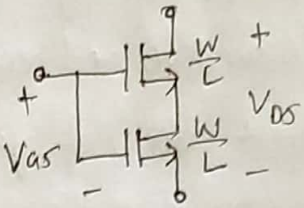
Assignment-1

① Based on Problem 2.16 of the Razavi's book:

Consider the structure shown in the following figure. Determine I_D as a function of V_{GS} and V_{DS} and prove that the structure can be viewed as a single transistor having an aspect ratio $W/(2L)$. Assume $\lambda = \gamma = 0$



case-1 M_1: Triode , M_2: Triode

$V_{OD1} = V_{GS} - V_{th}$,

$V_{OD2} = V_{GS} - V_x - V_{th}$

$V_{DS1} = V_x$, $V_{DS2} = V_{DS} - V_x$

$$I_{D1} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[2(V_{GS}-V_{th})V_x - V_x^2\right] \quad ①$$

$$I_{D2} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[2(V_{GS}-V_{th}-V_x)(V_{DS}-V_x) - (V_{DS}-V_x)^2\right]$$

$$I_{D1} = I_{D2}$$

$$2(V_{GS}-V_{th})V_x - V_x^2 = 2(V_{GS}-V_{th})V_{DS} + 2V_x^2 - 2V_x(V_{GS}-V_{th}) - 2(\cancel{V_x V_{DS}}) - V_{DS}^2 - V_x^2 + 2\cancel{V_x V_{DS}}$$

$$= 2\left[2(V_{GS}-V_{th})V_x - V_x^2\right] = 2(V_{GS}-V_{th})V_{DS} - V_{DS}^2 \quad ②$$

from eq ① & ②

$$I_{D1} = I_{D2} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\times\frac{1}{2}\left[2(V_{GS}-V_{th})V_{DS} - V_{DS}^2\right] \Rightarrow \frac{W}{2L} \text{ is in triode}$$

case-2 M_1: Triode , M_2: saturation

$$I_{D1} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[2(V_{GS}-V_{th})V_x - V_x^2\right] - ③$$

$$I_{D2} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_x-V_t)^2$$

$$I_{D1} = I_{D2}$$

$$V_x^2 - 2V_x(V_{GS}-V_{th}) + (V_{GS}-V_{th})^2 = 2(V_{GS}-V_{th})V_x - V_x^2$$

$$(V_{GS}-V_{th})^2 = 2\left[2(V_{GS}-V_{th})V_x - V_x^2\right] - ④$$

from eq ③ & ④

$I_{D_1} = I_{D_2} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \times \frac{1}{2} (V_{GS} - V_{th})^2 \Rightarrow \frac{W}{2L}$ is in saturation

as $V_{GS} - V_{th} \geq 0 \Rightarrow V_{GS} - V_x - V_{th} > 0 \Rightarrow V_{GS} - V_{th} > V_x$

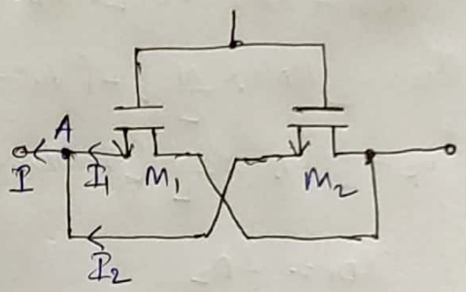
$V_{GS_1} - V_{th} > V_{DS_1} \Rightarrow M_1$ is in triode

It means the equivalent transistor is in saturation., if M_2 is in saturation and vice versa.

② Repeat part ① of question for the following structure (assuming both transistors have the same aspect ratio W/L) and show that the structure can be viewed as a single transistor having an aspect ratio of $2W/L$.)

At point A

$I_{D_1} + I_{D_2} = I$

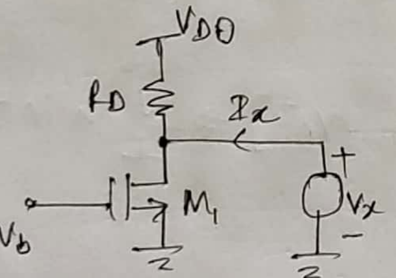


$\Rightarrow \frac{1}{2} \mu_n C_{ox} \frac{2W}{L} (V_{GS} - V_{th})^2$

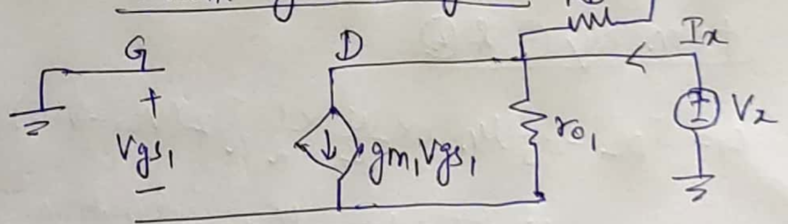
$\Rightarrow$ which equals the transistors to be a single transistor of 2W width

