# Assignment 2 Solution

**Question 1**

$$p(X) = \frac{exp(\beta_0 + \beta_1 X_1 + \beta_2 X_2)}{1 + exp(\beta_0 + \beta_1 X_1 + \beta_2 X_2)}$$

$$X_1 = hoursstudied, X_2 = undergradGPA$$

$$\widehat{\beta_0} = -6, \widehat{\beta_1} = 0.05, \widehat{\beta_2} = 1$$

(a) $X_1 = 40\ hours, X_2 = 3.5GPA$

$$p(X) = \frac{exp(\text{-}6 + 0.05X_1 + X_2)}{1 + exp(\text{-}6 + 0.05X_1 + X_2)}$$

$$= \frac{exp(\text{-}6 + 0.05{*}40 + 3.5)}{1 + exp(\text{-}6 + 0.05{*}40 + 3.5)}$$

$$= 37.75\%$$

(b) $X_1\ hours, X_2 = 3.5GPA$

$$p(X) = \frac{exp(\text{-}6 + 0.05X_1 + X_2)}{1 + exp(\text{-}6 + 0.05X_1 + X_2)}$$

$$0.50 = \frac{exp(\text{-}6 + 0.05X_1 + 3.5)}{1 + exp(\text{-}6 + 0.05X_1 + 3.5)}$$

$$0.50 = 0.50exp(\text{-}6 + 0.05X_1 + 3.5)$$

$$X_1 = 50hours$$

**Question 2**

(a) If the Bayes decision boundary is linear, we expect QDA to perform better on the training set because it's higher flexiblity will yield a closer fit. On the test set, we expect LDA to perform better than QDA because QDA could overfit the linearity of the Bayes decision boundary.

(b) If the Bayes decision bounary is non-linear, we expect QDA to perform better both on the training and test sets.

(c) We expect the test prediction accuracy of QDA relative to LDA to improve, in general, as the the sample size nn increases because a more flexibile method will yield a better fit as more samples can be fit and variance is offset by the larger sample sizes.

(d) False. With fewer sample points, the variance from using a more flexible method, such as QDA, would lead to overfit, yielding a higher test rate than LDA.

