EE2302 Foundations of Information and Data Engineering

Assignment 3 (Solution)

1.

a) We check the three conditions:

   i. If $m = 0$, then $m \times m = 0$. If $m \neq 0$, then $m \times m > 0$. Therefore, R is **reflexive**.

   ii. Suppose mRn. We only need to consider the case $m \neq n$. (The case where $m = n$ is the same as reflexivity.) Then, $mn > 0$, which implies that $nm > 0$. Therefore, R is **symmetric**.

   iii. Suppose mRn and nRp. We only need to consider the case where m, n, and p are distinct. (The other cases are the same as reflexivity or symmetry.) Then, $mn > 0$ and $np > 0$. Multiplying these two inequalities gives $mn^2p > 0$. Since $n \neq 0$ (for otherwise we cannot have $mn > 0$), we have $mp > 0$. Therefore, R is **transitive**.

b) There are three equivalence classes. They are

   i. $[1] = \{x \in Z \mid x > 0\}$,

   ii. $[-1] = \{x \in Z \mid x < 0\}$, and

   iii. $[0] = \{0\}$.

2. By the definition of congruences,

$a = kn + b$ for some integer k.

$c = hn + d$ for some integer h.

Multiplying them together, we obtain

$$\begin{aligned} ac &= (kn + b)(hn + d) \\ &= hkn^2 + hbn + kdn + bd \\ &= (hkn + hb + kd)n + bd \end{aligned}$$

Since $(hkn + hb + kd)$ is an integer, we have

$ac \equiv bd \pmod{n}$.

3. $R_1 = \{(a, a), (b, b)\}$, $R_2 = \{(a, a), (b, b), (a, b)\}$, $R_3 = \{(a, a), (b, b), (b, a)\}$.

4.

a) S is not an equivalence relation. It is not symmetric, since $x \geq y$ does not imply $y \geq x$.

b) T is an equivalence relation.

(reflexive): $x - x = 0$ is an integer

(symmetric): if $x - y$ is an integer, then $y - x = -(x - y)$ is also an integer.

(transitive): if $x - y$ and $y - z$ are integers, then $x - z = (x - y) - (y - z)$, which is a difference of two integers, is also an integer.

c) There is an equivalence class for each real number x, where $0 \leq x < 1$.

(Note: The answer is not unique. For example, $-0.5 \leq x < 0.5$ is also correct.)