EE2302 Foundations of Information and Data Engineering

Assignment 8 (Solution)

1. Consider two arbitrary matrices in the subset, $A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$ and $B = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}$, where $a_{12} = -a_{21}$ and $b_{12} = -b_{21}$.

   First, addition is closed, since $A + B = \begin{pmatrix} a_{11} + b_{11} & a_{12} + b_{12} \\ a_{21} + b_{21} & a_{22} + b_{22} \end{pmatrix}$ and $a_{12} + b_{12} = -(a_{21} + b_{21})$.
   Second, scalar multiplication is closed, since $cA = \begin{pmatrix} ca_{11} & ca_{12} \\ ca_{21} & ca_{22} \end{pmatrix}$ and $ca_{12} = -ca_{21}$.
   Hence, the subset is a subspace of all 2×2 real matrices.

2.
   a) No. Choose $x = (1,0)$, $y = (0,1)$, and $\alpha = \beta = 1$. Then, $f(\alpha x + \beta y) = 1$ but $\alpha f(x) + \beta f(y) = 2$, which shows that superposition fails.

   b) Yes. a is the vector whose first component is –1, the last component is 1, and all other components are 0, i.e., $a = (-1, 0, 0, \dots, 0, 1)$.

3.
   a) A vector (a, b, c) reflecting through the x-y plane becomes the vector $(a, b, -c)$. That means, the x-coordinate and the y-coordinate remain unchanged while the z-coordinate is multiplied by -1. The corresponding matrix is given by

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}.$$

   It can be checked that

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} a \\ b \\ -c \end{bmatrix}.$$

   b) Since only the x- and y-coordinates are rotated while the z-axis remain unchanged, the matrix must be of the form

$$\begin{bmatrix} ? & ? & 0 \\ ? & ? & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

where the 2×2 submatrix in the upper left corner is the rotation matrix with angle $= 90^o$. By the formula for the rotation matrix (given in the lecture notes), we obtain the transformation matrix as follows:

$$\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

4.

a) Given $a = (2, 1)$, we can obtain its inner product as 5. The projection matrix is given by

$$P = \frac{aa^T}{a^T a} = \frac{1}{5}\begin{bmatrix} 4 & 2 \\ 2 & 1 \end{bmatrix}.$$

b) The result is given by

$$Pb = \frac{1}{5}\begin{bmatrix} 4 & 2 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 5 \\ 5 \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \end{bmatrix}.$$

5.

a) It means that if the price of product 3 is increased, then the total profit will decrease.

b) You should choose product i, where $|\beta_i|$ is the largest. If $\beta_i > 0$, increase the price of product i by 1%. Otherwise, decrease the price by 1%.

c) You should choose products i and j, where $|\beta_i|$ and $|\beta_j|$ are the two largest. For each of the product, increase the price by 1% if the corresponding β-value is positive, or decrease the price by 1% otherwise.