Solution of Midterm question

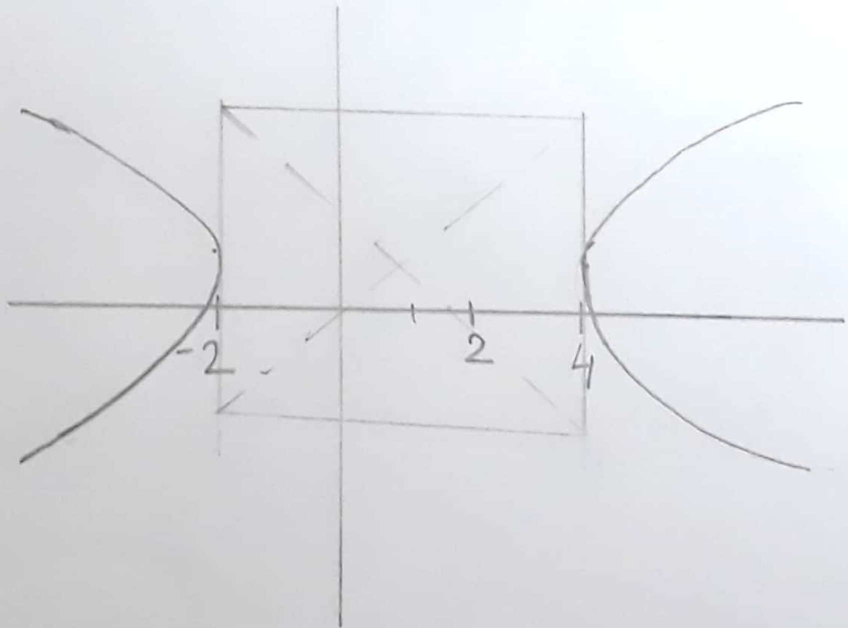
Solution 1:-

$$9x^2 - 16y^2 - 36x + 32y = 124$$

$$\Rightarrow 9(x^2 - 4x) - 16(y^2 - 2y) = 124$$

$$\Rightarrow 9(x-2)^2 - 16(y-1)^2 = 144$$

$$\Rightarrow \frac{(x-2)^2}{16} - \frac{(y-1)^2}{9} = 1$$



Centre is $(2,1)$ and $c = \sqrt{a^2+b^2} = 5$

Foci $(2 \pm 5, 1) = (7,1)$ and $(-3,1)$

Vertices $(6,1)$, $(-2,1)$

asymptotes, $y - 1 = \pm \frac{3}{4}(x-2)$

Solution 2:-

i) $f(x) = \log_2(4-x^2)$

$D: (-2, 2)$

$R: (-\infty, 2]$

ii) $g(x) = \log_2(8-x^3)$

$D: (-\infty, 2)$

$R: (-\infty, \infty)$

Solution 3:-

$$\frac{-7x+29}{(x+1)(x^2-4x+13)} = \frac{A}{x+1} + \frac{Bx+C}{x^2-4x+13}$$

$$\Rightarrow -7x+29 = A(x^2-4x+13) + (x+1)(Bx+C)$$

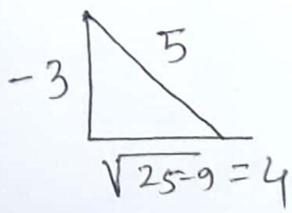
After solving, $A = 2,\ B = -2,\ C = 3$

Then,

$$\frac{-7x+29}{(x+1)(x^2-4x+13)} = \frac{2}{(x+1)} + \frac{-2x+3}{x^2-4x+13}$$

Solution 4:-

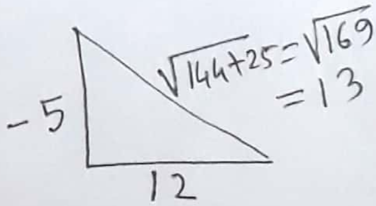
$$\cos\left(\sin^{-1}\left(-\frac{3}{5}\right) + \tan^{-1}\left(-\frac{5}{12}\right)\right)$$



Let,

$A = \sin^{-1}\left(-\frac{3}{5}\right),\ A \in \left(-\frac{\pi}{2}, 0\right)$

$\Rightarrow \sin A = -\frac{3}{5}$



$B = \tan^{-1}\left(-\frac{5}{12}\right),\ B \in \left(-\frac{\pi}{2}, 0\right)$

$\tan B = \frac{5}{12}$

Now,

$\cos A = \frac{4}{5}$,

$\cos B = \frac{12}{13}$

$\sin B = -\frac{5}{13}$

Now, $\cos(A+B) = \cos A \cos B - \sin A \sin B$

$= \frac{4}{5}\left(\frac{12}{13}\right) - \left(-\frac{3}{5}\right)\left(\frac{-5}{13}\right)$

$= \frac{33}{65}$

Solution 5:-

$\sin(2x + \pi/3) = \frac{1}{2} = \sin\left(\frac{\pi}{6}\right)$

$\Rightarrow 2x + \pi/3 = n\pi + (-1)^n \frac{\pi}{6}$

$\Rightarrow x = \frac{n}{2}\pi + (-1)^n \frac{\pi}{12} - \frac{\pi}{6}$

Another method:-

$2x + \frac{\pi}{3} = 2n\pi + \frac{\pi}{6}$ or $2x + \frac{\pi}{3} = 2n\pi + \frac{5\pi}{6}$

$\Rightarrow x = n\pi - \frac{\pi}{12}$ $\Rightarrow x = n\pi + \frac{\pi}{4}$