MA1200 Practice Exercise 9 Answers

1 (a) $${}_nC_{n-3}=\frac{n!}{(n-3)![n-(n-3)]!}=\frac{n!}{(n-3)!\,3!}=\frac{n(n-1)(n-2)(n-3)!}{(n-3)!\cdot 3\cdot 2\cdot 1}=\frac{n(n-1)(n-2)}{6}$$

(b) $${}_nC_{n-2}+{}_nC_{n-1}={}_nC_2+{}_nC_1=\frac{n!}{2!(n-2)!}+\frac{n!}{1!(n-1)!}$$
$$=\frac{n(n-1)(n-2)!}{2!(n-2)!}+\frac{n(n-1)!}{1!(n-1)!}=\frac{n(n-1)}{2}+n=\frac{n(n+1)}{2}$$

2. (a) $$\sum_{i=1}^{6}\left(i^2+1\right)=\left(1^2+1\right)+\left(2^2+1\right)+\left(3^2+1\right)+\left(4^2+1\right)+\left(5^2+1\right)+\left(6^2+1\right)$$
$$=2+5+10+17+26+37$$

(b) $$\sum_{r=4}^{7}\left[(-2)^r-5\right]=\left[(-2)^4-5\right]+\left[(-2)^5-5\right]+\left[(-2)^6-5\right]+\left[(-2)^7-5\right]$$
$$=11-37+59-133$$

(c) $$\sum_{r=7}^{n}\frac{r-1}{r}=\frac{7-1}{7}+\frac{8-1}{8}+\frac{9-1}{9}+\cdots+\frac{n-1}{n}$$
$$=\frac{6}{7}+\frac{7}{8}+\frac{8}{9}+\cdots+\frac{n-1}{n}$$

(d) $$2\sum_{r=0}^{n}\frac{n-r}{n+r}=2\left[\frac{n-0}{n+0}+\frac{n-1}{n+1}+\frac{n-2}{n+2}+\cdots+\frac{n-(n-1)}{n+(n-1)}+\frac{n-n}{n+n}\right]$$
$$=2\left[1+\frac{n-1}{n+1}+\frac{n-2}{n+2}+\cdots+\frac{1}{2n-1}\right]$$

(e) $$\sum_{r=1}^{8}3=\underbrace{3+3+\cdots+3}_{8\text{ terms}}=24$$

3. (a) $$\sum_{r=1}^{n-1}\frac{1}{3^r}$$

(b) $$\sum_{r=1}^{n}(a+d^r)$$

(c) $$\sum_{r=1}^{n}(2r-1)$$

4. (a) $$(2x-3)^4=\sum_{r=0}^{4}{}_4C_r(2x)^{4-r}(-3)^r=(2x)^4+4(2x)^3(-3)+6(2x)^2(-3)^2+4(2x)(-3)^3+(-3)^4$$
$$=16x^4-96x^3+216x^2-216x+81$$

(b) $$\left(z-\frac{1}{z}\right)^5=\sum_{r=0}^{5}{}_5C_r z^{5-r}\left(-\frac{1}{z}\right)^r$$
$$=z^5+5z^4\left(-\frac{1}{z}\right)+10z^3\left(-\frac{1}{z}\right)^2+10z^2\left(-\frac{1}{z}\right)^3+5z\left(-\frac{1}{z}\right)^4+\left(-\frac{1}{z}\right)^5$$
$$=z^5-5z^3+10z-\frac{10}{z}+\frac{5}{z^3}-\frac{1}{z^5}$$

(c) $\left(\frac{a}{2}+\frac{2}{a}\right)^6 = \sum_{r=0}^{6} {}_6C_r \left(\frac{a}{2}\right)^{6-r} \left(\frac{2}{a}\right)^r$

$$= \left(\frac{a}{2}\right)^6 + 6\left(\frac{a}{2}\right)^5\left(\frac{2}{a}\right) + 15\left(\frac{a}{2}\right)^4\left(\frac{2}{a}\right)^2 + 20\left(\frac{a}{2}\right)^3\left(\frac{2}{a}\right)^3 + 15\left(\frac{a}{2}\right)^2\left(\frac{2}{a}\right)^4 + 6\left(\frac{a}{2}\right)\left(\frac{2}{a}\right)^5 + \left(\frac{2}{a}\right)^6$$

$$= \frac{a^6}{64} + \frac{3}{8}a^4 + \frac{15}{4}a^2 + 20 + \frac{60}{a^2} + \frac{96}{a^4} + \frac{64}{a^6}$$

(d) $\left[\sqrt{5}(\cos\theta + i\sin\theta)\right]^4 = \left(\sqrt{5}\right)^4(\cos\theta + i\sin\theta)^4$

$$= 25\left[(\cos\theta)^4 + 4(\cos\theta)^3(i\sin\theta) + 6(\cos\theta)^2(i\sin\theta)^2 + 4(\cos\theta)(i\sin\theta)^3 + (i\sin\theta)^4\right]$$

$$= 25\left[\cos^4\theta + 4i\cos^3\theta\sin\theta + 6i^2\cos^2\theta\sin^2\theta + 4i^3\cos\theta\sin^3\theta + i^4\sin^4\theta\right]$$

(e) $\left(\frac{2x}{y} - \frac{y}{4x^2}\right)^5 = \sum_{r=0}^{5} {}_5C_r \left(\frac{2x}{y}\right)^{5-r}\left(-\frac{y}{4x^2}\right)^r$

$$= \left(\frac{2x}{y}\right)^5 + 5\left(\frac{2x}{y}\right)^4\left(-\frac{y}{4x^2}\right) + 10\left(\frac{2x}{y}\right)^3\left(-\frac{y}{4x^2}\right)^2 + 10\left(\frac{2x}{y}\right)^2\left(-\frac{y}{4x^2}\right)^3$$

$$+ 5\left(\frac{2x}{y}\right)\left(-\frac{y}{4x^2}\right)^4 + \left(-\frac{y}{4x^2}\right)^5$$

$$= \frac{32x^5}{y^5} - \frac{20x^2}{y^3} + \frac{5}{xy} - \frac{5y}{8x^4} + \frac{5y^3}{128x^7} - \frac{y^5}{1024x^{10}}$$

5. (a) $\left(\frac{1}{5} - 5x\right)^9 = \sum_{r=0}^{9} {}_9C_r \left(\frac{1}{5}\right)^{9-r}(-5x)^r$

The term in x^6 is ${}_9C_6\left(\frac{1}{5}\right)^3(-5x)^6$.

$\therefore$ Coefficient of the term in x^6 is ${}_9C_6\left(\frac{1}{5}\right)^3(-5)^6 = 10500$.

(b) $(2y-3)^7 = (-3+2y)^7 = \sum_{r=0}^{7} {}_7C_r(-3)^{7-r}(2y)^r$.

The 4th term in ascending powers of y (when $r = 3$) is ${}_7C_3(-3)^4(2y)^3$.

$\therefore$ The required coefficient is ${}_7C_3(-3)^4(2)^3 = 22680$.

(c) $\left(5z - \frac{3}{z}\right)^8 = \sum_{r=0}^{8} {}_8C_r(5z)^{8-r}\left(-\frac{3}{z}\right)^r$.

The constant term is the term when $8 - r = r$, i.e. $r = 4$.

$\therefore$ The required coefficient is ${}_8C_4(5)^4(-3)^4 = 3543750$.