MA1200 Exercise for Chapter 7 Techniques of Differentiation
Solutions

First Principle

1. a) To simplify the calculation, note $\frac{2x-3}{3x+4}=\frac{2}{3}-\frac{17}{3(3x+4)}$. Then

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{\frac{2}{3}-\frac{17}{3(3x+3h+4)}-\frac{2}{3}+\frac{17}{3(3x+4)}}{h}=\lim_{h\to 0}\frac{17(3x+3h+4-(3x+4))}{3h(3x+3h+4)(3x+4)}$$

$$=\lim_{h\to 0}\frac{17(3h)}{3h(3x+3h+4)(3x+4)}=\lim_{h\to 0}\frac{17}{(3x+3h+4)(3x+4)}=\frac{17}{(3x+4)^2}=f'(x)$$

b)

$$\lim_{h\to 0}\frac{\sqrt{2(x+h)+1}-\sqrt{2x+1}}{h}\times\frac{\sqrt{2(x+h)+1}+\sqrt{2x+1}}{\sqrt{2(x+h)+1}+\sqrt{2x+1}}$$

$$=\lim_{h\to 0}\frac{2h}{h[\sqrt{2(x+h)+1}+\sqrt{2x+1}]}=\frac{2}{2\sqrt{2x+1}}=\frac{1}{\sqrt{2x+1}}=f'(x)$$

Product/Quotient/Chain Rules

2. a) $28x^3-12x+1+\frac{1}{x^2}-\frac{6}{x^3}-\frac{6}{x^4}$

b) $y=\sqrt[3]{3}x^{2/3}$, $y'=\frac{2\sqrt[3]{3}}{3}x^{\frac{-1}{3}}$

c) $y=(25-x^2)^{1/2}$, so $y'=\frac{1}{2}(25-x^2)^{-\frac{1}{2}}(-2x)=\frac{-x}{\sqrt{25-x^2}}$

d) Quotient rule: $\frac{-12}{(3+2x)^2}$

e) Quotient rule: $\frac{8x-x^3}{(4-x^2)^{1.5}}$

f) Chain rule: $3000(1+2x-5x^2)^{2999}(2-10x)$

g) Product rule + Chain rule: $2(x^2+4)(2x)(2x^3-1)^3+(x^2+4)^2 3(2x^3-1)^2(6x^2)$

h) Chain rule: $6\sin^2(2x)\cos(2x)+15x^2\sin(x^3+1)$

i) Chain rule: $y'=\frac{1}{\ln(\ln x)}(\ln(\ln x))'=\frac{1}{(\ln(\ln x))(\ln x)}(\ln x)'=\frac{1}{x\ln x\ln(\ln x)}$

j) Product rule: $\ln x$
k) Chain rule: $\cot x$
l) Chain rule: $-\tan x$

m) $y=\ln 3+\ln x-x$. $y'=-1+\frac{1}{x}$

n) Chain rule: $-e^x\sin(e^x)$
o) Product rule: $y'=3x^2(e^x\cos x)+x^3e^x\cos x+x^3e^x(-\sin x)$

$=e^x(x^3\cos x+3x^2\cos x-x^3\sin x)$

p) Quotient rule + Product rule: $\dfrac{(3x^2-5x)e^x(2x+\sin x+2+\cos x)-e^x(2x+\sin x)(6x-5)}{(3x^2-5x)^2}$

q) Quotient rule: $\dfrac{(n-1)x^n-nx^{n-1}+1}{(x-1)^2}$ r) Quotient rule: $\dfrac{-x^2-4x+6}{(x^2-3x)^2}$

s) Product rule + Chain rule: $2\cos(2x)\cos(3x)-3\sin(2x)\sin(3x)$

t) Chain rule: $\cos(\ln[\cos(\ln(2x)+1)]+1)\dfrac{-\sin(\ln(2x)+1)}{\cos(\ln(2x)+1)}\dfrac{1}{x}$

3. a) Differentiate both sides:
$2x-2y(y')=0$ So $y'=x/y$
(We need to use Chain rule on the y^2term, and then we get a y')
b) Differentiate both sides:
$2x+y+x(y')+2y(y')=0$ So $y'=-\dfrac{2x+y}{x+2y}$
In the middle term we used Product rule.
c) Differentiate both sides:
$2(x^2+y^2)(2x+2y(y'))=4(y+xy')$ Collecting like terms, we get
$y'=\dfrac{(x^2+y^2)x-y}{x-(x^2+y^2)y}$

