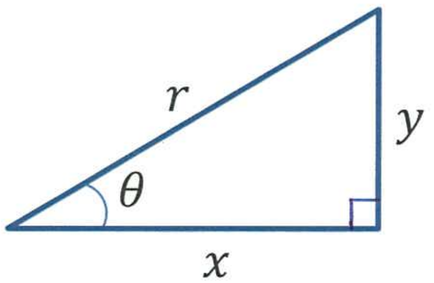
MA1200 CALCULUS AND BASIC LINEAR ALGEBRA I

LECTURE: CG1

Chapter 4
Trigonometric Functions
and Inverse Trigonometric Functions

Trigonometric Functions

In elementary trigonometry, the 3 basic trigonometric functions ($\sin\theta$, $\cos\theta$, $\tan\theta$) are defined as the ratios of sides of a right-angled triangle, and the angles θ are restricted to acute angles, i.e. $0° \le \theta < 90°$



y = Opposite side

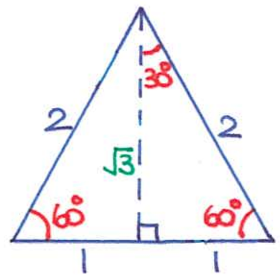
x = Adjacent side

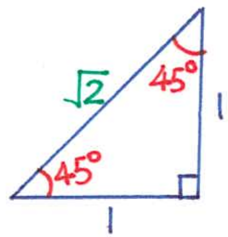
r = Hypotenuse

By definition, $\sin\theta = \dfrac{opp.}{hyp.} = \dfrac{y}{r}$, $\cos\theta = \dfrac{adj.}{hyp.} = \dfrac{x}{r}$ and $\tan\theta = \dfrac{opp.}{adj.} = \dfrac{y}{x}$.

The special angles of sine, cosine and tangent are summarized below.

	$30°$	$45°$	$60°$
sin	$\frac{1}{2}$	$\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
cos	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
tan	$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$	1	$\sqrt{3}$





The six trigonometric functions are **sine**, **cosine**, **tangent**, **cosecant**, **secant** and **cotangent**, which are written as **sin**, **cos**, **tan**, **csc** (or **cosec**), **sec** and **cot**, respectively.

They are defined as follows:

