

DS-100 Practice Final Questions

Fall 2017

Name: _____

Email address: _____

Student id: _____

Instructions:

- These are a random selection of previous final exam questions.
- This is not representative of the length of the final (its too long!).
- You may use a single page (two-sided) cheat sheet.

1 Loss Minimization

1. In a petri dish, yeast populations grow exponentially over time. In order to estimate the growth rate of a certain yeast, you place yeast cells in each of n petri dishes and observe the population y_i at time x_i and collect a dataset $\{(x_1, y_1), \dots, (x_n, y_n)\}$. Because yeast populations are known to grow exponentially, you propose the following model:

$$\log(y_i) = \gamma x_i \quad (1)$$

where γ is the growth rate parameter (which you are trying to estimate). We would like to derive the L_2 regularized estimator least squares estimator.

- (1) [4 Pts.] Write the *regularized least squares loss function* for γ under this model. Use λ as the regularization parameter.

Solution:

$$L(\gamma) = \frac{1}{n} \sum_{i=1}^n (\log(y_i) - \gamma x_i)^2 + \lambda \gamma^2 \quad (2)$$

- (2) [8 Pts.] Solve for the optimal γ as a function of the data and λ

Solution: Taking the derivative of the regularized loss function function:

$$\frac{\partial}{\partial \gamma} L(\gamma) = \frac{1}{n} \sum_{i=1}^n \frac{\partial}{\partial \gamma} (\log(y_i) - \gamma x_i)^2 + \frac{\partial}{\partial \gamma} \lambda \gamma^2 \quad (3)$$

$$= -\frac{2}{n} \sum_{i=1}^n (\log(y_i) - \gamma x_i) x_i + 2\lambda \gamma \quad (4)$$

$$= -\frac{2}{n} \sum_{i=1}^n \log(y_i) x_i + \frac{2\gamma}{n} \left(\lambda + \sum_{i=1}^n x_i^2 \right) \quad (5)$$

Setting the derivative equal to zero and solving for γ :

$$0 = -\frac{2}{n} \sum_{i=1}^n \log(y_i) x_i + \frac{2\gamma}{n} \left(\lambda + \sum_{i=1}^n x_i^2 \right) \quad (6)$$

$$\gamma \left(\lambda + \sum_{i=1}^n x_i^2 \right) = \sum_{i=1}^n \log(y_i) x_i \quad (7)$$

$$\gamma = \left(\lambda + \sum_{i=1}^n x_i^2 \right)^{-1} \sum_{i=1}^n \log(y_i) x_i \quad (8)$$

$$(9)$$

2. Suppose we observe a dataset $\{x_1, \dots, x_n\}$ of independent and identically distributed samples from the exponential distribution. Suppose we give you a "probability model" parameterized by λ :

$$f_\lambda(x) = \lambda e^{-\lambda x}$$

that estimates the probability of a particular data point. In addition we give you the "log-likelihood" loss function as the following:

$$L(\lambda) = -n \log(\lambda) + \lambda \sum_{i=1}^n x_i \quad (10)$$

Derive the parameter value λ that minimizes this loss function. **Circle your answer.**

Solution: Taking the derivative of the loss function with respect to the parameter λ we get:

$$\frac{\partial}{\partial \lambda} L(\lambda) = -n \frac{\partial}{\partial \lambda} \log(\lambda) + \frac{\partial}{\partial \lambda} \lambda \sum_{i=1}^n x_i \quad (11)$$

$$= -n \frac{1}{\lambda} + \sum_{i=1}^n x_i \quad (12)$$

$$(13)$$

To minimize the loss we set the above derivative equal to zero and solve:

$$0 = -n \frac{1}{\hat{\lambda}} + \sum_{i=1}^n x_i \quad (14)$$

$$\frac{1}{\hat{\lambda}} = \frac{1}{n} \sum_{i=1}^n x_i \quad (15)$$

$$\hat{\lambda} = \frac{n}{\sum_{i=1}^n x_i} \quad (16)$$

Thus loss minimizing estimate is:

$$\hat{\lambda} = \frac{n}{\sum_{i=1}^n x_i} = \left(\frac{1}{n} \sum_{i=1}^n x_i \right)^{-1} = \frac{1}{\mathbf{Mean}(x)} \quad (17)$$

3. Suppose we collect a dataset of n observations $\{x_1, \dots, x_n\}$ which we believe are drawn from a distribution with the following PDF:

$$f_\mu(x) = C \exp\left(-\frac{(x - \mu)^6}{6}\right) \quad (18)$$

where C is a constant that does not depend on μ . As before we are given the loss function:

$$L(\mu) = -n \log C + \frac{1}{6} \sum_{i=1}^n (x_i - \mu)^6 \quad (19)$$

- (1) [4 Pts.] Compute the derivative of the derivative of the loss with respect to μ .

