Solution of Assignment 2

Q1.

1. a) P{X$\geq$ 2} = P{$X = 2$} + P{$X = 3$} = 2/10 + 1/10 = 0.3.

b) P{X$<$ 4} =

$$\sum_{x=0}^{3} P\{X = x\} = 1.$$

c) Let Event A: Odd number of people at the ATM. P{A} = P{X= 1}+ P{X= 3} =0.4.

Q2:

$$F(x) = \int_{-\infty}^{\infty} f(x)\,\mathrm{d}x = \int_{-\infty}^{0} 0\,\mathrm{d}x + \int_{0}^{0.5} 4x\,\mathrm{d}x + \int_{0.5}^{1} (4-4x)\,\mathrm{d}x + \int_{-\infty}^{\infty} 0\,\mathrm{d}x$$

$$F(x) = \begin{cases} 0 & \text{if } -\infty < x < 0. \\ 2x^2 & \text{if } 0 \leq x \leq 0.5. \\ 4x - 2x^2 - 1 & \text{if } 0.5 < x \leq 1. \\ 1 & \text{if } x > 1. \end{cases}$$

$P\{0.2 < X \leq 0.6\}$ = F(0.6) - F(0.2) = 0.68-0.08 = 0.6.

Q3:

$$f(x) = \frac{dF(x)}{dx}$$

$$f(x) = \begin{cases} (2.5 \times 10^{2.5})(x^{-3.5}) & \text{if } x \geq 10. \\ 0 & \text{if } x < 10. \end{cases}$$

$$P\{5 < X \leq 15\} = F(15) - F(5) = 1 - (\frac{2}{3})^{2.5} - 0 = 0.6371.$$

$$P\{X > 20\} = 1 - P\{X \leq 20\} = 1 - F(20) = 1 - (1 - (\frac{1}{2})^{2.5}) = 0.1767.$$

Q4.

Let X be the random variable that denotes the number of cars driven by women. Let $p(x)$ be the probability that x $[X : 0, 1, 2 ... n]$ cars are driven by women out of a total of n selected cars where p is the probability of any car being driven by a woman. Then the PMF of X is given by

$p(x) = \binom{n}{x} p^x (1-p)^{n-x}$, where X is binomially distributed.

a) $P(X = 2) = \binom{5}{2}(0.55)^2(1-0.55)^3 = 0.2756$.

b) $P(X{>}1) = 1 - [P(0) + P(1)]$ = 1- $[\binom{5}{0}(0.55)^0(0.45)^5 + \binom{5}{1}(0.55)^1(0.45)^3] = 0.8687$.

Q5.

a) Let X be the number of trials until a message is succesfuly transmitted. If the probabiility of success for transmitting the message is is p then the PMF of number of trials till message is succesfully transmitted is a geometric distribution given by:
$p(x) = pq^{x\text{-}1}$ where $q = 1 - p$, $x = 1, 2, 3 \ldots$
a)$P(X = 3) =$

$$0.1 \times 0.9 \times 0.9 = 0.081.$$

b) The probability of k^{th} successful transmission in the n^{th} attempt is denoted by: $\binom{n-1}{k-1}p^k(1-p)^{n-k}$. This is a negative binomial distribution for X where X is the number of the trial in which the k^{th} successful transmission occurs. Therefore, the probability of successfully transmitting three messages in exactly five attempts is given by:
$\binom{5-1}{3-1}p^3(1-p)^{5-3}$,where $p = 0.1$. This is $\binom{4}{2}(0.1)^3(0.9)^2$ =0.00486 = 0.486%.

Q6.

a) The standard deviation of an exponential distribution is equal to its mean. Therefore the standard deviation of the time the student spends in the ISEN computer terminal is 20 mins.
b) $E[X] = 1/\lambda$. Therefore $\lambda = 0.05$.

$$P\{X \geq 80\} = 1 - P\{X \leq 80\} = 1 - F(80) = e^{-4} = .0183.$$

Q7.

Let $E[X]$ be the expectation of the weekly demand of propane gas.

$$E[X] = \int_{-\infty}^{\infty} x f(x)\,\mathrm{d}x = \int_1^2 (2x - \frac{2}{x})\ \mathrm{d}x = 3 - 2\log(2) = 1.614.$$

Similarly,

$$E[X^2] = \int_{-\infty}^{\infty} x^2 f(x)\,\mathrm{d}x = \int_1^2 (2x^2 - 2)\ \mathrm{d}x = \frac{8}{3} = 2.667.$$

$E[X] = 1.614$. $Var[X] = E[X^2] - E[X]^2 = 0.062$.

Q8.

Let μ_1 and μ_2 be the means of cork diameters produced by Machine 1 and 2 respectively. Let their standard deviations of the diameters be denoted by σ_1 and σ_2 respectively. For acceptable corks in the range $[2.9, 3.1]$, the standard normal Z_1 and Z_2 lie in the range $[(2.9 - \mu_1)/\sigma_1, (3.1 - \mu_1)/\sigma_1]$ and $[(2.9 - \mu_2)/\sigma_2, (3.1 - \mu_2)/\sigma_2]$ respectively. Z_1 and Z_2 lie in the range$[-1, 1]$ and $[-7, 3]$ respectively. Since the probability corresponding to the range of Z_2 is greater than that of Z_1, (99.8 percent vs. 68.2 percent), it is more likely for Machine 2 to produce an acceptable cork.
To be 90 percent certain that Machine 1 produces an acceptable cork, it should not produce a cork exceeding the maximum size or not meeting the minimum size more than 5 percent

of the time respectively. Therefore for a 95 percent confidence interval $z_1 = 1.645$, the range of corks can be in the interval $[3 - z_1\sigma_1, 3 + z_1\sigma_1]$. Therefore the range of diameters is $[2.8355, 3.1645], d = 0.1645$.