MA1200 MIDTERM EXAM FRIDAY 12:05 PM -1:05 PM, A B C D

Q1. (30 points) Write $9x^2 + 4y^2 - 18x + 8y = 23$ into the standard form, find foci, center, and vertices, and sketch the graph of it.

Q2. (15 points) Find the largest possible domain and the range of the following functions:

$$f(x) = \log_3(9 - x^2) \quad \text{and} \quad g(x) = \log_3(27 - x^3).$$

Q3.(20 points) Express $\dfrac{2x^2 + 21x + 37}{(x-1)(x^2+6x+13)}$ as partial fractions.

Q4.(20 points) Simplify $\sin(\sin^{-1}(-\frac{12}{13}) + \cos^{-1}(-\frac{4}{5}))$.
(Hint: $\sin(A + B) = \sin A \cos B + \cos A \sin B$, $3^2 + 4^2 = 5^2$, $5^2 + 12^2 = 13^2$))

Q5.(15 points) Solve $\cos(2x + \pi/3) = \sqrt{2}/2$ in radians.

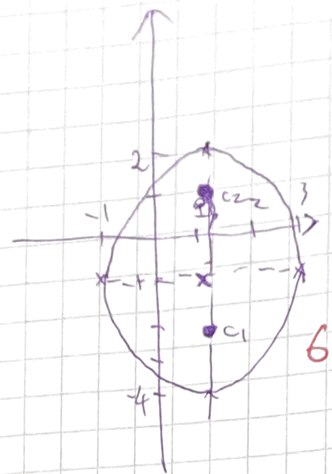
MA1200. Midterm Ver 1.

Q1 $9x^2+4y^2-18x+8y=23$

$9(x^2-2x)+4(y^2+2y)=23$
$\quad +1 \qquad\qquad +1 \qquad +9+4$

$9(x-1)^2+4(y+1)^2=36$

$\frac{(x-1)^2}{4}+\frac{(y+1)^2}{9}=1 \quad \leftarrow 10$



6

Center $(1,-1)$ 3 $\quad c=\sqrt{a^2-b^2}=\sqrt{5}$.

Foci $(1,-1\pm\sqrt{5})$ 7 $\quad$ Vertices $(1,-1\pm 3)=(1,2)$, $(1,-4)$ 4

$(1\pm 2,-1)=(3,-1)$, $(-1,-1)$

30

Q2 $f(x)=\log_3(9-x^2)$

domain: $9-x^2>0 \quad 9>x^2 \Rightarrow x\in(-3,3)$. 3 $\quad$ 8

$0<9-x^2\le 9 \Rightarrow f(x)\le\log_3 9=2$ 4 $\quad$ 15.

range $(-\infty,2]$ $\quad$ 8

$g(x)=\log_3(27-x^3)$
domain $27-x^3>0 \quad \{x<3\}=(-\infty,3)$ 3
range: $\mathbb{R}=(-\infty,\infty)$ 5

Q3 $\frac{2x^2+21x+37}{(x-1)(x^2+6x+13)}=\frac{A}{x-1}+\frac{Bx+C}{x^2+6x+13} \quad \leftarrow 7$

$x^2+6x+13$ can't be factorized since $6^2-4\cdot 13<0$

$2x^2+21x+37=A(x^2+6x+13)+(x-1)(Bx+C)$

V1 ②

$x=1 \Rightarrow 2+21+37 = A(20) \Rightarrow A=3$ 4

$x=0 \Rightarrow 37 = 13\cdot A - C = 39 - C$ 4

$C = 2$

Coefficient of $x^2 = 2 = A+B = 3+B \Rightarrow B=-1$ 4

thus $\cdot = \dfrac{-x+2}{x^2+6x+13} + \dfrac{3}{x-1}$. 1

$\underline{\underline{20}}$

Q4. $\sin\left(\sin^{-1}\left(-\frac{12}{13}\right) + \cos^{-1}\left(-\frac{4}{5}\right)\right)$

$A = \sin^{-1}\left(-\frac{12}{13}\right) \Rightarrow \sin A = -\frac{12}{13}, \; A \in \left(-\frac{\pi}{2}, 0\right)$

$\Rightarrow \cos A = \frac{5}{13}$ 7

$B = \cos^{-1}\left(-\frac{4}{5}\right) \Rightarrow \cos B = -\frac{4}{5}. \; B \in \left(\frac{\pi}{2}, \pi\right)$

$\sin B = 3/5.$ 7

$\sin(A+B) = \sin A \cos B + \cos A \sin B$ 6

$= \left(-\frac{12}{13}\right)\cdot\left(-\frac{4}{5}\right) + \frac{5}{13}\cdot\frac{3}{5} = \frac{48+15}{65} = \frac{63}{65}$

$\underline{\underline{20}}$

Q5 $\cos\left(2x+\frac{\pi}{3}\right) = \frac{\sqrt{2}}{2} = \cos\left(\frac{\pi}{4}\right)$

$\Rightarrow 2x+\frac{\pi}{3} = 2\pi n \pm \frac{\pi}{4}. \quad n \in \mathbb{Z}$ 7

"+" $2x+\frac{\pi}{3} = 2n\pi + \frac{\pi}{4} \Rightarrow 2x = 2n\pi - \frac{\pi}{12} \Rightarrow x = n\pi - \frac{\pi}{24}$ 4

"−" $2x+\frac{\pi}{3} = 2n\pi - \frac{\pi}{4} \Rightarrow 2x = 2n\pi - \frac{11}{12}\pi \Rightarrow x = n\pi - \frac{11}{24}\pi$ 4

15 $n \in \mathbb{Z}$