

DT convolution.

$$z[n] = x[n] * y[n]$$

$$= \sum_{k=-\infty}^{\infty} x[k] y[n-k]$$

$$= \sum_{k=-\infty}^{\infty} y[k] x[n-k]$$

ex 1) $x[n] = \{0.5, 2\}$



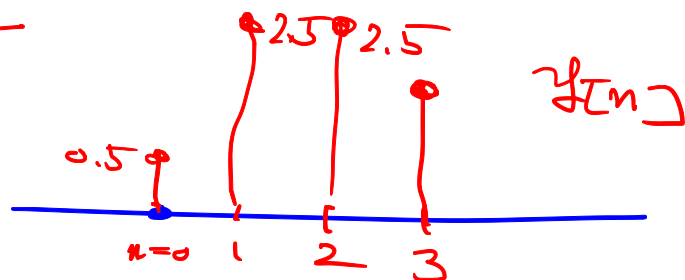
$h[n] = \{1, 1, 1\} \leftarrow h[0, 1, 2]$

$$y[n] = x[n] * h[n]$$

$$= \sum_{k=-\infty}^{\infty} x[k] h[n-k]$$

$$= x[0] h[n-0] + x[1] h[n-1]$$

$$= 0.5 h[n] + 2 h[n-1]$$



$$y[0] = 0.5 \cdot 1 + 2 \cdot \emptyset = 0.5$$

$$y[1] = 0.5 \cdot 1 + 2 \cdot 1 = 2.5$$

$$y[2] = 0.5 \cdot 1 + 2 \cdot 1 = 2.5$$

$$y[3] = 0.5 \cdot \emptyset + 2 \cdot 1 = 2$$

$$y[n] = 0$$

$$\text{if } n \geq 4 \text{ or } n \leq -1$$

Ex 2)

$$x[n] = \{ \underset{\uparrow}{2}, 1, 3 \}$$

$$h[n] = \frac{1}{2^n} u[n] \leftarrow \left[\text{if } n \geq 0, h[n] = \frac{1}{2^n} \right]$$

$$y[n] = \sum_{k=-\infty}^{\infty} x[k] h[n-k]$$

$$= x[0] h[n]$$

$$+ x[1] h[n-1]$$

$$+ x[2] h[n-2]$$

$$\left(\frac{1}{2^{n-2}} \right) u[n-2] \quad \checkmark$$

\Rightarrow

$$y[n] = 2 \cdot \underbrace{h[n]}_{n \geq 0} + \underbrace{h[n-1]}_{n \geq 1} + 3 \cdot \underbrace{h[n-2]}_{n \geq 2}$$

$$\checkmark y[0] = 2 \cdot \left(\frac{1}{2^0} u[0] \right) \Big|_{n=0} = 2$$

$$\checkmark y[1] = 2 \cdot \frac{1}{2^1} + \frac{1}{2^{1-1}} = 1 + 1 = 2$$

$$\checkmark y[2] = \frac{4}{2^{2-2}} = 4$$

$$y[n] = 2 \cdot \frac{1}{2^n} + \frac{1}{2^{n-1}} + 3 \cdot \frac{1}{2^{n-2}}$$

if $n \geq 2$

$$= \frac{1}{2^{n-2}} + \frac{3}{2^{n-2}} = \frac{4}{2^{n-2}}$$

$$\underline{h[n] \Rightarrow n=0, 1, \dots, N-1}$$

N terms

$$\boxed{x[n] \Rightarrow n=0, 1, \dots, M-1}$$

M terms.

$$\underline{y[n] = x[n] * h[n]}$$

$\hookrightarrow N+M-1$ terms

$$n=0, 1, \dots, N+M-2$$

$$\begin{bmatrix} y[0] \\ y[1] \\ y[2] \\ \vdots \\ y[N+M-2] \end{bmatrix} = \begin{bmatrix} x[0] & 0 & 0 & \dots & 0 \\ x[1] & x[0] & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x[M] & x[M-1] & \dots & x[0] & 0 \\ 0 & x[M] & \dots & x[M-1] & x[0] \\ \vdots & \vdots & \ddots & \vdots & \vdots \end{bmatrix} \begin{bmatrix} h[0] \\ h[1] \\ \vdots \\ h[N-1] \end{bmatrix}$$

$(N+M-1) \times N$
 $N \times 1$

$(N+M-1) \times 1$

$$= \begin{bmatrix} h[0] & 0 & \dots & 0 \\ h[1] & h[0] & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ h[N-1] & h[N-2] & \dots & h[N-1] \\ 0 & 0 & \dots & 0 \end{bmatrix} \begin{bmatrix} x[0] \\ x[1] \\ \vdots \\ x[M-1] \end{bmatrix}$$

$(N+M-1) \times M.$

$$M \times 1.$$

Ex 3)

$$x[n] = \{4, 2\}$$

$$N = 2$$

$$h[n] = \{4, 3, 2, 4\}$$

$$M = 4$$

$$y[n] = \{$$

$$\}^{N+M-1} = 5$$

$$\begin{bmatrix} y[0] \\ y[1] \\ y[2] \\ y[3] \\ y[4] \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 3 & 4 \\ 2 & 3 \\ 4 & 2 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} 4 \\ 2 \end{bmatrix}$$

5×1 5×2 2×1

$$= \begin{bmatrix} 16 + 0 \times 2 \\ \underline{3 \times 4} + \underline{4 \times 2} \\ \underline{2 \times 4} + \underline{3 \times 2} \\ \underline{4 \times 4} + \underline{2 \times 2} \\ \cancel{0 \times 4} + \underline{4 \times 2} \end{bmatrix} = \begin{bmatrix} 16 \\ 20 \\ 14 \\ 20 \\ 8 \end{bmatrix}$$

$$\begin{bmatrix} y_{20} \\ \vdots \\ y_{14} \end{bmatrix} = \begin{bmatrix} 4 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 \\ 2 & 4 & 0 & 0 \\ 0 & 2 & 4 & 0 \\ 0 & 0 & 2 & 4 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \\ 2 \\ 4 \end{bmatrix}$$

$$5 \times 1$$

$$(5 \times 4)$$

$$4 \times 1$$