MA1201 Session: A/B/C D Semester B (16/17)

1. (a) $\vec{a} = \vec{i} + 2\vec{j} + 3\vec{k}$, $\vec{b} = 2\vec{i} - 3\vec{k}$, $\vec{c} = -\vec{j} + 5\vec{k}$

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$$\vec{a}\cdot\vec{b}\times\vec{c} \underset{(2)}{=} \begin{vmatrix} 1 & 2 & 3 \\ 2 & 0 & -3 \\ 0 & -1 & 5 \end{vmatrix} \overset{R_1}{=} 1\begin{vmatrix} 0 & -3 \\ -1 & 5 \end{vmatrix} - 2\begin{vmatrix} 2 & -3 \\ 0 & 5 \end{vmatrix} + 3\begin{vmatrix} 2 & 0 \\ 0 & -1 \end{vmatrix}$$

$$= (0-3) - 2(10-0) + 3(-2-0)$$

$$= -3 - 20 - 6 = -29 \quad (6)$$

$$\text{Volume}(\vec{a},\vec{b},\vec{c}) = |\vec{a}\cdot\vec{b}\times\vec{c}| = |-29| = 29 \quad (2)$$

(b) $A(1,2,3)$, $B(-2,1,4)$, $C(3,-2,1)$

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$$\vec{AB} = \vec{OB} - \vec{OA} = (-2\vec{i} + \vec{j} + 4\vec{k}) - (\vec{i} + 2\vec{j} + 3\vec{k}) = -3\vec{i} - \vec{j} + \vec{k} \quad (2)$$

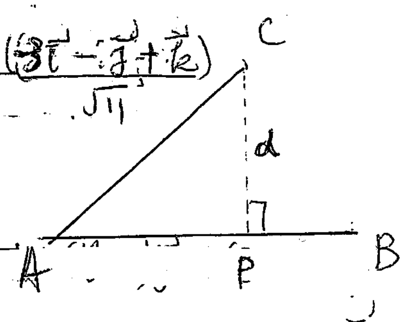
$$\vec{AC} = \vec{OC} - \vec{OA} = (3\vec{i} - 2\vec{j} + \vec{k}) - (\vec{i} + 2\vec{j} + 3\vec{k}) = 2\vec{i} - 4\vec{j} - 2\vec{k} \quad (2)$$

$$|\vec{AB}| = \sqrt{(-3)^2 + (-1)^2 + 1^2} = \sqrt{9+1+1} = \sqrt{11} \quad (1)$$

$$\vec{AP} = \operatorname{proj}_{\vec{AB}}\vec{AC} = \left(\vec{AC}\cdot\frac{\vec{AB}}{|\vec{AB}|}\right)\frac{\vec{AB}}{|\vec{AB}|} = \frac{(2)(-3) + (-4)(-1) + (-2)(1)}{\sqrt{11}}\;\frac{(-3\vec{i} - \vec{j} + \vec{k})}{\sqrt{11}}$$

$$= \frac{-6+4-2}{\sqrt{11}}\;\frac{-3\vec{i} - \vec{j} + \vec{k}}{\sqrt{11}} = \frac{-4}{11}(-3\vec{i} - \vec{j} + \vec{k})$$

$$= \frac{12}{11}\vec{i} + \frac{4}{11}\vec{j} - \frac{4}{11}\vec{k} \quad (3)$$



$$|\vec{AC}| = \sqrt{2^2 + (-4)^2 + (-2)^2} = \sqrt{4+16+4} = \sqrt{24} = 2\sqrt{6} \quad (1)$$

$$|\vec{AP}| = |\operatorname{proj}_{\vec{AB}}\vec{AB}| = \left|\vec{AC}\cdot\frac{\vec{AB}}{|\vec{AB}|}\right| = \left|\frac{-4}{\sqrt{11}}\right| = \frac{4}{\sqrt{11}} \quad (2)$$

$$d = \sqrt{|\vec{AC}|^2 - |\vec{AP}|^2} = \sqrt{24 - \left(\frac{4}{\sqrt{11}}\right)^2} = \sqrt{24 - \frac{16}{11}} = \sqrt{\frac{248}{11}} = 2\sqrt{\frac{62}{11}} \quad (2)$$

$$\vec{OP} = \vec{OA} + \vec{AP} = (\vec{i} + 2\vec{j} + 3\vec{k}) + \left(\frac{12}{11}\vec{i} + \frac{4}{11}\vec{j} - \frac{4}{11}\vec{k}\right) = \frac{23}{11}\vec{i} + \frac{26}{11}\vec{j} + \frac{29}{11}\vec{k} \;\therefore P = \left(\frac{23}{11}, \frac{26}{11}, \frac{29}{11}\right) \quad (2)$$

2 (a)

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$$\int \tan x\,dx = \int \frac{\sin x}{\cos x}\,dx \underset{(2)}{=} -\int \frac{d(\cos x)}{\cos x} \underset{(3)}{=} -\ln|\cos x|_{(4)} + C_{(1)}$$

(b)

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$$\int e^{-3x+1}\,dx = -\frac{1}{3}e^{-3x+1}_{(5)} + C_{(1)}$$

