Review on Ch.4-6 (Part 1)

Ch.4

- Low Pass Filter

$$\frac{V_o}{V_i} = \frac{1}{1 + j\omega/\omega_c}$$

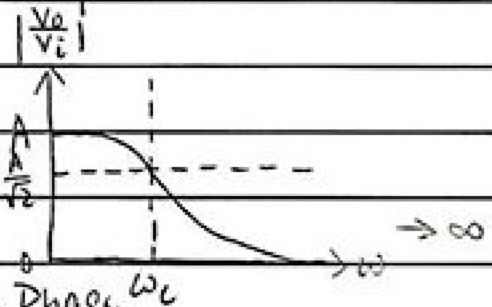
$A\angle P^\circ$ ← $\frac{V_o}{V_i}$; ω_c ← cut-off angular frequency

- Range: $f: 0 \to \infty$

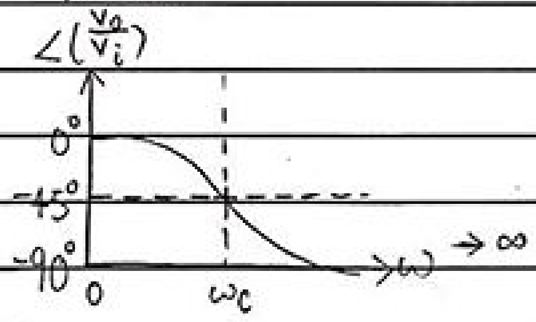
  $\left|\frac{V_o}{V_i}\right| : 1 \to 0$

  $\angle\left(\frac{V_o}{V_i}\right) : 0^\circ \to -90^\circ$

- Graph:
  - Amplitude



  - Phase



- High Pass Filter

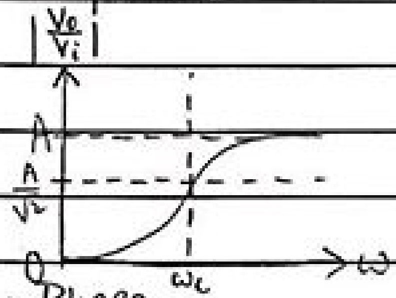
$$\frac{V_o}{V_i} = \frac{1}{1+\omega_c/j\omega}$$

- Range : $f : 0 \to \infty$

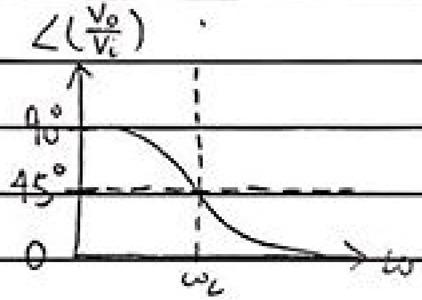
  $\left|\frac{V_o}{V_i}\right| : 0 \to 1$

  $\angle\left(\frac{V_o}{V_i}\right) : 90^\circ \to 0^\circ$

- Graph :
  - Amplitude



  - Phase



- RCL Series Filter (@Lab)

$$\frac{v_o}{v_i} = \frac{j\omega CR}{1-\omega^2 LC + j\omega CR}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

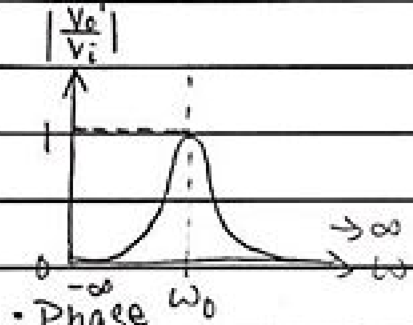
$$Q = \frac{\sqrt{L/C}}{R}$$

- Range: $\omega: -\infty \to \omega_0 \to \infty$

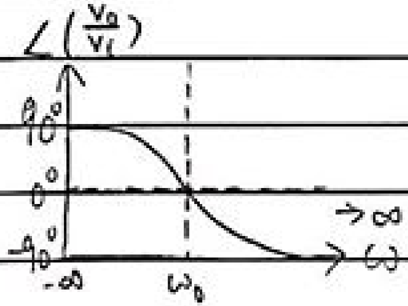
  $\left|\frac{v_o}{v_i}\right|: 0 \to 1 \to 0$

