Discrete Random Variables – Measuring Center

Cont'd

- Expected value (Mean)
  - Weighted average of all possible values of X
  - Corresponding probability is treated as weight

$$\mu = E(X) = \sum_{i=1}^{N} x_i P(X = x)$$

E.g. Toss 2 coins, count the number of heads

X = number of heads

Compute the expected value of X:

X	P(X)
0	0.25
1	0.50
2	0.25

$$\mu = x_1 P(X = x_1) + x_2 P(X = x_2) + x_3 P(X = x_3)$$

$$= (0)(0.25) + (1)(0.5) + (2)(0.25) = 1$$

Discrete Random Variables – Measuring Variation

Cont'd

- Variance
  - Weighted average squared deviation about the mean

$$\sigma^2 = \sum_{i=1}^{N} [x_i - E(X)]^2 P(X = x_i)$$

- Standard deviation
  - Square root of variance

$$\sigma = \sqrt{\sigma^2} = \sqrt{\sum_{i=1}^{N} [x_i - E(X)]^2 P(P = x_i)}$$

E.g. Toss 2 coins, count the number of heads

X = number of heads

Compute the variance of X:

X	P(X)
0	0.25
1	0.50
2	0.25

$$\sigma^2 = (x_1 - \mu)^2 P(X = x_1) + (x_2 - \mu)^2 P(X = x_2) + (x_3 - \mu)^2 P(X = x_3)$$

$$= (0 - 1)^2 (0.25) + (1 - 1)^2 (0.5) + (2 - 1)^2 (0.25) = 0.5$$

Discrete Random Variables – Exercise

Cont'd

- Assume the following table shows the return per $1,000 for an investment under different economic conditions

Return in amount, Y_i	Economic Condition	P (Y_i)
-$200	Recession	0.2
+ 50	Stable Economy	0.5
+ 350	Expanding Economy	0.3

- Compute the expected return and standard deviation

$$E(Y) = \mu_Y = (-200)(0.2) + (50)(0.5) + (350)(0.3) = \$90$$

$$\sigma_Y{}^2 = (-200 - 90)^2 (0.2) + (50 - 90)^2 (0.5) + (350 - 90)^2 (0.3) = 37{,}900$$

$$\sigma_Y = \sqrt{37{,}900} = \$194.68$$

Should you, from a statistical stand point, invest or not?

Binomial Distribution – Exercise

Cont'd

An experiment about the interest of going to the cinema is conducted in a secondary school. Five students are selected randomly. Assume the probability of going to cinema within a week is 0.1.

X = no. of students going to cinema out of 5 students

X follows Binomial distribution ($n = 5, \pi = 0.1$)

The probability of 3 students going to the cinema out of these 5 students:

$$P(X = 3) = \frac{n!}{x!\,(n - x)!} \pi^x (1 - \pi)^{(n-x)}$$

$$= \frac{5!}{3!(5-3)!} 0.1^3 (1 - 0.1)^{(5-3)}$$

$$= 0.0081$$

Binomial Distribution – Exercise

Cont'd

- What is the probability that there are 3 or more students going to the cinema within a week?

$$P(X \geq 3) = P(X = 3) + P(X = 4) + P(X = 5)$$
$$= 0.0081 + 0.00045 + 0.00001$$
$$= 0.00856$$

- What is the probability that there are less than 3 students going to the cinema?

$$P(X < 3) = P(X = 0) + P(X = 1) + P(X = 2)$$
$$= 1 - P(X \geq 3)$$
$$= 1 - 0.00856 = 0.99144$$

Binomial Distribution Mean and Standard Deviation

Binomial Probability Distribution:

x_i	$P(X=x_i)$
0	0.59049
1	0.32805
2	0.0729
3	0.0081
4	0.00045
5	0.00001

$$\mu = \sum x_i P(X = x_i)$$
$$= (0)(0.59049) + (1)(0.32805) + (2)(0.0729)$$
$$+ (3)(0.0081) + (4)(0.00045) + (5)(0.00001)$$
$$= 0.5$$

$$\sigma^2 = \sum (x_i - \mu)^2 P(X = x_i)$$
$$= (0\text{-}0.5)^2(0.59049) + (1\text{-}0.5)^2(0.32805) + (2\text{-}0.5)^2$$
$$(0.0729) + (3\text{-}0.5)^2(0.0081) + (4\text{-}0.5)^2(0.00045)$$
$$+ (5\text{-}0.5)^2(0.00001)$$
$$= 0.45$$

$$\rightarrow \sigma = 0.6708$$