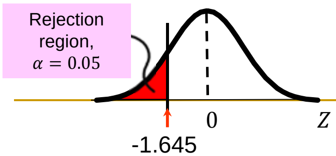
Z Test for the Population Mean (σ Known) – Exercise

Cont'd

H_0: $\mu \geq 368$
H_1: $\mu < 368$

At $\alpha = 0.05$
$n = 25$
Critical Value = -1.645
Reject H_0 if $Z < -1.645$



$$Z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} = \frac{364.5 - 368}{15/\sqrt{25}}$$
$$= -1.17$$

At $\alpha = 0.05$, do not reject H_0

There is no evidence that the true mean weight is less than 368 g

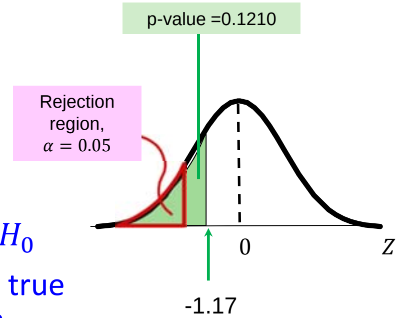
Z Test for the Population Mean (σ Known) – Exercise

Cont'd

$H_0: \mu \geq 368$
$H_1: \mu < 368$

$$Z = \frac{\bar{X}-\mu_0}{\sigma/\sqrt{n}} = \frac{364.5-368}{15/\sqrt{25}} = -1.17$$

p-value
$= P(Z \leq -1.17)$
$= 0.1210$



As p-value > α, do not reject H_0

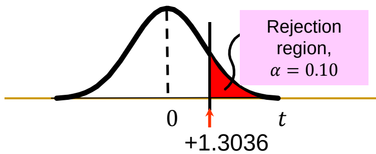
There is no evidence that the true mean weight is less than 368 g

t Test for the Population Mean (σ Unknown) – Exercise

Cont'd

$H_0: \mu \leq 1$
$H_1: \mu > 1$

At $\alpha = 0.10$
$n = 40$ $\quad df = 39$
Critical Value = $+1.3036$
Reject H_0 if $t > +1.3036$



$$t = \frac{\bar{X} - \mu_0}{S/\sqrt{n}} = \frac{1.03 - 1}{0.08/\sqrt{40}}$$
$$= 2.37$$

At $\alpha = 0.10$, reject H_0

There is evidence that the true mean amount is more than 1 L

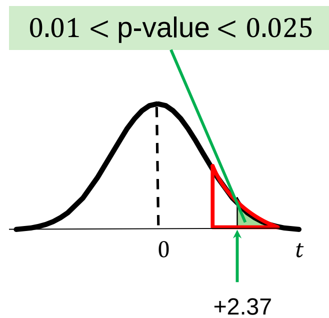
t Test for the Population Mean (σ Unknown) – Exercise

Cont'd

$H_0: \mu \leq 1$
$H_1: \mu > 1$

$$t = \frac{\bar{X}-\mu_0}{S/\sqrt{n}} = \frac{1.03-1}{0.08/\sqrt{40}} = 2.37$$

p-value
$= P(t \geq 2.37)$
$= (0.01, 0.025)$



As p-value < α, H_0 is rejected

There is evidence that the true mean amount is more than 1 L

Using Excel "T.DIST" function, the p-value is found to be 0.0114

Hypothesis Test – More Exercise

Cont'd

- Step 1: Define hypotheses

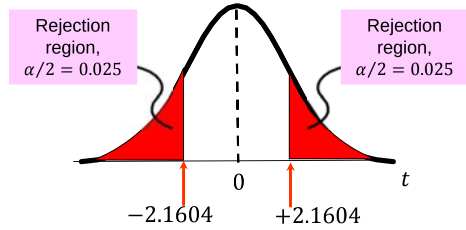
  $H_0: \mu = -0.545$

  $H_1: \mu \neq -0.545$
- Step 2: Collect data and identify rejection region(s)
  - Population distribution: Unknown
  - Sample size: 14
  - Any assumption needed? Yes
    - What is the assumption? Assume Normal population
    - Why? The sample size is too small to apply Central Limit Theorem
  - σ: unknown
  - Distribution to be used: t

Hypothesis Test – More Exercise

Cont'd

- Step 2: Collect data and identify rejection region(s)
  - Significance level: 0.05
  - Degrees of freedom: 13
  - Critical value(s): ± 2.1604
  - Decision rule: Reject H_0 if t < -2.1604 or t > +2.1604



Hypothesis Test – More Exercise

Cont'd

- Step 3: Compute test statistic
  - Test statistic = $t = \frac{\bar{X}-\mu_0}{S/\sqrt{n}} = \frac{-0.550-(-0.545)}{0.016/\sqrt{14}} = -1.17$
  - p-value = (0.20, 0.50)
- Step 4: Make statistical decision
  - Decision: At α = 0.05, do not reject H_0
  - Conclusion: There is insufficient evidence that the mean freezing point of the milk is not -0.545 °C

Hypothesis Test – More Exercise

Cont'd

- What would happened if the sample size is 144 rather than 14?
  - Assumed the sample mean and standard deviation remain unchanged
- Step 1: Define hypotheses

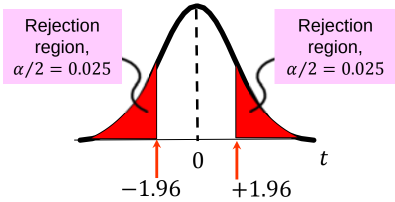
  $H_0: \mu = -0.545$

  $H_1: \mu \neq -0.545$

Hypothesis Test – More Exercise

Cont'd

- Step 2: Collect data and identify rejection region(s)
  - Population distribution: Unknown
  - Sample size: 144
  - Any assumption needed? No
    - What is the assumption? NA
    - Why? The sample size is large enough to apply Central Limit Theorem
  - σ: unknown
  - Distribution to use: t
  - Significance level: 0.05
  - Degrees of freedom: 143 $\approx \infty$
  - Critical value(s): ± 1.96
  - Decision rule: Reject H_0 if t < -1.96 or t > +1.96



Hypothesis Test – More Exercise

Cont'd

- Step 3: Compute test statistic
  - Test statistic = $t = \frac{\bar{X}-\mu_0}{S/\sqrt{n}} = \frac{-0.550-(-0.545)}{0.016/\sqrt{144}} = -3.75$
  - p-value < 0.01
- Step 4: Make statistical decision
  - Decision: At α = 0.05, reject H_0
  - Conclusion: There is sufficient evidence that the mean freezing point of the milk is not -0.545 °C