Solution: Taking the derivative:

$$\frac{\partial}{\partial \mu} L(\mu) = -\frac{\partial}{\partial \mu} n \log C + \frac{1}{6} \sum_{i=1}^n \frac{\partial}{\partial \mu} (x_i - \mu)^6 \quad (20)$$

$$= 0 - \sum_{i=1}^n (x_i - \mu)^5 \quad (21)$$

$$= - \sum_{i=1}^n (x_i - \mu)^5 \quad (22)$$

- (2) [3 Pts.] Because there is no closed form solution for μ in $\frac{\partial}{\partial \mu} L(\mu) = 0$, we would likely use gradient *descent* to approximately compute $\hat{\mu}$. Given the gradient function:

$$g(\mu) = \frac{\partial}{\partial \mu} \log L(\mu), \quad (23)$$

and a step size $\rho(t)$, what is the gradient descent update rule to go from $\mu^{(t)}$ to $\mu^{(t+1)}$? (Hint: your answer should contain only the variables $g(\mu^{(t)})$, $\mu^{(t)}$, $\mu^{(t+1)}$, and $\rho(t)$.)

Solution: Recall that the gradient points in the “uphill” direction. When we minimize we want to go the opposite direction (negative gradient). So the update rule would look like:

$$\mu^{(t+1)} \leftarrow \mu^{(t)} - \rho(t)g(\mu^{(t)}) \quad (24)$$

2 Wrangling and Querying Data

2.1 SQL

For the questions in this subsection, assume we have a massive database in the cloud with the following schema:

```
-- A simple digital media store database
CREATE TABLE media
  (mid integer PRIMARY KEY,
   name text, type char, year_released integer, length integer,
   buy_cost float, rent_cost float, avg_rating float);

CREATE TABLE customers
  (cid integer PRIMARY KEY,
   name text, joined date, nation_id integer,
   activity_level integer);

CREATE TABLE transactions
  (tid integer PRIMARY KEY,
   tdate date, item integer, customer integer,
   rent_or_buy integer, price_paid float, percent_viewed float,
   FOREIGN KEY (item) REFERENCES media,
   FOREIGN KEY (customer) REFERENCES customers);

CREATE VIEW stats AS
SELECT min(length) AS len_min, max(length) AS len_max,
       avg(length) AS len_mu, stddev(length) AS len_sigma,
       min(avg_rating) AS ar_min, max(avg_rating) AS ar_max,
       avg(avg_rating) AS ar_mu, stddev(avg_rating) AS ar_sigma
FROM media;
```

4. [4 Pts.] In the `media` table above, the `type` column encodes the type of media as a unique character code (e.g., 'S' for song, 'M' for movie, 'E' for episode, etc.). Suppose we wanted to modify the `stats` view to display the stats for each `type` of media. Which of the following are true? (Select *all* that apply.)

- A. We need to change the granularity of the view to be finer than it is above.
- B. We need to add a `GROUP BY type` clause to the view.
- C. It would be helpful to add `media.type` to the list of columns in the `SELECT` clause of the view.
- D. The modified view should have more rows than the original view above.
- E. None of the above.

5. [3 Pts.] Which of the following queries finds the ids of media that are 2 standard deviations longer than the mean length? (Select *only one*.)

A.

```
SELECT media.mid
FROM media, stats
WHERE media.mid = stats.mid
      AND media.length >= stats.len_mu
                           + 2*(stats.len_sigma);
```

B.

```
SELECT media.mid
FROM media, stats
WHERE media.length >= stats.len_mu
                           + 2*(stats.len_sigma);
```

C.

```
SELECT media.mid
FROM media
WHERE media.length >= avg(media.length)
                           + 2*stddev(media.length);
```

D. None of the above.

2.2 SQL Sampling

The `transactions` table has 30 million (30×10^6) rows. It is too large to load into the memory of our laptop. We will extract a sample from the database server to process on our laptop in Python.

```
SELECT *  
FROM transactions TABLESAMPLE Bernoulli(.0001);
```

6. [2 Pts.] In expectation, how many rows will there be in the answer to this query?

