Tutorial 9

Codes (with solution)

Question 1

You want to encode the information bits 1010101010101011 by a two-dimensional parity scheme. You put the information bits into a 4×4 array, and even parity is assumed.

a) Determine the parity bits.
b) When determining the parity bit at the bottom right corner, do you use column parity or row parity? Is there any inconsistency?

Q. 1 (solution)

a) The rightmost column and bottom row are the parity bits.

1	0	1	0	0
1	0	1	0	0
1	0	1	0	0
1	0	1	1	1
0	0	0	1	1

b) There won't be any inconsistency.

Question 2

- Suppose a transmission channel operates at 3 Mbps and has a bit error rate of 10^{-3}. Bit errors occur at random and are independent of each other. Suppose the (3, 1) repetition code is used. The receiver takes the three received bits and decides which bit was sent by taking the majority vote of the three bits.
  a) What is the generator matrix of this code?
  b) What is the parity-check matrix of this code?
  c) What is the effective data rate if this repetition code is used?
  d) Find the probability that the receiver makes a decoding error.

Q. 2 (solution)

a) $G = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$

b) $H = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

c) Code rate = 1/3, so Effective data rate = 1 Mbps

d) The receiver will make a decoding error if two or more bits are in error. Hence,

$$\Pr\{\text{decoding error}\} = 3(1-p)p^2 + p^3,$$

where $p = 10^{-3}$.

$$\begin{aligned}\Pr\{\text{decoding error}\} &= 2.997 \times 10^{-6} + 10^{-9} \\ &= 2.998 \times 10^{-6}\end{aligned}$$

Question 3

- Let C be the binary code with the generator matrix

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{bmatrix}.$$

a) List all the codewords of C.

b) What is the minimum distance of C?

c) What is the error detection capability of C?

d) What is the parity-check matrix of C?

Q.3 (solution)

a) There are eight codewords:
   - 0000000, 0010111, 0101011, 1001101,
   - 0111100, 1100110, 1011001, 1110001

b) The minimum distance is equals to the minimum weight of non-zero codewords. Hence, $d_{\min} = 4$.

c) It can detect up to 3 bit errors.

d) $$H = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

Question 4

- Consider the (7, 4) Hamming code with the following parity-check matrix:

$$H = \begin{bmatrix} 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}_{r\times n}$$

a) Draw a Venn diagram and indicate the data bits and parity bits.

b) If the received vector is 1001010, what are the decoded data bits?

Q.4 (solution)

- $H = \begin{bmatrix} 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}_{r \times n}$

The decoded data bits should be 1011.