Question 1 (10 marks)

The following figure shows (a) a 3-bit image of size 5-by-5 image in the square, with x and y coordinates specified, (b) a Laplacian filter.

y \ x	0	1	2	3	4
0	3	7	6	2	0
1	2	4	6	1	1
2	4	7	2	5	4
3	3	0	6	2	1
4	5	7	5	1	2

(a)

Laplacian filter

0	1	0
1	-4	1
0	1	0

(b)

Compute the following:

(a) The output of a 3 × 3 mean filter at (3,3).
(b) The output of a 3 × 3 median filter at (2,3).
(c) The output of the 3 × 3 Laplacian filter shown above at (1,3).
(d) Obtain the histogram of the image.
(e) Apply histogram equalization on the above image and calculate the histogram equalized image, and the new histograms.

a)

y \ x	0	1	2
0	2	5	4
1	6	2	1
2	5	2	1

mean: $\frac{2+5+4+6+2+1+5+2+1}{9}$

$= \frac{28}{9}$

$= 3.111$ (3 d.p.)

⇒

y \ x	0	1	2
0	*	*	*
1	*	3.111	*
2	*	*	*

b)

y \ x	0	1	2
0	7	2	5
1	0	6	2
2	7	5	1

median: 0 1 2 2 **5** 5 6 7 7

⇒

y \ x	0	1	2
0	*	*	*
1	*	5	*
2	*	*	*

c)

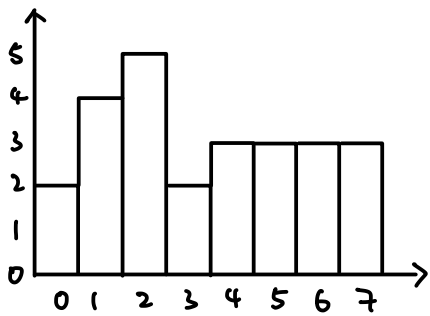
$$\begin{vmatrix} 4 & 7 & 2 \\ 3 & 0 & 6 \\ 5 & 7 & 5 \end{vmatrix} \quad \begin{vmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{vmatrix}$$

$= 4\times0 + 7\times1 + 2\times0 + 3\times1 + 0\times-4 + 6\times1 + 5\times0 + 7\times1 + 5\times0$

$= 0 + 7 + 0 + 3 + 0 + 6 + 0 + 7 + 0$

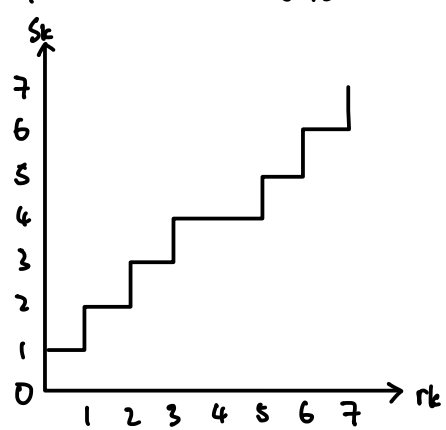
$= 23_{11}$

d)



e)

r_k	n_k	$pr(r_k) = n_k/5\times5$
0	2	0.08
1	4	0.16
2	5	0.2
3	2	0.08
4	3	0.12
5	3	0.12
6	3	0.12
7	3	0.12



