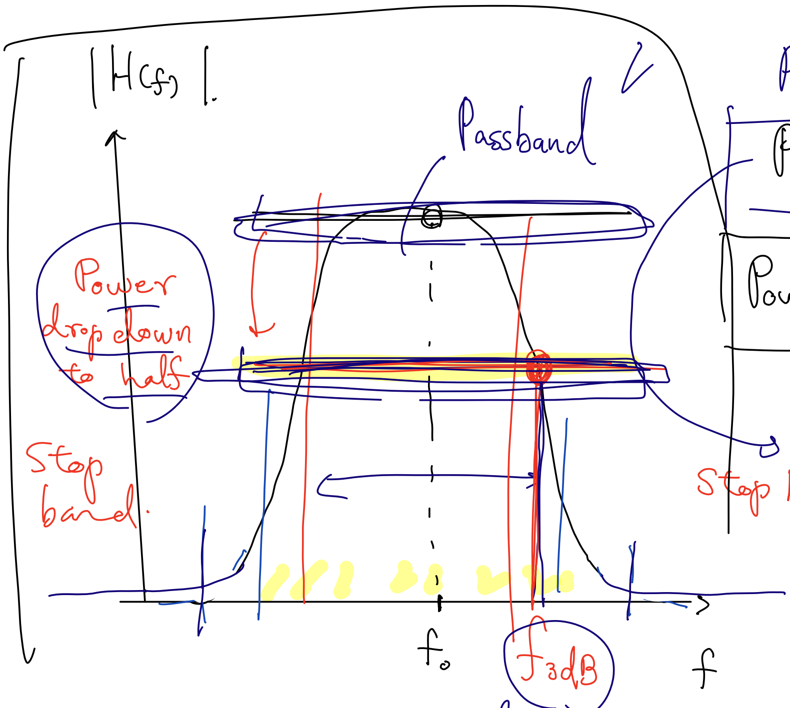


Power at peak $= |H(f_0)|^2$

Peak $|H(f_0)|$

Power $\propto |H(f)|^2$

Power $|H(f_{3dB})|^2 = \frac{|H(f_0)|^2}{2}$

Stop band

$$|H(f_{3dB})|^2 = \frac{|H(f_0)|^2}{2}$$

① 3-dB bandwidth f_{3dB}

(Half power bandwidth)

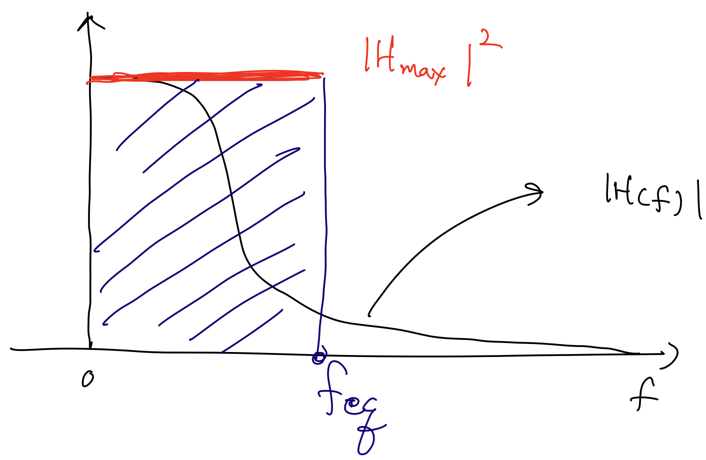
$$f_{3dB} \Rightarrow \boxed{|H(f_{3dB})| = \frac{|H(0)|}{\sqrt{2}}}$$

Usually $f_0 = 0$

$$10\log_{10}\left(\frac{P(f_{3dB})}{P(f_0)}\right) = 10\log_{10}\left(\frac{\cancel{|H(f_0)|^2}/2}{\cancel{|H(f_0)|^2}}\right).$$

$$= -10\log_{10}(2) = -3 \text{ dB}$$

② Equivalent bandwidth f_{eq}



Power of this filter $\qquad$ Power in that rect

$$\int_0^\infty |H(f)|^2 df = |H_{max}|^2 \cdot f_{eq}.$$

$$\boxed{f_{eq} = \frac{1}{|H_{max}|^2} \int_0^\infty |H(f)|^2 df.}$$

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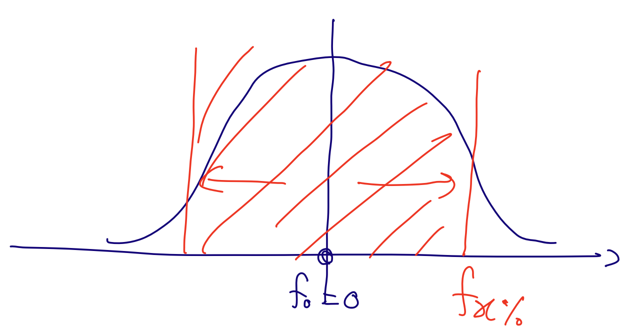
③ x percent Energy containment BW

$f_{x\%}$



$$x\% \times \left(\text{Power of this filter} = \int_{-\infty}^{\infty} |H(f)|^2 \, df \right) = \text{Power in the red interval} \; \int_{-f_{x\%}}^{f_{x\%}} |H(f)|^2 \, df$$

0.9

$$x\% \times \int_{-\infty}^{\infty} |H(f)|^2 \, df = \boxed{\int_{-f_{x\%}}^{f_{x\%}} |H(f)|^2 \, df}$$