Solution: $3000 = 30 \times 10^2$

7. [4 Pts.] Your friend Emily Engineer tells you to avoid Bernoulli sampling, and use the following query instead:

```
SELECT *  
FROM transactions  
LIMIT XX;
```

(where XX is replaced by the correct answer to the previous question). **Select all the true statements:**

- A. Emily's LIMIT query will probably run faster than the TABLESAMPLE query. For Emily's query, the database engine can simply access the first XX rows it finds in the table, and skip the rest.**

Solution: True. For reasoning above.

- B. Emily's query result may be biased to favor certain rows.**

Solution: True. The database will optimize for speed, which will likely favor clusters of records stored near each other.

- C. The output of the TABLESAMPLE query provides a hint about how many rows there are in the transactions table while Emily's LIMIT query does not.**

Solution: True. You can extrapolate from the sample size and the sample probability to predict the table size.

- D. Emily's LIMIT query may run fast, but it will swamp the memory on your laptop, since it doesn't sample the database.**

Solution: False. Emily's query will only return XX rows to the laptop.

- E. None of the above.
8. [2 Pts.] You will recall from Homework 5 that it is possible to do bootstrap sampling in SQL by constructing a `design` table with two columns. Each of the columns used in that scheme is described by a single choice below. **Identify the *two* correct choices:**
- A. A foreign key to the table being sampled.**
 - B. A `count` column to capture the number of tuples in each bootstrap sample.
 - C. An identifier to group rows together into bootstrap samples.**
 - D. A regularization column to prevent overfitting.

2.3 Pandas

For the questions in this subsection, assume that we have pandas dataframes with the same schemas as described in the previous section on SQL. That is, we have a `media` dataframe with columns `mid`, `name`, `type`, `year`, et cetera. Assume that the index column of each dataframe is meaningless—the primary key is represented as a regular column.

9. [3 Pts.] Consider the following code snippet:

```
def get_average_price_paid(join_method):
    return (customers
            .merge(transactions, how=join_method,
                  left_on='cid', right_on='customer')
            .loc[:, 'price_paid']
            .fillna(0)           # <- Important
            .mean()
    )

inner = get_average_price_paid('inner')
outer = get_average_price_paid('outer')
left = get_average_price_paid('left')
right = get_average_price_paid('right')
```

Assume that all item *prices are positive*, all `transactions` refer to valid customers in the `customers` table, but some customers may have no transactions.

- (1) How are `inner` and `outer` related? **Pick *one* best answer.**
- A. `inner < outer`
 - B. `inner ≤ outer`
 - C. `inner = outer`

D. $\text{inner} \geq \text{outer}$

E. $\text{inner} > \text{outer}$

(2) How are left and right related? **Pick one best answer.**

A. $\text{left} < \text{right}$

B. $\text{left} \leq \text{right}$

C. $\text{left} = \text{right}$

D. $\text{left} \geq \text{right}$

E. $\text{left} > \text{right}$

(3) How are left and outer related? **Pick one best answer.**

A. $\text{left} < \text{outer}$

B. $\text{left} \leq \text{outer}$

C. $\text{left} = \text{outer}$

D. $\text{left} \geq \text{outer}$

10. [3 Pts.] We wish to write a python expression to find the largest amount of money spent by one person on any single date. We will use the following code:

```
biggie = transactions.groupby(____) ['price_paid'].sum().max()
```

What should we be pass in as our groupby predicate? **Select only one answer.**

A. 'tdate'

B. 'customer'

C. ['item', 'tdate']

D. ['customer', 'tdate']

E. ['customer', 'item']

11. [6 Pts.] Fill in the following python code that finds the names of every customer who has spent over \$100.

```
merged = customers.merge(__A__, left_on=__B__, right_on=__C__)
grouped = merged.groupby(__D__).__E__()
names = grouped[__F__].index
```

Solution:

```
merged = customers.merge(transactions, \
                        left_on=cid, right_on=customer)
grouped = merged.groupby(cid).sum()
names = grouped[grouped.price_paid > 100].index
```

