<u>Ans. to Tut. 4</u>

<u>Qn 1</u>

VRP = (30, 30, 30)   **VPN** = $(\cos 30^0, \sin 30^o, 0)$ **VUP** = (0, 1, 0)

$$\mathbf{Z}_{VC} = |\mathbf{VPN}| = (\cos 30^o, \sin 30^o, 0)$$

$$\mathbf{VUP} \times \mathbf{VPN} = \begin{vmatrix} i & j & k \\ 0 & 1 & 0 \\ \cos 30^o & \sin 30^o & 0 \end{vmatrix} = (0, 0, -\cos 30^o)$$

$$\mathbf{X}_{VC} = |\mathbf{VUP} \times \mathbf{VPN}| = (0, 0, -1)$$

$$\mathbf{Y}_{VC} = \mathbf{Z}_{VC} \times \mathbf{X}_{VC} = (-\sin 30^o, \cos 30^o, 0)$$

$$\mathbf{M}_{C1 \leftarrow WC} = \begin{pmatrix} 0 & -\sin 30^o & \cos 30^o & 30 \\ 0 & \cos 30^o & \sin 30^o & 30 \\ -1 & 0 & 0 & 30 \\ 0 & 0 & 0 & 1 \end{pmatrix}^{-1}$$

<u>Qn 2</u>

$$\mathbf{M}_{C2 \leftarrow WC} = \mathbf{M}_{C2 \leftarrow C1} \mathbf{M}_{C1 \leftarrow WC}$$

$$= \begin{pmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}^{-1} \begin{pmatrix} 0 & -\sin 30^o & \cos 30^o & 30 \\ 0 & \cos 30^o & \sin 30^o & 30 \\ -1 & 0 & 0 & 30 \\ 0 & 0 & 0 & 1 \end{pmatrix}^{-1}$$

$$= \left[\begin{pmatrix} 0 & -\sin 30^o & \cos 30^o & 30 \\ 0 & \cos 30^o & \sin 30^o & 30 \\ -1 & 0 & 0 & 30 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \right]^{-1} \quad (\ (\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1} \ )$$

$$= \begin{pmatrix} 0 & -\sin 30^o & \cos 30^o & 30 \\ 0 & \cos 30^o & \sin 30^o & 30 \\ -1 & 0 & 0 & 28 \\ 0 & 0 & 0 & 1 \end{pmatrix}^{-1}$$

Alternatively, there is a much quicker method:

The second camera's **VRP** is (30, 30, 30) + 2 $\mathbf{X}_{VC1}$ = (30, 30, 30) + 2(0, 0, -1) = (30, 30, 28). Therefore

$$\mathbf{M}_{C2\leftarrow WC} = \begin{pmatrix} 0 & -\sin 30^o & \cos 30^o & 30 \\ 0 & \cos 30^o & \sin 30^o & 30 \\ -1 & 0 & 0 & 28 \\ 0 & 0 & 0 & 1 \end{pmatrix}^{-1}$$

<u>Qn 3</u>

$$(V_{px}, V_{py}, V_{pz}) = (\frac{\sqrt{3}}{2}, 0.5, -2) \qquad Z_{vp} = 0$$

Since

$$\mathbf{M}_{parallel} = \begin{pmatrix} 1 & 0 & -\frac{V_{px}}{V_{pz}} & z_{vp}\frac{V_{px}}{V_{pz}} \\ 0 & 1 & -\frac{V_{py}}{V_{pz}} & z_{vp}\frac{V_{py}}{V_{pz}} \\ 0 & 0 & 0 & z_{vp} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathbf{M}_{parallel} = \begin{pmatrix} 1 & 0 & \frac{\sqrt{3}}{4} & 0 \\ 0 & 1 & 0.25 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\tan\alpha = \frac{|V_{pz}|}{\sqrt{V_{px}^{\ 2} + V_{py}^{\ 2}}} = 2 \quad \Rightarrow \text{ Cabinet Projection}$$

Qn 4

a) $$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -100 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

OpenGL command

glortho (100, 300, 100, 300, 100, 1000)

$dnear = 100 \Rightarrow \; Z_{near} = -100$; $dfar = 1000 \Rightarrow Z_{far} = -1000$

b) Cavalier projection

$$(\mathrm{x_p}, \mathrm{y_p}, 1) = (\mathrm{X}, \mathrm{Y}, \mathrm{Z}) + \mathrm{t}(-1, 1, \sqrt{2})$$

Take the 3rd component,

$$\mathrm{t} = \frac{1}{\sqrt{2}} - \frac{\mathrm{Z}}{\sqrt{2}}$$

Take the 1st and 2nd components,

$$x_p = X - t = X + \frac{Z}{\sqrt{2}} - \frac{1}{\sqrt{2}}$$

$$y_p = Y + t = Y - \frac{Z}{\sqrt{2}} + \frac{1}{\sqrt{2}}$$

Writing out,

$$\mathbf{M} = \begin{pmatrix} 1 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ 0 & 1 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

c) **VRP** = (200, 200, 200) **VUP** = (0, 1, 0)

VPN = (200, 200, 200) – (0, 0, 0) = (200, 200, 200)

$$\mathbf{Z}_{\mathbf{VC}} = |\mathbf{VPN}| = \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$$

$$\mathbf{X}_{VC} = |\mathbf{VUP} \times \mathbf{VPN}| = \left\| \begin{matrix} i & j & k \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{matrix} \right\| = |(1,0,-1)|$$

$$\mathbf{Y}_{VC} = |\mathbf{Z}_{VC} \times \mathbf{X}_{VC}| = \left\| \begin{matrix} i & j & k \\ 1 & 1 & 1 \\ 1 & 0 & -1 \end{matrix} \right\| = |(-1,2,-1)|$$

$$\mathbf{M} = \begin{pmatrix} \mathbf{X}_{VC} & \mathbf{Y}_{VC} & \mathbf{Z}_{VC} & \mathbf{VRP} \\ 0 & 0 & 0 & 1 \end{pmatrix}^{-1} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 200 \\ 0 & \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 200 \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 200 \\ 0 & 0 & 0 & 1 \end{pmatrix}^{-1}$$

OpenGL command:

gluLookAt (200, 200, 200, 0, 0, 0, 0, 1, 0)

d) Denote 1 as the original camera, 2 as the rotated camera, and w as the world coordinate system. Wish to find $\mathrm{M}_{2\leftarrow W}$.

$$\mathbf{M}_{2\leftarrow W} = \mathbf{M}_{2\leftarrow 1}\mathbf{M}_{1\leftarrow W} = R_Z(-30^o)\mathbf{M}$$

$$= R_Z(30^0)^{-1} \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 200 \\ 0 & \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 200 \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 200 \\ 0 & 0 & 0 & 1 \end{pmatrix}^{-1}$$

$$= \left(\begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 200 \\ 0 & \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 200 \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 200 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{\sqrt{3}}{2} & -0.5 & 0 & 0 \\ 0.5 & \frac{\sqrt{3}}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \right)^{-1}$$

$$= \begin{pmatrix} \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} & 200 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} & 200 \\ -\frac{2}{\sqrt{6}} & 0 & \frac{1}{\sqrt{3}} & 200 \\ 0 & 0 & 0 & 1 \end{pmatrix}^{-1}$$