MA1200
Practice Exercise Chapter 2 Sets and Functions

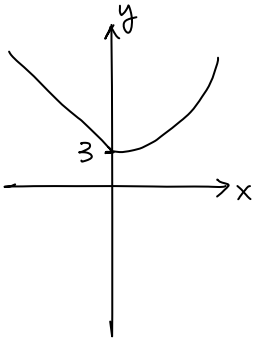
1. Let $A=\{x\in\mathbf{R}\mid -3<x\le 8\}$, $B=\{x\in\mathbf{R}\mid -11\le x<-3\}$, $C=\{x\in\mathbf{Z}\mid -11\le x<-3\}$ and $D=\{x\in\mathbf{R}\mid x>5\}$.

(a) Use a more concise interval notation to represent the above sets, if it is possible.

(b) Find each of the following sets. (Try expressing the set in as simplest form as possible.)
(i) $A\cap B$ (ii) $A\cup B$ (iii) $B\cup C$ (iv) $B\cap C$ (v) $A\cap D$ (vi) $A\cup D$

(c) Determine whether each of the following statements is correct or not.
(i) $A\subset\mathbf{Z}$ (ii) $C\subset\mathbf{Z}$ (iii) $\{3\}\subset\mathbf{Z}$ (iv) $\sqrt{2}\subset\mathbf{R}$
(v) $A\cap(7,8)\subset D$ (vi) $[4,12)\subset\mathbf{Z}$ (vii) $\{3,6\}\in\mathbf{Z}$ (viii) $\mathbf{R}\setminus(-\infty,5]=D$

2. In each of the following, the graph of a function is shown. Determine the largest possible domain and the largest possible range for each of the functions.

(a)


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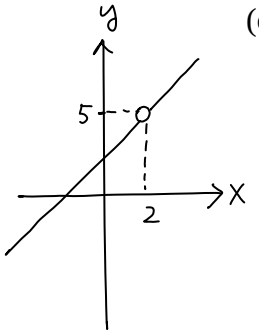

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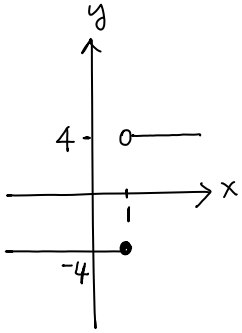

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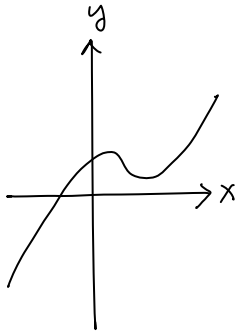

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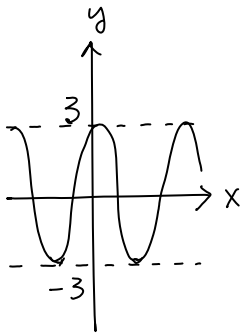
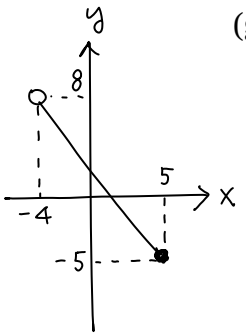
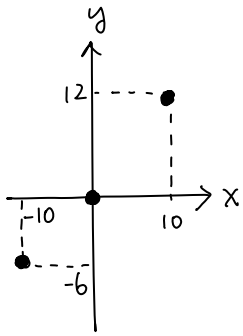
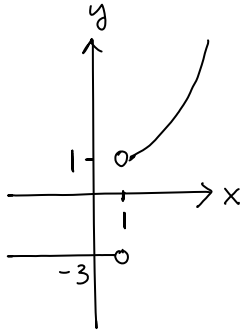


(f)


(g)


(h)


3. Determine the largest possible domain and the largest possible range for each of the following functions.

(a) $y=x^2-9$ (b) $y=4-x$ (c) $y=5$

(d) $y=x^3$ (e) $y=x^2-x-2$ (f) $y=\dfrac{5}{x-3}$

(g) $y=\begin{cases}x & -5\le x\le 2\\ x+1 & x>2\end{cases}$ (h) $y=\begin{cases}1 & -2\le x\le 2\\ x^2 & x>3\end{cases}$ (i) $y=|x|-3$

(j) $f(x)=\sqrt{25-x^2}$ (k) $f(x)=\sqrt{x^2-4x+8}$ (l) $f(x)=\sqrt{x^2-4x-21}$

4. Let $f(x)=\dfrac{3}{x-2}$ and $g(x)=\dfrac{6}{x-5}$ be two functions of *x*.

(a) Determine the domain of $f(x)$.

(b) Determine the domain of $g(x)$.

(c) Determine the domain of $f(x)+g(x)$.

5. Consider the function $f(x)=x-[x]$, where $[x]$ represents the greatest integer which is less than or equal to *x*.

(a) Plot $f(x)=x-[x]$.

(b) Find the largest possible domain and the largest possible range of $f(x)=x-[x]$.

6. Let $f(x)=\dfrac{5}{x-3}$ and $g(x)=x^2-6x+18$ be two functions of *x*.

(a) Determine the largest possible domain of $f(x)$.

(b) Determine the largest possible domain of $g(x)$.

(c) Determine $f(g(x))$ and its largest possible domain.

(d) Determine $g(f(x))$ and its largest possible domain.

(b) Find the largest possible domain and the largest possible range of $f(x)=x-[x]$.

7. Determine whether each of the following functions are even or odd or neither of them.

(a) $f(x)=\sin(2x)+5x^3$ (b) $f(x)=\tan x-3$

(c) $f(x)=\dfrac{\cos x}{x^2}$ (d) $f(x)=|-3x|+5$

8. Find the largest possible domain of each of the following functions:

(a) $y=f(x)=\dfrac{2x}{x^2-4x-5}$ (b) $\phi(x)=\dfrac{x^2-1}{x-1}$

9. Let $f(x)=x^3+2,\ g(x)=\dfrac{2}{x-1}$.

Find formulas for (a) $(f+g)(x)$, (b) $\left(\dfrac{g}{f}\right)(x)$, (c) $(g\circ f)(x)$, (d) $(f\circ g)(x)$ and state their largest possible domains.

10. State whether each of the following is an odd function, an even function or neither. Prove your statements or give counterexamples.

(a) The sum of two even functions

(b) The sum of two odd functions

(c) The product of two even functions

(d) The product of two odd functions

(e) The product of an even function and an odd function

11. Let F be any function whose domain contains $-x$ whenever it contains x.
Prove each of the following:
(a) $F(x) - F(-x)$ determines an odd function.
(b) $F(x) + F(-x)$ determines an even function.
(c) F can always be expressed as the sum of an odd function and an even function.

12. (a) Sketch the graph for $f(x) = \left(x^2 - 2x - 8\right)u_3(x)$, where $u_a(x) = \begin{cases} 0, & x < a \\ 1 & x \geq a \end{cases}$.
(b) Determine the largest possible domain of $f(x)$.
(c) Determine the largest possible range of $f(x)$.

13. (a) Sketch the graph for $f(x) = \left(u_\pi(x) - u_{2\pi}(x)\right)(3 + \sin x)$, where $u_a(x) = \begin{cases} 0, & x < a \\ 1 & x \geq a \end{cases}$.
(b) Determine the largest possible domain of $f(x)$.
(c) Determine the largest possible range of $f(x)$.