<u>Ans. to Tut 6</u>

<u>Qn 1</u>

a) $I_A = k_d I_l (\mathbf{N}_A \cdot \mathbf{L}) = (0.7)(1.0)[(0.6,0,0.8)\cdot(0,0,1)] = (0.7)(0.8) = 0.56$

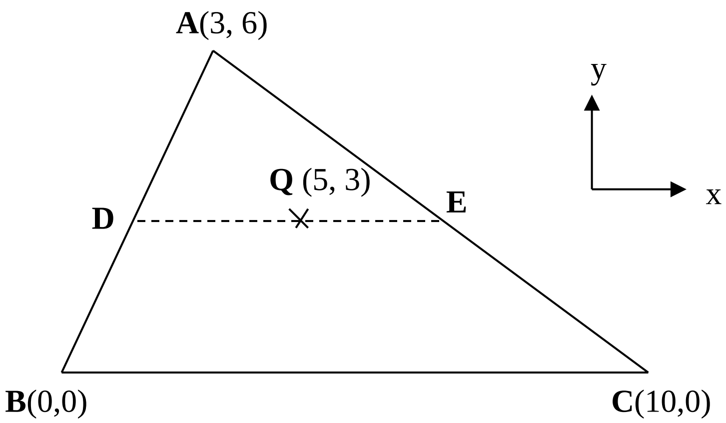
$I_B = 0.42 \qquad\qquad I_C = 0$

Revision (Dot/Scalar Product):

$\mathbf{A} = (a_1, a_2, a_3) \qquad \mathbf{B} = (b_1, b_2, b_3)$

$\mathbf{A}\cdot\mathbf{B} = a_1b_1 + a_2b_2 + a_3b_3 = |\mathbf{A}||\mathbf{B}|\cos\theta$ $\qquad\theta$ is the angle between vector A and B

The scanning direction is the x direction.



By linear interpolation, $\mathbf{D} = \frac{3}{6}\mathbf{A} + \frac{3}{6}\mathbf{B} = (1.5,\ 3)$ $\qquad$ E = (6.5, 3).

Gouraud shading interpolates the <u>intensities</u>.

$I_D = \frac{3}{6} I_A + \frac{3}{6} I_B = 0.49 \qquad I_E = 0.28$

$$I_Q = \frac{6.5-5}{6.5-1.5} I_D + \frac{5-1.5}{6.5-1.5} I_E = 0.343$$

b) Phong shading interpolates the <u>normals</u>

$\mathbf{N}_D = \frac{3}{6}\mathbf{N}_A + \frac{3}{6}\mathbf{N}_B = (0.7,\ 0,\ 0.7) \qquad \mathbf{N}_E = (0.6,\ 0.4,\ 0.4)$

$$\mathbf{N}_Q = \frac{1.5}{5}\mathbf{N}_D + \frac{3.5}{5}\mathbf{N}_E = (0.63,\ 0.28,\ 0.49)$$

Normalize to find the unit normal at Q,

$$\left|\mathbf{N}_Q\right| = (0.744845299,\ 0.331042355,\ 0.579324122)$$

Applying the illumination model at Q,

$$I_Q = k_d I_l (\left|\mathbf{N}_Q\right| \cdot \mathbf{L}) = 0.405526885$$

<u>Qn 2</u>

a) The ordinary form for finding the intensity of a point (x, y) lying along the line segment AB is

$$I_P = \frac{y-0}{6-0} I_A + \frac{6-y}{6-0} I_B \tag{1}$$

Let I_P be called I_y as it is a function of y.

As we scan from top to bottom, we scan with decreasing y values. Since we have calculated I_{y+1}, we can express the above in terms of it:

$$I_y = \frac{y-0}{6-0} I_A + \frac{6-y}{6-0} I_B = \frac{(y+1)-0}{6-0} I_A + \frac{6-(y+1)}{6-0} I_B - \frac{1}{6} I_A + \frac{1}{6} I_B = I_{y+1} + C \tag{2}$$

where C is a constant equal to $\frac{1}{6}(I_B - I_A)$. Eqn (2) is the incremental form.

b) Eqn (2) is much faster to compute than eqn (1).

Consider eqn (1) and (2). (6 – 0) is a constant. Therefore, it can be calculated once and for all and stored.

Thus in its general form, eqn (1) needs 3 +/- and 4 x/ /, whereas eqn (2) needs only 1 +/-, saving 2 addition/subtractions and 4 multiplications/divisions.

<u>Qn 3</u>

$$\mathbf{L} = \left|(100,70,1000) - (100,70,50)\right| = (0,\ 0,\ 1)$$

$$\mathbf{V} = \left|(150,120,100) - (100,70,50)\right| = (\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$$

$$I = k_a I_a + k_d I_l (\mathbf{N} \cdot \mathbf{L}) + W(\theta) I_l (\mathbf{V} \cdot \mathbf{R})^{n_s}$$

$$k_a = 0.2 \quad I_a = 0.4 \quad k_d = 0.6 \quad I_l = 2.0 \qquad \mathbf{N} = \left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right)$$

$$W(\theta) \text{ or } k_s = 0.3 \qquad n_s = 2$$

$$\mathbf{N}\cdot\mathbf{L} = \left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right)\cdot(0,0,1) = \frac{1}{\sqrt{2}}$$

$$\mathbf{R} = 2(\mathbf{N}\cdot\mathbf{L})\mathbf{N} - \mathbf{L}$$

$= (1,0,1) - (0,0,1) = (1,0,0)$ [Check: it should be a unit vector]

$$\mathbf{V}\cdot\mathbf{R} = \frac{1}{\sqrt{3}}$$

$$I = (0.2)(0.4) + (2.0)[(0.6)\left(\frac{1}{\sqrt{2}}\right) + (0.3)(\frac{1}{\sqrt{3}})^2] = 1.128528137$$