MA1201 Calculus and Basic Linear Algebra II

Problem Set 1 Basic Concept in Integration

Problem 1 Compute the following indefinite integrals:

(a) $\int \cos(3x+1)\,dx$

(b) $\int \left(\frac{1}{x^3} - \sqrt{x}\right) dx$

(c) $\int e^{1-x} dx$

(d) $\int \frac{1}{1+16x^2} dx$

(e) $\int \frac{1}{2x+1} dx$

(f) $\int \frac{1}{(2x+1)^2} dx$

Problem 2 (A bit harder) Compute the following indefinite integrals:

(a) $\int \frac{x^2 - x + 1}{x^2} dx$

(b) $\int \frac{2x^2}{x^2+1} dx$

(c) $\int \frac{e^{2x} + e^{x-3} + 1}{e^{x+1}} dx$

(d) $\int \sin 3x \sin 2x\,dx$

(e) $\int \cos^3 2x\,dx$

(f) $\int \frac{1}{(x-1)(2x-3)} dx$

(g) $\int \frac{3}{x^2 - 2x + 5} dx$

(h) $\int \frac{1}{2x^2 - 4x + 9} dx$

(i) $\int \frac{x+6}{(2x-1)^3} dx$

(j) $\int \tan^2 x\,dx$

(Hint: For (j), you need to use a trigonometric identity.)

Problem 3 Compute the following definite integrals:

(a) $\int_1^2 \frac{x-1}{3x^2} dx$

(b) $\int_{-1}^{1} \cos(3x+1)\,dx$

(c) $\int_0^1 (e^{2x+1} - e^{2x-1}) dx$

(d) $\int_0^{\frac{\pi}{2}} \sin^2 x\,dx$

(e) $\int_0^1 |2x-1| dx$

(f) $\int_{-\pi}^{\pi} |\sin x| dx$

(g) $\int_0^2 e^{1+|x-1|} dx$

(h) $\int_{-1}^{1} x^4 \sin^9 x\,dx$

(i) $\int_{-\pi}^{\pi} \frac{x^2 \sin^3 x}{1 + \cos^5 x} dx$

*(j) $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{x^2 \cos x + \cos x + \sin^3 x}{x^2 + 1} dx$

(Hint: For (h), (i),(j), you may check whether the given function is an odd function. It requires some extra trick to handle the integral in (j).)

Problem 4 Compute the following derivatives:

(a) $\frac{d}{dx} \int_3^x e^{2y^2+1} dy$

(b) $\frac{d}{dx} \int_{2x}^{x^2} \cos(y^2)\,dy$

Problem 5

(a) Using fundamental theorem of calculus, show that

$$\int_0^a f(x)dx = \int_0^a f(a-x)dx$$

(b) It is given that $g(x)$ is a periodic function with period 1 (i.e. $g(x+1) = g(x)$ for any x). Using fundamental theorem of calculus, show that

(i) $\int_0^4 g(x)dx = 4\int_0^1 g(x)dx$ (ii) $\int_0^1 g(3x)dx = \frac{1}{3}\int_0^3 g(x)dx$