

Hypothesis Testing: One-sample Inference

Lecture 3

Overview

- Null hypothesis and alternative hypothesis
- Type I error and type II error
- Hypothesis testing using one-sample inference, including:
 - calculation of t statistics, critical value and p-value for one-sided alternative
 - computation of t statistics, critical value and p-value for twosided alternative
- Calculate the power of a test and determine the appropriate sample size

Hypothesis Testing

- Hypothesis-testing framework: null and alternative hypothesis
- Hypothesis-testing: make decisions using probabilities methods, (not rely on subjective impressions)
 - uniform and consistent decision-making criterion
- one-sample problem: hypotheses about a single distribution
- two-sample problem: compare two different distributions

Hypothesis Testing

- The null hypothesis (H_o): hypothesis to be tested
- The alternative hypothesis (H₁): hypothesis contradicts the null hypothesis

$$H_0$$
: $\mu = \mu_0$ vs. H_1 : $\mu < \mu_0$

- Decisions: H₀ is true or H₁ is true
- All outcomes in a hypothesis testing situation: null hypothesis
- If we decide H₀ is true → accept H₀
 If we decide H₁ is true → H₀ is not true (reject H₀)

Table 7.1 Four possible outcomes in hypothesis testing

ecision		Tru	th
sion		$H_{\rm o}$	H_1
Dec	Accept H ₀	$H_{\scriptscriptstyle 0}$ is true and $H_{\scriptscriptstyle 0}$ is accepted	$H_{\scriptscriptstyle 1}$ is true and $H_{\scriptscriptstyle 0}$ is accepted
	Reject H₀	$H_{\scriptscriptstyle 0}$ is true and $H_{\scriptscriptstyle 0}$ is rejected	$H_{\scriptscriptstyle 1}$ is true and $H_{\scriptscriptstyle 0}$ is rejected

Hypothesis Testing

Given the Null Hypothesis Is True False Type I Correct Reject Error Decision Your Decision Based On a Random Sample Do Not Type II Correct Reject Decision Error

- Probability of a type I error:
 α : significance level of a test
- Probability of a type II error:
- β : function of μ and other factors
- **Power** of a test : $1 \beta = 1$ probability of a type II error = $Pr(rejecting H_0|H_1 true)$
- Objective of hypothesis testing: use statistical tests that make α and β as small as possible

Determination of Statistical Significance for Results from Hypothesis Tests

- Critical-value method: compute a test statistic and determine the outcome of a test by comparing the test statistic with a critical value determined by α (type I error)
- P-value: α level that we are not concerned between accepting or rejecting H₀

Figure 7.1 Graphic display of a p-value $0.4 \\
0.3 \\
0.2 \\
0.1$ 0.0 0.1 0.0 0.0Value

$$p = Pr(t_{n-1} \le t)$$

Determination of Statistical Significance for Results from Hypothesis Tests

Critical-value method

- 1) Compute test statistic t
- 2) Compare with critical value $t_{n-1,\alpha}$ at α level e.g. 0.05

$$H_0$$
: $\mu = \mu_0$ vs. H_1 : $\mu < \mu_0$

t < t_{n-1,0.05}
 Reject H₀

Result: statistically significant (p<0.05)

• $t \ge t_{n-1,0.05}$ Accept H_0

Result: not statistically significant (p≥0.05)

P-value method

- -Compute exact p-value
- If p < 0.05 Reject H₀

Result: statistically significant (p<0.05)

• if $p \ge 0.05$ Accept H_0

Result: not statistically significant (p≥0.05)

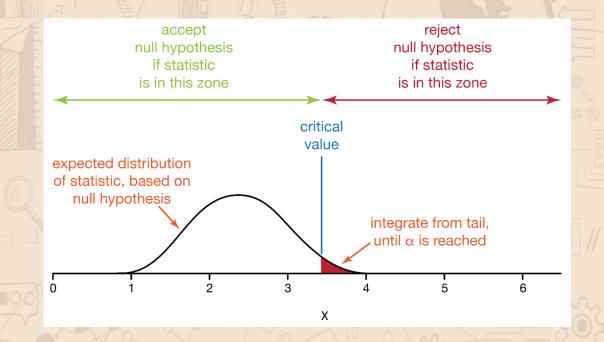
P-value

- P-value: probability of obtaining a test statistic <u>as extreme</u>
 as or more extreme than the actual test statistic obtained
 (null hypothesis is true)
 - If the null hypothesis is true → determine likelihood of getting me observed sample data
 - "If the null hypothesis is true, are your sample data unusual?"

Guidance for Judging the significance of a p-value:

- If $0.01 \le p < 0.05$: significant results
- If $0.001 \le p < 0.01$: highly significant results
- o If p < 0.001: very highly significant results
- \circ If p > 0.05: results are considered not statistically significant (NS)
- If $0.05 \le p < 0.1$: a trend toward statistical significance

One-Sample Test for the Mean of a Normal Distribution: One-Sided Alternatives



- Acceptance region: range of values of x that H₀ is accepted
- Rejection region: range of values of x that H₀ is rejected
- One-tailed test: values of the parameter being studied (i.e. μ) under the <u>alternative hypothesis</u> are allowed to be either > or < the values of the parameter under the <u>null hypothesis</u> (μ_0)

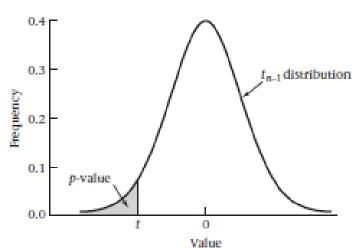
One-sample t Test for the Mean of a Normal Distribution with Unknown Variance (Alternative Mean < Null Mean)

• H_0 : $\mu = \mu_0$ vs. H_1 : $\mu < \mu_0$ * σ unknown* *with a significance level of α

$$t = (x - \mu_0)/(s/\sqrt{n})$$

- $t_{n-1,\alpha}$: critical value
- $t < t_{n-1,\alpha} \rightarrow$ Reject H₀ $t \ge t_{n-1,\alpha} \rightarrow$ accept H₀

FIGURE 7.1 Graphic display of a p-value

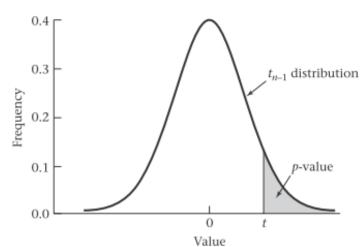


One-Sample *t* Test for the Mean of a Normal Distribution with Unknown Variance (Alternative Mean > Null Mean)

 H_0 : $\mu = \mu_0$ vs. H_1 : $\mu > \mu_0$ with a significance level of α $t = (x - \mu_0)/(s/\sqrt{n})$

- $t > t_{n-1,1-\alpha}$ \rightarrow reject H_0
- $t \le t_{n-1,1-\alpha}$ \rightarrow accept H_0
 - P-value = $Pr(t > t_{n-1})$

Figure 7.2 p-value for the one-sample t test when the alternative mean (μ_1) > null mean (μ_2)



Example on One-Sample t Test for the Mean of a Normal Distribution with Unknown Variance: Cardiovascular Disease: Pediatrics

- A current area of research interest is the familial aggregation of cardiovascular risk factors in general and lipid levels in particular.
- Suppose the "average" cholesterol level in children is 175 mg/dL. A group of men
 who have died from heart disease within the past year are identified, and the
 cholesterol levels of their offspring are measured.
- Two hypotheses are considered:
- (1) The average cholesterol level of these children is 175 mg/dL.
- (2) The average cholesterol level of these children is >175 mg/dL.
- Suppose the mean cholesterol level of 10 children whose fathers died from heart disease is 200 mg/dL and the sample standard deviation is 50 mg/dL.

Q: Test the hypothesis that the mean cholesterol level is higher in this group than in the general population.

