Tutorial 2

Functions

Q.1: Encoder of Even Parity

Encoding function f.

- Input: (b_1, b_2, b_3, b_4), where $b_i \in \{0, 1\}\ \forall i$
- Output: $(c_1, c_2, c_3, c_4, c_5)$, where $c_i \in \{0, 1\}\ \forall i$
  - $c_1 = b_1, c_2 = b_2, c_3 = b_3, c_4 = b_4,$
  - $c_1 + c_2 + c_3 + c_4 + c_5 = 0 \pmod 2$

a) What is the domain of f?
   - Hint: Use Cartesian product.

b) What is the co-domain of f?

c) What is the image of $(0, 1, 0, 0)$?

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d) What is the range of f ?

1) $\{0, 1\}^5$
2) $\{x \in \{0, 1\}^5 \mid x \text{ has an even number of 1s}\}$
3) $\{x \in \{0, 1\}^5 \mid x \text{ has an odd number of 1s}\}$

Q.2: Decoder of Even Parity

Decoding function g.

- Input: $(c_1, c_2, c_3, c_4, c_5)$, where $c_i \in \{0, 1\}\ \forall i$
- Output:
  - Either (b_1, b_2, b_3, b_4), where $b_i \in \{0, 1\}\ \forall i$
  - Or a special symbol e when an error is detected.

a) What is the image of $(0, 1, 0, 0, 1)$?

b) What is the image of $(1, 1, 0, 1, 0)$?

c) What is the domain of g?

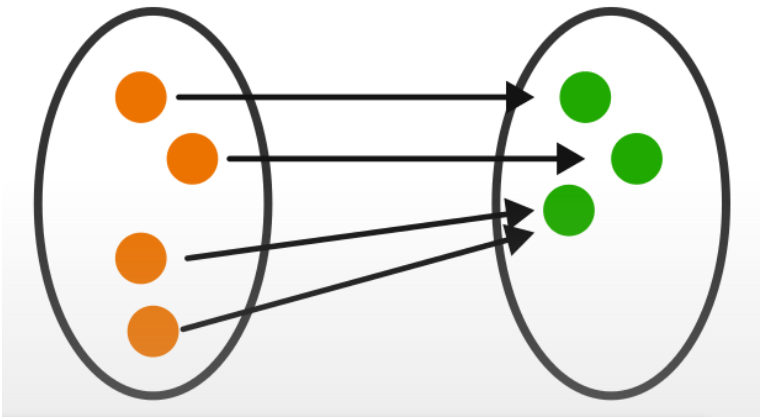
d) What is the co-domain of g?

- Hint: Don't forget the special symbol e.

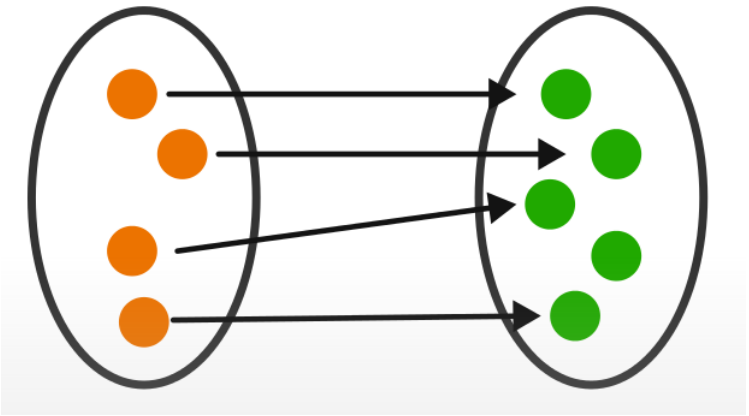
Q.3: Injection & Surjection

- Is it injection or surjection?

i)



ii)



a) i) is injection, ii) is surjection
b) i) is injection, ii) is also injection
c) i) is surjection, ii) is injection
d) i) is surjection, ii) is also surjection

Q.4: Composition of Onto Functions

- Suppose $f: X \to Y$ and $g: Y \to Z$ are both surjections.
- Is $g \circ f$ a surjection? Prove or disprove it.
  a) Yes
  b) No

Q.5: Comparison of Infinities

- Do the intervals (0,1) and (0, 2) have the same cardinality? Prove or disprove it.

  a) Yes
  b) No