Area of the region

Problem 1

(a) Find the area of the region bounded by the curve $y = 1 + e^{3x}$, x-axis, $x = 1$ and $x = 3$.

(b) Find the area of the region bounded by $y = \ln x$, x-axis, $x = \frac{1}{2}$ and $x = 2$.

(c) Find the area of the region bounded by $y = 1 + x^2$, $y = e^{-x}$, y-axis and $x = 2$.

(d) Find the area of the region bounded by $y = -x^2 + 2x + 1$, $x + y = 1$.

Problem 2

In this problem, we would like to find the area of the region bounded by the curves $y = e^{2x} - 3e^x - 1$ and $y = e^x - 4$ for $-2 \le x \le 2$. In order to find the area, it is important to compare the values between $f_1(x) = e^{2x} - 3e^x - 1$ and $f_2(x) = e^x - 4$ for $-2 \le x \le 2$ so that we can determine the "upper curve" and "lower curve". This problem will show you a general technique (which is taught in the lecture) to achieve this goal.

(a) Find all *critical points* by solving the equation $f_1(x) = f_2(x)$ for $-2 \le x \le 2$. These critical points are the points where the relative magnitude between $f_1(x)$ and $f_2(x)$ changes.

(b) With the critical points obtained in (a), we divide the interval $[-2, 2]$ into several small intervals with the critical points as the "cutoff" points. For each small interval, determine which function ($f_1(x)$ or $f_2(x)$) is larger.
(Hint: You may compare the values by simply substituting some value of x within the small interval.)

(c) Using the information obtained in (b), find the area of the region bounded by the curves $y = e^{2x} - 3e^x - 1$ and $y = e^x - 4$ for $-2 \le x \le 2$.

Problem 3

Using similar technique as in Problem 2, find the area of the region bounded by the curves $y = (x^2 - x + 1)e^x$ and $y = xe^x$ for $0 \le x \le 2$.

Volume

Problem 4

(a) Find the volume of the solid generated by rotating the region bounded by $y = \sin 3x$, x-axis for $0 \le x \le \pi$ about the x-axis for one complete revolution.

(b) Find the volume of the solid generated by rotating the region bounded by $y = 1 + \cos 3x$ and $y = 1 + \cos x$ for $0 \le x \le \frac{\pi}{2}$ about the x-axis for one complete revolution.

(c) Find the volume of the solid generated by rotating the region bounded by $y = e^{2x}$, x-axis, y-axis and $x = \ln 3$ about

(i) the x-axis for 1 complete revolution.
(ii) the y-axis for 1 complete revolution.
(iii) $y = -1$ for 1 complete revolution.
(iv) $x = -1$ for 1 complete revolution.

(d) Find the volume of the solid generated by rotating the region above $y = \frac{1}{2}$ and below $y = \sin x$ for $0 \le x \le \pi$ about

(i) the x-axis for 1 complete revolution.

(ii) the y-axis for 1 complete revolution.

(iii) the line $y = \frac{1}{2}$ for 1 complete revolution.

Arc Length

Problem 5

(a) Find the arc length of the curve $y = \ln(\sec x)$ for $0 \le x \le \frac{\pi}{4}$.

(b) Find the arc length of the curve $y = \frac{1}{3}x^{\frac{3}{2}}$ for $0 \le x \le 5$.

(c) Find the arc length of the curve $(y-1)^3 = \frac{9}{4}x^2$ for $0 \le x \le \frac{2}{3}(3)^{\frac{3}{2}}$.

(d) Find the arc length of the curve $y = \frac{a}{2}\left(e^{\frac{x}{a}} + e^{-\frac{x}{a}}\right)$ for $0 \le x \le a$ where $a > 0$.

(e) Find the arc length of the curve $\begin{cases} x(t) = \cos t + t\sin t \\ y(t) = \sin t - t\cos t \end{cases}$, $0 \le t \le \frac{\pi}{2}$.

(f) Find the arc length of the curve $\begin{cases} x(t) = t - \sin t \\ y(t) = 1 - \cos t \end{cases}$ for $0 \le t \le 2\pi$.

(g) Find the length of the parabolic arc with equation $\begin{cases} x(t) = at^2 \\ y(t) = 2at \end{cases}$ for $0 \le t \le a$, where $a > 0$.

Surface Area

Problem 6

(a) Find the surface area of the surface generated by rotating the region bounded by the curves $y = x^3$, x-axis, $x = 0$ and $x = 2$ about the x-axis for one complete revolution.

(b) Find the surface area of the surface generated by rotating the region in the first quadrant bounded by the curve $y^2 = 4 - x$ and the x-axis about the x-axis for 1 complete revolution.

(c) Find the surface area of the surface generated by rotating the region bounded by the curve $y = e^x$, $x = 0$, $x = 1$ and the x-axis about the x-axis for 1 complete revolution.
(Hint: The substitution $e^x = \tan\theta$ may be useful in computing the resulting integral. Try this when you get stuck.)

(d) Let R be the region bounded by the four straight lines $y = x$, $x + y = 4$, $y = x - 2$ and $x + y = 2$. Find the surface area of the surface obtained by rotating the region R about the x-axis for 1 complete revolution.

Miscellaneous Problems

Problem 7

Let R be the region bounded by $y = e^x$, $y = e^{-x}$ and the vertical line $x = 2$.

(a) Find the area of the region R.

(b) (i) Find the volume of the solid generated by rotating the region R about the x-axis.
(ii) Find the volume of the solid generated by rotating the region R about the y-axis.
(iii) Find the volume of the solid generated by rotating the region R about $y = -1$.

Problem 8

Let R be the region bounded by $y = x^2$ and $x = y^2$.

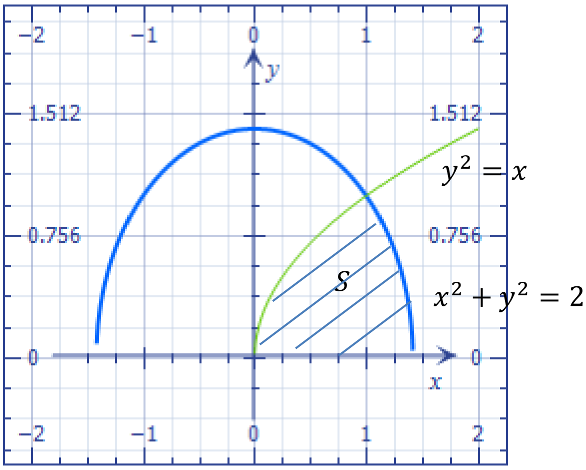
(a) Find the area of the region R.

(b) (i) Find the volume of the solid generated by rotating the region R about the x-axis.
(ii) Find the volume of the solid generated by rotating the region R about the y-axis.
(iii) Find the volume of the solid generated by rotating the region R about $y = -1$.

(c) Find the arc length of the boundary curve of the region R.

Problem 9

Let S be the region (in the first quadrant) bounded by a circle $x^2 + y^2 = 2$, $y^2 = x$ and the x-axis (as shown below)



(a) Find the area of the region S.

(b) (i) Find the volume of the solid generated by rotating the region S about the <u>x-axis</u>.

(ii) Find the volume of the solid generated by rotating the region S about the <u>y-axis</u>

(c) Find the surface area of the solid generated by rotating the region S about the <u>x-axis.</u>