## Question 1.1:

$$G' = (V \cup \{s, t\}, E')$$
$$E \subseteq E'$$
$$for\ all\ u \in V \begin{cases} (s, u) \in E' \\ (u, t) \in E' \end{cases}$$

$$w_{uv} = \begin{cases} 1 & (u, v) \in E \\ \deg(v), & u = s \\ c & v = t \end{cases}$$

$$suppose\ \exists S \subseteq V\ and\ \rho(S) \geq \lambda$$
$$\bar{S} = V \backslash S$$

$$\rho(S) = \frac{\sum_{(u,v)\in E(S)} w_{uv}}{|S|}$$
$$\frac{\sum_{(u,v)\in E(S)} w_{uv}}{|S|} \geq \lambda$$
$$\frac{2|E(S,S)|}{|S|} \geq \lambda$$
$$\sum_{u\in S} \deg(u) - |E(S,\bar{S})| \geq \lambda|S|$$
$$\sum_{u\in S} \deg(u) + \sum_{u\in \bar{S}} \deg(u) - \sum_{u\in \bar{S}} \deg(u) - |E(S,\bar{S})| \geq \lambda|S|$$
$$\sum_{u\in \bar{S}} \deg(u) + |E(S,\bar{S})| + \lambda|S| \leq 2|E|$$

$$cut\ value\ if\ S \begin{cases} = \emptyset, & 2|E| \\ \neq \emptyset, & \sum_{u\in \bar{S}} \deg(u) + |E(S,\bar{S})| + \lambda|S| \end{cases}$$

$$WC(\{s\} \cup S, \bar{S} \cup \{t\}) \leq \gamma$$
$$\frac{\sum_{(u,v)\in E', u\in\{s\}\cup S, v\in\bar{S}\cup\{t\}} w_{uv}}{|S|} \leq \frac{\gamma}{|S|}$$
$$\frac{\gamma}{|S|} \geq \rho(S)$$
$$\gamma \geq \lambda|S|$$

**Question 1.2:**

$$\rho(S) = \frac{c-1}{2}$$

$$\frac{\sum_{(u,v)\in E(S)} w_{uv}}{|S|} = \frac{c-1}{2}$$

$$\sum_{(u,v)\in E'} w_{uv} = \frac{|S|(c-1)}{2}$$

## Question 2

| $A_{ij}$ | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $j\backslash i$ | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** | **11** |
| **1** | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **2** | 1 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| **3** | 1 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| **4** | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| **5** | 0 | 1 | 1 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 |
| **6** | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 |
| **7** | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 1 | 1 | 0 |
| **8** | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 0 |
| **9** | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 1 |
| **10** | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 1 |
| **11** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 |

| $d_1$ | $d_2$ | $d_3$ | $d_4$ | $d_5$ | $d_6$ | $d_7$ | $d_8$ | $d_9$ | $d_{10}$ | $d_{11}$ | $\|E\|$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $=2$ | $=4$ | $=4$ | $=3$ | $=5$ | $=2$ | $=5$ | $=3$ | $=4$ | $=4$ | $=2$ | $=19$ |

| $d_i d_j / 2\|E\|$ | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $j\backslash i$ | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** | **11** |
| **1** | 2/19 | 4/19 | 4/19 | 3/19 | 5/19 | 2/19 | 5/19 | 3/19 | 4/19 | 4/19 | 2/19 |
| **2** | 4/19 | 8/19 | 8/19 | 6/19 | 10/19 | 4/19 | 10/19 | 6/19 | 8/19 | 8/19 | 4/19 |
| **3** | 4/19 | 8/19 | 8/19 | 6/19 | 10/19 | 4/19 | 10/19 | 6/19 | 8/19 | 8/19 | 4/19 |
| **4** | 3/19 | 6/19 | 6/19 | 9/38 | 15/38 | 3/19 | 15/38 | 9/38 | 6/19 | 6/19 | 3/19 |
| **5** | 5/19 | 10/19 | 10/19 | 15/38 | 25/38 | 5/19 | 25/38 | 15/38 | 10/19 | 10/19 | 5/19 |
| **6** | 2/19 | 4/19 | 4/19 | 3/19 | 5/19 | 2/19 | 5/19 | 3/19 | 4/19 | 4/19 | 2/19 |
| **7** | 5/19 | 10/19 | 10/19 | 15/38 | 25/38 | 5/19 | 25/38 | 15/38 | 10/19 | 10/19 | 5/19 |
| **8** | 3/19 | 6/19 | 6/19 | 9/38 | 15/38 | 3/19 | 15/38 | 9/38 | 6/19 | 6/19 | 3/19 |
| **9** | 4/19 | 8/19 | 8/19 | 6/19 | 10/19 | 4/19 | 10/19 | 6/19 | 8/19 | 8/19 | 4/19 |
| **10** | 4/19 | 8/19 | 8/19 | 6/19 | 10/19 | 4/19 | 10/19 | 6/19 | 8/19 | 8/19 | 4/19 |
| **11** | 2/19 | 4/19 | 4/19 | 3/19 | 5/19 | 2/19 | 5/19 | 3/19 | 4/19 | 4/19 | 2/19 |

