Tutorial 4 (with solution)

Numbers

Question 1: Divisibility

- For all integers a, b, and c, if $a|b$ and $a|c$, then $a|(b+c)$? Is it true? Prove or disprove it.
  a. Yes
  b. No

Q.1 (solution)

Yes.

Proof

By definition of divisibility,

$$b = ar \text{ and } c = as \text{ for some integers } r \text{ and } s.$$

By substitution,

$$b + c = ar + as = a(r + s).$$

Since $r + s$ is an integer, by definition of divisibility, $a|(b + c)$.

Q.E.D.

Question 2: Simple Proof

- Prove that the square of any odd integer has the form $8m + 1$ for some integer m.

Q.2 (solution)

Proof

Let n be an odd number, which can be written as $n = 2k + 1$ for some integer k.

Then,

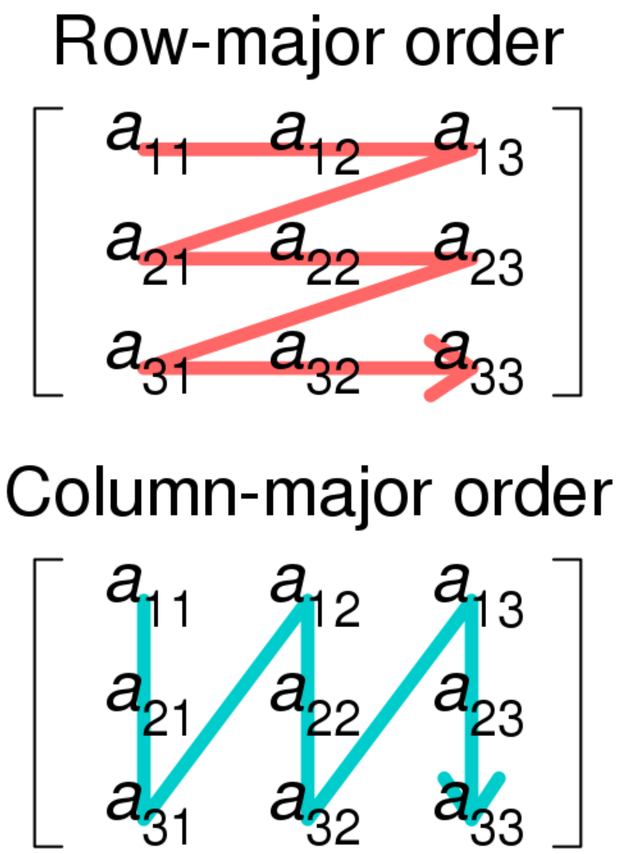
$$n^2 = (2k+1)(2k+1) = 4k^2 + 4k + 1$$
$$= 4k(k+1) + 1$$

Since either k or $k + 1$ is an even number, we have $n^2 = 8m + 1$ for some integer m.

Q.E.D.

Question 3: Data Storage

- In computing, row-major order and column-major order are methods for storing two-dimensional array in linear storage such as RAM or hard disk.



Question 3: Data Storage

- A matrix M has 3 rows and 4 columns.

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}$$

- The 12 entries in M are to be stored in row major order in locations 7,609 to 7,620 in a computer's memory.

a) Which location will a_{22} be stored in?

b) Write a formula (in i and j) for the location in which a_{ij} is stored.

c) Find formulas (in n) for r and s so that a_{rs} is stored in location 7,609 + n.

Q.3 (solution)

a) 7614.

b) $7609 + 4(i-1) + (j-1)$.

c) $r = 1 + (n \operatorname{div} 4)$ and $s = 1 + (n \bmod 4)$.

- Alternatively, we can write $r = 1 + \lfloor n/4 \rfloor$, where $\lfloor x \rfloor$ is called the floor function. It gives the greatest integer less than or equal to x as its output.

Question 4: Euclidean Algorithm

- Compute gcd(65432, 8642).

Q.4 (solution)

7	65432	8642	1
	60494	4938	
1	4938	3704	3
	3704	3702	
617	1234	2	
	1234		
	0		

Therefore,
$\gcd(65432, 8642) = 2.$

Question 5: Extended Euclidean Alg.

- Find a solution in integers to the equation

$$65432x + 8642y = \gcd(65432, 8642).$$

Q.5

7	65432	8642	1
	60494	4938	
1	4938	3704	3
	3704	3702	
617	1234	2	
	1234		
	0		

65432	**8642**		
1	0	65432	a
0	1	8642	b

Q.5 (solution)

65432	**8642**		
1	0	65432	a
0	1	8642	b
1	-7	4938	$c = a - 7b$
-1	8	3704	$d = -a + 8b$
2	-15	1234	$e = c - d$
-7	53	2	$f = d - 3e$

$$x = -7 \text{ and } y = 53$$