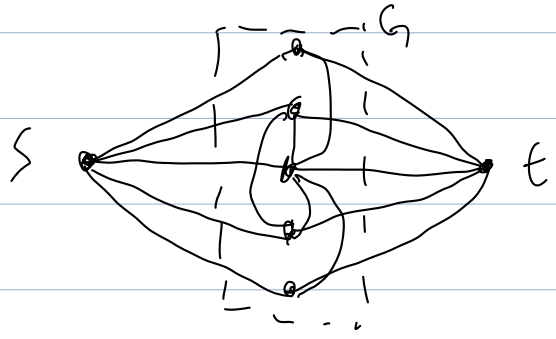
Q1.(1) Transform $G=(V,E)$ to $G'=\langle V \cup \{s,t\}, E' \rangle$

- $E \subseteq E'$
- $(s,u) \in E'$, $(u,t) \in E'$ for all $u \in V$
- E': $w_{uv}=1$ if $(u,v) \in E$

  $w_{uv} = \deg(v)$ if $u=s$, $w_{uv}=c$ if $v=t$



Suppose $\exists S \subseteq V$, $\rho(S) \geq \lambda$. $\bar{S} = V \setminus S$

$$\rho(S) = \frac{\sum_{(u,v) \in E(S)} w_{uv}}{|S|} \geq \lambda$$

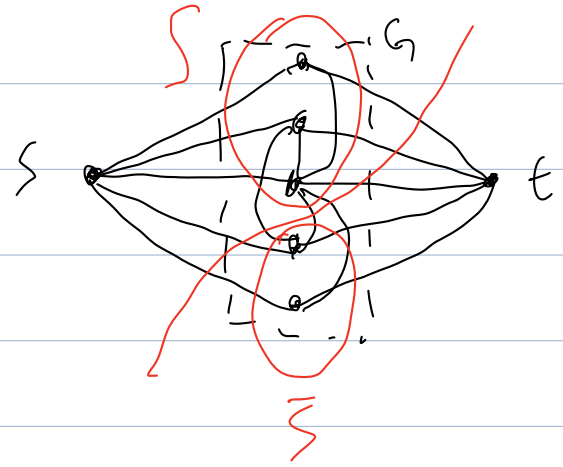
$$2|E(S,S)| \geq \lambda |S|$$

$$\sum_{u \in S} \deg(u) - |E(S,\bar{S})| \geq \lambda |S|$$

$$\sum_{u \in S} \deg(u) + \sum_{u \in \bar{S}} \deg(u) - \sum_{u \in \bar{S}} \deg(u) - |E(S,\bar{S})| \geq \lambda |S|$$

$$\sum_{u \in \bar{S}} \deg(u) + |E(S,\bar{S})| + \lambda |S| \leq 2|E|$$

a cut of $2|E|$ always exists, for $S = \emptyset$



$S \neq \emptyset$ gives cut of value:

$$\sum_{u \in \bar{S}} \deg(u) + |E(S,\bar{S})| - \lambda|S| \leq 2|E|$$

$= WC(\{s\} \cup S_1, (V - S_1) \cup \{t\}) \leq \gamma$, for $S = S_1$, $\bar{S} = S_2$, $\gamma = 2|E|$

ie. $p(S) \geq \lambda$ is equivalent to (s,t)-cut in G'

Q2.

A_{ij} =

	1	2	3	4	5	6	7	8	9	10	11
1	0	1	1	0	0	0	0	0	0	0	0
2	1	0	1	1	1	0	0	0	0	0	0
3	1	1	0	1	1	0	0	0	0	0	0
4	0	1	1	0	1	0	0	0	0	0	0
5	0	1	1	1	0	1	1	0	0	0	0
6	0	0	0	0	1	0	1	0	0	0	0
7	0	0	0	0	1	1	0	1	1	1	0
8	0	0	0	0	0	0	1	0	1	1	0
9	0	0	0	0	0	0	1	1	0	1	1
10	0	0	0	0	0	0	1	1	1	0	1
11	0	0	0	0	0	0	0	0	1	1	0