4. a) $\dfrac{dy}{dx}=\dfrac{\frac{dy}{dt}}{\frac{dx}{dt}}=\dfrac{9t^2}{4t}=\dfrac{9}{4}t=\dfrac{9}{4}$ when t = 1. When $t=1, (x,y)=(3,5)$. So

Tangent: $y-5=\dfrac{9}{4}(x-3)$, $y=2.25x-1.75$

Normal: $y-5=\dfrac{-4}{9}(x-3)$, $y=\dfrac{-4}{9}x+\dfrac{19}{3}$

(Recall: eqn of line is $y-y_0=(slope)(x-x_0)$; slope of tangent = dy/dx;
slope of normal $\times$ slope of tangent = -1)

b) We can do as in part a) to get $\dfrac{dy}{dx}=2\sqrt{t}+\dfrac{1}{t}=\dfrac{17}{4}$ when t = 4.

Or we can write $y=x^2-\frac{1}{x}$ and we sub in $x=2$. $(x,y)=(2,3.5)$

Tangent: $y=\dfrac{17}{4}x-5$, Normal: $y=-\dfrac{4}{17}x+\dfrac{135}{34}$.

5. a) Take log. Then use Chain rule on LHS and Product rule on RHS:
$\ln y=\cot x\ln(x+1)$ So $\frac{1}{y}y'=(-\csc^2 x)\ln(x+1)+\cot x\left(\frac{1}{x+1}\right)$

$y'=(x+1)^{\cot x}\left[-\csc^2 x\ln(x+1)+\dfrac{\cot x}{x+1}\right]$

b) $\ln y=e^x\ln(x^4+2x^2)$ So $\frac{1}{y}y'=e^x\ln(x^4+2x^2)+e^x\left(\frac{4x^3+4x}{x^4+2x^2}\right)$

$$y' = (x^4+2x^2)^{e^x}\left[e^x\ln(x^4+2x^2)+\frac{e^x(4x^3+4x)}{x^4+2x^2}\right]$$

c) $\ln u = (x^2+2)\ln(\sin 5x)$. Then $\frac{1}{u}u' = (2x)\ln(\sin 5x) + (x^2+2)\frac{5\cos 5x}{\sin 5x}$

Then $y' = (\sin 5x)^{x^2+2}[2x\ln(\sin 5x)+5(x^2+2)\cot 5x]+3$

6. a) $y' = 3(4)e^{4x}$ $\qquad\qquad$ $y'' = 3(4)^2e^{4x}$

So in general, $y^{(n)} = 3(4^n)e^{4x}$

b) Use the formula: If $u = a\sin(bx+c)$, then $u^{(n)} = a(b)^n\sin(bx+c+\frac{n\pi}{2})$

So $y^{(n)} = 5(6^n)\sin(6x-7+\frac{n\pi}{2})$

c) Write $y = 2(2x-1)^{-1}$. Then $y' = 2(-1)(2x-1)^{-2}2$

$y'' = 2(-1)(-2)(2x-1)^{-3}2^2$ $\quad$ $y^{(3)} = 2(-1)(-2)(-3)(2x-1)^{-4}2^3$

So $y^{(n)} = 2^{n+1}(-1)^n n!(2x-1)^{-n-1}$

(There is no $(-n)!$, need to write $(-1)^n n!$)

$$d)\ y^{(n)} = \begin{cases} \frac{11!}{(11-n)!}x^{11-n} - 6\frac{3!}{(3-n)!}x^{3-n} & 1\le n\le 3 \\ \frac{11!}{(11-n)!}x^{11-n} & \text{when } 4\le n\le 11 \\ 0 & 12\le n \end{cases}$$

(First we know if differentiate more than 11 times, all terms become 0; we also know if differentiate more than 3 times, only the x^{11} term remains. We differentiate a few times to guess the general form above)