**Question 3**

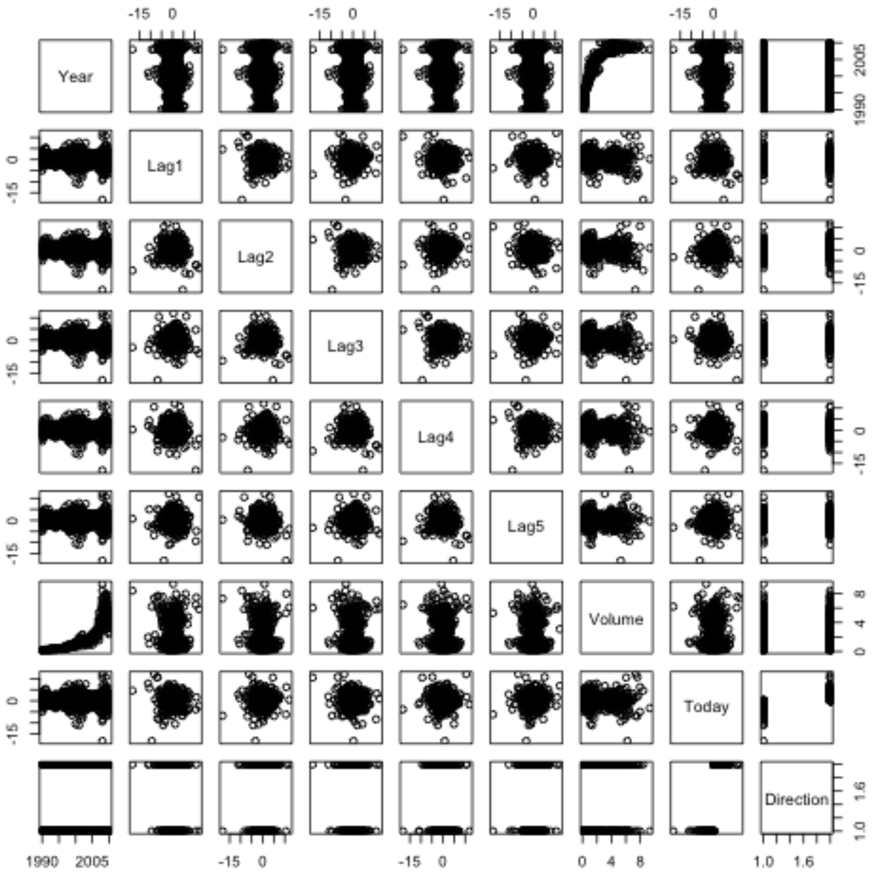
(a)

```
library(ISLR)
summary(Weekly)
```

```
##       Year           Lag1               Lag2               Lag3
##  Min.   :1990   Min.   :-18.195   Min.   :-18.195   Min.   :-18.195
##  1st Qu.:1995   1st Qu.: -1.154   1st Qu.: -1.154   1st Qu.: -1.158
##  Median :2000   Median :  0.241   Median :  0.241   Median :  0.241
##  Mean   :2000   Mean   :  0.151   Mean   :  0.151   Mean   :  0.147
##  3rd Qu.:2005   3rd Qu.:  1.405   3rd Qu.:  1.409   3rd Qu.:  1.409
##  Max.   :2010   Max.   : 12.026   Max.   : 12.026   Max.   :  12.026
##       Lag4              Lag5              Volume          Today
##  Min.   :-18.195   Min.   :-18.195   Min.   :0.087   Min.:    -18.195
##  1st Qu.: -1.158   1st Qu.: -1.166   1st Qu.:0.332   1st Qu.: -1.154
```

```
##  Median :  0.238   Median :  0.234   Median :1.003   Median :  0.241
##  Mean   :  0.146   Mean   :  0.140   Mean   :1.575   Mean   :  0.150
##  3rd Qu.:  1.409   3rd Qu.:  1.405   3rd Qu.:2.054   3rd Qu.:  1.405
##  Max.   : 12.026   Max.   : 12.026   Max.   :9.328   Max.   : 12.026
##  Direction
##  Down:484
##  Up  :605
##
```

```
Pairs(Weekly)
```




```
cor(Weekly[,-9])
```

```
##               Year      Lag1      Lag2      Lag3      Lag4      Lag5.    Volume
## Year    1.00000 -0.032289 -0.03339 -0.03001 -0.031128 -0.030519.   0.84194
## Lag1   -0.03229  1.000000 -0.07485  0.05864 -0.071274 -0.008183.  -0.06495
## Lag2   -0.03339 -0.074853  1.00000 -0.07572  0.058382 -0.072499.  -0.08551
## Lag3   -0.03001  0.058636 -0.07572  1.00000 -0.075396  0.060657.  -0.06929
## Lag4   -0.03113 -0.071274  0.05838 -0.07540  1.000000 -0.075675.  -0.06107
## Lag5   -0.03052 -0.008183 -0.07250  0.06066 -0.075675  1.000000.  -0.05852
## Volume  0.84194 -0.064951 -0.08551 -0.06929 -0.061075 -0.058517.   1.00000
## Today  -0.03246 -0.075032  0.05917 -0.07124 -0.007826  0.011013.  -0.03308
##             Today
## Year   -0.032460
## Lag1   -0.075032
## Lag2   -0.059167
## Lag3   -0.071244
## Lag4   -0.007826
## Lag5   -0.011013
## Volume -0.033078
## Today   1.000000
```

Year and Volume appear to have a relationship. No other patterns are discernible.

(b)

```
attach(Weekly)
glm.fit = glm(Direction ~ Lag1 + Lag2 + Lag3 + Lag4 + Lag5 + Volume, data =
          Weekly, family = binomial)
summary(glm.fit)
```

```
##
## Call:
## glm(formula = Direction ~ Lag1 + Lag2 + Lag3 + Lag4 + Lag5 +
##     Volume, family = binomial, data = Weekly)
##
## Deviance Residuals:
##    Min      1Q  Median      3Q     Max
## -1.695  -1.256   0.991   1.085   1.458
##
## Coefficients:
##             Estimate Std. Error z value Pr(>|z|)
## (Intercept)   0.2669     0.0859    3.11.  0.0019 **
## Lag1.        -0.0413     0.0264   -1.56.  0.1181
## Lag2.         0.0584     0.0269    2.18.  0.0296 *
## Lag3.        -0.0161     0.0267   -0.60.  0.5469
## Lag4.        -0.0278     0.0265   -1.05.  0.2937
## Lag5.        -0.0145     0.0264   -0.55.  0.5833
## Volume.      -0.0227     0.0369   -0.62.  0.5377
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##     Null deviance: 1496.2  on 1088  degrees of freedom
## Residual deviance: 1486.4  on 1082  degrees of freedom
## AIC: 1500
##
## Number of Fisher Scoring iterations: 4
```

Lag 2 appears to have some statistical significance with a Pr(>|z|) = 3%.

