1. Review of DC Circuit Analysis

1.1 Basic circuit laws and resistor networks

1.2 Circuit analysis methods

1.3 Equivalent circuit transformation and dependent sources

1.1 Basic circuit laws and resistor networks

- This unit is organized into 6 sections. The two sections cover the 3 basic laws in circuits:

I. Kirchhoff's Current and Voltage Law

II. Ohm's Law

- The remaining sections of this unit are just applications of these 3 laws together.

III. Electrical Power

IV. Sources, Short Circuit, Open Circuit

V. Resistive Networks

VI. Measuring Instruments

Alexander & Sadiku, "Fundamentals of Electric Circuits" 6th Edition Ch 1 & 2

I. Current and Kirchhoff's Current Law

- Charge is a fundamental electrical quantity
- Typically denoted by the symbol Q, and its SI unit is the Coulomb (C)
- The smallest amount of charge that exists is the charge that is carried by an electron:

$$Q_e = 1.602 \times 10^{-19}\ \text{C}$$

Always include units, otherwise there is no meaning

Current: Free electrons on the move

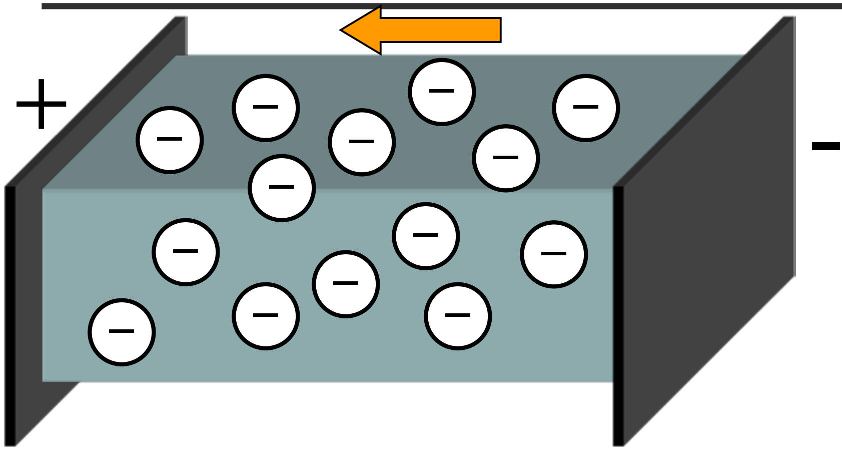


Fig 2a. A voltage is applied to move the charges

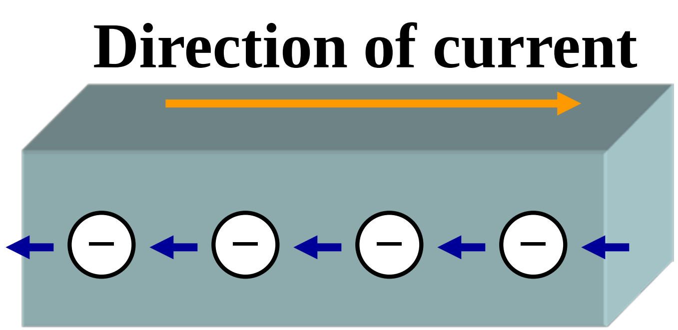


Fig 2b. Movement of electrons gives rise to current

Key Concept 1
For current to flow, there must be mobile charges

- Direction of electron flows is from negative to positive
- Direction of current flows (positive charges) is from positive to negative
- Typically denoted by the symbol, I
- SI unit is the Ampere (A)

Current must run in loops

- If a current flows out of a given point, then it must return to that point with the same amount
- Current must flow in loops
- What goes around comes around

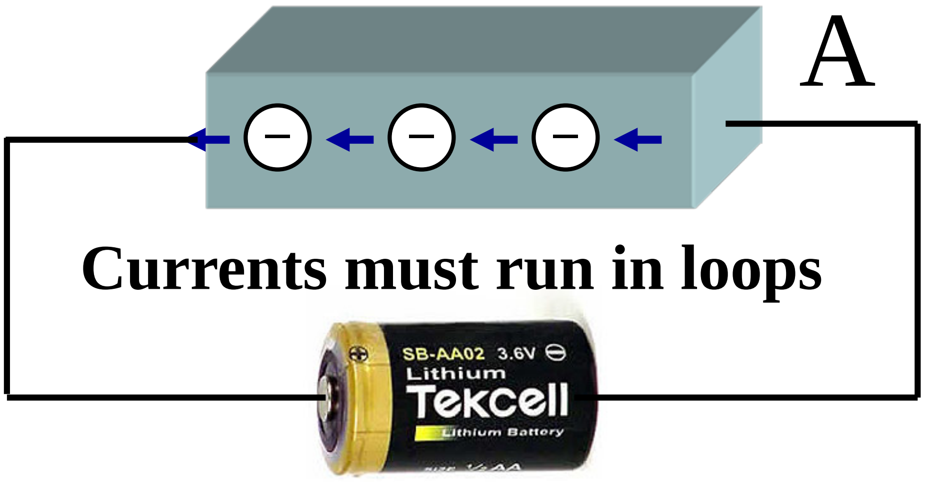


Fig 3. Current must run in loops

Key Concept 2
Currents must run in loops (otherwise charge is either destroyed or created which cannot happen)

Fundamental law for charge

- Current has to flow in closed loop
- No current flows if there is a break in the path
- Underlying physical law: Charge cannot be created or destroyed ("what goes in must also come out")
- This is the basis of Kirchhoff's Current Law

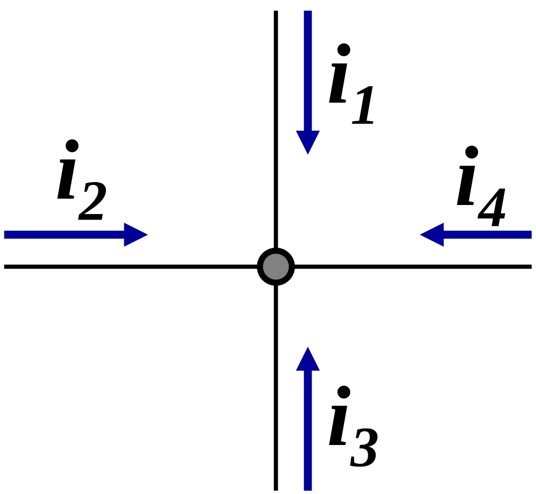


Fig 4.

Kirchhoff's current law

Sum of currents at a **node** must equal to zero:

$$i_1 + i_2 + i_3 + i_4 = 0$$

Kirchhoff's current law

Does the direction of current matter? **YES!!**

I_2 running **into** node

I_2 running **out of** node

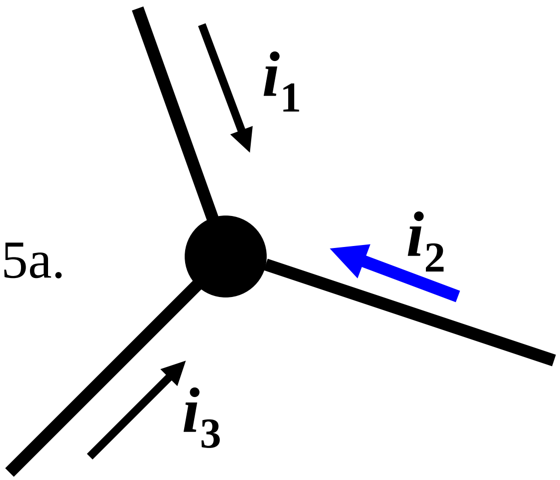


Fig 5a.

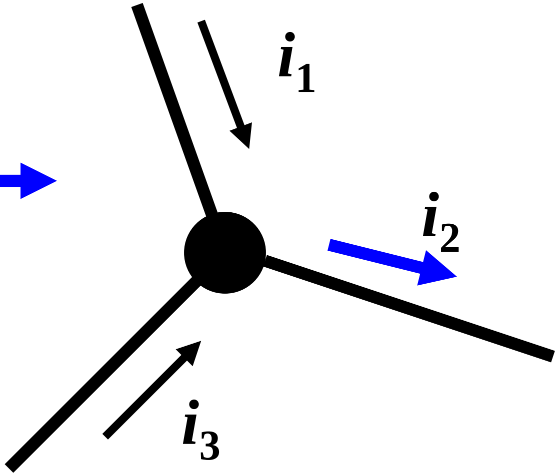


Fig 5b.

$$i_1 + i_2 + i_3 = 0$$

$$i_1 + i_2 + i_3 = 0 \text{ (WRONG)}$$

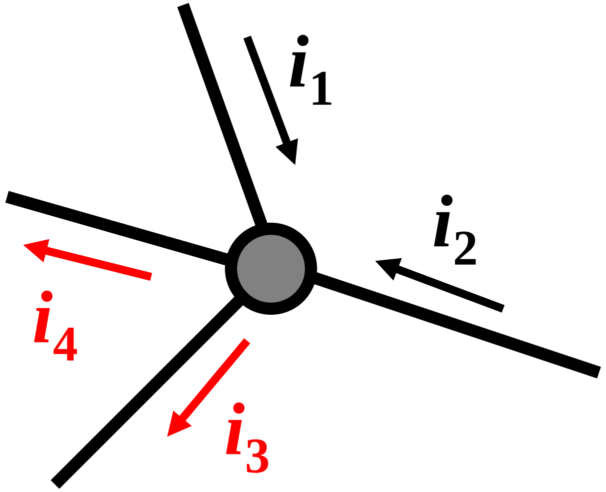
$$i_1 + i_3 = i_2 \text{ (CORRECT)}$$

Kirchhoff's current law

Sign convention when applying KCL

Currents flowing IN - Positive

Currents flowing OUT - Negative



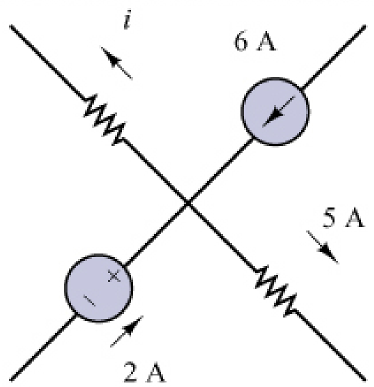
Entering: I_1 & I_2 (+ve)

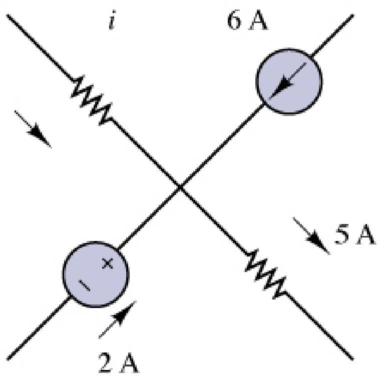
Exiting: I_3 & I_4 (-ve)

$$\boxed{i_1 + i_2 - i_3 - i_4 = 0}$$

Worked Example on KCL

Find the unknown current within each of the following circuit networks





$6A - 5A + 2A - i = 0$

➔ $i = 3A$

➔ $6A + 2A = 5A + i$

$6A - 5A + 2A + i = 0$

➔ $i = -3A$

Voltage and Kirchhoff's Voltage Law

- Energy is required to move electrons between 2 points
- This energy could come from the battery cell connected to the circuit
- As the current goes across the cell, energy is pumped in
- As the current goes through a light bulb, energy is consumed
- The amount of energy required to move a unit charge is given by the voltage
- SI unit is Voltage (V)

Potential Difference

$$v_x = v_A - v_B$$

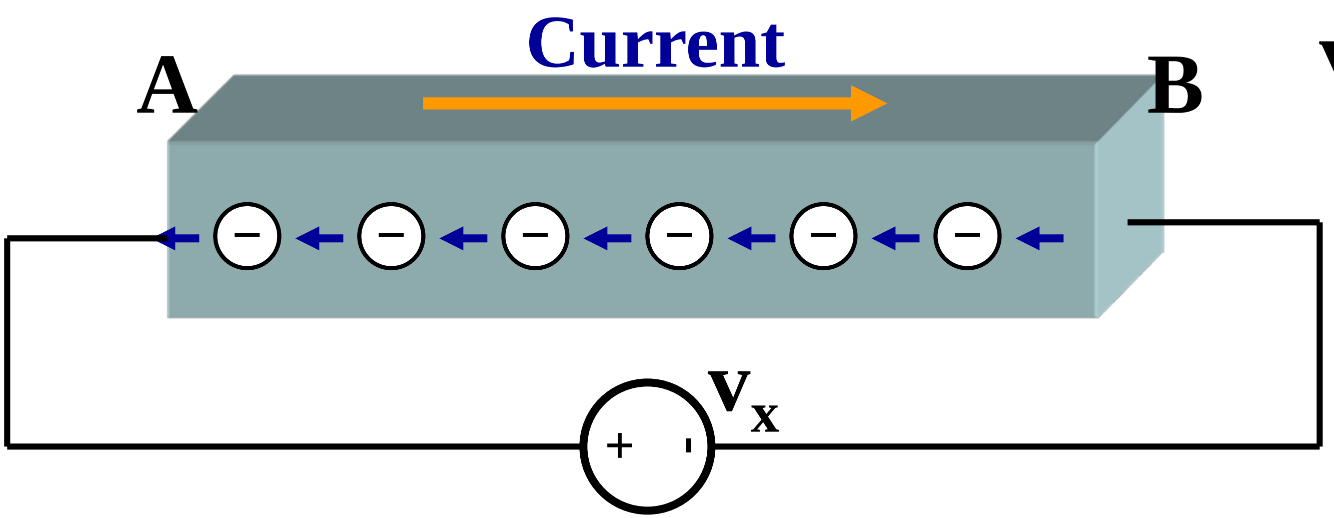


Fig 6

Sign Convention

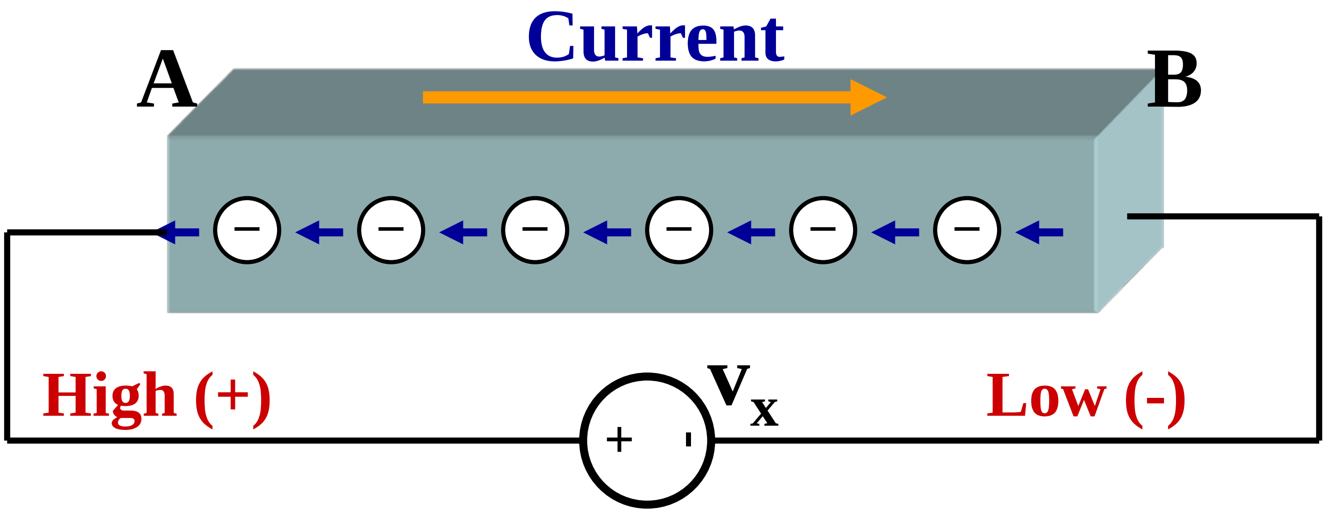


Fig 6

Sign Convention for Voltage

Voltage **rises** (negative to positive) along the direction of the current for an element that is **generating** power

