1 Maxima, Minima and Saddle points

Single variable Case: Given a single variable function

$$y = f(x).$$

- Stationary (Critical) points:

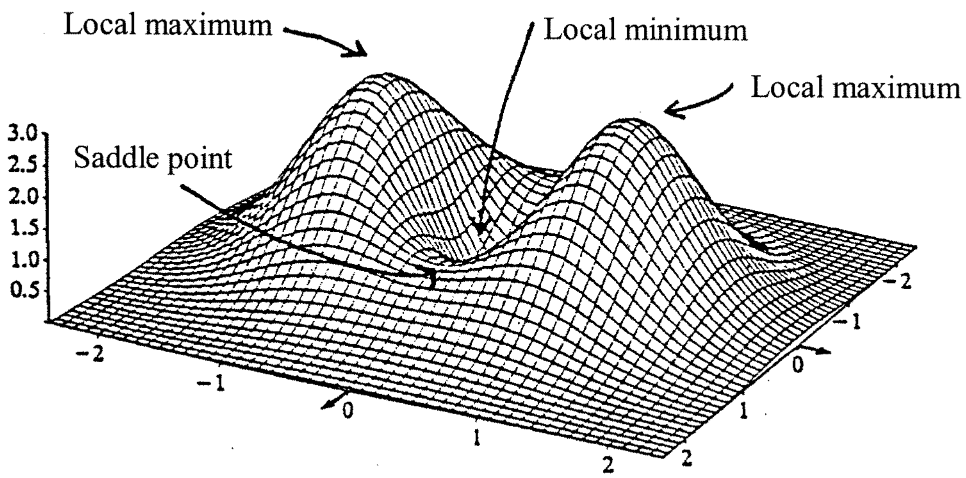
- Loc max, Loc min:

- Second order derivative test:

- Global(Absolute) Max, Global(Absolute) Min:

Two-variable Case: Given a two-variable function

$$z = f(x, y).$$



- **Stationary Points:**

- **Loc Maxima, Loc Minima:**

- **Saddle points:**

Question: How to find local maxma and local minima?

Question: Determine the nature of stationary points?

Example $z = y^2 - x^2$. Find extreme values of z.

Example $z = xy - x^2 - y^2 - 2x - 2y + 4$. Find extreme values of z.

we say f has a local maximum at $(a, b$ if for all points (x, y) near (a, b), $f(a, b) \geq f(x, y)$.

we say f has a local minimum at $(a, b$ if for all points (x, y) near (a, b), $f(a, b) \leq f(x, y)$.

For a given region Ω, f has global maximum at (a, b) if for any $(x, y) \in \Omega$, $f(a, b) \geq f(x, y)$.

For a given region Ω, f has global minimum at (a, b) if for any $(x, y) \in \Omega$, $f(a, b) \leq f(x, y)$.

How to find local maxmimum and local minimum for a given function?

Hence, if f takes local maximum or local minimum at (a, b), then (a, b) must be a stationary point. Be aware that the converse may not be true.

Definition A point (a, b) is called a saddle point if there exists two path l_1 and l_2 passing through (a, b) such that

$f(x, y)$ along l_1 has local maximum at (a, b),

$f(x, y)$ along l_1 has local minimum at (a, b).