Example on One-Sample t Test for the Mean of a Normal Distribution with Unknown Variance: Cardiovascular Disease: Pediatrics

Solution:

- Hypothesis: H_0 : $\mu = 175$ vs. H_1 : $\mu > 175$ at α level of .05
- H_0 is rejected if: $t > t_{n-1,1-\alpha} = t_{9,.95}$

$$t = \frac{200 - 175}{50/\sqrt{10}} = \frac{25}{15.81} = 1.58$$

- *Table:* $t_{9..95}$ = 1.833
- 1.833 > 1.58 → accept H0 at the 5% level of significance
- use p-value method:
 - exact *p*-value : p = Pr(t9 > 1.58)
 - -t9,.90 = 1.383 and t9,.95 = 1.833
 - because $1.383 < 1.58 < 1.833 \rightarrow .05 < p < .10$
 - using the pt function of R: exact *p*-value = Pr(t9 > 1.58) = 1 - pt (1.58,9) = .074 (*p*> .05)
- Conclusion: results are not statistically significant, and the null hypothesis is accepted
 - → mean cholesterol level of these children does not differ significantly from that of an average child



TABLE 5 Percentage points of the t distribution $(t_{dv})^a$

D	u								
Degrees of freedom, d	.75	.80	.85	.90	.95	.975	.99	.995	.9995
1	1.000	1.376	1.963	3.078	6.314	12.706	31.821	63.657	636.619
2	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925	31.598
3	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	12.924
4	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	8.610
5	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	6.869
6	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	5.959
7	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	5.408
8	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	5.041
9	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	4.781
10	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	4.587
11	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106	4.437
12	0.695	0.873	1.083	1.356	1.782	2.179	2.681	3.055	4.318
13	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	4.221
14	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	4.140
15	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947	4.073
16	0.690	0.865	1.071	1.337	1.746	2.120	2.583	2.921	4.015
17	0.689	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.965
18	0.688	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.922
19	0.688	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.883
20	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.850
21	0.686	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.819
22	0.686	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.792
23	0.685	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.767
24	0.685	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.745
25	0.684	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.725
26	0.684	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.707
27	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.690
28	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.674
29	0.683	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.659
30	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.646
40	0.681	0.851	1.050	1.303	1.684	2.021	2.423	2.704	3.551
60	0.679	0.848	1.046	1.296	1.671	2.000	2.390	2.660	3.460
120	0.677	0.845	1.041	1.289	1.658	1.980	2.358	2.617	3.373
00	0.674	0.842	1.036	1.282	1.645	1.960	2.326	2.576	3.291

^{*}The uth percentile of a t distribution with d degrees of freedom.

Source: Table 5 is taken from Table III of Fisher and Yates: "Statistical Tables for Biological, Agricultural and Medical Research," published by Longman Group Ltd., London (previously published by Oliver and Boyd Ltd., Edinburgh).

One-Sample Test for the Mean of a Normal Distribution: Two-Sided Alternatives

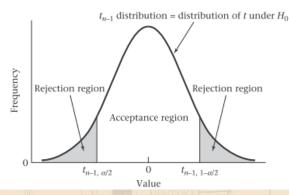
- Two-tailed test: values of the parameter being studied (μ) under H₁ are allowed to be either > or < the values of the parameter under H₀ (μ = μ ₀)
- Decision rule: reject H₀ if t is either too small or too large
 - o if t is either $< c_1$ or $> c_2$ for some constants $c_1, c_2 \rightarrow$ reject H_0
 - o if $c_1 \le t \le c_2$ → accept H_0

One-sample t Test for the Mean of a Normal Distribution with Unknown Variance (Two-Sided Alternative)

 H_0 : $μ=μ_0$ vs. H_1 : $μ ≠ μ_0$ with a significance level of α $t = (x - μ_0)/(s/√n)$

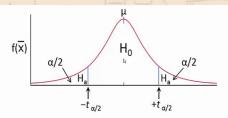
- If $|t| > t_{n-1,1-\alpha/2}$ \rightarrow reject H_0
- If $|t| < t_{n-1.1-\alpha/2}$ \rightarrow accept H_0

Figure 7.3 One-sample t test for the mean of a normal distribution (two-sided alternative)



$$t = \frac{\bar{x} - \mu}{s / \sqrt{n}}$$

- $\underline{\mu}$: Population mean
- X: Sample mean
- s: Sample standard deviation
- n: Sample size
- α : Type I error, the proportion of the time the incorrectly reject the null hypothesis
- The number of sample standard deviations (s) the sample mean (\overline{X}) is away from the population mean (μ)



Example on One-sample t Test for the Mean of a Normal Distribution with Unknown Variance: Cardiovascular Disease

- Suppose we want to compare fasting serum-cholesterol levels among recent Asian immigrants to the United States with typical levels found in the general U.S. population.
- Assumption: cholesterol levels in women ages 21–40 in the United States are approximately normally distributed with mean 190 mg/dL. It is unknown whether cholesterol levels among recent Asian immigrants are higher or lower than those in the general U.S. population.
- Let's assume that levels among recent female Asian immigrants are normally distributed with unknown mean μ.
- we wish to test H0: $\mu = \mu_0 = 190$ vs. H1: $\mu \neq \mu_0$. Blood tests are performed on 100 female Asian immigrants ages 21–40, and the mean level (\bar{x}) is 181.52 mg/dL with standard deviation = 40 mg/dL.

Q: Test the hypothesis that the mean cholesterol level of recent female Asian immigrants is different from the mean in the general U.S. population.

Example on One-sample t Test for the Mean of a Normal Distribution with Unknown Variance: Cardiovascular Disease

Solution:

Given that
$$t_{40,975}=2.021$$
 , $t_{60,975}=2.000$, $t_{120,975}=1.980$

We compute the test statistic:
$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{181.52 - 190}{40/\sqrt{100}} = \frac{-8.48}{4} = -2.12$$

- two-sided test with $\alpha = .05$: critical values are c1 = t99,.025, c2 = t99,.975.
- $t99,.975 < t60,.975 = 2.000 \rightarrow c2 < 2.000$
- $c1 = -c2 \rightarrow c1 > -2.000$
- $t = -2.12 < -2.000 < c1 \rightarrow$ reject H0 at the 5% level of significance
- Conclusion: the mean cholesterol level of recent Asian immigrants is significantly different from that of the general U.S. population



TABLE 5 Percentage points of the t distribution $(t_{dv})^a$

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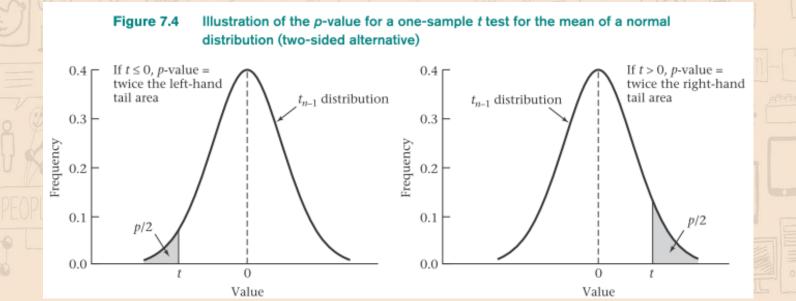
Source: Table 5 is taken from Table III of Fisher and Yates: "Statistical Tables for Biological, Agricultural and Medical Research," published by Longman Group Ltd., London (previously published by Oliver and Boyd Ltd., Edinburgh).

P-Value for the One-Sample t Test for the Mean of a Normal Distribution (Two-Sided Alternative)

Let
$$t = (x - \mu_0)/(s/\sqrt{n})$$

$$p = \begin{cases} 2 \times Pr(t_{n-1} \le t), & \text{if } t \le 0 \\ 2 \times [1 - Pr(t_{n-1} \le t)], & \text{if } t > 0 \end{cases}$$

- P-value: probability under H₀ of obtaining a test statistic as extreme as or more extreme than the observed test statistic
 - a two-sided H₁ is used
 - o **absolute value** of t: measures extremeness



Example on P-Value calculation for the One-Sample t Test

Q: Compute the *p*-value for the hypothesis test in the previous example (cholesterol levels).

Solution:

• Because t = -2.12: p-value for the test is twice the left-hand tail area, or

$$p = 2 \times Pr(t_{99} < -2.12) = 2 \times pt (-2.12,99) = .037$$
 (using pt function of R)

Conclusion: results are statistically significant with a p-value of .037

When is a one-sided test more appropriate than a two-sided test?

