Problem Set 7 Hint Sheet

Q1

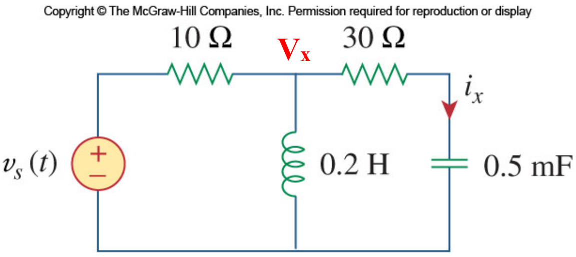
Convert source from sinusoid to phasor: 5 cos($10t$ + 40°) → $I_S = 5\angle 40°$ A
Note that 3Ω and 0.1F are in series ⇨ Impedance Z = 3 – j3Ω
Note that 4Ω, 0.2H and Z are in parallel
We can thus find current through Z using current divider rule:

$$I_o = \frac{1/Z}{(1/Z)+1/4+1/j2} I_S$$

Final step: Convert Io from phasor into sinusoid

Q2

Convert source from sinusoid to phasor: 20 sin(100t – 40°) → $V_S = 20\angle{-130°}$ V
Note that 30Ω and 0.5mF are in series ⇨ $Z_x = 30 - j20\ \Omega$
Suggested approach to solve: Apply nodal analysis at V_x (see figure below) and then use V_x to find I_x ($I_x = V_x/Z_x$)



Nodal equation:

$$\frac{V_x - V_s}{10} + \frac{V_x}{j20} + \frac{V_x}{30 - j20} = 0$$

⇨$V_x = 15.643\angle{-114.29°}$ V

Q3

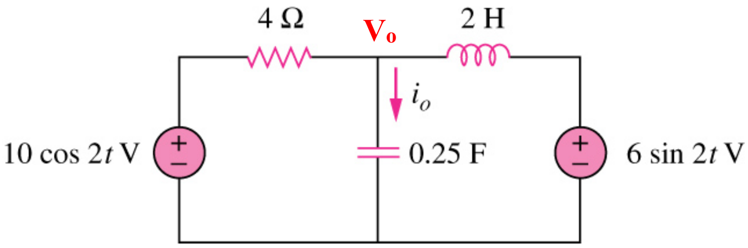
Apply nodal analysis at node V (i.e. voltage across current source)

$$\frac{V - (120\angle -15°)}{40 + j20} + \frac{V}{-j30} + \frac{V}{50} + 6\angle 30° = 0$$

Q4

Convert sources from sinusoid to phasor: 10 cos(2t) → $10\angle 0°$, 6 sin(2t) → $6\angle{-90°}$

Work out the impedance values for the inductor and capacitor: 2H → j4, 0.25F → -j2



Suggested approach: Apply nodal analysis at V_o (see figure above) and then use V_o to find I_o through the impedance of the 0.25F capacitor.

Nodal equation at node V_o:

$$\frac{V_o - 10\angle 0^\circ}{4} + \frac{V_o}{-j2} + \frac{V_o - 6\angle -90^\circ}{j4} = 0$$

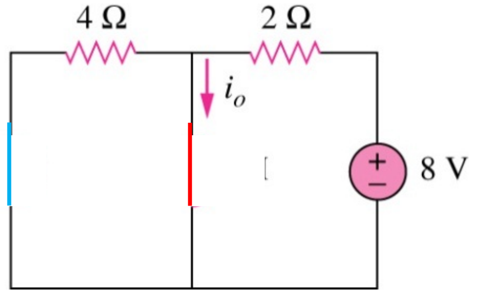
⇨ $V_o = 2.828\angle -45^\circ$

Q5

First analyse at DC

Replace 1H inductor with short circuit; Replace AC source with short circuit

Re-draw the circuit, which should look like this:

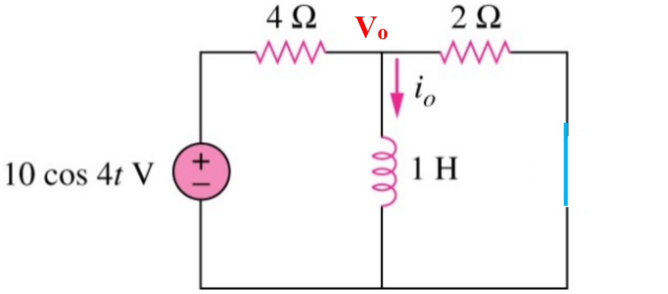


Observe that in the above circuit, 4Ω is being bypassed and all the current through 2Ω will flow through the inductor path (marked in red).

