Ans. to Tut 2

Qn 1

a)

$$\begin{pmatrix}1&0&0&0\\0&1&0&60\\0&0&1&0\\0&0&0&1\end{pmatrix}\begin{pmatrix}\cos 30^o&-\sin 30^o&0&0\\\sin 30^o&\cos 30^o&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}\begin{pmatrix}1.5&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}=\begin{pmatrix}1.5\cos 30^o&-\sin 30^o&0&0\\1.5\sin 30^o&\cos 30^o&0&60\\0&0&1&0\\0&0&0&1\end{pmatrix}$$

b)

```
glTranslatef (0.0, 60.0, 0.0);
glRotatef  (30.0, 0.0, 0.0, 1.0);
glScalef    (1.5, 1.0, 1.0);
object ( );
```

c) No, because matrix multiplications is non-commutative.

Another way to visualize the fact is by *thought experiment*: After

$\mathbf{T}(0,60,0)\mathbf{R}_Z(30^o)$

The origin (0, 0, 0) will be moved to (0, 60, 0). The origin is rotated about itself then translated.

After

$\mathbf{R}_z(30^o)\mathbf{T}(0,60,0)$

(0, 0, 0) will be moved to (-30, 51.96152423, 0). (0, 0, 0) is translated to (0, 60, 0), then orbital rotate about the origin by 30^o.

Qn 2

a)

Using rule 1, we use coordinate system 1 as the coordinate system. Every physical action is measured in this coordinate system.

$$\mathbf{M}_{2\leftarrow 1} = \mathbf{T}(4,2)^{-1} = \mathbf{T}(-4,-2) = \begin{pmatrix}1&0&-4\\0&1&-2\\0&0&1\end{pmatrix}$$

Check: $(0,0)^{(1)} = (-4,-2)^{(2)}$

Using rule 2, we use coordinate system 2 as the coordinate system

$$\boldsymbol{M}_{2\leftarrow 1} = \mathbf{T}(-4,-2) = \begin{pmatrix} 1 & 0 & -4 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{pmatrix}$$

b)

Using rule 1, we use coordinate system 2 as the coordinate system

$$\mathbf{M}_{3\leftarrow 2} = [\mathbf{T}(2,3)\mathbf{S}(0.5,0.5)]^{-1} = \mathbf{S}(2,2)\mathbf{T}(-2,-3)$$
$$= \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & -2 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 0 & -4 \\ 0 & 2 & -6 \\ 0 & 0 & 1 \end{pmatrix}$$

Useful identity: $(AB)^{-1} = B^{-1}A^{-1}$

Using rule 2, we use coordinate system 3 as the coordinate system

$$\mathbf{M}_{3\leftarrow 2} = \mathbf{T}(-4,-6)\boldsymbol{S}(2,2)$$
$$= \begin{pmatrix} 1 & 0 & -4 \\ 0 & 1 & -6 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 0 & -4 \\ 0 & 2 & -6 \\ 0 & 0 & 1 \end{pmatrix}$$

Check: $(0,0)^{(2)} = (-4,-6)^{(3)}$ $\quad$ $(2,0)^{(2)} = (0,-6)^{(3)}$

c)

It is awkward to find $(0,0)^{(3)}$ expressed in coordinate system 4. Hence it is more convenient to use rule 1. The position of $(0,0)^{(4)}$ is approximately $(6.3, 2)^{(3)}$

$$\mathbf{M}_{4\leftarrow 3} = [\mathbf{T}(6.3,2)\mathbf{R}(45^o)]^{-1} = \mathbf{R}(-45^o)\mathbf{T}(-6.3,-2)$$
$$= \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & -6.3 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} & -8.3/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} & 4.3/\sqrt{2} \\ 0 & 0 & 1 \end{pmatrix}$$

d) $\mathbf{M}_{4\leftarrow 1} = \mathbf{M}_{4\leftarrow 3}\mathbf{M}_{3\leftarrow 2}\mathbf{M}_{2\leftarrow 1} = \mathbf{R}(-45^o)\mathbf{T}(-6.3,-2)\mathbf{S}(2,2)\mathbf{T}(-2,-3)\mathbf{T}(-4,-2)$