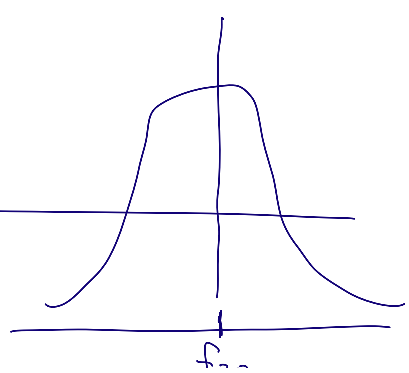
(Ex 4-8)

a) $h(t) = \omega_0 \boxed{e^{-\omega_0 t} u(t)}$, $\omega_0 = 2\pi f_0$

$$H(f) = \frac{\omega_0}{\omega_0 + j2\pi f} = \frac{2\pi f_0}{2\pi f_0 + j\,2\pi f}$$

$$= \frac{1}{1 + j(f/f_0)}$$

$$\boxed{|X| = \sqrt{X \cdot X^*}}$$



f_{3dB} : $\quad |H(f_{3dB})| = \sqrt{H(f)\cdot H(f)^*}$ LHS

$$= \frac{1}{\sqrt{\left(1+j\left(f/f_0\right)\right)\left(1-j\left(f/f_0\right)\right)}}$$

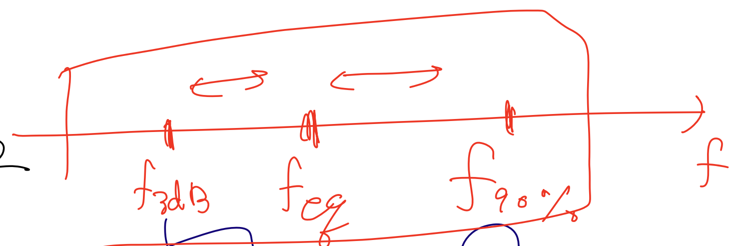
$$= \frac{1}{\sqrt{1+\left(f_{3dB}/f_0\right)^2}}$$

$$\frac{|H(0)|}{\sqrt{2}} = \frac{\frac{1}{\sqrt{1+0^2}}}{\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$\frac{1}{\sqrt{1+0^2}} \to 1$

RHS

$$1 + \left(\cancel{f_{3dB}/f_0}\right)^2 = 2$$



$$\left(\cancel{f_{3dB}/f_0}\right)^2 = 1$$

$$\boxed{f_{3dB} = f_0}$$

$$f_{eq} = f_0 \cdot \frac{\pi}{2} \simeq 1.57\, f_0$$

$$f_{90\%} = f_0 \cdot \tan(0.45\pi) \simeq 6.31\, f_0$$

$$\boxed{f_{eq}} = \frac{1}{|H_{max}|^2} \int_0^\infty |H(f)|^2 \, df$$

$$= \int_0^\infty \frac{1}{1+(f/f_0)^2} \, df \quad — \quad ①$$

$$= \boxed{\frac{f_0 \pi}{2}}$$

$$\boxed{\int_0^\infty \frac{1}{a^2+x^2} \, dx = \frac{\pi}{2|a|}}$$

In ①, $f/f_0 \rightarrow x$.

$$f = f_0 x.$$

$$df = f_0 \, dx$$

$$f_0 \int_0^\infty \frac{1}{1+x^2} \, dx = \frac{f_0 \pi}{2}$$

90% energy containment BW $f_{90\%}$

LHS

$$0.9 \cdot \boxed{\int_{-\infty}^{\infty} |H(f)|^2 \, df} = 0.9 \int_{-\infty}^{\infty} \frac{1}{1+(f/f_0)^2} \, df$$

$$= 0.9 \times 2 \underbrace{\int_0^{\infty} \frac{1}{1+(f/f_0)^2} \, df}$$

$$= 0.9 \times \cancel{2} \times \frac{f_0 \pi}{\cancel{2}} = \boxed{0.9 f_0 \pi}$$

RHS

$$\int_{-f_{90\%}}^{f_{90\%}} \frac{1}{1+(f/f_0)^2} \, df = 2 \underbrace{\int_0^{f_{90\%}} \frac{1}{1+(f/f_0)^2} \, df.}$$

$$\boxed{\int \frac{1}{a^2+x^2} \, dx = \frac{\tan^{-1}\left(\frac{x}{a}\right)}{a}}$$

$$\frac{f}{f_0} = x$$

$$df = f_0 \, dx$$

$$2f_0 \int_0^{\left(f_{90\%}/f_0\right)} \frac{1}{1+x^2} \, dx$$

RHS

$$= 2f_0 \left[\tan^{-1}(x) \Big|_0^{\left(f_{90\%}/f_0\right)} \right]$$

$$= 2f_0 \tan^{-1}\left(\frac{f_{90\%}}{f_0}\right).$$

$$0.45 \, \cancel{f_0} \, \pi. = \cancel{f_0} \tan^{-1}\left(\frac{f_{90\%}}{f_0}\right).$$

$$\tan(0.45\pi.) = \tan\left(\tan^{-1}\left(\frac{f_{90\%}}{f_0}\right)\right)$$

$$= \frac{f_{90\%}}{f_0}$$

$$f_{90\%} = f_0 \cdot \tan(0.45\pi)$$