Academic Honesty Pledge

I pledge that the answers in this exam/quiz are my own and that I will not seek or obtain an unfair advantage in producing these answers. Specifically,

- I will not plagiarize (copy without citation) from any source;
- I will not communicate or attempt to communicate with any other person during the exam/quiz; neither will I give or attempt to give assistance to another student taking the exam/quiz; and
- I will use only approved devices (e.g., calculators) and/or approved device models.

I understand that any act of academic dishonesty can lead to disciplinary action.

Student ID:____

Signature:____

Date:____

CITY UNIVERSITY OF HONG KONG

Module code & title	:	MA1200 Calculus and Basic Linear Algebra I
Session	:	Semester A, 2020–2021
Time allowed	:	Three hours

This paper has five pages (including this page).

Instruction to candidates:

1. This paper consists 11 questions.

2. Show all working.

3. Attempt ALL questions.

4. This is an online exam and you are required to reaffirm your academic honesty pledge.

5. There is a departmental hotline 3442 8646. In case you might need any help during the exam, please call the hotline and contact the course leader directly via email: shun.zhang@cityu.edu.hk or zoom chat with the zoom host. (zoom/email is preferred)

6. Submit your answers in pdf to the canvas assignment after the exam finished. Please be patient in case of possible network congestion.

This is a **closed-book** examination.

Materials, aid & instruments which students are permitted to use during the examination:

Non-programmable Calculator

Materials/aids other than those stated above are not permitted. Candidates will be subject to disciplinary action if any unauthorized aerials or aids are found to them.

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1. (a)(10 points) Write $x^2 - y^2 + 6x + 34 = 0$ into the standard form, find foci, center, and vertices, (asymptotes if it is a hyperbola), and sketch the graph of it.

(b)(3 points) Find the tangent line of the curve passing through the point $(9, 13)$.

2. (14 points) Differentiate with respect to x.

(a)(2 points) $\left(\dfrac{3x+5}{x+2}\right)^3$

(b)(2 points) $\sqrt{1+x^4} + \ln(1+x^4)$

(c)(3 points) $(\sin x)^{\sqrt{x}}$

(d)(3 points) $\tan^{-1}(\cosh x)$

(e)(4 points) $\dfrac{x^2 \sin^2 x}{e^{3x}\sqrt{x^2+2}}$

3. (11 points)Evaluate the following limits.

(a)(2 points) $\lim_{x\to\infty} \dfrac{2x+2}{\sqrt{x^2+x+4}}$

(b)(3 points) $\lim_{x\to 0} \dfrac{x - \ln(1+x)}{1 - \cos x}$

(c)(3 points) $\lim_{x\to 2^+} \dfrac{x^2-4}{x^2-5x+6}$

(d)(3 points) $\lim_{x\to 0}(e^x + 2x)^{\frac{1}{2x}}$

4.(6 points) Express $\dfrac{5x^2+7x+8}{(x+1)(x^2+2x+3)}$ as partial fractions.

5.(6 points) The function

$$f(x) = \begin{cases} \sqrt{e} & x \le 0, \\ (1+\frac{x}{a})^{1/x} & x > 0 \end{cases}$$

is continuous at $x = 0$. Find a.

6. (9 points) Let

$$f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & x \neq 0, \\ 0 & x = 0 \end{cases}.$$

(a) (4 points) Find $f'(0)$ by the definition of the derivative (the first principle).

(b) (5 points) Does $\lim_{x\to 0} f'(x)$ exist?

7. (5 points) The increasing function $f(x) = x^3 + 4x - 2$ has an inverse function $y = f^{-1}(x)$. Find the derivative $\dfrac{df^{-1}(x)}{dx}$ at $x = -2$.

8.(6 points) A curve has parametric equations: $x = t\sin(t)$ and $y = \cos(t)$, $t \in (-\pi, \pi)$ Find d^2y/dx^2 for $t = \pi/2$.

9.(15 points) Let $f(x) = \sinh(\sin^{-1} x)$, (where $\sinh x = (e^x - e^{-x})/2$ and $\cosh x = (e^x + e^{-x})/2$)

(3 points) (a) Show that $(1 - x^2)f''(x) - xf'(x) - f(x) = 0$.

(6 points) (b) Let n be a positive integer, show that

$$(1 - x^2)f^{(n+2)}(x) - (2n + 1)xf^{(n+1)}(x) - (n^2 + 1)f^{(n)}(x) = 0.$$

Hint, Leibnitz' rule: $(uv)^{(n)} = \sum_{r=0}^{n} C_r^n u^{(r)} v^{(n-r)}$, $C_r^n = \frac{n!}{(n-r)!r!}$

(6 points) (c) Find the Maclaurin series of $\sinh(\sin^{-1} x)$ as far as the terms in x^5.

10.(10 points) The cost of x products is

$$C = 25000 + 200x + \frac{1}{40}x^2.$$

(a) (5 points) To minimize the average cost (the cost of one product), how many products should we produce? (Find x)

(b) (5 points) If each product is sold at a price 500, how many products should we produce in order to maximize the profit?