$$\sin\theta = \frac{y}{r}$$

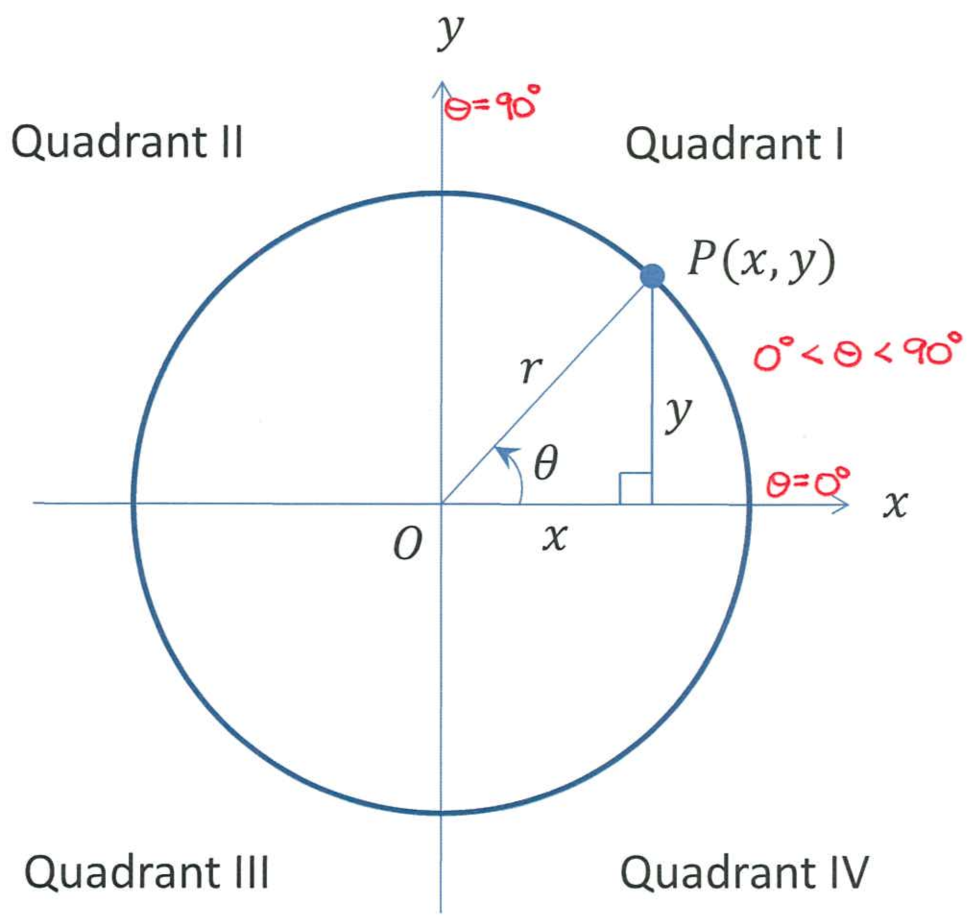
$$\cos\theta = \frac{x}{r}$$

$$\tan\theta = \frac{y}{x}\,, \quad \text{i.e.}\ \tan\theta = \frac{\sin\theta}{\cos\theta}$$

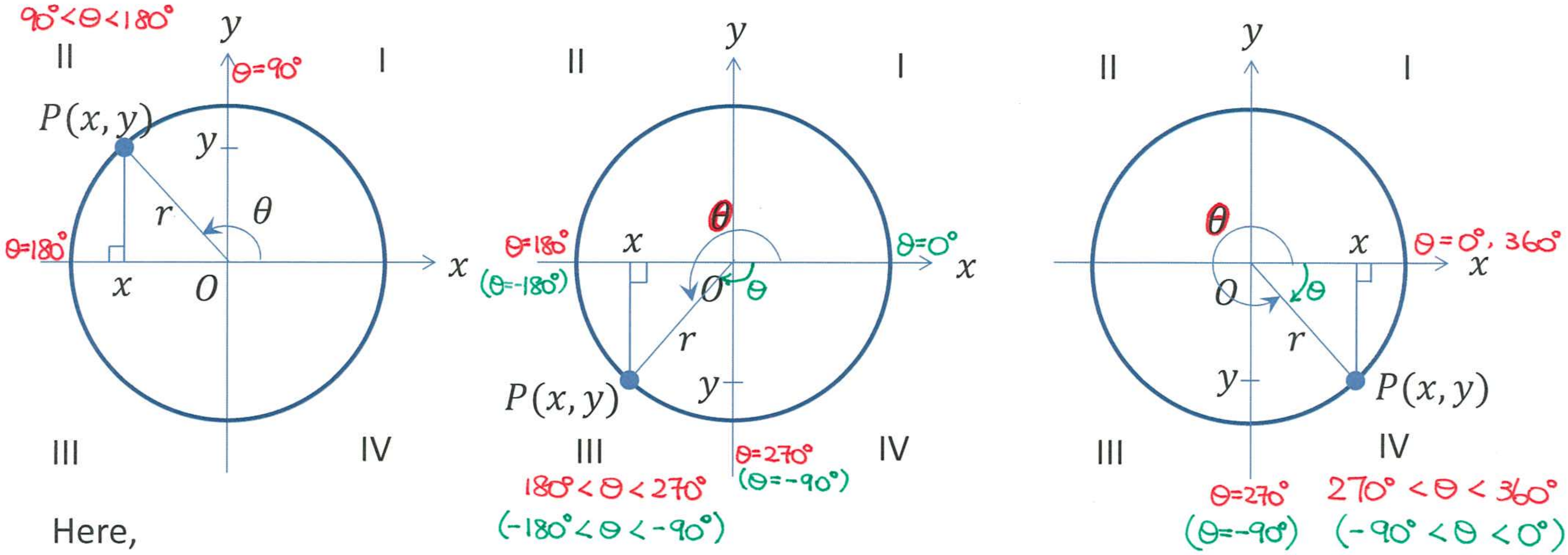
$$\csc\theta = \frac{r}{y}\,, \quad \text{i.e.}\ \csc\theta = \frac{1}{\sin\theta}$$

$$\sec\theta = \frac{r}{x}\,, \quad \text{i.e.}\ \sec\theta = \frac{1}{\cos\theta}$$

$$\cot\theta = \frac{x}{y}\,, \quad \text{i.e.}\ \cot\theta = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$$



The above results are also true when the point P lies in other quadrants.

