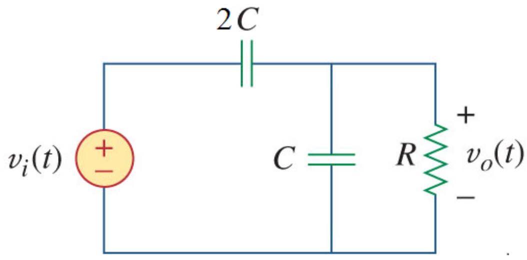
Student ID:__________________ Student Name:____________________________

For the circuit on the right,

a) Its frequency response follows the form:

$$\frac{\mathbf{V_o}}{\mathbf{V_i}} = \frac{A}{1+\omega_c/j\omega}$$

Find the value of A and derive the expression for ω_c in terms of the component symbols (e.g. R, C);



Note that general form of high pass filter in notes is $\frac{V_o}{V_i} = \frac{A(j\omega/\omega_c)}{1+j\omega/\omega_c} = \frac{A}{1+\omega_c/(j\omega)}$

$$\frac{V_o}{V_i} = \frac{Z_2}{Z_1+Z_2} = \frac{1}{1+\frac{Z_1}{Z_2}}$$

$$Z_1 = \frac{1}{2j\omega C}\ ,\ Z_2 = \frac{\frac{R}{j\omega C}}{\frac{1}{j\omega C}+R} = \frac{R}{1+j\omega RC} \ \rightarrow\ \frac{Z_1}{Z_2} = 2+\frac{2}{j\omega CR}$$

$$\frac{V_o}{V_i} = \frac{1}{\frac{3}{2}+\frac{1}{2j\omega CR}} = \frac{2/3}{1+\frac{1}{3CR}/j\omega}$$

$$\therefore \mathrm{A} = \frac{2}{3}\ , \omega_c = \frac{1}{3CR}$$

b) Determine $|V_o/V_i|$ when $\omega = 0$, $\omega = \omega_c$, and $\omega \rightarrow \infty$;

$$\left|\frac{V_o}{V_i}\right| = \left|\frac{A}{1+\frac{\omega_c}{j\omega}}\right| = \left|\frac{A}{1-\frac{\omega_c}{\omega}j}\right| = \frac{A}{\sqrt{1+\left(\frac{\omega_c}{\omega}\right)^2}}$$

$$\omega = 0, \left|\frac{V_o}{V_i}\right| = 0$$

$$\omega = \omega_c, \left|\frac{V_o}{V_i}\right| = \frac{A}{\sqrt{1+\left(\frac{\omega_c}{\omega}\right)^2}} = \frac{2/3}{\sqrt{2}} = \frac{\sqrt{2}}{3}$$

$$\omega \rightarrow \infty, \left|\frac{V_o}{V_i}\right| \rightarrow \mathrm{A} = \frac{2}{3}$$

c) Determine $\angle(V_o/V_i)$ when $\omega = 0$, $\omega = \omega_c$, and $\omega \rightarrow \infty$;

Denominator: $1 + j\left(-\frac{\omega_c}{\omega}\right)$

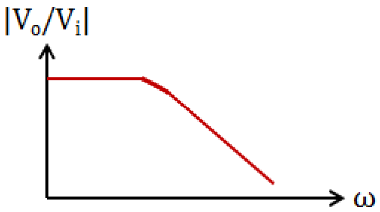
$$\angle\left({}^{V_o}\!/_{V_i}\right) = -\tan^{-1}\left(-\frac{\omega_c}{\omega}\right)$$

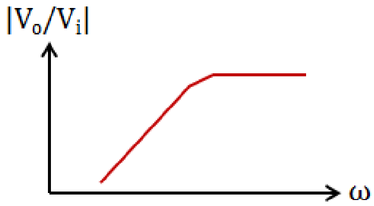
$$\omega = 0, \angle\left({}^{V_o}\!/_{V_i}\right) = -\tan^{-1}(-\infty) = \frac{\pi}{2}$$

$$\omega = \omega_c\,, \angle\left({}^{V_o}\!/_{V_i}\right) = -\tan^{-1}(-1) = \frac{\pi}{4}$$

$$\omega \rightarrow \infty, \angle\left({}^{V_o}\!/_{V_i}\right) = -\tan^{-1}(0) = 0$$

d) Circle the corresponding plot of $|V_o/V_i|$ vs ω from the following choices;





e) When $\omega = 10\omega_c$, and the amplitude of $V_i = 2$ V, estimate the amplitude of V_o in V.

When $\omega = 10\omega_c$, $\frac{V_0}{V_i} = \frac{A}{1+\frac{\omega_C}{j10\omega_C}} = \frac{A}{1+\frac{1}{10j}} = \frac{A}{1-0.1j}$

$$|V_o| = \frac{A}{\sqrt{1.01}}|V_i| = \frac{4}{3\sqrt{1.01}}\ V$$

Note that we may also approximate $\omega \rightarrow \infty$ when ω is much larger than ω_c, i.e. $\left|\frac{V_o}{V_i}\right| \rightarrow A = \frac{2}{3}$, $V_o = \frac{4}{3}$ V.