SDSC 3006 L02
Class 3. Classification

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Outline

- **Logistic regression**
- **LDA and QDA**
- **ROC curve**
- **KNN**

Logistic Regression

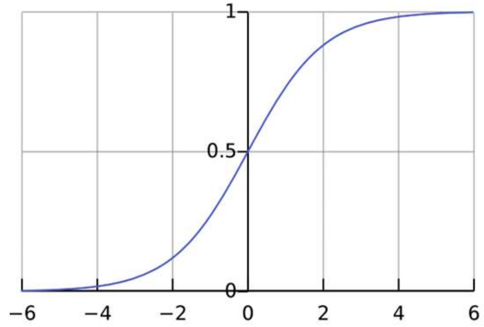
Preliminary

- Model structure

This X is an event

$$\log\left(\frac{p(X)}{1-p(X)}\right) = \beta_0 + \beta_1 X_1 + \cdots + \beta_p X_p,$$

$$p(X) = \frac{e^{\beta_0+\beta_1 X_1+\cdots+\beta_p X_p}}{1+e^{\beta_0+\beta_1 X_1+\cdots+\beta_p X_p}}.$$



Sigmoid function

- Parameters estimation method: maximum likelihood

$$\ell(\beta_0, \beta_1) = \prod_{i:y_i=1} p(x_i) \prod_{i':y_{i'}=0} (1 - p(x_{i'})).$$

- Accuracy of estimates: z-statistics (p-value)

Logistic regression help you to calculate a probability of X.

Preliminary

- Smarket data set in the ISLR2 package.
- Percentage returns for the S&P 500 stock index in 2001~2005.
- Predict the Direction (Up/Down) of the stock market on a day based on the values in the previous days (Lag1,…,Lag5), etc.
- Code:

```
library(ISLR2)
names(Smarket)
dim(Smarket)
attach(Smarket)
```

Step1-Training model

- Code of logistic model
  logistic.fit=glm(Direction~Lag1+Lag2+Lag3+Lag4+Lag5+Volume, data=Smarket, family=binomial)
  summary(logistic.fit)

- What is the meaning of Number of Fisher Scoring iterations?
  We use Fisher's Scoring algorithm(numerical method) here instead of calculating MLE.

Step2-predictions

- Get the prediction of probability:
  logistic.probs=predict(logistic.fit,type="response")
  ##print the first ten probabilities
  logistic.probs[1:10]

- Meaning of prob: prob of going up, this is what we calculate
  ##check dummy variable
  contrasts (Direction)

- Convert the prob to class:
  logistic.pred=rep("Down",1250) #create all ”down” array
  logistic.pred[logistic.probs>0.5]="Up" #set threshold 0.5

Step3-Accuracy

- Confusion matrix
  table(logistic.pred,Direction)

		Predicted condition	
	Total population = P + N	Positive (PP)	Negative (PN)
Actual condition	Positive (P)	True positive (TP), hit	False negative (FN), type II error, miss, underestimation
	Negative (N)	False positive (FP), type I error, false alarm, overestimation	True negative (TN), correct rejection

- calculate prediction accuracy: 0.5216
  #(507+145)/1250 or
  mean(logistic.pred==Direction)

- What this accuracy means?
  It appears that the logistic regression model is working a little better than random guessing. However, this result is misleading because we trained and tested the model on the same set. In other words,
  1 − 52.2% = 47.8% is the training error rate.
  Training error rate often underestimate the test error rate!

Cross validation

- Split the data set into a training set and a test set:
  For example, use data in year 2001~2004 for training, and data in year 2005 for test.

- Fit a logistic regression model using the training set

- Find the test error rate using the test set

Cross validation

##Step 1: Split data (2001~2004 for training, 2005 for test)

```
train=(Year<2005)
Smarket.2005=Smarket[!train,]
Direction.2005=Direction[!train]
```

##Step 2: Train model on training data

```
logistic.fit=glm(Direction~Lag1+Lag2+Lag3+Lag4+Lag5+Volume,
data=Smarket,family=binomial,subset=train)
```

##Step 3: Make Prediction on test data

```
logistic.probs=predict(logistic.fit,Smarket.2005,type="response")
logistic.pred=rep("Down", 252)
#length(Direction.2005)
logistic.pred[logistic.probs>0.5] = "Up”
```

##Step 4: Assess prediction accuracy

```
table(logistic.pred,Direction.2005)
mean(logistic.pred==Direction.2005)
```

Summary of Results

- Test error rate = 1–48% = 52% (Training error rate = 47.8%). Prediction of the logistic regression is even worse than random guessing!

