EE2302 Foundations of Information and Data Engineering

Assignment 2 (Solution)

1.

a) No, because the elements *c* and *d* map to the same element in the co-domain of *F*.
b) Yes, because all the elements in *Y* are images of some element in *X*.
c) {*e*, *f*, *g*}.
d) No, because the elements *a* and *b* map to the same element in the co-domain of G.
e) No, because the element *g* is not an image of any element in the domain of *G*.
f) {*e*, *f*}.

2.

a) Yes. Suppose that $g(n_1) = g(n_2)$ for some elements n_1 and n_2 in ***Z***. By definition of g, $5n_1 + 7 = 5n_2 + 7$, which implies that $n_1 = n_2$. Therefore, *g* is injective.

b) No. One possible counter example: Choose $0 \in \boldsymbol{Z}$. There does not exist $n \in \boldsymbol{Z}$ such that $g(n) = 5n + 7 = 0$, since the only root is -7/5, which is not in ***Z***. Therefore, *g* is not surjective.

3. Let x_1 and x_2 be elements in *X* such that $g \circ f(x_1) = g \circ f(x_2)$. That is equivalent to $g(f(x_1)) = g(f(x_2))$. Since *g* is injective, $f(x_1) = f(x_2)$. Since *f* is also injective, $x_1 = x_2$. Hence, $g \circ f$ is injective.

4. Yes. Define a function $f: (0,1) \to (1,100)$ such that $f(x) = 99x + 1$. Suppose $f(x_1) = f(x_2)$. Then $99x_1 + 1 = 99x_2 + 1$, which implies that $x_1 = x_2$. Hence, f is one to one. Given any $y \in (1, 100)$, there exists $x = \frac{y-1}{100} \in (0,1)$ such that $f(x) = y$. Hence, f is onto. Therefore, *f* is a one to one correspondence. Hence, the two sets have the same cardinality.