  $\angle\left(\frac{v_o}{v_i}\right): 90^\circ \to 0 \to -90^\circ$

- Graph:
  - Amplitude

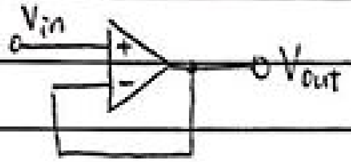


  - Phase



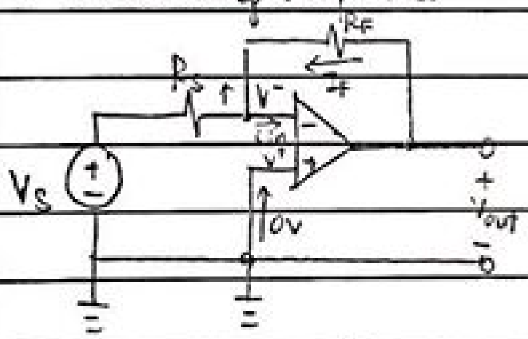
- Ideal Op Amp
  - $V^+ = V^-$
  - $I_{in} = 0A$
  - Open loop gain $A = \infty$
    $V_{out} = A(V_o - V_i)$
  - Input Resistance $R_{in} = \infty$
  - Output Resistance $R_{out} = 0$

- Source follower



Gain $= \frac{V_{out}}{V_{in}} = 1$

- Inverting amplifier



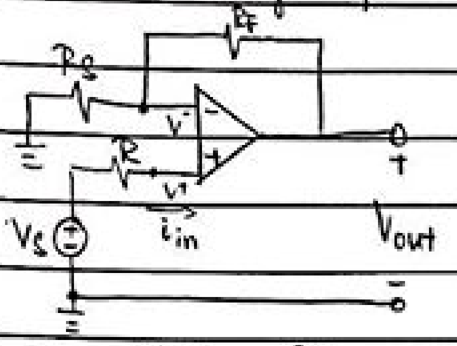
Source → V^-

Ground → V^+

Gain $= \frac{V_{out}}{V_{in}} = -\frac{R_F}{R_S}$

Output Voltage $V_{out} = -V_S \frac{R_F}{R_S}$

- Non-inverting amplifier



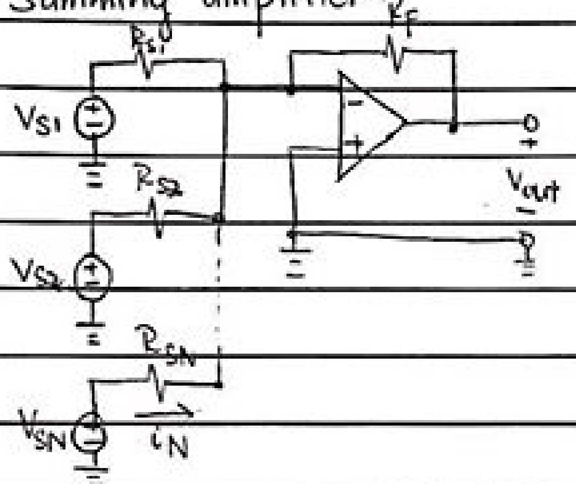
Source → V^+

Ground → V^-

Gain $= \frac{V_{out}}{V_S} = 1 + \frac{R_F}{R_S}$

Output Voltage $V_{out} = V_S \left(1 + \frac{R_F}{R_S}\right)$

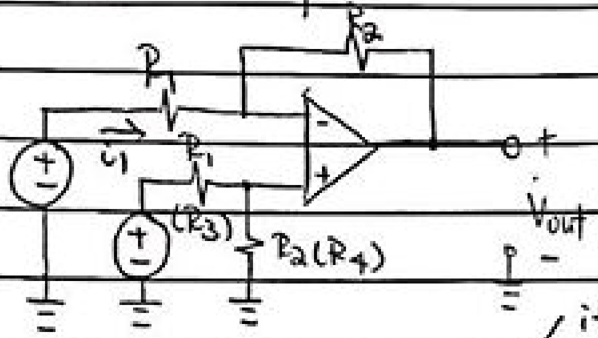