- One-sided test is easier to reject H_0 : sample mean falls in the expected direction from μ_0
- One-sided test is better: only alternatives on one side of the null mean are of interest or possible
 - o more power (easier to reject H₀ based on a finite sample if H₁ is true)
- Decision about using one-side or two-sided test should be made <u>before</u> the data analysis (or <u>before data collection</u>)
 - not to bias conclusions based on results of hypothesis testing
 - Do not change from a two-sided to a one-sided test after looking at the data

One-Sample z Test for the Mean of a Normal Distribution with Known Variance (Two-Sided Alternative)

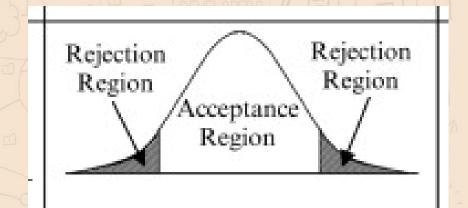
 H_0 : $\mu = \mu_0$ vs. H_1 : $\mu \neq \mu_0$ with a significance level of α

• standard deviation σ is known $z = (\overline{x} - \mu_0)/(\sigma/\sqrt{n})$

- If $z < z_{\alpha/2}$ or $z > z_{1-\alpha/2} \rightarrow \text{reject H}_0$
- If $z_{\alpha/2} \le z \le z_{1-\alpha/2} \rightarrow$ accept H_0
- Two-sided p-value calculation:

$$p = 2\Phi(z)$$
 if $z \le 0$

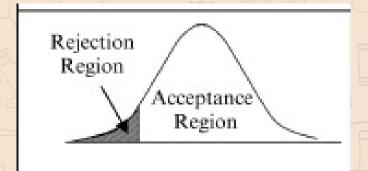
$$p = 2[1 - \Phi(z)] \text{ if } z > 0$$



One-Sample z Test for the Mean of a Normal Distribution with Known Variance (One-Sided Alternative)

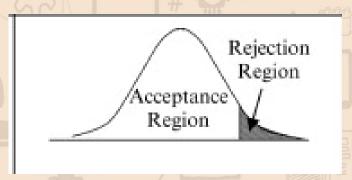
 H_0 : $\mu = \mu_0$ vs. H_1 : $\mu < \mu_0$ with a significance level of α

- standard deviation σ is known $z = (x \mu_0)/(\sigma/\sqrt{n})$
- If $z < z_{\alpha} \rightarrow reject H_0$
- if $z \ge z_{\alpha}$, \rightarrow accept H_0 is accepted
- p-value: $p = \Phi(z)$



 H_0 : $\mu = \mu_0$ vs. H_1 : $\mu > \mu_0$ with a significance level of α

- standard deviation σ is known $z = (x \mu_0)/(\sigma/\sqrt{n})$
- If $z > z_{1-\alpha} \rightarrow \text{reject } H_0$
- if $z \le z_{1-\alpha} \to \text{accept } H_0$
- p-value: $p = 1 \Phi(z)$.



Example on One-Sample z Test for the Mean of a Normal Distribution with Known Variance

Q: Consider the cholesterol data in the cholesterol example. Assume that the standard deviation is known to be 40 and the sample size is 200 instead of 100. Assess the significance of the results.

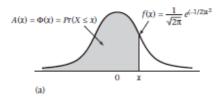
Solution:

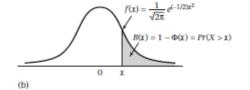
The test statistic :
$$z = \frac{181.52 - 190}{40/\sqrt{200}} = \frac{-8.48}{2.828} = -3.00$$

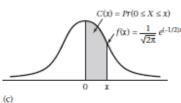
- Critical-value method with $\alpha = 0.05$
 - critical values are -1.96 and 1.96
- z = -3.00 < -1.96 -> reject H0 at a 5% level of significance
- two-sided p-value : $2 \times \Phi(-3.00) = .003$

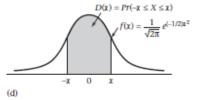


TABLE 3 The normal distribution









-				_
2,96	.9985	.0015	.4985	.9969
2.97	,9985	.0015	4985	.9970
2,98	,9986	.0014	4986	9971
2.99	,9986	0014	4986	9972
3,00	.9987	.0013	4987	.9973
3,01	.9987	.0013	4987	9974
3.02	,9987	.0013	4987	9975
3.03	,9988	.0012	4988	9976
3.04	.9988	.0012	.4988	9976
3.05	.9989	.0011	4989	.9977
3.06	.9989	,0011	.4989	9978
3.07	9989	.0011	4989	9979
3.08	.9990	.0010	,4990	.9979
3.09	9990	.0010	4990	9980
3,10	.9990	.0010	.4990	.9981
3.11	,9991	.0009	.4990	.9981
3,12	,9991	,0009	,4991	,9982
3.13	.9991	.0009	.4991	.9983
3.14	,9992	.0008	,4992	,9983
3.15	.9992	.0008	,4992	.9984
3.16	,9992	.0008	.4992	.9984
3.17	.9992	.0008	.4992	.9985
3.18	,9993	.0007	.4993	.9985
3.19	.9993	.0007	.4993	.9986
3,20	,9993	.0007	.4993	,9986
3.21	.9993	.0007	4993	.9987
3,22	,9994	.0006	,4994	.9987
3.23	,9994	.0006	.4994	.9988
3,24	,9994	,0006	,4994	,9988
3.25	.9994	.0006	.4994	.9988
3,26	.9994	,0006	,4994	,9989
3.27	.9995	.0005	4995	.9989
3,28	,9995	,0005	.4995	,9990
3.29	.9995	.0005	.4995	.9990
3,30	,9995	,0005	4995	.9990
3,31	.9995	.0005	.4995	.9991
3,32	,9995	,0005	4995	9991
3,33	.9996	.0004	4996	.9991
3,34	,9996	,0004	4996	,9992
3,35	.9996	.0004	,4996	.9992
3,36	,9996	.0004	4996	.9992
3,37	,9996	.0004	,4996	.9992
3,38	,9996	.0004	.4996	.9993
3,39	.9997	.0003	4997	.9993
3.40	.9997	.0003	.4997	.9993
3.42	.9997	.0003	4997	.9994
3,43	.9997	.0003	.4997	.9994
3,45	.9997	.0003	4997	.9994
3,46	.9997	.0003	.4997	.9995
3.47	.9997	.0003	.4997	.9995
3,48	.9997	.0003	.4997	.9995
	$\Phi(x) = P(X \le x)$			

The normal distribution (continued)

C°

ВÞ

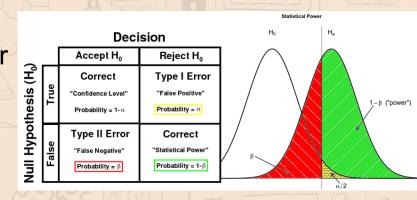
 A^*

x

 ${}^{*}A(x) = \Phi(x) = Pr(X \le x)$, where X is a standard normal distribution, ${}^{*}B(x) = 1 - \Phi(x) = Pr(X > x)$, where X is a standard normal distribution, ${}^{*}C(x) = Pr(0 \le X \le x)$, where X is a standard normal distribution. ${}^{*}D(x) = Pr(-x \le X \le x)$, where X is a standard normal distribution.

The Power of a Test

- how likely a statistically significant difference will be distinguished based on a finite sample size n
- probability that statistical test correctly rejects the null hypothesis
- Likelihood of a true positive result
- Probability of avoiding a Type II error



The power of the one-sided test (one-sample z test):

$$\Phi(z_{\alpha} + | \mu_0 - \mu_1 | \sqrt{n/\sigma}) = \Phi(-z_{1-\alpha} + | \mu_0 - \mu_1 | \sqrt{n/\sigma})$$

$$\Phi\!\left[-z_{1-\alpha/2} + \frac{\left(\mu_0 - \mu_1\right)\sqrt{n}}{\sigma}\right] + \Phi\!\left[-z_{1-\alpha/2} + \frac{\left(\mu_1 - \mu_0\right)\sqrt{n}}{\sigma}\right]$$

Approx. by:
$$\Phi(-z_{1-\alpha/2} + |\mu_0 - \mu_1|\sqrt{n/\sigma})$$

Example on Power Calculation:Cardiovascular Disease, Pediatrics

Q: Using a 5% level of significance and a sample of size 10, compute the power of the test for the cholesterol data example, with an alternative mean of 190 mg/dL, a null mean of 175 mg/dL, and a standard deviation (σ) of 50 mg/dL.