Leibniz's rule

7. If $y = f(x)g(x)$, then Leibniz' rule states that

$$y^{(n)} = \binom{n}{0}f^{(n)}g^{(0)} + \binom{n}{1}f^{(n-1)}g^{(1)} + \binom{n}{2}f^{(n-2)}g^{(2)} + \cdots + \binom{n}{n}f^{(0)}g^{(n)}$$

a) $y = x^3e^{2x} = g(x)f(x)$. Then putting into the formula above and noting that $f^{(n)}(x) = 2^ne^{2x}$ and $g^{(m)} = 0$ for $m \ge 4$, we get

$$y^{(n)} = \binom{n}{0}2^ne^{2x}x^3 + \binom{n}{1}2^{n-1}e^{2x}3x^2 + \binom{n}{2}2^{n-2}e^{2x}6x + \binom{n}{3}2^{n-3}e^{2x}6 + 0$$

$$y^{(n)} = \begin{cases} e^{2x}x^2(3+2x) & n=1 \\ 2e^{2x}x(2x^2+6x+3) & \text{when } n=2 \\ 2^{n-3}e^{2x}[2^3x^3+3(2^2)nx^2+3(2)n(n-1)x+n(n-1)(n-2)] & n\ge 3 \end{cases}$$

b) $y = (x^2-4x+7)\cos(2x-1) = g(x)f(x)$. So for $n \ge 2$,

$$y^{(n)} = \binom{n}{0}(x^2-4x+7)2^n\cos\left(2x-1+\frac{n\pi}{2}\right) + \binom{n}{1}(2x-4)2^{n-1}\cos\left(2x-1+\frac{(n-1)\pi}{2}\right)$$
$$+\binom{n}{2}(2)2^{n-2}\cos\left(2x-1+\frac{(n-2)\pi}{2}\right)+0$$

$$y^{(n)} = \begin{cases} (2x-4)\cos(2x-1) + 2(x^2-4x+7)\cos\left(2x-1+\frac{\pi}{2}\right) & \text{when } n=1 \\ 2^n \cos\left(2x-1+\frac{n\pi}{2}\right)(x^2-4x+7) & \\ \quad + n2^{n-1}\cos\left(2x-1+\frac{(n-1)\pi}{2}\right)(2x-4) & \\ \quad + n(n-1)2^{n-2}\cos\left(2x-1+\frac{(n-2)\pi}{2}\right) & \text{when } n \ge 2 \end{cases}$$

c) $y = (x^2)\frac{1}{1+x} = g(x)f(x)$. So for $n \ge 2$,

$$y^{(n)} = \binom{n}{0}(-1)^n n!\,(1+x)^{-n-1}x^2 + \binom{n}{1}(-1)^{n-1}(n-1)!\,(1+x)^{-n}2x + \binom{n}{2}(-1)^{n-2}(n-2)!\,(1+x)^{-n+1}2$$

$$y^{(n)} = (-1)^n n!\,(1+x)^{-n-1}x^2 + (-1)^{n-1}n!\,(1+x)^{-n}2x + (-1)^{n-2}n!\,(1+x)^{-n+1}$$

$$y^{(n)} = (-1)^n n!\,(1+x)^{-n-1}[x^2 + (-1)(1+x)2x + (1+x)^2]$$
$$= (-1)^n n!\,(1+x)^{-n-1}[x-(1+x)]^2$$
$$= (-1)^n n!\,(1+x)^{-n-1}$$

$$y^{(n)} = \begin{cases} (1+x)^{-2}x(2+x) & \text{when } n=1 \\ (-1)^n n!(1+x)^{-n-1} & \text{when } n \ge 2 \end{cases}$$

Miscellaneous

8. If $y = x\sin x$, prove that $x^2y'' - 2xy' + \left(2+x^2\right)y = 0$.

Proof:

$y = x\sin x \Rightarrow y' = \sin x + x\cos x \Rightarrow y'' = \cos x + \cos x - x\sin x = 2\cos x - x\sin x$.

