SDSC 2102 Statistical Methods and Data Analysis - Assignment 3

Question 1a:

$$\alpha = 0.02 \quad \sigma^2 = 0.64 \quad \sigma = 0.8$$
$$n = 16 \quad \bar{x} = 89.4$$

$$H_0: \mu = \mu_0$$
$$H_1: \mu \neq \mu_0$$

$$\begin{aligned} Z_{\frac{\alpha}{2}} &= Z_{\frac{0.02}{2}} \\ &= 2.325 \\ |Z_0| &= \left| \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} \right| \\ &= \left| \frac{89.4 - 90}{0.8 / \sqrt{16}} \right| \\ &= 3 \end{aligned}$$

$$\because 3 > 2.325$$
$$\therefore Reject\ H_0$$

Conclusion:

There is evidence that the paper is not aligned correctly.

Question 1b:

$$\begin{aligned} \mu &\in \left(\bar{x} - Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}, \bar{x} + Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} \right) \\ &= \left(89.4 - 2.325 \times \frac{0.8}{\sqrt{16}}, 89.4 + 2.325 \times \frac{0.8}{\sqrt{16}} \right) \\ &= (88.935, 89.865) \end{aligned}$$

$$smallest\ value\ \mu_0 = 88.935$$

Question 2:

$$\alpha = 0.01 \quad n = 25$$
$$\bar{x} = -20.7 \quad s^2 = 1.0 \quad s = 1$$

$$H_0: \mu = -20$$
$$H_1: \mu < -20$$

$$\begin{aligned} -t_{\alpha,n-1} &= -t_{0.01,25-1} \\ &= -2.492 \\ t_0 &= \frac{\bar{x} - \mu_0}{s/\sqrt{n}} \\ &= -3.5 \end{aligned}$$

$$\because -3.5 < -2.492$$
$$\therefore Reject\ H_0$$

Conclusion:

There is evidence that the average temperature is -20F or lower.

Question 3a:

$$\alpha = 0.1 \quad n_{S1} = 5 \quad n_{S2} = 7$$
$$\overline{x_{S1}} = 97.4 \quad s_{S1} = 8.877 \quad \overline{x_{S2}} = 110 \quad s_{S2} = 30.221$$

$$\begin{aligned}
\mu_{S1} &\in \left(\overline{x_{S1}} - t_{\frac{\alpha}{2},n_{S1}-1}\frac{s_{S1}}{\sqrt{n_{S1}}}, \overline{x_{S1}} + t_{\frac{\alpha}{2},n_{S1}-1}\frac{s_{S1}}{\sqrt{n_{S1}}}\right) \\
&= \left(97.4 - 2.132 \times \frac{8.877}{\sqrt{5}}, 97.4 + 2.132 \times \frac{8.877}{\sqrt{5}}\right) \\
&= (88.936, 105.864) \\
\mu_{S2} &\in \left(\overline{x_{S2}} - t_{\frac{\alpha}{2},n_{S2}-1}\frac{s_{S2}}{\sqrt{n_{S2}}}, \overline{x_{S2}} + t_{\frac{\alpha}{2},n_{S2}-1}\frac{s_{S2}}{\sqrt{n_{S2}}}\right) \\
&= \left(110 - 1.943 \times \frac{30.221}{\sqrt{7}}, 110 + 1.943 \times \frac{30.221}{\sqrt{7}}\right) \\
&= (87.806, 132.194)
\end{aligned}$$

Question 3b:

$$H_0: \mu_{S1} = \mu_{S2}$$
$$H_1: \mu_{S1} < \mu_{S2}$$

$$\begin{aligned}
S_p^2 &= \left(\frac{(n_{S1}-1)s_{S1}^2 + (n_{S2}-1)s_{S2}^2}{n_{S1}+n_{S2}-2}\right) \\
&= \left(\frac{(5-1) \times 8.877^2 + (7-1) \times 30.221^2}{5+7-2}\right) \\
&= 579.52 \\
S_p &= 24.073
\end{aligned}$$

$$\begin{aligned}
-t_{\alpha,n_{S1}+n_{S2}-2} &= -t_{0.1,10} \\
&= -1.372 \\
t_0 &= \frac{\overline{x_{S1}} - \overline{x_{S2}}}{S_p\sqrt{1/n_{S1} + 1/n_{S2}}} \\
&= \frac{97.4 - 110}{24.073\sqrt{1/5 + 1/7}} \\
&= -0.894
\end{aligned}$$

$$\because -0.894 > -1.372$$
$$\therefore Faile\ to\ reject\ H_0$$

Conclusion:

There is no evidence that the mean number of customers under the second student's design higher than that under the first student's design.