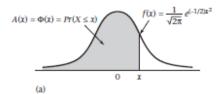
Solution:

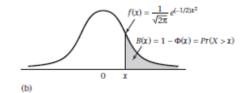
We have $\mu 0 = 175$, $\mu 1 = 190$, $\alpha = .05$, $\sigma = 50$, n = 10

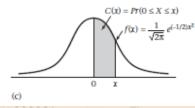
$$Power = \Phi \left[-1.645 + \frac{(190 - 175)\sqrt{10}}{50} \right] = \Phi \left(-1.645 + \frac{15\sqrt{10}}{50} \right)$$
$$= \Phi(-0.696) = 1 - \Phi(0.696) = 1 - 0.757 = 0.243$$

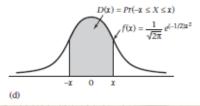
- Chance of finding a significant difference in this case is only 24%
- it is not surprising that a significant difference was not found the previous example because the sample size was too small

TABLE 3 The normal distribution









	X	A*	Вь	C:	D⁴
	1.56	.9406	.0594	.4406	.8812
	1.57	.9418	.0582	.4418	.8836
	1.58	.9429	.0571	.4429	.8859
	1.59	.9441	.0559	.4441	.8882
	1.60	.9452	.0548	.4452	.8904
	1.61	.9463	.0537	.4463	.8926
	1.62	.9474	.0526	.4474	.8948
	1.63	.9484	.0516	.4484	.8969
1	1.64	.9495	.0505	.4495	.8990
1	1.65	9505	.0495	.4505	.9011
	1.66	.9515	.0485	.4515	.9031
	1.67	.9525	.0475	.4525	.9051
	1.68	.9535	.0465	.4535	.9070
	1.69	.9545	.0455	.4545	.9090
	1.70	.9554	.0446	.4554	.9109
	1.71	.9564	.0436	.4564	.9127
	1.72	.9573	.0427	.4573	.9146
	1.73	.9582	.0418	.4582	.9164
	1.74	.9591	.0409	.4591	.9181
	1.75	.9599	.0401	.4599	.9199
	1.76	.9608	.0392	.4608	.9216
	1.77	.9616	.0384	.4616	.9233
	1.78	.9625	.0375	.4625	.9249
	1.79	.9633	.0367	.4633	.9265
	1.80	.9641	.0359	.4641	.9281
	1.81	.9649	.0351	.4649	.9297

TABLE 3 The normal distribution (continued)

TABLE 3	The nor	rmal distributi	on (continue	ed)
x	A=	BÞ	C=	D⁴
1.82	.9656	.0344	.4656	.9312
1.83	.9664	.0336	.4664	.9327
1.84	.9671	.0329	.4671	.9342
1.85	.9678	.0322	.4678	.9357
1.86	.9686	.0314	.4686	.9371
1.87	.9693	.0307	.4693	.9385
1.88	.9699	.0301	.4699	.9399
1.89	.9706	.0294	.4706	.9412
1.90	.9713	.0287	.4713	.9426
1.91	.9719	.0281	.4719	.9439
1.92	.9726	.0274	.4726	.9451
1.93	.9732	.0268	.4732	.9464
1.94	.9738	.0262	.4738	.9476
1.95	.9744	.0256	.4744	.9488
1.96	.9750	.0250	.4750	.9500
1.97	.9756	.0244	.4756	.9512
1.98	.9761	.0239	.4761	.9523
1.99	.9767	.0233	.4767	.9534
2.00	.9772	.0228	.4772	.9545
2.01	.9778	.0222	.4778	.9556
2.02	.9783	.0217	.4783	.9566
2.03	.9788	.0212	.4788	.9576
2.04	.9793	.0207	.4793	.9586
2.05	.9798	.0202	.4798	.9596
2.06	.9803	.0197 .0192	.4803 .4808	.9606 .9615
2.07	.9812	.0192	.4812	.9625
2.09	.9817	.0183	.4817	.9634
2.10	.9821	.0179	.4821	.9643
2.10	.9826	.0174	.4826	.9651
2.12	.9830	.0170	.4830	.9660
2.13	.9834	.0166	.4834	.9668
2.14	.9838	.0162	.4838	.9676
2.15	.9842	.0158	.4842	.9684
2.16	.9846	.0154	.4846	.9692
2.17	.9850	.0150	.4850	.9700
2.18	.9854	.0146	.4854	.9707
2.19	.9857	.0143	.4857	.9715
2.20	.9861	.0139	.4861	.9722
2.21	.9864	.0136	.4864	.9729
2.22	.9868	.0132	.4868	.9736
2.23	.9871	.0129	.4871	.9743
2.24	.9875	.0125	.4875	.9749
2.25	.9878	.0122	.4878	.9756
2.26	.9881	.0119	.4881	.9762
2.27	.9884	.0116	.4884	.9768
2.28	.9887	.0113	.4887	.9774
2.29	.9890	.0110	.4890	.9780
2.30	.9893	.0107	.4893	.9786
2.31	.9896	.0104	.4896	.9791
2.32	.9898	.0102	.4898 .4901	.9797 .9802
2.33	.9901 .9904	.0099	.4901	.9802
2.34	.9904	.0096	.4904	.9807
2.36	.9909	.0094	.4906	.9817
2.37	.9911	.0089	.4909	.9822
2.38	.9913	.0087	.4913	.9827
2.00	.0010	.0007	.4010	.0027

INSIGHTS

TABLE 3 The normal distribution

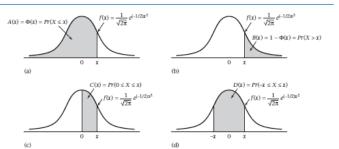
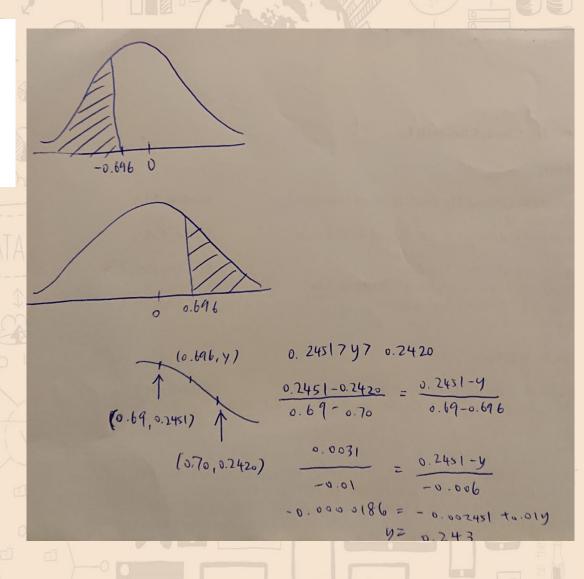


TABLE 3 The normal distribution (continued)

x	A*	B ^b	C+	D^{i}	т
_	^	ь			н
0,64	,7389	2611	,2389	.4778	ш
0.65	.7422	.2578	.2422	.4843	ш
38,0	.7454	.2546	2454	.4907	11
0.67	.7486	.2514	2486	.4971	П
88,0	7517	2483	2517	,5035	ш
96,0	.7549	.2451	2549	.5098	ш
,70	.7580	,2420	2580	.5161	ш
.71	.7611	12308	.2611	.5223	ш
0.72	.7642	2358	,2642	.5285	ш
).73	.7673	.2327	.2673	.5346	ш
0.74	.7703	2297	2703	5407	П
0.75	.7734	.2266	.2734	.5467	
),76	.7764	.2236	2764	.5527	
).77	.7793	.2207	.2793	.5587	ш
),78	.7823	.2177	2823	.5646	ш
.79	.7852	.2148	2852	.5705	ш
08,0	7881	2119	2881	5763	ш
,81	.7910	.2090	2910	.5821	ш
0.82	.7939	2061	2939	5878	п
0,83	.7967	.2033	.2967	.5935	ш
0.84	.7995	2005	2995	5991	ш
0.85	.8023	.1977	3023	.6047	ш
38,0	8051	.1949	3051	6102	ш
0.87	.8078	.1922	.3078	.6157	
88,0	8106	.1894	3106	6211	
0.89	8133	.1867	3133	.6265	
0,90	,8159	1841	3159	6319	
0.91	8186	1814	3186	6372	
0.00	8919	1700	9010	8494	



Example on Power Calculation: Obstetrics

Suppose we want to test the hypothesis that mothers with low socioeconomic status (SES) deliver babies whose birthweights are lower than "normal." To test this hypothesis, a list is obtained of birthweights from 100 consecutive, full-term, live-born deliveries from the maternity ward of a hospital in a low-SES area. The mean birthweight (x) is found to be 115 oz with a sample standard deviation (s) of 24 oz. Suppose we know from nationwide surveys based on millions of deliveries that the mean birthweight in the United States is 120 oz. Compute the power of the test for the birthweight data with an alternative mean of 115 oz and $\alpha = .05$, assuming the true standard deviation = 24 oz.

