MA1201, CE1, Review for Test (2021, SemB)

Chapter 4

1. (p. 4, 5, 6, 10, 17) Magnitude of vector $\vec{a}$:

$$|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$$

where $\vec{a} = a_1\vec{i} + a_2\vec{j} + a_3\vec{k}$.

2. (p. 9, 10) Change a vector $\vec{a}$ to a unit vector $\vec{n}$ with same direction:

$$\vec{n} = \frac{\vec{a}}{|\vec{a}|}$$

with $|\vec{a}| \neq 0$.

3. (p.21-38) Scalar Product:

$$\vec{a} \cdot \vec{b} = a_1b_1 + a_2b_2 + a_3b_3$$

where $\vec{a} = a_1\vec{i} + a_2\vec{j} + a_3\vec{k}$ and $\vec{b} = b_1\vec{i} + b_2\vec{j} + b_3\vec{k}$.

$$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$$

where $0 \leq \theta \leq \pi$ is the angle between two vectors (see figure on p.23).
If $\vec{a} \perp \vec{b}$, then $\theta = \pi/2$ and

$$\vec{a} \cdot \vec{b} = 0.$$

If $\vec{a} = \vec{b}$, then

$$\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{a} = |\vec{a}|^2.$$

4. (p. 39-62) Vector Product:

$$\vec{a} \times \vec{b} = (|\vec{a}||\vec{b}| \sin\theta)\vec{n},$$

where $0 \leq \theta \leq \pi$ is the angle between two vectors (see figure on p.23), and $\vec{n}$ is the unit vector perpendicular to both $\vec{a}$ and $\vec{b}$ (see figure on p.40). **Read the list on p.41.**
If $\vec{a}$ is parallel to $\vec{b}$, then $\theta = 0$ and

$$\vec{a} \times \vec{b} = 0.$$

5. (p.31- 38) Projection vector of $\vec{a}$ onto $\vec{b}$:

$$Proj_{\vec{b}}\vec{a} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|^2}\vec{b}.$$

6. (p.35-36) Distance from a point P to a line passing through A and B:

$$d = \sqrt{|\vec{AP}|^2 - |proj_{\vec{AB}}\vec{AP}|^2}.$$

7. (p.48-49) Distance from a point D to a plane containing three points A, B and C:

$$d = |proj_{\vec{n}}\vec{AD}|,$$

where $\vec{n} = \vec{AB} \times \vec{AC}$.
8. (p.52-53) Distance from a line passing through A and B to a line passing through C and D:

$$d = |proj_{\vec{n}}\vec{AD}|,$$

where $\vec{n} = \vec{AB} \times \vec{CD}$.
9 (p.45-46) Area of Triangle ABC:

$$Area = |\vec{AC} \times \vec{AB}|/2.$$

10 (p.45-46) Area of Parallelogram formed by $\vec{AB}$ and $\vec{AC}$ (see figure on p.45):

$$Area = |\vec{AC} \times \vec{AB}|.$$

(p. 47) If A, B and C are collinear, then

$$Area = |\vec{AC} \times \vec{AB}| = 0.$$

11 (p.57-59) Volume of Parallelepiped formed by A, B, C and D:

$$Volume = |\vec{AB} \cdot (\vec{AC} \times \vec{AD})|.$$

(p.60) If A, B, C and D are coplanar, then

$$Volume = |\vec{AB} \cdot (\vec{AC} \times \vec{AD})| = 0.$$

12 (p.44) Equation of a plane with a normal vector $\vec{n}$ and containing a point $A = (a_1, a_2, a_3)$:

$$\vec{n} \cdot ((x - a_1)\vec{i} + (y - a_2)\vec{j} + (z - a_3)\vec{k}) = 0$$

(Assignment question) Parametric equations for a line:

$$\vec{r}(t) = \vec{r}_0 + t\vec{v} \text{ for } -\infty < t < \infty$$

13 (p.64-72) Definition of Linear Dependence and Linear Independence:
$\vec{a}_1, \vec{a}_2, \vec{a}_3, ...\vec{a}_n$ are linearly dependent if there is a vector $\vec{a}_k$ which can be expressed as a linear combination of other vectors. If not, they are linearly independent.