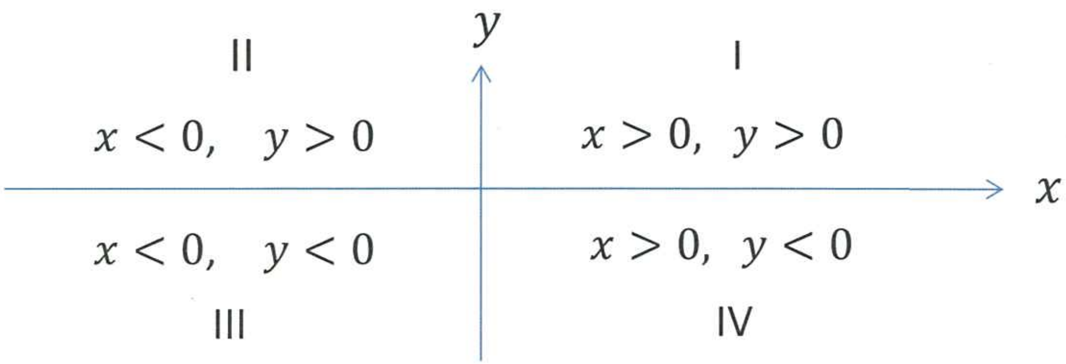


Here,

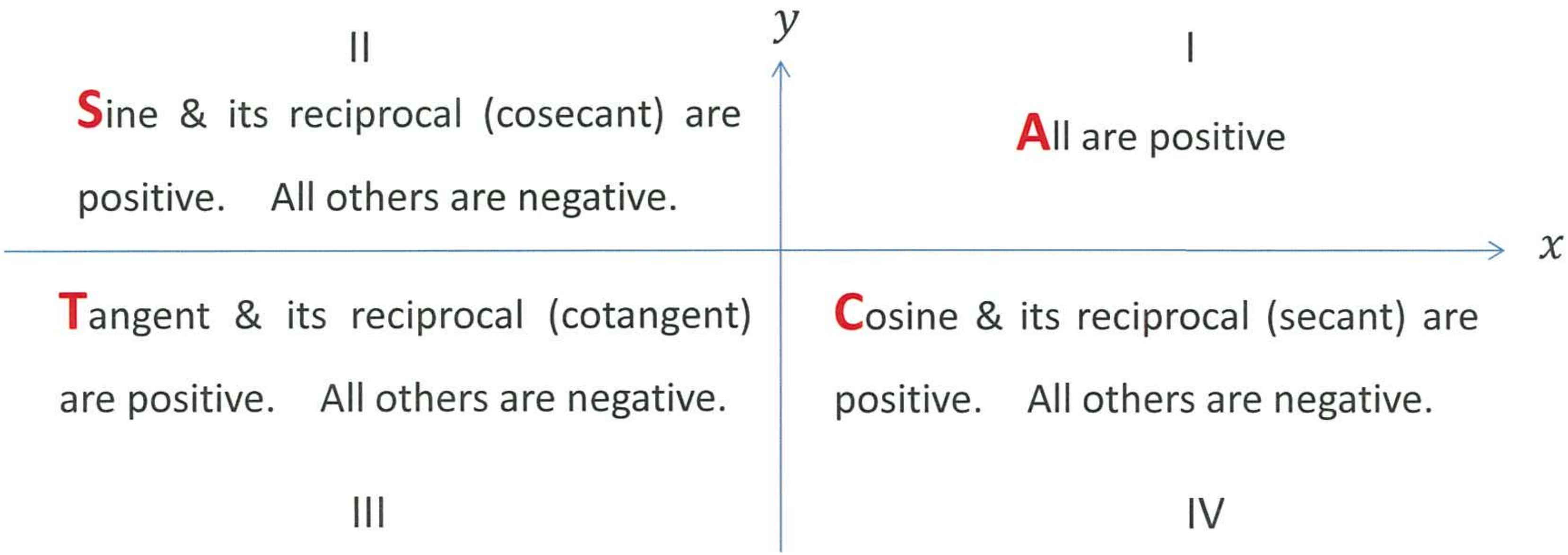
- P is a point in the xy-plane with Cartesian coordinates (x, y).
- θ is the angle measured from the positive x-axis to the line OP in anticlockwise direction.