Q: Assuming a sample size of 10 rather than 100, compute the power for the birthweight data with an alternative mean of 115 oz and $\alpha = .05$.

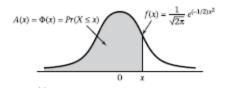
Solution:

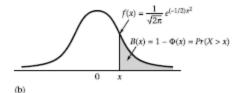
We have $\mu 0 = 120 \text{ } oz, \mu 1 = 115 \text{ } oz, \alpha = .05, \sigma = 24, n = 100.$

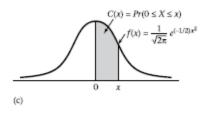
$$Power = \Phi \left[z_{.05} + \frac{(120 - 115)\sqrt{100}}{24} \right] = \Phi \left[-1.645 + \frac{5(10)}{24} \right]$$
$$= \Phi(0.428) = 0.669$$

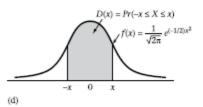
• There is about a 67% chance of detecting a significant difference using a 5% significance level with this sample size

TABLE 3 The normal distribution

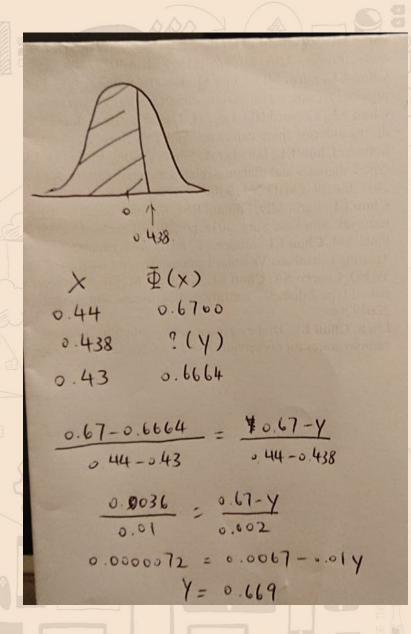








x	A*	Bb	C°	D ⁴	х	Α	В	С	D
0.0	,5000	,5000	.0	.0	0,32	6255	,3745	.1255	.2510
0,01	5040	.4960	,0040	.0080	0,33	6293	,3707	1293	,2586
0.02	.5080	.4920	.0080	.0160	0,34	6331	,3669	.1331	,2661
0,03	5120	.4880	,0120	.0239	0,35	.6368	,3632	.1368	,2737
0.04	.5160	.4840	.0160	.0319	0,36	.6406	.3594	.1406	.2812
0,05	5199	.4801	,0199	.0399	0,37	.6443	3557	.1443	,2886
0,06	5239	.4761	.0239	.0478	0.38	.6480	.3520	.1480	,2961
0,07	5279	4721	0279	,0558	0,39	8517	3483	1517	3035
80,0	.5319	.4681	.0319	.0638	0.40	6554	.3446	1554	,3108
0,09	5359	.4641	.0359	.0717	0,41	.6591	,3409	.1591	,3182
0.10	,5398	4602	.0398	.0797	0.42	.6628	3372	.1628	3255
0,11	5438	4562	,0438	.0876	0,43	,6884	,3336	1884	,3328
0.12	5478	4522	.0478	.0955	0.44	.6700	,3300	.1700	,3401
0,13	5517	.4483	,0517	,1034	0,45	,6736	3264	.1736	,3473
0.14	.5557	.4443	.0557	.1113	0.46	.6772	3228	.1772	3545
0,15	5596	.4404	,0596	,1192	0,47	8089,	3192	.1808	,3616
0.16	5636	.4364	.0636	.1271	0.48	.6844	,3156	.1844	,3688
0,17	5675	.4325	.0675	,1350	0,49	.6879	3121	.1879	,3759
0.18	5714	4286	.0714	.1428	0.50	6915	.3085	1915	3829
0,19	5753	.4247	,0753	.1507	0,51	6950	,3050	,1950	3899
0.20	5793	.4207	0793	.1585	0.52	6985	.3015	1985	3969
0,21	5832	,4168	,0832	.1663	0,53	7019	,2981	2019	,4039
0.22	.5871	4129	.0871	1741	0.54	7054	.2946	2054	4108
0,23	5910	.4090	.0910	.1819	0,55	7088	2912	2088	4177
0.24	5948	4052	.0948	.1897	0.56	7123	.2877	.2123	4245
0,25	5987	.4013	0987	.1974	0,57	.7157	,2843	,2157	,4313
0.26	6026	3974	1026	.2051	0.58	.7190	2810	.2190	,4381
0,27	,6064	,3936	1084	,2128	0,59	,7224	2776	2224	4448
0.28	6103	.3897	1103	.2205	0.60	.7257	2743	.2257	4515
0,29	,6141	3859	,1141	,2282	0,61	.7291	2709	,2291	,4581
0.30	6179	3821	1179	.2358	0.62	7324	2676	2324	4647
0,31	6217	3783	.1217	.2434	0,63	.7357	2643	2357	4713



Example on Power Calculation: Obstetrics

$$\mu 0 = 170 \text{ oz}, \mu 1 = 190, \alpha = .05, \sigma = 50, n = 10$$
:

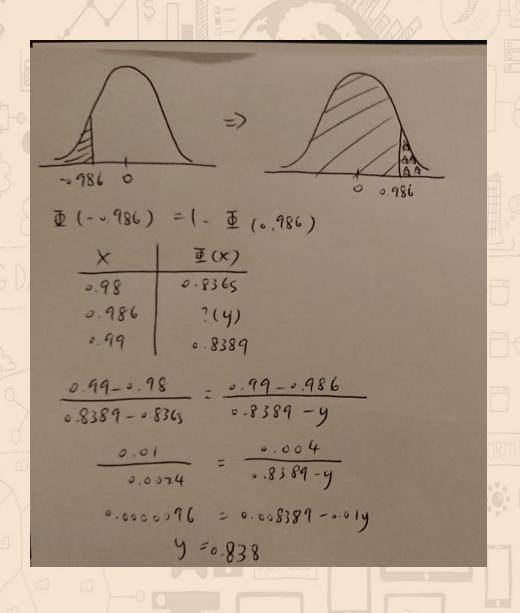
Power =
$$\Phi \left[z_{.05} + \frac{(120 - 115)\sqrt{10}}{24} \right] = \Phi \left[-1.645 + \frac{5\sqrt{10}}{24} \right] = \Phi(-0.986)$$

= 1 - 0.838 = 0.162

- there is only a 16% chance of finding a significant difference with a sample size of 10
- whereas there was a 67% chance with a sample size of 100
- → if 10 infants were sampled, we would have virtually no chance of finding a significant difference and would almost surely report a false-negative result

TABLE 3 The normal distribution (continued)

TABLE 3	The non	mal distributi	ion (continue	ed)
x	A*	Вь	C,	D ^a
0,64	7389	2611	,2389	.4778
0.85	7422	.2578	.2422	.4843
0,66	7454	.2546	.2454	.4907
0.67	7486	.2514	.2486	.4971
0.68	7517	.2483	.2517	5035
0.69	7549	.2451	.2549	.5098
0,70	7580	.2420	.2580	.5161
0.71	.7611	.2389	.2611	.5223
0.72	7642	.2358	.2642	.5285
0.73	.7673	.2327	.2673	.5346
0.74	7703	2297	.2703	.5407
0.75	.7734	.2266	.2734	.5467
0.76	7764	.2236	.2764	.5527
0.77	.7793	.2207	.2793	.5587
0,78	7823	.2177	2823	.5646
0.79	7852	.2148	.2852	.5705
08,0	7881	.2119	.2881	.5763
0.81	7910	.2090	.2910	.5821
0,82	7939	.2061	.2939	.5878
0.83	.7967	.2033	.2967	.5935
0.84	7995	.2005	.2995	5991
0.85	.8023	.1977	.3023	.6047
0,86	8051	.1949	3051	6102
0.87	.8078	.1922	.3078	.6157
88,0	8106	.1894	,3106	6211
0.89	8133	.1867	.3133	.6265
0,90	8159	1841	,3159	6319
0.91	8186	.1814	.3186	.6372
0.92	8212	.1788	3212	.6424
0.93	8238	.1762	.3238	.6476
),94	8264	.1736	3264	.6528
0.95	8289	.1711	.3289	.6579
0.96 0.97	8315	.1685	3315	.6629
	8340 8365	.1660 .1635	3340 3365	.6680 .6729
0,98 0,99	8389	1611	,3389	6778
1,00	.8413	.1587	,3413	6827
1.01	8438	1562	3438	6875
1,02	8461	1539	3461	6923
1.03	8485	1515	3485	6970
1,04	8508	1492	,3508	7017
1.05	8531	.1469	3531	7063
1,06	8554	.1446	3554	7109
1.07	8577	1423	3577	7154
1,08	8599	.1401	3599	7199
1.09	8621	.1379	3621	7243
1.10	8643	.1357	3643	7287
1.11	8665	.1335	3665	7330
1.12	8686	.1314	,3686	7373
1.13	8708	1292	.3708	7415
1,14	8729	.1271	3729	7457
1.15	8749	.1251	3749	7499
1,16	8770	.1230	3770	.7540
1.17	8790	.1210	.3790	.7580
1.18	8810	.1190	3810	.7620
1.19	8830	.1170	.3830	7660
	8849	1151	,3849	7699
1,20				
1.20	8869	.1131	.3869	7737