- Summing amplifier



Output Voltage $V_{out} = -\left(\frac{R_F}{R_{S1}} V_{S1} + \frac{R_F}{R_{S2}} V_{S2} + \cdots + \frac{R_F}{R_{SN}} V_{SN}\right)$

$= -R_F \left(\frac{V_{S1}}{R_{S1}} + \frac{V_{S2}}{R_{S2}} + \cdots + \frac{V_{SN}}{R_{SN}}\right)$

- Differential amplifier



if ($R_1 = R_3$ & $R_2 = R_4$)

Output Voltage $V_{out} = \frac{R_2}{R_1}(v_2 - v_1)$

$$= \frac{R_2(1+R_1/R_2)}{R_1(1+R_3/R_4)}V_2 - \frac{R_2}{R_1}V_1$$

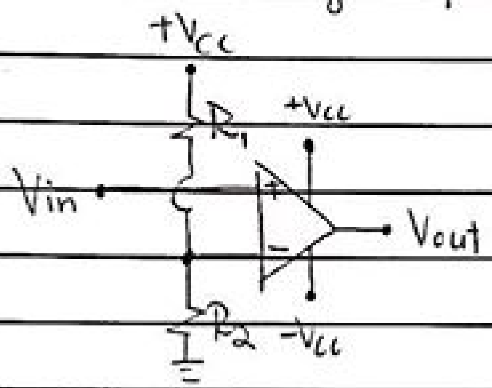
- Cascaded amplifiers

Application of Multiple Types of Amplifiers

Review on Ch.4-6 (Part 2)

Ch.6

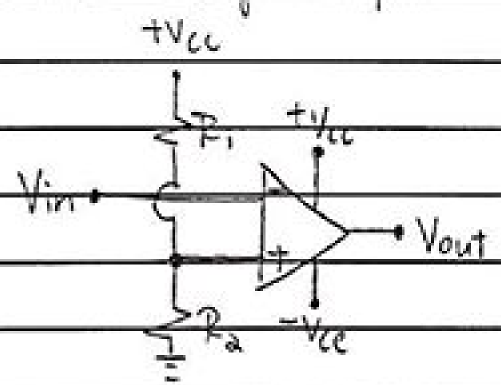
- Op-amp applications
  - Non-inverting comparator



$$V_{REF} = \frac{R_2}{R_1+R_2} V_{CC}$$

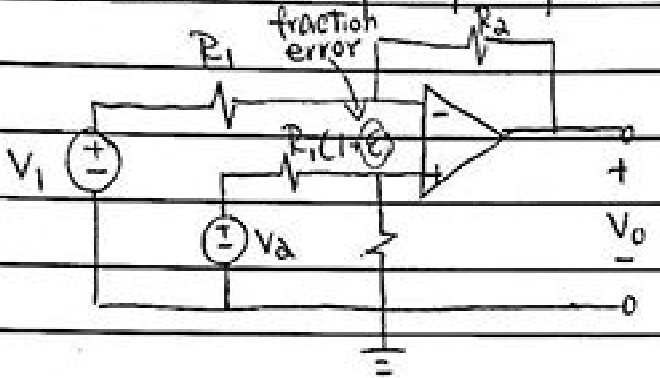
$$V_{out} = \begin{cases} +V_{CC} & V_{in} > V_{REF} \\ -V_{CC} & V_{in} < V_{REF} \end{cases}$$

