 increases → n increases

- The significance level is made smaller (α decreases) → n increases

- The required power increases (1- β increases) → n increases

- The absolute value of the distance between the null and alternative means ($|\mu_0 - \mu_1|$) increases → n decreases

Sample-Size Estimation When Testing for the Mean of a Normal Distribution

(Two-Sided Alternative)

H_0: $\mu = \mu_0$ vs. H_1: $\mu = \mu_1$, data ~ $N(\mu, \sigma^2)$

- What is the sample size for a one-sided test with significance level α and power 1- β ?

$$n = \frac{\sigma^2 \left(z_{1-\beta} + z_{1-\alpha/2}\right)^2}{\left(\mu_0 - \mu_1\right)^2}$$

<u>Sample-Size Estimation Based on CI Width</u>

- What is the mean of a normal distribution with sample variance s^2?
  - The two-sided 100% × (1 - α) CI for μ be no wider than L

$$n = 4z_{1-\alpha/2}^2 s^2 / L^2$$

Example on Sample Size Estimation: Obstetrics

Consider the birthweight data. Suppose that $\mu 0 = 120$ oz, $\mu 1 = 115$ oz, $\sigma = 24$, $\alpha = .05$, $1 - \beta = .80$.
Q: Using a one-sided test, compute the appropriate sample size needed to conduct the test.

Solution:

$$n = \frac{24^2(z_{.8} + z_{.95})^2}{25} = 23.04(0.84 + 1.645)^2 = 23.04(6.175) = 142.3$$

- sample size is always rounded up → sure to achieve at least the required level of power (in this case, 80%)
- a sample size of 143 is needed to have an 80% chance of detecting a significant difference at the 5% level (if the alternative mean is 115 oz and a one-sided test is used)

