- 1 *Solution.*

(a)
$$\int_1^2 (2x+3)^{\frac{1}{3}} dx = \frac{1}{2}\int_1^2 (2x+3)^{\frac{1}{3}} d(2x+3) = \frac{3}{8}(2x+3)^{\frac{4}{3}}|_1^2 = \frac{3}{8} * (7\sqrt[3]{7} - 5\sqrt[3]{5})$$

(b) Using the formula $\sin(\alpha)\cos(\beta) = \frac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)]$ for any real number α and β,

$$\int \sin(2x)\cos(5x)dx = \frac{1}{2}\int \sin(7x) - \sin(3x)dx = -\frac{1}{14}\cos(7x) + \frac{1}{6}\cos(3x).$$

(c) Using the integration by parts, it holds that

$$\begin{aligned}\int x^2 \tan^{-1} x dx &= \frac{1}{3}\int \tan^{-1} x d(x^3) = \frac{1}{3}x^3 \tan^{-1} x - \frac{1}{3}\int x^3 d(\tan^{-1} x) \\ &= \frac{1}{3}x^3 \tan^{-1} x - \frac{1}{3}\int \frac{x^3}{1+x^2}dx.\end{aligned}$$

Next, consider the

$$\int \frac{x^3}{1+x^2}dx = \frac{1}{2}\int \frac{x^2}{1+x^2}d(x^2) = \frac{1}{2}(x^2 - \ln|1+x^2|) + C.$$

By this,

$$\begin{aligned}\int x^2 \tan^{-1} x dx &= \frac{1}{3}x^3 \tan^{-1} x - \frac{1}{3}\int \frac{x^3}{1+x^2}dx \\ &= \frac{1}{3}x^3 \tan^{-1} x - \frac{1}{6}(x^2 - \ln|1+x^2|) + C.\end{aligned}$$

(d) Let $x = 2\tan t$, then $x^2 + 4 = 4(\tan^2 t + 1) = 4(\cos t)^{-2}$ and $dx = (\cos t)^{-2}dx$.

$$\int \frac{1}{(x^2+4)^{\frac{3}{2}}}dx = -8\int (\cos t)^3 (\cos t)^{-2} dt = -8\int \cos t dt = -8\sin t + C.$$

By $\tan t = \frac{x}{2}$ and thus $\sin t = \frac{x}{\sqrt{(x^2+4)}}$.

$$\int \frac{1}{(x^2+4)^{\frac{3}{2}}}dx = -8\frac{x}{\sqrt{(x^2+4)}} + C.$$

(e)

$$\begin{aligned}\int \frac{9x-7}{(x+2)(x^2-4x+13)}dx &= \int -\frac{1}{x+2} + \frac{x+3}{x^2-4x+13}dx \\ &= -\int \frac{1}{x+2}dx + \int \frac{x-4}{x^2-4x+13}dx + \int \frac{7}{x^2-4x+13}dx.\end{aligned}$$

Then

$$-\int \frac{1}{x+2}dx = -\ln|x+2| + C,$$

$$\int \frac{x-4}{x^2-4x+13}dx = \frac{1}{2}\int \frac{1}{x^2-4x+13}d(x^2-4x+13) = \frac{1}{2}\ln|x^2-4x+13| + C$$

and

$$\int \frac{1}{x^2-4x+13}dx = \frac{1}{3}\int \frac{1}{((x-2)/3)^2+1}d((x-2)/3) = \frac{1}{3}\tan^{-1}\frac{x-2}{3} + C.$$

By these three equalities,

$$\int \frac{9x-7}{(x+2)(x^2-4x+13)}dx = -\ln|x+2| + \frac{1}{2}\ln|x^2-4x+13| + \frac{7}{3}\tan^{-1}\frac{x-2}{3} + C.$$

- 2 *Solution.*

(a)

$$\begin{aligned} Area &= -\int_{-3}^{-2} x+2dx + \int_{-2}^{0} x+2dx + \int_{0}^{1} 2e^x dx \\ &= (-\frac{1}{2}x^2 - 2x)|_{-3}^{-2} + (\frac{1}{2}x^2+2x)|_{-2}^{0} + 2e^x|_0^1. \end{aligned}$$

