Compute the following integrals using suitable method. You may need to use method of substitution or integration by parts or both.

(a)
$$\int e^{2x} \sin(2e^x + 1) dx$$

(b)
$$\int_0^1 \sin(2\sqrt{x}) dx$$
(d)
$$\int \cos(\ln x) dx$$

$$\int_0^1 \ln(1+\sqrt[3]{x}) dx$$

(d)
$$\int_{0}^{3} \cos(\ln x) \, dx$$

(e)
$$\int_{0}^{2\pi} \sin 2x \ln(\sin x) dx$$

(f)
$$\int (x+1)\ln(x+3)\,dx$$

$$\int_{1}^{2} \frac{e^{2x}}{e^{x}-1} dx$$

(h)
$$\int_{0}^{\infty} x^3 \cos(3x^2) \sin(x^2) dx$$

method of substitute
$$\int e^{\lambda x} \sin(2e^{\lambda t} + 1) = \int \frac{dy}{dx} = 2e^{\lambda} = \int dx = \frac{1}{2e^{\lambda}} dy$$

of substitute $\int e^{\lambda x} \sin(2e^{\lambda t} + 1) \frac{1}{2e^{\lambda}} dy = \frac{1}{2} \int e^{\lambda} \frac{1}{2e^{\lambda}} \frac{dy}{dy} = \frac{$

let
$$u=y$$

 $dv=\frac{\sin y}{dy}=\frac{1}{\sin y}$ integration by parts

$$\int \frac{1}{x^3} \sin x \, dx$$

$$\int \frac{1}{x^3} \cos x \, dx$$

$$\int \frac{1}{x^3} \cos x \, dx$$

$$= \cos x \, dx$$

$$= e^{x^3} dx$$

$$= e^{x^3} dx.$$

$$\int_0^1 \ln(1+\sqrt[3]{x}) \, dx$$

Let
$$y=H^{3}Jx = \frac{1}{3x^{\frac{3}{2}}} = 3 dx = 3x^{\frac{3}{2}} dy$$
.
When $x=0$, $y=1$; $x=1$, $y=2$.
 $\int_{1}^{2} h(H^{3}Jx) [3x]^{\frac{3}{2}} dy = 3\int_{1}^{2} h(y-1)^{2} dy$.

(d)
$$\int \cos(\ln x) dx$$

$$\int y = \ln x = \frac{dy}{dx} = \frac{1}{x} \implies dx = x dy$$

$$\int ax \lim_{x \to y} x dy = \int ax y dy = \frac{1}{y} = \frac{1}{y$$

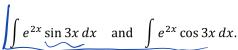
(g)
$$\int_{1}^{2} \frac{e^{2x}}{e^{x}-1} dx$$

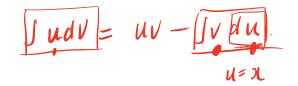
let $y = e^{x}-1 = 0$ $\frac{dy}{dx} = e^{x} = 0$ $\frac{dx}{dx} = \frac{1}{e^{x}} \frac{dy}{dy}$.

When $x = 1$, $y = e^{-1}$; $x = 2$, $y = e^{x}-1$.

 $\int_{e^{-1}}^{e^{x}-1} \frac{e^{x}}{e^{x}-1} \frac{1}{e^{x}} dy = \int_{e^{-1}}^{e^{x}-1} \frac{e^{x}}{e^{x}-1} dy$
 $= \int_{e^{-1}}^{e^{x}-1} \frac{1}{e^{x}} \frac{dy}{dy}$
 $= \int_{e^{-1}}^{e^{x}-1} \frac{1}{(1+y)} dy$.

(a) Compute the integrals





(b) Hence, compute the integrals

$$\int x e^{2x} \cos 3x \, dx.$$

(Hint: You need to eliminate x in the integrand so that you can compute the integral using the result of (a). Which technique should you use: Method of substitution and/or integration by parts?)

$$\int e^{ix} \sin 3x \, dx$$

Let
$$U = Sin 3x$$

$$dV = e^{ix}U = V = +e^{ix}$$

$$\int e^{ix} \sin 3x \, dx. = \frac{dv = e^{ix} tx}{dv = e^{ix} tx} = \frac{1}{2} e^{ix} \sin 3x \, dx$$

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$$= \frac{1}{2}e^{2x} \cdot \sin 3x - \frac{3}{4}e^{2x} \cos 3x - \frac{9}{4} \cdot \int e^{2x} \sin 3x \, dx.$$

=>
$$\frac{1}{4} \int e^{2x} \sin 3x dx = \frac{1}{2} e^{2x} \sin 3x - \frac{3}{4} e^{2x} \cos 3x + C$$

=)
$$\int e^{12} \sin 31 \, da = \frac{2}{13} e^{12} \sin 31 - \frac{3}{13} e^{12} \cos 31 + C$$
.

 $\int \pm e^{\lambda t} \cos \lambda dt = \pm \left[\frac{1}{13} e^{\lambda t} \sin \lambda t + \frac{1}{13} e^{\lambda t} \sin \lambda t - \int \frac{3}{13} e^{\lambda t} \sin \lambda t + \frac{1}{13} e^{\lambda t} \sin \lambda t - \frac{1}{13} e^{\lambda t} \sin \lambda t + \frac{1}{13} e^{\lambda t} \sin \lambda t - \frac{1}{13} e^{\lambda t} \sin \lambda t + \frac{1}{13} e^{\lambda t} \sin \lambda t - \frac{1}{13} e^{\lambda t} \sin \lambda t + \frac{1}{13} e^{\lambda t} \sin \lambda t - \frac{1}{13} e^{\lambda t} \sin \lambda t + \frac{1}{13} e^{\lambda t} \sin \lambda t - \frac{1}{13} e^{\lambda t}$

$$-\frac{3}{13}\int e^{2x}\sin 3x \, dx - \frac{2}{13}\int e^{2x}\cos 3x \, dx.$$

Let f(x) be a differentiable function on [a,b] such that $\int_a^b f(x)dx = 0$ and $\underline{f(a)} = f(b) = 1$. Find the value of $\int_a^b x f'(x)dx$.

Let
$$u = \lambda$$
, $dv = f(x)dx = \int V = f(x)$.

$$= \chi f(a) \Big|_{a}^{b} - \int_{a}^{b} f(a) d\lambda$$

$$= b f(b) - a f(a)$$

$$= \int_{a}^{b} f(a) d\lambda$$

$$=$$
 $b-\alpha$.

Let f(x) be a twice differentiable function on [0,1] such that f(0) = f(1) = 1 and $\int_0^1 f(x) dx = 1$.

Using integration by parts, find the value of

$$\int_0^1 x(1-x)f''(x)dx.$$

(Hint: The technique in Problem 5 may be useful.)

let
$$N = X(FX)$$
. $dV = \int_{0}^{\pi} (x) dx \Rightarrow V = \int_{0}^{\pi} (x) dx$

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