TABLE 3 The normal distribution *(continued)*

x	A[a]	B[b]	C[c]	D[d]	x	A	B	C	D
0.64	.7389	.2611	.2389	.4778	1.23	.8907	.1093	.3907	.7813
0.65	.7422	.2578	.2422	.4843	1.24	.8925	.1075	.3925	.7850
0.66	.7454	.2546	.2454	.4907	1.25	.8944	.1056	.3944	.7887
0.67	.7486	.2514	.2486	.4971	1.26	.8962	.1038	.3962	.7923
0.68	.7517	.2483	.2517	.5035	1.27	.8980	.1020	.3980	.7959
0.69	.7549	.2451	.2549	.5098	1.28	.8997	.1003	.3997	.7995
0.70	.7580	.2420	.2580	.5161	1.29	.9015	.0985	.4015	.8029
0.71	.7611	.2389	.2611	.5223	1.30	.9032	.0968	.4032	.8064
0.72	.7642	.2358	.2642	.5285	1.31	.9049	.0951	.4049	.8098
0.73	.7673	.2327	.2673	.5346	1.32	.9066	.0934	.4066	.8132
0.74	.7703	.2297	.2703	.5407	1.33	.9082	.0918	.4082	.8165
0.75	.7734	.2266	.2734	.5467	1.34	.9099	.0901	.4099	.8198
0.76	.7764	.2236	.2764	.5527	1.35	.9115	.0885	.4115	.8230
0.77	.7793	.2207	.2793	.5587	1.36	.9131	.0869	.4131	.8262
0.78	.7823	.2177	.2823	.5646	1.37	.9147	.0853	.4147	.8293
0.79	.7852	.2148	.2852	.5705	1.38	.9162	.0838	.4162	.8324
0.80	.7881	.2119	.2881	.5763	1.39	.9177	.0823	.4177	.8355
0.81	.7910	.2090	.2910	.5821	1.40	.9192	.0808	.4192	.8385
0.82	.7939	.2061	.2939	.5878	1.41	.9207	.0793	.4207	.8415
0.83	.7967	.2033	.2967	.5935	1.42	.9222	.0778	.4222	.8444
0.84	.7995	.2005	.2995	.5991	1.43	.9236	.0764	.4236	.8473
0.85	.8023	.1977	.3023	.6047	1.44	.9251	.0749	.4251	.8501
0.86	.8051	.1949	.3051	.6102	1.45	.9265	.0735	.4265	.8529
0.87	.8078	.1922	.3078	.6157	1.46	.9279	.0721	.4279	.8557
0.88	.8106	.1894	.3106	.6211	1.47	.9292	.0708	.4292	.8584
0.89	.8133	.1867	.3133	.6265	1.48	.9306	.0694	.4306	.8611
0.90	.8159	.1841	.3159	.6319	1.49	.9319	.0681	.4319	.8638
0.91	.8186	.1814	.3186	.6372	1.50	.9332	.0668	.4332	.8664
0.92	.8212	.1788	.3212	.6424	1.51	.9345	.0655	.4345	.8690
0.93	.8238	.1762	.3238	.6476	1.52	.9357	.0643	.4357	.8715
0.94	.8264	.1736	.3264	.6528	1.53	.9370	.0630	.4370	.8740
0.95	.8289	.1711	.3289	.6579	1.54	.9382	.0618	.4382	.8764
0.96	.8315	.1685	.3315	.6629	1.55	.9394	.0606	.4394	.8789
0.97	.8340	.1660	.3340	.6680	1.56	.9406	.0594	.4406	.8812
