Class Assignment 2 - Solution

1)

a) When t < 0, i(t) = 0 A.

b) When t < 0, v(t) = 20 V.

c)

When t > 0, the voltage source can be considered as replaced by "short circuit" as 20u(-t) = 0.

The Norton resistance at the capacitor terminal is

$$R_N = 5||10 = \frac{5(10)}{5+10} = \frac{10}{3}\Omega$$

and the time constant is

$$\tau = R_N C = \frac{2}{3}s$$

When $t \to \infty$, the capacitor is fully charged by the current source

$$v(\infty) = 3\left(\frac{10}{3}\right) = 10V$$

After the switch is closed, v(t) decreases from 20V to 10V ➔ Case B for v_c where $\Delta v = 10V$

$$v(t) = v(\infty) + \Delta v\, e^{-\frac{t-t_0}{\tau}} = 10 + 10\, e^{-\frac{t-0}{2/3}} = 10(1 + e^{-1.5t})\ V$$

For t > 0, the voltage across the 5Ω resistor is the same as that of capacitor in parallel. The current along the resistor is

$$i(t) = -\frac{v(t)}{5} = -2(1 + e^{-1.5t})\ A$$

NOTE

After solving voltage across capacitor by Case A or Case B, current along the capacitor could be evaluated from v. For t > 0, to find current along the capacitor:

$$i_c(t) = C\frac{dv}{dt} = C(-15e^{-1.5t}) = -3e^{-1.5t}\ A$$

OR by KCL :

$$i_c(t) = 3 - \frac{v(t)}{5} - \frac{v(t)}{10} = 3 - 2(1 + e^{-1.5t}) - (1 + e^{-1.5t}) = -3e^{-1.5t}\ A$$

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2.

When t < 0, i(t) = 0 A when both S1 and S2 are open.

For 0 < t < 2, we evaluate current i_1 along the inductor for S1 is closed and S2 is open.

The Norton resistance at the inductor terminal is

$$R_1 = 15 + 10 + 20 = 45\Omega$$

Time constant is

$$\tau_1 = \frac{L}{R_1} = \frac{5}{45} = \frac{1}{9}s$$

To find the current along the inductor, we assume that S2 remains open

$$i_1(\infty) = 6 \times \frac{15}{15 + 10 + 20} = 2A$$

The current along the inductor can be described with Case A as the current increases for 0 < t < 2:

$$i_1(t) = i_1(t_0) + \Delta i_1 \left(1 - e^{-\frac{t-t_0}{\tau_1}}\right) = 2(1 - e^{-9t})\ A$$

For t > 2, S2 is closed and we evaluate current i_2 along the inductor.

The Norton resistance at the inductor terminal is

$$R_2 = 15 + 10 = 25\Omega$$

Time constant is

$$\tau_2 = \frac{L}{R_2} = \frac{5}{25} = 0.2s$$

At $t \to \infty$, current along the inductor converges to

$$i_2(\infty) = 6 \times \frac{15}{15 + 10} = 3.6A$$

Therefore, for t > 2, i_2 increases and Case A applies

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$$i_2(t) = i_2(t_0) + \Delta i_2\left(1 - e^{-\frac{(t-t_0)}{\tau}}\right) = i_1(2) + \left(i_2(\infty) - i_1(2)\right)\left(1 - e^{-(t-2)/0.2)}\right)$$
$$= \left(3.6 - 1.6e^{-5(t-2)}\right)A$$

Therefore,

$$i(t) = \begin{cases} 0\ A & ,t < 0 \\ 2(1 - e^{-9t})\ A & ,0 < t < 2 \\ \left(3.6 - 1.6e^{-5(t-2)}\right)A & ,t > 2 \end{cases}$$

i(1) = 1.9998 A

i(3) = 3.5892 A