

Take Home Assignment MA2001 #2

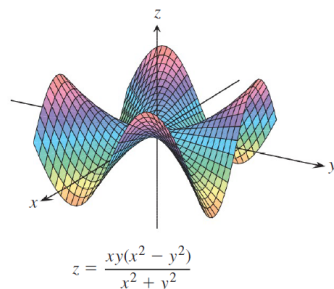
For each of the following questions, write down your solution with details of steps. Marks will not be given if only final answers are provided.

- Find and sketch the level curves $f(x, y) = c$ on the same set of coordinate axes for the given values of c . We refer to these level curves as a contour map.

(a) $f(x, y) = x^2 + y^2, c = 0, 1, 4, 9, 16, 25$

(b) $f(x, y) = xy, c = -9, -4, -1, 0, 1, 4, 9$

- Let $f(x, y) = \begin{cases} xy \frac{x^2 - y^2}{x^2 + y^2}, & \text{if } (x, y) \neq 0, \\ 0, & \text{if } (x, y) = (0, 0). \end{cases}$ The graph of f is shown as below.



- Show that $\frac{\partial f}{\partial y}(x, 0) = x$ for all x , and $\frac{\partial f}{\partial x}(0, y) = -y$ for all y .
 - Show that $\frac{\partial^2 f}{\partial y \partial x}(0, 0) \neq \frac{\partial^2 f}{\partial x \partial y}(0, 0)$.
- Let $w = f(r, s, t)$ with $r = g(x, y), s = h(x, y), t = k(x, y)$. Draw a branch diagram and write a Chain Rule formula for partial derivatives $\frac{\partial w}{\partial x}$ and $\frac{\partial w}{\partial y}$.
 - Laplace equations** Show that if $w = f(u, v)$ satisfies the Laplace equation $f_{uu} + f_{vv} = 0$ and if $u = (x^2 - y^2)/2$ and $v = xy$, then w satisfies the Laplace equation

$$w_{xx} + w_{yy} = 0.$$

5. Assuming the equation defines y as a differentiable function of x , use implicit differentiation find the value of dy/dx at the given point.

$$xe^y + \sin(xy) + y - \ln 2 = 0, \quad (0, \ln 2)$$

6. (a) Find the linearization $L(x, y)$ of the function

$$f(x, y) = (1/2)x^2 + xy + (1/4)y^2 + 3x - 3y + 4 \quad \text{at} \quad P_0(2, 2).$$

- (b) Then find an upper bound for the magnitude $|E|$ of the error in the approximation $f(x, y) \approx L(x, y)$ over the rectangle

$$R : |x - 2| \leq 0.1, |y - 2| \leq 0.1.$$

7. Find all the local maxima, local minima, and saddle points of the functions.

(a) $f(x, y) = x^2 - 2xy + 2y^2 - 2x + 2y + 1$

(b) $f(x, y) = e^y - ye^x$

8. Find the absolute maxima and minima of the function

$$T(x, y) = x^2 + xy + y^2 - 6x$$

on the rectangular domain: $0 \leq x \leq 5, \quad -3 \leq y \leq 3$.

9. Let $f(x, y) = \frac{(x - y)}{(x + y)}$. Find $D_{\mathbf{u}}f(-\frac{1}{2}, \frac{3}{2})$ if $\mathbf{u} = (1, 2)$, and the directions \mathbf{u} and the values of $D_{\mathbf{u}}f(-\frac{1}{2}, \frac{3}{2})$ for which

(a) $D_{\mathbf{u}}f(-\frac{1}{2}, \frac{3}{2})$ is largest (b) $D_{\mathbf{u}}f(-\frac{1}{2}, \frac{3}{2})$ is smallest

(c) $D_{\mathbf{u}}f(-\frac{1}{2}, \frac{3}{2}) = 0$ (d) $D_{\mathbf{u}}f(-\frac{1}{2}, \frac{3}{2}) = -2$

10. **Discovery Question.** Given the function $f(x, y)$ and the positive number ϵ . Show that there exists a $\delta > 0$ such that for all (x, y) ,

$$\sqrt{x^2 + y^2} < \delta \Rightarrow |f(x, y) - f(0, 0)| < \epsilon$$

(i) $f(x, y) = y/(x^2 + 1)$, $\epsilon = 0.05$

(ii) $f(x, y) = \frac{x^3 + y^4}{x^2 + y^2}$ and $f(0, 0) = 0$, $\epsilon = 0.02$

11. **Discovery Question. Changing voltage in a circuit** The voltage V in a circuit that satisfies the law $V = IR$ is slowly dropping as the battery wears out. At the same time, the resistance R is increasing as the resistor heats up. Use the equation

$$\frac{dV}{dt} = \frac{\partial V}{\partial I} \frac{dI}{dt} + \frac{\partial V}{\partial R} \frac{dR}{dt}$$

to find how the current is changing at the instant when $R = 600$ ohms, $I = 0.04$ amp, $dR/dt = 0.5$ ohm/sec, and $dV/dt = -0.01$ volt/sec.