0.98	.8365	.1635	.3365	.6729	1.57	.9418	.0582	.4418	.8836
0.99	.8389	.1611	.3389	.6778	1.58	.9429	.0571	.4429	.8859
1.00	.8413	.1587	.3413	.6827	1.59	.9441	.0559	.4441	.8882
1.01	.8438	.1562	.3438	.6875	1.60	.9452	.0548	.4452	.8904
1.02	.8461	.1539	.3461	.6923	1.61	.9463	.0537	.4463	.8926
1.03	.8485	.1515	.3485	.6970	1.62	.9474	.0526	.4474	.8948
1.04	.8508	.1492	.3508	.7017	1.63	.9484	.0516	.4484	.8969
1.05	.8531	.1469	.3531	.7063	1.64	.9495	.0505	.4495	.8990
1.06	.8554	.1446	.3554	.7109	1.65	.9505	.0495	.4505	.9011
1.07	.8577	.1423	.3577	.7154	1.66	.9515	.0485	.4515	.9031
1.08	.8599	.1401	.3599	.7199	1.67	.9525	.0475	.4525	.9051
1.09	.8621	.1379	.3621	.7243	1.68	.9535	.0465	.4535	.9070
1.10	.8643	.1357	.3643	.7287	1.69	.9545	.0455	.4545	.9090
1.11	.8665	.1335	.3665	.7330	1.70	.9554	.0446	.4554	.9109
1.12	.8686	.1314	.3686	.7373	1.71	.9564	.0436	.4564	.9127
1.13	.8708	.1292	.3708	.7415	1.72	.9573	.0427	.4573	.9146
1.14	.8729	.1271	.3729	.7457	1.73	.9582	.0418	.4582	.9164
1.15	.8749	.1251	.3749	.7499	1.74	.9591	.0409	.4591	.9181
1.16	.8770	.1230	.3770	.7540	1.75	.9599	.0401	.4599	.9199
1.17	.8790	.1210	.3790	.7580	1.76	.9608	.0392	.4608	.9216
1.18	.8810	.1190	.3810	.7620	1.77	.9616	.0384	.4616	.9233
1.19	.8830	.1170	.3830	.7660	1.78	.9625	.0375	.4625	.9249
1.20	.8849	.1151	.3849	.7699	1.79	.9633	.0367	.4633	.9265
1.21	.8869	.1131	.3869	.7737	1.80	.9641	.0359	.4641	.9281
1.22	.8888	.1112	.3888	.7775	1.81	.9649	.0351	.4649	.9297

Example on Sample Size Estimation: Cardiology

Suppose it is well known that propranolol lowers heart rate over 48 hours when given to patients with angina at standard dosage levels. A new study is proposed using a higher dose of propranolol than the standard one. Investigators are interested in estimating the drop in heart rate with high precision.

Q: Find the minimum sample size needed to estimate the change in heart rate (μ), if we require that the two-sided 95% CI for μ be no wider than 5 beats per minute and the sample standard deviation for change in heart rate equals 10 beats per minute.

