

CITY UNIVERSITY OF HONG KONG

---

Course code and title : MA1201 Calculus and Basic Linear Algebra II

Session : Semester B, 2017/2018

Time allowed : Three hours

---

This paper has **THREE** pages (including this cover page and the attached table).

---

Instructions to candidates:

1. Attempt all **SIX** questions in this paper.
  2. Start each question on a new page.
  3. Show all steps in details in order to get full credits.
- 

*This is a **closed-book** examination.*

*Candidates are allowed to use the following materials/aids:*

*Non-programmable calculators*

*Materials/aids other than those stated above are not permitted. Candidates will be subject to disciplinary action if any unauthorised materials or aids are found on them.*

**NOT TO BE TAKEN AWAY**

NOT TO BE TAKEN AWAY  
PUT FORWARDED TO LIB

**Question 1** [15]

Compute the following elementary integrals.

(a)  $\int \frac{e^{3x} - 3e^{-x-2}}{e^{x+1}} dx$  [5]

(b)  $\int x^3 \sec^2(x^4 + 2) dx$  [4]

(c)  $\int_0^2 |x - 1| dx$  [6]

**Question 2** [20]

Evaluate the following indefinite integrals.

(a)  $\int \frac{x^2}{\sqrt{9 - x^2}} dx$  [6]

(b)  $\int (x + 1) \tan^{-1} x dx$  [6]

(c)  $\int \frac{10x}{(x + 3)(x^2 + 4x + 13)} dx$  [8]

**Question 3** [15]

(a) Find the area of the region bounded by the parabola  $x = y^2 - 5$  and the straight line  $y = x - 1$ . [8]

(b) Calculate the area of the surface generated by rotating about the  $x$ -axis, the parametric curve  $x = \cos^2 t$ ,  $y = \sin^2 t$ ,  $0 \leq t \leq \frac{\pi}{2}$ . [7]

**Question 4** [15]

(a) Find the coordinates of the point  $R$  on the line segment between  $P(1, 2, 3)$  and  $Q(-3, 1, -2)$  such that  $2|\overrightarrow{PR}| = 3|\overrightarrow{QR}|$ . [6]

(b) Determine the equation of the plane containing  $A(-1, -2, -3)$ ,  $B(3, -1, 2)$  and  $C(1, 3, 0)$ . [9]

**Question 5** [15]

(a) Simplify the complex expression  $\left(\frac{1+i}{1-i}\right)^{2018}$  into the polar form. [5]

(b) Solve the complex equation  $(iz)^3 = 3 + \sqrt{3}i$  and list all the solutions in Euler's form with principal arguments. [10]

**Question 6** [20]

Given the matrix

$$A = \begin{pmatrix} 3 & 1 & -2 \\ -2 & 2 & 2 \\ 0 & -1 & -1 \end{pmatrix}.$$

(a) Calculate the determinant of  $A$  by the cofactor expansion and then compute  $|A^T A^{-2}|$ . [5]

(b) Reduce the matrix  $A$  into a row echelon form. [5]

(c) Find the inverse of  $A$  by the Gauss-Jordan elimination. [10]

**Brief Table of Integrals**

$\int x^p dx = \frac{x^{p+1}}{p+1} + C, \quad p \neq -1$	$\int \frac{1}{x} dx + C$
$\int e^x dx = e^x + C$	$\int \sec^3 x dx = \frac{1}{2} \sec x \tan x + \frac{1}{2} \ln  \sec x + \tan x  + C$
$\int \sin x dx = -\cos x + C$	$\int \cos x dx = \sin x + C$
$\int \sec^2 x dx = \tan x + C$	$\int \csc^2 x dx = -\cot x + C$
$\int \sec x \tan x dx = \sec x + C$	$\int \csc x \cot x dx = -\csc x + C$
$\int \sec x dx = \ln  \sec x + \tan x  + C$	$\int \csc x dx = -\ln  \csc x + \cot x  + C$
$\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + C$	$\int \frac{1}{1+x^2} dx = \tan^{-1} x + C$