Problem 1

(b) $$\int \left(\frac{1}{x^3} - \sqrt{x}\right) dx$$

(d) $$\int \frac{1}{1+16x^2} dx$$

(f) $$\int \frac{1}{(2x+1)^2} dx$$

(b). $$\int \left(\frac{1}{x^3} - \sqrt{x}\right) dx = \int \left(x^{-3} - x^{\frac{1}{2}}\right) dx = \frac{x^{-3+1}}{-3+1} - \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C.$$

$$= -\frac{x^{-2}}{2} - \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + C = -\frac{1}{2x^2} - \frac{2}{3}x^{\frac{3}{2}} + C.$$

(d) $$\int \frac{1}{1+16x^2} dx = \int \frac{1}{1+(4x)^2} dx = \frac{1}{4}\tan^{-1} 4x + C$$

$$\left\{ \begin{array}{l} \left(\tan^{-1}(x)\right)' = \dfrac{1}{1+x^2} \\ \int f(ax+b)\, dx = \dfrac{1}{a} F(ax+b) + C. \end{array} \right.$$

$$\downarrow$$

$$\tan^{-1}(ax+b)$$

(f) $$\int \frac{1}{(2x+1)^2} dx = \int \overset{ax+b}{(2x+1)}^{-2} dx = \overset{\frac{1}{a}}{\frac{1}{2}} \frac{(2x+1)^{-2+1}}{-2+1} + C$$

$$= -\frac{1}{2}\frac{1}{2x+1} + C.$$

Q2

(b) $\int \frac{2x^2}{x^2+1}dx$

(d) $\int \sin 3x \sin 2x\, dx$

(f) $\int \frac{1}{(x-1)(2x-3)}dx$

(h) $\int \frac{1}{2x^2-4x+9}dx$

(j) $\int \tan^2 x\, dx$

(b) $$\int \frac{2x^2}{x^2+1}dx = \int \frac{2x^2+2-2}{x^2+1}dx = \int 2\,dx - \int 2\cdot\frac{1}{1+x^2}dx$$

$$= 2x + C_1 - 2\tan^{-1}(x) + C_2$$

$$\frac{2x^2}{x^2+1} = \frac{2x^2+2-2}{x^2+1} = 2 - 2\cdot\frac{1}{1+x^2}$$

$$= 2x - 2\tan^{-1}(x) + C \quad = C_1 + C_2$$

$$(\tan^{-1}(x))' = \frac{1}{1+x^2}$$

(d) $$\int \sin 3x \sin 2x\, dx = \int -\frac{1}{2}\left[\cos(5x) - \cos x\right]dx$$

$$\sin A \sin B = -\frac{1}{2}\left[\cos(A+B) - \cos(A-B)\right]$$

$$(\sin x)' = \cos x$$

$$= -\frac{1}{2}\int \cos 5x\, dx + \frac{1}{2}\int \cos x\, dx$$

$$= -\frac{1}{2}\cdot\frac{1}{5}\sin(5x) + C_1 + \frac{1}{2}\sin x + C_2 = -\frac{1}{10}\sin(5x) + \frac{1}{2}\sin x + C$$

$$\int f(ax+b)\,dx = \frac{1}{a}F(ax+b)$$

(f) $$\int \frac{1}{(x-1)(2x-3)}dx$$

$$\frac{1}{(x-1)(2x-3)} = \frac{A}{x-1} + \frac{B}{2x-3} = -\frac{1}{x-1} + \frac{2}{2x-3}$$

$$1 = A(2x-3) + B(x-1) \qquad x = \frac{3}{2} \Rightarrow 1 = \frac{1}{2}B \Rightarrow B = 2$$

$x=1 \Rightarrow 1=-A \Rightarrow A=-1.$

$$\int \frac{1}{(x-1)(2x-3)}\,dx = \int -\frac{1}{x-1}\,dx + \int \frac{2}{2x-3}\,dx. \qquad (\ln|x|)' = \frac{1}{x}.$$

$(x-1)^{-1}$

$$= -\ln|x-1| + C_1 + 2\cdot\ln|2x-3| + C_2.$$

$$= -\ln|x-1| + 2\ln|2x-3| + C.$$

(h) $$\int \frac{1}{2x^2-4x+9}\,dx = \int \frac{1}{2(x-1)^2+7}\,dx.$$

$\frac{1}{1+(\ )^2}\,dx \to \tan^{-1}(x).$

$$= \int \frac{1}{7}\cdot\frac{1}{1+\left(\frac{\sqrt{2}}{\sqrt{7}}(x-1)\right)^2}\,dx. = \frac{1}{7}\left(\frac{1}{\frac{\sqrt{2}}{\sqrt{7}}}\tan^{-1}\left(\sqrt{\frac{2}{7}}(x-1)\right)+C_1\right)$$

$ax+b$

$\frac{\sqrt{2}}{\sqrt{7}}x - \frac{\sqrt{2}}{\sqrt{7}}$

$$= \frac{1}{\sqrt{14}}\tan^{-1}\left(\sqrt{\frac{2}{7}}x - \sqrt{\frac{2}{7}}\right) + C$$

$(\tan x)' = \sec^2 x.$

(j) $$\int \tan^2 x\,dx = \int (\sec^2 x - 1)\,dx = \int \sec^2 x\,dx - \int 1\,dx$$

