Example on Estimation of small changes

$$f(x,y) \simeq \boxed{f(a,b)} + (x-a)f_x(a,b) + (y-b)f_y(a,b)$$

$$f(x,y) - f(a,b) = \delta f$$

$$(x-a) = \delta x = h$$

$$(y-b) = \delta y = k$$

eg 10^6, 0.0001, 10, 0.4

$$\boxed{\underbrace{\delta f}_{\text{Total errors}} \simeq \underbrace{\frac{\partial f}{\partial x}(a,b)}_{\text{rate of change of } f \text{ w.r.t } x} \underbrace{\delta x}_{\text{error on } x} + \underbrace{\frac{\partial f}{\partial y}(a,b)}_{\text{rate of change of } f \text{ w.r.t } y} \underbrace{\delta y}_{\text{error on } y}}$$

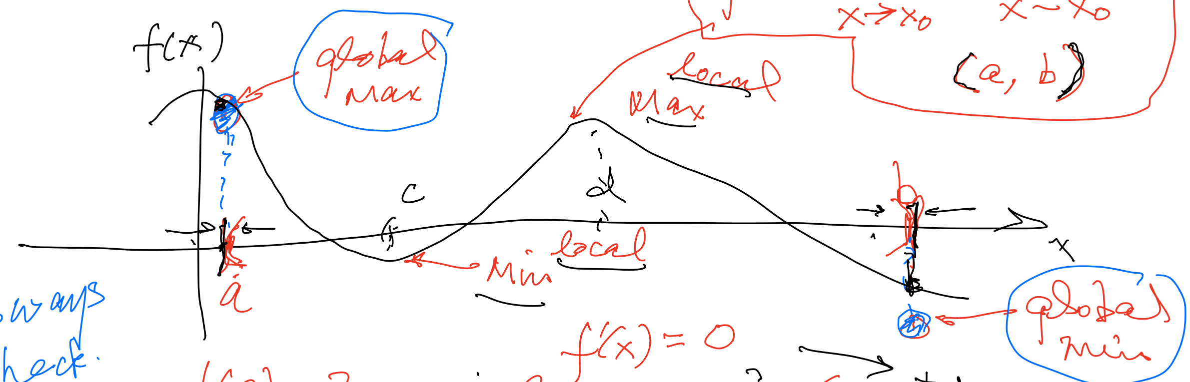
$$|\delta f| \le \left|\frac{\partial f}{\partial x}\right| |\delta x| + \left|\frac{\partial f}{\partial y}\right| |\delta y|$$

"Upper" bound of Total errors

Max and Min

$$f'(x) = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0}$$

(a, b)



Always check your boundary

$f(a) = ?$

$f(b) = ?$

$f'(x) = 0 \rightarrow$ stationary pts

min $\{ f''(c) > 0 ? \checkmark$

max $\{ f''(d) < 0 ? \checkmark$

Maxima and Minima

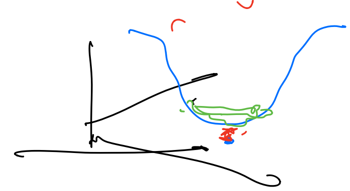
$f(x,y)$ has a "local" maximum at a point (a,b)

$f(x,y) - f(a,b) \le 0 \Rightarrow f(x,y) \le f(a,b)$

$f(x,y)$ has a "local" minimum at (a,b)

$f(x,y) - f(a,b) \ge 0 \Rightarrow f(x,y) \ge f(a,b)$

(a,b)



Stationary points

$$\begin{cases} \dfrac{\partial f}{\partial x} = 0 \\[2ex] \dfrac{\partial f}{\partial y} = 0 \end{cases}$$

solve for $(a_1,b_1), (a_2,b_2), \ldots, (a_n,b_n)$

necessary condition

f_{xx} f_{yy}

Sufficient Condition for checking

$$\Delta = f_{xx}(a,b) f_{yy}(a,b) - f_{xy}^2(a,b)$$

① $f_{xx}(a,b) > 0$ ✓ $\quad$ > 0

$\Rightarrow$ $f(x,y)$ at (a,b) has a local minimum

② $f_{xx}(a,b) < 0$ $\quad$ $\Delta > 0$

$\Rightarrow$ $f(x,y)$ at (a,b) has a local maximum

③ If $\Delta < 0$ $\Rightarrow$ Saddle point

④ If $\Delta = 0$, No Conclusion

further checking is needed

checking is needed around neighbouring points

could be max or min ?

$$\Delta = \begin{vmatrix} f_{xx}(a,b) & f_{xy}(a,b) \\ f_{xy}(a,b) & f_{yy}(a,b) \end{vmatrix}$$

From Taylor Thm

necessary condition ① ② (a,b) is the stationary point

** $f(x,y) = f(a,b) + h\frac{\partial f}{\partial x}(a,b) + k\frac{\partial f}{\partial y}(a,b)$

$+ \frac{1}{2!}\left[h^2 f_{xx}(a,b) + 2hk f_{xy}(a,b) + k^2 f_{yy}(a,b) + \right] \ldots$

$$f(x,y) - f(a,b) = \frac{1}{2!}\begin{bmatrix} h & k \end{bmatrix}\begin{bmatrix} f_{xx}(a,b) & f_{xy}(a,b) \\ f_{xy}(a,b) & f_{yy}(a,b) \end{bmatrix}\begin{bmatrix} h \\ k \end{bmatrix}$$

Δ

$$= \frac{1}{2!}\begin{bmatrix} h & k \end{bmatrix} \; Q \; \begin{bmatrix} h \\ k \end{bmatrix}$$

$h = (x-a)$
$k = (y-b)$

$\Delta = |Q| > 0$, $f_{xx} > 0$

$\Rightarrow$ (a,b) is min