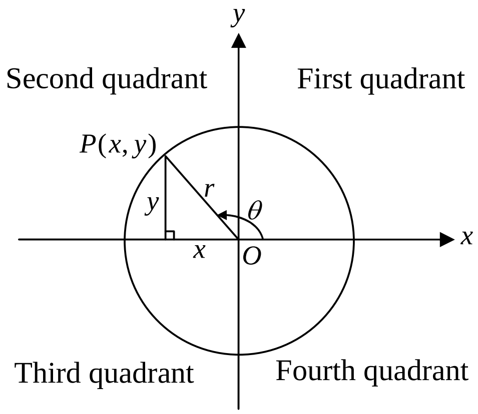
MA1200 Calculus and Basic Linear Algebra I
Chapter 4 Trigonometric Functions and Inverse Trigonometric Functions

1 Trigonometric Functions

1. In elementary trigonometry, the trigonometric functions are defined as the ratios of sides of a right-angled triangle and the angles are restricted to acute angles.

2. The six trigonometric functions are called **sine**, **cosine**, **tangent**, **cosecant**, **secant** and **cotangent**, which are written as sin, cos, tan, csc (or cosec), sec and cot respectively. We define their angles of any magnitude as follows:



$$\sin\theta = \frac{y}{r} \qquad \cos\theta = \frac{x}{r} \qquad \tan\theta = \frac{y}{x}$$

$$\csc\theta = \frac{r}{y} \qquad \sec\theta = \frac{r}{x} \qquad \cot\theta = \frac{x}{y}$$

Figure 1

where r is taken to be positive, and the signs of x and y depend on the quadrant in which the point P lies.

3. With the above definitions, the signs of the six trigonometric ratios in four quadrants are tabulated below.

	1st quadrant	2nd quadrant	3rd quadrant	4th quadrant
$\sin\theta$, $\csc\theta$	+	+	−	−
$\cos\theta$, $\sec\theta$	+	−	−	+
$\tan\theta$, $\cot\theta$	+	−	+	−

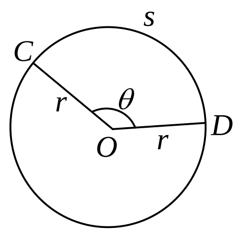
Remark We may indicate the positive functions in each quadrant with a diagram. The word CAST is the key for memory.

The followings are special angles of sine, cosine and tangent and have to be *memorized*:

	30°	45°	60°
sin	$1/2$	$\sqrt{2}/2$	$\sqrt{3}/2$
cos	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$
tan	$\sqrt{3}/3$	1	$\sqrt{3}$

Radian Measures

A radian, denoted by rad, is the measure of an angle subtended at the centre of a circle by an arc with length equal to its radius.



If the length of the arc CD is s and $\angle COD = \theta$, where θ is measure in radian, then $\boxed{\theta = \dfrac{s}{r}}$.

In particular, if $\theta = 180°$, then $s = \pi r$. It follows that $\theta = \dfrac{s}{r} = \dfrac{\pi r}{r} = \pi$, so that $\boxed{\pi \text{ rad } = 180°}$.

Example 1 (a) Convert the following angles to radians.

(i) $30°$ (ii) $45°$ (iii) $90°$ (iv) $210°$

(b) Convert the following angles to degrees.

(i) $\dfrac{5\pi}{12}$ rad (ii) $\dfrac{3\pi}{2}$ rad (iii) $\dfrac{5\pi}{6}$ rad (iv) $\dfrac{3\pi}{4}$ rad

Solutions

(a) (i) $30° = 30 \cdot \dfrac{\pi}{180}$ rad $= \dfrac{\pi}{6}$ rad (ii) $45° = 45 \cdot \dfrac{\pi}{180}$ rad $= \dfrac{\pi}{4}$ rad

(iii) $90° = 90 \cdot \dfrac{\pi}{180}$ rad $= \dfrac{\pi}{2}$ rad (iv) $210° = 210 \cdot \dfrac{\pi}{180}$ rad $= \dfrac{7\pi}{6}$ rad