Solution:

$$n = \frac{4(z_{.975})^2(10)^2}{(5)^2} = \frac{4(1.96)^2(100)}{25} = 61.5$$

We have α =.05, s=10,L=5

- 62 patients need to be studied

TABLE 3 The normal distribution *(continued)*

x	A^a	B^b	C^c	D^d	x	A	B	C	D
1.82	.9656	.0344	.4656	.9312	2.39	.9916	.0084	.4916	.9832
1.83	.9664	.0336	.4664	.9327	2.40	.9918	.0082	.4918	.9836
1.84	.9671	.0329	.4671	.9342	2.41	.9920	.0080	.4920	.9840
1.85	.9678	.0322	.4678	.9357	2.42	.9922	.0078	.4922	.9845
1.86	.9686	.0314	.4686	.9371	2.43	.9925	.0075	.4925	.9849
1.87	.9693	.0307	.4693	.9385	2.44	.9927	.0073	.4927	.9853
1.88	.9699	.0301	.4699	.9399	2.45	.9929	.0071	.4929	.9857
1.89	.9706	.0294	.4706	.9412	2.46	.9931	.0069	.4931	.9861
1.90	.9713	.0287	.4713	.9426	2.47	.9932	.0068	.4932	.9865
1.91	.9719	.0281	.4719	.9439	2.48	.9934	.0066	.4934	.9869
1.92	.9726	.0274	.4726	.9451	2.49	.9936	.0064	.4936	.9872
1.93	.9732	.0268	.4732	.9464	2.50	.9938	.0062	.4938	.9876
1.94	.9738	.0262	.4738	.9476	2.51	.9940	.0060	.4940	.9879
1.95	.9744	.0256	.4744	.9488	2.52	.9941	.0059	.4941	.9883
1.96	.9750	.0250	.4750	.9500	2.53	.9943	.0057	.4943	.9886
1.97	.9756	.0244	.4756	.9512	2.54	.9945	.0055	.4945	.9889
1.98	.9761	.0239	.4761	.9523	2.55	.9946	.0054	.4946	.9892
1.99	.9767	.0233	.4767	.9534	2.56	.9948	.0052	.4948	.9895
2.00	.9772	.0228	.4772	.9545	2.57	.9949	.0051	.4949	.9898
2.01	.9778	.0222	.4778	.9556	2.58	.9951	.0049	.4951	.9901
2.02	.9783	.0217	.4783	.9566	2.59	.9952	.0048	.4952	.9904
2.03	.9788	.0212	.4788	.9576	2.60	.9953	.0047	.4953	.9907
2.04	.9793	.0207	.4793	.9586	2.61	.9955	.0045	.4955	.9909
2.05	.9798	.0202	.4798	.9596	2.62	.9956	.0044	.4956	.9912
2.06	.9803	.0197	.4803	.9606	2.63	.9957	.0043	.4957	.9915
2.07	.9808	.0192	.4808	.9615	2.64	.9959	.0041	.4959	.9917
2.08	.9812	.0188	.4812	.9625	2.65	.9960	.0040	.4960	.9920
2.09	.9817	.0183	.4817	.9634	2.66	.9961	.0039	.4961	.9922
2.10	.9821	.0179	.4821	.9643	2.67	.9962	.0038	.4962	.9924
2.11	.9826	.0174	.4826	.9651	2.68	.9963	.0037	.4963	.9926
2.12	.9830	.0170	.4830	.9660	2.69	.9964	.0036	.4964	.9929
2.13	.9834	.0166	.4834	.9668	2.70	.9965	.0035	.4965	.9931
2.14	.9838	.0162	.4838	.9676	2.71	.9966	.0034	.4966	.9933
2.15	.9842	.0158	.4842	.9684	2.72	.9967	.0033	.4967	.9935
2.16	.9846	.0154	.4846	.9692	2.73	.9968	.0032	.4968	.9937
2.17	.9850	.0150	.4850	.9700	2.74	.9969	.0031	.4969	.9939
2.18	.9854	.0146	.4854	.9707	2.75	.9970	.0030	.4970	.9940
2.19	.9857	.0143	.4857	.9715	2.76	.9971	.0029	.4971	.9942
2.20	.9861	.0139	.4861	.9722	2.77	.9972	.0028	.4972	.9944
2.21	.9864	.0136	.4864	.9729	2.78	.9973	.0027	.4973	.9946
2.22	.9868	.0132	.4868	.9736	2.79	.9974	.0026	.4974	.9947
2.23	.9871	.0129	.4871	.9743	2.80	.9974	.0026	.4974	.9949
2.24	.9875	.0125	.4875	.9749	2.81	.9975	.0025	.4975	.9950
2.25	.9878	.0122	.4878	.9756	2.82	.9976	.0024	.4976	.9952
2.26	.9881	.0119	.4881	.9762	2.83	.9977	.0023	.4977	.9953
2.27	.9884	.0116	.4884	.9768	2.84	.9977	.0023	.4977	.9955
2.28	.9887	.0113	.4887	.9774	2.85	.9978	.0022	.4978	.9956
2.29	.9890	.0110	.4890	.9780	2.86	.9979	.0021	.4979	.9958
2.30	.9893	.0107	.4893	.9786	2.87	.9979	.0021	.4979	.9959
2.31	.9896	.0104	.4896	.9791	2.88	.9980	.0020	.4980	.9960
2.32	.9898	.0102	.4898	.9797	2.89	.9981	.0019	.4981	.9961
2.33	.9901	.0099	.4901	.9802	2.90	.9981	.0019	.4981	.9963
2.34	.9904	.0096	.4904	.9807	2.91	.9982	.0018	.4982	.9964
2.35	.9906	.0094	.4906	.9812	2.92	.9982	.0018	.4982	.9965
2.36	.9909	.0091	.4909	.9817	2.93	.9983	.0017	.4983	.9966
2.37	.9911	.0089	.4911	.9822	2.94	.9984	.0016	.4984	.9967
2.38	.9913	.0087	.4913	.9827	2.95	.9984	.0016	.4984	.9968

