# Tutorial 1 (with solution)

## Sets

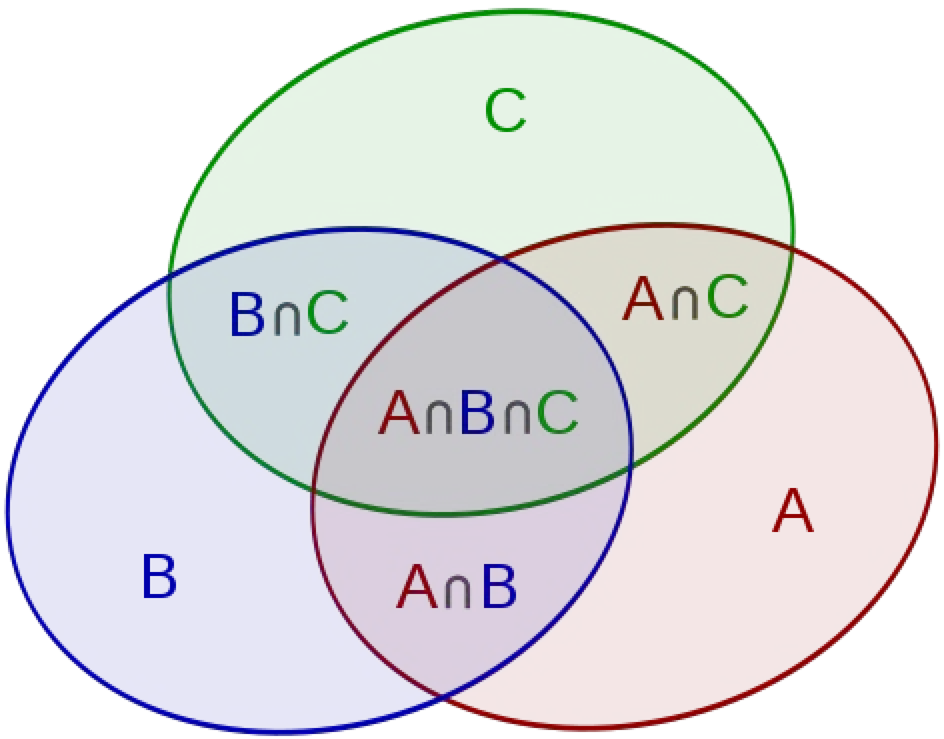
# Question 1: Inclusion & Exclusion

- What is the formula for $|A \cup B \cup C|$?

a) $|A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$
b) $|A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + 3|A \cap B \cap C|$
c) $|A| + |B| + |C| - 2|A \cap B| - 2|A \cap C| - 2|B \cap C| + 3|A \cap B \cap C|$
d) $|A| + |B| + |C| - 3|A \cap B| - 3|A \cap C| - 3|B \cap C| + 3|A \cap B \cap C|$

# Q.1 Solution

$$|A|+|B|+|C|-|A\cap B|-|A\cap C|-|B\cap C|+|A\cap B\cap C|$$



# Question 2: Subset Relationship

Let $A = \{n \in \mathbf{Z} \mid n = 5r \text{ for some integer } r\}$

and $B = \{m \in \mathbf{Z} \mid m = 20s \text{ for some integer } s\}$.

i. Is $A \subseteq B$ ?

ii. Is $B \subseteq A$ ?

a) Both are true.
b) Both are false.
c) (i) is true while (ii) is false
d) (i) is false while (ii) is true

# Q.2 Solution

a) No.

- This can be proved by a counter-example.
- For example, $5 \in A$ (since $5 = 5r$, where $r = 1$).
- But 5 cannot be written as $20s$, where s is an integer.
- So, 5 is not an element of $B$.
- Therefore, $A \nsubseteq B$.

b) Yes.

- Let $n \in B$, so $n = 20s$, where $s$ is an integer.
- Since $n = 20s = 5(4s)$, where $4s$ is an integer, $n \in A$.
- Therefore, $B \subseteq A$.

# Question 3: Power Set

“If $A$ and $B$ are two sets with the same power set, then $A = B$.“

Is the above statement true?

a) Yes
b) No
c) Cannot be determined

Justify your answer.

# Q.3 Solution

- The statement is true.

**Proof:** We prove it by contraposition.

Suppose $A \neq B$. Then there exists an element $x$ which belongs to one set but not the other.

Without loss of generality, assume $x \in A$ but $x \notin B$.

- (Otherwise, reverse the role of $A$ and $B$.)

Then $\{x\} \in \mathcal{P}(A)$ but $\{x\} \notin \mathcal{P}(B)$.

Hence, $\mathcal{P}(A) \neq \mathcal{P}(B)$.

*Q.E.D.*

# Q.4 Cartesian Product

- Consider two nonempty sets $A$ and $B$.

- Is it true that $A \times B \neq B \times A$?

  a) Yes
  b) No
  c) Cannot be determined

  Justify your answer.

# Q.4 Solution

- It cannot be determined.

- If $A = B$, then $A \times B = B \times A$.

- If $A \neq B$, then $A \times B \neq B \times A$.
  - For example, $A = \{a\}$ and $B = \{1, 2\}$.
  - $A \times B = \{(a, 1), (a, 2)\}$.
  - $B \times A = \{(1, a), (2, a)\}$.

# Question 5: Set Equality

Is it true that $B = C$, where

$$B = \{y \in \mathbf{Z} \mid y = 18b - 2 \text{ for some integer } b\},$$

and

$$C = \{z \in \mathbf{Z} \mid z = 18c + 16 \text{ for some integer } c\}?$$

a) Yes
b) No
c) Cannot be determined

Justify your answer.

# Q.5 Solution

Yes, it is true. The proof consists of two parts.

**Part 1, Prove that $\boldsymbol{B \subseteq C}$:**

- Let $y$ be an element of $B$, so $y = 18b - 2$ for some integer $b$.
- We can re-write it as
$$y = 18b - 2 = 18(b - 1) + 16.$$
- Since $b - 1$ is an integer, $y \in C$.
- Therefore, $B \subseteq C$.

# Q.5 Solution

**Part 2, Prove that $\boldsymbol{C \subseteq B}$:**

- Let $z$ be an element of $C$, so $z = 18c + 16$, for some integer $c$.
- We can re-write it as

$$z = 18c + 16 = 18(c + 1) - 2.$$

- Since $c + 1$ is an integer, $z \in B$.
- Therefore, $C \subseteq B$.

Combining the two parts, we conclude that $B = C$.

*Q.E.D.*