(b) (i) $\dfrac{5\pi}{12}$ rad $= \dfrac{5\pi}{12} \cdot \dfrac{180°}{\pi} = 75°$ (ii) $\dfrac{3\pi}{2}$ rad $= \dfrac{3\pi}{2} \cdot \dfrac{180°}{\pi} = 270°$

(iii) $\dfrac{5\pi}{6}$ rad $= \dfrac{5\pi}{6} \cdot \dfrac{180°}{\pi} = 150°$ (iv) $\dfrac{3\pi}{4}$ rad $= \dfrac{3\pi}{4} \cdot \dfrac{180°}{\pi} = 135°$

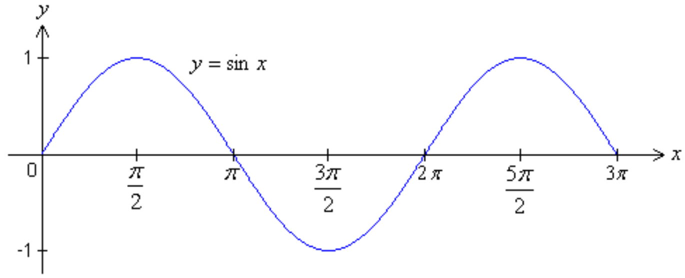
A Graph of the Trigonometric Functions

The graphs of $f(x) = \sin x$, $f(x) = \cos x$ and $f(x) = \tan x$ are drawn.

(a) $f(x) = \sin x$

Its domain is **R** and its range is [–1, 1]. Its period is 2π.

Since $f(-x) = \sin(-x) = -\sin x$, therefore, $f(x) = \sin x$ is odd.

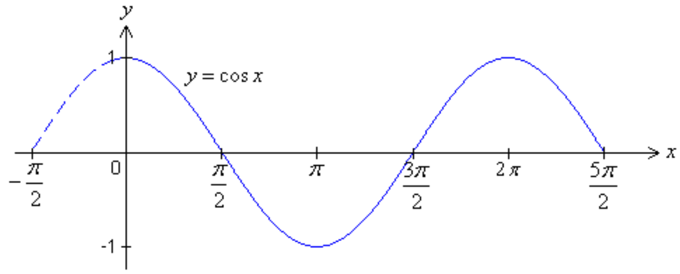


$y = \sin x$

(b) $f(x) = \cos x$

Its domain is **R** and its range is [–1, 1]. Its period is 2π.

Since $f(-x) = \cos(-x) = \cos x$, therefore, $f(x) = \cos x$ is even.

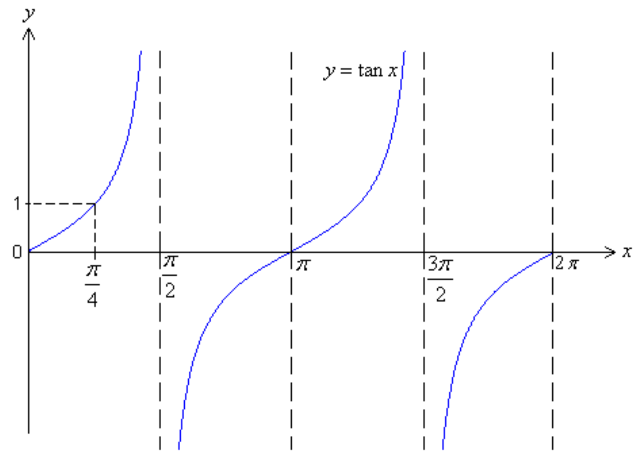


$y = \cos x$

(c) $f(x) = \tan x$

Its range is **R**. Its period is π. Question: What is the domain of $f(x) = \tan x$?

Since $f(-x) = \tan(-x) = -\tan x$, therefore, $f(x) = \tan x$ is odd.