(c)

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$$\int_{-1}^{2} |x|\,dx = \int_{-1}^{0}(-x)\,dx + \int_{0}^{2} x\,dx \underset{(4)}{=} -\frac{x^2}{2}\Big|_{-1}^{0} + \frac{x^2}{2}\Big|_{0}^{2} \quad (4)$$

$$= -\frac{1}{2}[0^2 - (-1)^2] + \frac{1}{2}[2^2 - 0^2]$$

$$= -\frac{1}{2}(-1) + \frac{1}{2}(4) = \frac{1}{2} + 2 = \frac{5}{2} \quad (6)$$

3 (a) substitution $x = 2\sin\theta$ (2), $dx = 2\cos\theta \, d\theta$ (2)

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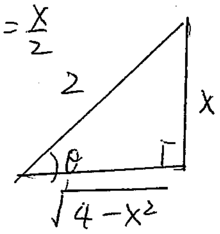
$$(4-x^2)^2 = (4 - 4\sin^2\theta)^2 = [4(1-\sin^2\theta)]^2 = 16\cos^4\theta \quad (2)$$

$$\int \frac{1}{(4-x^2)^2}dx = \int \frac{2\cos\theta}{16\cos^4\theta}d\theta = \frac{1}{8}\int \frac{1}{\cos^3\theta}d\theta \underset{(2)}{=} \frac{1}{8}\int \sec^3\theta \, d\theta$$

$$= \frac{1}{8}\left\{\frac{1}{2}\sec\theta\tan\theta + \frac{1}{2}\ln|\sec\theta + \tan\theta|\right\}_{(2)} + C \quad (1)$$

$$= \frac{1}{16}\sec\theta\tan\theta + \frac{1}{16}\ln|\sec\theta + \tan\theta| + C$$

$\sin\theta = \frac{x}{2}$



$$= \frac{1}{16}\frac{2}{\sqrt{4-x^2}}\frac{x}{\sqrt{4-x^2}} + \frac{1}{16}\ln\left|\frac{2}{\sqrt{4-x^2}} + \frac{x}{\sqrt{4-x^2}}\right| + C$$

$$= \frac{1}{8}\frac{x}{4-x^2} + \frac{1}{16}\ln\left|\frac{2+x}{\sqrt{4-x^2}}\right| + C \quad (4)$$

(b) $\int x^2\tan^{-1}x\,dx = \int \tan^{-1}x \, d\left(\frac{x^3}{3}\right)_{(2)} \overset{IP}{=} \frac{x^3}{3}\tan^{-1}x - \frac{1}{3}\int x^3 \, d(\tan^{-1}x)_{(2)}$

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$$= \frac{x^3}{3}\tan^{-1}x - \frac{1}{3}\int \frac{x^3}{1+x^2}dx \quad (2)$$

$$= \frac{x^3}{3}\tan^{-1}x - \frac{1}{3}\int \left[x - \frac{x}{1+x^2}\right]dx_{(2)} = \frac{x^3}{3}\tan^{-1}x - \frac{x^2}{6} + \frac{1}{6}\int \frac{d(1+x^2)}{1+x^2}$$

$$= \frac{x^3}{3}\tan^{-1}x - \frac{1}{6}x^2_{(1)} + \frac{1}{6}\ln|1+x^2|_{(2)} + C \quad (1)$$

(c) Resolve partial fractions

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$$\frac{-18}{(x+1)(x^2-4x+13)} = \frac{A}{x+1} + \frac{Bx+C}{x^2-4x+13} \quad (2)$$

$$\Rightarrow \quad -18 = A(x^2-4x+13) + (Bx+C)(x+1)$$

let $x=-1$: $-18 = A(1+4+13) + 0 = 18A \Rightarrow A = -1$ (2)

Compare coefficient x^2: $0 = A + B \Rightarrow B = -A = 1$ (2)

Compare constant term: $-18 = 13A + C \Rightarrow C = -18 - 13A = -18 + 13 = -5$ (2)

$$\int \frac{-18}{(x+1)(x^2-4x+13)}dx = \int \frac{-1}{x+1}dx + \int \frac{x-5}{x^2-4x+13}dx$$

$$= -\int \frac{d(x+1)}{x+1} + \int \frac{x-2}{x^2-4x+13}dx - 3\int \frac{1}{(x-2)^2+9}dx \quad (2)$$

$$= -\ln|x+1|_{(2)} + \frac{1}{2}\int \frac{d(x^2-4x+13)}{x^2-4x+13} - \frac{3}{9}\int \frac{1}{\left(\frac{x-2}{3}\right)^2+1}dx \quad (1)$$

$$= -\ln|x+1| + \frac{1}{2}\ln|x^2-4x+13|_{(2)} - \frac{1}{3}\frac{1}{\frac{1}{3}}\tan^{-1}\left(\frac{x-2}{3}\right) + C$$

$$= -\ln|x+1| + \frac{1}{2}\ln|x^2-4x+13| - \tan^{-1}\left(\frac{x-2}{3}\right)_{(2)} + C \quad (1)$$