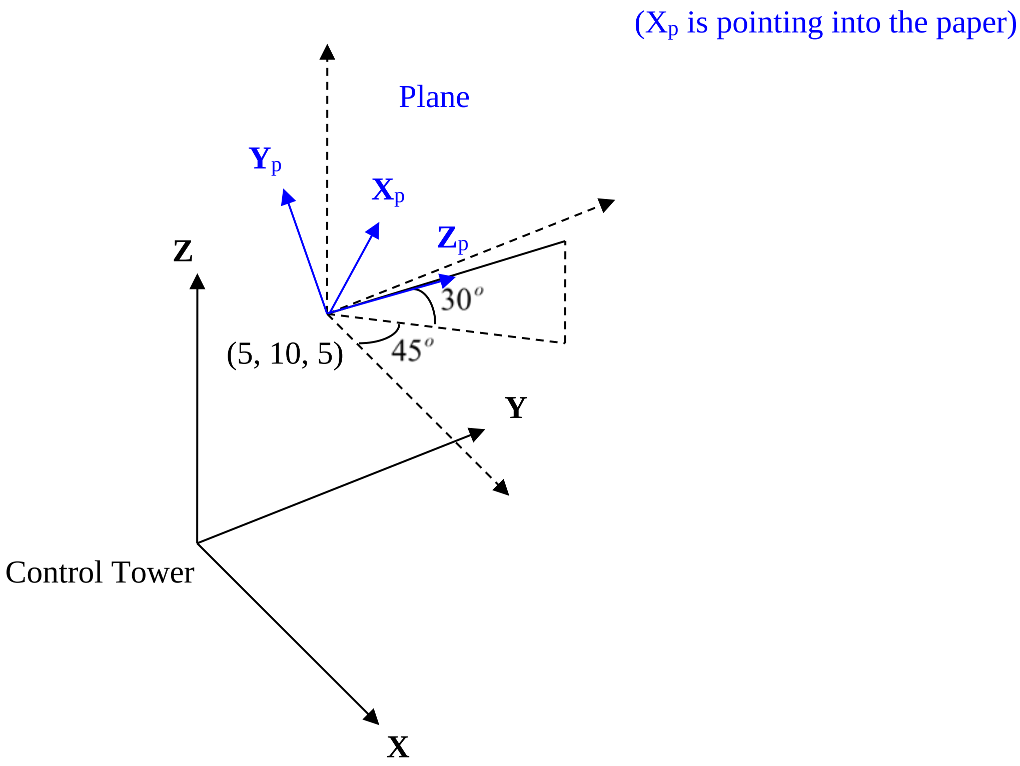
$$= \mathbf{R}(-45^o)\mathbf{T}(-6.3,-2)\mathbf{S}(2,2)\mathbf{T}(-6,-5)$$

$$\mathbf{M}_{1\leftarrow 4} = \mathbf{M}_{4\leftarrow 1}{}^{-1} = \mathbf{T}(6,5)\mathbf{S}(0.5,0.5)\mathbf{T}(6.3,2)\mathbf{R}(45^o)$$

$$= \begin{pmatrix} 0.5 & 0 & 6 \\ 0 & 0.5 & 5 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 6.3 \\ 1/\sqrt{2} & 1/\sqrt{2} & 2 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0.5/\sqrt{2} & -0.5/\sqrt{2} & 9.15 \\ 0.5/\sqrt{2} & 0.5/\sqrt{2} & 6 \\ 0 & 0 & 1 \end{pmatrix}$$

Useful Identity: $[ABC\ldots\mathbf{Z}]^{-1} = \mathbf{Z}^{-1}\ldots\mathbf{C}^{-1}\mathbf{B}^{-1}\mathbf{A}^{-1}$

Qn 3



$\mathbf{X}$, $\mathbf{Y}$, $\mathbf{Z}$, $\mathbf{X}_p$, $\mathbf{Y}_p$, $\mathbf{Z}_p$ are unit vectors.

