

Transform pair	Signal	Transform	ROC
1	$\delta(t)$	1	All s
2	$u(t)$	$\frac{1}{s}$	$\Re\{s\} > 0$
3	$-u(-t)$	$\frac{1}{s}$	$\Re\{s\} < 0$
4	$\frac{t^{n-1}}{(n-1)!} u(t)$	$\frac{1}{s^n}$	$\Re\{s\} > 0$
5	$-\frac{t^{n-1}}{(n-1)!} u(-t)$	$\frac{1}{s^n}$	$\Re\{s\} < 0$
6	$e^{-\alpha t} u(t)$	$\frac{1}{s + \alpha}$	$\Re\{s\} > -\alpha$
7	$-e^{-\alpha t} u(-t)$	$\frac{1}{s + \alpha}$	$\Re\{s\} < -\alpha$
8	$\frac{t^{n-1}}{(n-1)!} e^{-\alpha t} u(t)$	$\frac{1}{(s + \alpha)^n}$	$\Re\{s\} > -\alpha$
9	$-\frac{t^{n-1}}{(n-1)!} e^{-\alpha t} u(-t)$	$\frac{1}{(s + \alpha)^n}$	$\Re\{s\} < -\alpha$
10	$\delta(t - T)$	e^{-sT}	All s
11	$[\cos \omega_0 t] u(t)$	$\frac{s}{s^2 + \omega_0^2}$	$\Re\{s\} > 0$
12	$[\sin \omega_0 t] u(t)$	$\frac{\omega_0}{s^2 + \omega_0^2}$	$\Re\{s\} > 0$
13	$[e^{-\alpha t} \cos \omega_0 t] u(t)$	$\frac{s + \alpha}{(s + \alpha)^2 + \omega_0^2}$	$\Re\{s\} > -\alpha$
14	$[e^{-\alpha t} \sin \omega_0 t] u(t)$	$\frac{\omega_0}{(s + \alpha)^2 + \omega_0^2}$	$\Re\{s\} > -\alpha$
15	$u_n(t) = \frac{d^n \delta(t)}{dt^n}$	s^n	All s
16	$u_{-n}(t) = \underbrace{u(t) * \cdots * u(t)}_{n \text{ times}}$	$\frac{1}{s^n}$	$\Re\{s\} > 0$

Laplace Transform Pairs

Section	Property	Signal	Laplace Transform	ROC
		$x(t)$ $x_1(t)$ $x_2(t)$	$X(s)$ $X_1(s)$ $X_2(s)$	R R_1 R_2
9.5.1	Linearity	$ax_1(t) + bx_2(t)$	$aX_1(s) + bX_2(s)$	At least $R_1 \cap R_2$
9.5.2	Time shifting	$x(t - t_0)$	$e^{-st_0} X(s)$	R
9.5.3	Shifting in the s-Domain	$e^{s_0 t} x(t)$	$X(s - s_0)$	Shifted version of R (i.e., s is in the ROC if $s - s_0$ is in R)
9.5.4	Time scaling	$x(at)$	$\frac{1}{ a } X\left(\frac{s}{a}\right)$	Scaled ROC (i.e., s is in the ROC if s/a is in R)
9.5.5	Conjugation	$x^*(t)$	$X^*(s^*)$	R
9.5.6	Convolution	$x_1(t) * x_2(t)$	$X_1(s)X_2(s)$	At least $R_1 \cap R_2$
9.5.7	Differentiation in the Time Domain	$\frac{d}{dt} x(t)$	$sX(s)$	At least R
9.5.8	Differentiation in the s-Domain	$-tx(t)$	$\frac{d}{ds} X(s)$	R
9.5.9	Integration in the Time Domain	$\int_{-\infty}^t x(\tau) d(\tau)$	$\frac{1}{s} X(s)$	At least $R \cap \{\operatorname{Re}\{s\} > 0\}$
Initial- and Final-Value Theorems				
9.5.10	If $x(t) = 0$ for $t < 0$ and $x(t)$ contains no impulses or higher-order singularities at $t = 0$, then			
	$x(0^+) = \lim_{s \rightarrow \infty} sX(s)$			
	If $x(t) = 0$ for $t < 0$ and $x(t)$ has a finite limit as $t \rightarrow \infty$, then			
	$\lim_{t \rightarrow \infty} x(t) = \lim_{s \rightarrow 0} sX(s)$			