$\tan^2 x = \sec^2 x - 1.$

$$= \tan x + C_1 - x + C_2$$

$$= \tan x - x + C.$$

(b) $\displaystyle\int_{-1}^{1} \cos(3x+1)\,dx$

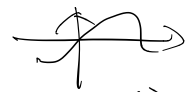
(d) $\displaystyle\int_{0}^{\frac{\pi}{2}} \sin^2 x\,dx$

(f) $\displaystyle\int_{-\pi}^{\pi} |\sin x|\,dx$

(h) $\displaystyle\int_{-1}^{1} x^4 \sin^9 x\,dx$

*(j) $\displaystyle\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{x^2\cos x + \cos x + \sin^3 x}{x^2+1}\,dx$

Q3



odd function
$\sin(-x) = -\sin x$

$\frac{1}{a}\sin(ax+b)$

(b) $$\int_{-1}^{1} \cos(3x+1)\,dx = \frac{1}{3}\sin(3x+1)\Big|_{-1}^{1} = \frac{1}{3}\sin 4 - \frac{1}{3}\sin(-2)$$

$$\int f(x)\,dx = F(x)\Big|_a^b \qquad = \frac{1}{3}(\sin 4 + \sin 2)$$

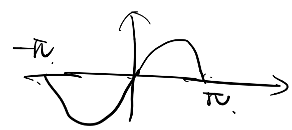
(d) $$\int_0^{\frac{\pi}{2}} \sin^2 x\,dx = \int_0^{\frac{\pi}{2}} -\frac{1}{2}(\cos(2x) - \cos 0)\,dx = \int_0^{\frac{\pi}{2}} \frac{1}{2}\,dx - \int_0^{\pi} \frac{1}{2}\cos 2x\,dx$$

$$= \frac{1}{2}x\Big|_0^{\frac{\pi}{2}} - \frac{1}{4}\sin 2x\Big|_0^{\frac{\pi}{2}} = \frac{1}{2}\cdot\frac{\pi}{2} - 0 - \left(\frac{1}{4}\sin\pi - \frac{1}{4}\sin 0\right)$$

$$= \frac{\pi}{4}$$

$\sin A \sin B = -\frac{1}{2}[\cos(A+B) - \cos(A-B)]$

(f) $$\int_{-\pi}^{\pi} |\sin x|\,dx$$



$$|\sin x| = \begin{cases} \sin x & \text{if } \sin x \geq 0 \Rightarrow 0 \leq x \leq \pi \\ -\sin x & \text{if } \sin x < 0 \Rightarrow -\pi \leq x \leq 0 \end{cases}$$

$$\int_{-\pi}^{\pi} |\sin x|\,dx = \int_0^{\pi} \sin x\,dx - \int_{-\pi}^{0} \sin x\,dx$$

$$= -\cos x\Big|_0^{\pi} - (-\cos x)\Big|_{-\pi}^{0}$$

$$= -\cos\pi + \cos 0 - (-\cos 0 + \cos(-\pi))$$

$$= 1 + 1 + 1 + 1 = 4.$$

(h) $$\int_{-1}^{1} x^4 \sin^9 x\,dx$$

$$f(-x) = (-x)^4 \sin^9(-x) = x^4(-\sin x)^9 = -x^4\sin^9 x = -f(x)$$

$\sin(-x) = -\sin x$

$x^4\sin^9 x$ is an odd function.

$f(x)$ is odd, $f(-x) = -f(x)$.

$\int_{-a}^{a} f(x)\,dx = 0.$

$$\int_{-1}^{1} x^4 \sin^9 x\,dx = 0.$$

*(j) $$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{x^2\cos x + \cos x + \sin^3 x}{x^2+1}\,dx$$

$$= \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\cos x(x^2+1)+\sin^3 x}{x^2+1}\,dx = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \cos x\,dx + \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \underbrace{\frac{\sin^3 x}{x^2+1}}_{\text{odd.}}\,dx.$$

$$f(-x) = \frac{\sin^3(-x)}{(-x)^2+1} = \frac{(-\sin x)^3}{x^2+1} = \frac{-\sin^3 x}{x^2+1} = -f(x)$$

$$= \sin x \Big|_{-\frac{\pi}{4}}^{\frac{\pi}{4}} = \sin\left(\frac{\pi}{4}\right) - \sin\left(-\frac{\pi}{4}\right) = 2\cdot\sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}\cdot 2 = \sqrt{2}.$$

Q4. (b) $$\frac{d}{dx}\int_{2x}^{x^2} \cos(y^2)\,dy$$

$$F(y) = \int \cos(y^2)\,dy \Rightarrow \frac{dF(y)}{dy} = \cos(y^2)$$

$$\int_{2x}^{x^2} \cos(y^2)\,dy = F(x^2) - F(2x)$$

$$\frac{d}{dx}\int_{2x}^{x^2} \cos(y^2)\,dy = \frac{d}{dx}\left(F(x^2) - F(2x)\right) = \underbrace{\frac{dF(x^2)}{dx^2}}_{y=x^2}\cdot\frac{dx^2}{dx} - \underbrace{\frac{dF(2x)}{d2x}}_{y=2x}\cdot\frac{d2x}{dx}.$$

$$= \frac{dF(y)}{dy}\cdot 2x - \frac{dF(y)}{dy}\cdot 2.$$

$$= \cos(x^4)\cdot 2x - \cos(4x^2)\cdot 2$$

$$= 2x\cos(x^4) - 2\cos(4x^2)$$