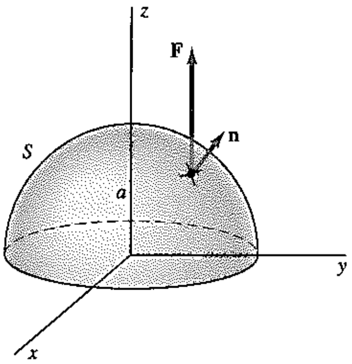
MA2001 Assignment for Chapter 2 Vector Integral Calculus

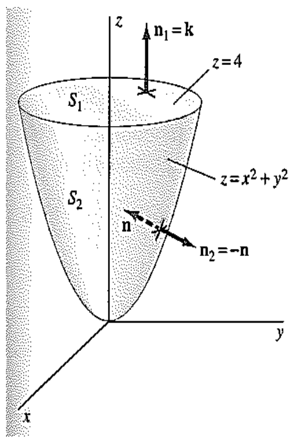
1. Find the work done in moving a particle from $(0,0)$ to $(1,1)$ in the force field $\vec{F}=\left(xy+2y^2\right)\vec{i}+\left(3x^2+y\right)\vec{j}$ along the paths, (a) $y=x^2$; (b) $y=x$; (c) the y-axis and then $y=1$. What work is done if the particle moves from $(0,0)$ to $(1,1)$ along path (b) and returns to the origin along path (a)?

2. Prove that the vector field $\vec{F}=\left(3x^2-y\right)\vec{i}+\left(2yz^2-x\right)\vec{j}+2y^2z\vec{k}$ is conservative, but not solenoidal. Hence find a scalar function $f(x,y,z)$ such that $F=\nabla f$ and evaluate $\int_C \vec{F}\cdot d\vec{r}$ along any curve C joining the point $(0,0,0)$ to the point $(1,2,3)$.

3. Calculate the flux $\iint_S \vec{f}\cdot\vec{n}dS$ where $\vec{f}=v_0\vec{k}$ and S is the hemispherical surface of radius a with equation $z=\sqrt{a^2-x^2-y^2}$ and with outer unit normal vector $\vec{n}$.



4. Find the flux of the vector field $\vec{f}=x\vec{i}+y\vec{j}+3\vec{k}$ out of S, where S is the closed surface of the region bounded by the paraboloid $z=x^2+y^2$ and the plane $z=4$. (i.e. to find $\iint_S \vec{F}\cdot d\vec{S}$.)



5. Consider the magnetic field $\vec{B}=(x+2)\vec{i}+(1-3y)\vec{j}+2z\vec{k}$ and evaluate the total magnetic flux through each of the faces of the cube bounded by the planes $x=0,\ x=1,\ y=0,\ y=1,\ z=0,\ z=1$. Check that your result is consistent with the divergence theorem.

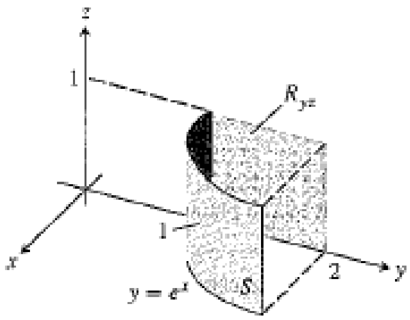
6. Use the divergence theorem to show that $\iint_S \left(x^2+y+z\right) dS = \frac{4}{3}\pi$ where W is the solid ball $x^2+y^2+z^2 \le 1$ and S is its boundary.

7. Verify the divergence theorem for the vector field $\vec{F} = (8+z)\,\vec{j} + z^2\vec{k}$ and the region bounded by the planes $z=0,\ z=6,\ x=2,\ y=0$ and the surface $y^2 = 8x$ in the first octant.

8. Verify Stokes's theorem for the vector field $\vec{F} = (x-y)\,\vec{i} + 2z\,\vec{j} + x^2\vec{k}$ where S is the cone $z = \sqrt{x^2+y^2}$ for $x^2+y^2 \le 4$.

9. Verify Stokes's theorem by evaluating both sides of $\iint_S \nabla \times \vec{F} \cdot \vec{n} dS = \oint_C \vec{F} \cdot d\vec{r}$ for the vector field $\vec{F} = (2x-y)\vec{i} - yz^2\vec{j} - y^2z\vec{k}$ where S is the curved surface of the hemisphere $x^2+y^2+z^2 = 16,\ z \le 0$ and C is its boundary.

10. Let S be the portion of the cylinder $y = e^x$ in the first octant that projects parallel to the x-axis onto the rectangle $R_{yz} : 1 \le y \le 2,\ 0 \le z \le 1$ in the yz-plane. Let $\vec{n}$ be the unit vector normal to S that points away from the yz-plane. Find the flux of the field $\vec{F}(x,y,z) = -2\vec{i} + 2y\vec{j} + z\vec{k}$ across S in the direction of $\vec{n}$.



11. Let $\vec{F} = \left(y^2-z^2\right)\vec{i} + \left(z^2-x^2\right)\vec{j} + \left(x^2-y^2\right)\vec{k}$. Use Stoke's Theorem to calculate $\oint_C \vec{F} \cdot d\vec{r}$ where C is the path which is the intersection of the plane $x+y+z=2$ and the faces of a parallelepiped bounded by the planes $x=0,\ x=1,\ y=0,\ y=1,\ z=0,\ z=2$. The direction of the path C is anticlockwise when looking from the positive direction of x-axis.

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