③ Calculate the output resistance (V_x/I_x) of following circuits. Assume $\lambda \neq 0$ and $\gamma \neq 0$

ⓐ



Small signal Analysis



$V_{gs_1} = 0$

Applying KCL in output node.

$I_x = \frac{V_x}{r_{o1}} + \frac{V_x}{R_D}$

$\boxed{\frac{V_x}{I_x} = \frac{r_{o1} R_D}{r_{o1} + R_D}}$

or $\boxed{\frac{V_x}{I_x} = r_{o1} \| R_D}$

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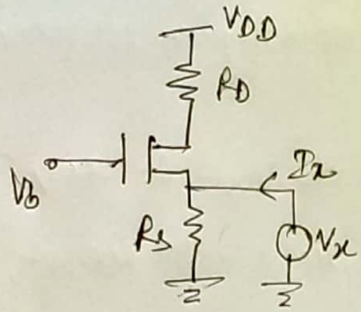
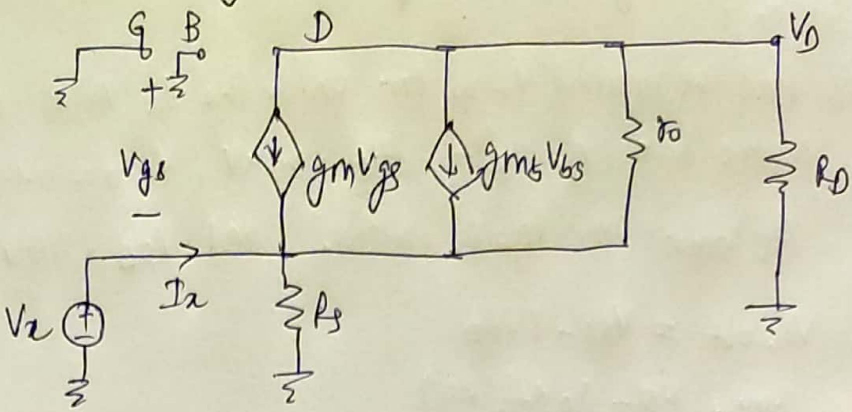
(b)



Small signal Analysis



$V_{gs} = -V_x$

$g_m V_{gs} = -g_m V_x$

$g_{mb} V_{bs} = g_{mb}(0 - V_x) = -g_{mb} V_x$

Apply KCL at drain terminal:

$$\frac{V_D}{R_D} + \frac{V_D - V_x}{r_o} + g_m V_{gs} + g_{mb} V_{bs} = 0$$

$$\frac{V_D}{R_D} + \frac{V_D - V_x}{r_o} - g_m V_x - g_{mb} V_x = 0$$

$$V_D\left[\frac{1}{R_D} + \frac{1}{r_o}\right] = V_x\left[\frac{1}{r_o} + g_m + g_{mb}\right] \quad \text{—(1)}$$

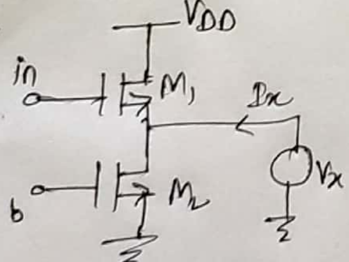
Apply KCL at source terminal

$$I_x = \frac{V_x}{R_s} + \frac{V_D}{R_D} = \frac{V_x}{R_s} + \frac{1}{R_D}\left[\frac{\frac{1}{r_o} + g_m + g_{mb}}{\frac{1}{R_D} + \frac{1}{r_o}}\right] V_x$$