Factors affecting the power

E.g. 2-sided one-sample test: $\Phi(-z_{1-\alpha/2} + |\mu_0 - \mu_1|\sqrt{n/\sigma})$

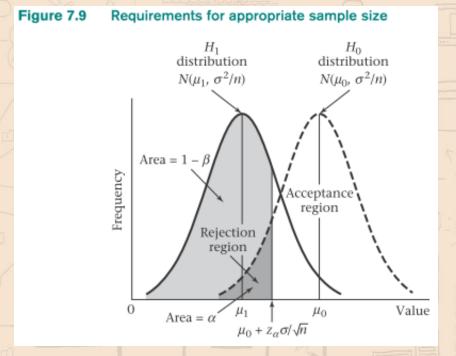
- If the significance level is made smaller (α decreases) → z_α increases → power decreases
- If the alternative mean is shifted farther away from the null mean $(|\mu_0 \mu_1| \text{ increases}) \rightarrow \text{ power increases}$
- If σ of the distribution of individual observations increases (σ increases) → power decreases
- If the sample size increases (n increases) → power increases

Sample-Size Determination: One-Sided Alternatives

$$H_0$$
: $\mu = \mu_0$ vs. H_1 : $\mu = \mu_1$, data ~ $N(\mu, \sigma^2)$

• What is the same size for a one-sided test with significance level α and probability of detecting a significant difference = 1- β ?

$$n = \frac{\sigma^2 (z_{1-\beta} + z_{1-\alpha})^2}{(\mu_0 - \mu_1)^2}$$



Factors Affecting the Sample Size (n)

$$n = \frac{\sigma^2 \left(z_{1-\beta} + z_{1-\alpha} \right)^2}{\left(\mu_0 - \mu_1 \right)^2}$$

- $\rightarrow \sigma^2$ increases \rightarrow n increases
- The significance level is made smaller (α decreases) \rightarrow n increases
- ightharpoonup The required power increases (1- β increases) ightharpoonup n increases
- The absolute value of the distance between the null and alternative means ($|\mu_0 \mu_1|$) increases \rightarrow n decreases

Sample-Size Estimation When Testing for the Mean of a Normal Distribution (Two-Sided Alternative)

$$H_0$$
: $\mu = \mu_0$ vs. H_1 : $\mu = \mu_1$, data ~ $N(\mu, \sigma^2)$

• What is the sample size for a one-sided test with significance level α and power 1- β ?

$$n = \frac{\sigma^2 \left(z_{1-\beta} + z_{1-\alpha/2} \right)^2}{\left(\mu_0 - \mu_1 \right)^2}$$

Sample-Size Estimation Based on CI Width

- What is the mean of a normal distribution with sample variance s²?
 - The two-sided 100% × (1 α) CI for μ be no wider than L

$$n = 4z_{1-\alpha/2}^2 s^2/L^2$$

Example on Sample Size Estimation: Obstetrics

Consider the birthweight data. Suppose that $\mu 0 = 120$ oz, $\mu 1 = 115$ oz, $\sigma = 24$, $\alpha = .05$, $1 - \beta = .80$.

Q: Using a one-sided test, compute the appropriate sample size needed to conduct the test.

Solution:

$$n = \frac{24^2(z_{.8} + z_{.95})^2}{25} = 23.04(0.84 + 1.645)^2 = 23.04(6.175) = 142.3$$

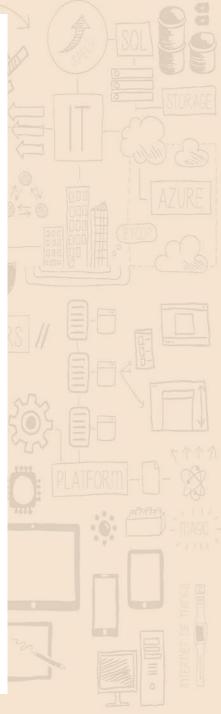
- sample size is always rounded up → sure to achieve at least the required level of power (in this case, 80%)
- a sample size of 143 is needed to have an 80% chance of detecting a significant difference at the 5% level (if the alternative mean is 115 oz and a one-sided test is used)

TAT
х
0,6
0.66
0.6
0.89
0.7
0.73
0.74
0.70
0,78
0.79
0.8
0,83
0.8
0.8
0.89
0.9
0.9
0.9
0.99
0.9
0.99
1.00
1.03
1,04
1.00
1.03
1.09
1.11
1.13
1.14 1.18
1.16 1.13 1.18
1.18
1,20
1.21 1.25

TABLE 3	The normal distribution /	continued)