Voltage **drops** (positive to negative) along the direction of the current for an element that is **consuming** power

Kirchhoff's Voltage Law (KVL)

- KVL states that the sum of voltages around a loop must equal to zero
- If a voltage starts at any given point, no gain or loss of any voltage when it returns to the same point
- Whatever energy is pumped into the loop must also be consumed

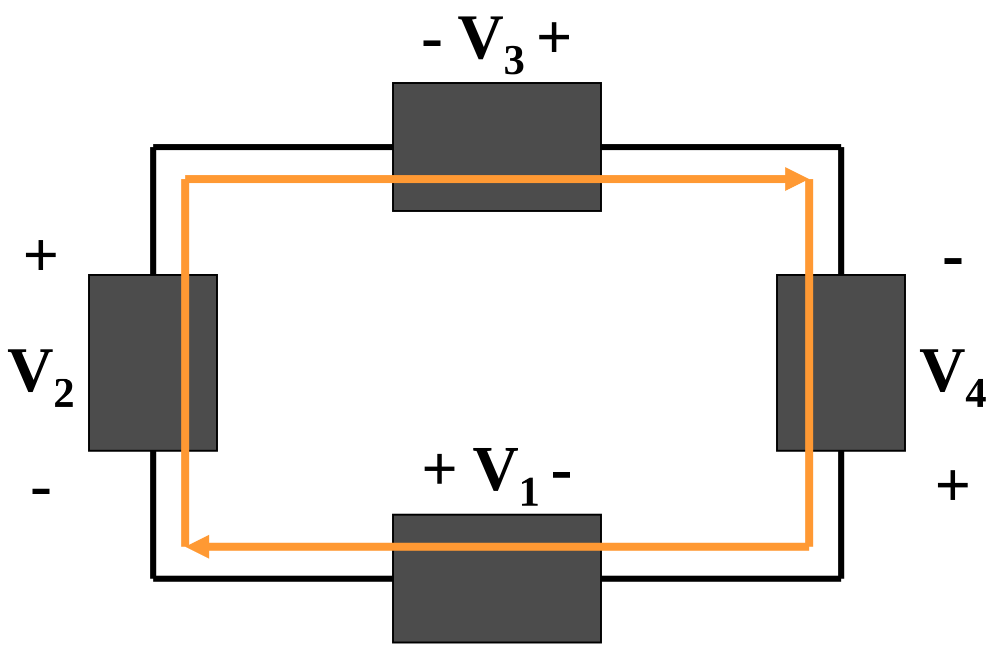


Fig 7: Voltages around a closed loop

Kirchhoff's voltage law

Net voltage around a closed circuit is zero:

$$\mathbf{v_1 + v_2 + v_3 + v_4 = 0}$$

Kirchhoff's voltage law

- First, you need to define the direction
- V_1 and V_3 are voltage rises (positive) while V_2 is a voltage drop (negative)
- In the other direction, V_2 is voltage rise (positive), while V_1 and V_3 are voltage drops (negative)

  $V_2 - V_1 - V_3 = 0$

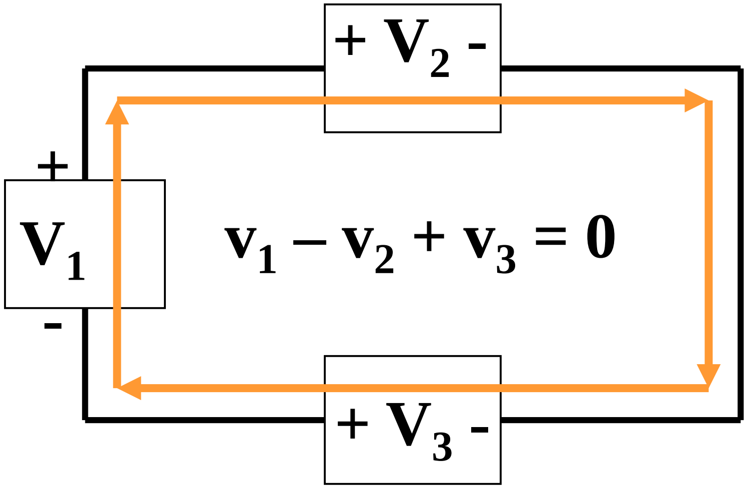


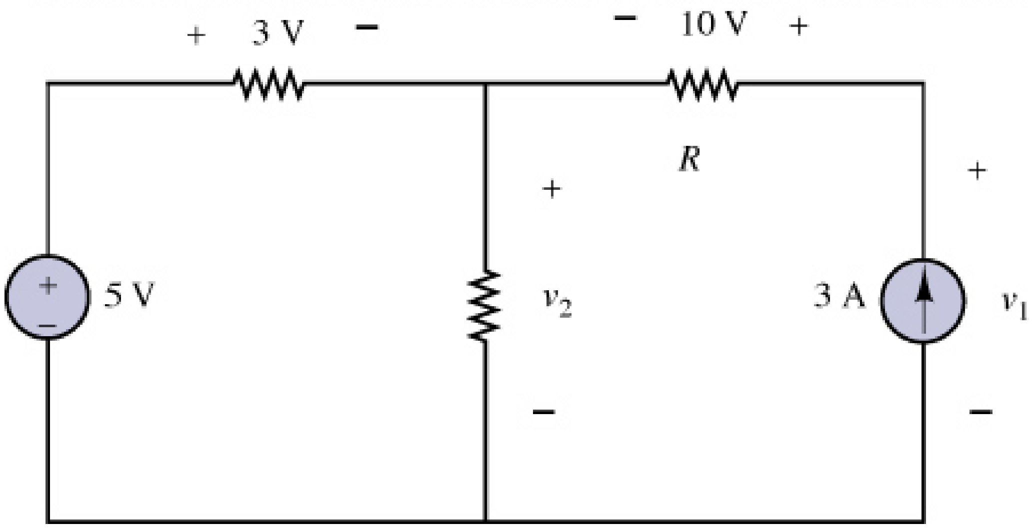
Fig 8: Defined voltage drops and rises

Sign Convention for KVL
Voltage RISE – Positive
Voltage DROP – Negative

Worked Example on KVL

Apply KVL to find voltage V_1 and V_2

$V_1 = 12V$, $V_2 = 2V$



II. Resistance and Ohm's Law

- When current flows through a conductor, it will always experience some resistance
- Resistance is given by the change in voltage over change in current
- SI unit is Ohm (Ω)
- If the voltage is linear with current, then resistance is said to be linear and obeys Ohm's law

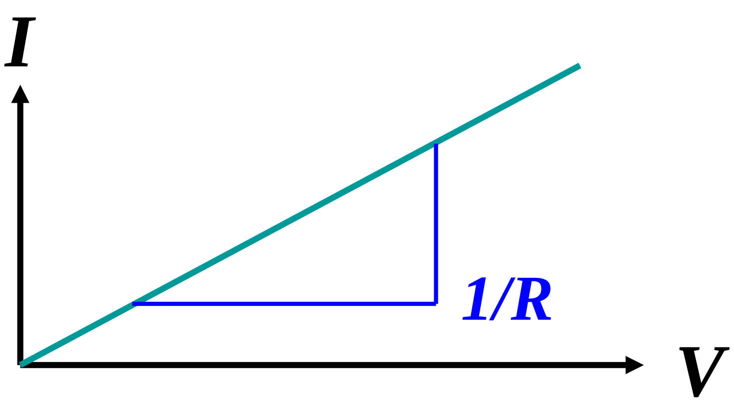


Fig 9a: Linear resistance that obeys Ohm's law

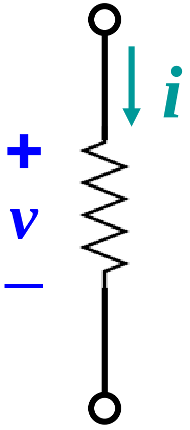


Fig 9b

Ohm's law is given as:

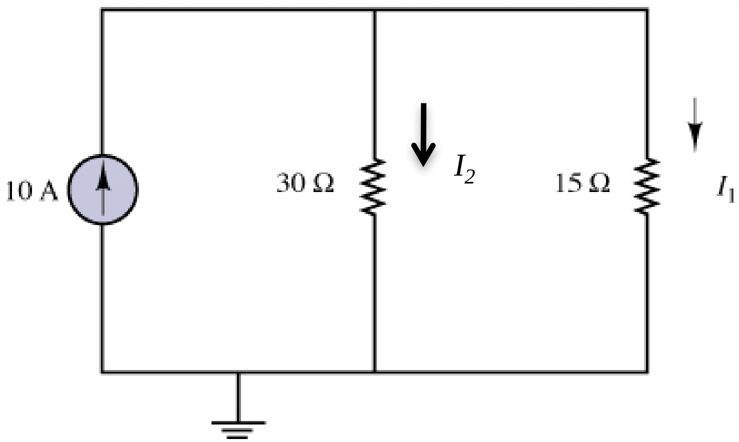
$V = IR$

Important point to note:

If an unknown current is defined to flow from A to B, the voltage at A is assumed to be higher than B.

Worked Example Applying 3 Laws

Find the current through the 15Ω resistor



KCL: $I_1 + I_2 = 10A$

Ohm's:

$15I_1 = V_{15\Omega}$ (1); $30I_2 = V_{30\Omega}$ (2)

KVL: $V_{15\Omega} = V_{30\Omega}$

Therefore, $2I_2 = I_1$

Solving for the variables:

$I_2 = 3.33A, I_1 = 6.67A$

III. Electrical Power

- A voltage drop in the direction of the current indicates the power is consumed
- Conversely, a voltage rise in the direction of the current indicates that power is generated
- If the voltage across a resistor is V, and current through it is I, then the power consumed is given by:

  $P = VI$

  Ohm's law → $P = I^2R$

  Ohm's law → $P = V^2/R$

Unit: Watt (W)

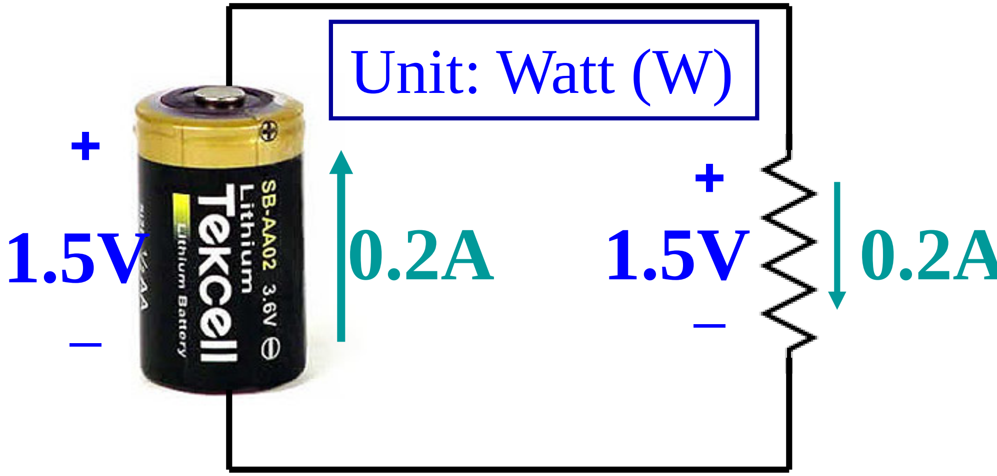


Fig 10: Power generated and consumed in a circuit

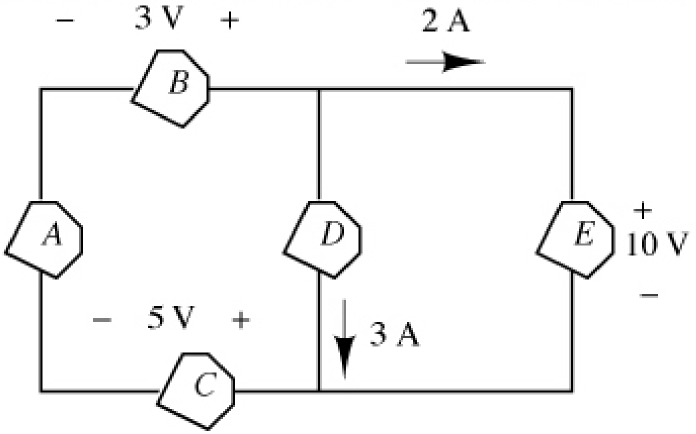
Power generated by source

MUST EQUAL

Power dissipated in the load

Worked Example on Power

Determine which components are absorbing power and which are delivering power



1) Find the voltage drops across each element

Apply KVL,

Loop 2: D = 10V

Loop 1: 10V - 3V - VA + 5V = 0⇒VA = 12V

2) Find all branch currents

Apply KCL: I = 5A

Delivering: A (60W), B (15W)

Consuming: C (25W), D (30W), E (20W)