(p.67) For three-dimensional case, $\vec{a}_1$ and $\vec{a}_2$ are linearly independent if and only if $\vec{a}_1 \times \vec{a}_2 \neq \vec{0}$.

For three-dimensional case, $\vec{a}_1$, $\vec{a}_2$ and $\vec{a}_3$ are linearly independent if and only if $(\vec{a}_1 \times \vec{a}_2) \cdot \vec{a}_3 \neq 0$.

(p.71) In general, $\vec{a}_1, \vec{a}_2, \vec{a}_3, ...\vec{a}_n$ are linearly dependent if and only if the system $x_1\vec{a}_1 + x_2\vec{a}_2 + ... + x_n\vec{a}_n$ has only trivial solution $x_1 = x_2 = ... = x_n = 0$.

Important questions in problem set 4: 1, 5, 11, 16, 18, 19, 20.

Chapter 1 and 2

1. Remember Product to Sum formula and Compound angle formula (see p.22 in Chapter 1).

2. Learn how to solve the following types of integrals a) $\int x \sin(2x)\cos(2x)dx$;

b) $\int \frac{1+x^2}{(x-1)^2(x^2+x+3)}dx$;

c) $\int_0^9 \frac{1}{2\sqrt{x}+1}dx$;

d) $\frac{d}{dx}\int_x^{x^2} \sin(y^2)dy$;

e) $\int_{-\pi/3}^{\pi/3} \frac{x^3\tan(x)\sin(x)}{x^2+\cos(x)}dx$.

f) $\int_0^1 |2x-1|dx$

g) $\int \sec^3(x)dx$

3. Reduction Formula (p.67-89)

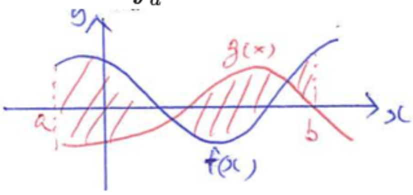
Important questions in problem set 1: 2, 3, 4.

Important questions in problem set 2: 2, 3, 8, 16.

Chapter 3

1. (p. 4-17) Area of the region bounded by the curves $y = f(x)$ and $y = g(x)$:

$$Area = \int_a^b |f(x) - g(x)|dx.$$

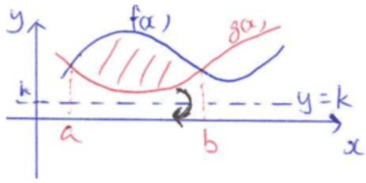


If the area enclosed by the curves $y = f(x)$ and x-axis (g(x)=0):

$$Area = \int_a^b |f(x)|dx.$$

2. (p. 18-27, 30-31) Volume of the solid formed by rotating an area between $y = f(x)$ and $y = g(x)$ about $y = k$ ($f(x) > g(x)$ and $y = k$ not cut the region):

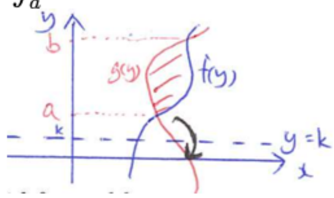
$$V_x = \pi \int_a^b (f(x) - k)^2 - (g(x) - k)^2 dx.$$



$f(x) \geq g(x)$ in $[a, b]$

(shell method, p.32) Volume of the solid formed by rotating an area between $x = f(y)$ and $x = g(y)$ about $y = k$ ($f(y) > g(y)$ and $y = k$ not cut the region):

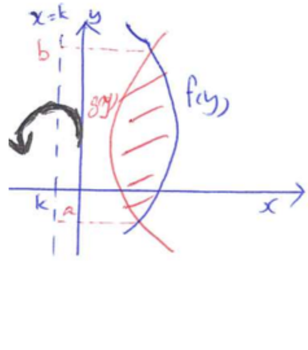
$$V_x = 2\pi \int_a^b (f(y) - g(y))|y - k|dy.$$



