<u>Ans to Tut. 8</u>

<u>Qn 1</u>

a) Since the mapping is assumed to be linear,

$$\theta = a_u s + b_u t + c_u$$
$$Z = a_v s + b_v t + c_v$$

$s = 0$ maps to $\theta = 0$. This gives

$$0 = b_u t + c_u \qquad (1)$$

for any t.

$s = 1$ maps to $\theta = 2\pi$. This gives

$$2\pi = a_u + b_u t + c_u \qquad (2)$$

for any t.

$t = 0$ maps to $Z = -4$. This gives

$$-4 = a_v s + c_v \qquad (3)$$

for any s.

$t = 1$ maps to $Z = 4$. This gives

$$4 = a_v s + b_v + c_v \qquad (4)$$

for any s.

Using eqn (1) and (2) gives $a_u = 2\pi$.

Since eqn (1) is satisfied for any t, $b_u = 0$. Hence $c_u = 0$.

Using eqn (3) and (4) gives $b_v = 8$.

Since eqn (3) is satisfied for any s, $a_v = 0$. Hence $c_v = -4$.

Expressed in matrix form:

$$\begin{pmatrix} \theta \\ Z \end{pmatrix} = \begin{pmatrix} 2\pi & 0 \\ 0 & 8 \end{pmatrix} \begin{pmatrix} s \\ t \end{pmatrix} + \begin{pmatrix} 0 \\ -4 \end{pmatrix} \tag{5}$$

b) Since the cylinder is physically rotated by 30^o,

$$\mathbf{M}_{\text{mod}elview} = \mathbf{R}_y(+30^o) = \begin{pmatrix} \cos 30^o & 0 & \sin 30^o & 0 \\ 0 & 1 & 0 & 0 \\ -\sin 30^0 & 0 & \cos 30^o & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{6}$$

Projection vector $(V_{px}, V_{py}, V_{pz}) = (1,1,-2)$. As the view plane is the x-y plane, $Z_{vp} = 0$.

$$\mathbf{M}_{projection} = \begin{pmatrix} 1 & 0 & -V_{px}/V_{pz} & Z_{vp}(V_{px}/V_{pz}) \\ 0 & 1 & -V_{py}/V_{pz} & Z_{vp}(V_{py}/V_{pz}) \\ 0 & 0 & 0 & Z_{vp} \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0.5 & 0 \\ 0 & 1 & 0.5 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{7}$$

c) The parametric equation of the cylinder is

$$\begin{aligned} X &= 3\cos\theta + 3 \\ Y &= 3\sin\theta + 5 \end{aligned} \tag{8}$$

$$\begin{pmatrix} x \\ y \\ 0 \\ 1 \end{pmatrix} = \mathbf{M}_{projection}\mathbf{M}_{\text{mod}elview} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

Put in eqn (5) – (8),

$$\begin{pmatrix} x \\ y \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0.5 & 0 \\ 0 & 1 & 0.5 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos 30^o & 0 & \sin 30^o & 0 \\ 0 & 1 & 0 & 0 \\ -\sin 30^0 & 0 & \cos 30^o & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 3\cos(2\pi s) + 3 \\ 3\sin(2\pi s) + 5 \\ 8t - 4 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} \cos 30^o - 0.5\sin 30^o & 0 & \sin 30^o + 0.5\cos 30^o & 0 \\ -0.5\sin 30^o & 1 & 0.5\cos 30^o & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 3\cos(2\pi s) + 3 \\ 3\sin(2\pi s) + 5 \\ 8t - 4 \\ 1 \end{pmatrix}$$

$$x = (\cos 30^o - 0.5\sin 30^o)(3\cos(2\pi s) + 3) + (\sin 30^o + 0.5\cos 30^o)(8t - 4)$$
$$y = (-0.5\sin 30^o)(3\cos(2\pi s) + 3) + 3\sin(2\pi s) + 5 + 0.5\cos 30^o (8t - 4)$$

Simplifying,

$$x = 1.848076211 \cos(2\pi s) + 7.464101615 (t) - 1.883974596 \quad (9)$$
$$y = -0.75 \cos(2\pi s) + 3 \sin(2\pi s) + 3.464101615 (t) + 2.517949193 \quad (10)$$

Using eqn (9),

$$t = \frac{x + 1.883974596 - 1.848076211\cos(2\pi s)}{7.464101615}$$

Now, we need to express s as a function of x and y.

(9) – 2.154700538 (10) :

$$x - 2.154700538y = 3.464101615 \cos(2\pi s) - 6.464101615 \sin(2\pi s) - 7.309401078 \quad (11)$$

Let

$$A\cos B = 3.464101615$$
$$A\sin B = 6.464101615$$

$$A = \sqrt{(3.464101615)^2 + (6.464101615)^2} = 7.333799131$$
$$B = 61.81321457^o \text{ or } 1.078844115 \text{ rad}$$

Put into eqn (11),

$$x - 2.154700538y + 7.309401078 = 7.333799131 \cos(2\pi s + 1.078844115)$$

Simplifying,

$$s = \frac{\cos^{-1}(0.136354975x - 0.293804139y + 0.996673204) - 1.078844115}{2\pi}$$

d) Pixel order scanning has advantage over texture scanning because it makes sure that every pixel is mapped to a texture, thus ensuring that there is no hole in the image.

e) Objects are represented as quadrilateral mesh. Each quadrilateral can in turn be divided into two triangles. For each vertex of a projected triangle in the image, it is simple to find the mapping that maps to the corresponding texture location. Then use interpolation to find the texture for pixels inside the triangle.

<u>Qn 2</u>

$$\alpha = a_{11}s + a_{12}t + b_1$$
$$\beta = a_{21}s + a_{22}t + b_2$$

$(s,t) = (0,0)$ maps to $(\alpha,\beta) = (0,0) \Rightarrow (b_1, b_2) = (0,0)$
$(s,t) = (1,0)$ maps to $(\alpha,\beta) = (\pi/2, 0) \Rightarrow (a_{11}, a_{21}) = (\pi/2,0)$
$(s,t) = (1,1)$ maps to $(\alpha,\beta) = (\pi/4, \pi/4)$

$$\Rightarrow \frac{\pi}{4} = \frac{\pi}{2} + a_{12} \implies a_{12} = -\frac{\pi}{4} \qquad \frac{\pi}{4} = a_{22}$$

Hence,

$$\alpha = \frac{\pi}{2}s - \frac{\pi}{4}t$$
$$\beta = \frac{\pi}{4}t$$