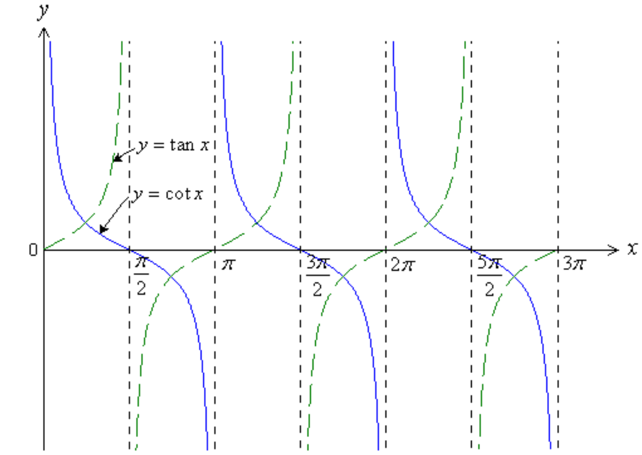
$y = \tan x$

Questions: For each of the functions below, (a) plot its graph; (b) find its domain and range; (c) find its period.

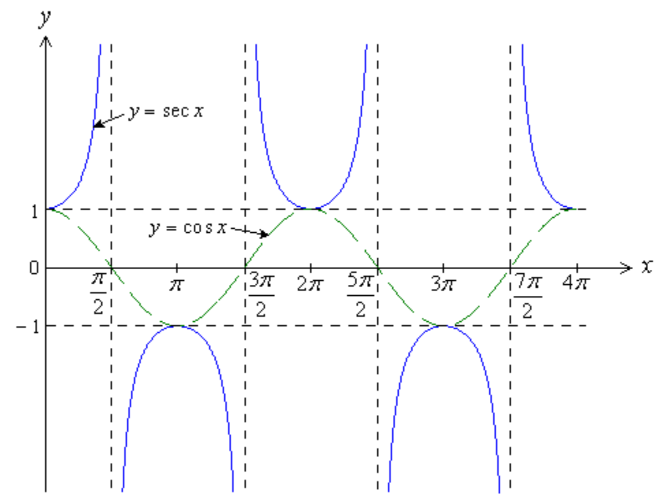
(i) $f(x) = \sin 2x$ (ii) $f(x) = \sin \dfrac{x}{3}$ (iii) $f(x) = \cos 3x$ (iv) $f(x) = \cos \dfrac{x}{5}$

(v) $f(x) = \tan \dfrac{x}{2}$ (vi) $f(x) = 2\sin\left(x - \dfrac{\pi}{4}\right) + 1$ (vii) $f(x) = \cos\left(2x - \dfrac{\pi}{2}\right)$

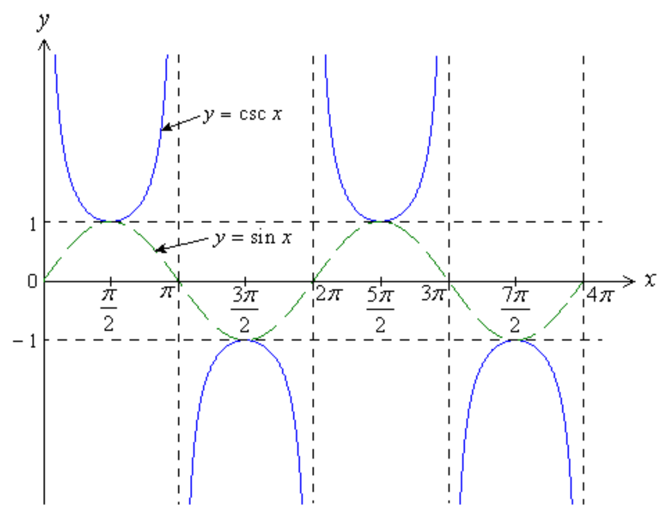
The graphs of $y = \cot x$, $y = \sec x$ and $y = \csc x$ are also drawn.



