EE2302 Foundations of Information and Data Engineering

Assignment 9 (Solution)

1.

a) The codewords are

0 0 0 0 0
1 0 1 1 0
0 1 0 1 1
1 1 1 0 1

b) Yes. It can be checked that the following two conditions hold:

i. Closed under Addition: It is straightforward to check that the sum of any two codewords is equal to another codeword. For example, 10110 + 01011 = 11101.

ii. Closed under Scalar Multiplication: Multiplying 0 to any codeword c gives 00000, which is a codeword. Multiplying 1 to any codeword c gives c itself, which is, of course, a codeword.

c) The minimum distance d_{min} is 3. It can correct one error bit.

d) The generator matrix $G = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \end{bmatrix}$.

The parity check matrix $H = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$.

e) The syndrome is $s = yH^T = [1 \quad 0 \quad 0 \quad 1 \quad 0]\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = [1 \quad 0 \quad 0]$.

f) The first component is non-zero and the last two components are zero, which indicate that the first parity-check equation is in error while the last two parity-check equations are without error. Since only c_3 occurs in the first parity-check equation but not in the last two equations, the error bit is c_3.

2.

a) Yes. It is systematic because the information bit is embedded in the corresponding codeword.

b) $G = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \end{bmatrix}$; $H = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix}$. (The answer for H is not unique.)

c) For this code, the code rate is 1/5 and the minimum distance is 5 (which can correct two error bits).
For the code in Q.1, the code rate is 2/5 and the minimum distance is 3 (which can correct one error bit).
Therefore, this code is less efficient in sending information, but has a better error correction capability.