⇨$I_o = 8V/2\Omega$

Next analyse at AC

Replace DC source with short circuit. Re-draw the circuit, which should look like this:



Convert source from sinusoid to phasor: $10\angle 0^\circ$

Find impedance of IH inductor: j4

Next, solve for I_o. Suggestions to solve for I_o:

Option 1) Find voltage across 1 H by using voltage divider rule noting that 2Ω and 1H are in parallel (impedance: 1.6 + j0.8), the combination of which is in series with 4Ω:

$$V_o = \left(\frac{1.6 + j0.8}{4 + 1.6 + j0.8}\right)10$$

Option 2) Find the voltage across 1 H by applying nodal analysis at node V_o:

$$\frac{V_o - 10}{4} + \frac{V_o}{j4} + \frac{V_o}{2} = 0$$

In the above two options, once V_o is known ($3.16\angle 18.44^\circ$ V), I_o can then be derived using the impedance of 1 H (j4).

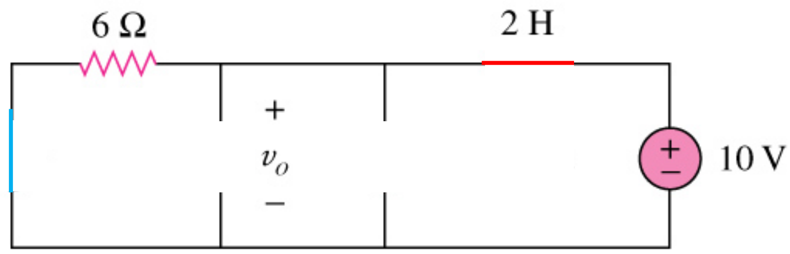
Option 3) Convert the source and 4 Ω from a Thevenin form to a Norton form (by applying source transformation). This will result in a current source ($2.5\angle 0°$ A) and all 3 elements in parallel (i.e. 4 || 2 || j4). We can then use current divider rule to find I_o:

Remember to convert your solution from a phasor back into a sinusoid. Then add it to the solution at DC such that final answer as two terms.

Q6

First analyse at DC

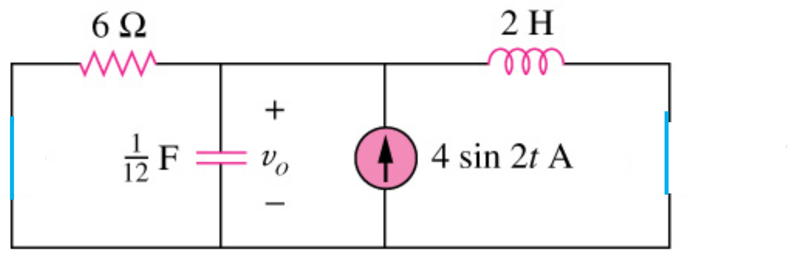
Replace inductor with short circuit; Replace capacitor with open circuit
Replace AC current source with open circuit; Replace AC voltage source with short circuit
Redraw the circuit, which should look like this:



It should be evident that v_o is simply 10 V

Then analyze at 2 rad/s

Replace both the 3 rad/s AC and DC voltage sources with short circuits
Work out the impedance of the capacitor and inductor at 2 rad/s: 2 H → j4 Ω, 1/12 F → -j6 Ω
Redraw the circuit, which should look like this:



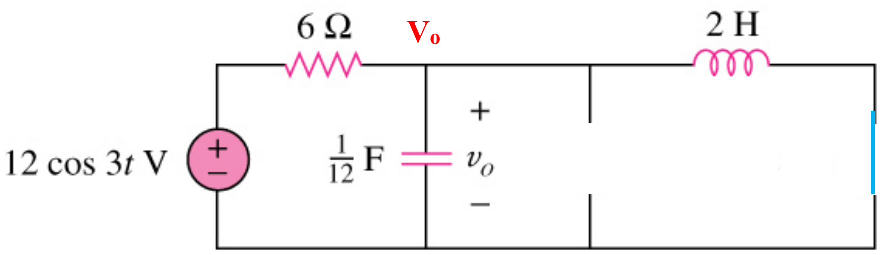
Note that all 3 elements are in parallel with the current source. Hence we can find the total impedance of all 3 elements and use it to find v_o:
$1/Z = 1/6 + 1/j4 + 1/(-j6) = 1/6 - j/12$ S

In this example, there is only one source at 2 rad/s. So we can actually skip converting from sine to cosine as long as we remember that we are using sine as the reference instead of cosine. The final answer for the term at 2 rad/s will then be expressed as a sine.

