Tutorial 2 (with solution)

Functions

Q.1: Encoder of Even Parity

Encoding function *f*.

- Input: (b_1, b_2, b_3, b_4), where $b_i \in \{0, 1\}\ \forall i$
- Output: $(c_1, c_2, c_3, c_4, c_5)$, where $c_i \in \{0, 1\}\ \forall i$
  - $c_1 = b_1, c_2 = b_2, c_3 = b_3, c_4 = b_4,$
  - $c_1 + c_2 + c_3 + c_4 + c_5 = 0 \pmod 2$

a) What is the domain of f?
   - Hint: Use Cartesian product.

b) What is the co-domain of f?

c) What is the image of $(0, 1, 0, 0)$?

Q.1: Encoder of Even Parity

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  - $c_1 = b_1, c_2 = b_2, c_3 = b_3, c_4 = b_4,$
  - $c_1 + c_2 + c_3 + c_4 + c_5 = 0 \pmod 2$

d) What is the range of f ?

1) $\{0, 1\}^5$
2) $\{x \in \{0, 1\}^5 \mid x$ has an even number of 1s $\}$
3) $\{x \in \{0, 1\}^5 \mid x$ has an odd number of 1s $\}$

Q.1: Encoder of Even Parity

a) $\{0,1\}\times\{0,1\}\times\{0,1\}\times\{0,1\}$

- It can also be succinctly written as $\{0,1\}^4$.

b) $\{0,1\}^5$

c) $(0,1,0,0,1)$

d) $\{x \in \{0,1\}^5 \mid x \text{ has an even number of 1s}\}$

Q.2: Decoder of Even Parity

Decoding function g.

- Input: $(c_1, c_2, c_3, c_4, c_5)$, where $c_i \in \{0, 1\}$ $\forall i$
- Output:
  - Either (b_1, b_2, b_3, b_4), where $b_i \in \{0, 1\}$ $\forall i$
  - Or a special symbol e when an error is detected.

a) What is the image of $(0, 1, 0, 0, 1)$?

b) What is the image of $(1, 1, 0, 1, 0)$?

c) What is the domain of g?

d) What is the co-domain of g?

  - Hint: Don't forget the special symbol e.

Q.2: Decoder of Even Parity

a) $(0, 1, 0, 0)$

b) e

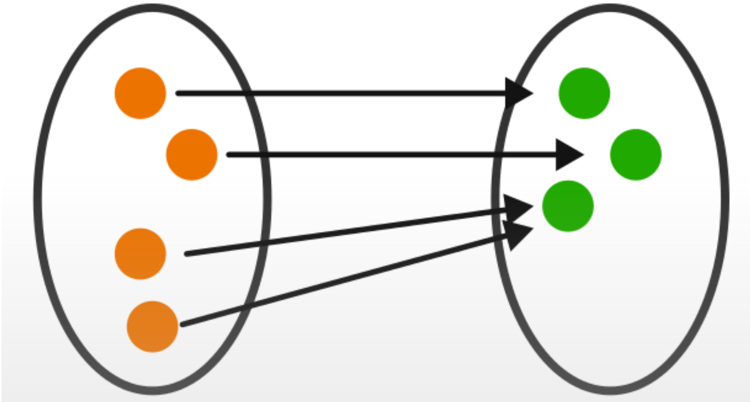
c) $\{0, 1\}^5$

d) $\{0, 1\}^4 \cup \{e\}$

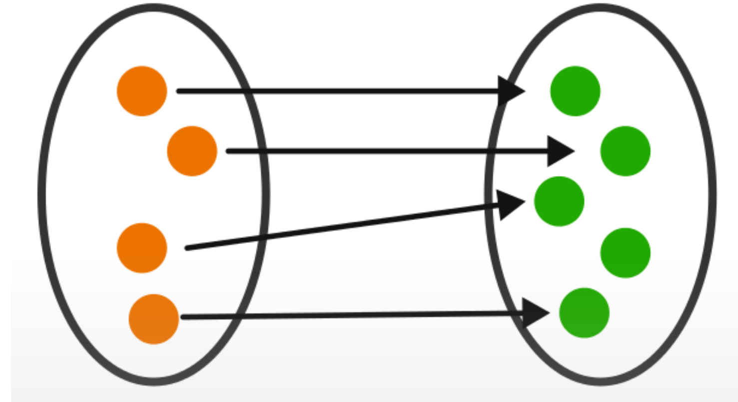
Q.3: Injection & Surjection

- Is it injection or surjection?