$|E| = 19$

$d_1 = 2$

$d_2 = 4$

$d_3 = 4$

$d_4 = 3$

$d_5 = 5$

$d_6 = 2$

$d_7 = 5$

$d_8 = 3$

$d_9 = 4$

$d_{10} = 4$

$d_{11} = 2$

$B_{ij} =$

	1	2	3	4	5	6	7	8	9	10	11
1	$\frac{-2}{19}$	$\frac{15}{19}$	$\frac{15}{19}$	$\frac{-3}{19}$	$\frac{-5}{19}$	$\frac{-2}{19}$	$\frac{-5}{19}$	$\frac{-3}{19}$	$\frac{-4}{19}$	$\frac{-4}{19}$	$\frac{-2}{19}$
2	$\frac{15}{19}$	$\frac{-8}{19}$	$\frac{11}{19}$	$\frac{13}{19}$	$\frac{9}{19}$	$\frac{-4}{19}$	$\frac{-10}{19}$	$\frac{-6}{19}$	$\frac{-8}{19}$	$\frac{-8}{19}$	$\frac{-4}{19}$
3	$\frac{15}{19}$	$\frac{11}{19}$	$\frac{-8}{19}$	$\frac{13}{19}$	$\frac{9}{19}$	$\frac{-4}{19}$	$\frac{-10}{19}$	$\frac{-6}{19}$	$\frac{-8}{19}$	$\frac{-8}{19}$	$\frac{-4}{19}$
4	$\frac{-3}{19}$	$\frac{13}{19}$	$\frac{13}{19}$	$\frac{-9}{38}$	$\frac{23}{38}$	$\frac{-3}{19}$	$\frac{-15}{38}$	$\frac{-9}{38}$	$\frac{-6}{19}$	$\frac{-6}{19}$	$\frac{-3}{19}$
5	$\frac{-5}{19}$	$\frac{9}{19}$	$\frac{9}{19}$	$\frac{23}{38}$	$\frac{-25}{38}$	$\frac{14}{19}$	$\frac{13}{38}$	$\frac{-15}{38}$	$\frac{-10}{19}$	$\frac{-10}{19}$	$\frac{-5}{19}$
6	$\frac{-2}{19}$	$\frac{-4}{19}$	$\frac{-4}{19}$	$\frac{-3}{19}$	$\frac{14}{19}$	$\frac{-2}{19}$	$\frac{14}{19}$	$\frac{-3}{19}$	$\frac{-4}{19}$	$\frac{-4}{19}$	$\frac{-2}{19}$
7	$\frac{-5}{19}$	$\frac{-10}{19}$	$\frac{-10}{19}$	$\frac{-15}{38}$	$\frac{13}{38}$	$\frac{14}{19}$	$\frac{-25}{38}$	$\frac{23}{38}$	$\frac{9}{19}$	$\frac{9}{19}$	$\frac{-5}{19}$
8	$\frac{-3}{19}$	$\frac{-6}{19}$	$\frac{-6}{19}$	$\frac{-9}{38}$	$\frac{-15}{38}$	$\frac{-3}{19}$	$\frac{23}{38}$	$\frac{-9}{38}$	$\frac{13}{19}$	$\frac{13}{19}$	$\frac{-3}{19}$
9	$\frac{-4}{19}$	$\frac{-8}{19}$	$\frac{-8}{19}$	$\frac{-6}{19}$	$\frac{-10}{19}$	$\frac{-4}{19}$	$\frac{9}{19}$	$\frac{13}{19}$	$\frac{-8}{19}$	$\frac{11}{19}$	$\frac{15}{19}$
10	$\frac{-4}{19}$	$\frac{-8}{19}$	$\frac{-8}{19}$	$\frac{-6}{19}$	$\frac{-10}{19}$	$\frac{-4}{19}$	$\frac{9}{19}$	$\frac{13}{19}$	$\frac{11}{19}$	$\frac{-8}{19}$	$\frac{15}{19}$
11	$\frac{-2}{19}$	$\frac{-4}{19}$	$\frac{-4}{19}$	$\frac{-3}{19}$	$\frac{-5}{19}$	$\frac{-2}{19}$	$\frac{-5}{19}$	$\frac{-3}{19}$	$\frac{15}{19}$	$\frac{15}{19}$	$\frac{-2}{19}$

