Why we need differentiation?

We consider the following problem:

In 1931, the HK population is about 864,117

In 1999, the HK population is about 6,840,000

Question:

What is the population growth per year?

Simple Answer:

$$The\ population\ growth = \frac{\overbrace{6{,}840{,}000 - 864{,}117}^{change\ in\ population}}{\underbrace{1999 - 1931}_{duration\ (time\ horizon)}} = 87{,}881/year$$

Is it a correct estimation?

The number given above is just the *average population growth*.

In fact, the population growth is different in different time due to various conditions (economic conditions, wars, culture, personality etc.)

Year	Population
1931	864,117
1941	1,600,000
1945	750,000
1947	1,750,000
1951	2,013,000
1961	3,133,131
1971	3,950,000
1981	4,986,560
1991	5,647,114

In order to have a better estimation on the population growth at certain time, one needs to shorten the time horizon (say 10 years, 5 years, 1 year, 3 month …. so that the occasion is small.   Then we can obtain a better estimation on the population growth

$$\underbrace{\frac{P(t+10)-P(t)}{10}}_{less\ accurate} \Rightarrow \frac{P(t+5)-P(t)}{5} \Rightarrow \frac{P(t+1)-P(t)}{1} \Rightarrow \underbrace{\frac{P\left(t+\frac{1}{4}\right)-P(t)}{\frac{1}{4}}}_{more\ accurate}$$

(*Note: $P(t)$ is the population at year t.)
In the limiting case when time horizon Δt is close to 0, the corresponding estimation should be the most accurate one since every condition between t and $t+\Delta t$ is essentially identical and the population growth should be uniform (grows at same "speed", i.e.

$$\lim_{\Delta t \to 0} \frac{P(t+\Delta t)-P(t)}{\Delta t}$$

This number is commonly used to describe the "rate of change" of population.