Problem 3

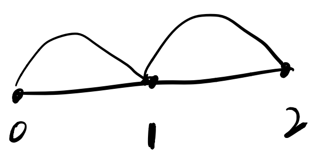
[redacted], find the area of the region bounded by the curves $y = (x^2 - x + 1)e^x$ and $y = xe^x$ for $0 \le x \le 2$.

1°. $f(x) = (x^2 - x + 1)e^x$, $g(x) = xe^x$.

$f(x) = g(x) \iff (x^2 - x + 1)\cancel{e^x} = x\cancel{e^x}$

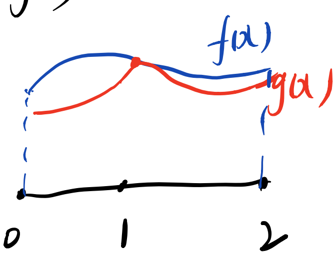
$x^2 - 2x + 1 = 0$

$\Rightarrow x = 1$



2°. $x \in [0,1]$, $x = 0$, $f(0) = 1 > 0 = g(0)$

$x \in [1,2]$, $x = 2$, $f(2) = 3e^2 > 2e^2 = g(2)$



$$\int_0^2 |f(x) - g(x)|\,dx = \int_0^2 f(x) - g(x)\,dx.$$

$$= \int_0^2 (x^2 - x + 1)e^x - xe^x\,dx$$

$$= \int_0^2 (x-1)^2 e^x\,dx$$

integration by parts

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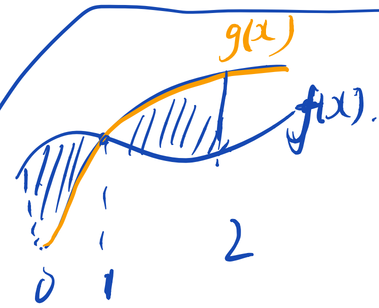


$$\int_0^2 |f(x) - g(x)|\,dx$$

$$= \int_0^1 f(x) - g(x)\,dx + \int_1^2 g(x) - f(x)\,dx$$

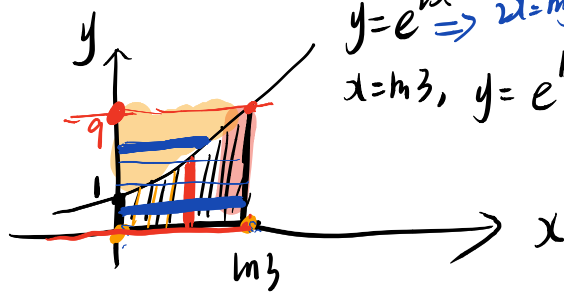
(c) Find the volume of the solid generated by rotating the region bounded by $y = e^{2x}$, x-axis, y-axis and $x = \ln 3$ about

(i) the x-axis for 1 complete revolution.

(ii) the y-axis for 1 complete revolution.

(iii) $y = -1$ for 1 complete revolution.

(iv) $x = -1$ for 1 complete revolution.



$y = e^{2x} \Rightarrow 2x = \ln y \Rightarrow x = \frac{1}{2}\ln y$

$x = \ln 3,\ y = e^{\ln 9} = 9$

(i) $V_x = \int_a^b \pi [f(x)]^2 \, dx$ $\quad \pi r^2$

$$\int_0^{\ln 3} \pi \cdot (e^{2x})^2 \, dx$$

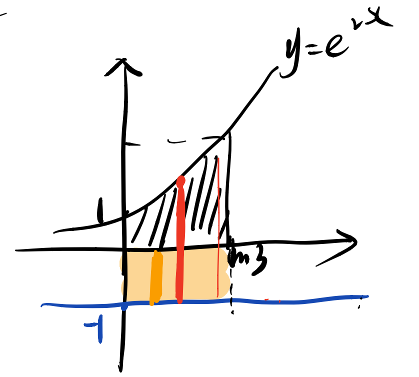
$y = e^{2x}$

$x = \frac{1}{2}\ln y$

(ii) $V_y = \int_c^d \pi \cdot [g(y)]^2 \, dy$

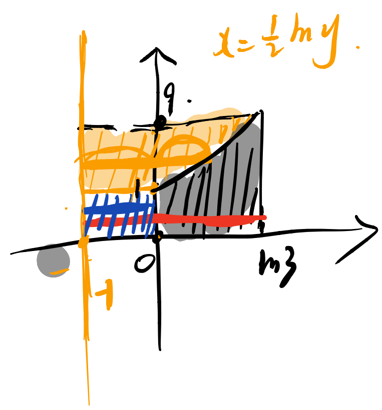
$$\int_0^9 \pi \cdot (\ln 3)^2 \, dy - \int_1^9 \pi \left(\frac{1}{2}\ln y\right)^2 dy$$

(iii) $\int_0^{\ln 3} \pi [e^{2x}+1]^2 \, dx - \int_0^{\ln 3} \pi \cdot 1^2 \, dx$



(iv) $\int_0^9 \pi \cdot [\ln 3 + 1]^2 \, dy - \int_1^9 \pi \left[\frac{1}{2}\ln y + 1\right]^2 dy$

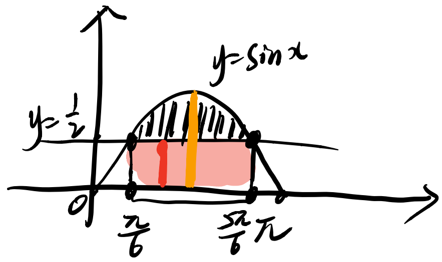
$$- \int_0^1 \pi \cdot 1^2 \, dy$$



(d) Find the volume of the solid generated by rotating the region above $y=\frac{1}{2}$ and below $y=\sin x$ for $0\le x\le \pi$ about

(i) the x-axis for 1 complete revolution.

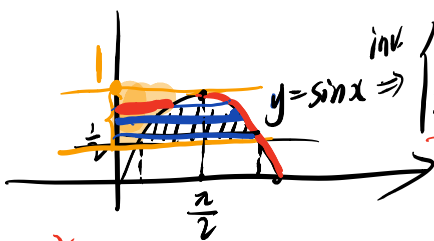
$\sin x=\frac{1}{2} \Rightarrow x=\frac{\pi}{6}$ or $\frac{5\pi}{6}$



$$\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \pi\cdot(\sin x)^2\,dx-\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \pi\cdot\left(\frac{1}{2}\right)^2 dx$$

(ii) the y-axis for 1 complete revolution.

(iii) the line $y=\frac{1}{2}$ for 1 complete revolution.



$y=\sin x \Rightarrow$ inv $\begin{cases} x\in[\frac{\pi}{2},\pi], & x=\pi-\sin^{-1}y \\ x\in[0,\frac{\pi}{2}], & x=\sin^{-1}y \end{cases}$

(ii). $$\int_{\frac{1}{2}}^{1} \pi\,[\pi-\sin^{-1}y]^2\,dy-\int_{\frac{1}{2}}^{1}\pi\cdot(\sin^{-1}y)^2\,dy$$

(shell method). $$V_y=\int_a^b 2\pi\,x\,f(x)\,dx.$$

$$V_y=\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} 2\pi x\cdot\sin x\,dx \quad \text{(shell method)}$$

(iii). $$\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \pi\left[\sin x-\frac{1}{2}\right]^2 dx$$