

SDSC 2102 Statistical Methods and Data Analysis
Assignment 2

Due: Feb 27, 2022 (Sunday) @10:00 pm

1. Sensor data from a particular ATM reveals that the number of people at the ATM when probed at random times is a random variable X that takes four values 0, 1, 2 and 3. It was also found that the PMF $p(x) = P(X = x)$ for $x = 0, 1, 2, 3$ is

$$p(x) = \frac{4 - x}{10}$$

- (a) What is the probability there are two or more people at the ATM when probed?
- (b) What is the probability there are fewer than 4 people at the ATM when probed?
- (c) What is the probability there are an odd number of people at the ATM when probed?

2. The PDF of a continuous random variable X with a triangular distribution is given by

$$f(x) = \begin{cases} 4x & \text{if } 0 \leq x \leq 0.5 \\ 4 - 4x & \text{if } 0.5 < x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

- (a) Find the CDF
- (b) Find $P(0.2 < X \leq 0.6)$

3. The file size of documents on web pages are often characterized by the Pareto distribution. A particular continuous random variable with a Pareto distribution has the following CDF

$$F(x) = \begin{cases} 1 - \left(\frac{10}{x}\right)^{2.5} & \text{if } x \geq 10 \\ 0 & \text{otherwise} \end{cases}$$

- (a) Find the PDF
- (b) Find $P(5 < X \leq 15)$
- (c) Find $P(X > 20)$

4. From historical information it is known that the probability that the driver of an SUV (Sport Utility Vehicle, like the Ford Explorer or the Jeep Cherokee) is a woman is 0.55. Five SUVs travelling in a Los Angeles street are going to be randomly selected.

- (a) What is the probability that exactly 2 out of the 5 SUVs selected will be driven by a woman?
- (b) What is the probability that more than 1 out of the 5 SUVs selected will be driven by a woman?

5. In a large local-area-network (LAN), a message is successfully transmitted if it does not encounter any “collisions”. These collisions are due to other users trying to transmit messages at the same time. If a collision occurs another transmission is attempted at a later time. These attempts continue until the message is successfully transmitted without any collisions. Whenever a message transmission is attempted, the probability of collision is 90%.

- (a) What is the probability that a certain message is transmitted at exactly the 3rd attempt?
- (b) What is the probability that it would take a total of exactly 5 attempts to transmit 3 messages?

6. The amount of time a student spends at a terminal in a CityU computer room is exponentially distributed with mean 20 minutes.

- (a) What is the standard deviation of the time a student would spend at a terminal?
- (b) What is the probability that a student would spend more than 80 minutes at a terminal?

7. The weekly demand for propane gas (in 1000's of gallons) from a particular facility is a continuous random variable X with PDF

$$f(x) = \begin{cases} 2\left(1 - \frac{1}{x^2}\right) & \text{if } 1 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

- (a) Find $E(X)$
- (b) Find $Var(X)$

8. There are two machines available for cutting corks intended for use in bottles. The first produces corks with diameters that are normally distributed with mean 3 cm and standard deviation 0.1 cm. The second machine produces corks with diameters that have a normal distribution with mean 3.04 cm and standard deviation 0.02 cm. Acceptable corks have diameters between 2.9 cm and 3.1 cm.

- (a) Which machine is more likely to produce an acceptable cork?
- (b) What should the acceptable range for cork diameters be (from $3 - d$ to $3 + d$ cm) to be 90% certain for the first machine to produce an acceptable cork?