(c)

```
glm.probs = predict(glm.fit, type = "response")
glm.pred = rep("Down", length(glm.probs))
glm.pred[glm.probs > 0.5] = "Up"
table(glm.pred, Direction)
```

```
##         Direction
## glm.pred Down  Up
##     Down   54  48
##     Up    430 557
```

Percentage of currect predictions: (54+557)/(54+557+48+430) = 56.1%. Weeks the market goes up the logistic regression is right most of the time, 557/(557+48) = 92.1%. Weeks the market goes up the logistic regression is wrong most of the time 54/(430+54) =11.2%.

(d)

```
train = (Year < 2009)
Weekly.0910 = Weekly[!train, ]
glm.fit = glm(Direction ~ Lag2, data = Weekly, family = binomial, subset =
train)
glm.probs = predict(glm.fit, Weekly.0910, type = "response")
glm.pred = rep("Down", length(glm.probs))
glm.pred[glm.probs > 0.5] = "Up"
Direction.0910 = Direction[!train]
table(glm.pred, Direction.0910)
```

```
##         Direction.0910
##  glm.pred Down Up
##      Down    9  5
```

```
##         Up   34 56
```

```
mean(glm.pred == Direction.0910)
```

```
## [1] 0.625
```

(e)

```
library(MASS)
lda.fit = lda(Direction ~ Lag2, data = Weekly, subset = train)
lda.pred = predict(lda.fit, Weekly.0910)
table(lda.pred$class, Direction.0910)
```

```
##        Direction.0910
##         Down Up
##   Down     9  5
##   Up      34 56
```

```
mean(lda.pred$class == Direction.0910)
```

```
## [1] 0.625
```

(f)

```
qda.fit = qda(Direction ~ Lag2, data = Weekly, subset = train)
qda.class = predict(qda.fit, Weekly.0910)$class
table(qda.class, Direction.0910)
```

```
##        Direction.0910
## qda.class Down Up
##       Down    0  0
##         Up   43 61
```

```
mean(qda.class == Direction.0910)
```

```
## [1] 0.5865
```

A correctness of 58.7% even though it picked Up the whole time!

(g)

```
library(class)
train.X = as.matrix(Lag2[train])
test.X = as.matrix(Lag2[!train])
train.Direction = Direction[train]
set.seed(1)
knn.pred = knn(train.X, test.X, train.Direction, k = 1)
table(knn.pred, Direction.0910)
```

```
##         Direction.0910
## knn.pred Down Up
##     Down   21 30
##       Up   22 31
```

```
mean(knn.pred == Direction.0910)
```

```
## [1] 0.5
```

(h) Logistic regression and LDA methods provide similar test error rates.

**Question 4**

(a) $1\text{-}1/n$
(b) $1\text{-}1/n$

(c) As we are using sampling with replacement to generate the bootstrap sample, the selection probabilities are independent; so Probability(Observation j is not the first bootstrap observation, Observation j is not the second bootstrap observation, …, Observation j is not the *n*th bootstrap observation) = Probability(Observation j is not the first bootstrap observation) * Probability(Observation j is not the second bootstrap observation) * … * Probability(Observation j is not the *n*th bootstrap observation).

(d) $For\ n = 5, 1 - (1 - 1/n)^n = 1 - (1 - 1/5)^5 = 67.23\%$

**Question 5**

(a) k-fold cross-validation is implemented by taking the set of n observations and randomly splitting into k non-overlapping groups. Each of these groups acts as a validation set and the remainder as a training set. The test error is estimated by averaging the k resulting MSE estimates.

(b) i. The validation set approach is conceptually simple and easily implemented as you are simply partitioning the existing training data into two sets. However, there are two drawbacks: (1.) the estimate of the test error rate can be highly variable depending on which observations are included in the training and validation sets. (2.) the validation set error rate may tend to overestimate the test error rate for the model fit on the entire data set.

ii. LOOCV is a special case of k-fold cross-validation with k = n. Thus, LOOCV is the most computationally intense method since the model must be fit n times. Also, LOOCV has higher variance, but lower bias, than k-fold CV.

