EE2302 Foundations of Information and Data Engineering

Assignment 1 (Solution)

1.

a. $C \subseteq D$ Proof: *[We will show that every element of C is in D.]* Suppose n is any element of C. Then $n = 6r - 5$ for some integer r. Let $s = 2r - 2$. Then s is an integer (because products and differences of integers are integers), and

$$3s + 1 = 3(2r - 2) + 1 = 6r - 6 + 1 = 6r - 5,$$

which equals n. Thus n satisfies the condition for being in D. Hence, every element in C is in D.

b. $D \not\subseteq C$ because there are elements of D that are not in C. For example, 4 is in D because $4 = 3 \cdot 1 + 1$. But 4 is not in C because if it were, then $4 = 6r - 5$ for some integer r, which would imply that $9 = 6r$, or, equivalently, that $r = 3/2$, and this contradicts the fact that r is an integer.

2. Proof: We need to prove both $B \subseteq C$ and $C \subseteq B$.
   i. Let m be an element of B, so there is an integer b such that $m = 10b - 3 = 10(b - 1) + 7$. Since $(b - 1)$ is an integer, by the definition of C, m is an element of C. Therefore, $B \subseteq C$.
   ii. Let n be an element of C, so there is an integer c such that $n = 10c + 7 = 10(c + 1) - 3$. Since $(c + 1)$ is an integer, be the definition of B, n is an element of B. Therefore, $C \subseteq B$.

3. domain = $\mathbb{N} \cup \{0\}$, range = $\{0, 1, 2, \dots, 9\}$.

4. The description "the smallest integer not describable in fewer than twelve English words" contains only eleven English words. If n is well defined, then it is describable in only eleven English words, which is a contradiction.

5. No. Suppose there were a computer program P that had as output a list of all computer programs that do not list themselves in their output. Consider the following two cases:
   a) If P lists itself as output, then it would be on the output list of P, which means that P would not list itself in its output. A contradiction.
   b) If P does not list itself as output, then it would be a member of the list of all programs that do not list themselves in their output, and this list is exactly the output of P. Hence, P would list itself as output. Again, a contradiction.

   In both cases, the assumption of the existence of such a program P is contradictory, and so no such program exists.