Q1-

a). $T_0 = 3. \quad \omega_0 = \frac{2\pi}{3}$

$$c_0 = \frac{1}{3}\int_{-1}^{1} x(t)\,dt = 1$$

$$c_k = \frac{1}{3}\int_{-1}^{1} x(t)\, e^{-jk\omega_0 t}\,dt = \frac{1}{3}\left\{ 2\int_{-1}^{0} e^{-jk\omega_0 t}\,dt + \int_{0}^{1} e^{-jk\omega_0 t}\,dt \right\}$$

$$= \frac{1}{3}\left\{ \frac{2}{(-jk\omega_0)} e^{-jk\omega_0 t}\Big|_{-1}^{0} + \frac{1}{(-jk\omega_0)} e^{-jk\omega_0 t}\Big|_{0}^{1} \right\}$$

$$= \frac{1}{3}\left\{ \frac{2}{-jk\omega_0}\Big(1 - \underbrace{e^{jk\omega_0}}_{=(-j)^k}\Big) + \frac{1}{-jk\omega_0}\Big(\underbrace{e^{-jk\omega_0}}_{=j^k} - 1\Big) \right\}$$

$$= \frac{j}{3k\omega_0}\left\{ 2(1-(-j)^k) + (j^k - 1) \right\}$$

$$= \frac{j}{2k\pi}\left\{ 2 + (j^k - 2(-j)^k) \right\}$$

$$\Rightarrow \begin{cases} \text{for } k = 4m+1, & c_k = -\frac{3}{2k\pi} \\ k = 4m+2, & c_k = \frac{j}{2k\pi} \\ k = 4m+3, & c_k = \frac{3}{2k\pi} \\ k = 4m, & c_k = \frac{-j}{2k\pi} \end{cases}$$

where m = integer

$$a_0 = 2c_0 = 2$$

$$a_k = 2\,Re(c_k)$$

$$= \begin{cases} \text{for } k = 4m+1, & a_k = -\frac{3}{k\pi} \\ k = 4m+2, & a_k = \emptyset \\ k = 4m+3, & a_k = \frac{3}{k\pi} \\ k = 4m, & a_k = \emptyset \quad 0 \end{cases}$$

$$b_k = -2\,Im((c_k))$$

$$= \begin{cases} \text{for } k = 4m+1, & b_k = \emptyset \\ k = 4m+2, & b_k = -\frac{1}{k\pi} \\ k = 4m+3, & b_k = \emptyset \\ k = 4m, & b_k = \frac{1}{k\pi} \quad 1/(k * pi) \end{cases}$$

Q1 -

b) $T_0 = 0.2 \quad \omega_0 = \frac{2\pi}{T_0} = 10\pi.$

$$C_0 = \frac{1}{0.2}\int_{0-\varepsilon}^{0.2-\varepsilon} x_2(t)\,dt = \frac{1}{0.2}(1-1) = \phi$$

$$C_k = \frac{1}{0.2}\int_{0-\varepsilon}^{0.2-\varepsilon} x_2(t)\, e^{-jk\omega_0 t}\,dt$$

$$= 5\left[e^{-jk\omega_0 t}\Big|_{t=0} - e^{-jk\omega_0 t}\Big|_{t=0.1} \right]$$

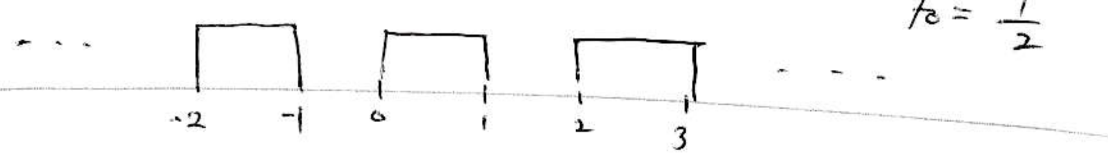
$$= 5\left[1 - e^{-jk\pi}\right] = 5\left(1-(-1)^k\right) = \begin{cases} \text{for even } k=2m, & C_k = \phi \\ \text{odd } k=2m+1, & C_k = 10 \end{cases}$$

$$\begin{cases} a_0 = 2C_0 = \phi \\ a_k = 2\,Re(C_k) = \begin{cases} \text{for even } k, & a_k = \phi \\ \text{odd } k, & a_k = 20. \end{cases} \\ b_k = -2\,Im(C_k) = \phi \end{cases}$$

Q 2.

a). $x(t)$ looks as follows, where $T_0 = 2$. $\omega_0 = \frac{2\pi}{T_0} = \pi$.

$f_0 = \frac{1}{2}$



Fundamental period = 2, Fundamental frequency = 0.5, and Fundamental Angular frequency = π.

Q2. a) This type of sequence has been derived in Ex 3-2) in the lecture note. Hence.

$$C_0 = \frac{1}{2}$$

$$C_k = \begin{cases} 0 & \text{for even } k = 2m \\ \dfrac{1}{j\pi(2m+1)} & \text{odd } k = 2m+1 \end{cases}$$

$$a_0 = 2C_0 = 1, \quad a_k = 2\,Re(C_k) = \emptyset$$

$$b_k = -2\,Im(C_k) = \begin{cases} \emptyset & \text{for even } k = 2m \\ \dfrac{2}{(2m+1)\pi} & \text{odd } k = 2m+1 \end{cases}$$

It is ok without derivation as long as the anwser is correct.

