Solution of Assignment 3

Q1.

(a)

$H_0: \mu = 90$ vs. $H_1: \mu \neq 90$

Given $\mu_0 = 90, n = 16, \sigma = \sqrt{0.64} = 0.8, \bar{x} = 89.4 \Rightarrow z\ test$

Given $\alpha = 0.02$, $Z_{\alpha/2} = 2.326$

Method 1:

Find the confidence interval:

$\bar{x} - Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} = 89.4 - 2.326 \times \frac{0.8}{4} = 88.93$

$$\bar{x} + Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} = 89.4 + 2.326 \times \frac{0.8}{4} = 89.87$$

So the 98% confidence interval is $(88.93,\ 89.87)$. Since 90 is not in the confidence interval, we reject H_0 and conclude that the printer prints crooked.

Method 2:

Calculate the test statistic:

$$Z_0 = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{89.4 - 90}{0.8/4} = -3.0$$

Since $|Z_0| = 3 > Z_{\alpha/2}$, we reject H_0 and conclude that the printer prints crooked.

(b)

To fail to reject H_0,

$Z_0 = \frac{\bar{x}-\mu_0}{\sigma/\sqrt{n}} = Z_{\alpha/2} \Rightarrow \mu_0 = \bar{x} - Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} = 89.4 - 2.326 \times \frac{0.8}{4} = 88.93$, which is the lower bound of the 98% confidence interval.

Q2.

$H_0: \mu = -20$ vs. $H_1: \mu < -20$

Given $\mu_0 = -20, n = 25, s = 1, \bar{x} = -20.7 \Rightarrow one - sided\ t\ test$

Given $\alpha = 0.01$, $t_{\alpha,n-1} = 2.492$

Method 1:

Find the confidence interval:

$$\bar{x} + t_{\alpha,n-1} \frac{s}{\sqrt{n}} = -20.7 + 2.492 \times \frac{1}{5} = -20.2016$$

So the 99% upper bound confidence interval is $(-\infty, -20.2016)$. Since –20 is not in the confidence interval, we reject H_0 and conclude that the freezer can do the job.

Method 2:

Calculate the test statistic:

$$t_0 = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{-20.7 - (-20)}{1/5} = -3.5$$

Since $t_0 < -t_{\alpha,n-1}$, we reject H_0 and conclude that the freezer can do the job.

Q3.

(a)

Calculate confidence interval for Student 1:

First calculate the sample mean and sample variance:

Student 1: $\bar{x}_1 = 97.4$, $s_1^2 = 78.8$, $n_1 = 5$

Given $\alpha = 0.10, t_{\alpha/2,n_1-1} = 2.132$

$$\bar{x} - t_{\frac{\alpha}{2},n_1-1}\frac{s_1}{\sqrt{n}} = 88.93$$

$$\bar{x} + t_{\frac{\alpha}{2},n_1-1}\frac{s_1}{\sqrt{n}} = 105.86$$

So the 90% confidence interval for Student 1 is $(88.93,\ 105.86)$.

Calculate confidence interval for Student 2:

First calculate the sample mean and sample variance:

Student 2: $\bar{x}_2 = 110$, $s_2^2 = 913.3$, $n_1 = 7$

Given $\alpha = 0.10, t_{\alpha/2,n_2-1} = 1.943$

$$\bar{x} - t_{\frac{\alpha}{2},n_2-1}\frac{s_1}{\sqrt{n}} = 87.8059$$

$$\bar{x} + t_{\frac{\alpha}{2},n_2-1}\frac{s_1}{\sqrt{n}} = 132.1941$$

So the 90% confidence interval for Student 2 is $(87.8059, 132.1941)$.

(b)

$H_0: \mu_1 = \mu_2$ vs. $H_1: \mu_1 < \mu_2$

Unknow population variance and equal variance$\Rightarrow$ $one - sided\ t\ test$

Given

$\bar{x}_1 = 97.4$, $s_1^2 = 78.8$, $n_1 = 5$

$\bar{x}_2 = 110$, $s_2^2 = 913.3$, $n_1 = 7$

$$S_p^2 = \frac{(n_1 - 1)\cdot s_1^2 + (n_2 - 1)\cdot s_2^2}{n_1 + n_2 - 2} = 579.5 \Rightarrow S_p = 24.07$$

$\alpha = 0.1$, $t_{\alpha,n_1+n_2-2} = t_{0.1,10} = 1.372$

Method 1

Find the 90% confidence interval for $\mu_1 - \mu_2$:

$$\bar{x}_1 - \bar{x}_2 + t_{\alpha,n_1+n_2-2}s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}} = 6.74$$

So the confidence interval is $(-\infty, 6.74)$.

Since 0 is in the confidence interval, we fail to reject H_0. We conclude that there is no significant evidence that the mean number of customers under the second student's design is higher than that under the first student's design.

Method 2

Calculate the test statistic

$$t_0 = \frac{\bar{x}_1 - \bar{x}_2}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = -0.894$$

Since $t_0 = -0.894 > -t_{\alpha, n_1+n_2-2} = -1.372$, we fail to reject H_0. We conclude that there is no significant evidence that the mean number of customers under the second student's design is higher than that under the first student's design.