

# EE3210 Signals & Systems

Due on 11:00 AM, May 15, 2021

## Homework #2, 3

1. Total mark is 200 points ( $= 20$  points per problem  $\times 10$  problems)
2. Submission due by 11:00 AM, May 15, 2021. **We will not accept late submission.**
3. Online submission through Canvas
  - Scan or taking a photo of your answer sheet, then upload to Canvas

## Problem 1

Let's consider LTI systems described by the following differential equations. Derive the frequency responses  $H(f)$ , the corresponding impulse response  $h(t)$ , and the step responses using Fourier transform.

a)

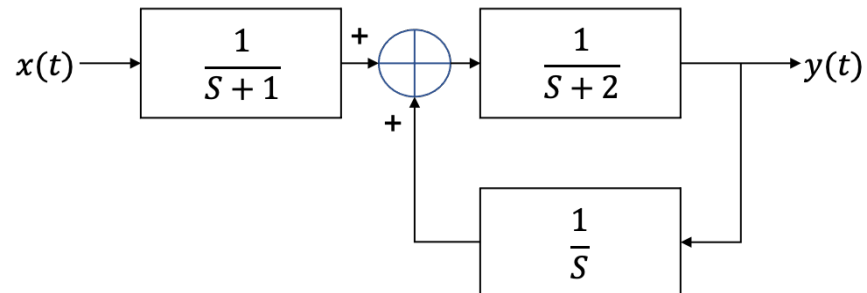
$$\frac{dy(t)}{dt} + 2y(t) = x(t) + \frac{dx(t)}{dt}$$

b)

$$\frac{d^2y(t)}{dt^2} + 6\frac{dy(t)}{dt} + 8y(t) = 2x(t)$$

## Problem 2

Determine the transfer function (or system function)  $H(s)$  of the following system model.



### Problem 3

Let us consider LTI system with a following transfer function  $H(s)$

$$H(s) = \frac{4}{(S+5)(S+3)(S-2)}.$$

- a) Indicate all possible ROC that can be associated with the given  $H(s)$ .
- b) For each ROC, specify whether the associated system is stable and/or causal.

**Problem 4**

Find the discrete time Fourier transform of the following signals

a)

$$x[n] = -a^n u[-n-1], \quad a \text{ is real}$$

b)

$$x[n] = u[n] - u[n-N]$$

## Problem 5

Prove the following four properties of discrete time Fourier transform based on the definition of DTFT

$$X(f) = \sum_{n=-\infty}^{n=\infty} x[n]e^{-j2\pi fn}$$

a) Time shifting property

$$x[n - n_0] \leftrightarrow e^{-j2\pi fn_0} X(f)$$

b) Frequency shifting property

$$e^{j2\pi f_0 n} x[n] \leftrightarrow X(f - f_0)$$

c) Differentiation

$$nx[n] \leftrightarrow \frac{j}{2\pi} \cdot \frac{dX(f)}{df}$$

**Problem 6**

Derive the Fourier transform of the following signals.

a)

$$x(t) = \text{sinc}^2(4t)$$

b)

$$x(t) = \text{sgn}(t)$$

c)

$$x(t) = e^{-2t}u(t)$$

d)

$$x(t) = \text{rect}\left(\frac{t}{3}\right)$$

**Problem 7**

- a) Derive (bilateral) Laplace transform of  $x(t) = e^{-5t}u(t)$  and define the ROS.
- b) Derive (bilateral) Laplace transform of  $x(t) = tu(t)$  and define the ROS.
- c) Derive (bilateral) Laplace transform of  $x(t) = x_1(t - 4)$  where  $x_1(t) = e^{-3t}u(t)$
- d) Find the inverse Laplace transform of the following  $X(s)$

$$X(s) = \frac{S^2 + 2S + 5}{(S + 3)(S + 5)^2}, \quad \text{Re}(S) > -3$$



**Problem 8**

Derive the Z-transform of  $x[n] = r^n \sin(w_0 n) u[n]$ .

**Problem 9**

- a) Find the Z-transform of the following sequence

$$x[n] = \left\{ 5, 3, \underset{\uparrow}{4}, 1, 0, 2 \right\}$$

- b) Find the Z-transform of the following signal

$$x[n] = \frac{1}{3} \cdot \frac{1}{4^n} u[n] - \frac{2}{2^n} u[n] + \frac{8}{3} u[n]$$

- c) Find the inverse Z-transform of the following  $X(z)$

$$X(z) = \log \left( \frac{1}{1-z} \right), \quad |z| < 1$$

- d) Find the inverse Z-transform of the following  $X(z)$

$$X(z) = \frac{3}{z-4}, \quad |z| > 4$$

**Problem 10**

Solve the following difference equation using Unilateral Z-transform

$$y[n] - 5y[n-1] + 6y[n-2] = 3x[n-1] + 5x[n-2],$$

where the initial conditions are given by

$$y[-1] = \frac{11}{6}, \quad y[-2] = \frac{37}{36},$$

and the input is  $x[n] = 2^{-n}u[n]$ .