The Relationship Between Hypothesis Testing and Confidence Intervals (Two-Sided Case)

H_0: $\mu = \mu_0$ vs. H_1: $\mu \neq \mu_0$

- The two-sided $100\% \times (1 - \alpha)$ CI for μ does not contain $\mu_0 \rightarrow$ Reject H_0
- The two-sided $100\% \times (1 - \alpha)$ CI for μ does contain $\mu_0 \rightarrow$ Accept H_0

Summary

- Null (H_0) and alternative (H_1) hypotheses

- Type I error (α), type II error (β), and the power ($1-\beta$) of a hypothesis test

- P-value of a hypothesis test and the distinction between on-sided and two-sided tests

- Hypothesis testing:
  -critical value method
  -p-value method

- Estimating appropriate sample size as determined by the pre-specified null and alternative hypotheses and type I and type II errors

FIGURE 7.18 Flowchart for appropriate methods of statistical inference

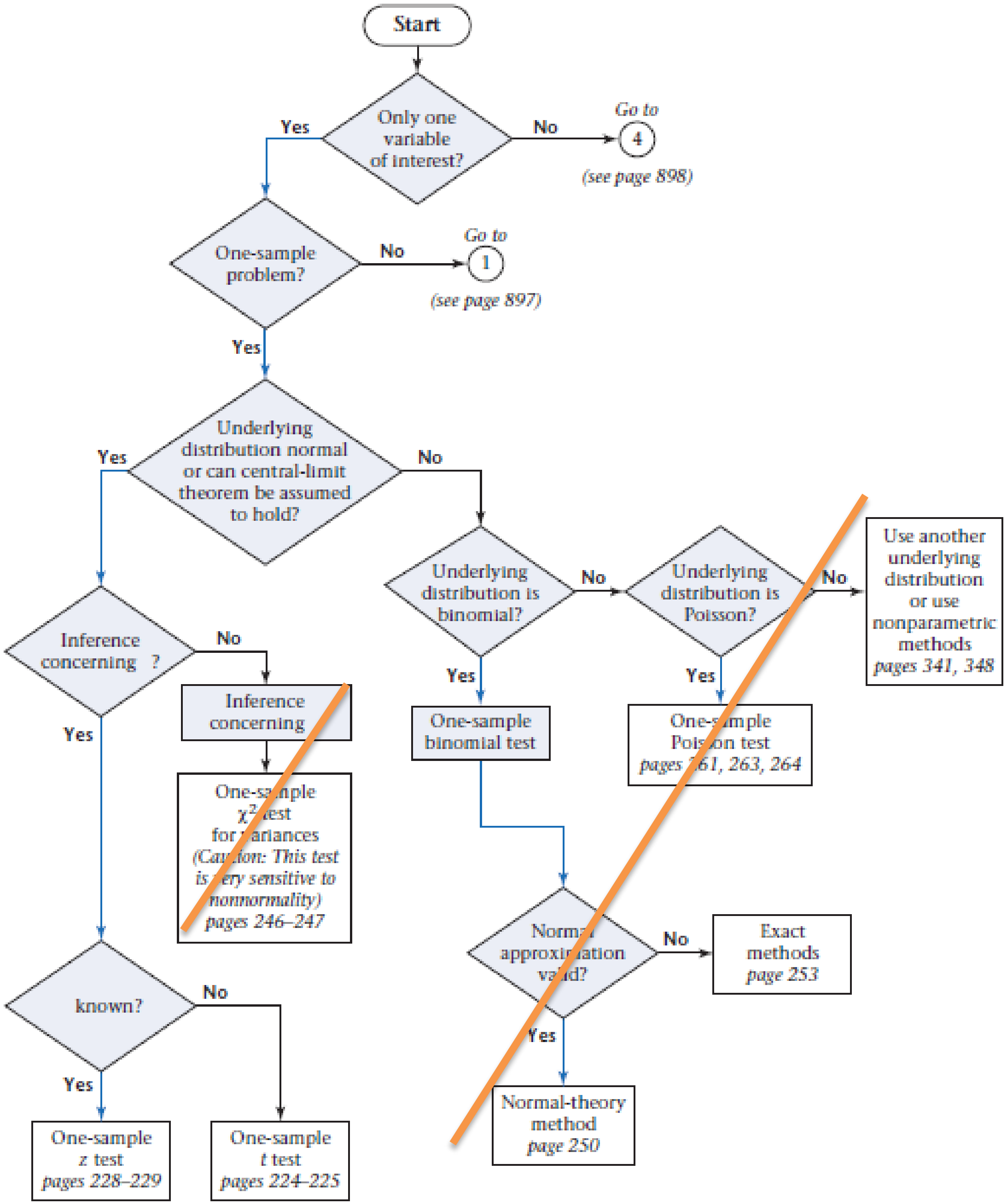


TABLE 3 The normal distribution *(continued)*

x	A[a]	B[b]	C[c]	D[d]	x	A	B	C	D
1.82	.9656	.0344	.4656	.9312	2.39	.9916	.0084	.4916	.9832
1.83	.9664	.0336	.4664	.9327	2.40	.9918	.0082	.4918	.9836
1.84	.9671	.0329	.4671	.9342	2.41	.9920	.0080	.4920	.9840
1.85	.9678	.0322	.4678	.9357	2.42	.9922	.0078	.4922	.9845
1.86	.9686	.0314	.4686	.9371	2.43	.9925	.0075	.4925	.9849
1.87	.9693	.0307	.4693	.9385	2.44	.9927	.0073	.4927	.9853
1.88	.9699	.0301	.4699	.9399	2.45	.9929	.0071	.4929	.9857
1.89	.9706	.0294	.4706	.9412	2.46	.9931	.0069	.4931	.9861
1.90	.9713	.0287	.4713	.9426	2.47	.9932	.0068	.4932	.9865
1.91	.9719	.0281	.4719	.9439	2.48	.9934	.0066	.4934	.9869
1.92	.9726	.0274	.4726	.9451	2.49	.9936	.0064	.4936	.9872
1.93	.9732	.0268	.4732	.9464	2.50	.9938	.0062	.4938	.9876
1.94	.9738	.0262	.4738	.9476	2.51	.9940	.0060	.4940	.9879
1.95	.9744	.0256	.4744	.9488	2.52	.9941	.0059	.4941	.9883
1.96	.9750	.0250	.4750	.9500	2.53	.9943	.0057	.4943	.9886
1.97	.9756	.0244	.4756	.9512	2.54	.9945	.0055	.4945	.9889
1.98	.9761	.0239	.4761	.9523	2.55	.9946	.0054	.4946	.9892
1.99	.9767	.0233	.4767	.9534	2.56	.9948	.0052	.4948	.9895
2.00	.9772	.0228	.4772	.9545	2.57	.9949	.0051	.4949	.9898
2.01	.9778	.0222	.4778	.9556	2.58	.9951	.0049	.4951	.9901
