

Question 1(i)

$$\begin{aligned}
f(S_1, S_2) &= 1 - JS(S_1, S_2) \\
&= 1 - \frac{|S_1 \cap S_2|}{|S_1 \cup S_2|} \\
&= \frac{|S_1 \cup S_2| - |S_1 \cap S_2|}{|S_1 \cup S_2|} \\
&\because |S_1 \cup S_2| - |S_1 \cap S_2| \geq 0 \\
&\therefore f(S_1, S_2) \geq 0
\end{aligned}$$

$$\begin{aligned}
f(S_2, S_1) &= 1 - JS(S_2, S_1) \\
&= 1 - \frac{|S_2 \cap S_1|}{|S_2 \cup S_1|} \\
&= 1 - \frac{|S_1 \cap S_2|}{|S_1 \cup S_2|} \\
&= f(S_1, S_2) \\
&\therefore f(S_1, S_2) = f(S_2, S_1) \geq 0
\end{aligned}$$

Question 1(ii)

$$\begin{aligned}
f(S_1, S_1) &= 1 - JS(S_1, S_1) \\
&= 1 - \frac{|S_1 \cap S_1|}{|S_1 \cup S_1|} \\
&= 1 - \frac{|S_1|}{|S_1|} \\
&= 0
\end{aligned}$$

$$\begin{aligned}
f(S_1, S_2) &= 1 - \frac{|S_1 \cap S_2|}{|S_1 \cup S_2|} \\
0 &= 1 - \frac{|S_1 \cap S_2|}{|S_1 \cup S_2|} \\
1 &= \frac{|S_1 \cap S_2|}{|S_1 \cup S_2|} \\
&\because |S_1 \cup S_2| = |S_1 \cap S_2| \\
&\therefore S_1 = S_2
\end{aligned}$$

Question 1(iii)

$$\begin{aligned}
f(S_1, S_3) &= 1 - JS(S_1, S_3) \\
&= 1 - \frac{|S_1 \cap S_3|}{|S_1 \cup S_3|}
\end{aligned}$$

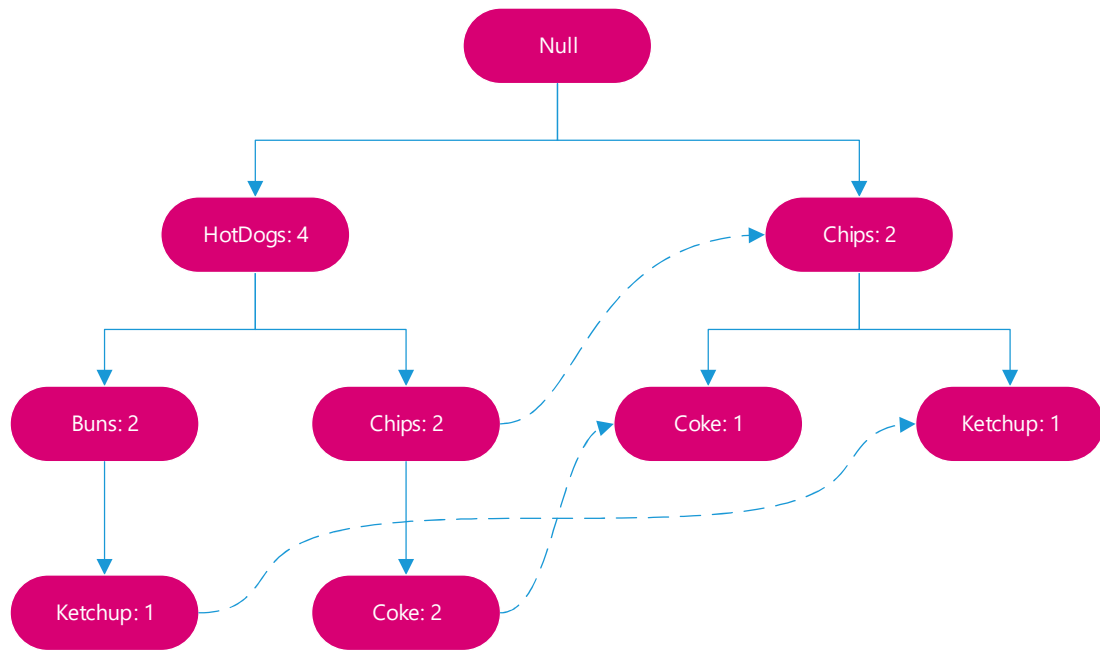
$$\begin{aligned}
f(S_1, S_2) + f(S_2, S_3) &= 1 - JS(S_1, S_2) + 1 - JS(S_2, S_3) \\
&= 2 - \frac{|S_1 \cap S_2|}{|S_1 \cup S_2|} - \frac{|S_2 \cap S_3|}{|S_2 \cup S_3|}
\end{aligned}$$

$$\begin{aligned}
0 &\leq JS(A, B) \leq 1 \\
0 &\leq 1 - f(A, B) \leq 1 \\
0 &\leq f(A, B) \leq 1
\end{aligned}$$

$$\begin{aligned}
&\therefore 0 \leq f(S_1, S_3) \leq 1 \\
&\therefore 0 \leq f(S_1, S_2) + f(S_2, S_3) \leq 2 \\
&\therefore f(S_1, S_3) \leq f(S_1, S_2) + f(S_2, S_3)
\end{aligned}$$