  - Inverting comparator



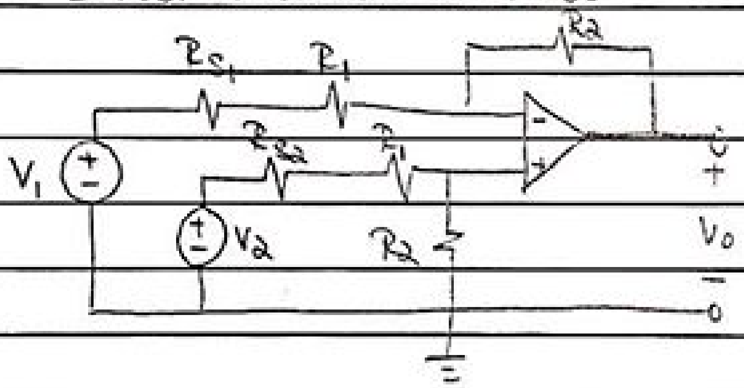
$$V_{out} = \begin{cases} -V_{CC} & V_{in} > V_{REF} \\ +V_{CC} & V_{in} < V_{REF} \end{cases}$$

- Real-life Problems
  - Differential op amp in practice



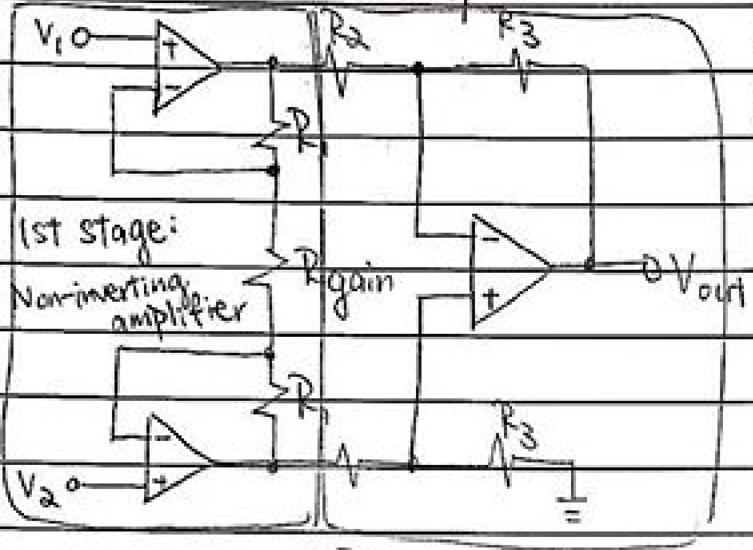
$$V_o = \frac{R_2}{R_1}(V_2 - V_1) - \frac{\epsilon R_2}{R_1(1+\epsilon) + R_2} V_2$$

- Effect of source resistance



$$V_o = \left(\frac{R_2}{R_1 + R_{S1}}\right)\left[\left(\frac{R_2 + R_1 + R_{S1}}{R_1 + R_2 + R_{S2}}\right) V_2 - V_1\right]$$

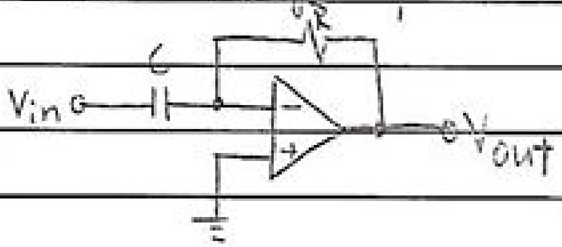
• Instrumentation amplifier



2nd stage:
Differential amplifier

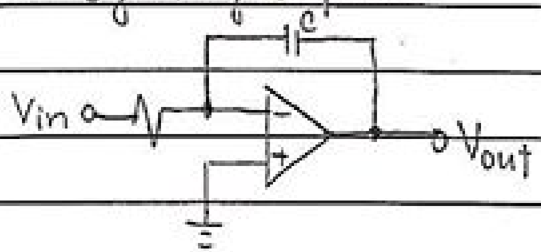
$$V_{out} = \left(1 + \frac{2R_1}{R_{gain}}\right)\left(\frac{R_3}{R_2}\right)(V_2 - V_1)$$

