

**Instruction:** Indicate carefully the above course session\* you register and hand in your answer script together with this question paper as a cover page. Marks will not be recorded without the question paper or with the wrong session you attend or indicate.

1. (a) Compute the volume of the solid by revolving the region bounded by the parabolas  $x = 3y^2 - 2$  and  $x = y^2$  about the  $x$ -axis. [15]
- (b) Find the surface area of the solid by revolving the Astroid:  $x = \cos^3 t$ ,  $y = \sin^3 t$ ,  $0 \leq t \leq 2\pi$ , about the  $y$ -axis. [18]
2. (a) Let  $A$  be a point in the Argand diagram representing the complex number  $z_A = -1 + \sqrt{3}i$ . Determine the resulting complex number  $z_B$  in the Cartesian form by rotating  $OA$   $75^\circ$  along the clockwise direction and sketched in length by five times to  $OB$ . [15]
- (b) Solve  $\frac{1}{i}z^3 = 1 + i$  and list all solution in the Polar form with principal arguments. [18]
3. (a) Let  $A = \begin{pmatrix} -1 & 2 & 3 \\ 0 & 2 & 0 \\ 2 & -1 & -4 \end{pmatrix}$ . Evaluate  $|A^3| + |AA^T| - 2|A^{-1}|$ . [10]
- (b) Consider the system of linear equations as follows.
$$\begin{aligned} 2x + 3y - z + w &= 1 \\ 8x + 12y - 5z + 8w &= 3 \\ -2x - 4y + 3z - 4w &= -3 \end{aligned}$$
  - (i) Solve the above linear system by the Gaussian elimination. [19]
  - (ii) Write down the corresponding homogeneous system explicitly and provide a non-trivial solution from (i) without resolving the homogeneous system. [5]

- END -

### Brief Table of Integrals

$\int x^p dx = \frac{x^{p+1}}{p+1} + C, \quad p \neq -1$	$\int \frac{1}{x} dx = \ln x  + C$
$\int e^x dx = e^x + C$	$\int \sec^3 x dx = \frac{1}{2} \sec x \tan x + \frac{1}{2} \ln  \sec x + \tan x  + C$
$\int \sin x dx = -\cos x + C$	$\int \cos x dx = \sin x + C$
$\int \sec^2 x dx = \tan x + C$	$\int \csc^2 x dx = -\cot x + C$
$\int \sec x \tan x dx = \sec x + C$	$\int \csc x \cot x dx = -\csc x + C$
$\int \sec x dx = \ln  \sec x + \tan x  + C$	$\int \csc x dx = -\ln  \csc x + \cot x  + C$
$\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + C$	$\int \frac{1}{1+x^2} dx = \tan^{-1} x + C$

Not to be taken away