11.(5 points) Use the mean value theorem to show that

$$\pi/4 + 10/221 < \tan^{-1}(1.1) < \pi/4 + 0.05.$$

Hint: let $f(x) = \tan^{-1}(x)$ and consider the interval $[1, 1.1]$.

Mean value theorem: Let $f(x)$ be a continuous function on the closed interval $[a, b]$, and differentiable on the open interval (a, b), where $a < b$. Then there exists some $c \in (a, b)$ such that $f(b) - f(a) = f'(c)(b - a)$.

Short Table of Derivatives of $y = f(u)$ with respect to x, where u is a function of x

Functions, $y = f(u)$	Derivative of y with respect to x
$y = c$, where c is a constant.	$\frac{dy}{dx} = 0$
$y = cu$, where c is a constant.	$\frac{dy}{dx} = c\frac{du}{dx}$
$y = u^p$, where p is a constant.	$\frac{dy}{dx} = pu^{p-1}\frac{du}{dx}$
$y = u + v$	$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$
$y = uv$	$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$
$y = \frac{u}{v}$	$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
$y = f(u)$, where u is a function of x.	$\frac{dy}{dx} = \frac{d f(u)}{du} \cdot \frac{du}{dx}$, the chain rule
$y = \log_a u$, $a > 0$.	$\frac{dy}{dx} = \frac{1}{u}\log_a e\frac{du}{dx}$
$y = a^u$, $a > 0$.	$\frac{dy}{dx} = a^u \log_e a\frac{du}{dx}$
$y = e^u$	$\frac{dy}{dx} = e^u\frac{du}{dx}$
$y = u^v$	$\frac{dy}{dx} = vu^{v-1}\frac{du}{dx} + u^v \log_e u\frac{dv}{dx}$
$y = \sin u$	$\frac{dy}{dx} = \cos u\frac{du}{dx}$
$y = \cos u$	$\frac{dy}{dx} = -\sin u\frac{du}{dx}$
$y = \tan u$	$\frac{dy}{dx} = \sec^2 u\frac{du}{dx}$
$y = \cot u$	$\frac{dy}{dx} = -\mathrm{cosec}^2 u\frac{du}{dx}$
$y = \sec u$	$\frac{dy}{dx} = \sec u \tan u\frac{du}{dx}$
$y = \mathrm{cosec}\, u$	$\frac{dy}{dx} = -\mathrm{cosec}\, u \cot u\frac{du}{dx}$
$y = \sin^{-1} u$	$\frac{dy}{dx} = \frac{1}{\sqrt{1-u^2}}\frac{du}{dx}$
$y = \cos^{-1} u$	$\frac{dy}{dx} = \frac{-1}{\sqrt{1-u^2}}\frac{du}{dx}$
$y = \tan^{-1} u$	$\frac{dy}{dx} = \frac{1}{1+u^2}\frac{du}{dx}$

Functions, $y = f(u)$	**Derivative of** y **with respect to** x
$y = \cot^{-1} u$	$\frac{dy}{dx} = -\frac{1}{1+u^2}\frac{du}{dx}$
$y = \sec^{-1} u$	$\frac{dy}{dx} = \frac{1}{\lvert u\rvert\sqrt{u^2-1}}\frac{du}{dx}$
$y = \operatorname{cosec}^{-1} u$	$\frac{dy}{dx} = -\frac{1}{\lvert u\rvert\sqrt{u^2-1}}\frac{du}{dx}$
$y = \sinh u$	$\frac{dy}{dx} = \cosh u\frac{du}{dx}$
$y = \cosh u$	$\frac{dy}{dx} = \sinh u\frac{du}{dx}$
$y = \tanh u$	$\frac{dy}{dx} = \operatorname{sech}^2 u\frac{du}{dx}$
$y = \coth u$	$\frac{dy}{dx} = -\operatorname{cosech}^2 u\frac{du}{dx}$
$y = \operatorname{sech} u$	$\frac{dy}{dx} = -\operatorname{sech} u \tanh u\frac{du}{dx}$
$y = \operatorname{cosech} u$	$\frac{dy}{dx} = -\operatorname{cosech} u \coth u\frac{du}{dx}$
$y = \sinh^{-1} u$	$\frac{dy}{dx} = \frac{1}{\sqrt{1+u^2}}\frac{du}{dx}$
$y = \cosh^{-1} u$	$\frac{dy}{dx} = \frac{1}{\sqrt{u^2-1}}\frac{du}{dx}$
$y = \tanh^{-1} u$	$\frac{dy}{dx} = \frac{1}{1-u^2}\frac{du}{dx}$
$y = \coth^{-1} u$	$\frac{dy}{dx} = \frac{1}{1-u^2}\frac{du}{dx}$
$y = \operatorname{sech}^{-1} u$	$\frac{dy}{dx} = -\frac{1}{u\sqrt{1-u^2}}\frac{du}{dx}$
$y = \operatorname{cosech}^{-1} u$	$\frac{dy}{dx} = -\frac{1}{\lvert u\rvert\sqrt{u^2+1}}\frac{du}{dx}$

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