x	A*	Вь	C°	D^{a}	x	Α	В	С	D
),64	,7389	.2611	,2389	.4778	1,23	.8907	,1093	,3907	,7813
0.65	7422	2578	2422	4843	1,24	8925	.1075	3925	.7850
38,0	7454	.2546	.2454	4907	1,25	.8944	.1056	3944	.7887
0.67	7486	2514	2486	4971	1,26	8962	.1038	3962	.7923
0,68	7517	2483	2517	.5035	1,27	.8980	,1020	.3980	,7959
0.69	7549	2451	2549	.5098	1,28	8997	.1003	3997	.7995
0,70	7580	.2420	,2580	,5161	1,29	.9015	.0985	4015	.8029
0.71	7611	2389	.2611	5223	1,30	9032	.0968	4032	.8064
0.72	7642	2358	,2642	.5285	1,31	9049	.0951	4049	,8098
0.73	7673	.2327	.2673	.5346	1,32	9066	.0934	4066	.8132
),74	7703	2297	2703	,5407	1,33	9082	.0918	4082	.8165
.75	7734	.2286	2734	.5467	1,34	9099	.0901	4099	.8198
),76	7764	2236	2764	5527	1,35	,9115	.0885	4115	.8230
.77	7793	2207	2793	.5587	1,36	9131	.0869	4131	.8262
,78	7823	.2177	.2823	.5646	1,37	.9147	.0853	4147	.8293
.79	7852	2148	2852	5705	1,38	9162	.0838	4162	8324
0,80	7881	2119	,2881	5763	1,39	,9177	.0823	4177	.8355
.81	7910	.2090	2910	5821	1,40	9192	.0808	4192	.8385
,82	.7939	2061	.2939	5878	1,41	9207	.0793	4207	.8415
).83	7067	.2033	2967	5935	1.42	9222	.0778	4222	8444
.84	,7995	,2005	2995	5991	1,43	9236	.0764	4236	8473
0.85	,8023	.1977	3023	6047	1.44	.9251	.0749	4251	8501
),86	,8051	1949	,3051	6102	1.45	9265	.0735	4265	.8529
0,87	,8078	.1922	3078	.6157	1.46	9279	.0735	4279	.8557
88,0	8106	.1894	,3106	6211	1,47	.9292	.0708	4292	.8584
.89	8133	.1867	3133	.6265	1.48	.9306	.0694	4306	.8611 .8638
.90	8159	1841	3159	6319	1.49	.9319	.0881	.4319	
0.91	,8186	.1814	,3186	.6372	1.50	.9332	.0668	.4332	.8664
.92	8212	.1788	3212	.6424	1.51	.9345	.0655	.4345	.8690
0.93	.8238	.1762	.3238	.6476	1.52	.9357	.0643	.4357	.8715
),94	8264	.1736	,3264	.6528	1,53	.9370	.0630	.4370	.8740
.95	.8289	.1711	.3289	.6579	1.54	.9382	.0618	.4382	.8764
,96	8315	.1685	3315	.6629	1.55	.9394	.0606	.4394	.8789
.97	.8340	.1660	.3340	.6680	1.56	.9408	.0594	.4408	.8812
98,0	8365	.1635	3365	6729	1,57	.9418	.0582	.4418	.8836
.99	,8389	.1611	,3389	.6778	1.58	.9429	.0571	.4429	.8859
,00	8413	.1587	,3413	6827	1,59	.9441	.0559	.4441	8882
.01	8438	.1562	.3438	.6875	1.60	.9452	.0548	.4452	.8904
.02	8461	.1539	,3461	6923	1,61	.9463	.0537	.4463	.8926
.03	,8485	.1515	.3485	.6970	1.62	.9474	.0526	4474	.8948
.04	8508	.1492	.3508	.7017	1.63	.9484	.0516	.4484	,8969
.05	.8531	.1469	.3531	.7063	1.64	.9495	.0505	.4495	.8990
.06	8554	.1446	.3554	7109	1,65	.9505	.0495	.4505	.9011
.07	.8577	.1423	.3577	.7154	1,66	.9515	.0485	.4515	.9031
.08	8599	.1401	,3599	7199	1,67	.9525	.0475	.4525	9051
.09	8621	.1379	.3621	.7243	1.68	.9535	.0465	4535	.9070
.10	8643	.1357	,3643	.7287	1,69	.9545	.0455	.4545	,9090
.11	,8665	.1335	.3665	.7330	1.70	.9554	.0446	.4554	.9109
.12	,8686	.1314	,3686	.7373	1.71	9564	.0436	4564	9127
.13	.8708	.1292	.3708	.7415	1.72	.9573	.0427	.4573	.9146
.14	8729	1271	3729	.7457	1.73	9582	.0418	.4582	.9164
.15	.8749	.1251	3749	.7499	1.74	.9591	.0409	.4591	.9181
.16	8770	.1230	3770	.7540	1.75	9599	.0401	.4599	9199
.17	.8790	.1210	3790	.7580	1.76	.9608	.0392	.4608	.9216
.18	8810	.1190	3810	.7620	1,77	9616	.0384	4616	,9233
.19	8830	.1170	.3830	.7660	1.78	9625	.0375	4625	.9249
.20	8849	1151	3849	.7699	1,79	9633	.0367	4633	,9265
.21	8869	1131	3869	.7737	1,80	.9641	.0359	4641	.9281
.22	8888	.1112	3888	.7775	1,81	9649	.0351	4649	,9297

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Example on Sample Size Estimation: Cardiology

Suppose it is well known that propranolol lowers heart rate over 48 hours when given to patients with angina at standard dosage levels. A new study is proposed using a higher dose of propranolol than the standard one. Investigators are interested in estimating the drop in heart rate with high precision.

Q: Find the minimum sample size needed to estimate the change in heart rate (μ), if we require that the two-sided 95% CI for μ be no wider than 5 beats per minute and the sample standard deviation for change in heart rate equals 10 beats per minute.

Solution:

$$n = \frac{4(z_{.975})^2(10)^2}{(5)^2} = \frac{4(1.96)^2(100)}{25} = 61.5$$

We have $\alpha = .05$, s=10,L=5

62 patients need to be studied

TABLE 3	The norm	na distributi	on (continu	ed)						^^		
x	A*	Вb	C.	D ¹	ж	А	В	С	D	TOD .		
1,82 1,83	9656 9664	.0344	4656 4664	.9312 .9327	2,39 2,40	.9916 .9918	.0084	.4916 .4918	.9832 .9836	5		
1.84	9671	.0329	.4671	9342	2.41	,9920	.0080	.4920	.9840	/ Ī		
1,85 1,86	9678 9686	.0322	.4678 .4686	.9357 .9371	2.42	.9922 .9925	.0078	.4922 .4925	.9845 .9849			
1.87 1.88	.9693 .9699	.0307	.4693 .4699	.9385 .9399	2.44 2.45	.9927 .9929	.0073 .0071	.4927 .4929	.9853 .9857			
1.89	.9708	.0294	4708	.9412	2.46	.9931	.0069	4931	.9861			
1.90 1.91	.9713 .9719	.0287	.4713 .4719	.9426 .9439	2.47 2.48	.9932 .9934	.0068 6900	.4932 .4934	.9865 .9869			
1.92	9726	.0274	4726	.9451	2,49	,9936	.0064	.4936	.9872			
1.93 1.94	.9732 .9738	.0268	.4732 .4738	.9464 .9476	2.50 2.51	.9938 .9940	,0062	.4938 .4940	.9876 .9879			
1.95	0744	.0256	4744	.9488	2.52	.9941	.0059	4941	.9883			
	.9750 .9758	.0250	.4750 .4756	.9500 .9512	2.53 2.54	.9943 .9945	,0057 ,0055	.4943 .4945	,9886 ,9889			
1.98	9761	.0239	4761	.9523	2,55	,9946	.0054	.4946	.9892			
1.99	.9767 .9772	.0233	.4767 .4772	.9534 .9545	2.56 2.57	.9948 .9949	.0052 .0051	.4948 .4949	.9895 .9898	on lon		
2.01	.9778	.0222	.4778	.9556	2.58	.9951	.0049	.4951	.9901			
2.02	9783 9788	.0217	4783 4788	.9566 .9576	2.59 2.60	.9952 .9953	.0048	.4952 .4953	.9904 .9907			
2.04	9793 9798	.0207	.4793 .4798	.9586	2,61	,9955	.0045 .0044	.4955 .4956	.9909 .9912	-		
2.05 2.06	9803	.0202	4803	.9596 .9606	2.62 2.63	.9956 .9957	.0043	4957	9915			
2.07 2.08	9808 9812	.0192 .0188	.4808 .4812	.9615 .9625	2.64 2.65	.9959 .9960	.0041	.4959 .4960	.9917 .9920			
2.09	.9817	.0183	.4817	.9834	2.66	.9961	.0039	.4961	.9922			
2.10	.9821 .9826	.0179 .0174	4821 4826	.9643 .9651	2.67 2.68	.9962 .9963	.0038	.4962 .4963	.9924 .9926			
2.12	9830	0170	4830	.9660	2,69	.9964	.0036	.4964	9929			
2.13 2.14	.9834 .9838	.0166	.4834 .4838	.9668 .9676	2.70 2.71	.9965 ,9966	.0035 .0034	.4965 .4966	.9931 .9933	523		
2.15	9842	.0158	4842	.9884	2.72	.9967	.0033	4967	9935	500		
2.16 2.17	9846 9850	.0154	.4846 .4850	.9692 .9700	2.73 2.74	.9968 .9969	.0032	.4968 .4969	.9937 .9939			
2.18	9854	.0146	.4854	.9707	2.75	.9970	.0030	.4970	.9940			
2.19 2.20	.9857 .9861	.0143	.4857 .4861	.9715 .9722	2.76 2.77	.9971 .9972	.0029	.4971 .4972	9942			
2.21	.9864	.0136	4864	.9729	2.78	.9973	.0027	.4973	.9946			
2,22	.9868 .9871	.0132	.4868 .4871	.9736 .9743	2.79 2.80	.9974	,0026 ,0026	4974	.9947 .9949			
2.24 2.25	9875 9878	.0125	.4875 .4878	.9749 .9756	2,81 2,82	.9975 .9976	.0025 .0024	4975 4976	.9950 .9952			
2,26	9881	0119	4881	.9762	2,83	.9977	.0023	4977	9953			
2.27 2.28	9884 9887	.0116	.4884 .4887	.9768 .9774	2.84 2.85	.9977 .9978	.0023	.4977 .4978	.9955 .9956			
2.29	.9890	.0110	.4890	.9780	2.86	.9979	.0021	.4979	.9958			
2,30 2,31	.9893 .9896	.0107	.4893 .4896	.9786 .9791	2.87 2.88	.9979 .9980	,0021	.4979 .4980	.9959 .9960			
2,32	.9898	0102	4898	.9797	2,89	.9981	.0019	.4981	.9961			
2,33 2,34	.9901 .9904	.0099	.4901 .4904	.9802 .9807	2.90 2.91	.9981 .9982	.0019	.4981 .4982	.9963 .9964			
2.35	9906	.0094	4906	.9812	2.92	.9982	.0018	4982	.9965			
2,36 2,37	.9909 .9911	.0091	.4909 .4911	.9817 .9822	2.93 2.94	.9983 .9984	,0017 ,0016	.4983 .4984	.9966 .9967			
2,38	9913	.0087	4913	.9827	2,95	.9984	,0016	4984	.9968			
							4	20	/			