Terminology: Branch and Node

BRANCH: Any **path** of a circuit with 2 TERMINALS connected to it

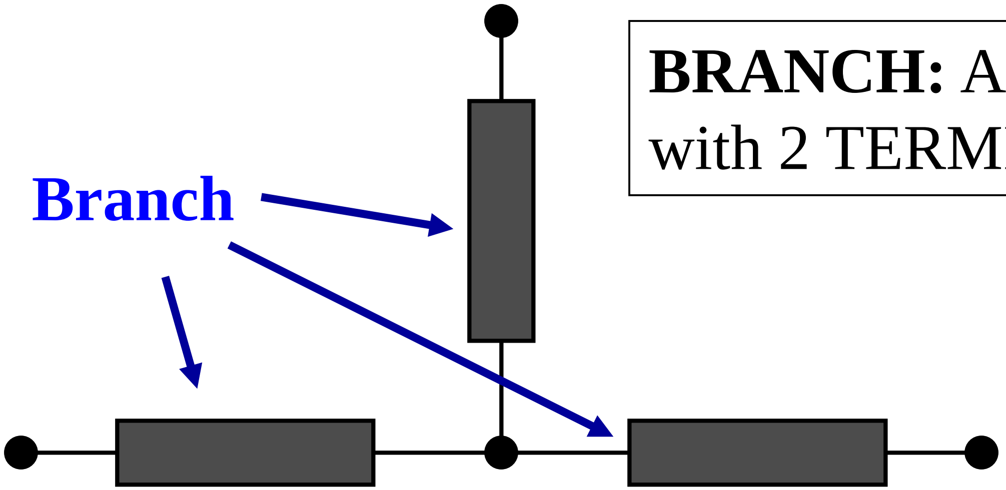


Fig 11a: 3 branches

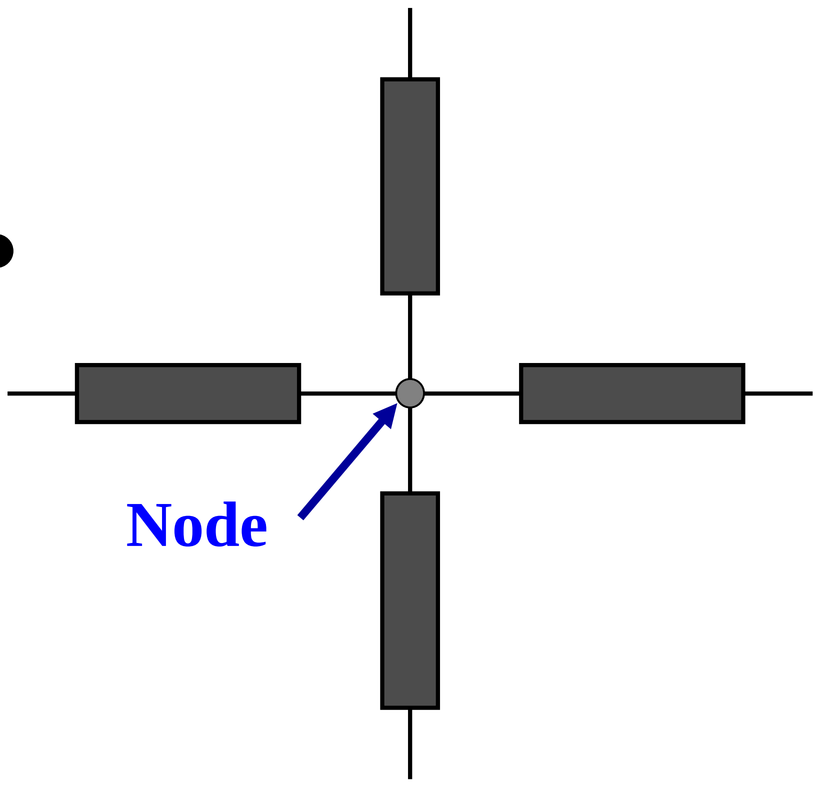


Fig 11b: 1 node

NODE: **Junction** of 2 or more branches

Terminology: Loop and Mesh

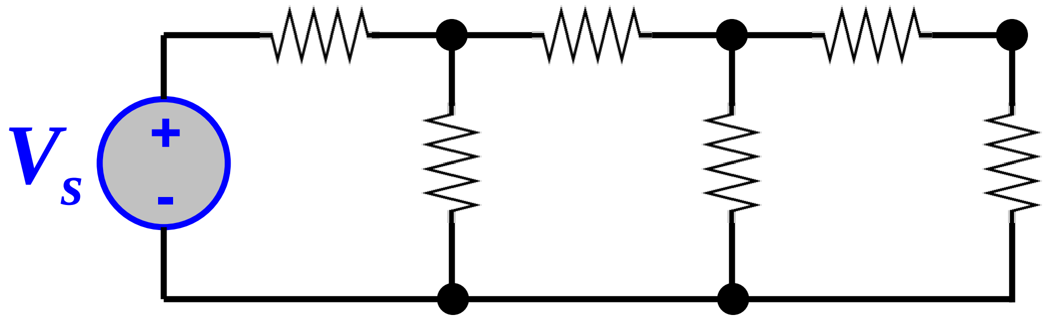


Fig 12: Loops (6) and meshes (3)

- **Loop**: Any closed connection of branches
- **Mesh**: Loop that does not contain any loops
- In Fig 12, there are 6 loops and 3 meshes.
- We will use the term mesh more than loop in this course

IV. Sources, Short Circuit and Open Circuit

- Loads (e.g. resistors) consume power
- Sources on the other hand deliver power

Ideal Voltage Source

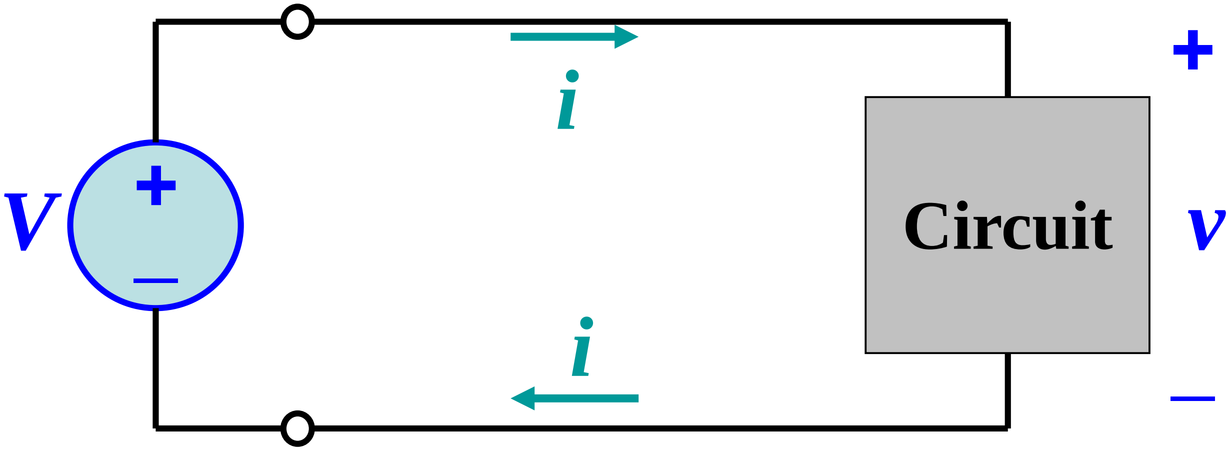


Fig 13a: Ideal voltage source in a circuit

- The purpose of a voltage source is to keep the voltage across its terminals unchanged
- Current through a voltage source is allowed to change in order to maintain the voltage at V with reference to Fig 13a

Ideal Current Source

Ideal Current Source

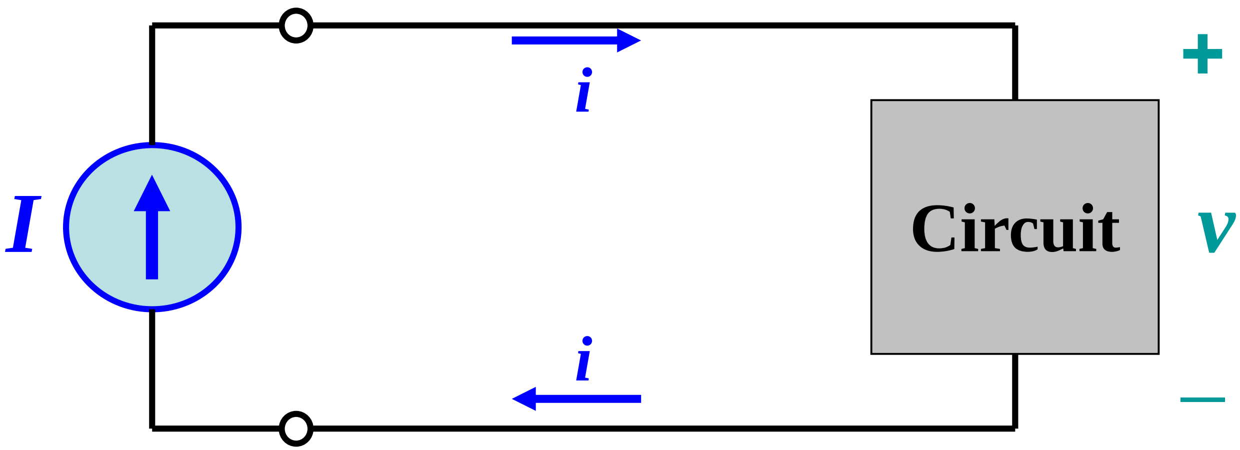


Fig 13b: Ideal current source in a circuit

- The purpose of a current source is to keep the current flowing through it unchanged
- Voltage across a current source is allowed to change in order to maintain the current at I with reference to Fig 13b

Recognize the differences between an ideal voltage and current source in their functions and symbols.

Short Circuit

- A short circuit means connecting 2 or more terminals together so that the voltage between them is zero
- It is typically associated with currents rather than voltage, e.g. short circuit current

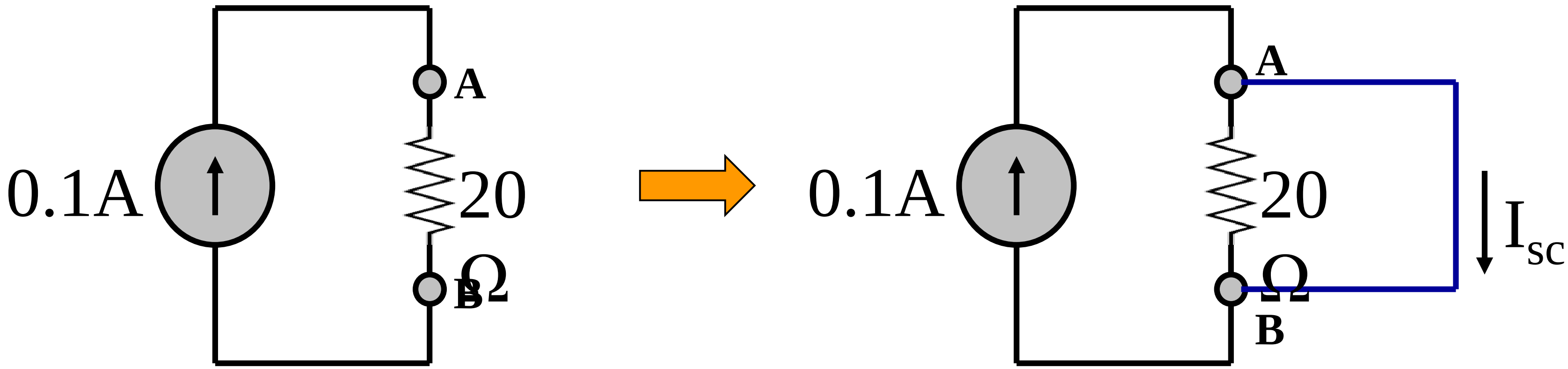


Fig 14a: Short circuiting the resistor

- All the current from the source flows through the short circuit, bypassing the resistor
- Short circuit current I_{SC} = 0.1 A

Open Circuit

- A open circuit means no extra connections are imposed across two terminals
- It is typically associated with voltage rather than current, e.g. open circuit voltage

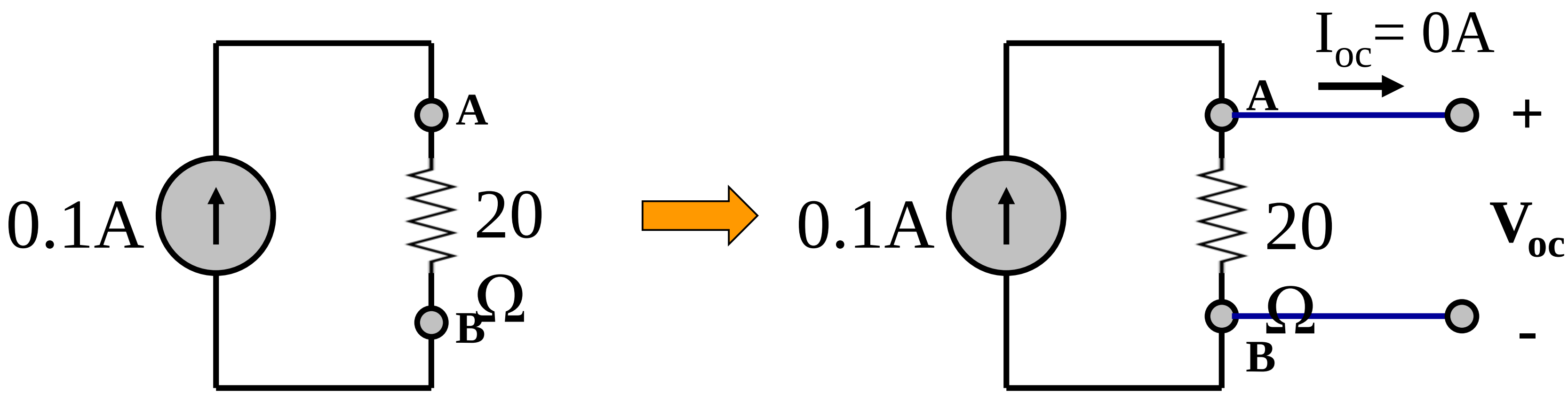
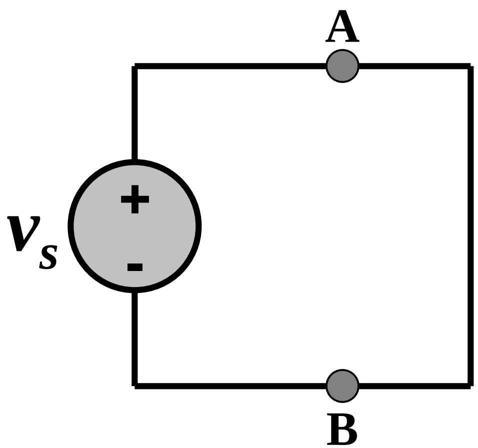


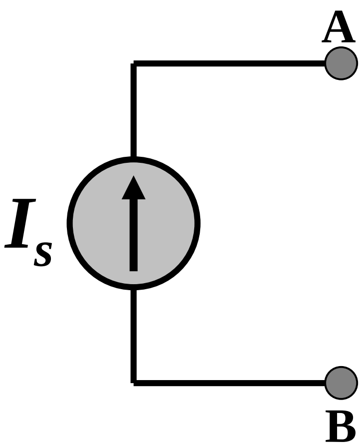
Fig 14b: Open circuiting the resistor

- In Fig 14b, the open circuit voltage of the resistor is simply the voltage across the resistor
- Open circuit Voltage V_{OC} = 2 V

Self-contradictory circuits



What is the voltage across A and B?