$y = \cot x$



$y = \sec x$



$y = \csc x$

B Elementary formulae

I. Review

$\sin^2\theta + \cos^2\theta = 1$	$1 + \tan^2\theta = \sec^2\theta$	$1 + \cot^2\theta = \csc^2\theta$		
$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$	$\cot\theta = \dfrac{\cos\theta}{\sin\theta}$	$\csc\theta = \dfrac{1}{\sin\theta}$	$\sec\theta = \dfrac{1}{\cos\theta}$	$\cot\theta = \dfrac{1}{\tan\theta}$
$\sin(-\theta) = -\sin\theta$	$\cos(-\theta) = \cos\theta$	$\tan(-\theta) = -\tan\theta$	$\pi \text{ rad} = 180°$	

Example 2 Prove the following identities.

(a) $\sin x\,(\tan x + \cot x) = \sec x$

(b) $\dfrac{\tan x}{1+\tan^2 x} = \sin x \cos x$

Solutions

(a) $\sin x\,(\tan x + \cot x) = \sin x\left(\dfrac{\sin x}{\cos x} + \dfrac{\cos x}{\sin x}\right) = \sin x\left(\dfrac{\sin^2 x + \cos^2 x}{\sin x \cos x}\right) = \dfrac{1}{\cos x} = \boxed{\sec x}$.

(b) $\dfrac{\tan x}{1+\tan^2 x} = \dfrac{\frac{\sin x}{\cos x}}{\sec^2 x} = \dfrac{\sin x}{\cos x}\cdot\cos^2 x = \boxed{\sin x \cos x}$.

II. Trigonometric Functions of (90°n ± θ) or ($\frac{n\pi}{2}$ ± θ)

To remember the formulae, always remember "Odd changed Even unchanged, sign refers to CAST."

Trigonometric Functions of (90° ± θ) or ($\frac{\pi}{2}$ ± θ)

	$90° - \theta$ (or $\frac{\pi}{2} - \theta$)	$90° + \theta$ (or $\frac{\pi}{2} + \theta$)
sin	$\cos\theta$	$\cos\theta$
cos	$\sin\theta$	$-\sin\theta$
tan	$\cot\theta$	$-\cot\theta$

Question: How about csc(90° ± θ), sec(90° ± θ) and cot(90° ± θ)?

Trigonometric Functions of (180° ± θ) or (π ± θ)

	$180° - \theta$ (or $\pi - \theta$)	$180° + \theta$ (or $\pi + \theta$)
sin	$\sin\theta$	$-\sin\theta$
cos	$-\cos\theta$	$-\cos\theta$
tan	$-\tan\theta$	$\tan\theta$

Question: How about csc(180° ± θ), sec(180° ± θ) and cot(180° ± θ)?

Trigonometric Functions of (360° ± θ) or (2π ± θ)

	$360° - \theta$ (or $2\pi - \theta$)	$360° + \theta$ (or $2\pi + \theta$)
sin	$-\sin\theta$	$\sin\theta$
cos	$\cos\theta$	$\cos\theta$
tan	$-\tan\theta$	$\tan\theta$

Question: How about csc($360° \pm \theta$), sec($360° \pm \theta$) and cot($360° \pm \theta$)?

Important: You can remember easily whether a –ve sign should be resulted by realizing whether the value obtained from the function should be –ve in the corresponding quadrant.

Example 3 Evaluate the following.

(a) $\sin(-30°)$ (b) $\tan(-135°)$ (c) $\cos\left(\frac{5}{4}\pi\right)$ (d) $\sec\left(-\frac{\pi}{3}\right)$

Solutions

(a) $\sin(-30°) = -\sin 30° = -\frac{1}{2}$

(b) $\tan(-135°) = -\tan 135° = -\tan(180° - 45°) = -(-\tan 45°) = 1$

(c) $\cos\left(\frac{5}{4}\pi\right) = \cos\left(\pi + \frac{\pi}{4}\right) = -\cos\left(\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

(d) $\sec\left(-\frac{\pi}{3}\right) = \dfrac{1}{\cos\left(-\frac{\pi}{3}\right)} = \dfrac{1}{\cos\left(\frac{\pi}{3}\right)} = 2$

Example 4 Simplify the following expressions.

