EE3009 Tutorial 6 (Solution)

Question 1

a) Its transmitted signal is 1, -1, 1, 1, 1, -1, 1, 1, 1, -1, 1, 1, 1, -1, 1, 1.

b) The decoded output for the first bit is:

[(0)(1) + (-2)(-1) + (0)(1) + (2)(1) + (0)(1) + (0)(-1) + (2)(1) + (2)(1)] / 8

= [0 + 2 + 0 + 2 + 0 + 0 + 2 + 2] / 8

= 1

Similarly, we can check that the second bit is also equal to 1.

c) The correlation (or normalized inner product) between the two signature sequences is given by

[(1)(1) + (1)(-1) + (1)(1) + (-1)(1) + (1)(1) + (-1)(-1) + (-1)(1) + (-1)(1)] / 8

= [1 – 1 + 1 – 1 + 1 + 1 – 1 – 1] / 8

= 0

Hence, they are orthogonal.

Question 2

The maximum throughput is $\dfrac{Q}{Q/R + d} = \dfrac{QR}{Q + dR}$

Question 3

a) Throughput of A = 0.6 (1-0.3) = 0.42

b) Throughput of B = 0.3 (1-0.6) = 0.12

Question 4

a)

$$E(p) = Np(1-p)^{N-1}$$
$$E'(p) = N(1-p)^{N-1} - Np(N-1)(1-p)^{N-2}$$
$$= N(1-p)^{N-2}((1-p) - p(N-1))$$
$$= N(1-p)^{N-2}(1-pN)$$

$$E'(p) = 0 \Rightarrow p^* = \frac{1}{N}$$

The case where $p = 1$ should be rejected, since when $p = 1$, the throughput is equal to zero. You should also check that the point is indeed a maximum.

b)

$$E(p^*) = N\frac{1}{N}(1-\frac{1}{N})^{N-1} = (1-\frac{1}{N})^{N-1} = \frac{(1-\frac{1}{N})^N}{1-\frac{1}{N}}$$

$$\lim_{N\to\infty}(1-\frac{1}{N}) = 1 \qquad \lim_{N\to\infty}(1-\frac{1}{N})^N = \frac{1}{e}$$

Thus $\lim_{N\to\infty} E(p^*) = \frac{1}{e}$.

c) Slotted ALOHA has a higher maximum throughput because its probability of collision is lower. (Details can be found in the lecture notes.)