Tutorial 4

Qn 1

In nature, there are some un-explained correlation phenomenon, for example, the EPR paradox (https://en.wikipedia.org/wiki/EPR_paradox). At other times, two random variables are correlated, but in very complicated ways that are difficult to model or analyze, for example, the trends of two stocks.

Consider two fair dice. Assume that the two die are correlated in a mysterious way with a covariance of 1.2. Calculate

a) the expected value

b) the standard deviation

of the sum of the two dice.

Qn 2

There are 151 different first generation Pokémon that can be caught in Hong Kong. Suppose a Pokémon appears at random,

a) how many types of Pokémon will you expect to catch in 100 successful catches?

b) how many catches is expected before all Pokémon is collected?

[Hint: See the Coupon Collector Problem (https://en.wikipedia.org/wiki/Coupon_collector%27s_problem)]

c) Discuss whether your estimate in b) is reasonable.

Qn 3

Find the "expectation of life at birth by sex" from the Hong Kong Census and Statistics Department (http://www.censtatd.gov.hk/hkstat/hkif/index.jsp. See under "Health")

Use the latest data. You may either use the male figure or the female figure in the question below.

a) Provide an estimate of the probability that a person will live to 100 years old using Markov's inequality.

b) If the standard deviation of the expectation of life at birth is 5 years, find the probability that a person will live to more than ± 20 years from the expected life at birth.

c) Cantelli's inequality is a generalization of Chebyshev's inequality to one sided situations (https://en.wikipedia.org/wiki/Cantelli%27s_inequality). Assume the standard deviation of the expectation of life at birth is 5 years, estimate the probability that a person will live to 100 years old using Cantelli's inequality.

d) Comment on your estimates. Also, suggest how you can estimate the expectation of life at birth 2 years later.

e) Consider the population pyramid in Hong Kong (https://www.bycensus2016.gov.hk/en/bc-population-pyramid.html).

   Also, reference the sex ratio at birth (https://en.wikipedia.org/wiki/Human_sex_ratio)

   Suggest a reason why the sex ratio in Hong Kong is dropping.