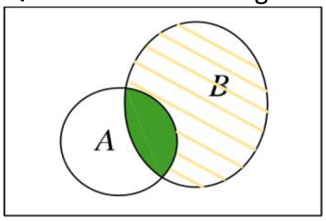
Solution to Assignment 1

Q1: Use the Venn diagram to help analyse the problem:



Let's consider two events: A and $B \cap A^C$ (shaded).
Since these two events are disjoint, i.e., $A \cap (B \cap A^C) = \emptyset$,
$P(A \cup (B \cap A^C)) = P(A) + P(B \cap A^C)$ (by Probability Axiom 4)
$\Rightarrow P(A \cup B) = P(A) + P(B \cap A^C)$ (1)

The following has been proved in class:
Since the two events $B \cap A^C$ (shaded) and $B \cap A$ (green) are disjoint,
i.e., $(B \cap A^C) \cap (B \cap A) = \emptyset$,
$P\big((B \cap A^C) \cup (B \cap A)\big) = P(B \cap A^C) + P(B \cap A)$ (by Probability Axiom)
$\Rightarrow P(B) = P(B \cap A^C) + P(B \cap A)$
$\Rightarrow P(B \cap A^C) = P(B) - P(B \cap A)$ (2)

Plugging (2) to (1):
$P(A \cup B) = P(A) + P(B) - P(B \cap A)$

Q2:
Number of ways to pick 3 cards in a pack of 52 cards is =

$$\binom{52}{3} = \frac{52!}{49!3!}.$$

Number of ways to pick 3 cards but no spades in a pack of 52 cards is =

$$\binom{13}{0}\binom{39}{3} = \frac{39!}{36!3!}.$$

Therefore the probability of picking no spades =

$$\binom{39}{3} / \binom{52}{3} = 0.414.$$

Thus the probability of picking atleast 1 spade = 1 - 0.414= 0.586.

Q3.

Let A and B be the events that a randomly selected parent listens to CDs and radio respectively;
Total number of people= 100; $P(A \cap B) = 0.5$; $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.8 + 0.6 - 0.5 = 0.9$;
therefore the number of people listening to neither radio or CD = 100-90 = 10.

a) $90/100 = 0.9$.

b) $10/100 = 0.1$.

c) P$(B \setminus A \cap B)$ OR P$(B \cap A')$ = (60-50)/100 = 0.1.

Q4.

a) False. because sum of probabilites is greater than 1.

b) True. P(ball with 'b') = (number of blue and black balls)/total number of balls = $10/15 = 2/3$.

c) False. P(picking both balls with 'b') = P(picking 2 blue balls) + P(picking a black and a blue ball) + P(picking 2 black balls).

$$\binom{6}{2}/\binom{15}{2} + \binom{6}{1}\binom{4}{1}/\binom{15}{2} + \binom{4}{2}/\binom{15}{2}$$

Alternately, one can think of having 10 'b' colored balls out of the total of 15 balls in the urn. The probabiity of picking 2 balls starting with letter 'b' is the same as the probabllity of picking one ball and picking the next without replacement. This is given by (10/15) (9/14)$\neq$ 1/5.

Q5.

Let Q_1 : Event that a quarter is picked in the first trial; D_2: Event that a dime is picked in the second trial; D_3: Event that a dime is picked in the third trial.
P$(Q_1 \cap D_2 \cap D_3) = P(D_3|Q_1 \cap D_2) \times P(D_2|Q_1) \times P(Q_1)$.
$=(4/11) \times (5/10) \times (4/9) = 8/99 = 0.0808$.

Q6.

Let W: Event that only white balls are picked; Y_x : Event that the number x is rolled on the die.

$$P(W) = \sum_{x=1}^{6} P(W|Y_x) \times P(Y_x) = 1/6 \times \sum_{x=1}^{6} P(W|Y_x) = (1/6) \times \sum_{x=1}^{5} \binom{5}{x}/\binom{15}{x}.$$

P(W) = 0.0758. **Note: P$(W|Y_6)$ = 0 as there are only 5 white balls in the urn.**

$$P(Y_3|W) = \frac{P(W|Y_3) \times P(Y_3)}{P(W)} = (\binom{5}{3}/\binom{15}{3}) \times (1/6) \div 0.0758 \ = 0.048.$$

Q7.

Let W: Event that the resigning employee is a woman; X_N : Event that the employee works in store 'N'.
P(W) = P$(W|X_A) \times P(X_A) + P(W|X_B) \times P(X_B) + P(W|X_C) \times P(X_C)$.
P$(X_A) = 50/(50 + 75 + 100) = 0.22$. Similarly $P(X_B), P(X_C) = 0.33$ and 0.44 respectively.
Therefore,$P(W) = 0.5 \times 0.22 + 0.6 \times 0.33 + 0.7 \times 0.44 = 0.622$.

$$P(X_C|W) = \frac{P(W|X_C) \times P(X_C)}{P(W)} = \frac{(0.7) \times (0.44)}{0.622} = 0.5.$$