

- Theorem. Greedy algorithm is optimal.
- Proof:
 - We compare the solution obtained from greedy algorithm with an optimal solution.
 - Let $G = i_1, i_2, \dots, i_k$ denote the set of jobs selected by greedy.
 - Let $Opt = j_1, j_2, \dots, j_n$ denote the set of jobs in the optimal solution.
 - The set of jobs are mutually compatible and the number of jobs is the largest.
 - Without loss of generality, we assume that $i_1 = j_1, i_2 = j_2, \dots, i_r = j_r$ and $i_{r+1} \neq j_{r+1}$, where r could be 0, 1, 2,
 - Job i_{r+1} finishes before (or at the same time of) j_{r+1} due to our greedy algorithm.

We consider the following solution:

$Opt_2 = i_1 = j_1, i_2 = j_2, \dots, i_r = j_r, i_{r+1}, j_{r+2}, \dots$

Opt_2 and Opt have the same number of jobs.

Thus, Opt_2 is also an optimum solution.

Now, greedy solution G and Opt_2 have $r+1$ choices in common.

Repeat the process, we can conclude that G and an optimum solution have all the choices in common.