Hamiltonian Volterra

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\begin{aligned}
\dot{x}_j &= \epsilon_j x_j + \frac{1}{\beta_j}\sum_k a_{jk} x_j x_k \\
\dot{x}_j &= \epsilon_j x_j + \sum_k a_{jk} x_j x_k \\
a_{jk} &= -a_{kj} \\
Q_j &\equiv \int_0^t x_j(\tau) d\tau \\
\ddot{Q}_j &= \epsilon_j Q_j + \sum_{k=1}^n a_{jk} \dot{Q}_j \dot{Q}_k \\
H &\equiv \sum \epsilon_j Q_j - \dot{Q}_j \\
\dot{H} &= 0 \\
P_j &\equiv \log \dot{Q}_j - \frac{1}{2}\sum a_{jk} Q_k \\
H &= \sum \epsilon_j Q_j - \sum e^{P_j + \frac{1}{2}\sum a_{jk} Q_k} \\
\dot{P}_j &= \frac{\partial H}{\partial Q_j} \\
\dot{Q}_j &= -\frac{\partial H}{\partial P_j} \\
I_j &\equiv P_j - \frac{1}{2}\sum a_{jk} Q_k - \epsilon_j t \\
\dot{I}_j &= 0 \\
\{I_j, I_k\} &= a_{jk}
\end{aligned}
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http://www.math.illinois.edu/~ruiloja/Meus-papers/HTML/equadiff.pdf