What is the current through the source?

Prefixes

- As engineers, it is important that we replace exponents with prefixes

- For example:
  0.00215 A → 2.15 mA
  never 2.15×10^{-3}A

- Make sure you memorize the prefixes from nano (n) to mega (M)

Prefixes: Memorize and **apply** them!

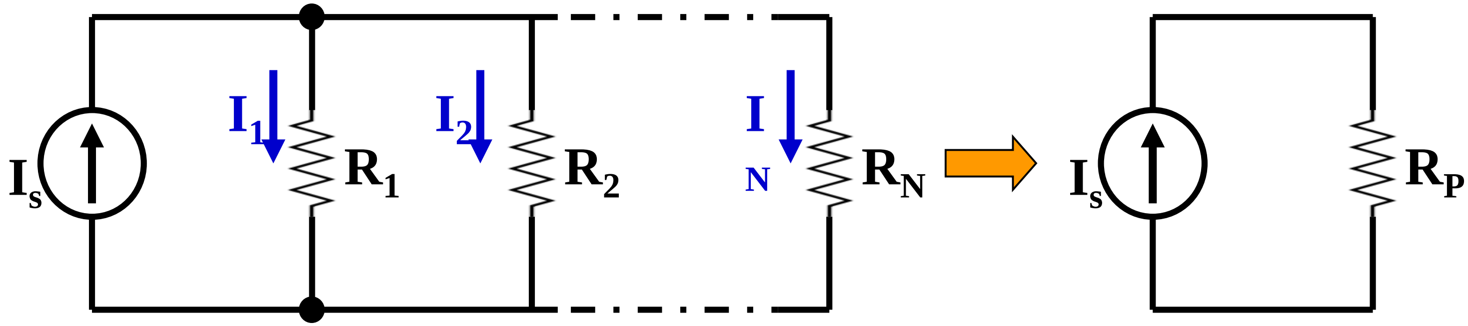
tera	T	10^{12}
giga	G	10^{9}
mega	M	10^{6}
kilo	k	10^{3}
milli	m	10^{-3}
micro	μ	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}
femto	f	10^{-15}

V. Resistive Networks

- Resistors are either arranged in parallel or in series or as combination of both

Parallel Network

Fig 15a: Parallel resistive network



Equivalent Resistance *Rp*:

$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots + \frac{1}{R_N}$$

Current Divider Rule

$$I_k = \frac{R_P}{R_k} I_S$$

- Current (I_s) from the source will have to be shared between the resistors (R_k) in each branch

Series network (Highlights)

Series Network

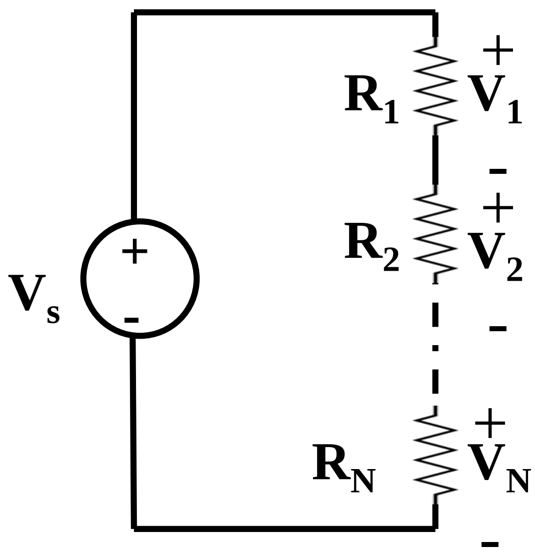
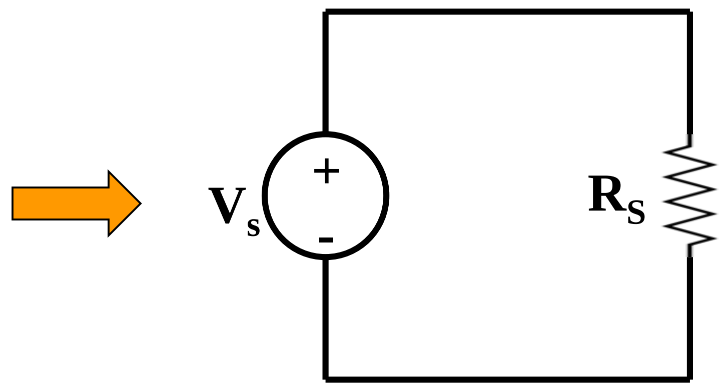


Fig 15b: Series resistive network



Equivalent Resistance *Rp*:

$$R_S = R_1 + R_2 + \ldots + R_N$$

Voltage Divider Rule

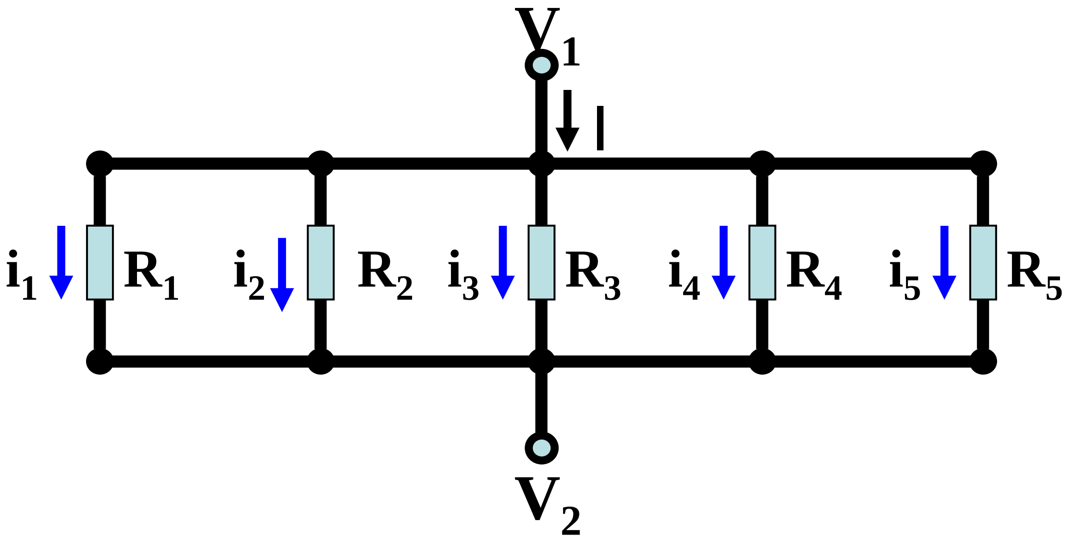
$$V_k = \frac{R_k}{R_S} V_S$$

- Total voltage drop across all the resistors (V_s) in series is the sum total of voltage (V_k) drops across each resistor

Worked Example on Current Divider Rule

Rank the currents from largest to smallest if $\mathbf{R_2 > R_4 > R_1 > R_5 > R_3}$

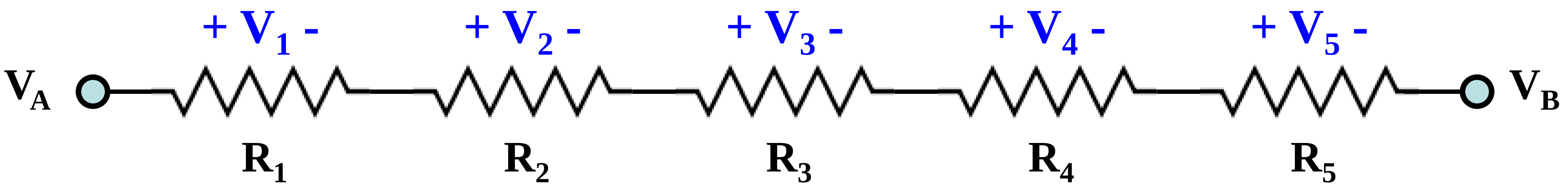
$I_3 > I_5 > I_1 > I_4 > I_2$



Worked Example on Voltage Divider Rule

Rank the potential difference from largest to smallest
if $\mathbf{R_2 > R_4 > R_1 > R_5 > R_3}$

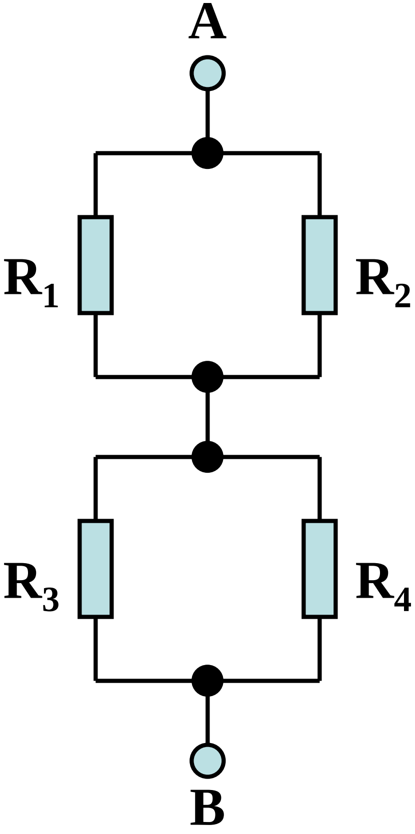
$V_2 > V_4 > V_1 > V_5 > V_3$



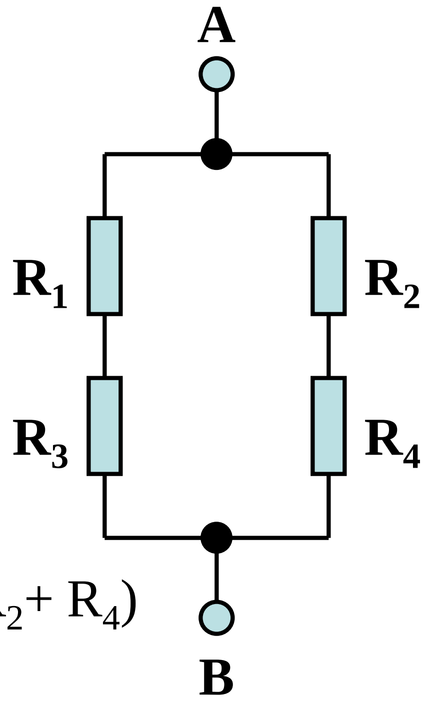
Worked Example 1 on Resistive Networks

Find the effective resistance across A and B

$R_1 = R_4 = 1\Omega$, $R_2 = R_3 = 3\Omega$



R_{AB}
$= (R_1 \| R_2) + (R_3 \| R_4)$
$= 1\|3 + 3\|1$
$= 1.5\Omega$



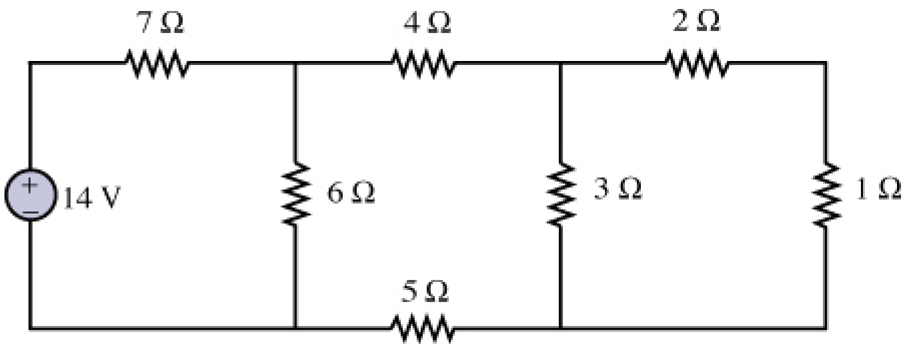
R_{AB}
$= (R_1 + R_3) \| (R_2 + R_4)$
$= 4\|4$
$= 2\Omega$

Worked Example 2 on Resistive Networks

What is the equivalent resistance seen by the source?

What is power supplied by the source?

R_{eq} = 10.818 Ω
P = 18.1 W



VI. Measuring Instruments - Voltmeter

- Voltmeter measures voltage across a circuit element

Fig 16: Image of typical voltage with a measuring range up to 30V

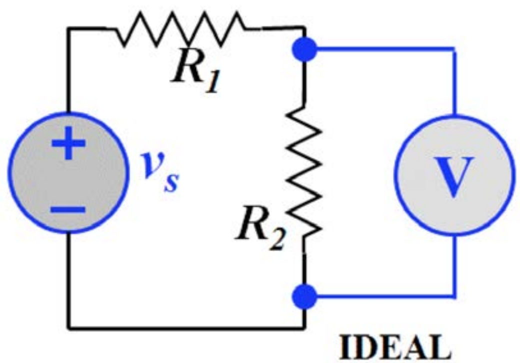


Fig 17a: Placement of an ideal voltmeter to measure the PD across R_2

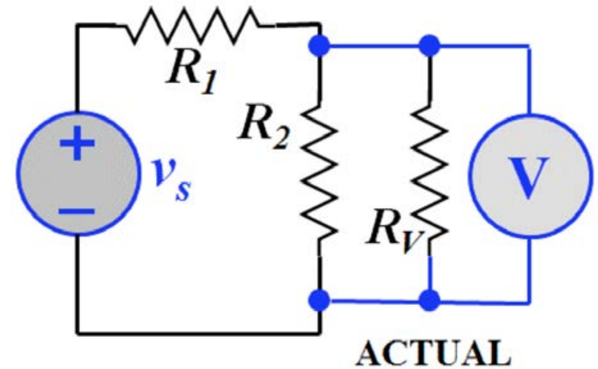


Fig 17b: Circuit model of how a real voltmeter behaves, which looks like a large parallel resistor (R_v)c

How would you place the voltmeter in the circuits?

