Tutorial 7

Vectors

Cauchy-Schwarz Inequality

$$|a^T b| \leq \|a\|\|b\|$$

- To apply this inequality, the key is to appropriately choose the two vectors a and b.

Question 1

- Show that for any angle θ,

$$|\cos^2\theta - \sin^2\theta| \leq 1.$$

Question 2

- Let $a_1, a_2, \ldots, a_n$ be real numbers. Show that

$$\left(\frac{1}{n}\sum_{i=1}^{n} a_i\right)^2 \leq \frac{1}{n}\sum_{i=1}^{n} a_i^2$$

$(\text{Average})^2 \leq$ Average of the Squares

or equivalently,

$|\mathbf{avg}(a)| \leq \mathbf{rms}(a)$

Question 3

- Let $a_1, a_2, \ldots, a_n$ be positive. Show that

$$a_1 + a_2 + \cdots + a_n \leq \frac{a_1^2}{a_2} + \frac{a_2^2}{a_3} + \cdots + \frac{a_n^2}{a_1}.$$

Question 4

- The triangle inequality is given by

$$\|a + b\| \leq \|a\| + \|b\|$$

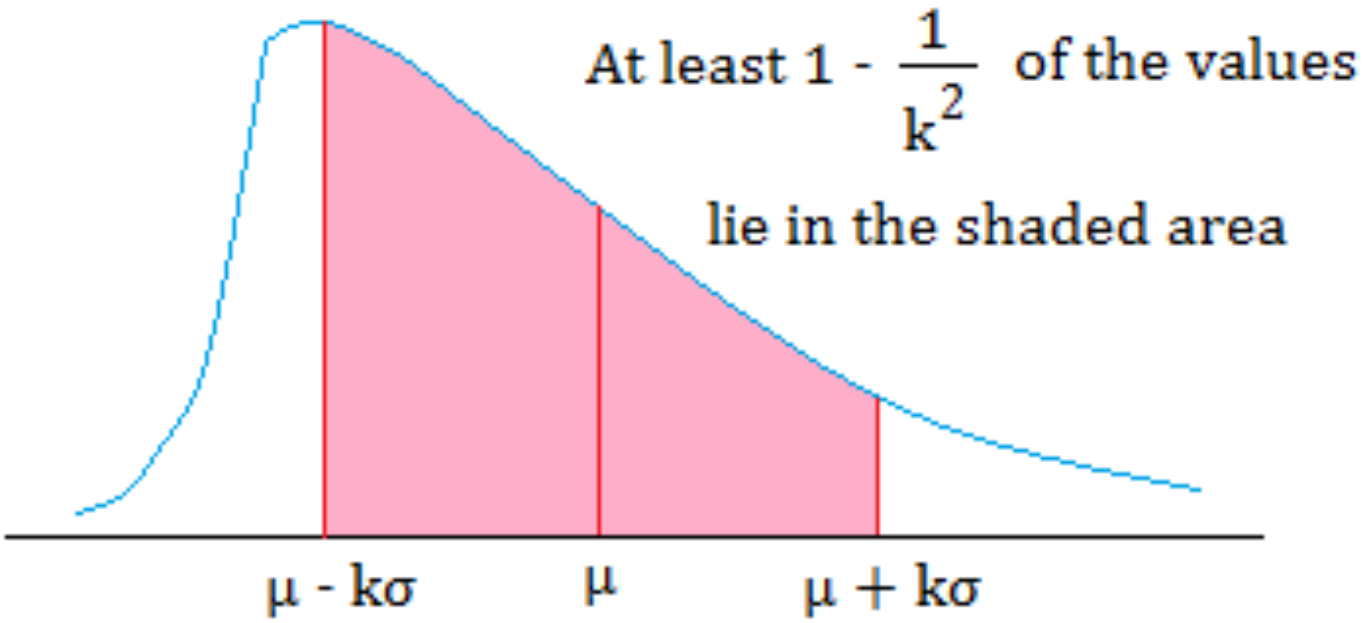
- When does it hold with equality?

Chebyshev's Inequality for Data Set

The proportion of entries of x that satisfy

$$|x_i - \mu| \geq m\sigma$$

is less than or equal to $\frac{1}{m^2}$.



The proportion of entries of x that satisfy

$$|x_i - \mu| < m\sigma$$

is greater than or equal to $1 - \frac{1}{m^2}$.

- Chebyshev's Theorem Explained (22 min)
  - https://www.youtube.com/watch?v=OM0K22pmkuY
  - Watch the first 10.5 min during tutorial.

- Type 1 Application
  - Determine the percentage p for a given interval (a, b).

- Type 2 Application
  - Determine an interval given the percentage p.

Question 5

- Consider the marks of the students obtained in Test 1.
- The mean and the standard deviation are 53 and 21, respectively.
- At least what percentage of students obtain marks between 21.5 and 84.5?