(a) $1 + \tan^2(270° - \theta)$ (b) $\dfrac{\cos(360° - A)\sin(90° - A)\tan(A - 180°)}{\sin(-A)\sin(180° + A)\cot(-A)}$

(c) $\dfrac{\cos\left(A - \frac{3\pi}{2}\right)\cot(A - \pi)}{\tan\left(A + \frac{\pi}{2}\right)}$

Solutions

(a) $1 + \tan^2(270° - \theta) = 1 + [\tan(270° - \theta)]^2 = 1 + [\cot\theta]^2 = 1 + \cot^2\theta = \csc^2\theta$

(b) $\dfrac{\cos(360° - A)\sin(90° - A)\tan(A - 180°)}{\sin(-A)\sin(180° + A)\cot(-A)} = \dfrac{\cos A \cos A \tan[-(180° - A)]}{(-\sin A)(-\sin A)(-\cot A)}$

$= \dfrac{-\cos^2 A \tan(180° - A)}{-\sin^2 A} \cdot \tan A = \dfrac{-\cos^2 A \tan A}{\sin^2 A} \cdot \tan A = -1$

(c) $\dfrac{\cos\left(A - \frac{3\pi}{2}\right)\cot(A - \pi)}{\tan\left(A + \frac{\pi}{2}\right)} = \dfrac{\cos\left(\frac{3\pi}{2} - A\right)[-\cot(\pi - A)]}{-\cot A} = \dfrac{(-\sin A)(\cot A)}{-\cot A} = \sin A$

2 Inverse Trigonometric Functions

The *inverse sine* (or *arcsine*) *function*, denoted by $\sin^{-1}$ (or arcsin), is defined by

$$y = \sin^{-1} x \quad \text{if and only if} \quad x = \sin y$$

for $-1 \le x \le 1$ and $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$.

The *inverse cosine* (or *arccosine*) *function*, denoted by $\cos^{-1}$ (or arccos), is defined by

$$y = \cos^{-1} x \quad \text{if and only if} \quad x = \cos y$$

for $-1 \le x \le 1$ and $0 \le y \le \pi$.

The *inverse tangent* (or *arctangent*) *function*, denoted by $\tan^{-1}$ (or arctan), is defined by

$$y = \tan^{-1} x \quad \text{if and only if} \quad x = \tan y$$

for every real number x and for $-\frac{\pi}{2} < y < \frac{\pi}{2}$.

Question: What is the domain and range for $y = \sin^{-1} x$, $y = \cos^{-1} x$ and $y = \tan^{-1} x$?

Remarks:

1. Note that $\sin^{-1} x \ne \frac{1}{\sin x}$ (similarly for $\cos^{-1} x$ and $\tan^{-1} x$.)
2. The ranges of the inverse trigonometric functions are known as the *principal ranges*.

Question: Find the values of each of the following.

(a) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$ (b) $\sin^{-1}\left(\sin\left(30^\circ\right)\right)$ (c) $\sin^{-1}\left(\sin\left(390^\circ\right)\right)$

(d) $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ (e) $\cos^{-1}\left(\cos\left(-45^\circ\right)\right)$ (f) $\sin^{-1}\left(\cos\left(390^\circ\right)\right)$

(g) $\tan^{-1}\left(-\sqrt{3}\right)$ (h) $\tan^{-1}\left(\tan\left(225^\circ\right)\right)$

3 Trigonometric Identities

A. Formulae for sin($A \pm B$), cos($A \pm B$) and tan($A \pm B$)

Please note that $\sin(A \pm B) \neq \sin A \pm \sin B$ and $\cos(A \pm B) \neq \cos A \pm \cos B$.

e.g. $\sin(30° + 60°) = \sin 90° = 1$

but $\sin 30° + \sin 60° = \frac{1}{2} + \frac{\sqrt{3}}{2}$

$\neq 1$

In general, we have the following **Compound Angle Formulae**.

$$\sin(A - B) = \sin A \cos B - \cos A \sin B$$
$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$
$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$
$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

Example 5 Simplify the following expressions.