- Connected in **parallel** with the element being measured

In Fig 17a, voltmeter has no effect on original circuit.
In Fig 17b, voltmeter will steal current from R_2. This will change the voltage measured across R_2. **Thus, the PD measured with a voltmeter is lower than the true value.**

Worked Example on Voltmeters

Referring to Fig 17b, if V_s = 5V, and R_V = 1MΩ, find the voltage across R_2 when

(i) $R_1 = R_2 = 1k\Omega$ V = 2.5V

(ii) $R_1 = R_2 = 1M\Omega$ V = 1.67V

Rv || R2 = 0.999 kOhm

R2 / (R1 +R2) = 0.4998

0.4998 * Vs = 2.499

Rv || R2 = 0.5 MOhm

R2 / (R1 + R2) = 0.3333

0.3333 * Vs = 1.66666

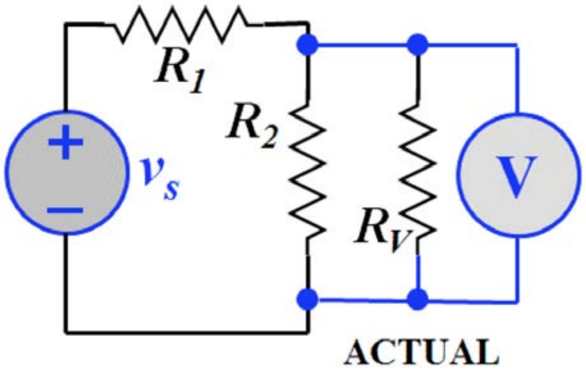


Fig 17b

Ammeter

- Ammeter measures current

Fig 18: Image of typical current with a measuring range up to 20A

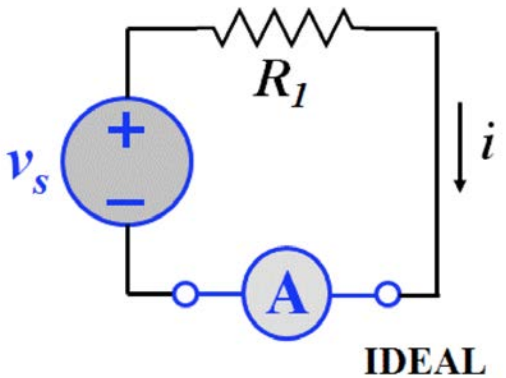


Fig 19a: Placement of an ideal ammeter to measure current through R_1

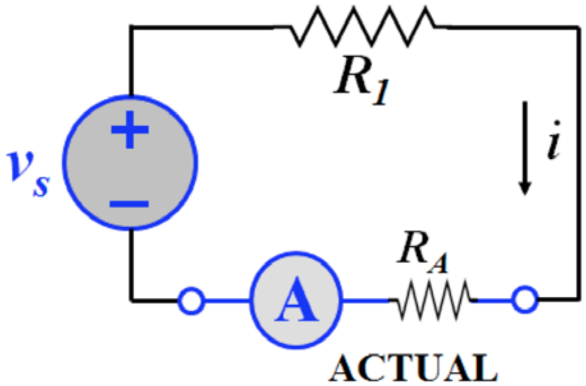


Fig 19b: Circuit model of how a real ammeter behaves, which looks like a small series resistor (R_A)

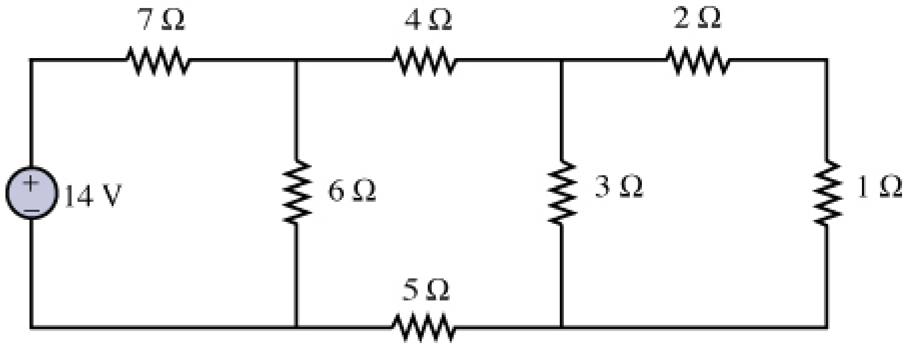
How would you place the ammeter in the circuits?

- Connected in **series** with the element being measured

In Fig 19a, ammeter has no effect on original circuit.
In Fig 19b, ammeter contains some resistance (R_A), adding to the total resistance, lowering the measured current. **Thus, the current measured with an ammeter is lower than the true value.**

Ammeter in Circuit

How would you place the ammeter in the circuits of the previous worked examples?



Ammeter Example

Referring to Fig 19b, if V_s = 5V, and R_A = 1Ω, find the loop current when

(i)R_1 = 1kΩ I = 4.995 mA

(ii)R_1 = 2Ω I = 1.67A

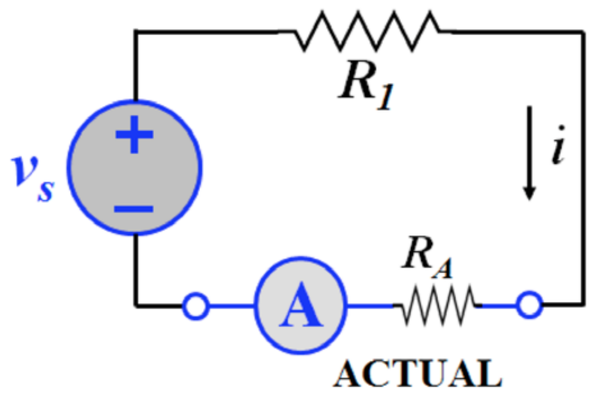


Fig 19b

1.2 Circuit Analysis Methods

- It is important to apply the 3 basic laws in circuit theory to analyze and design circuits
- This unit is organized into the following 3 sections
  - > Nodal voltage analysis (application of KCL and Ohm's law)
  - > Mesh current analysis (application of KVL and Ohm's law)
  - > Superposition

Alexander & Sadiku, "Fundamentals of Electric Circuits" 5th Edition Ch 3, 4.3

I. Nodal Voltage Analysis (NVA)

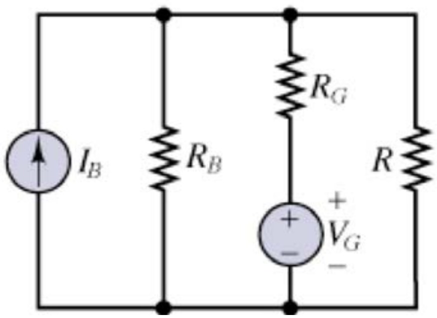
KCL & Ohm’s law in action

- The method of nodal voltage analysis is an application of KCL and Ohm’s law together
- The unknown variables that you will solve for are **node voltages**
- We will apply KCL at a mode and express the unknown currents as unknown node voltages

Example on how to do NVA

Find the voltage across the current source, V_S:

Given: $I_B = 12A$, $V_G = 12V$, $R_G = 0.3\Omega$, $R_B = 1\Omega$, $R = 0.23\Omega$



Apply KCL at V_S:

$$I_B = \frac{V_S}{R_B} + \frac{V_S - V_G}{R_G} + \frac{V_S}{R}$$

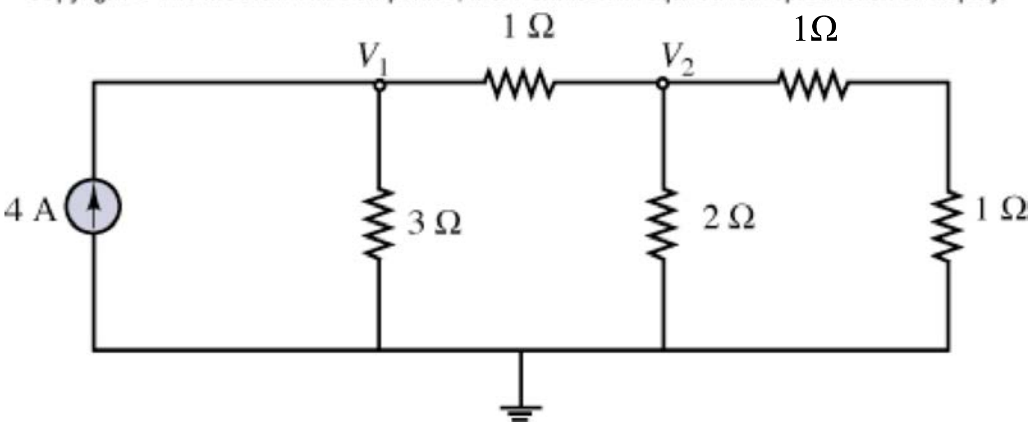
$$12 = \frac{V_S}{1} + \frac{V_S - 12}{0.3} + \frac{V_S}{0.23}$$

$$V_S = 5.99V$$

- Note that the negative terminal of V_G is used as a reference (0V) in the example
- The value of V_S is also referenced to this node
- The value of the reference node is not important since we are only interested in the voltage difference

Worked Example on NVA 1

Use nodal voltage analysis to find V_1 and V_2. (Answer: V_1 = 4.8V, V_2 = 2.4V)



First method:

-Apply KCL at V_1 (1)

-Apply KCL at V_2 (2)

-Solve equations (1) and (2)

Alternate method:

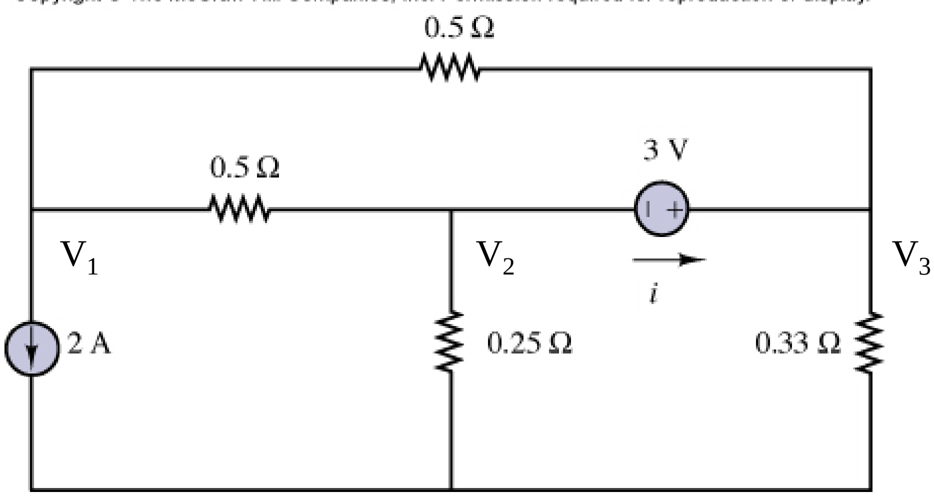
-Combine all the resistors to find V_1 first

-Then combine the 3 resistors on the right followed by voltage divider rule to find V_2

Worked Example on NVA 2

Use nodal voltage analysis to find the current through the 3V source.

(Answer: i = 8.31 A)



Apply KCL at V_1:

$$2 = \frac{V_2 - V_1}{0.5} + \frac{V_3 - V_1}{0.5}$$

$$V_2 + V_3 - 2V_1 = 1 \quad (1)$$

Apply KCL at V_2:

$$\frac{V_1 - V_2}{0.5} = \frac{V_2}{0.25} + i$$

$$2V_1 - 6V_2 = i \quad (2)$$

Apply KCL at V_3:

$$\frac{V_3}{0.33} + \frac{V_3 - V_1}{0.5} = i$$

$$\frac{166}{33} V_3 - 2V_1 = i \quad (3)$$

Since a new variable is introduced in (2), we need one more equation.

$$V_3 - V_2 = 3 \quad (4)$$

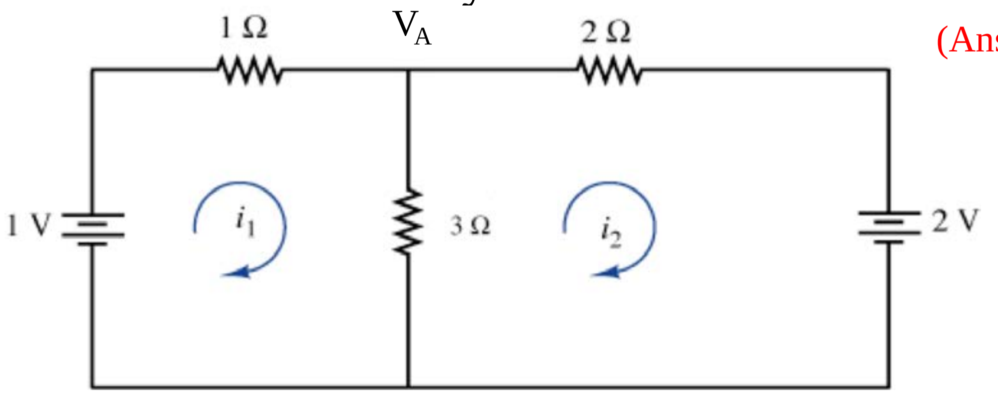
II. Mesh Current Analysis (MCA)

KVL & Ohm's law in action

- The method of mesh current analysis is an application of KVL and Ohm's law together
- The unknown variables that you will solve for are **mesh currents**
- From KVL, the sum of voltage drops and rise must equal zero
- The voltage differences are expressed as the current going through each branch in the mesh

Example on how to do MCA

Use mesh current analysis to find the current through the 3Ω resistor.



