MA1200 (CG1) Review on Ch.1-5

Ch.1

* Straight line : $y = mx + c$ or $\dfrac{y-y_0}{x-x_0} = m$

* m_1 = slope of line L_1

  m_2 = slope of line L_2

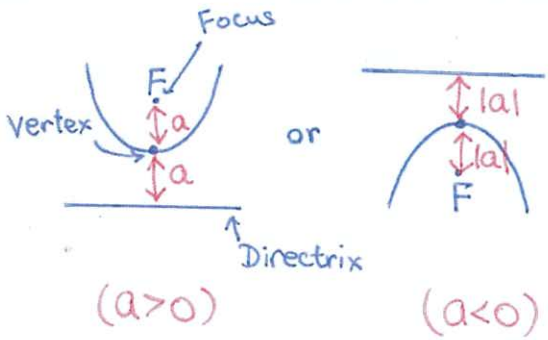
  $L_1 \parallel L_2 \Rightarrow m_1 = m_2$

  $L_1 \perp L_2 \Rightarrow m_1 = -\dfrac{1}{m_2}$

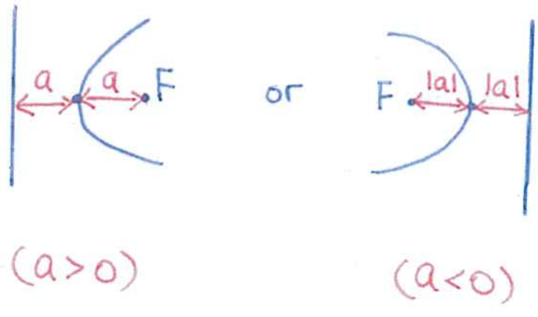
* Conic sections

① Parabola:

$(x-h)^2 = 4a(y-k)$



$(y-k)^2 = 4a(x-h)$

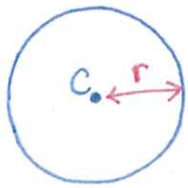


Vertex ?

Directrix ?

Focus ?

② Circle : $(x-h)^2+(y-k)^2=r^2$

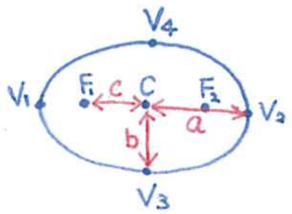


Centre : (h, k)

radius : r

③ Ellipse :

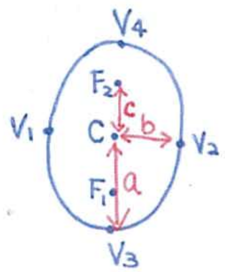
$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$$



"Fat ellipse"

$(a>b)$

$$\frac{(y-k)^2}{a^2}+\frac{(x-h)^2}{b^2}=1$$



"Thin ellipse"

$(a>b)$

$$\boxed{c=\sqrt{a^2-b^2}}$$

Centre ?

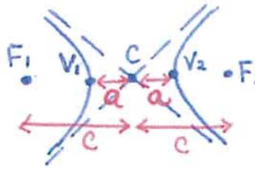
4 Vertices ?

2 Foci ?

④ Hyperbola:

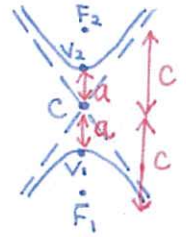
$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

"East-West Openings"



$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

"North-South Openings"



$$\boxed{c = \sqrt{a^2+b^2}}$$

Centre ?

2 Foci ?

2 Vertices ?

2 Asymptotes ?

* $Ax^2 + Cy^2 + Dx + Ey + F = 0$

Classify the type of conic section by completing the square.

* Translation of points and functions

$$\left.\begin{array}{l}\text{Point } (x_0, y_0) \\ \\ \text{function } y=f(x)\end{array}\right\} \xrightarrow[\text{right and } k \text{ units upward}]{\text{Translated } h \text{ units to the}} \left\{\begin{array}{l}(x_0+h,\ y_0+k) \\ \\ y-k = f(x-h)\end{array}\right.$$

Ch. 2

* Largest possible domain : Dom(f)
  - Set of all possible input (x) values

  Largest possible range : Ran(f)
  - set of all actual output (y) values

  Ran(f) depends on Dom(f) and f.

