Chapter 3. Multiple Integral

1 Single-Variable Case:

• Definition:

• Interpretation:

• Computation:

$$\int_{a}^{b} f(x)dx = F(b) - F(a),$$

where F is an antiderivative of f, i.e. F'(x) = f(x).

- 2 Two-Variable Case (Double Integral):
- 2.1 Definition and Interpretation:

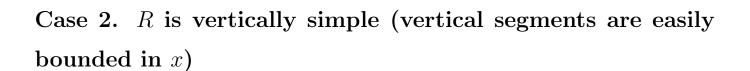
2.2 Computation of Double Integrals:

Case 1: R is an rectangle:

Example Use iterated integral in two different orders to evaluate

$$\iint_{R} (2xy + y^2) \, dx dy$$

with $R = [1, 2] \times [0, 1]$.



Example Evaluate $\iint_R 2x^2y dx dy$, with R as a region bounded by $x = 1, x = 3, y = x^2$ and $y = -x^2 + 18$.

Exercise Evaluate $\iint_R xy^2 dx dy$, with R as a region bounded by x = 1, y = 0 and $y = x^2$.

Case 3. R is horizontally simple (horizontal segments are easily bounded in y)

Example Evaluate

$$\iint_R xy^2 \ dxdy$$

, with R as a region bounded by $x = y^2$ and $x = -y^2 + 1$.

Example Show that

$$\int_0^1 \int_x^{\sqrt{x}} f(x,y) \ dy \ dx = \int_0^1 \int_{y^2}^y f(x,y) \ dx \ dy.$$