

## Exercise on Positive Definite

1. Determine whether each of the following quadratic forms in two variables is positive or negative definite or semidefinite, or indefinite.
  - a.  $x^2 + 2xy$ .
  - b.  $-x^2 + 4xy - 4y^2$
  - c.  $-x^2 + 2xy - 3y^2$ .
  - d.  $4x^2 + 8xy + 5y^2$ .
  - e.  $-x^2 + xy - 3y^2$ .
  - f.  $x^2 - 6xy + 9y^2$ .
  - g.  $4x^2 - y^2$ .
  - h.  $(1/2)x^2 - xy + (1/4)y^2$ .
  - i.  $6xy - 9y^2 - x^2$ .
2. Determine whether each of the following quadratic forms in three variables is positive or negative definite or semidefinite, or indefinite.
  - a.  $-x^2 - y^2 - 2z^2 + 2xy$
  - b.  $x^2 - 2xy + xz + 2yz + 2z^2 + 3zx$
  - c.  $-4x^2 - y^2 + 4xz - 2z^2 + 2yz$
  - d.  $-x^2 - y^2 + 2xz + 4yz + 2z^2$
  - e.  $-x^2 + 2xy - 2y^2 + 2xz - 5z^2 + 2yz$
  - f.  $y^2 + xy + 2xz$
  - g.  $-3x^2 + 2xy - y^2 + 4yz - 8z^2$
  - h.  $2x^2 + 2xy + 2y^2 + 4z^2$
3. Consider the quadratic form  $2x^2 + 2xz + 2ayz + 2z^2$ , where  $a$  is a constant. Determine the definiteness of this quadratic form for each possible value of  $a$ .
4. Determine the values of  $a$  for which the quadratic form  $x^2 + 2axy + 2xz + z^2$  is positive definite, negative definite, positive semidefinite, negative semidefinite, and indefinite.
5. Consider the matrix

$$\begin{Bmatrix} a & 1 & b \\ 1 & -1 & 0 \\ b & 0 & -2 \end{Bmatrix}.$$

Find conditions on  $a$  and  $b$  under which this matrix is negative definite, negative semidefinite, positive definite, positive semidefinite, and indefinite. (There may be no values of  $a$  and  $b$  for which the matrix satisfies some of these conditions.)

6. Show that the matrix

$$\begin{Bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{Bmatrix}$$

is not positive definite.