| $B_{ij}$ | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $j\backslash i$ | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** | **11** |
| **1** | - 2/19 | 15/19 | 15/19 | - 3/19 | - 5/19 | - 2/19 | - 5/19 | - 3/19 | - 4/19 | - 4/19 | - 2/19 |
| **2** | 15/19 | - 8/19 | 11/19 | 13/19 | 9/19 | - 4/19 | - 10/19 | - 6/19 | - 8/19 | - 8/19 | - 4/19 |
| **3** | 15/19 | 11/19 | - 8/19 | 13/19 | 9/19 | - 4/19 | - 10/19 | - 6/19 | - 8/19 | - 8/19 | - 4/19 |
| **4** | - 3/19 | 13/19 | 13/19 | - 9/38 | 23/38 | - 3/19 | - 15/38 | - 9/38 | - 6/19 | - 6/19 | - 3/19 |
| **5** | - 5/19 | 9/19 | 9/19 | 23/38 | - 25/38 | 14/19 | 13/38 | - 15/38 | - 10/19 | - 10/19 | - 5/19 |
| **6** | - 2/19 | - 4/19 | - 4/19 | - 3/19 | 14/19 | - 2/19 | 14/19 | - 3/19 | - 4/19 | - 4/19 | - 2/19 |
| **7** | - 5/19 | - 10/19 | - 10/19 | - 15/38 | 13/38 | 14/19 | - 25/38 | 23/38 | 9/19 | 9/19 | - 5/19 |
| **8** | - 3/19 | - 6/19 | - 6/19 | - 9/38 | - 15/38 | - 3/19 | 23/38 | - 9/38 | 13/19 | 13/19 | - 3/19 |
| **9** | - 4/19 | - 8/19 | - 8/19 | - 6/19 | - 10/19 | - 4/19 | 9/19 | 13/19 | - 8/19 | 11/19 | 15/19 |
| **10** | - 4/19 | - 8/19 | - 8/19 | - 6/19 | - 10/19 | - 4/19 | 9/19 | 13/19 | 11/19 | - 8/19 | 15/19 |
| **11** | - 2/19 | - 4/19 | - 4/19 | - 3/19 | - 5/19 | - 2/19 | - 5/19 | - 3/19 | 15/19 | 15/19 | - 2/19 |

## Question 3

| Iteration 1 | Distance to Centroid 1 | Distance to Centroid 2 | Distance to Centroid 3 | Assign to | | x | y |
|---|---|---|---|---|---|---|---|
| A1 | 0 | 3.6056 | 8.0623 | Centroid 1 | Centroid 1 | 2 | 10 |
| A2 | 5 | 4.2426 | 3.1623 | Centroid 3 | Centroid 2 | 6 | 6 |
| A3 | 8.4853 | 5 | 7.2801 | Centroid 2 | Centroid 3 | 1.5 | 3.5 |
| A4 | 3.6056 | 0 | 7.2111 | Centroid 2 | | | |
| A5 | 7.0711 | 3.6056 | 6.7082 | Centroid 2 | | | |
| A6 | 7.2111 | 4.1231 | 5.3852 | Centroid 2 | | | |
| A7 | 8.0623 | 7.2111 | 0 | Centroid 3 | | | |
| A8 | 2.2361 | 1.4142 | 7.6158 | Centroid 2 | | | |

| Iteration 2 | Distance to Centroid 1 | Distance to Centroid 2 | Distance to Centroid 3 | Assign to | | x | y |
|---|---|---|---|---|---|---|---|
| A1 | 0 | 5.6569 | 6.5192 | Centroid 1 | Centroid 1 | 3 | 9.5 |
| A2 | 5 | 4.1231 | 1.5811 | Centroid 3 | Centroid 2 | 6.5 | 5.25 |
| A3 | 8.4853 | 2.8284 | 6.5192 | Centroid 2 | Centroid 3 | 1.5 | 3.5 |
| A4 | 3.6056 | 2.2361 | 5.7009 | Centroid 2 | | | |
| A5 | 7.0711 | 1.4142 | 5.7009 | Centroid 2 | | | |
| A6 | 7.2111 | 2 | 4.5277 | Centroid 2 | | | |
| A7 | 8.0623 | 6.4031 | 1.5811 | Centroid 3 | | | |
| A8 | 2.2361 | 3.6056 | 6.0415 | Centroid 1 | | | |