• Differentiating amplifier



$$V_{out} = -RC\left(\frac{dV_{in}}{dt}\right)$$

• Integrating amplifier



$$V_{out} = -\frac{1}{RC}\int V_{in}\,dt$$

- Decibels (dB)

Gain $= 10 \log_{10} \frac{P_{out}}{P_{in}}$ dB

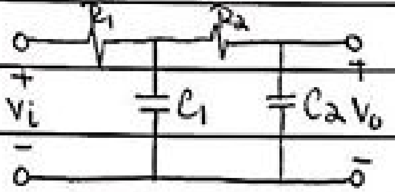
$= 20 \log_{10} \frac{V_{out}}{V_{in}}$ dB $\quad (P = \frac{V^2}{R})$

e.g. Low pass filter: $0 \to \infty$

Gain: $1 \to \frac{1}{\sqrt{2}} \to 0$ $\quad \omega_c$ $\qquad$ Factor: $\times 10 \to +20$dB

0dB $\to$ -3dB $\to$ $-\infty$ dB $\qquad$ $\div 10 \to -20$dB

- Passive 2nd order low pass filter



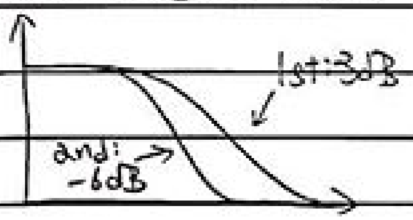
$$\frac{V_o}{V_i}(j\omega) = \left(\frac{1}{1 + j\omega/\omega_{c1}}\right)\left(\frac{1}{1 + j\omega/\omega_{c2}}\right)$$

$$= \frac{1}{[1 + j(\omega/\omega_c)^2]} \quad (\text{if } \omega_{c1} = \omega_{c2} = \omega_c)$$

$\omega_{c1} = \frac{1}{R_1 C_1}$ $\qquad f_{c1} = \frac{\omega_{c1}}{2\pi}$

$\omega_{c2} = \frac{1}{R_2 C_2}$ $\qquad f_{c2} = \frac{\omega_{c2}}{2\pi}$

Gain: -3dB $\to$ -6dB



• Active 1st order low pass filter

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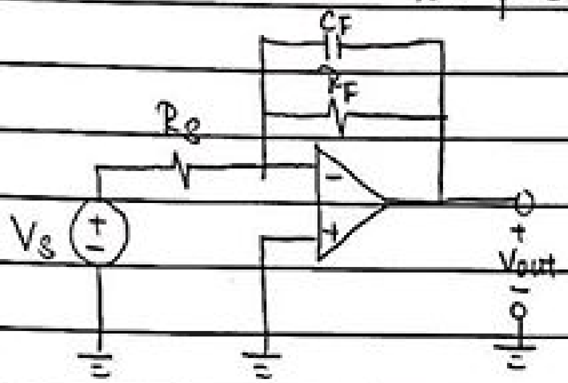
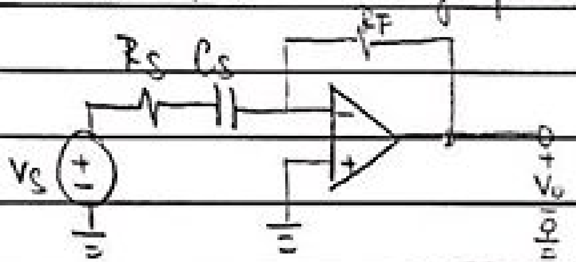


$$\frac{V_{out}}{V_S}(j\omega) = -\frac{(R_F/R_S) \leftarrow G}{1 + j\omega(C_F R_F) \leftarrow \frac{1}{\omega_c}}$$

