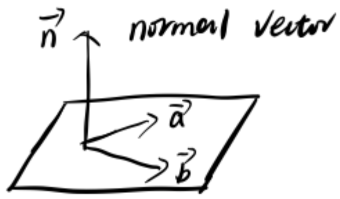
1°. Scalar product.

$$\vec{a}\cdot\vec{b} = a_1b_1 + a_2b_2 + a_3b_3 .$$

$$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta \iff \cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|} .$$

$$\vec{a}\perp\vec{b} \iff \theta = 90^\circ,\ \cos\theta = 0 \iff \vec{a}\cdot\vec{b} = 0 .$$

2°. Vector product.



$$|\vec{a}\times\vec{b}| = |\vec{a}||\vec{b}|\sin\theta .$$

$$\vec{a}\parallel\vec{b} \iff \theta = 0^\circ \text{ or } 180^\circ,\ \sin\theta = 0 \iff \vec{a}\times\vec{b} = 0 .$$

Some applications:
- area of parallelogram / triangle.
- colinear.
- find the (shortest) distance between a point a plane / two non-intersecting straight line.

3°. Triple scalar product.

$$\vec{a}\cdot(\vec{b}\times\vec{c}) .$$

Volume of parallelepiped: $V = |\vec{a}\cdot(\vec{b}\times\vec{c})|$

$\vec{a}$, $\vec{b}$ and $\vec{c}$ coplanar $\iff V = 0$.

4°. Triple vector product.

$$\vec{a}\times(\vec{b}\times\vec{c}) \neq (\vec{a}\times\vec{b})\times\vec{c} .$$

Part D: Linear Independence of vectors

Linear independence of <u>two vectors</u> $\vec{a}$ and $\vec{b}$:

$\Leftrightarrow$ $\vec{a} \parallel \vec{b} \Leftrightarrow \vec{a}\times\vec{b}=0$

Linear independence of <u>three vectors</u> $\vec{a}, \vec{b}$ and $\vec{c}$:

$\Leftrightarrow$ $\vec{a}, \vec{b}$ and $\vec{c}$ are coplanar $\Leftrightarrow |\vec{a}\cdot(\vec{b}\times\vec{c})|=0$

Theorem: (1) $\vec{a}_1, \vec{a}_2, \cdots, \vec{a}_n$ are linearly independent

$\Leftrightarrow$ the equation $\lambda_1\vec{a}_1+\lambda_2\vec{a}_2+\cdots+\lambda_n\vec{a}_n=0$ has only trivial solution.

$\Leftrightarrow$ $\lambda_1=\lambda_2=\cdots=\lambda_n=0$

(2) $\vec{a}_1, \vec{a}_2, \cdots, \vec{a}_n$ are linearly dependent

$\Leftrightarrow$ ..... has other solutions $\lambda_1, \cdots, \lambda_n$.

Problem 22

Find the value of m such that the following sets of vectors are *linearly dependent*.

$$\vec{a}=(1-m)\vec{i}+6\vec{j}+5\vec{k}, \qquad \vec{b}=2\vec{i}-m\vec{j}, \qquad \vec{c}=-5m\vec{j}+5\vec{k}.$$

Problem 23

Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three non-zero vectors. Show that

(a) $(\vec{a}-\vec{b}) \times (\vec{a}+\vec{b}) = 2(\vec{a} \times \vec{b})$.

(b) If $\vec{a}$ and $\vec{b}$ are perpendicular, then $|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}|$.

(c) If $\vec{a}$ and $\vec{b}$ are parallel, then the vectors $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ are also parallel. (Hint: If two vectors are parallel, what is the angle between them? What can you say about the vector product of these two vectors?)

(d) $\tan\theta = \frac{|\vec{a}\times\vec{b}|}{\vec{a}\cdot\vec{b}}$ where θ is the angle between $\vec{a}$ and $\vec{b}$.

(e) $|\vec{a}\times\vec{b}|^2 = |\vec{a}|^2|\vec{b}|^2 - (\vec{a}\cdot\vec{b})^2$.

Summary of the important integral of some elementary functions

$f(x)$	Antiderivative $F(x)$ $F'(x) = f(x)$	Integral $F(x) = \int f(x)dx$
x^a, a is real	$F(x) = \begin{cases} \frac{x^{a+1}}{a+1} & \text{if } a \neq -1 \\ \ln\lvert x\rvert & \text{if } a = -1 \end{cases}$	$\begin{cases} \frac{x^{a+1}}{a+1} + C & \text{if } a \neq -1 \\ \ln\lvert x\rvert + C & \text{if } a = -1 \end{cases}$
e^x	$F(x) = e^x$	$e^x + C$
$\sin x$	$F(x) = -\cos x$	$-\cos x + C$
$\cos x$	$F(x) = \sin x$	$\sin x + C$
$\tan x$	$F(x) = -\ln\lvert\cos x\rvert$ or $F(x) = \ln\lvert\sec x\rvert$	$-\ln\lvert\cos x\rvert + C$ or $\ln\lvert\sec x\rvert + C$
$\sec^2 x$	$F(x) = \tan x$	$\tan x + C$
$\frac{1}{1+x^2}$	$F(x) = \tan^{-1} x$	$\tan^{-1} x + C$
$\frac{1}{\sqrt{1-x^2}}$	$F(x) = \sin^{-1} x$	$\sin^{-1} x + C$

Problem 2 (A bit harder) Compute the following indefinite integrals:

(a) $\displaystyle\int \frac{x^2 - x + 1}{x^2} dx$

(b) $\displaystyle\int \frac{2x^2}{x^2 + 1} dx$

(c) $\displaystyle\int \frac{e^{2x} + e^{x-3} + 1}{e^{x+1}} dx$

(d) $\displaystyle\int \sin 3x \sin 2x \, dx$

(e) $\displaystyle\int \cos^3 2x \, dx$

(f) $\displaystyle\int \frac{1}{(x-1)(2x-3)} dx$

(g) $\displaystyle\int \frac{3}{x^2 - 2x + 5} dx$

(h) $\displaystyle\int \frac{1}{2x^2 - 4x + 9} dx$