(Answer: $i_{3\Omega}$ = 4/11A)

-There are 2 meshes and hence 2 mesh currents

-Apply KVL to each of these 2 meshes

Apply KVL to mesh 1:

$$1 = i_1(1+3) - 3i_2$$

$$4i_1 - 3i_2 = 1 \qquad (1)$$

Useful tip:

- keep voltages of sources on one side of the equation, and keep voltages of resistors on the other side

Apply KVL to mesh 2:

Follow the defined direction of the mesh current

$$-2 = i_2(2+3) - 3i_1$$

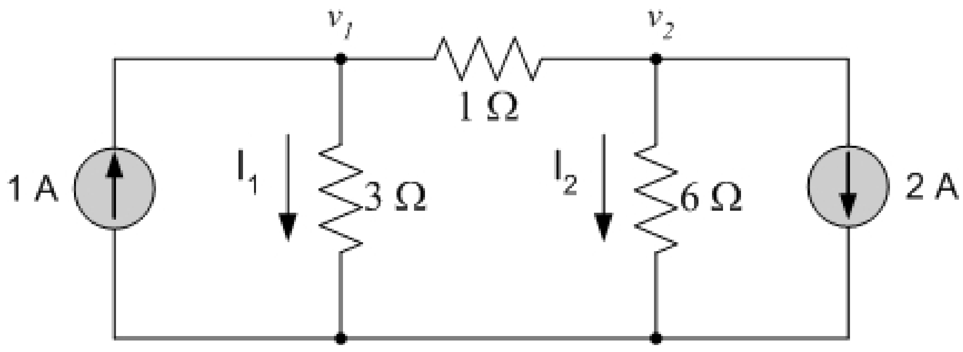
$$5i_2 - 3i_1 = -2 \qquad (2)$$

Now, solve equation (1) and (2):

$\mathbf{i_{3\Omega} = i_1 - i_2}$

Worked Example on MCA 1

Use mesh current analysis to find currents I_1 and I_2. (Answer: $I_1 = I_2 = -0.5A$)



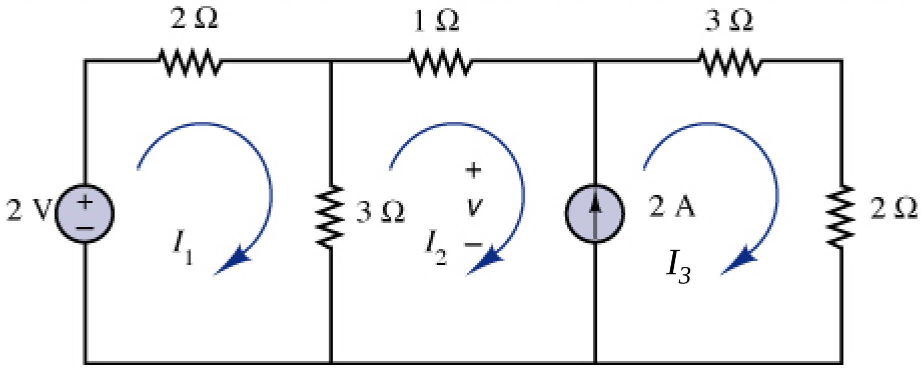
-You see 3 meshes, but only the middle mesh is unknown, the other 2 are defined by the current sources.

-For simplicity, defined the direction of known mesh currents in the same direction as the sources.

Worked Example on MCA 2

Use mesh current analysis to find the voltage across the current source.

(Answer: V = 3.89 V)



-In the process of deriving equations (2) and (3), a new variable V is introduced.

-Hence, we need one more equation, (4)

Apply KVL at mesh 1:

$2 = I_1(2+3) - I_2(3)$

$\Rightarrow 5I_1 - 3I_2 = 2 \qquad (1)$

Apply KVL at mesh 2:

$-V = I_2(3+1) - I_1(3)$

$\Rightarrow 4I_2 - 3I_1 = -V \qquad (2)$

Apply KVL at mesh 3:

$V = I_3(3+2)$

$\Rightarrow 5I_3 = V \qquad (3)$

One more equation:

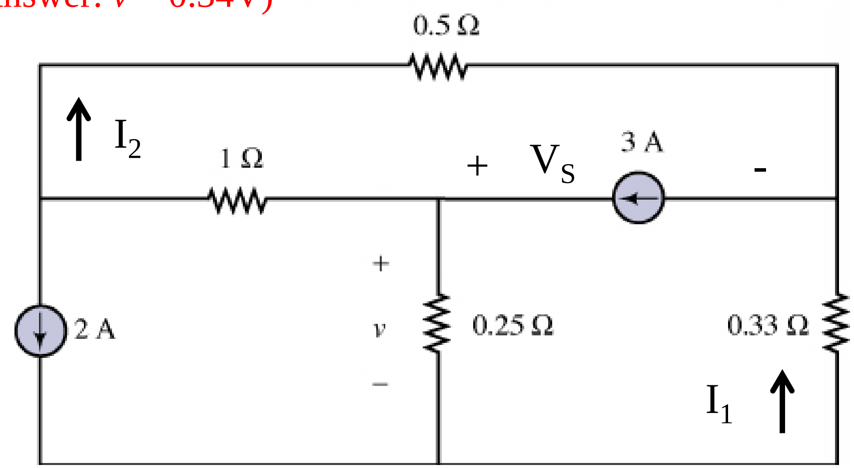
$I_3 - I_2 = 2A \qquad (4)$

Solve for V, verify by V = $(I_3)(3+2)$

Worked Example on MCA 3

Use mesh current analysis, find the voltage across the 0.25 Ω resistor.

(Answer: v = 0.34V)



Method:

-Find the current through 0.25Ω resistor first, then use this to find the voltage.

-Although there are 3 meshes, the bottom left mesh is already known (2A going anti-clockwise).

Apply KVL at mesh 1:

$V_s = I_2(0.5 + 1) + (2)(1)$

⇨ $V_s = 1.5I_2 + 2$ (1)

Apply KVL at mesh 2:

$V_s = I_1(0.25 + 0.33) - (2)(0.25)$

⇨ $V_s = 0.58I_1 - 0.5$ (2)

Consider the current source:

$I_1 + I_2 = 3$ (3)

Solve for I_1 and I_2

Current through 0.25 Ω resistor: $I_1 - 2$

$V = (I_1 - 2)(0.25)$

Quick note on choosing NVA and MCA

How do you choose between NVA and MCA?

-Your choice should not be due to level of familiarity between the methods.

- One consideration is whichever is simpler to use for a given circuit.

- How then do you decide what is simpler?

- For a start, having fewer equations certainly makes solving easier.

III. Superposition

The method

- This method applies only to circuits that have **<u>multiple sources</u>**
- In such case, it can come in handy (or not)
- In a circuit with multiple sources, superposition considers the current or voltage associated with a given branch for one of the sources, while turning the rest off

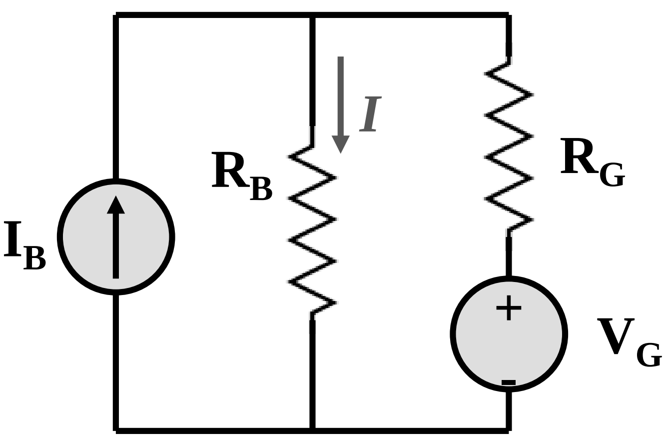


Fig 1: Basic circuit with 2 sources

Aim: Find the current through R_B.

1. **Find the current (I_1) through R_B when only I_B is present (V_G is removed)**
2. **Find the current (I_2) through R_B when only V_G is present (I_B is removed)**
3. **The net current $I = I_1 + I_2$**

How do we “remove” a current source or voltage source?

<u>Disabling sources</u>

Voltage source: If the voltage source does not exist, the voltage across the terminals would be zero. It looks like a short circuit. (see Fig 2a)

Current source: If the current source does not exist, the current through it would be zero. It looks like an open circuit. (see Fig 2b)

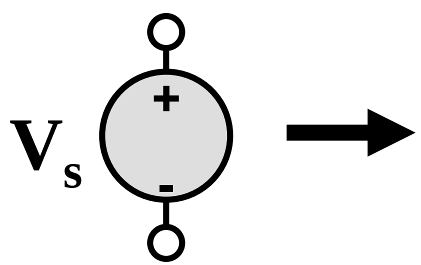


Fig 2a: Disable a voltage source by Replacing with a short circuit.

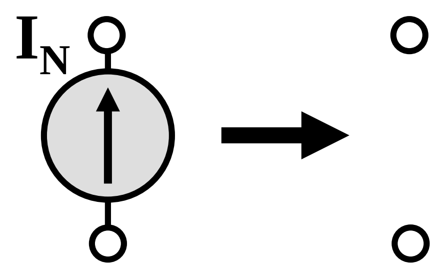


Fig 2b: Disable a current source by Replacing with an open circuit.

Apply the tips to Fig 1

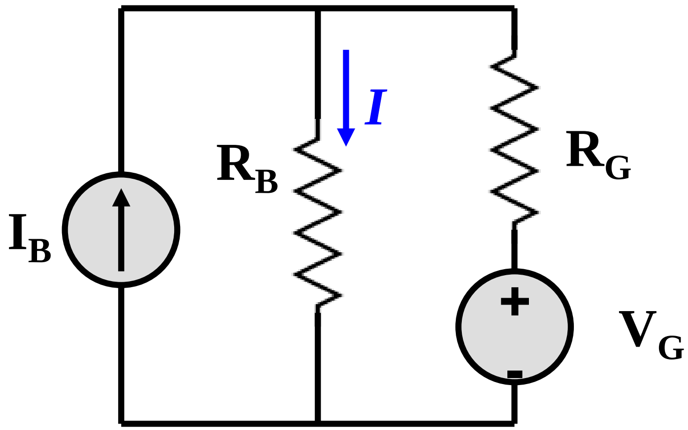
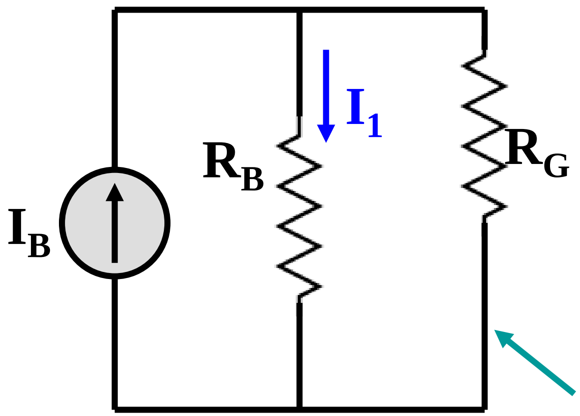


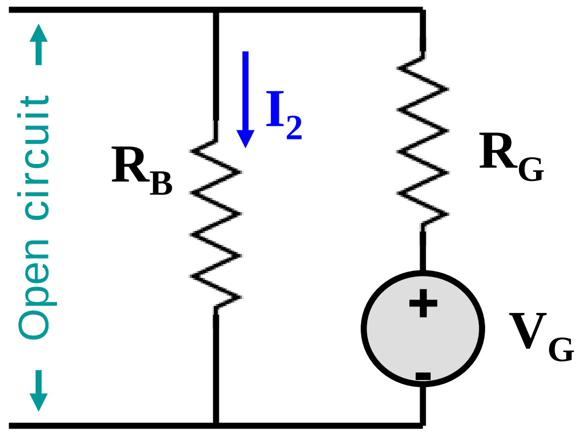
Fig 1: Basic circuit with 2 sources

Disabling $V_{G,}$ $V_G = 0$



$$\boxed{I = I_1 + I_2}$$

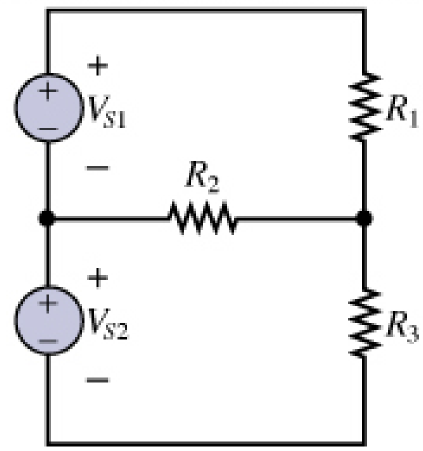
Disabling I_B , $I_B = 0$



Worked Example on Superposition

Determine the current through R_1 using superposition.

Given R_1 = 560Ω, R_2 = 3.5kΩ, R_3 = 810Ω, $V_{S2} = V_{S1}$ = 90V



a. First, short V_{S1}

Voltage across R_1:

$$V_{R1a} = \frac{R_1 \| R_2}{(R_1 \| R_2) + R_3} V_{S2}$$

$$= \left(\frac{560 \| 3500}{(560 \| 3500) + 810} \right) 90$$

$$= 33.61V$$

Current through R_1:

$$I_{R1a} = V_{R1a} / R_{1a}$$
$$= 33.61 / 560$$
$$= 0.06A$$

Worked Example on Superposition

b. Next, short V_{S2}

Current through R_1:

$$I_{R1b} = \frac{V_{S1}}{R_1 + \left(R_2 \parallel R_3\right)} = \frac{90}{560 + \left(3500 \parallel 810\right)}$$
$$= 0.074A$$

Take careful note of the directions defined for each of these currents.