12. [4 Pts.] We wish to find years where the average `price_paid` (over all time) for products released in that year is greater than the average `price_paid` across all transactions; from those years we want to return the *earliest* (smallest). We have the following code:

```
merged = transactions.merge(media, left_on="item", \
                             right_on="mid")
mean_price = merged.groupby("year_released") \
                  .mean().price_paid.mean() # Line A
by_year = merged.groupby("year_released").count() # Line B
is_greater = by_year[by_year.price_paid > mean_price] # Line C
result = is_greater.sort_index(ascending=False).index[0] # Line D
```

Some of these lines need to be modified in order for the code to work properly. We have suggested replacements for each line below. Which lines need to be *replaced*? **Select all that apply.**

- A. `mean_price = merged.price_paid.mean()`
- B. `by_year = merged.groupby("year_released").mean()`
- C. `is_greater = by_year.where(by_year.price_paid > mean_price)`
- D. `result = is_greater.sort_index(ascending=True).index[0]`
- E. All the lines are correct.

3 Feature Engineering

For this question you were given the following sales data and asked to build a model to predict units sold based on the *the product attributes* to guide the design of future products.

ProdID	Name	Desc	Price	Category	Units Sold
13	Errorplane	"A truly uncaught exception ..."	404.00	Toy	9
42	Rock Kit	"Launch into minerology with ..."	123.45	Toys	1
54	Punative Jokes	"Jokes that will get you fined ..."	1.00	Books	30

...

13. Write down a reasonable schema for this data.

Solution:

```
Products (prodId INTEGER,  
          name  VARCHAR,  
          desc  VARCHAR,  
          price REAL,  
          cat   CHARACTER(20),  
          sold  INTEGER);
```

14. Suppose we are interested in building a linear predictive model. For each of the columns indicate which (one or more) of the feature transformations could be appropriate.

- (1) The ProdID column:

- A. Drop the column
- B. One-Hot Encoding
- C. Leave as is

Solution: Because we are trying to predict sales of future products and each product is likely to have a unique product id, this feature is not likely to be helpful and may harm predictions.

- (2) The Name column:

- A. The length of the text in characters
- B. One-Hot Encoding
- C. Bag-of-words Encoding
- D. Leave as is

Solution: The length of the name could be helpful in predicting how well product sell. Perhaps long names are hard to remember? A one-hot encoding would not likely be helpful unless we are going to make new products with the same name as old products. A bag-of-words encoding could be helpful if for example certain words like `kit` implied a decrease or increase in sales.

(3) The `Desc` column:

- A. The length of the text in characters**
- B. One-Hot Encoding
- C. Bag-of-words Encoding**
- D. bi-gram Encoding**
- E. Leave as is

Solution: This is similar to the Name but given longer post a bi-gram featurization may also be helpful.

(4) The `Price` column:

- A. The length of the text in characters
- B. One-Hot Encoding
- C. Bag-of-words Encoding
- D. Convert the price to an indicator indicating if it is less than 19.99.**
- E. Leave as is**

Solution: As a floating point number we could leave the price as is. We might however also compare the price to a few thresholds (e.g., 19.99 or 199.99) to test if the price is in some critical sales ranges.

(5) The `Category` column:

- A. The length of the text in characters.
- B. One-Hot Encoding**
- C. Bag-of-words Encoding
- D. N-Gram Encoding
- E. Leave as is

Solution: Categorical data is typically best represented in a one-hot encoding.

15. It might be reasonable to assume that the relationship between units sold and price differs for each category (e.g., an expensive toy might be less likely to sell than expensive jewelry). Which of the following feature functions might capture this intuition?

- A. $\phi(\text{category}, \text{price}) = \text{category} + \text{price}$
- B. $\phi(\text{category}, \text{price}) = \text{price} \times \text{category}$
- C. $\phi(\text{category}, \text{price}) = \text{OneHot}(\text{category}) + \text{price}$
- D. $\phi(\text{category}, \text{price}) = \text{price} \times \text{OneHot}(\text{category})$

Solution: This will result in a vector of length k , where k is the number of distinct categories. Each element will be equal to the price for examples that match the category, and 0 otherwise. So the learned coefficient for each element is the slope for prices for that category. Most of the other choices would result in type errors, since it doesn't make sense to add or multiply a category (a string) by any number. Choice F would run, but it would only allow predictions for each category to vary by additive factors; each category would get a different *intercept* rather than a different *slope on price*.

