SDSC 2102 Statistical Methods and Data Analysis - Assignment 1

Question 1a:

$$P[X \geq 2] = \frac{4-2}{10} + \frac{4-3}{10}$$
$$= 0.3$$

Question 1b:

$$P[X < 4] = \frac{4-0}{10} + \frac{4-1}{10} + \frac{4-2}{10} + \frac{4-3}{10}$$
$$= 1$$

Question 1c:

$$P[X\ is\ odd] = \frac{4-1}{10} + \frac{4-3}{10}$$
$$= 0.4$$

Question 2a:

$$F(x)|_{x<0} = 0$$
$$F(x)|_{0\leq x\leq 0.5} = \int_0^x 4t dt$$
$$= 2x^2$$
$$F(x)|_{0.5<x\leq 1} = \int_{0.5}^x (4-4t) dt$$
$$= -2x^2 + 4x - \frac{3}{2}$$
$$F(x)|_{x>1} = 1$$

$$\therefore F(x) = \begin{cases} 0, & x < 0 \\ 2x^2, & 0 \leq x \leq 0.5 \\ -2x^2 + 4x - \frac{3}{2}, & 0.5 < x \leq 1 \\ 1, & x > 1 \end{cases}$$

Question 2b:

$$P(0.2 < X \leq 0.6) = F(0.6) - F(0.2)$$
$$= \left(-2 \times 0.6^2 + 4 \times 0.6 - \frac{3}{2}\right) - (2 \times 0.2^2)$$
$$= 0.18 - 0.08$$
$$= 0.1$$

Question 3a:

$$f(x)|_{x\geq 10} = \frac{d\left(1-\left(\frac{10}{x}\right)^{2.5}\right)}{dx}$$
$$= \frac{5\times 10^{\frac{5}{2}}}{2\times x^{\frac{7}{2}}}$$
$$f(x) = \begin{cases} \frac{5\times 10^{\frac{5}{2}}}{2\times x^{\frac{7}{2}}}, & x \geq 10 \\ 0, & otherwise \end{cases}$$

Question 3b:

$$P(5 < X \leq 15) = 0 + P(10 \leq X \leq 15)$$
$$= F(15) - F(10)$$
$$= 0.6371 - 0$$
$$= 0.6371$$

Question 3c:

$$P(X > 20) = 1 - F(20)$$
$$= 1 - 0.8232$$
$$= 0.1768$$

Question 4a:

$$probability = \binom{5}{2} \times 0.55^2 \times 0.45^3$$
$$= 0.2757$$

Question 4b:

$$probability = 1 - 0.45^5 - \binom{5}{1} \times 0.55^1 \times 0.45^4$$
$$= 0.8688$$

Question 5a:

$$probability = 0.9^2 \times 0.1$$
$$= 0.081$$

Question 5b:

$$probability = \binom{4}{2} \times 0.9^2 \times 0.1^3$$
$$= 0.00486$$

Question 6a:

$$\begin{aligned}\lambda &= \frac{1}{20} \\ &= 0.05 \\ \sigma &= \sqrt{\frac{1}{0.05^2}} \\ &= 20\end{aligned}$$

Question 6b:

$$\begin{aligned}P(X > 80) &= 1 - F(80) \\ &= 1 - (1 - e^{-0.05 \times 80}) \\ &= 0.01832\end{aligned}$$

Question 7a:

$$\begin{aligned}E(X) &= \int_1^2 x \times 2\left(1 - \frac{1}{x^2}\right) dx \\ &= 3 - 2ln(2) \\ &= 1.614\end{aligned}$$

Question 7b:

$$\begin{aligned}Var(X) &= E(X^2) - E(X)^2 \\ &= \int_1^2 x^2 \times 2\left(1 - \frac{1}{x^2}\right) dx - \big(3 - 2ln(2)\big)^2 \\ &= \frac{8}{3} - \big(3 - 2ln(2)\big)^2 \\ &= 0.06262\end{aligned}$$

Question 8a:

$$\begin{aligned}P_{first}\left(\frac{2.9 - 3}{0.1} \le Z \le \frac{3.1 - 3}{0.1}\right) &= \Phi(1) - \Phi(-1) \\ &= 0.84134 - 0.15866 \\ &= 0.68268 \\ P_{second}\left(\frac{2.9 - 3.04}{0.02} \le Z \le \frac{3.1 - 3.04}{0.02}\right) &= \Phi(3) - \Phi(-7) \\ &= 0.99865 - 1.2881 \times 10^{-1} \\ &= 0.99865\end{aligned}$$

$$\therefore \textit{The second machine is more likely.}$$

Question 8b:

$$\Phi\left(\frac{d}{\sigma}\right) = \frac{1 + 0.9}{2}$$
$$\Phi\left(\frac{d}{0.1}\right) = 0.95$$
$$\frac{d}{0.1} = 1.645$$
$$d = 0.1645$$

$$\therefore \textit{acceptable range is } (2.836, 3.165)$$