Finally,

$I_{R1} = I_{R1a} + I_{R1b} = 0.134A$

1.3 Equivalent Circuit Transformation and Dependent Sources

- One port network
  - > Thevenin equivalent circuit
  - > Norton equivalent circuit
  - > Source transformation
- Dependent sources
  - > Dependent voltage source
  - > Dependent current source
- Maximum power transfer

Alexander & Sadiku, "Fundamentals of Electric Circuits" 5th Edition Ch 4.4 – 4.6, 4.8

One-Port Network

Introduction

- A **one port network** is simply a **two terminal device** (See Fig below)

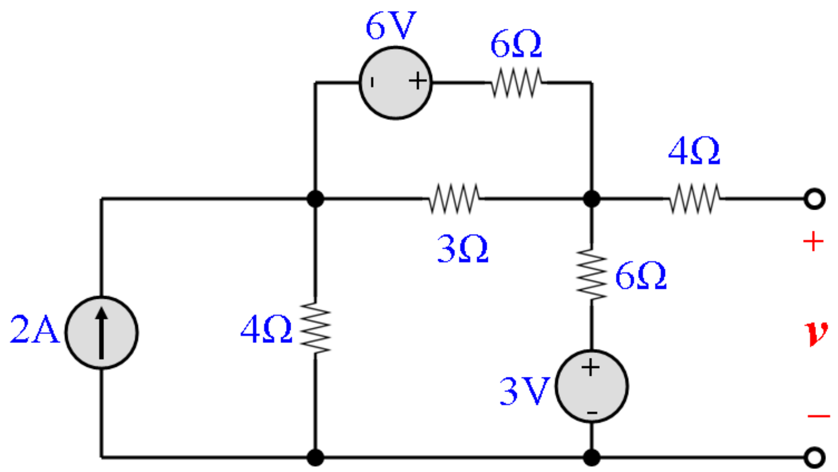


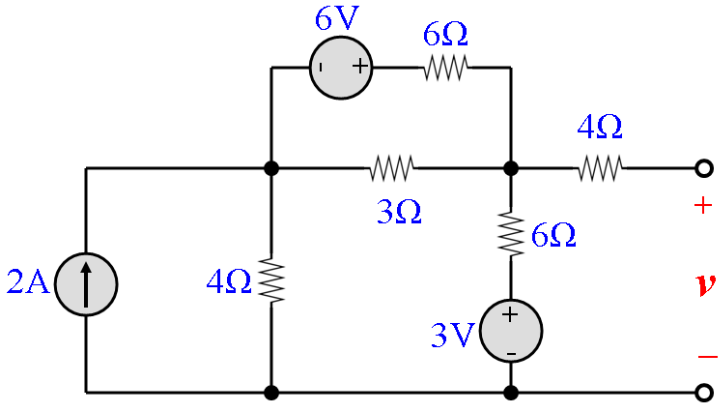
Fig 1: An example of a one port network

- Given a number of different resistor loads, if you were asked to find the current and voltage at the terminals for each resistor, you would have to recalculate the whole circuit
- A different load will give you different output current & voltage

One-Port Network

Simpler analysis

- Transform any 2 terminal circuit into a circuit that is as simple as having 1 resistor and 1 source (see Fig 2)



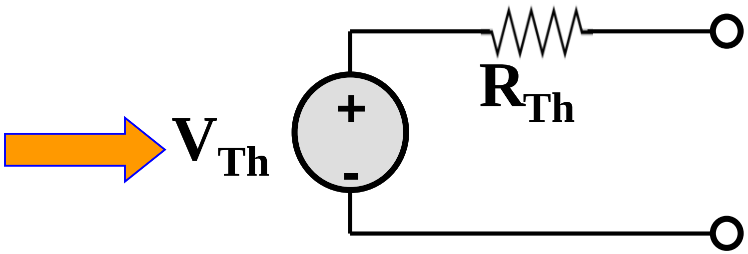


Fig 2: Transformed Thevenin equivalent circuit

The first part of this unit is organized into the following 3 sections:

- Thevenin equivalent
- Norton equivalent
- Source transformation

I. Thevenin

Definition

- Any one-port network composed of **ideal** voltage and current sources, and **linear** resistors, can be represented by an equivalent circuit consisting of an ideal **voltage source** V_{Th} in **series** with an equivalent **resistance** R_{Th}

Deriving the Thevenin equivalent circuit

Step 1: Remove the load from the rest of the one port network. This is the first most fundamental step.

Step 2: Find the equivalent resistance R_T. Disable all ideal sources then find the resistance across the terminals.

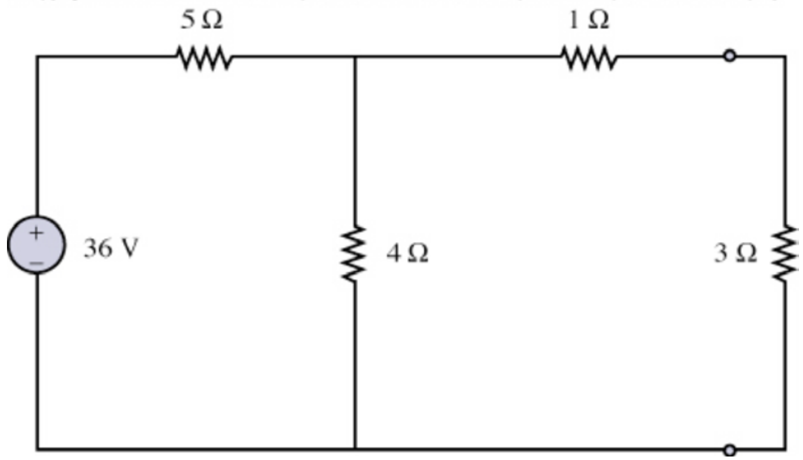
For voltage source – replace with short circuit

For current source – replace with open circuit

Step 3: Find the Thevenin voltage source. The Thevenin voltage source is equal to the open circuit voltage seen across the terminals (with no load). Solve for this voltage using any preferred method (NVA, MCA, superposition).

Worked Example on Thevenin 1

Derive the Thevenin equivalent circuit seen by the 3Ω load.

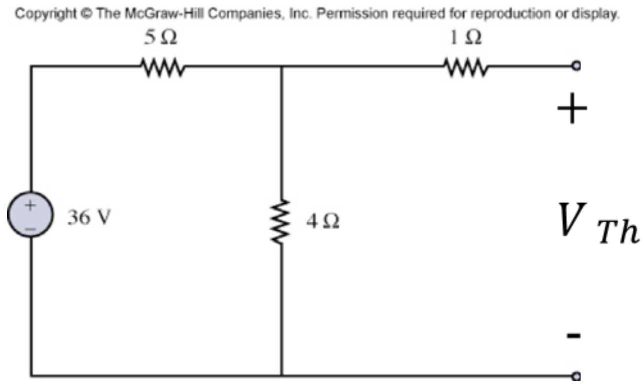


Steps:

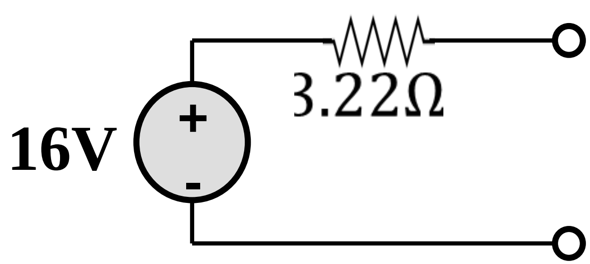
1. Remove the 3Ω load
2. Find R_{Th} (disable the voltage source)
3. Find V_{Th} (with no load)
4. Draw the Thevenin circuit

Worked Example on Thevenin 1

- Step 1:

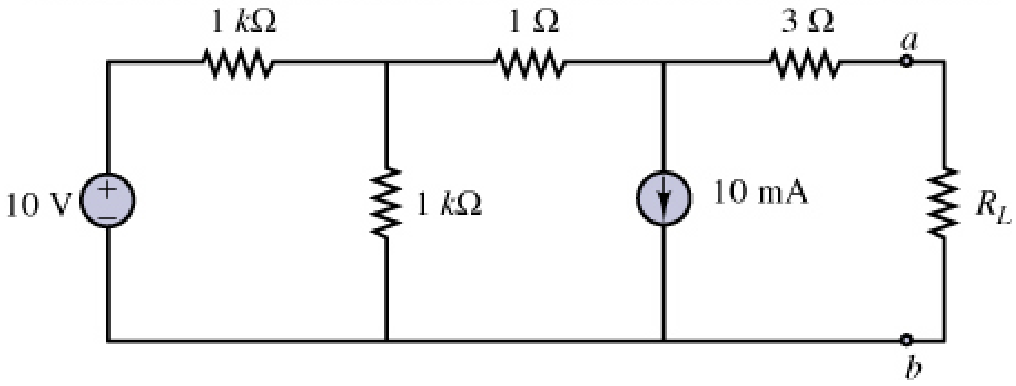


- Step 2: $R_{Th} = 1 + 5||4 = 3.22\Omega$
- Step 3: $V_{Th} = 36 \times \frac{4}{5+4} = 16V$
- Step 4:



Worked Example on Thevenin 2

Derive the Thevenin equivalent circuit as seen by the resistor R_L.

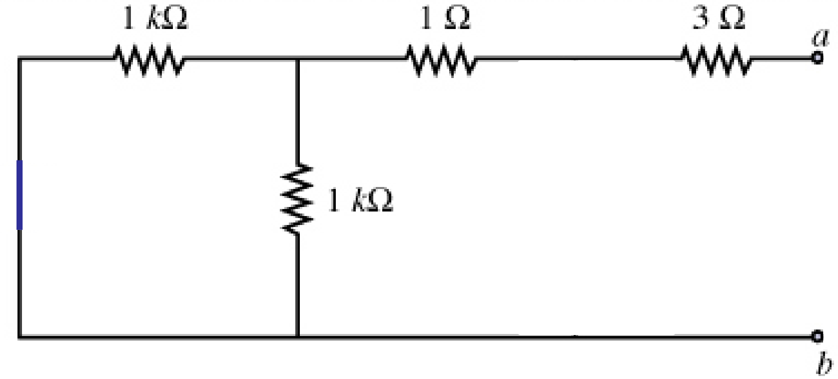


Steps:

1. Remove the load
2. Find R_{Th} (disable the voltage and current sources)
3. Find V_{Th} (with no load)
4. Draw the Thevenin circuit

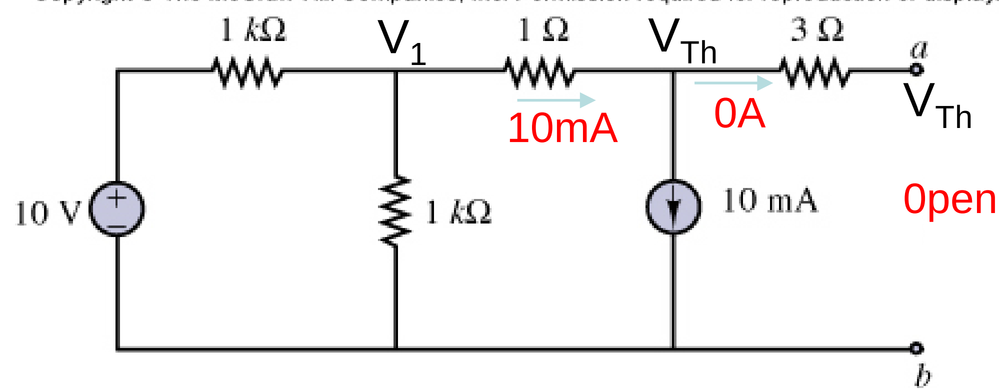
Worked Example on Thevenin 2

- Step 1 & 2:



$$R_{Th} = 1 + 3 + 1000||1000 = 504\Omega$$

- Step 3:



(i) $\frac{V_1 - V_{Th}}{1} = 10 \times 10^{-3}$

(ii) $\frac{10 - V_1}{1000} = \frac{V_1}{1000} + 10 \times 10^{-3}$

From (i), $V_{Th} = V_1 - 10 \times 10^{-3}$

From (ii), $10 - V_1 = V_1 + 10 \Rightarrow 2V_1 = 0 \Rightarrow V_1 = 0V$

Therefore, $V_{Th} = -10\ mV$

II. Norton

Definition

- Any one-port network composed of **ideal** voltage and current sources, and **linear** resistors, can be represented by an equivalent circuit consisting of an ideal **current source** I_N in **parallel** with an equivalent **resistance** R_N

Deriving the Norton equivalent circuit

Steps 1 and 2 for deriving the Norton equivalent circuit are exactly the same as that for Thevenin.

Step 3: Find the Norton current source. The Norton current source is equal to the short circuit current seen across the terminals (with no load). Solve for this voltage using any preferred method (NVA, MCA, superposition).

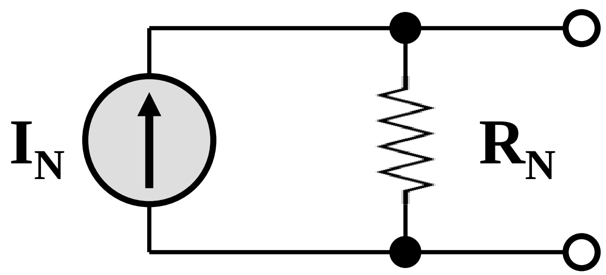
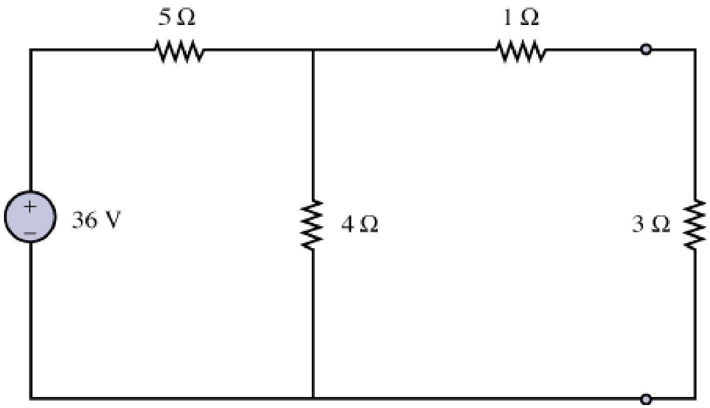
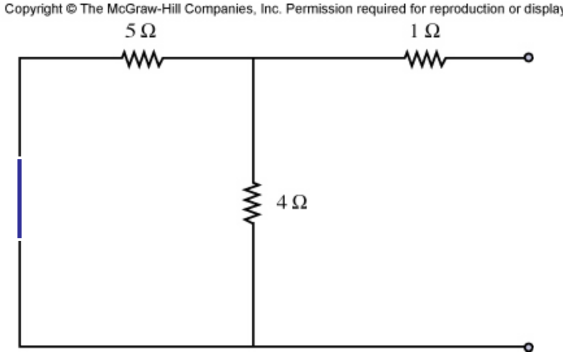


Fig 3: Transformed Norton equivalent circuit

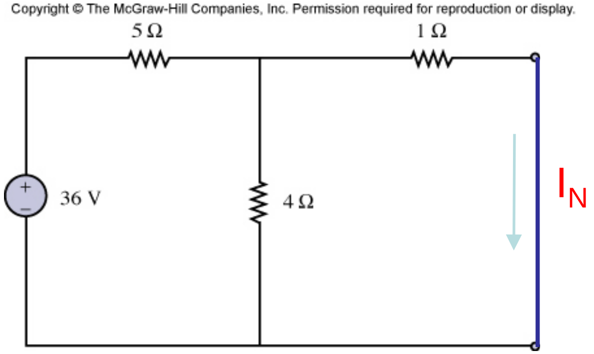
Worked Example on Norton 1

Derive the Norton equivalent circuit seen by the 3Ω load.