Using sine as the reference only for 2 rad/s,
$V_o = Z*4\angle 0° = 21.45\angle 26.56°$ V
⇨Convert to sinusoid: 21.45 sin(2t + 26.56°) V

Finally analyze at 3 rad/s

Replace DC voltage source with a short circuit; Replace the 2 rad/s AC current source with an open circuit.
Work out the impedance of the capacitor and inductor at 3 rad/s: 2 H → j6 Ω, 1/12 F → -j4 Ω
Redraw the circuit, which should look like this:



Note that the above circuit is exactly the same as the AC analysis of the previous problem and we so we can use the same three options to find V_o.

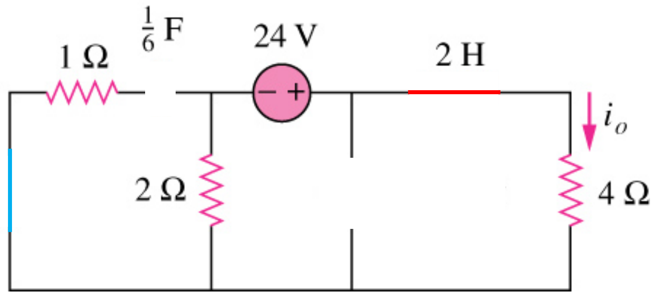
Q7

First analyze at DC

Replace 1 rad/s voltage source with short circuit; Replace 3 rad/s current source with open circuit

Replace capacitor with open circuit and inductor with short circuit

Redraw the circuit, which should look like this:



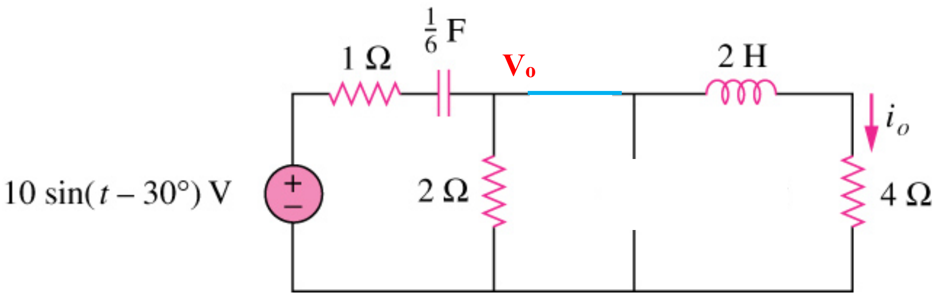
Total loop resistance appearing across 24V DC voltage source = 6Ω

Then analyze at 1 rad/s

Replace 3 rad/s current source with open circuit; Replace DC voltage source with short circuit

Work out impedance of capacitor and inductor at 1 rad/s

Redraw the circuit, which should look like this:



Note that the above circuit is exactly the same as the AC analysis of the Q5 and we so we can use the same three options to find V_o.

Applying nodal analysis at node V_o (using sine as reference),

$$\frac{V_o - 10\angle -30^\circ}{1 - j6} + \frac{V_o}{4 + j2} + \frac{V_o}{2} = 0$$

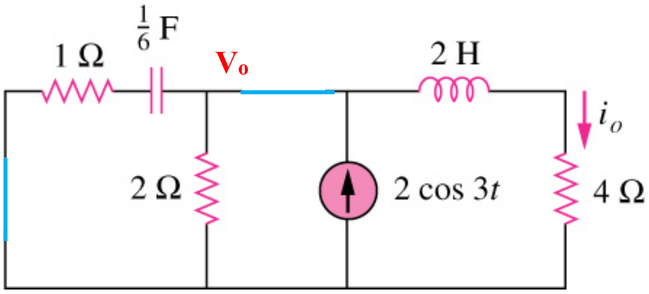
Solving: $V_o = 2.254\ \angle 45.67^\circ$ V

$I_o = V_o/(4 + j2) = 0.504\ \angle 19.1^\circ$ A (1 rad/s)

Remember to convert your solution from a phasor back into a sinusoid.

Finally analyze at 3 rad/s

Replace 1 rad/s voltage source and DC voltage source with short circuits;
Work out impedance of capacitor and inductor at 3 rad/s: L → j6, C → -j2
Redraw the circuit, which should look like this:



Note that the above circuit is exactly the same as the AC analysis of the Q6 at 2 rad/s and we once again have 3 branches in parallel with the current source. Hence we can find i_o using current divider rule.

Total impedance of 3 branches: 1/Z = 1/2 + 1/(1 – j2) + 1/(4 + j6) S

Current divider rule:

$$I_o = \left(\frac{1/(4+j6)}{1/2 + 1/(1-j2) + 1/(4+j6)} \right) 2 \text{ (3 rad/s)}$$

Remember to convert your solution from a phasor back into a sinusoid. Then put together all 3 terms such that final answer has 3 different frequency components.