**Question 6**

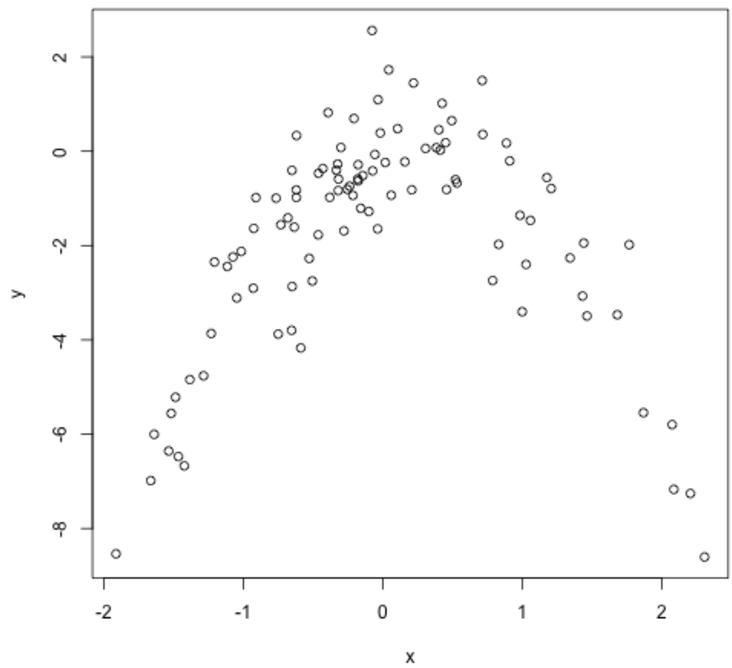
(a)

```
Set.seed(1)
y = rnorm(100)
x = rnorm(100)
y = x - 2 * x^2 + rnorm(100)
```

$$n = 100, p = 2\ and\ Y = X - 2X^2 + \epsilon$$

(b)

```
plot(x,y)
```



Quadratic plot. from about -2 to 2. from about -8 to 2.

(c)

```
library(boot)
Data = data.frame(x, y)
set.seed(1)
# i.
glm.fit = glm(y ~ x)
cv.glm(Data, glm.fit)$delta
```

```
## [1] 5.891 5.889
```

```
# ii.
glm.fit = glm(y ~ poly(x, 2))
cv.glm(Data, glm.fit)$delta
```

```
## [1] 1.087 1.086
```

```
# iii.
glm.fit = glm(y ~ poly(x, 3))
cv.glm(Data, glm.fit)$delta
```

```
## [1] 1.103 1.102
```

```
# iv.
glm.fit = glm(y ~ poly(x, 4))
cv.glm(Data, glm.fit)$delta
```

```
## [1] 1.115 1.114
```

(d)

```
set.seed(10)
# i.
glm.fit = glm(y ~ x)
cv.glm(Data, glm.fit)$delta
```

```
## [1] 5.891 5.889
```

```
# ii.
glm.fit = glm(y ~ poly(x, 2))
cv.glm(Data, glm.fit)$delta
```

```
## [1] 1.087 1.086
```

```
# iii.
glm.fit = glm(y ~ poly(x, 3))
cv.glm(Data, glm.fit)$delta
```

```
## [1] 1.103 1.102
```

```
# iv.
glm.fit = glm(y ~ poly(x, 4))
cv.glm(Data, glm.fit)$delta
```

```
## [1] 1.115 1.114
```

Exact same, because LOOCV will be the same since it evaluates n folds of a single observation.

(e) The quadratic polynomial had the lowest LOOCV test error rate. This was expected because it matches the true form of Y.

(f)

```
summary(glm.fit)
```

```
##
## Call:
```

```
## glm(formula = y ~ poly(x, 4))
##
## Deviance Residuals:
##     Min       1Q   Median       3Q      Max
## -2.8913  -0.5244   0.0749   0.5932   2.7796
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   -1.828      0.104  -17.55   <2e-16 ***
## poly(x, 4)1    2.316      1.041    2.22    0.029 *
## poly(x, 4)2  -21.059      1.041  -20.22   <2e-16 ***
## poly(x, 4)3   -0.305      1.041   -0.29    0.770
## poly(x, 4)4   -0.493.     1.041   -0.47    0.637
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' '
1
##
## (Dispersion parameter for gaussian family taken to be 1.085)
##
##     Null deviance: 552.21  on 99  degrees of freedom
## Residual deviance: 103.04  on 95  degrees of freedom
## AIC: 298.8
##
## Number of Fisher Scoring iterations: 2
```

p-values show statistical significance of linear and quadratic terms, which agrees with the CV results.