(b)

$$\begin{aligned} L_{arc} &= \int_0^\pi \sqrt{x'^2+y'^2}dt \\ &= \int_0^\pi \sqrt{(1-\cos t)^2 + (\sin t)^2}dt \\ &= \int_0^\pi \sqrt{1-2\cos t + (\cos t)^2 + (\sin t)^2}dt \\ &= \int_0^\pi \sqrt{2-2\cos t}dt \\ &= \int_0^\pi \sqrt{2(1-\cos(2*\frac{t}{2}))}dt \\ &= \int_0^\pi 2\sin\frac{t}{2}dt \\ &= 4. \end{aligned}$$

- 3 *Solution.*

(a) Vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are $(-2,-1,1)$ and $(-1,1,-4)$ respectively. The angle $\angle BAC$ is

$$\angle BAC = \cos^{-1}\frac{\overrightarrow{AB}\cdot\overrightarrow{AB}}{|\overrightarrow{AB}||\overrightarrow{AC}|} = \cos^{-1}(-\frac{\sqrt{3}}{6}).$$

(b) Let a vector be $\overrightarrow{v} = (a,b,c)$ which is perpendicular to the plane $\prod$. Then by $\overrightarrow{AB}\cdot\overrightarrow{v} = 0$ and $\overrightarrow{AC}\cdot\overrightarrow{v} = 0$, one can obtain a system

$$\begin{aligned} -2a - b + c &= 0 \\ -a + b - 4c &= 0, \end{aligned}$$

with the solution $(a, b, c) = (-1, 3, 1)c$ for any real number c. Here, choose $c = 1$ and $\overrightarrow{v} = (-1, 3, 1)$. So a unit vector perpendicular to the plane can be $\overrightarrow{n} = \frac{\overrightarrow{v}}{|\overrightarrow{v}|} = \frac{1}{\sqrt{11}}(-1, 3, 1)$.

(c) Let the equation of the plane be $Ix + Jy + Kz + L = 0$. The linear system regarded to these three points on the plane is

$$\begin{aligned} 3I - 2J + K + L &= 0 \\ I - 3J + 2K + L &= 0 \\ 2I - J - 3K + L &= 0. \end{aligned}$$

The solution is $I = -\frac{1}{8}L$, $J = \frac{3}{8}L$ and $K = \frac{1}{8}L$. The equation of the plane turns out to be

$$-\frac{1}{8}Lx + \frac{3}{8}Ly + \frac{1}{8}Lz + L = 0,$$

where the real number L is not zero. Therefore, the equation is

$$-x + 3y + z + 8 = 0,$$

if choose $L = 1$. The distance of the given point $D = (x_D, y_D, z_D) = (-4, -1, 2)$ to the plane is

$$d = \frac{-x_D + 3y_D + z_D + 8}{\sqrt{1 + 9 + 1}} = \sqrt{11}.$$

- 4 *Solution.*

(a)

$$\frac{1 - i}{1 + i} = \frac{(1 - i)^2}{(1 + i)(1 - i)} = \frac{(1 - i)^2}{2} = -i.$$

So

$$\left(\frac{1 - i}{1 + i}\right)^{2019} = (-i)^{2019} = -i.$$

Then $Re\left(\frac{1-i}{1+i}\right)^{2019} = 0$ and $Im\left(\frac{1-i}{1+i}\right)^{2019} = -1$ and

$$Arg\left(\left(\frac{1 - i}{1 + i}\right)^{2019}\right) = arg\left(\left(\frac{1 - i}{1 + i}\right)^{2019}\right) + 2k\pi = -\frac{\pi}{2} + 2k\pi,$$

where k is an integer.