$$S_0 = T(r_0) = (2^3-1)\sum_{j=0}^{0} pr(r_j)$$

$$= 7\times(0.08) = 0.56 \to 1$$

$\Rightarrow$ $S_1 = 7\times(0.08+0.16) = 1.68 \to 2$

$S_2 = 7\times(0.08+0.16+0.2)$

$= 3.08 \to 3$

$S_3 = 7\times(0.08+0.16+0.2+0.08)$

$= 3.64 \to 4$

$S_4 = 7\times(0.08+0.16+0.2+0.08+0.12)$

$= 4.48 \to 4$

$S_5 = 7\times(0.08+0.16+0.2+0.08+0.12$

$+ 0.12) = 5.32 \to 5$

$S_6 = 7\times(0.08+0.16+0.2+0.08+0.12$

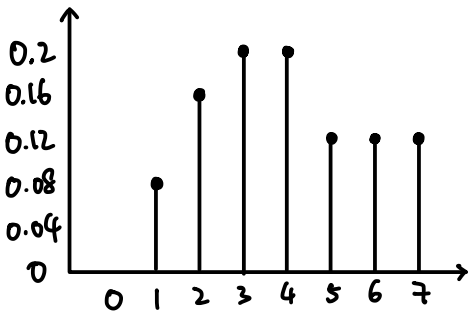
$+ 0.12+ 0.12) = 6.16 \to 6$

$S_7 = 7\times(0.08+0.16+0.2+0.08+0.12$

$+ 0.12 + 0.12 + 0.12)$

$= 7 \to 7$

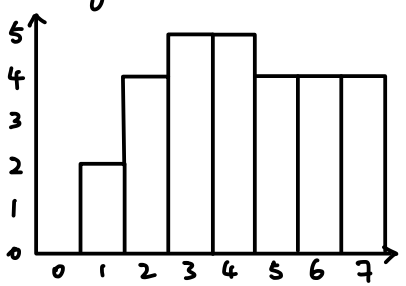
Histogram Equalization:



New image :

y \ x	0	1	2	3	4
0	4	7	6	3	1
1	3	4	6	2	2
2	4	7	3	5	4
3	4	1	6	3	2
4	5	7	5	2	3

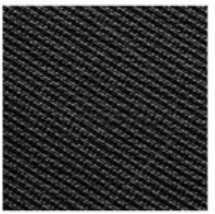
New histogram:



Question 2 (10 marks)
Consider the following images,

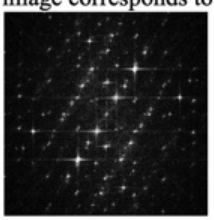

(a)

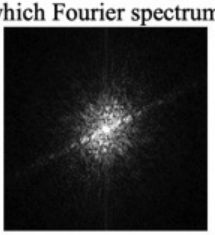

(b)

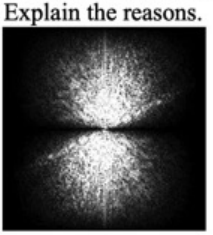

(c)

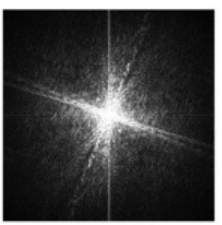

(d)

The modulus of the 2D DFT (followed by fftshift) of these images is shown below. Which image corresponds to which Fourier spectrum? Explain the reasons.


(1)


(2)


(3)


(4)

1 - c Diagonal shape
2 - a Vertical and horizontal general gray level change
3 - d Significantly change of gray level
4 - b The edges of the photo will have bright result in the Fourier spectrum with perpendicular result

Question 3 (10 marks)

Suppose that you form a lowpass spatial filter that average the four immediate neighbors of a point (x,y), but excludes the point itself.

(a) Find the equivalent filter H(u,v) in the frequency domain.
(b) Show that your result is a lowpass filter.

a) $g(x,y) = \frac{1}{4}[f(x,y+1) + f(x+1,y) + f(x-1,y) + f(x,y-1)]$

$f(x-x_0, y-y_0) \Leftrightarrow F(u,v)\, e^{-j2\pi(ux_0/M + vy_0/N)}$

$\Rightarrow G(u,v) = \frac{1}{4}\left(e^{j2\pi v/N} + e^{j2\pi u/M} + e^{-j2\pi u/M} + e^{-j2\pi v/N}\right) F(u,v)$

$= H(u,v)\, F(u,v)$

$\Rightarrow H(u,v) = \frac{1}{2}[\cos(2\pi u/M) + \cos(2\pi v/N)]$

b) When consider only one variable,

$\cos(2\pi u/M) \Rightarrow \cos(2\pi(u-M/2)/M)$

it starts at 1 when $u = \frac{M}{2}$

-1 when $u = M$

$\because$ The amplitude of the filter decrease as the frequency of the function increase

$\therefore$ It is a low-pass filter