| Iteration 3 | Distance to Centroid 1 | Distance to Centroid 2 | Distance to Centroid 3 | Assign to | | x | y |
|---|---|---|---|---|---|---|---|
| A1 | 1.1180 | 6.5431 | 6.5192 | Centroid 1 | Centroid 1 | 3.6667 | 9 |
| A2 | 4.6098 | 4.5069 | 1.5811 | Centroid 3 | Centroid 2 | 7 | 4.3333 |
| A3 | 7.4330 | 1.9526 | 6.5192 | Centroid 2 | Centroid 3 | 1.5 | 3.5 |
| A4 | 2.5 | 3.1325 | 5.7009 | Centroid 1 | | | |
| A5 | 6.0208 | 0.5590 | 5.7009 | Centroid 2 | | | |
| A6 | 6.2650 | 1.3463 | 4.5277 | Centroid 2 | | | |
| A7 | 7.7621 | 6.3885 | 1.5811 | Centroid 3 | | | |
| A8 | 1.1180 | 4.5069 | 6.0415 | Centroid 1 | | | |

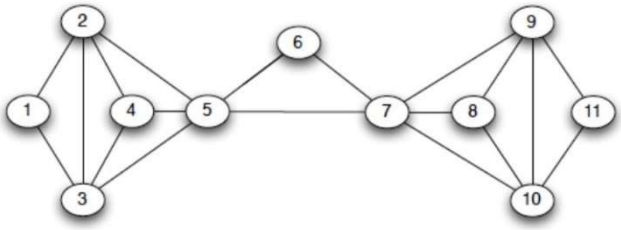
| Iteration 4 | Distance to Centroid 1 | Distance to Centroid 2 | Distance to Centroid 3 | Assign to | | x | y |
|---|---|---|---|---|---|---|---|
| A1 | 1.9437 | 7.5572 | 6.5192 | Centroid 1 | Centroid 1 | 3.6667 | 9 |
| A2 | 4.3333 | 5.0442 | 1.5811 | Centroid 3 | Centroid 2 | 7 | 4.3333 |
| A3 | 6.6165 | 1.0541 | 6.5192 | Centroid 2 | Centroid 3 | 1.5 | 3.5 |
| A4 | 1.6667 | 4.1767 | 5.7009 | Centroid 1 | | | |
| A5 | 5.2068 | 0.6667 | 5.7009 | Centroid 2 | | | |
| A6 | 5.5176 | 1.0541 | 4.5277 | Centroid 2 | | | |
| A7 | 7.4907 | 6.4377 | 1.5811 | Centroid 3 | | | |
| A8 | 0.3333 | 5.5478 | 6.0415 | Centroid 1 | | | |

The new centroids are:

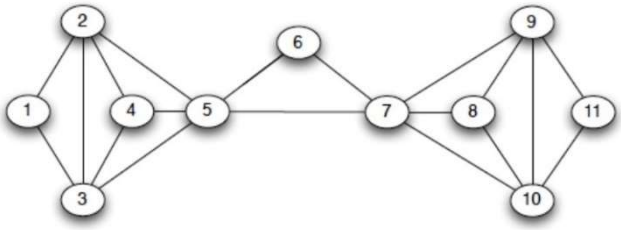
$$C1: \{A1, A4, A8\}\ centre\ at\ (3.6667,9)$$
$$C2: \{A3, A5, A6\}\ centre\ at\ (7,4.333)$$
$$C3: \{A2, A7\}\ centre\ at\ (1.5,3.5)$$

