MA1200 Practice Exercise 1 **Coordinate Geometry and Conic Section**

Completing the square

(With reference to the identity $(a+b)^2 \equiv a^2+2ab+b^2$ and $(a-b)^2 \equiv a^2-2ab+b^2$.)

1. Express each of the following expression in the form $(x+a)^2+b$ or $(y+a)^2+b$, where a and b are constants.

(a) $x^2+12x-3$ (b) $y^2-18y+5$

(c) x^2+6x (d) y^2+9y+1

(e) x^2-7x+4 (f) y^2-21y

2. Express each of the following expression in the form $(kx+a)^2+b$ or $(ky+a)^2+b$, where k, a and b are constants.

(a) $4x^2+24x-9$ (b) $9y^2-18y+5$

(c) $25x^2+80x+25$ (d) $3y^2+2\sqrt{3}y+15$

(e) $5+2x^2-6\sqrt{2}x$ (f) $12\sqrt{7}y+7y^2+21$

3. Express each of the following equation of parabola to its standard form. (i.e. $(y-k)^2=4p(x-h)$ or $(x-h)^2=4p(y-k)$.) Sketch its graph.

(a) $y^2-6y-24x+9=0$ (b) $x^2+4x-20y+24=0$

(c) $x^2-8x-12y+22=0$ (d) $y^2-2y+8x+25=0$

(e) $2y^2+4y+24x-46=0$ (f) $4x^2-4x-48y-47=0$

4. Determine the location(s) where the parabola cuts the x-axis. (The formula $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$ is useful for finding roots of the quadratic equation $ax^2+bx+c=0$.)

(a) $y=x^2+2x-5$ (b) $y=4x^2+12x+9$

(c) $y=3x^2+4x+10$

5. Express each of the following equation of circle to its standard form. (i.e. $(x-h)^2+(y-k)^2=r^2$.) Sketch its graph.

(a) $x^2+y^2-2x+8y+8=0$ (b) $x^2+y^2-2\sqrt{2}x-2\sqrt{3}y-20=0$

(c) $4x^2+4y^2+4x-8y-59=0$ (d) $4x^2+4y^2-16x+12y+21=0$

Polar Coordinates

6. Convert each of the following points from polar coordinates to rectangular coordinates.

(a) $(3, 120^\circ)$ (b) $(7, -30^\circ)$ (c) $(6, -135^\circ)$ (d) $(2, 90^\circ)$

7. Convert each of the following points from rectangular coordinates to polar coordinates. Express θ in the range $-180^\circ < \theta \le 180^\circ$.

(a) $P(6, 2\sqrt{3})$ (b) $P(-1, \sqrt{3})$ (c) $P(-4\sqrt{3}, -4)$ (d) $P(2\sqrt{3}, -2)$

8. Express each of the following equation of ellipse to its standard form. (i.e. $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ or $\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$.) Sketch its graph with the coordinates of vertices clearly shown. What are the coordinates of the Foci?

(a) $4x^2 + 36y^2 - 144 = 0$ (b) $169x^2 + 25y^2 - 4225 = 0$

(c) $4x^2 + 9y^2 - 8x - 36y + 4 = 0$ (d) $25x^2 + y^2 - 150x + 2y + 201 = 0$

9. Express each of the following equation of hyperbola to standard form. (i.e. $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ or $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$.) Sketch its graph with the coordinates of vertices clearly shown.

(a) $16x^2 - 25y^2 + 400 = 0$ (b) $9x^2 - 25y^2 - 225 = 0$

(c) $16x^2 - 9y^2 - 32x - 36y - 164 = 0$ (d) $5x^2 - 4y^2 + 10x + 8y + 21 = 0$

10. Classify the type of conic section described by each of the following equations using completing the squares. Write the equation in the standard form of the respective conic section.

(a) $4x^2 + 4y^2 - 4x + 12y - 6 = 0$ (b) $y^2 + 4x - 6y + 11 = 0$

(c) $4x^2 - y^2 + 2y - 5 = 0$ (d) $x^2 + 4y^2 - 2x + 24y + 33 = 0$

11. Write each equation in terms of a rotated $x'y'$-system using the angle of rotation θ. Write the equation involving x' and y' in standard form.

(a) $x^2 - 4xy + y^2 - 3 = 0$, $\theta = 45^\circ$ (b) $23x^2 + 26\sqrt{3}xy - 3y^2 - 144 = 0$, $\theta = 30^\circ$

12. Classify the type of conic section described by each of the following equations without using completing the squares.

(a) $4x^2 - 9y^2 - 8x - 36y - 68 = 0$ (b) $4x^2 + 4y^2 + 12x + 4y + 1 = 0$

(c) $x^2 + 4xy - 2y^2 - 1 = 0$ (d) $34x^2 - 24xy + 41y^2 - 25 = 0$

13. Determine the location(s) where the two curves meet in each case.

(a) $4x^2 + y^2 = 4$ and $x + y = 3$ (b) $4x^2 + y^2 = 4$ and $2x - y = 2$

(c) $x^2 + y^2 = 1$ and $x^2 + 9y^2 = 9$