- It is not surprising, since the stock market is too random to predict.

- Maybe removing some predictors which have high p-values can improve the prediction performance.

LDA and QDA

Keys

- Smarket data set in the ISLR2 package.
- Predict the Direction (Up/Down) of the stock market.
- Split the data set into training data and test data.
- Apply function lda() and qda() in the MASS library
- Next are the steps of LDA, You can change the words lda to qda to implement QDA

Steps of LDA

- ## Step 1 - Obtain dataset and Split it

```
library(ISLR2)
library(MASS)
attach(Smarket)
train=(Year<2005)
Smarket.2005 = Smarket[!train,]
Direction.2005 = Direction[!train]
```

- ## Step 2 - Train model and predict

```
lda.fit=lda(Direction~Lag1+Lag2, data=Smarket, subset=train)
lda.pred = predict(lda.fit,Smarket.2005)
names(lda.pred)
#see what prediction contains
##lda.pred$class
##lda.pred$posterior
```

Step

- ## Step 3 - Calculate prediction accuracy
```
lda.class = lda.pred$class
table(lda.class,Direction.2005)
mean(lda.class==Direction.2005)
```

- ## Step 4 - Change threshold (Extra)
```
lda.class = rep("Down",length(Direction.2005))
lda.class[lda.pred$posterior[,2]>0.49] = "Up"
table(lda.class,Direction.2005)
mean(lda.class==Direction.2005)
```

ROC curve

ROC curve

- Smarket data set in the ISLR2 package.

- Why we draw ROC curve: to compare the performance between methods.

- Compare two methods: logistic regression, LDA

- Method: Write a function roc.curve() which calculate and print the ROC curve for a given method.

ROC curve of Logistic Regression

```
library(ISLR2)
attach(Smarket)
##fit logistic regression to all data (2001~2005)
LR.fit = glm(Direction~Lag1+Lag2+Lag3,family=binomial,data=Smarket)
##predict probability of "UP"
LR.pred = predict(LR.fit,type="response")
```

ROC curve of Logistic Regression

```
## Calculate FPR and TPR under a given threshold
roc.curve=function(s,print=FALSE){
Ps=(LR.pred>s)*1
FP=sum((Ps==1)*(Direction==“Down”))/sum(Direction==“Down”)
TP=sum((Ps==1)*(Direction==“Up”))/sum(Direction==“Up”)
if(print==TRUE){
   print(table(Observed=Direction,Predicted=Ps))
}
 vect=c(FP,TP)
names(vect)=c("FPR","TPR")
return(vect)
}

threshold=0.5
roc.curve(threshold,print=TRUE)
## Plot ROC curve
ROC.curve=Vectorize(roc.curve)
M.ROC=ROC.curve(seq(0,1,by=0.01))
plot(M.ROC[1,],M.ROC[2,],col="grey",lwd=2,type="l",xlab="False positive rate",ylab="True
positive rate")
```

ROC curve of LDA

```
library(ISLR2)
attach(Smarket)
## fit model to all data
library(MASS)
LDA.fit = lda(Direction~Lag1+Lag2+Lag3,data=Smarket)
## predict probabilities of training data
LDA.pred0 = predict(LDA.fit,type="response")
LDA.pred = LDA.pred0$posterior[,2]
```

ROC curve of LDA

```
## Calculate FPR and TPR under a given threshold

roc.curve=function(s,print=FALSE){
Ps=(LDA.pred>s)*1 FP=sum((Ps==1)*(Direction=="Down"))/sum(Direction=="Down")
TP=sum((Ps==1)*(Direction=="Up"))/sum(Direction=="Up")
if(print==TRUE){
print(table(Observed=Direction,Predicted=Ps))
}
vect=c(FP,TP)
names(vect)=c("FPR","TPR")
return(vect) }
threshold=0.5
roc.curve(threshold,print=TRUE)

## Plot ROC Curve
ROC.curve=Vectorize(roc.curve)
M.ROC=ROC.curve(seq(0,1,by=0.01))
plot(M.ROC[1,],M.ROC[2,],col="blue",lwd=2,type="l",xlab="False positive rate",ylab="True
positive rate")
```

