

## Tutorial 1

### Qn 1

An experiment consists of tossing a coin three times.

- a) What is the sample space of this experiment?
- b) Which event corresponds to the experiment resulting in more heads than tails?

### Qn 2

Consider a group of 20 people. If everyone shakes hands with everyone else, how many handshakes take place?

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### Qn 3

Prove the binomial theorem using a combinatorial argument:

$$(x + y)^n = \sum_{k=0}^n \binom{n}{k} x^k y^{n-k}$$

### Qn 4

A gardener has two red flowers, four yellow flowers and five white flowers. He wishes to plant them in a row. How many different ways can he do that?

(There is a hint at the end of this tutorial.)

### Qn 5

The inclusion-exclusion principle for two sets is

$$P(E \cup F) = P(E) + P(F) - P(EF)$$

- a) The union bound is immediately derived from the principle. It is used in complicated situations (e.g. Probably Approximately Correct (PAC) learning) to derive simple but very useful probability bounds. Write down the union bound.
- b) Write down the inclusion-exclusion principle for three sets.
- c) Write down the principle for  $n$  sets.

#### Qn 6

You have two books. You have a probability of 0.3 of liking book A, and a probability of 0.4 of liking book B. You have a probability of 0.2 of liking both book A and B. What is the probability that you like neither book?

#### Qn 7

Probability is readily applied in games of chances.

Poker is played with a 52 card deck (13 cards of 4 suits). A player's hand is composed of 5 cards.

A 'royal flush' is the most difficult hand to obtain. It consists of Ace, King, Queen, Jack and 10, all of the same suit.

A 'pair' is the hand of the lowest value. It contains a simple pair of cards of the same value. An example is Ace Ace Four Jack Nine.

When the cards are dealt at the beginning of the game,

- a) what is the probability of obtaining a "royal flush"?
- b) what is the probability of obtaining a "pair"?

#### Qn 8

The game of 'Toto' is a football match outcome guessing game played in some European countries. There are 13 football matches and hence 13 entries. In each entry, you can tick it is a 'home win', 'draw' or 'away win'.

The prize money depends on how many of your guesses are correct.

If you are just guessing,

- a) what is the probability that all your 13 guesses are correct?
- b) what is the probability that 11 of your guesses are correct?

Qn 9

Mark 6 consists of 49 numbers. 6 numbers are randomly drawn without replacement. Then an extra number is drawn without replacement.

4<sup>th</sup> price is won when you have guessed right on 4 numbers as well as the extra number.

Calculate the probability of winning the 4<sup>th</sup> price.

Hint: Qn 5

The collection of two red, four yellow, and four white flowers is often called a multiset.  
multiset