(a) $\cos(x + y)\cos y + \sin(x + y)\sin y$ (b) $\frac{\cos^2 A - \sin^2 A}{\cos 3A \cos A + \sin 3A \sin A}$

Solutions

(a) $\cos(x + y)\cos y + \sin(x + y)\sin y = \cos[(x + y) - y] = \cos x$

(b) $$\frac{\cos^2 A - \sin^2 A}{\cos 3A \cos A + \sin 3A \sin A} = \frac{\cos A \cos A - \sin A \sin A}{\cos 3A \cos A + \sin 3A \sin A}$$
$$= \frac{\cos(A + A)}{\cos(3A - A)}$$
$$= \frac{\cos 2A}{\cos 2A}$$
$$= 1$$

Example 6 Prove that $\sin(30° + x) + \cos(60° + x) - \cos x = 0$

Solution

L.H.S. $= \sin(30° + x) + \cos(60° + x) - \cos x$

$= [\sin 30° \cos x + \cos 30° \sin x] + [\cos 60° \cos x - \sin 60° \sin x] - \cos x$

$= \frac{1}{2}\cos x + \frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x - \frac{\sqrt{3}}{2}\sin x - \cos x$

$= 0$

= R.H.S.

$\therefore$ $\sin(30° + x) + \cos(60° + x) - \cos x \equiv 0$

Example 7 Find the value of the following in surd form.

(a) $\sin 15°$ (b) $\cos\frac{5\pi}{12}$

Solutions

(a) $\sin 15° = \sin(45° - 30°)$

$= \sin 45° \cos 30° - \cos 45° \sin 30°$

$= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) = \frac{\sqrt{6}-\sqrt{2}}{4}$

(b) Note that $\frac{5\pi}{12} = \frac{\pi}{4} + \frac{\pi}{6}$. Therefore,

$$\cos\frac{5\pi}{12} = \cos\left(\frac{\pi}{4}+\frac{\pi}{6}\right) = \cos\frac{\pi}{4}\cos\frac{\pi}{6} - \sin\frac{\pi}{4}\sin\frac{\pi}{6}$$

$$= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) = \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \frac{\sqrt{6}-\sqrt{2}}{4}$$

B. Double and half angle formulae

Double angle formulae

Note that $\sin 2A = \sin(A + A)$

$= \sin A \cos A + \cos A \sin A$

$= 2\sin A \cos A$

and $\cos 2A = \cos(A + A)$

$= \cos A \cos A - \sin A \sin A$

$= \cos^2 A - \sin^2 A$

So, we have

$$\boxed{\begin{aligned} \sin 2A &= 2\sin A\cos A \\ \cos 2A &= \cos^2 A - \sin^2 A \end{aligned}}$$

Half angle formulae

Note that

$$\cos A = \cos^2\frac{A}{2} - \sin^2\frac{A}{2} = \cos^2\frac{A}{2} - \left(1-\cos^2\frac{A}{2}\right) = 2\cos^2\frac{A}{2} - 1$$

$$\cos A = \cos^2\frac{A}{2} - \sin^2\frac{A}{2} = \left(1-\sin^2\frac{A}{2}\right) - \sin^2\frac{A}{2} = 1 - 2\sin^2\frac{A}{2}$$

So, we have

$$\boxed{\begin{aligned} \cos^2\frac{A}{2} &= \frac{1}{2}(1+\cos A) \\ \sin^2\frac{A}{2} &= \frac{1}{2}(1-\cos A) \end{aligned}}$$

Example 8 Prove the following identities**.**

(a) $2\sin^2\frac{x}{2}\cdot\tan x=\tan x-\sin x$ (b) $\sin^2\theta\cos^2\theta=\frac{1}{8}-\frac{1}{8}\cos 4\theta$

Solutions

(a) L.H.S. $= 2\sin^2\frac{x}{2}\cdot\tan x = (1-\cos x)\cdot\tan x$

$= \tan x-\cos x\cdot\frac{\sin x}{\cos x} = \tan x-\sin x$ = R.H.S.