Step 1&2: $R_N = 1 + 4||5 = 1 + \frac{4(5)}{4+5} = 3.22\ \Omega$

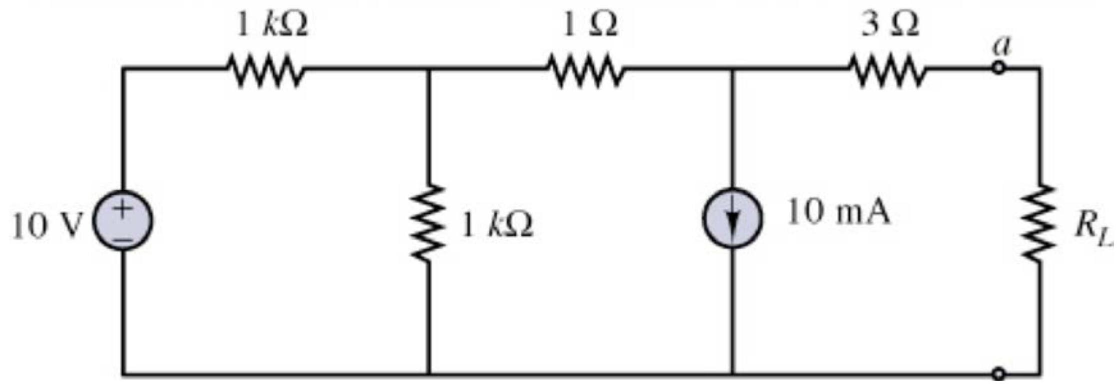


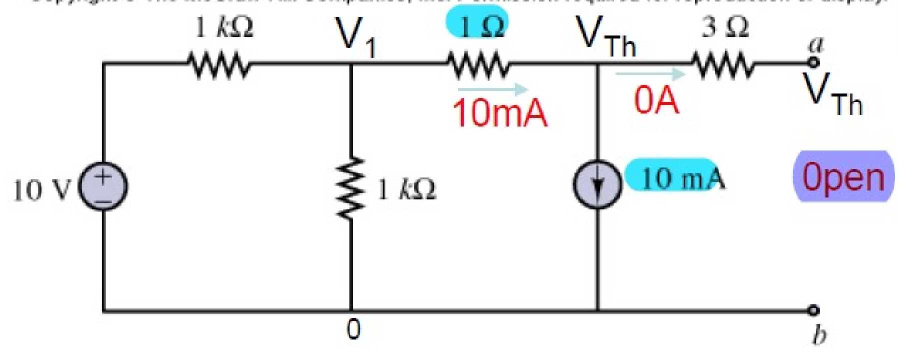
Step 3: $I_N = \frac{\frac{36\left(\frac{4(1)}{4+1}\right)}{5+\frac{4}{5}}}{1} = 4.966\ A$

Worked Example on Norton 2

Derive the Norton equivalent circuit seen by the load R_L.

Answer: $R_N = 504\ \Omega$, $I_N = -19.8\ \mu A$





- Step 3:

(i) $\frac{V_1 - V_{Th}}{1} = 10 \times 10^{-3}$

V1 - 0

(ii) $\frac{10 - V_1}{1000} = \frac{V_1}{1000} + 10 \times 10^{-3}$

From (i), $V_{Th} = V_1 - 10 \times 10^{-3}$

From (ii), $10 - V_1 = V_1 + 10 \Rightarrow 2V_1 = 0 \Rightarrow V_1 = 0V$

Therefore, $V_{Th} = -10\ mV$

III. Source Transformation

Definition

- This trick rests on the fact that the Thevenin and Norton forms are equivalent to each other and therefore are also interchangeable

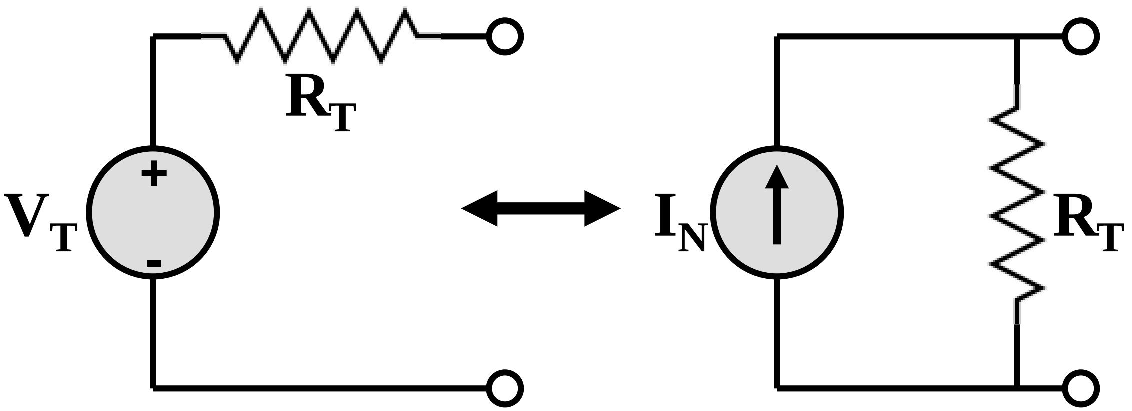


Fig 4a: Thevenin equivalent circuit Fig 4b: Norton equivalent circuit

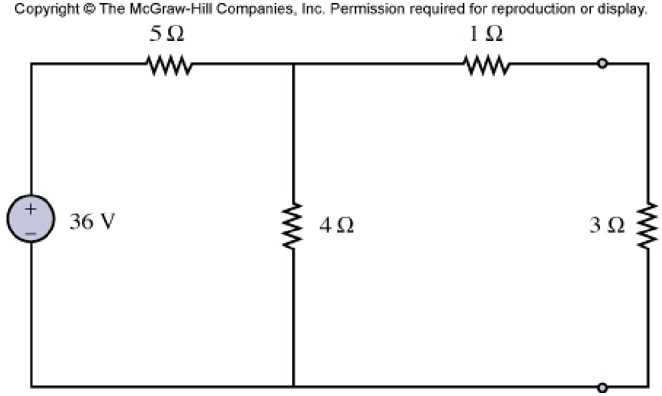
We can transform between the two equivalent circuits, observing each time that:

$$V_T = I_N R_T$$

- Rather than transform the whole circuit in one go to Thevenin or Norton equivalent forms, we can instead transform part of the circuit
- The process of merge transform and merge again can be repeated

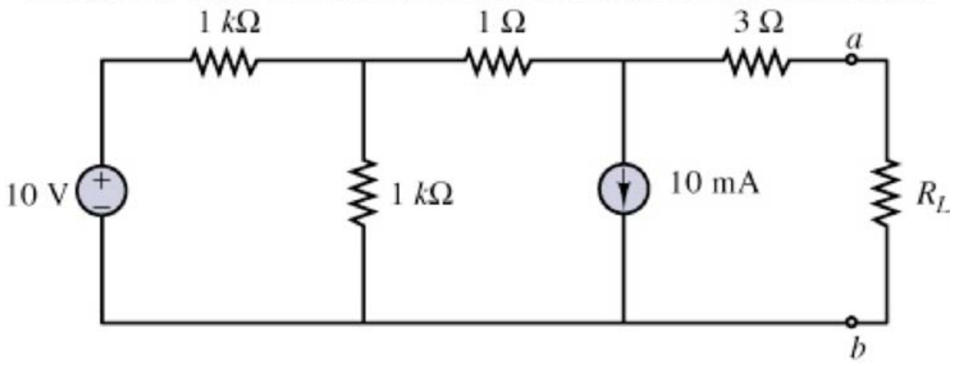
Worked Example on Source Transformation 1

Derive either of the equivalent circuit forms seen by the 3Ω load.



Worked Example on Source Transformation 2

Derive either of the equivalent circuit forms seen by the load R_L.



III. Dependent Sources

- All the sources we have come across are independent source
  - For a voltage source, the voltage maintained across the source is fixed
  - For a current source, the current through the source is fixed
- There also exist dependent sources in circuit theory
- Unlike independent sources, the value of a dependent source is not predetermined, but is set by the current or voltage through a specific branch

Symbol

- The symbol for an independent source is a circle
- For a dependent source, the symbol is a diamond

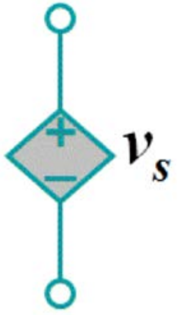


Fig 5a: Dependent voltage source

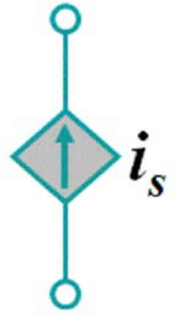
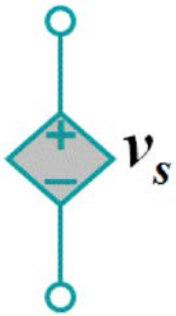


Fig 5b: Dependent current source

Dependent Sources

<u>Dependent Voltage Source</u>

v_S is the voltage source across the terminals.



Current Controlled Voltage Source

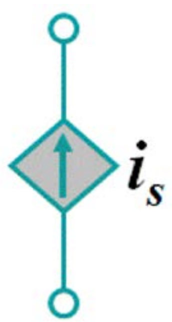
For example, $v_S = 5i_X$. The current i_X is not the current through the source but belongs to another branch.

Voltage Controlled Voltage Source

For example, $v_S = 7v_X$. The voltage v_X is not the source voltage. As v_X changes, so also v_S according to the above relations.

<u>Dependent Current Source</u>

i_S is the current source through the terminals.



Current Controlled Current Source

For example, $i_S = 5i_X$. The current i_X is not the current through the source but belongs to another branch.

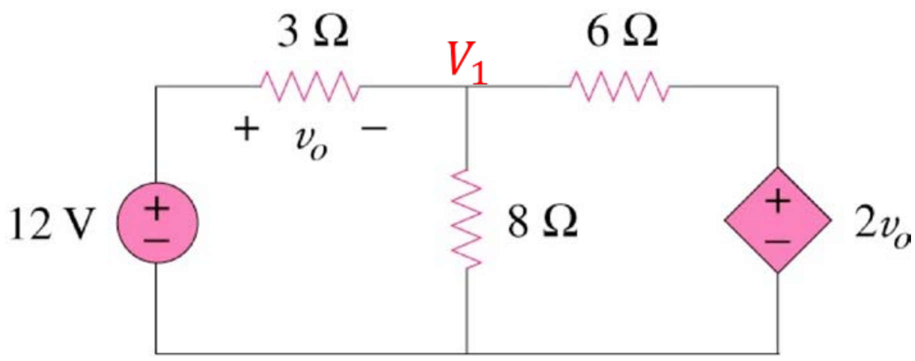
Voltage Controlled Current Source

For example, $i_S = 7v_X$. The voltage v_X is not the source voltage. As v_X changes, so also i_S according to the above relations.

Worked Example on Dependent Source 1

Calculate v_o in the circuit.

Answer: $v_o = 3.65$ V



Hints:

1. The dependent source here is a voltage controlled voltage source
2. Apply KCL

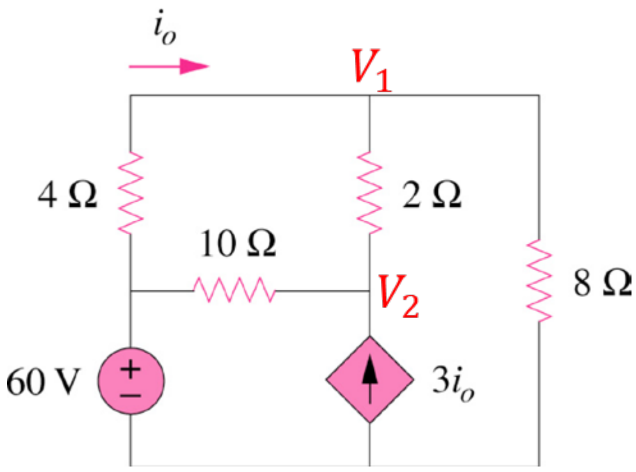
(i) $V_1 = 12 - v_0$

(ii) $\frac{v_0}{3} + \frac{2v_0 - V_1}{6} = \frac{V_1}{8}$

Worked Example on Dependent Source 2

Calculate i_o in the circuit.

Answer: $i_o = 1.73$ A



Hints:

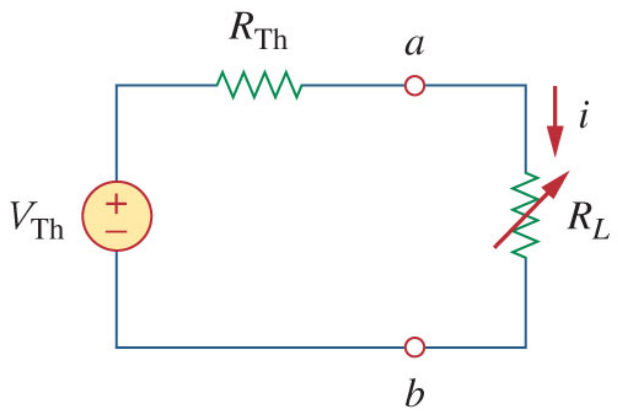
1. The dependent source here is a current controlled current source
2. Apply KCL

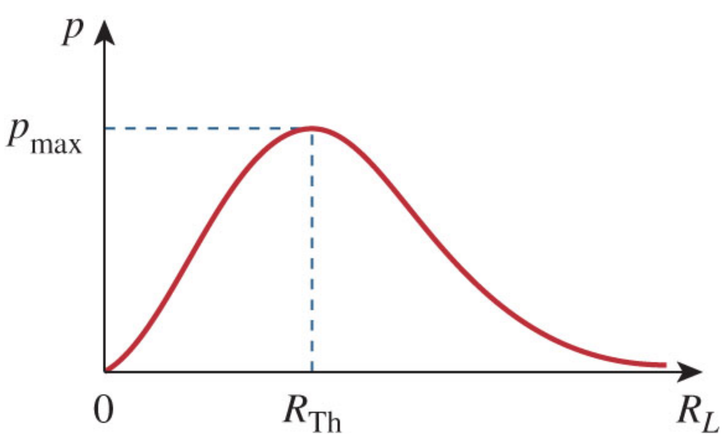
(i) $i_0 = \frac{60-V_1}{4}$

(ii) $i_0 = \frac{V_1-V_2}{2} + \frac{V_1}{8}$

(iii) $\frac{60-V_2}{10} + 3i_0 = \frac{V_2-V_1}{2}$

Maximum Power Transfer





$$p = i^2 R_L = \left(\frac{V_{Th}}{R_{Th} + R_L}\right)^2 R_L$$

$$\frac{dp}{dR_L} = 0 \longrightarrow R_L = R_{Th}$$

$$p_{max} = \frac{{V_{Th}}^2}{4R_{Th}}$$