$\omega_c = \frac{1}{R_F C_F}$ $\quad f_c = \frac{\omega_c}{2\pi}$

Passband gain $G = \frac{R_F}{R_S}$

• Active 1st order high pass filter



$$\frac{V_{out}}{V_S}(j\omega) = -\frac{(R_F/R_S) \leftarrow G}{1 + (1/j\omega(C_S R_S)) \leftarrow \frac{1}{\omega_c}}$$

$\omega_c = \frac{1}{R_S C_S}$ $\quad f_c = \frac{\omega_c}{2\pi}$

Passband Gain $G = \frac{R_F}{R_S}$

• Active Bandpass Filter

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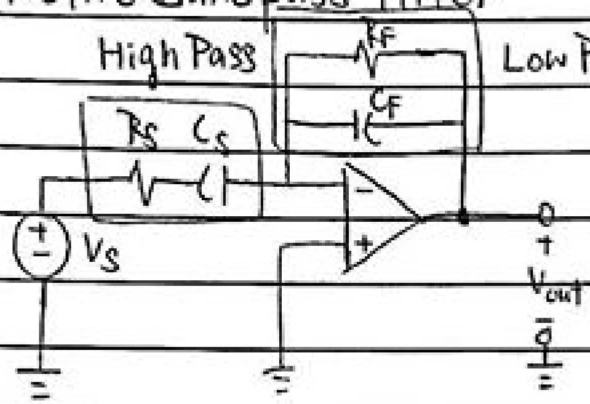
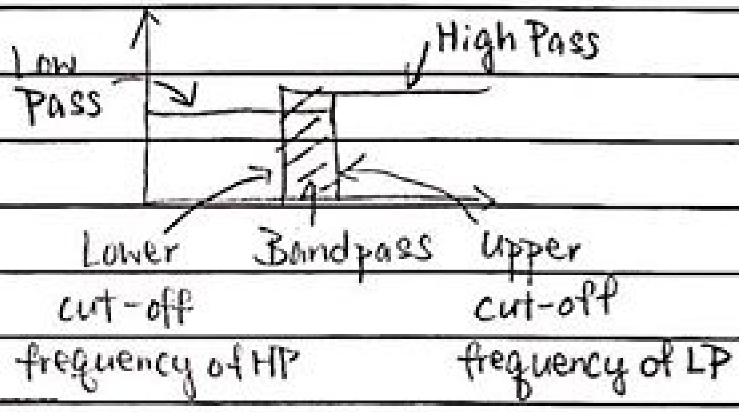


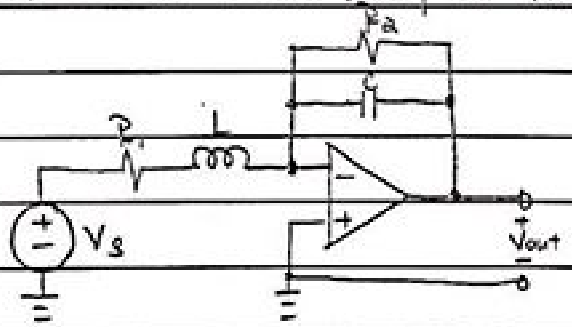


$$\frac{V_{out}}{V_S}(j\omega) = -\frac{-R_F/R_S}{(1+1/j\omega C_S R_S)(1+j\omega C_F R_F)}$$

$$= -\frac{-R_F/R_S}{(1+\omega_{HP}/j\omega)(1+j\omega/\omega_{LP})}$$

- Lower cut-off frequency $\omega_{HP} = \frac{1}{C_S R_S}$ $f_{HP} = \frac{\omega_{HP}}{2\pi}$
- Upper cut-off frequency $\omega_{LP} = \frac{1}{C_F R_F}$ $f_{LP} = \frac{\omega_{LP}}{2\pi}$
- Passband gain $G = \frac{-R_F}{R_S}$