## Question 4.1

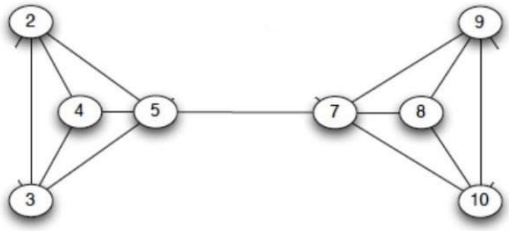
K = 1



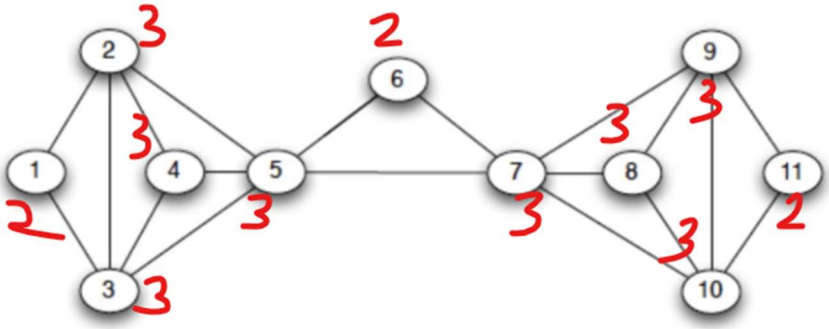

K = 2




K = 3




Core number of each node



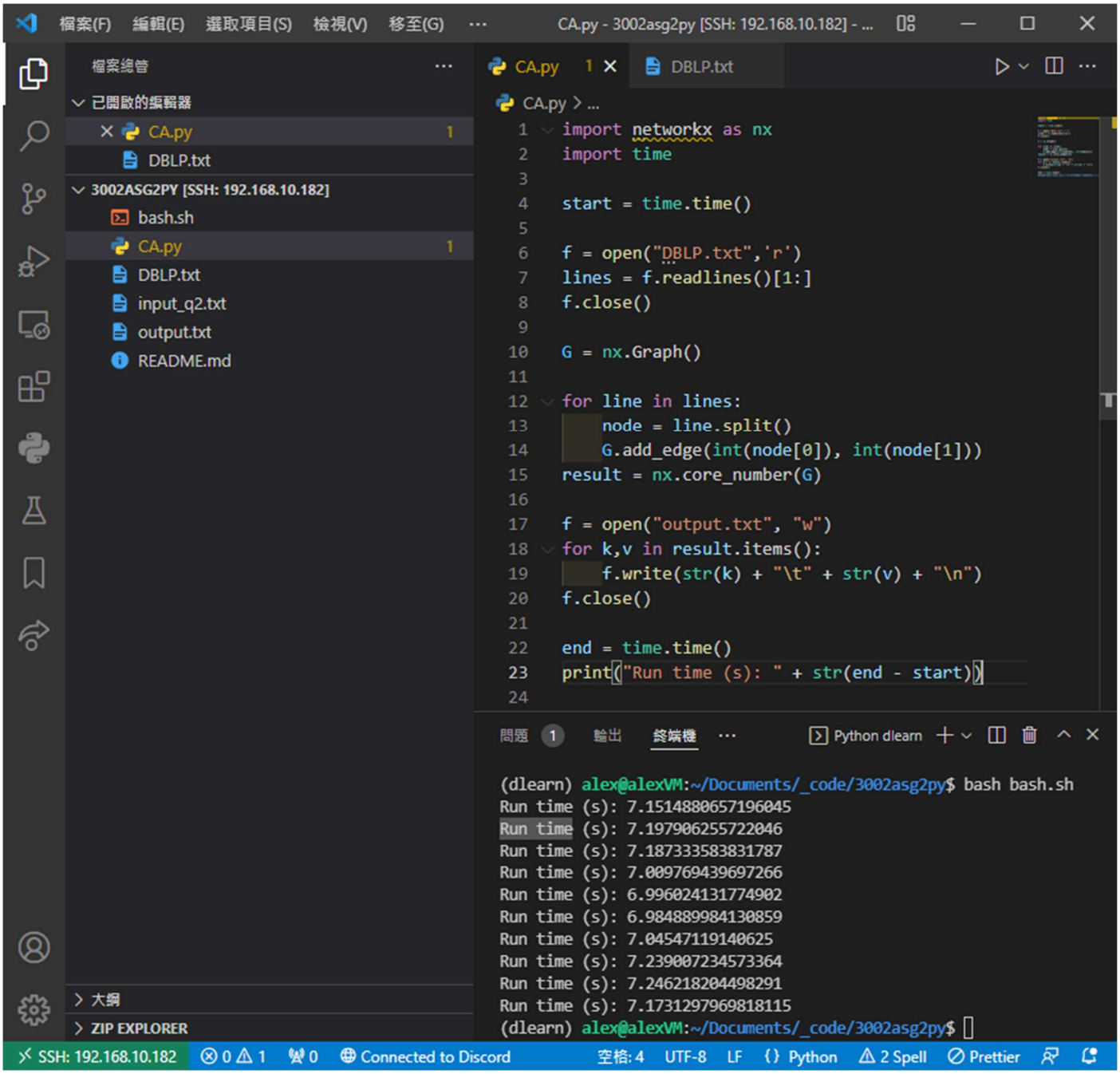
## Question 4.2:

Input: undirected graph G = (V, E)

Output: array of number of core number for each node

1. Set a degree map for each node u in G, MapD, key: u, value: degree
2. Sort MapD by degree
3. Set a core number array for each node, coreD
4. Set G' <- G
5. For k <- from 1 to n
    1. If (G' empty) break from loop
    2. For all MapD [value] =< k
        1. G' <- G'\u
    3. Recalculate MapD
    4. For all MapD [value]! = 0
        1. coreD [v]++
6. Return coreD

## Question 4.3:




Running time in second = (6.985, 7.246)