$\therefore \quad 2\sin^2\frac{x}{2}\cdot\tan x\equiv\tan x-\sin x$

(b) L.H.S. $= \sin^2\theta\cos^2\theta = (\sin\theta\cos\theta)^2$

$= \left(\frac{1}{2}\sin 2\theta\right)^2 = \frac{1}{4}\sin^2 2\theta$

$= \frac{1}{4}\cdot\frac{1}{2}(1-\cos 4\theta) = \frac{1}{8}-\frac{1}{8}\cos 4\theta$ = R.H.S.

$\therefore \quad \sin^2\theta\cos^2\theta\equiv\frac{1}{8}-\frac{1}{8}\cos 4\theta$

C. Sum-to-product and product-to-sum formulae

Using the formulae for $\sin(A\pm B)$ and $\cos(A\pm B)$, we have the following.

$$\sin A\cos B=\frac{1}{2}[\sin(A+B)+\sin(A-B)]$$
$$\cos A\sin B=\frac{1}{2}[\sin(A+B)-\sin(A-B)]$$
$$\cos A\cos B=\frac{1}{2}[\cos(A+B)+\cos(A-B)]$$
$$\sin A\sin B=-\frac{1}{2}[\cos(A+B)-\cos(A-B)]$$

These are called the **Product-to-Sum Formulae**.

$$\sin x+\sin y=2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$
$$\sin x-\sin y=2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$
$$\cos x+\cos y=2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$
$$\cos x-\cos y=-2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

These are called the **Sum-to-Product Formulae**.

Example 9 Prove the following identities.

(a) $\dfrac{\sin 4\theta + \sin 2\theta}{\cos 4\theta + \cos 2\theta} = \tan 3\theta$ (b) $\dfrac{\sin(x+y)+\sin(x-y)}{\sin(x+y)-\sin(x-y)} = \tan x \cot y$

Solutions

(a) L.H.S. $= \dfrac{\sin 4\theta + \sin 2\theta}{\cos 4\theta + \cos 2\theta}$

$$= \frac{2\sin\dfrac{4\theta+2\theta}{2}\cos\dfrac{4\theta-2\theta}{2}}{2\cos\dfrac{4\theta+2\theta}{2}\cos\dfrac{4\theta-2\theta}{2}} = \frac{2\sin 3\theta\cos\theta}{2\cos 3\theta\cos\theta}$$

$= \tan 3\theta$

= R.H.S.

$\therefore \quad \dfrac{\sin 4\theta + \sin 2\theta}{\cos 4\theta + \cos 2\theta} \equiv \tan 3\theta$

(b) L.H.S. $= \dfrac{\sin(x+y)+\sin(x-y)}{\sin(x+y)-\sin(x-y)}$

$$= \frac{2\sin\left[\dfrac{(x+y)+(x-y)}{2}\right]\cos\left[\dfrac{(x+y)-(x-y)}{2}\right]}{2\cos\left[\dfrac{(x+y)+(x-y)}{2}\right]\sin\left[\dfrac{(x+y)-(x-y)}{2}\right]}$$

$= \dfrac{2\sin x\cos y}{2\cos x\sin y}$

$= \tan x\cot y$

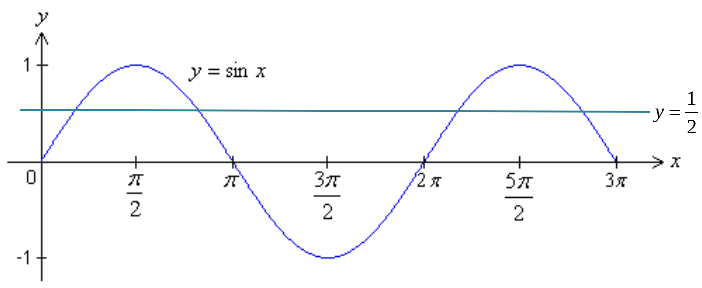
= R.H.S.

