Session: **Name:** **Student ID:**

MA 1201 Semester B 2020/21

Midterm Exam (E/F/G/H, 100 mins)

Academic honesty pledge for the online midterm assessment:
I pledge that the answers in this exam are my own and that I will not seek or obtain an unfair advantage in producing these answers. Specifically,

- I will not plagiarize (copy without citation) from any source;
- I will not communicate or attempt to communicate with any other person during the exam; neither will I give or attempt to give assistance to another student taking the exam;
- I will use only approved devices (e.g., calculators) and/or approved device models;
- I understand that any act of academic dishonesty can lead to disciplinary action.

Name: Student ID:

Signature:

or

Please write "I pledge to follow the Rules on Academic Honesty and understand that violations may lead to severe penalties" onto the first examination answer sheet.

Session: **Name:** **Student ID:**

Instructions:

- Please show your work. Unsupported answers will receive **NO** credits.
- Make sure you write down the correct lecture session (E/F/G/H) you have registered for, together with your full name and student ID on the front page of your answer script.
- Exams submitted to wrong lecture sessions will **NOT** be graded and will receive **0 POINTS**.

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1. (25 points) Let $A(1,2,0)$, $B(-1,3,0)$, $C(-1,2,-1)$, and $D(0,1,1)$ be four points in $\mathbb{R}^3$. Using vector method:

(a) (8 points) Find the area of the triangle $\triangle ABC$.

(b) (9 points) Find the equation of the plane that contains A, B, and C.

(c) (8 points) Find the volume of the parallelepiped with AB, AB, and AD as its adjacent sides.

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2. (50 points) Evaluate the following integrals.

(a) (7 points) $\int e^{3x+2}\,dx$.

(b) (8 points) $\int_1^3 \frac{1}{1+|x-2|}\,dx$.

(c) (10 points) $\int \mathrm{e}^{-x}\sin(5x)\,dx$.

(d) (10 points) $\int \frac{dx}{\sin x\cos x}$.

(e) (15 points) $\int \frac{x^3-3x^2+6x-2}{(x-1)(x^2-2x+2)}\,dx$.

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3. (25 points)

(a) (15 points) Find the volume of the solid generated by revolving the region in the first quadrant bounded from above by $y=a(1-\cos x)$ for $0\le x\le 2\pi$, from below by the x-axis, about the y-axis.

(b) (10 points) Find the length of the curve $y=\frac{x^2}{4}-\frac{\ln x}{2}$, $1\le x\le e$.

— THE END —

Useful Elementary Integrals

Constant and powers

1. $\int k\,dx = kx + C.$

2. $\int x^n\,dx = \begin{cases} \dfrac{x^{n+1}}{n+1} + C, & n \neq -1 \\ \ln|x| + C, & n = -1 \end{cases}.$

Exponentials

3. $\int \mathrm{e}^x\,dx = \mathrm{e}^x + C.$

4. $\int a^x\,dx = \dfrac{a^x}{\ln a} + C,\ a \neq 1,\ a > 0.$

Trigonometric functions

5. $\int \sin x\,dx = -\cos x + C.$

6. $\int \cos x\,dx = \sin x + C.$

7. $\int \sec^2 x\,dx = \tan x + C.$

8. $\int \csc^2 x\,dx = -\cot x + C.$

9. $\int \sec x \tan x\,dx = \sec x + C.$

10. $\int \csc x \cot x\,dx = -\csc x + C.$

11. $\int \tan x\,dx = \ln|\sec x| + C.$

12. $\int \cot x\,dx = \ln|\sin x| + C.$

13. $\int \sec x\,dx = \ln|\sec x + \tan x| + C.$

14. $\int \csc x\,dx = \ln|\csc x - \cot x| + C.$

Algebraic functions

15. $\int \dfrac{1}{1+x^2}\,dx = \tan^{-1} x + C.$

16. $\int \dfrac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1} x + C.$

Hyperbolic functions

17. $\int \sinh x\,dx = \cosh x + C.$

18. $\int \cosh x\,dx = \sinh x + C.$

Useful Trigonometric Identities

Pythagorean identities

1. $\sin^2\theta + \cos^2\theta = 1$.

2. $1 + \tan^2\theta = \sec^2\theta$.

3. $1 + \cot^2\theta = \csc^2\theta$.

Double-angle formulas

4. $\sin 2\theta = 2\sin\theta\cos\theta$.

5. $\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$.

Half-angle formulas

6. $\sin^2\theta = \dfrac{1}{2}(1 - \cos 2\theta)$.

7. $\cos^2\theta = \dfrac{1}{2}(1 + \cos 2\theta)$.

Compound-angle formulas

8. $\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$.

9. $\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$.

10. $\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$.

Sum-to-product formulas

11. $\sin A + \sin B = 2\sin\dfrac{A+B}{2}\cos\dfrac{A-B}{2}$.

12. $\sin A - \sin B = 2\cos\dfrac{A+B}{2}\sin\dfrac{A-B}{2}$.

13. $\cos A + \cos B = 2\cos\dfrac{A+B}{2}\cos\dfrac{A-B}{2}$.

14. $\cos A - \cos B = -2\sin\dfrac{A+B}{2}\sin\dfrac{A-B}{2}$.

Product-to-sum formulas

15. $\sin A\cos B = \dfrac{1}{2}\big[\sin(A+B) + \sin(A-B)\big]$.

16. $\cos A\sin B = \dfrac{1}{2}\big[\sin(A+B) - \sin(A-B)\big]$.

17. $\cos A\cos B = \dfrac{1}{2}\big[\cos(A+B) + \cos(A-B)\big]$.

18. $\sin A\sin B = -\dfrac{1}{2}\big[\cos(A+B) - \cos(A-B)\big]$.

Euler's formulas

19. $e^{\pm i\theta} = \cos\theta \pm i\sin\theta$.

20. $e^{i\theta} + e^{-i\theta} = 2\cos\theta$, $\cos\theta = \dfrac{1}{2}(e^{i\theta} + e^{-i\theta})$.

21. $e^{i\theta} - e^{-i\theta} = 2i\sin\theta$, $\sin\theta = \dfrac{1}{2i}(e^{i\theta} - e^{-i\theta})$.

Remark. Formulas of the form $A \pm B = C \pm D$ contain two separate formulas

$$A + B = C + D, \qquad \text{and} \qquad A - B = C - D.$$

Likewise, formulas of the form $A \pm B = C \mp D$ contain two separate formulas

$$A + B = C - D, \qquad \text{and} \qquad A - B = C + D.$$