

Tutorial 10

Groups

Q.1 Group or Not?

Is each of the following cases a group?

- a) Integers under addition
- b) Even numbers under addition
- c) Odd numbers under addition
- d) Integers under multiplication
- e) Multiples of 7 under addition
- f) Complex numbers under addition
- g) Complex numbers under multiplication
- h) 2×2 real matrices under addition
- i) 2×2 real matrices under multiplication

Pause and think:

https://www.youtube.com/watch?v=qvx9TnK85bw&list=PLi01XoE8jYoi3SgnnGorR_XOW3IcK-TP6&index=10

Q.2 Abelian or not?

□ Let G be the set of 2×2 real matrices with non-zero determinant.

a) Is $\langle G, + \rangle$ a group? If so, is it an Abelian group?

b) Is $\langle G, \times \rangle$ a group? If so, is it an Abelian group?

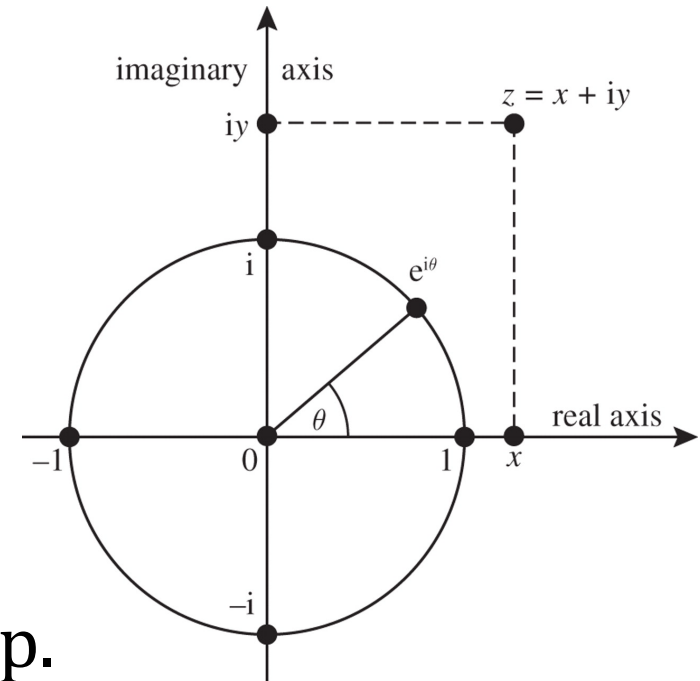
Q.3 Unit Circle on Complex Plane

- Consider the set of complex numbers on the unit circle:

$$H = \{z \in \mathbb{C}: |z| = 1\}.$$

- Denote multiplication by \times .

- e.g. $(1 + 2i)(3 - i)$
 $= (3 + 2) + (6 - 1)i$
 $= 5 + 5i.$



- a) Show that $\langle H, \times \rangle$ forms a group.
- b) Does it have a subgroup of order 3? Why?

Q.4 Binary Linear Code

- Recall that a binary linear code C is a subset of \mathbb{B}^n .
- It is defined by the encoding function $f: \mathbb{B}^k \rightarrow \mathbb{B}^n$, where $f(u) = uG$ and G is the generator matrix.
- Is C a subgroup of \mathbb{B}^n ?