Useful Elementary Integrals

Constant and powers

1. $\int k\,dx = kx + C.$

2. $\int x^n\,dx = \begin{cases} \dfrac{x^{n+1}}{n+1} + C, & n \neq -1 \\ \ln|x| + C, & n = -1 \end{cases}.$

Exponentials

3. $\int \mathrm{e}^x\,dx = \mathrm{e}^x + C.$

4. $\int a^x\,dx = \dfrac{a^x}{\ln a} + C,\ a \neq 1,\ a > 0.$

Trigonometric functions

5. $\int \sin x\,dx = -\cos x + C.$

6. $\int \cos x\,dx = \sin x + C.$

7. $\int \sec^2 x\,dx = \tan x + C.$

8. $\int \csc^2 x\,dx = -\cot x + C.$

9. $\int \sec x \tan x\,dx = \sec x + C.$

10. $\int \csc x \cot x\,dx = -\csc x + C.$

11. $\int \tan x\,dx = \ln|\sec x| + C.$

12. $\int \cot x\,dx = \ln|\sin x| + C.$

13. $\int \sec x\,dx = \ln|\sec x + \tan x| + C.$

14. $\int \csc x\,dx = \ln|\csc x - \cot x| + C.$

Algebraic functions

15. $\int \dfrac{1}{1+x^2}\,dx = \tan^{-1} x + C.$

16. $\int \dfrac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1} x + C.$

Hyperbolic functions

17. $\int \sinh x\,dx = \cosh x + C.$

18. $\int \cosh x\,dx = \sinh x + C.$

Useful Trigonometric Identities

Pythagorean identities

1. $\sin^2\theta + \cos^2\theta = 1$.

2. $1 + \tan^2\theta = \sec^2\theta$.

3. $1 + \cot^2\theta = \csc^2\theta$.

Double-angle formulas

4. $\sin 2\theta = 2\sin\theta\cos\theta$.

5. $\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$.

Half-angle formulas

6. $\sin^2\theta = \dfrac{1}{2}(1 - \cos 2\theta)$.

7. $\cos^2\theta = \dfrac{1}{2}(1 + \cos 2\theta)$.

Compound-angle formulas

8. $\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$.

9. $\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$.

10. $\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$.

Sum-to-product formulas

11. $\sin A + \sin B = 2\sin\dfrac{A+B}{2}\cos\dfrac{A-B}{2}$.

12. $\sin A - \sin B = 2\cos\dfrac{A+B}{2}\sin\dfrac{A-B}{2}$.

13. $\cos A + \cos B = 2\cos\dfrac{A+B}{2}\cos\dfrac{A-B}{2}$.

14. $\cos A - \cos B = -2\sin\dfrac{A+B}{2}\sin\dfrac{A-B}{2}$.

Product-to-sum formulas

15. $\sin A\cos B = \dfrac{1}{2}\big[\sin(A+B) + \sin(A-B)\big]$.

16. $\cos A\sin B = \dfrac{1}{2}\big[\sin(A+B) - \sin(A-B)\big]$.

17. $\cos A\cos B = \dfrac{1}{2}\big[\cos(A+B) + \cos(A-B)\big]$.

18. $\sin A\sin B = -\dfrac{1}{2}\big[\cos(A+B) - \cos(A-B)\big]$.

Euler's formulas

19. $e^{\pm i\theta} = \cos\theta \pm i\sin\theta$.

20. $e^{i\theta} + e^{-i\theta} = 2\cos\theta$, $\cos\theta = \dfrac{1}{2}(e^{i\theta} + e^{-i\theta})$.

21. $e^{i\theta} - e^{-i\theta} = 2i\sin\theta$, $\sin\theta = \dfrac{1}{2i}(e^{i\theta} - e^{-i\theta})$.

Remark. Formulas of the form $A \pm B = C \pm D$ contain two separate formulas

$$A + B = C + D, \qquad \text{and} \qquad A - B = C - D.$$

Likewise, formulas of the form $A \pm B = C \mp D$ contain two separate formulas

$$A + B = C - D, \qquad \text{and} \qquad A - B = C + D.$$