(b)

$$\begin{aligned} Z^2 &= \left[-2 * \left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) \right]^{\frac{1}{2}} \\ &= \sqrt{2}i * \exp\left(\frac{\pi}{6}i\right) \\ &= \sqrt{2}\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) \\ &= \sqrt{2}\exp\left(\frac{2\pi}{3}i\right). \end{aligned}$$

By this, $z = 2^{1/4}\exp(\frac{\pi}{3}i) = 2^{1/4} * (\frac{\sqrt{3}}{2} + \frac{1}{2}i)$ with $Rez = 2^{1/4} * \frac{\sqrt{3}}{2}$ and $Imz = 2^{1/4} * \frac{1}{2}$

$$Arg(z) = arg(z) + 2k\pi = \frac{\pi}{3} + 2k\pi.$$

- 5 *Solution.*

$$|A| = \begin{vmatrix} 3 & 1 & -2 \\ -3 & 3 & 3 \\ 0 & 2 & 2 \end{vmatrix} = \begin{vmatrix} 3 & 1 & -2 \\ 0 & 4 & 1 \\ 0 & 2 & 2 \end{vmatrix} = 3 * \begin{vmatrix} 4 & 1 \\ 2 & 2 \end{vmatrix} = 3 * (8-2) = 18.$$

Besides, due to the fact that A is nonsingular, $1 = |I| = |(A^{-1})A| = |A^{-1}| \cdot |A|$. Hence, $|A^{-1}| = |A|^{-1} = 1/18$. By this, $|A^{\intercal} A^{-3}| = |A^{\intercal}| \cdot |A^{-3}| = |A^{\intercal}| \cdot |A^{-1}|^3 = |A^{\intercal}| \cdot |A|^{-3} = |A| \cdot |A|^{-3} = |A|^{-2} = 1/18^2$.

- 6 *Solution.*

(1) Let

$$A = \begin{pmatrix} 1 & -2 & 3 & -4 \\ -2 & 3 & -4 & 10 \\ 1 & -1 & 2 & -3 \end{pmatrix}.$$

The augmented matrix of the linear system is

$$B = \left(\begin{array}{cccc|c} 1 & -2 & 3 & -4 & 1 \\ -2 & 3 & -4 & 10 & 2 \\ 1 & -1 & 2 & -3 & 3 \end{array}\right).$$

$$\left(\begin{array}{cccc|c} 1 & -2 & 3 & -4 & 1 \\ -2 & 3 & -4 & 10 & 2 \\ 1 & -1 & 2 & -3 & 3 \end{array}\right) \to \left(\begin{array}{cccc|c} 1 & -2 & 3 & -4 & 1 \\ 0 & -1 & 2 & 2 & 4 \\ 0 & 1 & -1 & 1 & 2 \end{array}\right) \to \left(\begin{array}{cccc|c} 1 & -2 & 3 & -4 & 1 \\ 0 & -1 & 2 & 2 & 4 \\ 0 & 0 & 1 & 3 & 6 \end{array}\right)$$
$$\to \left(\begin{array}{cccc|c} 1 & 0 & -1 & -8 & -7 \\ 0 & -1 & 2 & 2 & 4 \\ 0 & 0 & 1 & 3 & 6 \end{array}\right) \to \left(\begin{array}{cccc|c} 1 & 0 & -1 & -8 & -7 \\ 0 & -1 & 0 & -4 & -8 \\ 0 & 0 & 1 & 3 & 6 \end{array}\right) \to \left(\begin{array}{cccc|c} 1 & 0 & 0 & -5 & -1 \\ 0 & 1 & 0 & 4 & 8 \\ 0 & 0 & 1 & 3 & 6 \end{array}\right).$$

The solution $(x, y, z, w) = (5, -4, -3, 1)w + (-1, 8, 6, 0)$, where w is any real number.

(2) The homogeneous system is

$$\begin{aligned} x - 2y + 3z - 4w &= 0 \\ -2x + 3y - 4z + 10w &= 0 \\ x - y + 2z - 3w &= 0. \end{aligned}$$

By (1), the nontrivial solution of this homogeneous system is $(x, y, z, w) = (5, -4, -3, 1)w$, where the real number w is not equal to zero.