Q3. Iteration 1

	Distance to centroid			assignments
	C1	C2	C3	
A1	0	$\sqrt{13}$	$\sqrt{65}$	C1
A2	$\sqrt{25}$	$\sqrt{18}$	$\sqrt{10}$	C3
A3	$\sqrt{72}$	$\sqrt{25}$	$\sqrt{53}$	C2
A4	$\sqrt{13}$	0	$\sqrt{52}$	C2
A5	$\sqrt{50}$	$\sqrt{13}$	$\sqrt{45}$	C2
A6	$\sqrt{52}$	$\sqrt{17}$	$\sqrt{29}$	C2
A7	$\sqrt{65}$	$\sqrt{52}$	0	C3
A8	$\sqrt{5}$	$\sqrt{2}$	$\sqrt{58}$	C2

C1 centroid = (2, 10)

C2 centroid = (6, 6)

C3 centroid = (1.5, 3.5)

Iteration 2

	Distance to centroid			assignments
	C1	C2	C3	
A1	0	$\sqrt{32}$	$\sqrt{42.5}$	C1
A2	$\sqrt{25}$	$\sqrt{17}$	$\sqrt{2.5}$	C3
A3	$\sqrt{72}$	$\sqrt{8}$	$\sqrt{42.5}$	C2
A4	$\sqrt{13}$	$\sqrt{5}$	$\sqrt{32.5}$	C2
A5	$\sqrt{50}$	$\sqrt{2}$	$\sqrt{32.5}$	C2
A6	$\sqrt{52}$	$\sqrt{4}$	$\sqrt{20.5}$	C2
A7	$\sqrt{65}$	$\sqrt{41}$	$\sqrt{2.5}$	C3
A8	$\sqrt{5}$	$\sqrt{13}$	$\sqrt{36.5}$	C1

C1 centroid = (3, 9.5)
C2 centroid = (6.5, 5.25)
C3 centroid = (1.5, 3.5)

Iteration 3

	Distance to centroid			assignments
	C1	C2	C3	
A1	$\sqrt{1.25}$	$\sqrt{42.8}$	$\sqrt{42.5}$	C1
A2	$\sqrt{21.2}$	$\sqrt{20.3}$	$\sqrt{2.5}$	C3
A3	$\sqrt{55.3}$	$\sqrt{3.8}$	$\sqrt{42.5}$	C2
A4	$\sqrt{6.3}$	$\sqrt{9.81}$	$\sqrt{32.5}$	C1
A5	$\sqrt{36.3}$	$\sqrt{0.31}$	$\sqrt{32.5}$	C2
A6	$\sqrt{39.3}$	$\sqrt{1.81}$	$\sqrt{20.5}$	C2
A7	$\sqrt{60.3}$	$\sqrt{40.8}$	$\sqrt{2.5}$	C3
A8	$\sqrt{1.25}$	$\sqrt{20.3}$	$\sqrt{36.5}$	C1

C1 centroid = (3.67, 9)
C2 centroid = (7, 4.33)
C3 centroid = (1.5, 3.5)

