

Q1. Suppose you are given that $\begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}$ is an eigenvector of

$$A = \begin{bmatrix} a & -2 & 0 \\ -2 & 1 & 0 \\ b & 0 & 3 \end{bmatrix}.$$

- (a) [10 marks] Find the constants a and b in A ;
- (b) [10 marks] Find all eigenvalues and corresponding linear independent eigenvectors of A ;
- (c) [10 marks] Orthogonally diagonalize A if possible; (Hint: constructing an orthogonal matrix P such that $A = PDP^{-1}$ with diagonal matrix D)
- (d) [10 marks] Determine A is positive or negative definite or semidefinite, or indefinite, and explain.
- (e) [10 marks] Compute A^9 . [Hint: using its diagonalization].

Q2. Consider the function

$$f(x, y) = x^3 + 2xy - 3y^3.$$

- (a) [10 marks] Find the first and second order partial derivatives of f at $P(1, 2)$;
- (b) [10 marks] Find the **linear** and **quadratic** approximation of f at $P(1, 2)$;
- (c) [10 marks] Find the stationary points of f and determine their nature.
- (d) [10 marks] Find the gradient vector of f at $(1, 2)$;
- (e) [10 marks] Determine the direction \vec{u} along which f decreases most rapidly at $(1, 2)$;

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