2.02	.9783	.0217	.4783	.9566	2.59	.9952	.0048	.4952	.9904
2.03	.9788	.0212	.4788	.9576	2.60	.9953	.0047	.4953	.9907
2.04	.9793	.0207	.4793	.9586	2.61	.9955	.0045	.4955	.9909
2.05	.9798	.0202	.4798	.9596	2.62	.9956	.0044	.4956	.9912
2.06	.9803	.0197	.4803	.9606	2.63	.9957	.0043	.4957	.9915
2.07	.9808	.0192	.4808	.9615	2.64	.9959	.0041	.4959	.9917
2.08	.9812	.0188	.4812	.9625	2.65	.9960	.0040	.4960	.9920
2.09	.9817	.0183	.4817	.9634	2.66	.9961	.0039	.4961	.9922
2.10	.9821	.0179	.4821	.9643	2.67	.9962	.0038	.4962	.9924
2.11	.9826	.0174	.4826	.9651	2.68	.9963	.0037	.4963	.9926
2.12	.9830	.0170	.4830	.9660	2.69	.9964	.0036	.4964	.9929
2.13	.9834	.0166	.4834	.9668	2.70	.9965	.0035	.4965	.9931
2.14	.9838	.0162	.4838	.9676	2.71	.9966	.0034	.4966	.9933
2.15	.9842	.0158	.4842	.9684	2.72	.9967	.0033	.4967	.9935
2.16	.9846	.0154	.4846	.9692	2.73	.9968	.0032	.4968	.9937
2.17	.9850	.0150	.4850	.9700	2.74	.9969	.0031	.4969	.9939
2.18	.9854	.0146	.4854	.9707	2.75	.9970	.0030	.4970	.9940
2.19	.9857	.0143	.4857	.9715	2.76	.9971	.0029	.4971	.9942
2.20	.9861	.0139	.4861	.9722	2.77	.9972	.0028	.4972	.9944
2.21	.9864	.0136	.4864	.9729	2.78	.9973	.0027	.4973	.9946
2.22	.9868	.0132	.4868	.9736	2.79	.9974	.0026	.4974	.9947
2.23	.9871	.0129	.4871	.9743	2.80	.9974	.0026	.4974	.9949
2.24	.9875	.0125	.4875	.9749	2.81	.9975	.0025	.4975	.9950
2.25	.9878	.0122	.4878	.9756	2.82	.9976	.0024	.4976	.9952
2.26	.9881	.0119	.4881	.9762	2.83	.9977	.0023	.4977	.9953
2.27	.9884	.0116	.4884	.9768	2.84	.9977	.0023	.4977	.9955
2.28	.9887	.0113	.4887	.9774	2.85	.9978	.0022	.4978	.9956
2.29	.9890	.0110	.4890	.9780	2.86	.9979	.0021	.4979	.9958
2.30	.9893	.0107	.4893	.9786	2.87	.9979	.0021	.4979	.9959
2.31	.9896	.0104	.4896	.9791	2.88	.9980	.0020	.4980	.9960
2.32	.9898	.0102	.4898	.9797	2.89	.9981	.0019	.4981	.9961
2.33	.9901	.0099	.4901	.9802	2.90	.9981	.0019	.4981	.9963
2.34	.9904	.0096	.4904	.9807	2.91	.9982	.0018	.4982	.9964
2.35	.9906	.0094	.4906	.9812	2.92	.9982	.0018	.4982	.9965
2.36	.9909	.0091	.4909	.9817	2.93	.9983	.0017	.4983	.9966
2.37	.9911	.0089	.4911	.9822	2.94	.9984	.0016	.4984	.9967
2.38	.9913	.0087	.4913	.9827	2.95	.9984	.0016	.4984	.9968