$\therefore \quad \dfrac{\sin(x+y)+\sin(x-y)}{\sin(x+y)-\sin(x-y)} \equiv \tan x\cot y$

4 Trigonometric Equations and Their Solutions

A trigonometric equation is an equation that contains a trigonometric expression with a variable, such as $\cos x$. The values that satisfy such an equation are its solutions.

Consider the trigonometric equation $\sin x = \dfrac{1}{2}$.



It can be observed that the solutions of $\sin x = \frac{1}{2}$ in $[0, 2\pi)$ are

$$x = \frac{\pi}{6} \qquad \text{and} \qquad x = \pi - \frac{\pi}{6} = \frac{5\pi}{6}.$$

Since the graph of $y = \sin x$ is periodic of 2π, any multiple of 2π added to the two values above are still the solutions of the equation. We can express the solutions for the equation $\sin x = \frac{1}{2}$ as

$$x = n\pi + (-1)^n \frac{\pi}{6}. \quad (n \text{ is any integer})$$

This is the *general solution* for the equation.

The following lists the general solutions for the given trigonometric equations.

Trigonometric Equation	General Solution
$\sin\theta = k$, where $-1 \le k \le 1$	$\theta = n\pi + (-1)^n \alpha$, where n is any integer, $\alpha = \sin^{-1} k$ and $-\frac{\pi}{2} \le \alpha \le \frac{\pi}{2}$.
$\cos\theta = k$, where $-1 \le k \le 1$	$\theta = 2n\pi \pm \alpha$, where n is any integer, $\alpha = \cos^{-1} k$ and $0 \le \alpha \le \pi$.
$\tan\theta = k$	$\theta = n\pi + \alpha$, where n is any integer, $\alpha = \tan^{-1} k$ and $-\frac{\pi}{2} < \alpha < \frac{\pi}{2}$.

<u>Example 10</u>

Solve each of the following equations.

(a) $\sin\frac{x}{3} = \frac{\sqrt{3}}{2}$ (b) $2\cos^2 x + \cos x - 1 = 0$

(c) $\cos 2x + 3\sin x - 2 = 0$, $0 \le x < 2\pi$.

Solutions

(a) Notice that $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$, so that

$$\frac{x}{3} = n\pi + (-1)^n \frac{\pi}{3} \qquad \therefore \quad x = 3n\pi + (-1)^n \pi, \text{ where } n \text{ is any integer.}$$

(b)

$$\begin{aligned} 2\cos^2 x + \cos x - 1 &= 0 \\ (2\cos x - 1)(\cos x + 1) &= 0 \\ 2\cos x - 1 &= 0 \quad \text{or} \quad \cos x + 1 = 0 \\ \cos x &= \frac{1}{2} \quad \text{or} \quad \cos x = -1 \\ x &= 2n\pi \pm \frac{\pi}{3} \quad \text{or} \quad x = 2n\pi \pm \pi \end{aligned}$$

$\therefore$ The solution is $x = 2n\pi \pm \frac{\pi}{3}$ or $x = (2n+1)\pi$, where n is any integer.

(c) Consider $\cos 2x + 3\sin x - 2 = 0$.

$$
\begin{aligned}
1 - 2\sin^2 x + 3\sin x - 2 &= 0 \\
2\sin^2 x - 3\sin x + 1 &= 0 \\
(2\sin x - 1)(\sin x - 1) &= 0
\end{aligned}
$$

$$2\sin x - 1 = 0 \quad \text{or} \quad \sin x - 1 = 0$$

$$\sin x = \frac{1}{2} \quad \text{or} \quad \sin x = 1$$

$$x = n\pi + (-1)^n \frac{\pi}{6} \quad \text{or} \quad x = n\pi + (-1)^n \frac{\pi}{2}$$

Now, as $0 \le x < 2\pi$, the solution is

$$x = \frac{\pi}{6}, \quad x = \pi - \frac{\pi}{6} = \frac{5\pi}{6}, \quad x = \frac{\pi}{2}$$