Tutorial 3 (with solution)

Relations

Q.1 Properties of Relation

Let $A = \{1, 2, 3, 4\}$. Define a relation R on A by

$$R = \{(1,2), (1,3), (2,3), (4,4)\}.$$

a) Is it reflexive?
b) Is it symmetric?
c) Is it antisymmetric?
d) Is it transitive?

Q.1 (solution)

a) No
b) No
c) Yes
d) Yes

Q.2 Properties of Relation

Let A be the set of all lines in the 2-dimensional plane. Let R be the relation on A defined by

$$l_1 R l_2 \text{ iff } l_1 \text{ is perpendicular to } l_2.$$

a) Is it reflexive?
b) Is it symmetric?
c) Is it antisymmetric?
d) Is it transitive?

Q.2 (solution)

a) No
b) Yes
c) No
d) No

Q.3 Equivalence Relation

Let S be the set of all digital logic circuit with two inputs and one output.

Let R be defined on S as follows:

$c_1 R c_2$ iff c_1 has the same input/output table as c_2.

a) *Is R* an equivalence relation ? Why?

1. Yes
2. No

Q.3 Equivalence Relation

Let S be the set of all digital logic circuit with two inputs and one output.

Let R be defined on S as follows:

$c_1 R c_2$ iff c_1 has the same input/output table as c_2.

b) How many distinct equivalence classes are there?

1. 2^4
2. 2^3
3. 2^2

Q.3 Equivalence Relation

Let S be the set of all digital logic circuit with two inputs and one output.

Let R be defined on S as follows:

$c_1 R c_2$ iff c_1 has the same input/output table as c_2.

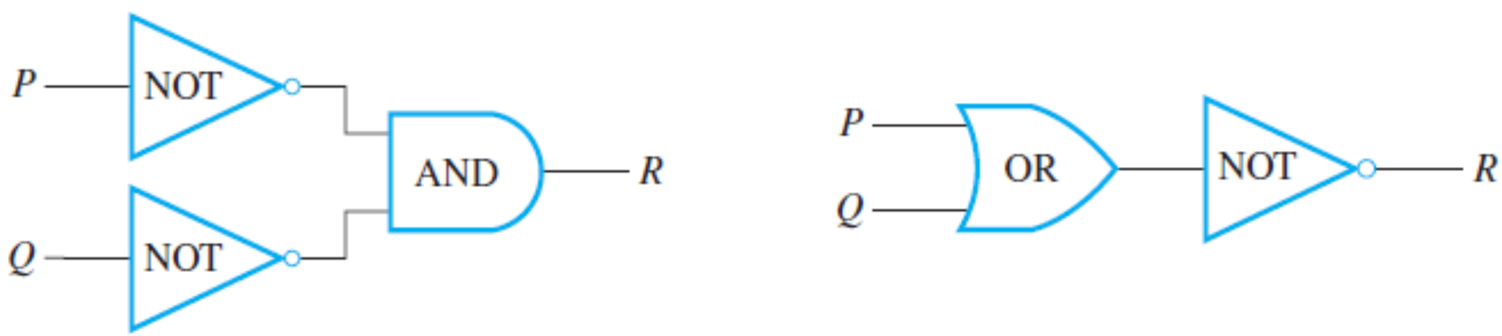
c) Find two different circuits that are in the same equivalence class.

Q.3 (solution)

a) Check the three defining conditions. Details omitted.

b) There are $2^4 = 16$ equivalence classes.

c) An example:



De Morgan's laws

Q.4 Partial Order

Let $A = \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\}$.
Consider the relation R on A defined as

$$xRy \text{ iff } x \text{ is a factor of } y.$$

a) Is R a partial order? Why?
   1. Yes
   2. No
b) List all maximal elements.
c) List all minimal elements.

Q.4 (solution)

a) Yes. Check the three defining conditions of partial order. Details omitted.
b) Maximal elements: 8, 9, 10, 11, 12, 13, 14, 15
c) Minimal elements: 2, 3, 5, 7, 11, 13

Q.5 Congruence

Let $a \equiv b \pmod{n}$ and $c \equiv d \pmod{n}$.

Is $a + c \equiv b + d \pmod{n}$ right? Prove or disprove it.

1. Yes
2. No

Q.5 (solution)

- By the definition of congruences,

$$a = kn + b \text{ for some integer } k.$$

$$c = hn + d \text{ for some integer } h.$$

- Adding the two equations,

$$\begin{aligned} a + c &= kn + hn + b + d \\ &= (k + h)n + (b + d). \end{aligned}$$

- Since $(k + h)$ is an integer,

$$a + c \equiv b + d \pmod{n}.$$

Q.E.D.

Q.5 (a question from one student)

Is $a + c(mod\ n) \equiv a(\text{mod } n)$ + c(mod n) right?

Yes. The following is the proof.

- By the definition of congruences,($a \equiv b$ (mod n) and $c \equiv d$ (mod n))

$$a = kn + b \text{ for some integer } k.$$

$$c = hn + d \text{ for some integer } h.$$

- Then, $\quad a + c = kn + hn + b + d$

$$= (k + h)n + (b + d).$$

$$\equiv b + d\ (\text{mod } n).$$

Q.5 (a question from one student)

- While a(mod n) + c(mod n)$= b + d \equiv b + d \pmod{n}$. Thus, $a + c(mod\ n) \equiv a(\text{mod}\ n)$ + c(mod n) .

Q.E.D.

More about Module n

“A fundamental fact about congruence modulo *n* is that if you first perform an addition, subtraction, or multiplication on integers and then reduce the result modulo *n*, you will obtain the same answer as if you had first reduced each of the numbers modulo *n*, per- formed the operation, and then reduced the result modulo *n*. “ ------Chapter 8, S. S. Epp, *Discrete Mathematics with Applications*, 4th ed., Brooks Cole, 2010.