EE2302 Foundations of Information and Data Engineering

Assignment 7 (Solution)

1.

a) $\mathbf{1}^T x$

b) Let a be a 100-vector, where $a_i = 0$ for $i = 1, 2, \ldots 18$ and $a_i = 1$ for $i = 19, 20, \ldots, 100$. Then the answer is $a^T x$.

c) Let b be a 100-vector, where $b_i = i - 1$ for $i = 1, 2, \ldots, 100$. Then the answer is $b^T x / \mathbf{1}^T x$.

2.

a) The distance between x and y is

$$\|x - y\| = \sum_{i=1}^{n} (x_i - y_i)^2.$$

We want to minimize the distance. If $x_i \geq 0$, obviously we should simply let y_i be equal to x_i. But if $x_i < 0$, we cannot do so because y_i must be non-negative (as stated in the question). To minimize $(x_i - y_i)^2$, the best we can do is to let y_i be zero. Hence,

$$y_i = \begin{cases} x_i & \text{if } x_i \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

b) By the definition of z, we obtain

$$z_i = y_i - x_i = \begin{cases} 0 & \text{if } x_i \geq 0, \\ -x_i & \text{otherwise.} \end{cases}$$

Since each component of z is non-negative, z is a non-negative vector.

c) To determine the inner product, we separate it into two summations, depending on whether x_i is non-negative or not. If $x_i \geq 0$, then $z_i = 0$. Otherwise, $z_i = -x_i$. Hence,

$$z^T y = \sum_i z_i y_i = \sum_{i:\, x_i \geq 0} 0 \cdot x_i + \sum_{i:\, x_i < 0} -x_i \cdot 0 = 0.$$

3. Choose $b = \mathbf{1}$. Then $a^T b = a_1 + a_2 + \cdots + a_n \leq \|a\| \|b\| = \sqrt{a_1^2 + a_2^2 + \cdots a_n^2} \sqrt{n}$. Squaring both sides, we obtain

$$(a_1 + a_2 + \cdots + a_n)^2 \leq n(a_1^2 + a_2^2 + \cdots a_n^2).$$