Exercise in Lecture 6

Suppose $T(2^k) \leq T(2^{k-1})$+C for k= 1, 2, …, and T(1)=C.
Prove that $T(2^k) = O(k)$

$$\begin{aligned} T(2^k) &\leq T(2^{k-1}) + C \\ &\leq T(2^{k-2}) + 2C \\ &\leq T(2^{k-3}) + 3C \\ &\leq T(2^{k-k}) + kC \\ &= C + kC \\ T(2^k) &= O(C + kC) \\ &= O(k) \end{aligned}$$