Unit 8

Linearity

Remark: Questions 1 and 2 belong to Unit 7.

Question 1: Vector Space

Consider the set of all binary n-vectors, $\{0,1\}^n$

- Addition of two vectors is defined by

$$(x_1, \ldots, x_n) + (y_1, \ldots, y_n) = (x_1 + y_1, \ldots, x_n + y_n),$$

  where the addition of two bits is defined by modulo-2 addition (i.e., logical XOR).

- Scalar multiplication is defined by

$$c(x_1, \ldots, x_n) = (cx_1, \ldots, cx_n), \quad \text{for } c \in \{0,1\},$$

  where multiplication of two bits is defined by usual multiplication (i.e., $0 \cdot 0 = 0 \cdot 1 = 0$ and $1 \cdot 1 = 1$).

Is it a vector space?

Question 2: Subspace

The set of all real polynomials (with usual addition and scalar multiplication) is a vector space.

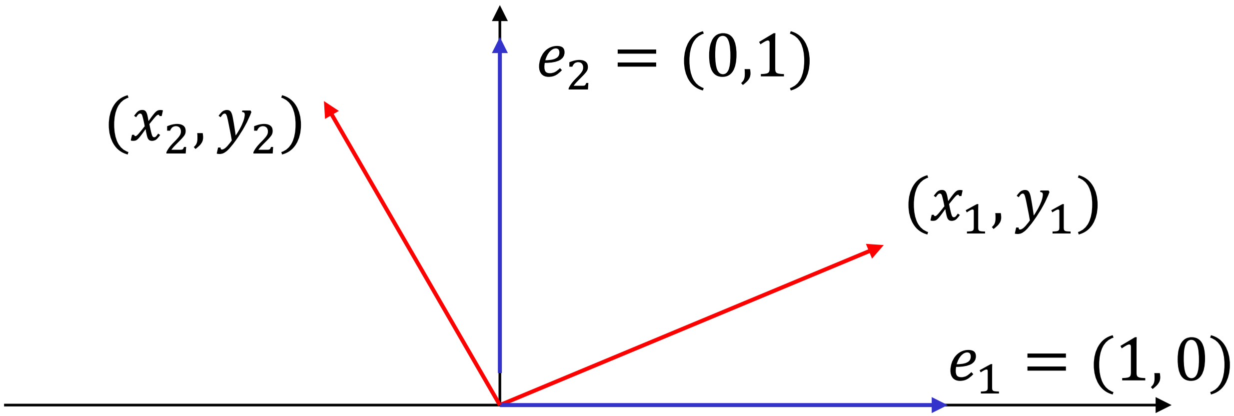
- The degree of a polynomial is the highest of the degrees of its monomials (individual terms) with ***non-zero*** coefficients.

Is each of the following sets its subspaces? Why?

a) The set of all real polynomials with degree less than n;

b) The set of all real polynomials with degree equal to n.

Question 3: Rotation



Consider anti-clockwise rotations of e_1 and e_2 by 30^o.

a) Find (x_1, y_1) and (x_2, y_2).

b) Consider an arbitrary vector $v = (x, y)$. Express v as a linear combination of e_1 and e_2.

c) What is the resultant vector after rotating v by 30^o?

d) What is the corresponding rotation matrix?

Question 4: Projection

Consider the straight line $y = \frac{x}{2}$ in the 2-dimensional space.

a) Find the matrix that projects any vector to the above line.
b) Hence, find the projection of (3, 2) onto the above line.

Question 5: Line Fitting

There are three data points given:

(0, 2), (1, 1), and (3, 2).

a) Find the best line (in the sense of minimum RMS) that fits the three points and passes through the origin.

b) Find the predicted value at $x = 2$.