  (θ is positive if the angle is measured in anticlockwise direction; and it is negative if the angle is measured in clockwise direction.)
- $r = \sqrt{x^2 + y^2}$ (> 0) is the distance from the origin O to the point P.

The signs of x and y depend on the quadrant in which the point P lies.



By using the definitions of the six trigonometric functions and also the fact that r is always positive, the signs of the trigonometric functions can be deduced and the results are summarized by the **CAST rule**:



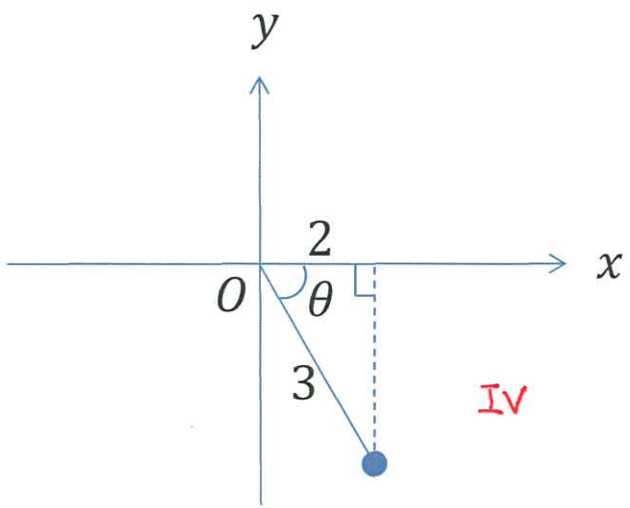
Example 1

adj.

If $\cos\theta = \frac{2}{3}$ and θ is in Quadrant IV, find $\tan\theta$.

hyp.

Solution



Since θ is in Quadrant IV, x must be positive and y must be negative.

$\cos\theta = \frac{x}{r}$ is the ratio of x to r.

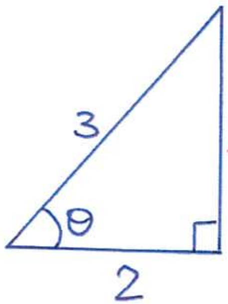
Take $x = 2$ and $r = 3$. Then $y = -\sqrt{r^2 - x^2} = -\sqrt{3^2 - 2^2} = -\sqrt{5}$.

Thus, $\tan\theta = \frac{y}{x} = \frac{-\sqrt{5}}{2}$.

Method 2:

Draw a right-angled triangle first.

$\cos\theta = \frac{2}{3}$



$\sqrt{3^2-2^2} = \sqrt{5}$ by Pythagoras' Theorem

$\because$ θ is in Quad. IV

$\therefore$ $\tan\theta < 0$

$\therefore$ $\tan\theta = -\frac{\sqrt{5}}{2}$ ← opp. ← adj.

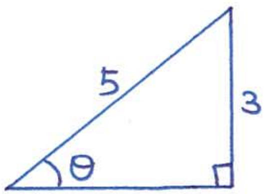
Example 2

If $\sin\theta = -\frac{3}{5}$ and $180^\circ < \theta < 270^\circ$, find $\sec\theta$ and $\cot\theta$.

opp. (3), hyp. (5), Quad. III

Solution

Consider positive lengths first :



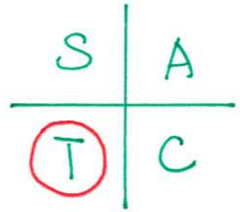
$\sqrt{5^2-3^2}$
$= \sqrt{16}$
$= 4$

$\because$ θ is in Quad. III

$\therefore$ $\sec\theta < 0$ & $\cot\theta > 0$

$\therefore$ $\sec\theta = -\frac{5}{4}$ ← hyp. ← adj.

& $\cot\theta = \frac{4}{3}$ ← adj. ← opp.



Remark: If we find $\theta = \sin^{-1}(-\frac{3}{5}) \approx -36.87^\circ$ by using calculator,
then $\sec\theta = \frac{1}{\cos\theta} = \frac{1}{0.8} = \frac{5}{4}$ Which is wrong!! Why?