Chapter 3. Multiple Integral

1 Single-Variable Case:

- Definition:

- Interpretation:

- Computation:

$$\int_a^b f(x)dx = F(b) - F(a),$$

where F is an antiderivative of f, i.e. $F'(x) = f(x)$.

2 Two-Variable Case (Double Integral):

2.1 Definition and Interpretation:

2.2 Computation of Double Integrals:

Case 1: R is an rectangle:

Example Use iterated integral in two different orders to evaluate

$$\iint_R \left(2xy + y^2\right) dxdy$$

with $R = [1, 2] \times [0, 1]$.

Case 2. R **is vertically simple (vertical segments are easily bounded in** x**)**

Example Evaluate $\iint_R 2x^2 y dx dy$, with R as a region bounded by $x = 1, x = 3, y = x^2$ and $y = -x^2 + 18$.

Exercise Evaluate $\iint_R xy^2 dx dy$, with R as a region bounded by $x = 1, y = 0$ and $y = x^2$.

Case 3. R **is horizontally simple (horizontal segments are easily bounded in** y **)**

Example Evaluate

$$\iint_R xy^2 \, dxdy$$

, with R as a region bounded by $x = y^2$ and $x = -y^2 + 1$.

Example Show that

$$\int_0^1 \int_x^{\sqrt{x}} f(x,y)\, dy\, dx = \int_0^1 \int_{y^2}^{y} f(x,y)\, dx\, dy.$$