i)



ii)



a) i) is injection, ii) is surjection

b) i) is injection, ii) is also injection

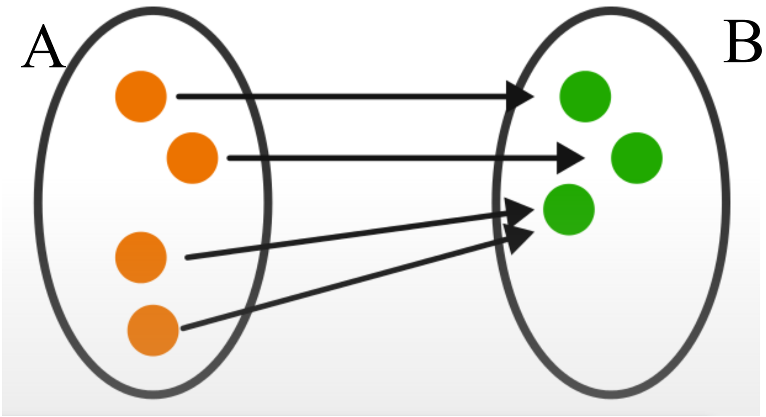
c) i) is surjection, ii) is injection

d) i) is surjection, ii) is also surjection

Q.3: Injection & Surjection

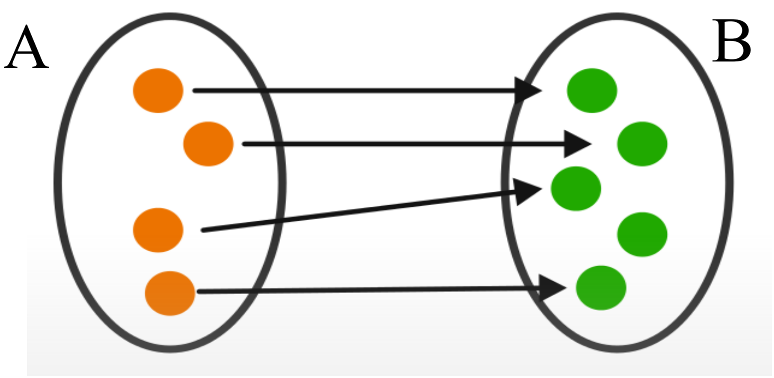
i) Surjection

- as each element in B is mapped from one or more elements in A.



ii) Injection

- as each element in A mapped to one distinct element in B.



Q.4: Composition of Onto Functions

- Suppose $f: X \to Y$ and $g: Y \to Z$ are both surjections.
- Is $g \circ f$ a surjection? Prove or disprove it.
  a) Yes
  b) No

Q.4: Composition of Onto Functions

Proof: $f: X \to Y$ and $g: Y \to Z$

Let z be an arbitrary chosen element in Z.

By definition of surjection, there must be an element $y \in Y$ such that $g(y) = z$.

Since f is also a surjection, there is an element $x \in X$ such that $f(x) = y$.

Hence, there is an element $x \in X$ such that $g\big(f(x)\big) = g(y) = z$.

Thus, $g\big(f(x)\big)$ is a surjection. *Q.E.D.*

Q.5: Comparison of Infinities

- Do the intervals $(0,1)$ and $(0, 2)$ have the same cardinality? Prove or disprove it.

a) Yes

b) No

Q.5: Comparison of Infinities

Proof:

Define $f: (0,1) \to (0,2)$ such that $f(x) = 2x$.

If $f(x_1) = f(x_2)$, then $2x_1 = 2x_2$, which implies that $x_1 = x_2$. Hence, $f(n)$ is one to one.

Given any $y \in (0, 2)$, let $x = y/2$, so $x \in (0,1)$ and $f(x) = y$. Hence, $f(n)$ is onto.

Therefore, f is a one-to-one correspondence.

Hence, the two sets have the same cardinality.

Q.E.D.