So

$x^2y'' - 2xy' + (2+x^2)y = x^2(2\cos x - x\sin x) - 2x(\sin x + x\cos x) + (2+x^2)x\sin x$

$= x^3(-\sin x + \sin x) + x^2(2\cos x - 2\cos x) + x(-2\sin x + \sin x) = 0$

9. If $u = \sqrt{ax^2+2bx+c}$, prove that $\frac{d}{dx}(xu) = \frac{2ax^2+3bx+c}{u}$.

Proof:

$$\frac{du}{dx} = \frac{d}{dx}\left(\sqrt{ax^2+2bx+c}\right) = \frac{d}{dx}\left((ax^2+2bx+c)^{\frac{1}{2}}\right) = \frac{1}{2}(ax^2+2bx+c)^{-\frac{1}{2}}(2ax+2b) = \frac{ax+b}{(ax^2+2bx+c)^{\frac{1}{2}}} = \frac{ax+b}{u}$$

Then

$$\frac{d}{dx}(xu) = x\frac{du}{dx} + u = x\frac{ax+b}{u} + u = \frac{ax^2+bx+u^2}{u} = \frac{ax^2+bx+ax^2+2bx+c}{u} = \frac{2ax^2+3bx+c}{u}$$

*10. Find the values of a and b (in terms of c) such that $f'(c)$ exists, where $f(x)=\begin{cases}\frac{1}{|x|} & \text{if } |x|>c \\ ax+b & \text{if } |x|\le c\end{cases}$.

Solution:

$$f(x)=\begin{cases}\frac{1}{|x|} & \text{if } |x|>c \\ ax+b & \text{if } |x|\le c\end{cases} \Rightarrow f(x)=\begin{cases}\frac{1}{x} & \text{if } x>c \\ ax+b & \text{if } -c\le x\le c \\ \frac{1}{-x} & \text{if } x<-c\end{cases}$$

In order that $f'(c)$ exists, $f(x)$ must be continuous at $x=c$.

That is, $\lim_{x\to c^+} f(x)=\lim_{x\to c^+}\frac{1}{x}=\frac{1}{c}=\lim_{x\to c^-} f(x)=ac+b=f(c)$.

$$\lim_{x\to c^+}\frac{f(x)-f(c)}{x-c}=\lim_{x\to c^+}\frac{\frac{1}{x}-(ac+b)}{x-c}=\lim_{x\to c^+}\frac{\frac{1}{x}-\frac{1}{c}}{x-c}=\lim_{x\to c^+}\frac{\frac{c-x}{xc}}{x-c}=-\lim_{x\to c^+}\frac{\frac{x-c}{xc}}{x-c}=-\lim_{x\to c^+}\frac{1}{xc}=-\frac{1}{c^2}.$$

$$\lim_{x\to c^-}\frac{f(x)-f(c)}{x-c}=\lim_{x\to c^-}\frac{ax+b-(ac+b)}{x-c}=\lim_{x\to c^-}\frac{a(x-c)}{x-c}=a.$$

That $f'(c)$ exists means $\lim_{x\to c^+}\frac{f(x)-f(c)}{x-c}=-\frac{1}{c^2}=\lim_{x\to c^-}\frac{f(x)-f(c)}{x-c}=a$.

Therefore, $a=-\frac{1}{c^2}$ .and put in $ac+b=\frac{1}{c}$.

We have $-\frac{1}{c}+b=c^2\Rightarrow b=c^2+\frac{1}{c}=\frac{c^3+1}{c}$. So $b=-c^2$. Finally, we have $\begin{cases}a=-\frac{1}{c^2} \\ b=\frac{c^3+1}{c}\end{cases}$

11. A function y of x is defined by the equation $\sin(x-y)=m\sin y$. Express y explicitly in terms of x.

Hence, or otherwise, show that $\frac{dy}{dx}=\frac{1+m\cos x}{1+2m\ \cos x+m^2}$.

Proof:

$$\sin(x-y)=m\sin y\Rightarrow \sin x\cos y-\cos x\sin y=m\sin y\Rightarrow \sin x\cos y=(m+\cos x)\sin y$$

$$\Rightarrow\frac{\sin y}{\cos y}=\frac{\sin x}{m+\cos x}\Rightarrow \tan y=\frac{\sin x}{m+\cos x}\Rightarrow y=\tan^{-1}\left(\frac{\sin x}{m+\cos x}\right)$$

Then

$$\frac{dy}{dx}=\frac{d\left[\tan^{-1}\left(\frac{\sin x}{m+\cos x}\right)\right]}{dx}=\frac{\frac{d\left(\frac{\sin x}{m+\cos x}\right)}{dx}}{1+\left(\frac{\sin x}{m+\cos x}\right)^2}=\frac{\frac{(m+\cos x)\cos x+\sin^2 x}{(m+\cos x)^2}}{\frac{(m+\cos x)^2+\sin^2 x}{(m+\cos x)^2}}=\frac{1+m\cos x}{1+2m\cos x+m^2}$$

