Standardizing Sampling Distribution of Proportion – Example

Cont'd

- Given π = population proportion of depositors with multiple accounts $= 0.4$
- As $n = 200 > 30, n\pi = 80 > 5, n(1-\pi) = 120 > 5$
  → The sampling distribution of p follows Normal distribution approximately, i.e. $p \sim N(\mu_p, {\sigma_p}^2)$

$$P(p < 0.3)$$
$$= \mathrm{P}\left(\mathrm{Z} < \frac{0.3-0.4}{\sqrt{\frac{0.4(1-0.4)}{200}}}\right) = P(Z < -2.89)$$
$$= 0.0019$$

Confidence Interval Estimate for the Proportion – Example

Cont'd

For these data, $p = \frac{95}{200} = 0.475$

As $n = 200 > 30, np = 95 > 5, n(1-p) = 105 > 5$

→ The sampling distribution of p follows Normal distribution approximately

95% confidence interval (C.I.) for π

$$p \pm Z_{\alpha/2}\sqrt{\frac{p(1-p)}{n}} = 0.475 \pm 1.96\sqrt{\frac{0.475(1-0.475)}{200}}$$
$$= [0.406, 0.544]$$

We are 95% confident that the population proportion of depositors having RMB deposit account is between 0.406 and 0.544

Determining Sample Size for the Proportion – Example

Cont'd

$$\pi = \frac{22}{10000} = 0.0022$$

$$n = \frac{(Z_{\alpha/2})^2 \pi(1-\pi)}{E^2} = \frac{(2.575)^2 0.0022(1-0.0022)}{0.001^2}$$
$$= 14555.28 \cong 14556$$

Round Up

Test of Hypothesis for the Proportion – Exercise

Cont'd

$H_0: \pi = 0.80$
$H_1: \pi \neq 0.80$

$n = 45 > 30$
$np = 39 > 5$
$n(1-p) = 6 > 5$
$\therefore p \sim N$ approximately

At $\alpha = 0.05$

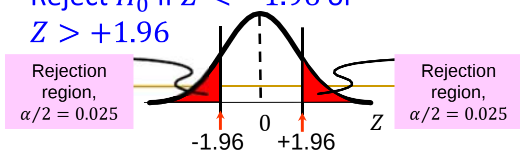
Critical Value = ± 1.96

Reject H_0 if $Z < -1.96$ or $Z > +1.96$

$$Z = \frac{p - \pi_0}{\sqrt{\frac{\pi_0(1-\pi_0)}{n}}} = \frac{\frac{39}{45} - 0.80}{\sqrt{\frac{0.80(1-0.80)}{45}}}$$
$$= 1.118$$

At $\alpha = 0.05$, do not reject H_0

There is insufficient evidence that the true proportion of customers to be served within 5 minutes upon their arrival is not 80%



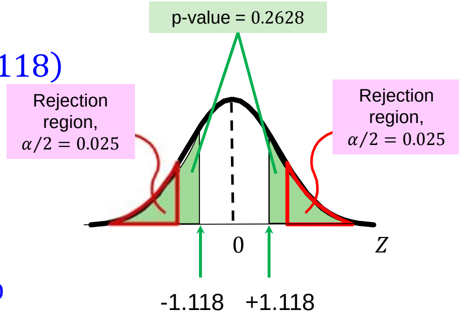
Test of Hypothesis for the Proportion – Exercise

Cont'd

$H_0: \pi = 0.80$
$H_1: \pi \neq 0.80$

$$Z = \frac{p - \pi_0}{\sqrt{\frac{\pi_0(1-\pi_0)}{n}}} = \frac{\frac{39}{45} - 0.80}{\sqrt{\frac{0.80(1-0.80)}{45}}} = 1.118$$

p-value
$= P(Z \leq -1.118) + P(Z \geq 1.118)$
$= 2 \times P(Z \leq -1.118)$
$= 2 \times 0.1314$
$= 0.2628$



As p-value > α, do not reject H_0

There is insufficient evidence that the true proportion of customers to be served within 5 minutes upon their arrival is not 80%