HW1. Solutions

Q1. Line "L" passing through $(-1,4)$, $\perp$ $x+2y+3=0$.

Sol: $x+2y=3 \Rightarrow y=-\frac{1}{2}x+\frac{3}{2}$.

$\Rightarrow$ slope of L is 2 $\Rightarrow$ $\boxed{y-4=2(x+1) \text{ or } y=2x+6}$

Q2. $x^2-9y^2+2x+36y-44=0$

$\Rightarrow x^2+2x+1-1-9(y^2-4y+4)+36-44=0$

$(x+1)^2-9(y-2)^2=9$

$\Rightarrow \boxed{\frac{(x+1)^2}{9}-(y-2)^2=1}$, hyperbola, center $(-1,2)$

$c^2=a^2+b^2=10 \Rightarrow \boxed{c=\sqrt{10}}$

foci: $\underline{(-1-\sqrt{10},2) \quad (-1+\sqrt{10},2)}$

$\underline{\frac{x+1}{3}=\pm(y-2)}$ asymptotes

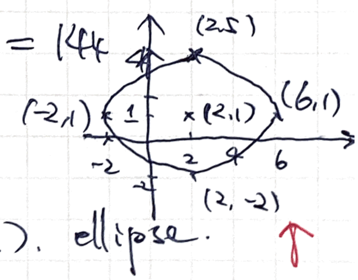
let $y=0$. $x+1=3\pm 6 \Rightarrow x=1\pm 6$. $\boxed{x=7 \text{ or } x=-5}$
(asy cuts x-axis).

Q3. $9x^2+16y^2-36x-32y-92=0$

$9(x^2-4x+4)+16(y^2-2y+1)=92+36+16=144$

$9(x-2)^2+16(y-1)^2=144=12^2$

$\boxed{\frac{(x-2)^2}{4^2}+\frac{(y-1)^2}{3^2}=1}$ center $(2,1)$. ellipse.



$a^2=b^2+c^2 \Rightarrow c=\sqrt{7}$. foci $\underline{(2-\sqrt{7},1). \quad (2+\sqrt{7},1)}$

Q4. $f(x) = 3x-2$. $g(x) = \frac{1}{x-2}$

a). $y = 3x-2 \Rightarrow 3x = y+2 \Rightarrow x = \frac{y}{3} + \frac{2}{3} \Rightarrow y = f^{-1}(x) = \frac{x}{3} + 2/3$

domain & range : $(-\infty, \infty)$. or $\mathbb{R}$.

b) $(g \circ f)(x) = g(f(x)) = \frac{1}{3x-2-2} = \frac{1}{3x-4}$

domain $3x-4 \neq 0$, $x \neq 4/3 \Rightarrow \mathbb{R} \setminus \{\frac{4}{3}\}$ or $(-\infty, \frac{4}{3}) \cup (\frac{4}{3}, \infty)$

range. $\frac{1}{z}$, $\neq 0$ for $z \neq 0$

$\Rightarrow$ range : $\mathbb{R} \setminus \{0\}$ or $(-\infty, 0) \cup (0, \infty)$

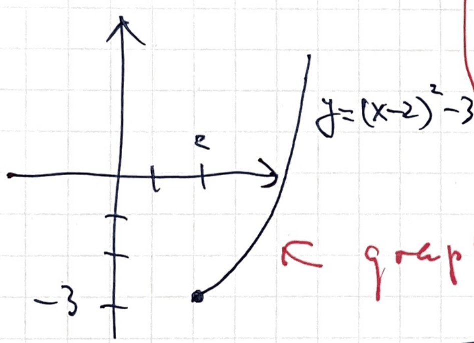
Q5 $y = (x-2)^2 - 3$. $x \in [2, \infty)$

$y+3 = (x-2)^2$

$\Rightarrow x-2 = \sqrt{y+3}$. (since $x \geq 2$)

$\Rightarrow \boxed{y = 2 + \sqrt{x+3}.}$ domain $[-3, \infty)$

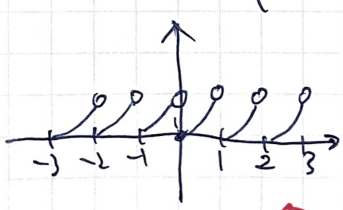
also range of $y = f(x)$.



← graph $y = f(x)$

Q6. $f(x) = (x - [x])^2$.

$$y = [x] = \begin{cases} \vdots & \\ 1 & x \in [1,2) \\ 0 & x \in [0,1) \\ -1 & x \in [-1,0) \\ \vdots & \end{cases} \Rightarrow y = (x-[x])^2 = \begin{cases} (x-2)^2 & x \in [2,3) \\ (x-1)^2 & x \in [1,2) \\ x^2. & x \in [0,1) \\ (x+1)^2 & x \in [-1,0) \\ (x+2)^2 & x \in [-2,-1) \\ (x+3)^2 & x \in [-3,-2) \end{cases}$$



range $[0, 1)$

periodic with $T = 1$.

MA1200 TAKE HOME PROBLEM SET 1

The following is the first take-home assignment of MA1200, which counts 3 point3 of total 100 of your final score of the course.

Please submit it via canvas in a pdf file (you can handwrite the answers and take photos by your phone, then make it into a pdf file, see for example, https://www.wikihow.com/Convert-JPG-to-PDF) for how to combine jpg files to a pdf; you can also do it by note-taking apps on an iPad or an Surface)

Q1. Find the equation of the straight line through $P(-1, 4)$ perpendicular to the line L, $x + 2y + 3 = 0$.

Q2. (a) Show that the equation $x^2 - 9y^2 + 2x + 36y - 44 = 0$ represents a hyperbola whose center is at the point $C(-1, 2)$

(b) Find the coordinates of the foci of the hyperbola, the equations of its asymptotes, and the coordinates of the points where the asymptotes cut the x-axis.

Q3. (a) Show that the equation $9x^2 + 16y^2 - 36x - 32y - 92 = 0$ represents an ellipse whose center is at the point $C(2, 1)$

(b) Find the coordinates of the foci of the ellipse

(c) Sketch the graph of the ellipse.

Q4.

$$f(x) = 3x - 2, \text{ for } x \in \mathbb{R} \quad \text{and} \quad g(x) = \frac{1}{x-2} \text{ for } x \in \mathbb{R}\backslash\{2\}$$

(a) Find the inverse function $f^{-1}(x)$ and state the largest possible domain and the range.

(b) Find $(g \circ f)(x)$, and state the largest possible domain and the range.

Q5. (a) Let $f(x) = (x - 2)^2 - 3$ for $x \in [2, \infty)$, sketch its graph

(b) Find the inverse of $f(x)$ and state its largest possible domain.

Q5. Let $f(x) = (x - [x])^2$, $x \in \mathbb{R}$, where $[x]$ is the greatest integer not greater than x

(a) Sketch $y = f(x)$ for $-3 \leq x \leq 3$.

(b) Find the range of $f(x)$

(c) Is $f(x)$ a periodic function of x?

The assignment is due on 23:59 of September 27, Sunday.

You will lose 1 point for each day of late submission. All submissions after the midnight of September 30 will be marked as 0.

Date: September 18, 2020.