12. Let $y=(x+1)^{\cot x}$, find $\frac{dy}{dx}$.

Solution:

$$y=(x+1)^{\cot x}\Rightarrow \ln y=\cot x\ln(x+1)\Rightarrow \frac{\frac{dy}{dx}}{y}=-\csc x\ln(x+1)+\cot x\left(\frac{1}{x+1}\right)$$

$$\Rightarrow \frac{dy}{dx}=y\left(-\csc x\ln(x+1)+\frac{\cot x}{x+1}\right)=-(x+1)^{\cot x}\csc x\ln(x+1)+\frac{\cot x}{x+1}(x+1)^{\cot x}$$

13. Show that $f'(x)=0$, where $f(x)=\tan^{-1}x+\tan^{-1}\frac{1}{x}$.

Solution:

$$f(x)=\tan^{-1}x+\tan^{-1}\frac{1}{x}\Rightarrow f'(x)=\frac{1}{1+x^2}+\frac{1}{1+\frac{1}{x^2}}\frac{d\left(\frac{1}{x}\right)}{dx}=\frac{1}{1+x^2}-\frac{x^2}{1+x^2}\frac{1}{x^2}=\frac{1}{1+x^2}-\frac{1}{1+x^2}=0.$$

14. Consider the parametric curve $\begin{cases}x=t^2+2\\ y=t^3\end{cases}$ where $-\infty<t<\infty$.

Find $\frac{dy}{dx},\frac{d^2y}{dx^2}$ and the equation of the tangent line to the curve at the point $(3,1)$.

Solution:

Method 1:

$$\begin{cases}x=t^2+2\\ y=t^3\end{cases}\Rightarrow x=y^{\frac{2}{3}}+2\Rightarrow y=(x-2)^{\frac{3}{2}}.$$

Then

$$y=(x-2)^{\frac{3}{2}}\Rightarrow \frac{dy}{dx}=\frac{3}{2}(x-2)^{\frac{1}{2}},\frac{d^2y}{dx^2}=\left(\frac{1}{2}\right)\frac{3}{2}(x-2)^{-\frac{1}{2}}=\frac{3}{4}(x-2)^{-\frac{1}{2}}$$

Method 2:

$$\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}\Rightarrow \frac{d^2y}{dx^2}=\frac{d}{dx}\left(\frac{dy}{dx}\right)=\frac{d}{dx}\left(\frac{\frac{dy}{dt}}{\frac{dx}{dt}}\right)=\frac{d}{dt}\left(\frac{\frac{dy}{dt}}{\frac{dx}{dt}}\right)\frac{dt}{dx}=\left[\frac{\frac{dx}{dt}\frac{d^2y}{dt^2}-\frac{dy}{dt}\frac{d^2x}{dt^2}}{\left(\frac{dx}{dt}\right)^2}\right]\frac{dt}{dx}$$

$$\begin{cases}x=t^2+2\\ y=t^3\end{cases}\Rightarrow\begin{cases}\frac{dx}{dt}=2t\\ \frac{dy}{dt}=3t^2\end{cases}\Rightarrow\begin{cases}\frac{d^2x}{dt^2}=2\\ \frac{d^2y}{dt^2}=6t\end{cases}.\quad \text{So } \frac{dy}{dx}=\frac{3t^2}{2t}=\frac{3}{2}t$$

$$\frac{d^2y}{dx^2}=\left[\frac{\frac{dx}{dt}\frac{d^2y}{dt^2}-\frac{dy}{dt}\frac{d^2x}{dt^2}}{\left(\frac{dx}{dt}\right)^2}\right]\frac{dt}{dx}=\left(\frac{2t(6t)-2(3t^2)}{4t^2}\right)\frac{dt}{dx}=\frac{3}{2}\frac{dt}{dx}$$

In addition, $x=t^2+2 \Rightarrow t=(x-2)^{\frac{1}{2}}$ also $x=t^2+2 \Rightarrow 1=2t\frac{dt}{dx} \Rightarrow \frac{dt}{dx}=\frac{1}{2t}$ $(t \neq 0)$.