Iteration 4

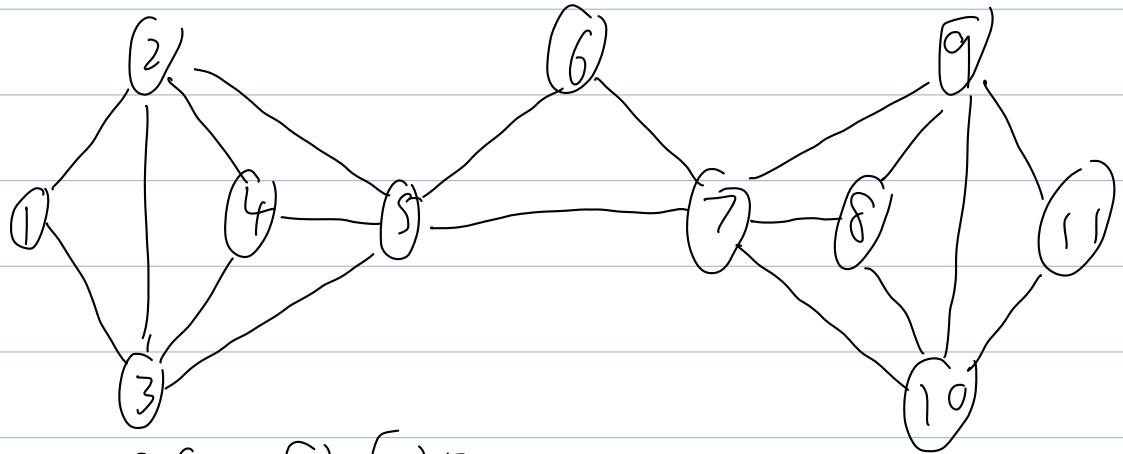
	Distance to centroid			assignments
	C1	C2	C3	
A1	$\sqrt{3.8}$	$\sqrt{57.1}$	$\sqrt{42.5}$	C1
A2	$\sqrt{18.8}$	$\sqrt{25.4}$	$\sqrt{2.5}$	C3
A3	$\sqrt{43.7}$	$\sqrt{1.11}$	$\sqrt{42.5}$	C2
A4	$\sqrt{2.77}$	$\sqrt{17.5}$	$\sqrt{32.5}$	C1
A5	$\sqrt{27.1}$	$\sqrt{0.45}$	$\sqrt{32.5}$	C2
A6	$\sqrt{30.4}$	$\sqrt{1.11}$	$\sqrt{20.5}$	C2
A7	$\sqrt{56.1}$	$\sqrt{41.4}$	$\sqrt{2.5}$	C3
A8	$\sqrt{0.11}$	$\sqrt{30.8}$	$\sqrt{36.5}$	C1

C1 centroid = (3.67, 9)
C2 centroid = (7, 4.33)
C3 centroid = (1.5, 3.5)

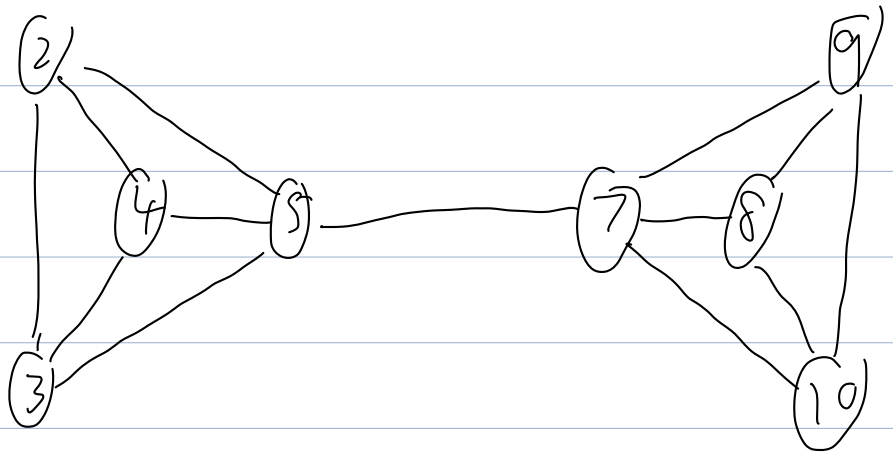
The centroids are same as the last iteration so it is converged.

C1 : {A1, A4, A8}
C2 : {A3, A5, A6}
C3 : {A2, A7}

Q4. (1).



Degree = 2 : {1, 6, 11}



After remove ①, ⑥, ⑪, there is no nodes degree ≤ 2

degree $= 3$: {②, ③, ④, ⑧, ⑨, ⑩}

After remove degree ≥ 3 nodes, only ⑤, ⑦ left, so they are all in 3-core.

∴ Core number 2 : ①, ⑥, ⑪

Core number 3 : ②, ③, ④, ⑤, ⑦, ⑧, ⑨, ⑩

(2).