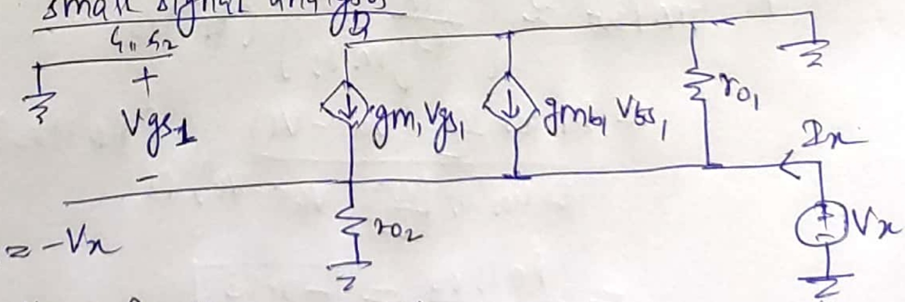
$$I_x = V_x\left[\frac{1}{R_s} + \frac{\frac{1}{r_o} + g_m + g_{mb}}{1 + \frac{R_D}{r_o}}\right]$$

$$\frac{V_x}{I_x} = \frac{R_s R_D + R_s r_o}{r_o + R_D + R_s + (g_m + g_{mb}) r_o R_s}$$

(c)



small signal analysis



$V_{gs} = -V_x$, $V_{bs_1} = 0 - V_x = -V_x$

KCL

$$I_x = \frac{V_x}{r_{o2}} + \frac{V_x}{r_{o1}} - g_{m1} V_{gs_1} - g_{mb_1} V_{bs_1}$$

$$= \frac{V_x}{r_{o2}} + \frac{V_x}{r_{o1}} + g_{m1} V_x + g_{mb_1} V_x$$

$$I_x = V_x\left[\frac{1}{r_{o1}} + \frac{1}{r_{o2}} + g_{m1} + g_{mb_1}\right]$$

$$\boxed{\frac{V_x}{I_x} = \frac{1}{g_{m1} + g_{mb_1} + \frac{1}{r_{o1}} + \frac{1}{r_{o2}}}}$$

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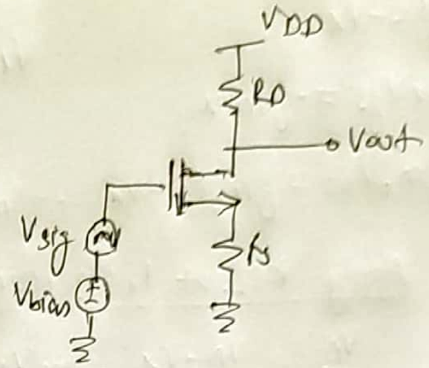
(4) In the following ckt, assuming that the transistor is operating in saturation region:

(a) Find the required V_{bias} for which the dc value of the V_{out} is 1.44V

Assume $\lambda = 0$, $\eta = 1 V^{1/2}$, $2\phi_F = 0.64V$, $V_{th0} = 0.4V$, $\mu_n C_{ox} = 800\ \mu A/V^2$, $\left(\frac{W}{L}\right)_{nmos} = 20$, $R_D = R_S = 0.5 k\Omega$, and $V_{DD} = 1.8V$



$V_{dc\text{-}out} = V_{dd} - I_d R_D$

$1.44 = 1.8 - I_D \times (\frac{1}{2} k\Omega)$

$I_d = (1.8 - 1.44)^2 mA$

$= 0.36 \times 2 = \boxed{0.72 mA}$

$I_d = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{th})^2$

$V_{th} = V_{th0} + \gamma\left(\sqrt{2\phi_F + V_{SB}} - \sqrt{2\phi_F}\right) = 0.6V$

$V_{GS} = V_{bias} - I_D R_D = V_{bias} - 0.36$

$0.72 \times 10^{-3} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{bias} - 0.36 - V_{th})^2$

$\boxed{V_{bias} = 1.26V}$

(b) Is the assumption that the transistor is in saturation region?

$V_{GD} = 1.26 - 1.44 \quad < V_{th}$

saturation

(c) Find the small-signal gain V_{out}/V_{sig}

As $G_m R_{out} \approx \left[\frac{g_m}{1 + g_m R_s}\right] (R_D \| r_o)$

$= \frac{\mu_n C_{ox} \frac{W}{L} (V_{GS} - V_t)}{1 + \mu_n C_{ox} \left(\frac{W}{L}\right) (V_{GS} - V_t) R_S} (R_D \| r_o)$

$= \frac{R_D \| r_o}{R_S} = \boxed{-0.53}$

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