## Question 2

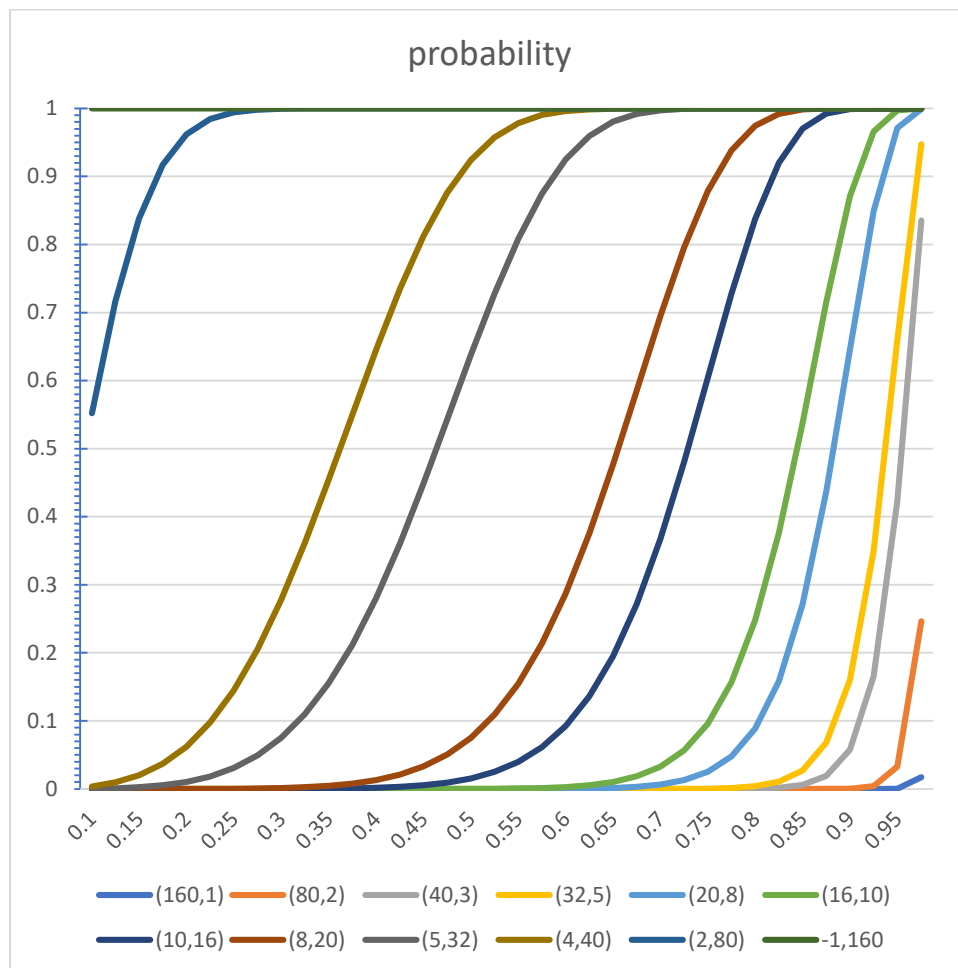
Item	Count
HotDogs	4
Chips	4
Coke	3
Buns	2
Ketchup	2



Question 3:

$$p = 1 - (1 - s^r)^b$$

r	160	80	40	32	20	16	10	8	5	4	2	1
b	1	2	4	5	8	10	16	20	32	40	80	160
0.1	0	0	0	0	0	0	1.6E-09	2E-07	0.00032	0.003992	0.552477	1
0.2	0	0	0	0	8.35E-14	6.55E-11	1.64E-06	5.12E-05	0.010189	0.062043	0.961832	1
0.3	0	0	0	0	2.79E-10	4.3E-08	9.45E-05	0.001311	0.074901	0.277703	0.999471	1
0.4	0	0	0	9.23E-13	8.8E-08	4.29E-06	0.001676	0.013026	0.280623	0.645602	0.999999	1
0.5	0	0	3.64E-12	1.16E-09	7.63E-06	0.000153	0.015511	0.075293	0.637945	0.924343	1	1
0.6	0	0	5.35E-09	3.98E-07	0.000292	0.002818	0.09248	0.287358	0.925009	0.996121	1	1
0.7	0	8.11E-13	2.55E-06	5.52E-05	0.006366	0.03274	0.367748	0.695026	0.997228	0.999983	1	1
0.8	0	3.53E-08	0.000532	0.003955	0.088596	0.248371	0.837553	0.974599	0.999997	1	1	1
0.85	5.09E-12	4.51E-06	0.005996	0.027264	0.271119	0.537691	0.97004	0.998275	1	1	1	1
0.9	4.77E-08	0.000437	0.057826	0.160292	0.645488	0.871185	0.998951	0.999987	1	1	1	1



Pair  $(r = 10, b = 16)$  is chosen, because this pair has high probability when  $t \geq 0.85$  compared with other pairs while it has low probability when  $t \leq 0.85$  compared with other pairs.