IMPORTANT: The answers are provided with the view to deepen your understanding of the topic and to stimulate more in depth discussions. They should not be shared with others or put on the web. The questions and answers may be revised in the next course delivery.

Answers to Tutorial 8

Qn 1

The weight of the mice has a distribution with mean $\mu_0 = 32$ and standard deviation $\sigma = 4$

$\bar{X} = 30.4 \quad n = 25$

Assume that $\bar{X}$ has a normal distribution with $\mu_0 = 32$ and standard deviation $\sigma/\sqrt{n} = 4/\sqrt{25}$. Converting to standard normal distribution

$$Z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} = \frac{30.4 - 32}{4/\sqrt{25}} = -2$$

Level of significance $\alpha = 0.05$

We set up the hypothesis test

Null hypothesis $\mu = \mu_0$

Alternative hypothesis $\mu \neq \mu_0$

We reject the hypothesis if

$$|Z| > z_{\alpha/2} = z_{0.025} = 1.96$$

Since $|Z| = 2 > 1.96$, the hypothesis is rejected.

Alternatively,

p-value = $P\{|Z| > 2\} = 2P\{Z > 2\} = 2(1 - 0.9772) = 0.0456$

Since the p-value is smaller than 0.05, the hypothesis is rejected.

Hence the scientist concludes that the assistant does not choose the mice at random.

Qn 2

$n = 64 \quad \sigma = 20 \quad \mu_0 = 50$

We test the hypothesis

Null hypothesis $\mu = \mu_0$

Alternative hypothesis $\mu \neq \mu_0$

By the central limit theorem, $\bar{X}$ is approximately normal with mean $\mu_0 = 50$ and standard deviation $\sigma/\sqrt{n} = 20/\sqrt{64}$

$$Z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} = \frac{\bar{X} - 50}{20/\sqrt{64}}$$

a) $\bar{X} = 52.5,\ Z = 1$

p-value = $P\{|Z| > 1\} = 2P\{Z > 1\} = 2(1 - 0.8413) = 0.3174$

b) $\bar{X} = 55.0,\ Z = 2$

p-value = $P\{|Z| > 2\} = 2P\{Z > 2\} = 2(1 - 0.9772) = 0.0456$

c) $\bar{X} = 57.5,\ Z = 3$

p-value = $P\{|Z| > 3\} = 2P\{Z > 3\} = 2(1 - 0.9987) = 0.0026$

Qn 3

Since the producer specifies that the mean lifetime of the battery is at least 240 hours, we should set the critical region such that we would reject the hypothesis if $\bar{X}$ is much less than 240 hours. Thus we wish to test the hypothesis

Null hypothesis $\mu = \mu_0$

Alternative hypothesis $\mu < \mu_0$

where $\mu_0 = 240$

Since the distribution is normal, the sample size is small and the variance is unknown, we use the t-distribution

$\bar{X} = 237.0556$ $S = 11.27972$ $n = 18$

$$T = \frac{\bar{X} - \mu_0}{S/\sqrt{n}} = \frac{237.0556 - 240}{11.27972/\sqrt{18}} = -1.107477068$$

At level of significance $\alpha = 0.05$,

$T_{0.05,17} = 1.740$

Since

$T > -T_{0.05,17}$

the hypothesis is not rejected. The testing result is consistent with the hypothesis that the battery life is at least 240 hours.

You can also observe that 8 out of 18 batteries have a life of at least 240 hours. Hence it is unlikely to be able to reject the hypothesis. Later, we'll see how we can use a sign test to formalize the argument.

Qn 4

$\bar{X} = 11.17$ $\quad \bar{Y} = 11.9875$ $\quad \sigma_x = \sqrt{0.09} = 0.3$ $\quad \sigma_y = \sqrt{0.16} = 0.4$ $\quad n = 10$
$m = 8$

We wish to test the hypothesis

Null hypothesis $\quad H_0$: $\mu_x = \mu_y$

Alternative hypothesis $\quad H_1$: $\mu_x \neq \mu_y$

Assume the two distributions are normal. Since the two variances are known, we can use

$$|Z| = \frac{|\bar{X} - \bar{Y}|}{\sqrt{\frac{{\sigma_x}^2}{n} + \frac{{\sigma_y}^2}{m}}}$$

$= 4.800525295$

At level of confidence $\alpha = 0.05$

$z_{0.025} = 1.96$

Since $|Z| > z_{0.025}$, the hypothesis is rejected. The two lakes are not equally contaminated.

Qn 5

$\bar{X} = 10.57167$ $\quad \bar{Y} = 10.54429$ $\quad S_x = 0.160302$ $\quad S_y = 0.042762$ $\quad n = 6$ $\quad m = 7$

We wish to test the hypothesis

Null hypothesis H_0: $\mu_x = \mu_y$

Alternative hypothesis H_1: $\mu_x \neq \mu_y$

Assume the two populations have normal distributions and equal variances. Since the variance is unknown, the t-test with $(n+m-2)$ d.f. is used.

$$T = \frac{\bar{X} - \bar{Y}}{\sqrt{S_p{}^2(\frac{1}{n} + \frac{1}{m})}}$$

$$S_p{}^2 = \frac{(n-1)S_x{}^2 + (m-1)S_y{}^2}{n+m-2} = 0.012677744$$

$T = 0.437084512$

At level of significance $\alpha = 0.05$

$t_{0.025,11} = 2.201$

Since $|T| < t_{0.025,11}$, the hypothesis is not rejected. The evidence is consistent with the hypothesis that the mean viscosity of the two brands is equal.