EE3009 Tutorial 12 (Solution)

Problem 1

a) $H(X)$
$= -0.49\log_2 0.49 - 0.26\log_2 0.26 - 0.12\log_2 0.12 - 2(0.04\log_2 0.04) - 0.03\log_2 0.03 - 0.02\log_2 0.02$
$= 2.013$

b) Code length = 3, since $2^3 > 7 > 2^2$.

c) The Shannon-Fanos code is shown below:

A	B	C	D	E	F	G
0	10	110	11100	11101	11110	11111

Remark: The answer is not unique, since it doesn't matter which branch is assigned 0 and which one is assigned 1. On the other hand, the codeword length for each symbol should be the same as the code shown in the above table.

L = 0.49 (1) + 0.26 (2) + 0.12 (3) + [0.04+0.04+0.03+0.02]*5 = 2.02

d) The Huffman code is shown below:

A	B	C	D	E	F	G
1	01	001	00011	00010	00001	00000

L = 0.49 (1) + 0.26 (2) + 0.12 (3) + [0.04+0.04+0.03+0.02]*5 = 2.02

e) The entropy calculated in (a) is a lower bound. The average code length of Shannon-Fanos code and Huffman code are the same and very close to the entropy in this example. If a fixed-length code is used, then almost one more bit is used for each symbol on average.

Problem 2

a) No, it can't. The code is not prefix-free.
b) Yes, it can.
c) No, it can't. The code is not optimal, as 110 could be replaced by 11 and the code remains prefix-free.
d) No, it can't. The code is not optimal, as the code can be replaced by {0, 1}.

Problem 3

Max. pulse rate = 2 M pulses/sec.
Each pulse represents 3 bits.
Bit rate = 6 Mbps.

Problem 4

a) $10 \log_{10} 2 = 3$ dB
b) $10 \log_{10} 2^n = 3n$ dB

Problem 5

Case 1:
SNR (dB) = 20 • SNR = $10^{20/10} = 100$
Capacity = (1 MHz) $\log_2(1 + 100) = 6.66$ Mbps

Case 2:
SNR (dB) = 40 • SNR = $10^{40/10} = 10000$
Capacity = (1 MHz) $\log_2(1 + 10000) = 13.29$ Mbps

Problem 6

Let P be the signal power and N be the noise power.
$10 \log_{10} (P / N) = 20$ implies that $P / N = 100$.

After 4 repeaters, the SNR is $P / (4N) = 25$.
This is equal to $10 \log_{10} (25) = 13.98$ dB.