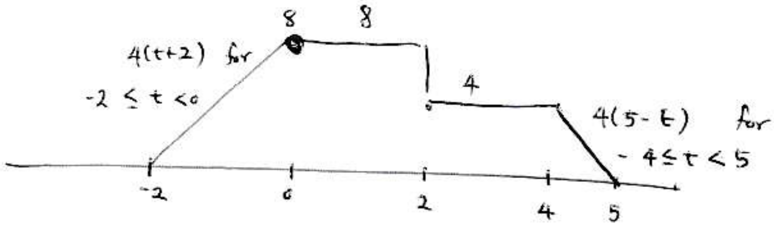
b) $\omega_0 = \pi$.

$$x(t) = \sum_{k=-\infty}^{\infty} C_k e^{jk\pi t}$$

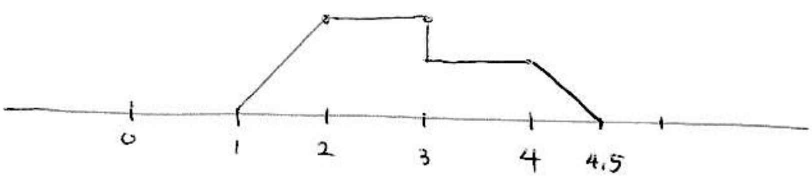
$$= \frac{1}{2} e^{-\frac{j\pi}{4}} e^{-j3\pi t} + 2 + e^{j\pi t} + \frac{1}{2} e^{\frac{j\pi}{4}} e^{j3\pi t}.$$

Q 3 -

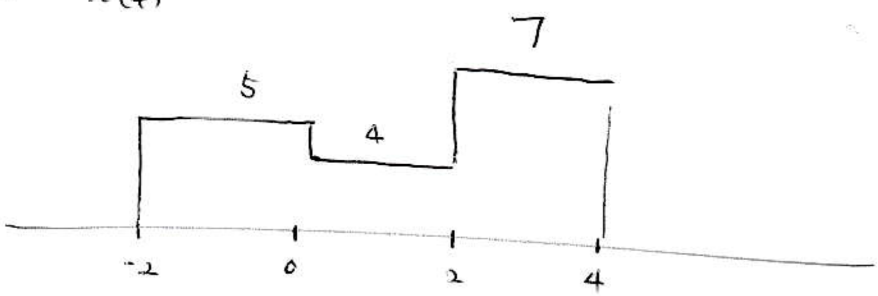
a) $x(t)$



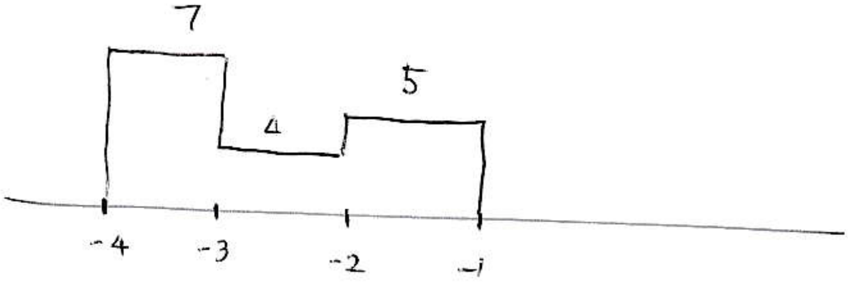
$x(2t-4)$



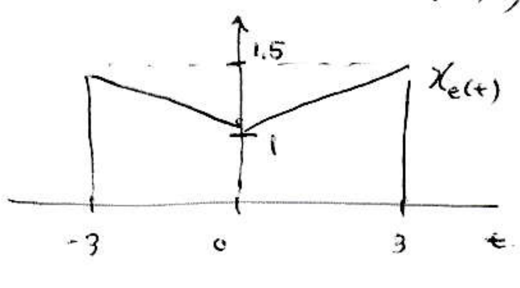
b) $x(t)$

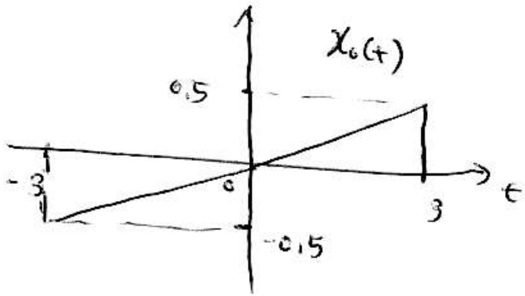


$x(-2t-4)$



c) $x_e(t) = \frac{1}{2}\left(x(t) + x(-t)\right)$ even part , $x_o(t) = \frac{1}{2}\left(x(t) - x(-t)\right)$





Q4 -

a) $h(t) = \delta(t-7)$

Causal as $h(t)=0$ for $t<0$

b) $h(t) = \int_{-\infty}^{t} \delta(\tau-7)d\tau = u(t-7)$

Causal as $h(t)=0$ for $t<0$

c) $h(t) = \int_{-\infty}^{t}\left[\int_{-\infty}^{\sigma} \delta(\tau-7)d\tau\right]d\sigma = \int_{-\infty}^{t} u(\sigma-7)\,d\tau$

$= \begin{cases} t-7 & \text{if } t \geq 7 \\ 0 & \text{otherwise} \end{cases} = r(t-7)$: ramp function

Causal as $h(t)=0$ for $t<0$

Q5 -

a) Noncausal, Stable

b) Causal, Non-Stable

c) Causal, Stable

d) Causal, Non-Stable

e) Non Causal, Stable

f) Non-causal, Stable

g) Causal, Stable