

## EE2302 Foundations of Information and Data Engineering

### Assignment 8

**Due: 11 pm, Nov 11**

Full Mark: 20 points

1. (4 marks) The set of all  $2 \times 2$  real matrices in the form of  $A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$  is a vector space. Consider the subset of  $2 \times 2$  real matrices which satisfy  $a_{12} = -a_{21}$ . Does it form a subspace? Prove or disprove it.
2. (4 points) Determine whether each of the following scalar-valued functions  $n$ -vectors is linear. If it is linear, give its inner product representation, i.e., an  $n$ -vector  $a$  for which  $f(x) = a^T x$  for all  $x$ . If it is not linear, give specific  $x, y, \alpha, \beta$  for which superposition fails, i.e.,

$$f(\alpha x + \beta y) \neq \alpha f(x) + \beta f(y).$$

- a)  $f(x) = \max_k x_k$
  - b)  $f(x) = x_n - x_1$
3. (4 points) What 3 by 3 matrices represent the transformations that
    - a) reflect every vector through the  $x$ - $y$  plane?
    - b) rotate the  $x$ - $y$  plane through  $90^\circ$ , leaving the  $z$ -axis alone?
  4. (2 points) Consider the 2-dimensional space and the projection of  $b$  on the line through  $a$ , where  $a = (2, 1)$ .
    - a) Determine the corresponding projection matrix.
    - b) Suppose  $b = (5, 5)$ . Determine the result after the projection.
  5. (6 points) A business sells  $n$  products, and is considering changing the price of one of the products to increase its total profits. A business analyst develops a regression model that (reasonably accurately) predicts the total profit when the product prices are changed, given by  $\hat{P} = \beta^T x + P$ , where the  $n$ -vector  $x$  denotes the fractional change in the product prices,  $x_i = (p_i^{new} - p_i)/p_i$ . Here  $P$  is the profit with the current prices,  $\hat{P}$  is the predicted profit with the changed prices,  $p_i$  is the current (positive) price of product  $i$ , and  $p_i^{new}$  is the new price of product  $i$ .
    - a) What does it mean if  $\beta_3 < 0$ ? (And yes, this can occur.)
    - b) Suppose that you are given permission to change the price of *one* product, by up to 1%, to increase total profit. Which product would you choose, and would you increase or decrease the price? By how much?
    - c) Repeat (b), assuming you are allowed to change the price of two products, each by up to 1%.