$f(y) \geq g(y)$ in $[a, b]$

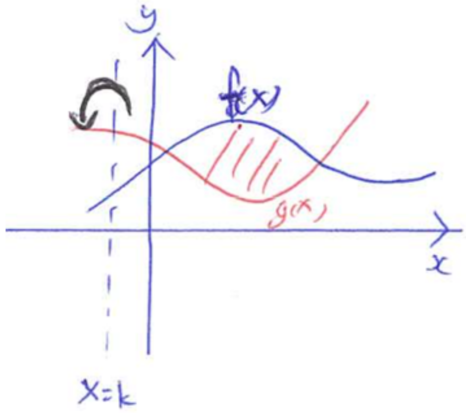
2.5. (p. 28-29) Volume of the solid formed by rotating an area between $x = f(y)$ and $x = g(y)$ about $x = k$ ($f(y) > g(y)$ and $x = k$ not cut the region):

$$V_y = \pi \int_a^b (f(y) - k)^2 - (g(y) - k)^2 dy.$$



(shell method) Volume of the solid formed by rotating an area between $y = f(x)$ and $y = g(x)$ about $y = k$ ($f(x) > g(x)$ and $x = k$ not cut the region):

$$V_y = 2\pi \int_a^b (f(x) - g(x))|x - k|dx.$$



3. (p.39-46) Arc length of a curve $y = f(x)$:

$$L = \int_a^b \sqrt{1 + [f'(x)]^2}dx$$

4. (p. 47-64) Area of surface generated by rotating $y = f(x)$ about $y = k$ ($f(x) > k$):

$$A = 2\pi \int_a^b (f(x) - k)\sqrt{1 + [f'(x)]^2}dx.$$

5. (p.65-76) Problems in parametric equations (see table on p.77-78).

Important questions in problem set 3: 1, 2, 4, 5, 6, 9.

MA1201, CE1, Chapters 1-4, (2021, SemB)

Total: 100 points. Time: 60 min.

1. (15 points) Let $A = (1, 2, 1), B = (1, 1, 2), C = (1, 2, 3)$ be three points on a plane π.
a) Find the angle $\angle BAC$;
b) Determine a unit vector perpendicular to the plane π;
c) Find the shortest distance from $D = (2, 3, 1)$ to the plane π.

2. (15 points) Let $A = (0, 0, -3), B = (2, 1, 5), C = (1, 2, 0)$ and $D = (1, 1, 1)$ be four points in the plane
a) Find the volume of the parallelepiped with A, B, C and D as adj. vertices;
b) Let $E = (x, 1, 0)$ be a point such that A, B, C and E are coplanar. Find the value of x.

3. (20 points) Evaluate the following integrals or derivatives:

a) $\int x \sin(2x) \cos(2x) dx$;
b) $\int \frac{1-x^2}{(x-1)^2(x^2+x+3)} dx$;
c) $\int_0^9 \frac{1}{2\sqrt{x}+1} dx$;
d) $\frac{d}{dx} \int_x^{x^2} \sin(y^2) dy$;
e) $\int_{-\pi/3}^{\pi/3} \frac{x^3 \tan(x) \sin(x)}{x^2+\cos(x)} dx$.

4. (20 points) Consider $I_n = \int x^n e^{mx} dx$, where m, n are integers and $m, n \geq 0$.
a) Show that

$$I_n = \frac{1}{m} x^n e^{mx} - \frac{n}{m} I_{n-1}, \ n \geq 1.$$

b) Using (a), find the value of

$$\int_0^1 x^3 e^{4x} dx.$$

5. (30 points)
a) Calculate the volume of the solid generated by rotating the region bounded by $y = 6 - 3x^2$ and $y = 3$ about the horizontal line $y = 1$.
b) Compute the arc length of the cycled: $x = t - \cos t$, $y = 1 - \sin t$, $0 \leq t \leq 2\pi$.

End of Test