

1. (2 points) Find a value of x that solves the following simultaneous congruences:

$$x \equiv 3 \pmod{37} \text{ and } x \equiv 5 \pmod{87}.$$

2. (3 points) Find a value of x that solves the following simultaneous congruences:

$$x \equiv 5 \pmod{7}, x \equiv 2 \pmod{12} \text{ and } x \equiv 8 \pmod{13}.$$

3. Consider the use of RSA cipher. The public key of Bob is $N = 55$ and $e = 3$.

- a) (1 point) Alice wants to send the message "HELLO" to Bob. Translate the message into its numeric equivalent (i.e. $A = 1, B = 2, \dots$).

- b) (2 points) Encrypt the message in (a) letter by letter. Show your steps.

(Remark: In practice, individual letters of the alphabet are grouped together in blocks during encryption so that deciphering cannot be accomplished through knowledge of frequency patterns of letters or words.)

- c) (2 points) Suppose Bob receives a message from Claire. The ciphertext of the message is 08 05 15. Decrypt it and translate the result into letters of the alphabet to discover the message. Show your steps.