IMPORTANT: The answers are provided with the view to deepen your understanding of the topic and to stimulate more in depth discussions. They should not be shared with others or put on the web. The questions and answers may be revised in the next course delivery.

Answers to Tutorial 1

Qn 1

a) {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

b) {HHH, HHT, HTH, THH}

Qn 2

$$\frac{(20)(19)}{2} = 190$$

Qn 3

Consider $(x + y)(x + y) \cdots (x + y)$. For the $x^k y^{n-k}$ term, one has to select k 'x' from n positions and the 'y' are selected from the remaining positions. There are $\binom{n}{k}$ ways to do it.

Qn 4

There are 11 flowers. If one considers all the flowers are distinct. Hence there are 11! permutations. However, for each particular arrangement in a row, the 2! permutations of red flowers gives the same arrangement. Similarly, the 4! permutations of yellow flowers and 5! permutations of white flowers also give the same arrangement. Hence the number of different arrangements are

$$\frac{11!}{(2!)(4!)(5!)} = 6930$$

Qn 5

a) $P(E \cup F) \leq P(E) + P(F)$

b)

$$P(E \cup F \cup G) = P(E) + P(F) + P(G) - P(E \cap F) - P(E \cap G) - P(F \cap G) + P(E \cap F \cap G)$$

c)

$$P\left(\bigcup_{i=1}^{n} E_i\right) = \sum_{i=1}^{n} P(E_i) - \sum_{i<j} P\left(E_i \cap E_j\right) + \sum_{i<j<k} P\left(E_i \cap E_j \cap E_k\right) - \cdots - (-1)^n P(\bigcap_{i=1}^{n} E_i)$$

Qn 6

Let A be the event that you like book A, and B be the event that you like book B.

$P(A) = 0.3 \;\; P(B) = 0.4 \;\;\; P(AB) = 0.2$

$P(A \cup B) = P(A) + P(B) - P(AB) = 0.3 + 0.4 - 0.2 = 0.5$

The event that you like neither book is $(A \cup B)^c = A^c B^c$.

$P(A^c B^c) = 1 - 0.5 = 0.5$

Qn 7

a)

There are $\binom{52}{5} =$ 2598960 different 5-card hands.

$P(royal\ flush) = \frac{4}{2598960} = 1.539077169 \times 10^{-6} \approx 0.0000015$

(about one in a million chance)

b)

We denote the hand by AABCD, where AA corresponds to the equal-value pair of cards. Think for instance that A is 'ace'. There are $\binom{4}{2} = 6$ ways of choosing. As there are 13 distinct values, the pair of cards can be chosen in $6 \times 13 = 78$ ways. The remaining three cards of the hand will be drawn out of the remaining 12 values, with each card being of any of the 4 suits. There are $\binom{12}{3}$ combinations of the values of the three cards. Since each of them can come from any of the 4 suits, we have

$$\binom{12}{3} \times 4 \times 4 \times 4 = 14080$$

different possibilities. Thus

$$P(pair) = 78 \times \frac{14080}{2598960} = 0.422569027 \approx 0.423$$

This is almost the probability of getting a 'head' in a fair coin flip (i.e. 0.5), or you can expect at least get one pair in a little more than 2 games.

The other probabilities of the Poker game can be found in https://en.wikipedia.org/wiki/Poker_probability

Qn 8

$P(13\ hits) = \left(\frac{1}{3}\right)^{13} = 6.272254772 \times 10^{-7} \approx 0.000000627$

(about 0.5 millionth)

For 11 hits, the problem is equivalent to selecting 11 positions out of 13 positions. There are $\binom{13}{11}$ ways. Hence

$$P(11\ hits) = \binom{13}{11}\left(\frac{1}{3}\right)^{11}\left(\frac{2}{3}\right)^{2} = 1.95694348 \times 10^{-4} \approx 0.0002$$

(about 1 in 5000)

Qn 9

There are $\binom{49}{6}$ possible scenarios. There are $\binom{6}{4}$ ways of drawing the 4 correct numbers. There are $\binom{42}{1}$ ways to draw the incorrect number and there is one way to draw the extra number.

Hence the probability is

$$\frac{\binom{6}{4}\binom{42}{1}}{\binom{49}{6}} = 4.50520802 \times 10^{-5}$$

Clarifications on Mark Six rule:

In Mark Six, people fill in **6 numbers only** in his entry form. Thus, for the fourth price, 4 out of 6 numbers are correct, the 5^{th} number is incorrect, and the 6^{th} number must be the extra number.