EE3210 Signals & Systems

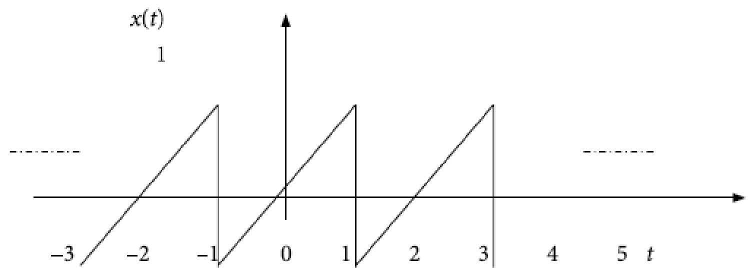
Due on 11:59 PM, December 7th, 2021

Homework #2, 3

1. Total mark is 200 points (= 20 points per problem $\times$ 10 problems)
2. Submission due by December 7th, Mid-night, 2021. We will not accept late submission.
3. Online submission through Canvas
   - Scan or taking a photo of your answer sheet, then upload to Canvas

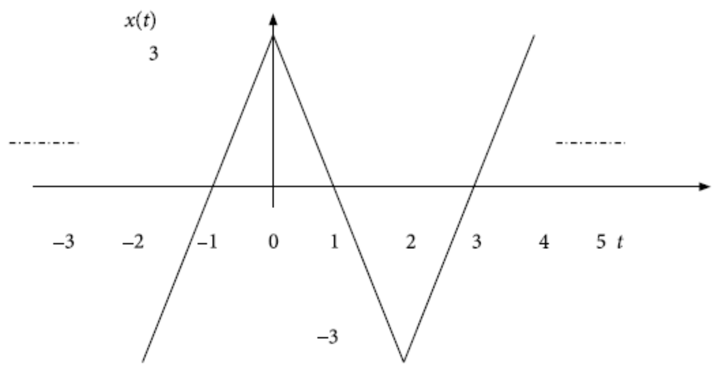
Problem 1

Derive the complex and trigonometric FS representation of the following periodic signal $x(t)$.



Problem 2

a) Derive the complex and trigonometric FS representation of the following periodic signal $x(t)$.



b) Use the trigonometric FS representation of $x(t)$ to prove the following equality

$$\frac{\pi^2}{8} = 1 + \frac{1}{9} + \frac{1}{25} + \ldots$$

Problem 3

a) Derive the FT of the following signal $x(t)$, which is given by

$$x(t) = e^{(1+2t)} u\left(-t+2\right)$$

b) Find the inverse FT of $X(f)$, which is given by

$$X(f) = \begin{cases} 2\cos(2\pi f) & \text{for} \quad |f| \le \dfrac{1}{2} \\ 0 & \text{for} \quad |f| > \dfrac{1}{2} \end{cases}$$

Problem 4

Consider a continuous time LTI systems described by the following differential equation.

$$\frac{d^2 y(t)}{dt^2} + 5\frac{dy(t)}{dt} + 6y(t) = \frac{dx(t)}{dt} + 4x(t)$$

Derive the frequency response $H(f)$ and the corresponding impulse response $h(t)$. Furthermore, derive the system output for input signal $x(t) = e^{-4t}u(t)$.

Problem 5

Consider the following filters with impulse response $h(t)$.

$$h(t) = w_0 \; e^{-w_0 t} u\left(t\right)$$

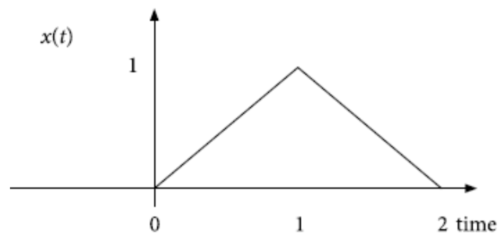
For the given filter, derive the 3-dB bandwidth $f_{3\text{ dB}}$, equivalent bandwidth f_{eq}, and 80 percent energy containment bandwidth $f_{90\%}$, respectively.

Problem 6

a) Find the Laplace Transform (LT) and its corresponding ROC of the following signal.

$$x(t) = \cos^2(t)\, u(t)$$

b) Find the LT and its corresponding ROC of the following signal.



c) Find the LT of the following signal.

$$x(t) = e^{2t}u(t) * tu(t)$$

d) Find the inverse LT of the following $X(s)$ for ROC given by $-2 < \mathrm{Re}(s) < -1$.

$$X(s) = \frac{-5s-7}{(s+1)(s-1)(s+2)}$$

Problem 7

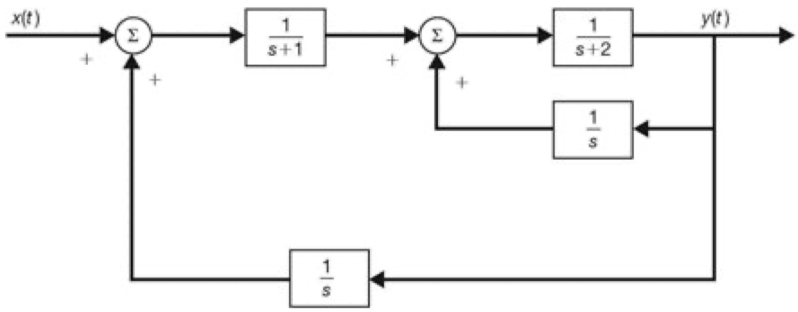
Consider a causal, stable LTI system described by the following expression.

$$\frac{d^2y(t)}{dt^2} + 5\frac{dy(t)}{dt} + 6y(t) = \frac{d^2x(t)}{dt^2} + 8\frac{dx(t)}{dt} + 13x(t)$$

Use LT to find the transfer function $H(s)$ and its corresponding impulse response $h(t)$.

Problem 8

Determine the overall transfer function $H(s)$ for the system diagram illustrated below.



Problem 9

a) Find the Z-transform of the following sequence and its corresponding ROC.

$$x[n] = \left\{5, 3, \underset{\uparrow}{-2}, 0, 4, -3\right\}$$

b) Find the Z-transform and its corresponding ROC for the following sequences

$$x_1[n] = \left(\frac{1}{3}\right)^n u[n] + \left(\frac{1}{2}\right)^n u[-n-1],$$
$$x_2[n] = \left(\frac{1}{2}\right)^n u[n] + \left(\frac{1}{3}\right)^n u[-n-1]$$

c) Find the inverse Z-transform of the following $X(z)$

$$X(z) = Z^2\left(1 - \frac{1}{2}z^{-1}\right)\left(1 - z^{-1}\right)\left(1 + 2z^{-1}\right), \quad 0 < |z| < \infty$$

Problem 10

a) Find the Z-transform of the following sequence and its corresponding ROC.

$$x[n] = \begin{cases} 1 & \text{for} \quad n < 0, \\ 0.5^n & \text{for} \quad n \geq 0 \end{cases}$$

b) Calculate the convolution of the two sequences using Z-transform

$$x_1[n] = \left\{ \underset{\uparrow}{1}, -2, 1 \right\}, \quad x_2[n] = \left\{ \underset{\uparrow}{1}, 1, 1, 1, 1, 1 \right\}$$

c) Find the inverse Z-transform of the following $X(z)$ for three different ROCs.

$$X(z) = \frac{Z + 1}{3Z^2 - 4Z + 1}$$

(i) Find $x[n]$ if ROC is $|Z| > 1$

(ii) Find $x[n]$ if ROC is $|Z| < \frac{1}{3}$

(iii) Find $x[n]$ if ROC is $\frac{1}{3} < |Z| < 1$