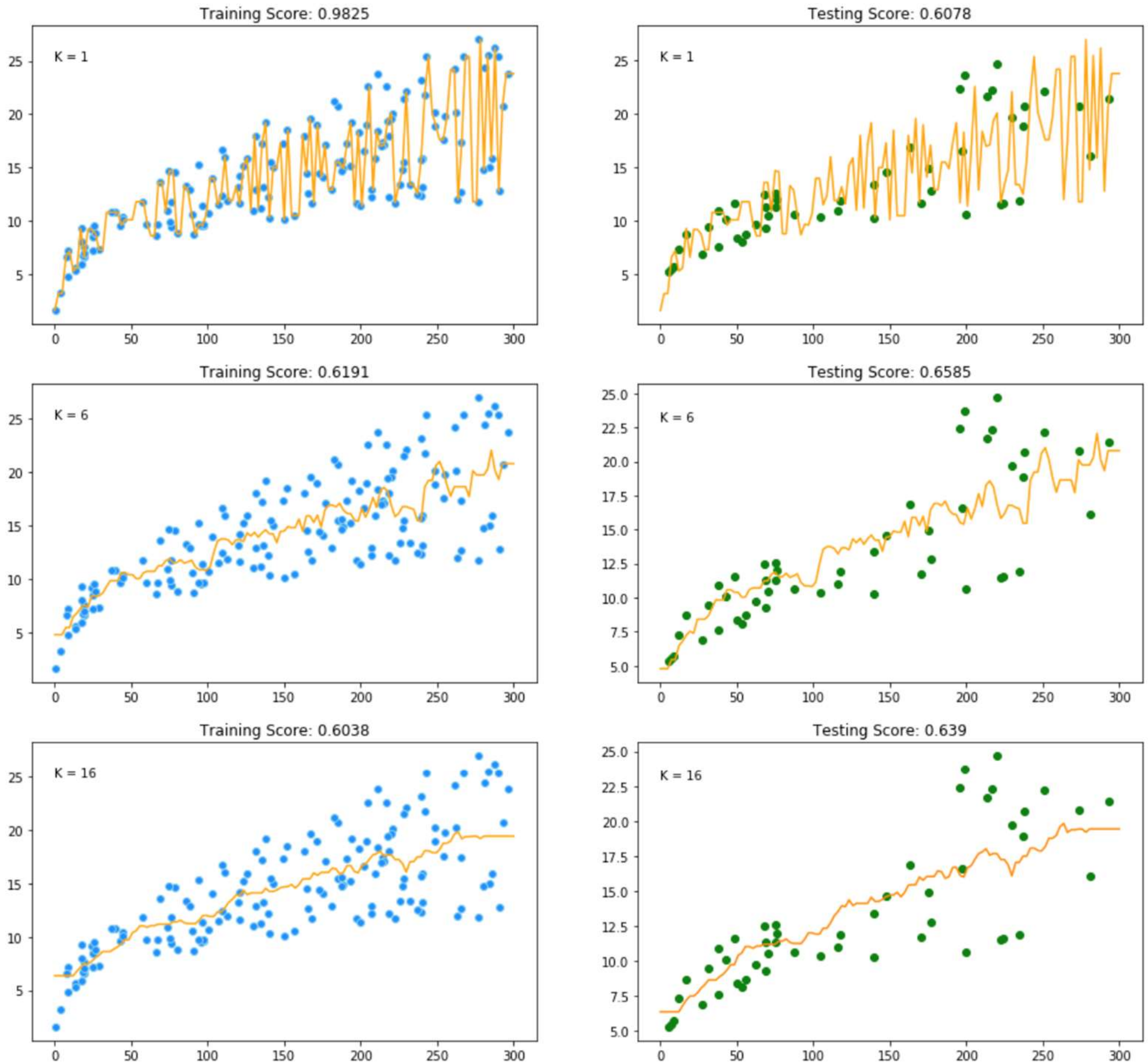
KNN

Notice in KNN

- Use the knn() function in the class library, which requires 4 inputs (training observation, test observation, training response, number of K).

- This function does not follow the two-step (first model fitting, then prediction) approach; it generates predictions using a single command.

A picture example of KNN



Left: Training dataset with KNN regressor **Right:** Testing dataset with same KNN regressors. Image by Sangeet Aggarwal

Code of KNN

```
library(ISLR2)
attach(Smarket)
library(class)
train=(Year<2005)
train.X=cbind(Lag1,Lag2)[train,] #training data of observation
test.X=cbind(Lag1,Lag2)[!train,] #test data of observation
train.Direction=Direction[train] #training data of response
knn.pred1=knn(train.X,test.X,train.Direction,k=1)
table(knn.pred1,Direction.2005)
mean(knn.pred1==Direction.2005) #predict accuracy

##change k to get different results
knn.pred2=knn(train.X, test.X, train.Direction, k=3)
table(knn.pred2,Direction.2005)
mean(knn.pred2==Direction.2005)
```