• Active 2nd order low pass filter



$$\frac{V_{out}}{V_S}(j\omega) = -\frac{R_2/R_1}{(1+j\omega C R_2)(1+j\omega L/R_1)}$$

$$= -\frac{R_2/R_1}{(1+j\omega/\omega_c)^2} \quad \left(\text{if } \omega_c = \frac{1}{CR_2} = \frac{R_1}{L}\right)$$

Passband Gain $G = \frac{R_F}{R_S} = \frac{R_2}{R_1}$ $f_c = \frac{\omega_c}{2\pi}$

- Multiple Feedback Narrow Band Filter

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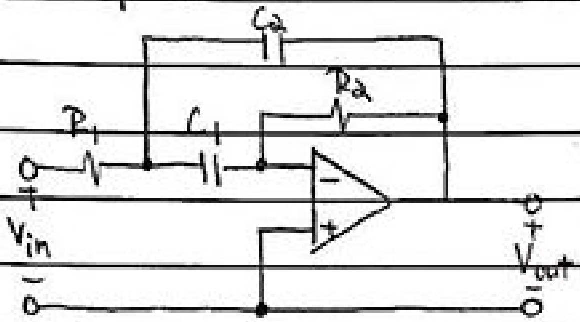


Remarks: Fourier Transform $\rightarrow H(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-2\pi j \omega t} \, dt$

Laplace Transform $\rightarrow H(s) = \int_{-\infty}^{\infty} x(t) e^{-st} \, dt$

Complex $s = \sigma + j\omega$

$$H(s) = \frac{V_{out}}{V_{in}} = \frac{-\frac{1}{R_1 C_2} s}{s^2 + \left(\frac{1}{R_2 C_1} + \frac{1}{R_2 C_2}\right) s + \frac{1}{R_1 R_2 C_1 C_2}}$$

$$= \frac{\frac{\omega_0}{Q} K s}{s^2 + \left(\frac{\omega_0}{Q}\right) s + \omega_0^2}$$

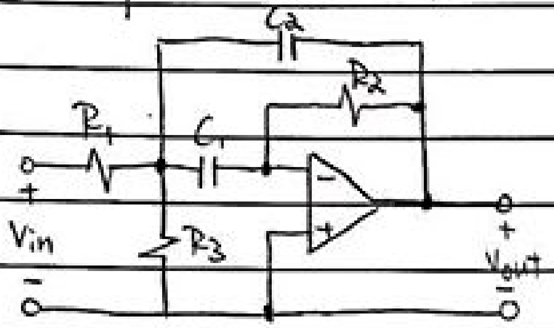
$\omega_0 = \frac{1}{C\sqrt{R_1 R_2}}$ $\quad f_0 = \frac{\omega_0}{2\pi}$

$Q = \frac{1}{2}\sqrt{\frac{R_2}{R_1}}$

$K = -2Q^2$

$= -\frac{R_2}{2R_1}$

• Multiple Feedback Narrow Band Filter (General Form) #cityueenotes@ig



$$H(s) = \frac{V_{out}}{V_{in}} = \frac{-\frac{1}{R_1 C_2} s}{s^2 + \left(\frac{1}{R_2 C_1} + \frac{1}{R_2 C_2}\right) s + \frac{1}{R_2 C_1 C_2}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)}$$

