Student ID:

Question 1 (10 marks)
The following figure shows (a) a 3-bit image of size 5-by-5 image in the square, with x and y coordinates specified, (b) a Laplacian filter.

y \ x	0	1	2	3	4
0	3	7	6	2	0
1	2	4	6	1	1
2	4	7	2	5	4
3	3	0	6	2	1
4	5	7	5	1	2

(a)

Laplacian filter

0	1	0
1	-4	1
0	1	0

(b)

Compute the following:

(a) The output of a 3 × 3 mean filter at (3,3).
(b) The output of a 3 × 3 median filter at (2,3).
(c) The output of the 3 × 3 Laplacian filter shown above at (1,3).
(d) Obtain the histogram of the image.
(e) Apply histogram equalization on the above image and calculate the histogram equalized image, and the new histograms.

Solution:

(a) 1/9*(2+5+4+6+2+1+5+1+2)
(b) 5 [0,1,2,2,5,5,6,7,7]
(c) -4*0+7+3+6+7=23
(d)

Frequency	2	4	5	2	3	3	3	3
Intensity	0	1	2	3	4	5	6	7

(e)
Histogram equalization

r_k	n_r	$p_r(r_k)=n_r/MN$	$T_r=(L-1)P_r(r_k)$
0	2	2/25	7*3/25 =0.84 ->1
1	4	4/25	7*(2+4)/ 25 =1.68 ->2
2	5	5/25	7*(2+4+5)/ 25 =3.08 ->3
3	2	2/25	7*13/25 =3.64 ->4
4	3	3/25	7*16/25 =4.48 ->4
5	3	3/25	7*19/25 =5.52 ->5
6	3	3/25	7*22/25 =6.16 ->6
7	3	3/25	7*25/25 =7 ->7

The new image:

4	7	6	3	1
3	4	6	2	2
4	7	3	5	4
4	1	6	3	2
5	7	5	2	3

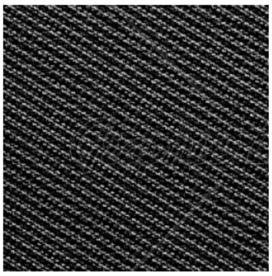
New histogram:

Frequency	0	2	4	5	5	3	3	3
Intensity	0	1	2	3	4	5	6	7

Question 2 (10 marks)
Consider the following images,

(a)

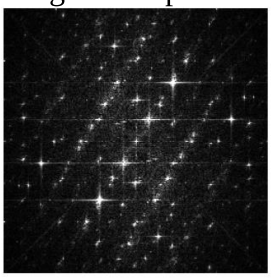
(b)



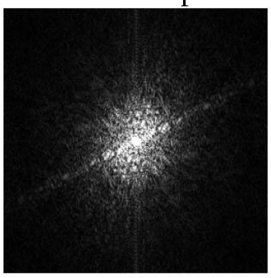
(c)

(d)

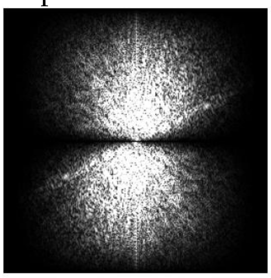
The modulus of the 2D DFT (followed by fftshift) of these images is shown below. Which image corresponds to which Fourier spectrum? Explain the reasons.



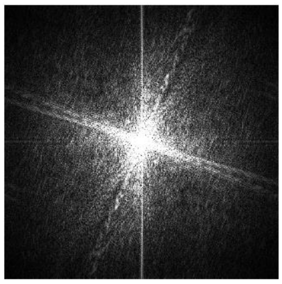
(1)



(2)



(3)



（4）

Solution:

image a) –> spectrum 2): a slowly varying image has essentially low frequency contents.
image b) −> spectrum 4): strong directional features result in orthogonal lines in Fourier.
image c) −> spectrum 1): a periodic pattern results in isolated points in Fourier.
image d) −> spectrum 3): a fastly varying image has higher frequency contents, with limited low-frequency contents.

Question 3 (10 marks)
Suppose that you form a lowpass spatial filter that average the four immediate neighbors of a point (x,y), but excludes the point itself.

(a) Find the equivalent filter H(u,v) in the frequency domain.
(b) Show that your result is a lowpass filter.

Solution:

(a) The spatial average is

$$g(x,y) = \frac{1}{4}\left[f(x,y+1) + f(x+1,y) + f(x-1,y) + f(x,y-1)\right]$$

$$\begin{aligned} G(u,v) &= \frac{1}{4}\left[e^{j2\pi v/N} + e^{j2\pi u/M} + e^{-j2\pi u/M} + e^{-j2\pi v/N}\right] F(u,v) \\ &= H(u,v)F(u,v), \end{aligned}$$

Therefore

$$H(u,v) = \frac{1}{2}\left[\cos(2\pi u/M) + \cos(2\pi v/N)\right]$$

(b) To see that this is a lowpass filter, it helps to express the preceding equation in the form of our familiar centered functions:

$$H(u,v) = \frac{1}{2}\left[\cos(2\pi[u - M/2)/M) + \cos(2\pi[v - N/2]/N)\right].$$

Consider one variable for convenience. As u ranges from 0 to M, the value of $\cos(2\pi[u-M/2)/M)$ starts at -1, peaks at 1 when $u = M/2$ (the center of the filter) and then decreases to -1 again when $u = M$. Thus, we see that the amplitude of the filter decreases as a function of distance from the origin of the centered filter, which is the characteristic of a lowpass filter. A similar argument is easily carried out when considering both variables simultaneously.

Question 4 (10 marks)

Please refer the page 42-45 of lecture notes EE 4211_2B_2020, utilize spatial enhancement methods to enhance the images (in the attachment), and write codes for the task.

Question 5 (10 marks)

Please refer the page 46-48 of lecture notes EE 4211_3B_2020, utilize image enhancement methods in frequency domain to enhance the images (in the attachment), and write codes for the task.