As a result, $\frac{dy}{dx}=\frac{3}{2}t=\frac{3}{2}(x-2)^{\frac{1}{2}}$, $\frac{d^2y}{dx^2}=\frac{3}{2}\frac{dt}{dx}=\frac{3}{2}\left(\frac{1}{2t}\right)=\frac{3}{2}\left(\frac{1}{2(x-2)^{\frac{1}{2}}}\right)=\frac{3}{4}(x-2)^{-\frac{1}{2}}$

The slope of the tangent line to the curve at the point $(3,1)=\frac{dy}{dx}\Big|_{x=3}=\frac{3}{2}$.

Therefore, the equation of the tangent line to the curve at the point $(3,1)$ is: $y-1=\frac{3}{2}(x-3)$

15. By Leibnitz's theorem on repeated differentiation, find the nth derivatives of the function $y = e^x \cos 2x$.
Solution:
By Leibniz's theorem on repeated differentiation, we have

$$\frac{d^n}{dx^n}(fg)=g\frac{d^n f}{dx^n}+\sum_{i=1}^{n-1}{}_nC_i\frac{d^i g}{dx^i}\frac{d^{n-i}f}{dx^{n-i}}+f\frac{d^n g}{dx^n}=\sum_{i=0}^{n}{}_nC_i\frac{d^i g}{dx^i}\frac{d^{n-i}f}{dx^{n-i}}, \text{ where } \frac{d^0 f}{dx^0}\equiv f,\ \frac{d^0 g}{dx^0}\equiv g,$$

${}_nC_r \equiv \frac{n!}{r!(n-r)!}$, $r=0,1,\cdots,n-1,n,$ and $0!=1$.

Now, $f(x)=e^x \Rightarrow \frac{d^i f}{dx^i}=e^x,\ i=0,1,2,\cdots$.

$g(x)=\cos 2x \Rightarrow \frac{d^{2i}g}{dx^{2i}}=(-1)^i 2^{2i}\cos 2x,\ \frac{d^{2i+1}g}{dx^{2i+1}}=(-1)^{i+1}2^{2i+1}\sin 2x,\ i=0,1,2,\cdots$.

So

$$\frac{d^n}{dx^n}(e^x\cos 2x)=\sum_{i=0}^{n}{}_nC_i\frac{d^i(\cos 2x)}{dx^i}\frac{d^{n-i}(e^x)}{dx^{n-i}}$$

If $n=2m$ where m is a nonnegative integer, then

$$\frac{d^n}{dx^n}(e^x\cos 2x)=\sum_{i=0}^{n}{}_nC_i\frac{d^i(\cos 2x)}{dx^i}\frac{d^{n-i}(e^x)}{dx^{n-i}}$$

$$=\sum_{i=0}^{m}{}_nC_{2i}\frac{d^{2i}(\cos 2x)}{dx^{2i}}\frac{d^{n-2i}(e^x)}{dx^{n-2i}}+\sum_{i=0}^{m-1}{}_nC_{2i+1}\frac{d^{2i+1}(\cos 2x)}{dx^{2i}}\frac{d^{n-2i-1}(e^x)}{dx^{n-2i}}$$

$$=\sum_{i=0}^{m}{}_nC_{2i}(-1)^i 2^{2i}(\cos 2x)e^x+\sum_{i=0}^{m-1}{}_nC_{2i+1}(-1)^{i+1}2^{2i+1}(\sin 2x)e^x$$

If $n=2m+1$, where m is a nonnegative integer, then

$$\frac{d^n}{dx^n}(e^x\cos 2x)=\sum_{i=0}^{n}{}_nC_i\frac{d^i(\cos 2x)}{dx^i}\frac{d^{n-i}(e^x)}{dx^{n-i}}$$

$$=\sum_{i=0}^{m}{}_nC_{2i}\frac{d^{2i}(\cos 2x)}{dx^{2i}}\frac{d^{n-2i}(e^x)}{dx^{n-2i}}+\sum_{i=0}^{m}{}_nC_{2i+1}\frac{d^{2i+1}(\cos 2x)}{dx^{2i}}\frac{d^{n-2i-1}(e^x)}{dx^{n-2i}}$$

$$=\sum_{i=0}^{m}{}_nC_{2i}(-1)^i 2^{2i}(\cos 2x)e^x+\sum_{i=0}^{m}{}_nC_{2i+1}(-1)^{i+1}2^{2i+1}(\sin 2x)e^x$$