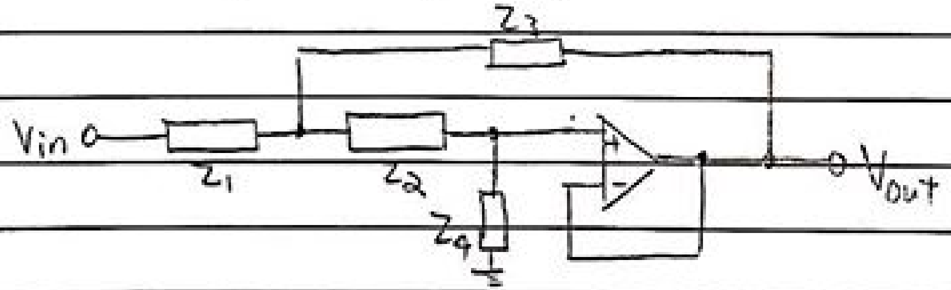
$$= \frac{\frac{\omega_0}{Q} K s}{s^2 + \left(\frac{\omega_0}{Q}\right) s + \omega_0^2}$$

$$\omega_0 = \frac{1}{C}\sqrt{\frac{1}{R_2}\left(\frac{1}{R_1} + \frac{1}{R_3}\right)} \qquad f_0 = \frac{\omega_0}{2\pi}$$

$$Q = \frac{1}{2}\sqrt{R_2\left(\frac{1}{R_1} + \frac{1}{R_3}\right)}$$

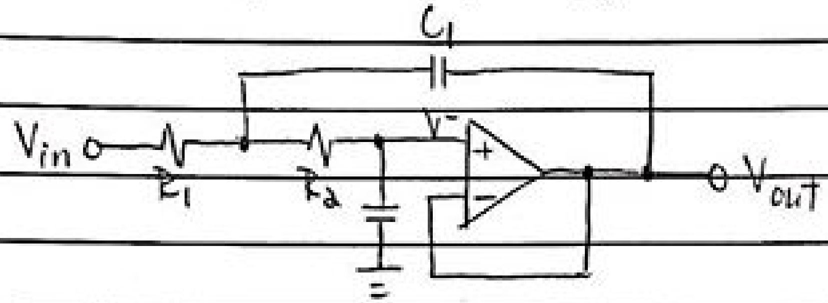
$$K = -\frac{R_2}{2R_1}$$

• Sallen-Key Filter Topology



$$H(j\omega) = \frac{V_{out}}{V_{in}} = \frac{Z_3 Z_4}{Z_1 Z_2 + Z_3(Z_1 + Z_2) + Z_3 Z_4}$$

- Sallen-Key Filter Topology : Low Pass Filter #cityueenotes@ig

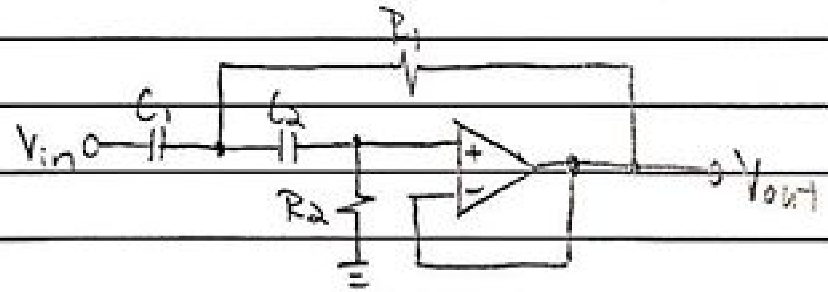


$$H(s)=\frac{\omega_0^2}{s+s\frac{\omega_0}{Q}+\omega_0^2}$$

$$\omega_0=\frac{1}{\sqrt{R_1R_2C_1C_2}} \qquad f_0=\frac{\omega_0}{2\pi}$$

$$Q=\frac{\sqrt{R_1R_2C_1C_2}}{C_2(R_1+R_2)}$$

- Sallen-key Filter Topology : High Pass Filter



$$H(s)=\frac{\omega_0^2}{s+s\frac{\omega_0}{Q}+\omega_0^2}$$

$$\omega_0=\frac{1}{\sqrt{R_1R_2C_1C_2}} \qquad f_0=\frac{\omega_0}{2\pi}$$

$$Q=\frac{\sqrt{R_1R_2C_1C_2}}{R_1(C_1+C_2)}$$