* Composite function : $(f \circ g)(x) = f(g(x))$

* Odd function : $f(-x) = -f(x)$ for all $x \in \text{Dom}(f)$

  symmetric about origin

  Even function : $f(-x) = f(x)$ for all $x \in \text{Dom}(f)$

  symmetric about y-axis

* Periodic function : $f(x+T) = f(x)$ for all $x \in \text{Dom}(f)$

  period

* Absolute value function :

$$|f(x)| = \begin{cases} f(x) & \text{if } f(x) \geq 0 \\ -f(x) & \text{if } f(x) < 0 \end{cases}$$

* One-to-one function $\Longrightarrow$ inverse function exists

* Find $f^{-1}(x)$.

  $\text{Dom}(f^{-1}) = \text{Ran}(f)$

  $\text{Ran}(f^{-1}) = \text{Dom}(f)$

Ch.3

* Partial fractions

① Check that it's a proper rational function.
If it's improper, use long division first.

② Factorize its denominator.

③ Write down the form of partial fractions.

E.g. $$\frac{5x+3}{(x+1)(x-2)} = \frac{A}{x+1} + \frac{B}{x-2}$$

E.g. $$\frac{5x+3}{(x+1)^2(x-2)} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{x-2}$$

E.g. $$\frac{5x+3}{(x^4-16)(x^2+2x+5)} = \frac{5x+3}{(x^2-4)(x^2+4)(x^2+2x+5)}$$

Cannot be factorized

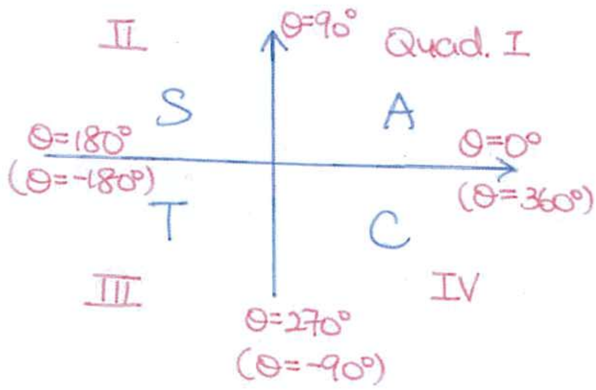
$$= \frac{5x+3}{(x-2)(x+2)(x^2+4)(x^2+2x+5)}$$

$$= \frac{A}{x-2} + \frac{B}{x+2} + \frac{Cx+D}{x^2+4} + \frac{Ex+F}{x^2+2x+5}$$

④ Find the unknowns.

Ch. 4

* CAST rule



* π radians $= 180°$

* Trigonometric identities

* Principal ranges of inverse trigonometric functions:

$\sin^{-1}$: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\cos^{-1}$: $[0, \pi]$

$\tan^{-1}$: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

* Solving trigonometric equations

① $\sin x = k$

$\Rightarrow x = n\pi + (-1)^n \alpha$, where $\alpha = \sin^{-1} k$, for $n \in \mathbb{Z}$

② $\cos x = k$

$\Rightarrow x = 2n\pi \pm \alpha$, where $\alpha = \cos^{-1} k$, for $n \in \mathbb{Z}$

③ $\tan x = k$

$\Rightarrow x = n\pi + \alpha$, where $\alpha = \tan^{-1} k$, for $n \in \mathbb{Z}$

Ch. 5

* Exponential function : $f(x) = b^x$ $(b>0, b\neq 1)$

$Dom(f) = \mathbb{R}$

$Ran(f) = (0, \infty)$

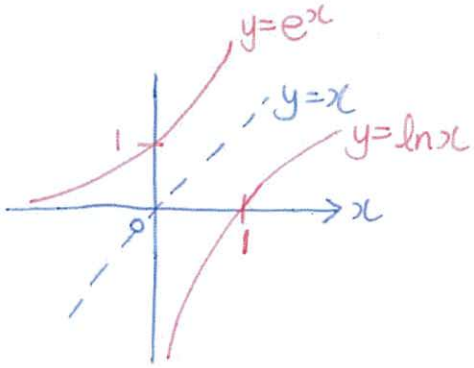
* Logarithmic function : $g(x) = \log_b x$ $(b>0, b\neq 1)$

$Dom(g) = (0, \infty)$

$Ran(g) = \mathbb{R}$

* $y = b^x \Leftrightarrow x = \log_b y$

*



* Properties of log :

$\ln(ab) = \ln a + \ln b$ $(a, b>0)$

$\ln(\frac{a}{b}) = \ln a - \ln b$ $(a, b>0)$

$\ln(a^b) = b \ln a$ $(a>0)$

* Solving equations involving logarithmic or exponential functions

* Hyperbolic functions : $\sinh x = \frac{1}{2}(e^x - e^{-x})$

$\cosh x = \frac{1}{2}(e^x + e^{-x})$