$\mathbf{Z}_p$ is in the direction of $(\cos 45^o, \sin 45^o, \tan 30^o)$. Normalizing,

$$\mathbf{Z}_p = \frac{\sqrt{3}}{2}(\cos 45^o, \sin 45^o, \tan 30^o)$$

$$\mathbf{M}_{CT\leftarrow P} = \mathbf{M}_{P\leftarrow CT}{}^{-1} = \begin{pmatrix} \mathbf{X}_p & \mathbf{Y}_p & \mathbf{Z}_p & \begin{matrix}5\\10\\5\end{matrix} \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \left|\mathbf{Z}\times\mathbf{Z}_p\right| & \mathbf{Z}_p\times\mathbf{X}_p & \mathbf{Z}_p & \begin{matrix}5\\10\\5\end{matrix} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathbf{Z}\times\mathbf{Z}_p = \begin{vmatrix} i & j & k \\ 0 & 0 & 1 \\ \frac{\sqrt{3}\cos 45^0}{2} & \frac{\sqrt{3}\sin 45^o}{2} & \frac{\sqrt{3}\tan 30^o}{2} \end{vmatrix} = \left(-\frac{\sqrt{3}\sin 45^o}{2}, \frac{\sqrt{3}\cos 45^o}{2}, 0\right)$$

$$X_p = \left|Z\times Z_p\right| = (-\sin 45^o, \cos 45^o, 0)$$

$$\mathbf{Y}_p = \mathbf{Z}_p\times\mathbf{X}_p = \frac{\sqrt{3}}{2}\begin{vmatrix} i & j & k \\ \cos 45^o & \sin 45^0 & \tan 30^o \\ -\sin 45^o & \cos 45^o & 0 \end{vmatrix} = (-0.35355339, -0.35355339, 0.866025403)$$

Hence

$$\mathbf{M}_{CT\leftarrow P} = \begin{pmatrix} -\sin 45^0 & -0.35355339 & \frac{\sqrt{3}}{2}\cos 45^o & 5 \\ \cos 45^o & -0.35355339 & \frac{\sqrt{3}}{2}\sin 45^o & 10 \\ 0 & 0.866025403 & \frac{\sqrt{3}}{2}\tan 30^o & 5 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

<u>Alternative Method</u>

CT : Control Tower
P : Plane

$$\mathbf{M}_{CT\leftarrow P} = \mathbf{M}_{P\leftarrow CT}{}^{-1}$$

But $\mathbf{M}_{P\leftarrow CT} = \left[\mathbf{T}(5,10,5)\mathbf{R}_z(135^o)\mathbf{R}_x(60^o)\right]^{-1}$

Using coordinate system CT, the plane (as an object) is first aligned with the CT coordinate system. (Note that when correctly aligned, the under tray of the plane is facing you.) We wish to find the transformation (an action) that transforms it to the current plane configuration. The transformation needed is

$\left[\mathbf{T}(5,10,5)\mathbf{R}_z(135^o)\mathbf{R}_x(60^o)\right]$

$\mathbf{R}_x(60^o)$ will rotate the plane about the **X** axis such that after rotating, the plane is at an inclination of 30^o, with no banking, facing south. Note that the **X** axis is that of the coordinate system of CT. (The coordinate system is fixed. It will not be changed after performing an action.)

Now we need to rotate the plane, keeping its inclination, such that it is facing north east. This is achieved by rotating about the **Z** axis by 135^o, i.e., $\mathbf{R}_z(135^o)$. Note that the **Z** axis is that of the coordinate system of CT. (The coordinate system is fixed. It will not be changed after performing an action.)

Now the plane is at the origin, inclines 30^o, and facing north east. Now we wish to translate it such that it is positioned at (5, 10, 5). This is achieved by $\mathbf{T}(5,10,5)$.

Therefore

$$\mathbf{M}_{CT\leftarrow P} = \mathbf{M}_{P\leftarrow CT}{}^{-1} = [[\mathbf{T}(5,10,5)\mathbf{R}_z(135^o)\mathbf{R}_x(60^o)]^{-1}]^{-1} = \mathbf{T}(5,10,5)\mathbf{R}_z(135^o)\mathbf{R}_x(60^o)$$

<u>Notes</u>

If you were asked to position the plane as described using OpenGL, here's the program:

```
glTranslatef (5, 10, 5);
glRotatef    (135, 0, 0, 1);
glRotatef    (60,   1, 0, 0);
plane ( );
```