

CS3335 Tutorial 5

Question 1. Consider the sorting problem. Suppose that the list is:
3, 2, 1, 5, 8, 9, 10, 4, 7, 6, 12, and 11. Use the merge sort algorithm to sort the list in increasing order. Show the intermediate steps.

Answer:

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Divide:  3, 2, 1, 5, 8, 9      | 10, 4, 7, 6, 12, 11.
Divide:  3, 2, 1,   | 5, 8, 9 | 10, 4, 7   | 6, 12, 11
Divide:  3, 2   | 1 | 5, 8 | 9 | 10, 4   | 7   | 6, 12   | 11
Divide:  3 | 2   | 1 | 5 | 8 | 9 | 10 | 4 | 7   | 6 | 12 | 11
Merge:   2 3     | 1 | 5 8 | 9 | 4 10   | 7   | 6 12   | 11
Merge:   1 2 3     | 5 8 9 | 4 7 10     | 6 11 12
Merge:   1 2 3 5 8 9      | 4 6 7 10 11 12
Merge:   1 2 3 4 5 6 7 8 9 10 11 12
  
```

Question 2. For the same list: 3, 2, 1, 5, 8, 9, 10, 4, 7, 6, 12, and 11. Suppose we have sorted the two halves as list1: 1, 2, 3, 5, 8, 9; and list2: 4, 6, 7, 10, 11, 12. Calculate the number of inversions with one number in list1 and the other number in list2 using $O(n)$ operations.

Answer: Merge and count

Initial: $i=6$

↓	↓
1, 2, 3, 5, 8, 9;	4, 6, 7, 10, 11, 12.

Sorted:

Step 1: $i= 5$

↓	↓
1, 2, 3, 5, 8, 9;	4, 6, 7, 10, 11, 12.

Sorted: 1

Step 2: i= 4

 ↓
 1, 2, 3, 5, 8, 9;

 ↓
 4, 6, 7, 10, 11, 12.

Sorted: 1, 2

Step 3: i= 3

 ↓
 1, 2, 3, 5, 8, 9;

 ↓
 4, 6, 7, 10, 11, 12.

Sorted: 1, 2, 3

Step 4: i= 3

 ↓
 1, 2, 3, 5, 8, 9;

 ↓
 4, 6, 7, 10, 11, 12.

Inversions:

3

Sorted: 1, 2, 3, 4

Step 5: i= 2

 ↓
 1, 2, 3, 5, 8, 9;

 ↓
 4, 6, 7, 10, 11, 12.

Inversions:

3

Sorted: 1, 2, 3, 4, 5

Step 6: i= 2

 ↓
 1, 2, 3, 5, 8, 9;

 ↓
 4, 6, 7, 10, 11, 12.

Inversions:

3 2

Sorted: 1, 2, 3, 4, 5, 6

Step 7: i= 2

 ↓
 1, 2, 3, 5, 8, 9;

 ↓
 4, 6, 7, 10, 11, 12.

Inversions:

3 2 2

Sorted: 1, 2, 3, 4, 5, 6, 7

Step 8: i= 1

↓

1, 2, 3, 5, 8, 9;

↓

4, 6, 7, 10, 11, 12.

Inversions: 3 2 2

Sorted: 1, 2, 3, 4, 5, 6, 7, 8

Step 9: i= 0

↓

1, 2, 3, 5, 8, 9;

↓

4, 6, 7, 10, 11, 12.

Inversions: 3 2 2

Sorted: 1, 2, 3, 4, 5, 6, 7, 8, 9

Step 10: i= 0

↓

1, 2, 3, 5, 8, 9;

↓

4, 6, 7, 10, 11, 12.

Inversions: 3 2 2 0

Sorted: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10

Step 11: i= 0

↓

1, 2, 3, 5, 8, 9;

↓

4, 6, 7, 10, 11, 12.

Inversions: 3 2 2 0 0

Sorted: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11

Step 12: i= 0

↓

1, 2, 3, 5, 8, 9;

↓

4, 6, 7, 10, 11, 12.

Inversions: 3 2 2 0 0 0

Sorted: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12

The number of inversions between the first half and the second half is
 $3+2+2+0+0+0 = 7$.