CITY UNIVERSITY OF HONG KONG

Course code and title	:	MA1201 Calculus and Basic Linear Algebra II
Session	:	Semester B, 2016/2017
Time allowed	:	Three hours

This paper has **THREE** pages (including this cover page and the attached table).

Instructions to candidates:

1. Attempt all **SIX** questions in this paper.
2. Start each question on a new page.
3. Show all steps in details to get full credits.

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*This is a **closed-book** examination.*

Candidates are allowed to use the following materials/aids:

Non-programmable calculators

Materials/aids other than those stated above are not permitted. Candidates will be subject to disciplinary action if any unauthorised materials or aids are found on them.

NOT TO BE TAKEN AWAY

Question 1

Compute the following elementary integrals.

(a) $\int \dfrac{2x+\sqrt{x}-\dfrac{3}{x^{2/3}}}{\sqrt[3]{x}}\,dx$ [5]

(b) $\int \dfrac{x^2+2}{x+2}\,dx$ [5]

(c) $\int_0^{\pi/3} \sin^3 x\,dx$ [5]

Question 2

Evaluate the following indefinite integrals.

(a) $\int \dfrac{1}{\left(x^2-4\right)^{3/2}}\,dx$ [6]

(b) $\int \dfrac{1}{x^2}\ln x\,dx$ [5]

(c) $\int \dfrac{9x^2}{(x-2)(x^2+2x+10)}\,dx$ [8]

Question 3

(a) Find the volume of the solid generated by revolving the region bounded by the straight lines $2y=x+4$, $y=x$ and $x=0$ about the y-axis. [8]

(b) Find the arc length of the parametric curve $x=\cos t+t\sin t$, $y=\sin t-t\cos t$, $0\le t\le 2\pi$. [8]

Question 4

(a) Find the volume of the parallelepiped with three adjacent edges $\vec{a}=\vec{i}-2\vec{j}+3\vec{k}$, $\vec{b}=-2\vec{i}+5\vec{k}$ and $\vec{c}=3\vec{j}-4\vec{k}$. [5]

(b) Determine the shortest distance from $P(1,2,-4)$ to the plane containing $A(0,1,-2)$, $B(2,-3,1)$ and $C(3,-2,0)$. [1 0]

Question 5

(a) Simplify the complex number $\dfrac{1+2i}{3-4i}-\dfrac{3-2i}{5i}$ into the Cartesian form. [5]

(b) Solve $iz^3=\sqrt{3}-i$ and list all solutions in Euler's form with principal arguments. [10]

Question 6

(a) Find the inverse of the matrix $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$. [5]

(b) Given a system of linear equations as follows.

$$\begin{aligned} 2x - y - 5w &= 1 \\ x - y + 2z &= 2 \\ -3x + y + 2z + 10w &= 0 \end{aligned}$$

(i) Solve the above linear system by the Gaussian elimination. [1 0]

(ii) Write down the corresponding homogeneous system explicitly and provide a largest possible set of linearly independent solution vectors to the homogeneous system from (b)(i) without resolving it. [5]

- END -

Brief Table of Integrals

$\int x^p\,dx = \dfrac{x^{p+1}}{p+1} + C, \quad p \neq -1$	$\int \dfrac{1}{x}\,dx = \ln\lvert x\rvert + C$
$\int e^x\,dx = e^x + C$	$\int \sec^3 x\,dx = \dfrac{1}{2}\sec x \tan x + \dfrac{1}{2}\ln\lvert \sec x + \tan x\rvert + C$
$\int \sin x\,dx = -\cos x + C$	$\int \cos x\,dx = \sin x + C$
$\int \sec^2 x\,dx = \tan x + C$	$\int \csc^2 x\,dx = -\cot x + C$
$\int \sec x \tan x\,dx = \sec x + C$	$\int \csc x \cot x\,dx = -\csc x + C$
$\int \sec x\,dx = \ln\lvert \sec x + \tan x\rvert + C$	$\int \csc x\,dx = -\ln\lvert \csc x + \cot x\rvert + C$
$\int \dfrac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1} x + C$	$\int \dfrac{1}{1+x^2}\,dx = \tan^{-1} x + C$