TABLE 10.2 SOME COMMON z-TRANSFORM PAIRS

Signal	Transform	ROC
1. $\delta[n]$	1	All z
2. $u[n]$	$\frac{1}{1 - z^{-1}}$	$ z > 1$
3. $-u[-n - 1]$	$\frac{1}{1 - z^{-1}}$	$ z < 1$
4. $\delta[n - m]$	z^{-m}	All z , except 0 (if $m > 0$) or ∞ (if $m < 0$)
5. $\alpha^n u[n]$	$\frac{1}{1 - \alpha z^{-1}}$	$ z > \alpha $
6. $-\alpha^n u[-n - 1]$	$\frac{1}{1 - \alpha z^{-1}}$	$ z < \alpha $
7. $n\alpha^n u[n]$	$\frac{\alpha z^{-1}}{(1 - \alpha z^{-1})^2}$	$ z > \alpha $
8. $-n\alpha^n u[-n - 1]$	$\frac{\alpha z^{-1}}{(1 - \alpha z^{-1})^2}$	$ z < \alpha $
9. $[\cos \omega_0 n]u[n]$	$\frac{1 - [\cos \omega_0]z^{-1}}{1 - [2 \cos \omega_0]z^{-1} + z^{-2}}$	$ z > 1$
10. $[\sin \omega_0 n]u[n]$	$\frac{[\sin \omega_0]z^{-1}}{1 - [2 \cos \omega_0]z^{-1} + z^{-2}}$	$ z > 1$
11. $[r^n \cos \omega_0 n]u[n]$	$\frac{1 - [r \cos \omega_0]z^{-1}}{1 - [2r \cos \omega_0]z^{-1} + r^2 z^{-2}}$	$ z > r$
12. $[r^n \sin \omega_0 n]u[n]$	$\frac{[r \sin \omega_0]z^{-1}}{1 - [2r \cos \omega_0]z^{-1} + r^2 z^{-2}}$	$ z > r$

TABLE 10.1 PROPERTIES OF THE z-TRANSFORM

Section	Property	Signal	z-Transform	ROC
		$x[n]$ $x_1[n]$ $x_2[n]$	$X(z)$ $X_1(z)$ $X_2(z)$	R R_1 R_2
10.5.1	Linearity	$ax_1[n] + bx_2[n]$	$aX_1(z) + bX_2(z)$	At least the intersection of R_1 and R_2
10.5.2	Time shifting	$x[n - n_0]$	$z^{-n_0}X(z)$	R , except for the possible addition or deletion of the origin
10.5.3	Scaling in the z-domain	$e^{j\omega_0 n}x[n]$ $z_0^n x[n]$ $a^n x[n]$	$X(e^{-j\omega_0}z)$ $X\left(\frac{z}{z_0}\right)$ $X(a^{-1}z)$	R $z_0 R$ Scaled version of R (i.e., $ a R =$ the set of points $\{ a z\}$ for z in R)
10.5.4	Time reversal	$x[-n]$	$X(z^{-1})$	Inverted R (i.e., R^{-1} = the set of points z^{-1} , where z is in R)
10.5.5	Time expansion	$x_{(r)}[n] = \begin{cases} x[r], & n = rk \\ 0, & n \neq rk \end{cases}$ for some integer r	$X(z^r)$	$R^{1/r}$ (i.e., the set of points $z^{1/r}$, where z is in R)
10.5.6	Conjugation	$x^*[n]$	$X^*(z^*)$	R
10.5.7	Convolution	$x_1[n] * x_2[n]$	$X_1(z)X_2(z)$	At least the intersection of R_1 and R_2
10.5.7	First difference	$x[n] - x[n - 1]$	$(1 - z^{-1})X(z)$	At least the intersection of R and $ z > 0$
10.5.7	Accumulation	$\sum_{k=-\infty}^n x[k]$	$\frac{1}{1 - z^{-1}}X(z)$	At least the intersection of R and $ z > 1$
10.5.8	Differentiation in the z-domain	$nx[n]$	$-z \frac{dX(z)}{dz}$	R
10.5.9	Initial Value Theorem If $x[n] = 0$ for $n < 0$, then $x[0] = \lim_{z \rightarrow \infty} X(z)$			