The Relationship Between Hypothesis Testing and Confidence Intervals (Two-Sided Case)

 H_0 : $\mu = \mu_0$ vs. H_1 : $\mu \neq \mu_0$

- The two-sided 100% × (1 α) CI for μ does not contain $\mu_0 \rightarrow \text{Reject } H_0$
- The two-sided 100% × (1 α) CI for μ does contain $\mu_0 \rightarrow$ Accept H_0

Summary

- Null (H₀) and alternative (H₁) hypotheses
- Type I error (α), type II error (β), and the power (1-β) of a hypothesis test
- P-value of a hypothesis test and the distinction between on-sided and two-sided tests
- Hypothesis testing:
 - -critical value method
 - -p-value method
- Estimating appropriate sample size as determined by the pre-specified null and alternative hypotheses and type I and type II errors

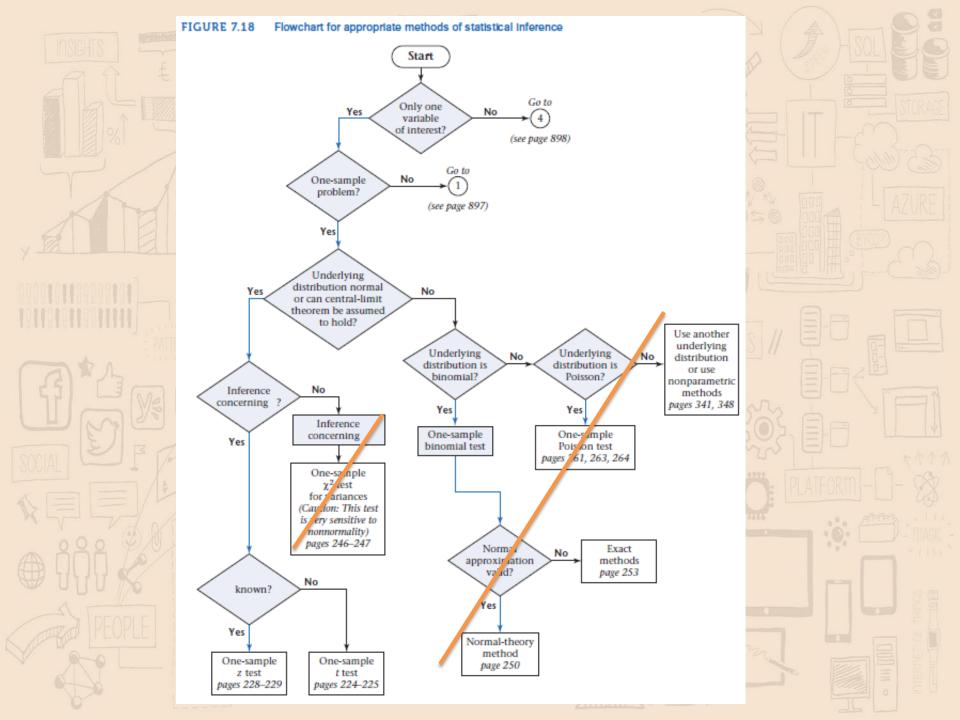


TABLE 3 The normal distribution (coefficiency) ** A* B* C* D* ** A* B* C* D* ** 182 6956												
A	17/7/		-	-1.45-1.75-1	/	4					1 A	
1.82 9660 DA44 A868 9317 2.30 9818 DO52 1818 0925 1818 1818 1818 1818 1818 1818 1818 18	I INSIGHTS I	TABLE 3	The non	mal distribut	ion (continu	ed)						
1.38		X	A*	Be	C=	D⁴	X	Α	В	С	D	
1.38		1.82	.9656	.0344	.4656	.9312	2.39	.9916	.0084	.4916	.9832	
1.85 9878 0.352 A678 9375 2.42 9922 0.078 A692 9945 18.8 9888 0.034 A688 9375 2.43 9922 0.077 A693 9845 18.9 9848 0.034 A688 9375 2.43 9929 0.071 A693 9845 18.9 9848 0.034 A688 9375 2.43 9929 0.071 A693 9845 18.9 9708 0.034 A788 9.812 2.46 9331 0.008 A693 9895 1.80 9708 0.034 A788 9.812 2.46 9331 0.008 A693 9895 1.80 9708 0.034 A788 9.812 2.46 9331 0.008 A693 9895 1.80 9708 0.034 A788 9.812 2.47 9.9932 0.088 A693 9895 9.87 9.87 9.87 9.87 9.87 9.87 9.87 9.87		1.83					2.40					6
1 88												TO STAPACE
1.87												
1.88												
1.91 9718 0.281 4719 3439 4.28 0.48 .9863 0.086 4.934 9.8865 1.52 0.722 0.724 4719 3439 4.28 0.9868 0.084 4.938 9.8878 1.52 0.722 0.724 4718 3439 4.73 0.088 0.084 0.088 0.084 4.938 9.878 1.52 0.722 0.724 4718 9.34 0.088 0.088 0.084 4.938 9.878 1.52 0.724 0.725 0.082 0.082 0.082 0.084 0.082 0.0												
1.91 9719 0.0281 4719 9.439 2.48 .9639 0.064 .9368 .9672 1.93 9719 0.0274 4726 9.439 .9669 0.064 .4936 .9672 1.93 9739 0.0268 4732 .9464 2.20 .9638 0.062 .4938 .9676 1.93 9739 0.0268 4732 .9464 2.20 .9638 0.062 .4938 .9676 1.93 9730 0.026 .4734 .9488 .9500 0.026 .4941 .9689 1.968 1.969 9700 0.026 .4750 .9500 0.253 .9633 0.007 .4943 .9866 1.969 1.969 0.0244 .4756 .9512 .224 .9946 .0005 .4940 .9869 1.969 .9761 0.028 .4764 .9489 .9500 0.23 .9943 0.007 .4943 .9868 1.99 9761 0.028 .4764 .9002 .253 .9943 0.007 .4943 .9868 1.99 9761 0.028 .4764 .9002 .254 .9946 .0004 .4946 .9869 .9869 1.99 9778 0.022 .4778 .9058 .258 .9961 .0048 .4962 .9961 .9869 1.99 9778 0.022 .4778 .9058 .259 .9962 .258 .9961 .0048 .4962 .9961 .9964 .203 .9788 .0217 .4788 .9068 .259 .9962 .258 .9961 .0048 .4962 .9961 .203 .9789 .0217 .4788 .9068 .258 .9961 .0048 .4962 .9961 .9062 .203 .9788 .0217 .4788 .9068 .258 .9962 .258 .9961 .0048 .4962 .9961 .9062 .203 .9983 .0017 .4808 .9618 .258 .9962 .258 .9962 .0044 .4968 .9969 .9062 .208 .9912 .008 .9												
1.92 9728 0274 4728 9401 2.20 9938 0.0064 4398 9972 1.93 9738 0.026 4739 9401 0.006 4940 9979 1.93 9740 0.026 4739 9401 0.006 4940 9979 1.93 9740 0.026 4739 9401 0.006 4940 9979 1.93 9740 0.006 4744 9478 2.21 9490 9401 0.006 4941 9889 1.93 9740 0.006 4744 9478 9488 9478 9478 9478 9478 9478												
1.94 9798 0.096 4738 9478 2.01 9840 0.006 4840 98978 1.186 9790 0.006 4700 0.006 4841 9883 1.186 97700 0.006 4700 0.006 4700 0.006 4840 0.0073 4843 9889 1.189 97700 0.006 4700 0.006 4700 0.006 4840 0.0073 4843 9890 1.189 9770 0.006 4700 0.006 4840 0.0073 4843 9890 1.189 9770 0.006 4700 0.006 4840 0.0074 4840 0.006 4840 0.0074 4840 0.0074 4840 0.0074 4840 0.0074 4840 0.0074 0.00												
1.96												
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