IMPORTANT: The answers are provided with the view to deepen your understanding of the topic and to stimulate more in depth discussions. They should not be shared with others or put on the web. The questions and answers may be revised in the next course delivery.

Answers to Tutorial 5

Qn 1

$$\binom{7}{7}(0.75)^7 + \binom{7}{6}(0.75)^6(0.25) + \binom{7}{5}(0.75)^5(0.25)^2 = 0.756408691$$

Qn 2

A word is either typed correctly or mistyped. Assume that the number of typed words is large and the probability of mistyped words is low. The assumptions fit the requirement of Poisson distribution.

Repeat the typing tests a few times. Set $\lambda =$ average no. of mistyped words.

Qn 3

Assume it is a Poisson distribution.

$$P\{X \geq 1\} = 1 - P\{X = 0\} = 1 - e^{-3}\frac{3^0}{0!} = 1 - e^{-3} \approx 0.9502$$

Qn 4

a) Let X denote the time in minutes past 7 a.m. that the passenger arrives at the stop. Since X is a uniform random variable over the interval (0, 30), the passage has to wait less than 5 minutes if he arrives between 7:10 and 7:15 or between 7:25 and 7:30.

$$P\{10 < X < 15\} + P\{25 < X < 30\} = \frac{5}{30} + \frac{5}{30} = \frac{1}{3}$$

b) Similarity, he has to wait for at least 12 minutes if he arrives between 7:00 and 7:03 or between 7:15 and 7:18.

$$P\{0 < X < 3\} + P\{15 < X < 18\} = \frac{3}{30} + \frac{3}{30} = \frac{1}{5}$$

Qn 5

a) This situation can be modelled by the binomial random variable.

$$P\{X = 5\} = \binom{20}{5}(0.1)^5(0.9)^{15} = 3.1921361\%$$

b) This situation can be modelled by the negative binomial random variable.

$$P\{X = 5\} = \binom{19}{4}(0.1)^5(0.9)^{15} = 0.798034028\ \%$$

c) Let X_i be the random variable denoting the number of tournaments played before winning the i^{th} tournament after $(i-1)$ tournaments have been won.

$$E[X_1 + \cdots + X_r] = E[X_1] + \cdots + E[X_r]$$

Since

$$E[X_i] = \frac{1}{p}$$

$$E[X] = \frac{r}{p} = \frac{5}{0.1} = 50$$

The player is expected to play 50 tournaments.

Qn 6

The hypergeometric random variable is used.

Let E be the event that the buyer accepts the lot.

$$P(E) = \frac{\binom{1}{0}\binom{99}{3}}{\binom{100}{3}} = 0.97$$

So he would reject 3% of the lots he inspected.