- E. $\phi(\text{category}, \text{price}) = \text{category} \times \text{OneHot}(\text{price})$
- F. $\phi(\text{category}, \text{price}) = \text{Concatenate}(\text{OneHot}(\text{category}), \text{price})$

4 Feature Engineering 2

For this problem we collected the following data on the new social networking app *UFace*.

PostID	UTC Time	Text	Num. Responses	State
3	08:10 PM	"Checkout my breakfast ..."	2	VA
13	11:00 AM	"Studied all night for ..."	5	CA
14	12:04 PM	"Hello world!"	0	NY
17	11:35 PM	"That exam was lit ..."	42	CA

...

16. Suppose we are interested in predicting the number of responses *for future posts*. For each of the columns, indicate which (**one or more**) of the given feature transformations could be informative. **Select *all* that apply.**

(1) [2 Pts.] The `PostID` column:

- A. Drop the column**
- B. One-Hot encoding
- C. Leave as is

(2) [2 Pts.] The `Time` column:

- A. Take the hour as a float**
- B. One-Hot encoding
- C. Bag-of-words encoding
- D. Time since midnight in seconds**

(3) [2 Pts.] The `Text` column:

- A. The length of the text**
- B. One-Hot encoding
- C. Bag-of-words encoding**
- D. Leave as is

(4) [2 Pts.] The `State` column:

- A. The length of the text
- B. One-Hot encoding**
- C. Bag-of-words encoding
- D. Leave as is

17. [4 Pts.] Suppose we believe that people are more likely to respond to tweets in the *afternoon* (roughly from hours 13 to 17). Which of the following feature functions would help capture this intuition? Assume that the function **localHour** takes a time and a state as its arguments and returns the hour of the day (in 24-hour time) in the state's time zone. Also assume that any boolean-valued feature is encoded as 0 (false) or 1 (true). **Select all that apply.**

- A. $\phi(\text{time}, \text{state}) = \text{localHour}(\text{time}, \text{state})$
- B. $\phi(\text{time}, \text{state}) = 13 < \text{localHour}(\text{time}, \text{state}) < 17$
- C. $\phi(\text{time}, \text{state}) = \exp(-(\text{localHour}(\text{time}, \text{state}) - 15)^2)$
- D. $\phi(\text{time}, \text{state}) = \exp(\text{localHour}(\text{time}, \text{state}) - 15)$
- E. None of the above.

18. [2 Pts.] Given the following text from a BigData Borat post:

“Data Science is statistics on a Mac.”

Which of the following is the *bi-gram* encoding *including stop-words*? (Select *only one*.)

- A. $\{('data', 1), ('science', 1), ('statistics', 1), ('mac', 1)\}$
- B. $\{('data science', 1), ('science statistics', 1), ('statistics mac', 1)\}$
- C. $\{('data science', 1), ('science is', 1), ('is statistics', 1), ('statistics on', 1), ('on a', 1), ('a mac', 1)\}$
- D. $\{('data science', 1), ('is statistics', 1), ('on a', 1), ('mac', 1)\}$

5 Least Squares Regression and Regularization

19. **Binary Features** You are part of a team that is analyzing data from a clinical trial. Let X be a full-column-rank $n \times 2$ design matrix with the following columns:

1. X_0 is a column of 1s. This generates an intercept term.
2. X_1 is a binary treatment indicator vector taking on values 0 or 1. $X_{i1} = 1$ means that y_i represents the response of a treated individual. $X_{i1} = 0$ means that y_i represents the response of an untreated individual

You propose the following linear model:

$$y_i = \beta_0 + \beta_1 X_{i1} + \epsilon_i$$

You solve for $\hat{\beta}_0$ and $\hat{\beta}_1$ (estimates of β_0 and β_1 , respectively) by minimizing the residual sum of squares. Show that

$$\hat{\beta}_1 = \bar{y}_T - \bar{y}_C,$$

where:

- \bar{y}_T is the average response of all the treated individuals, and
- \bar{y}_C is the average response over all untreated or “control” individuals.

Hint: Think about the meaning of $\sum_{i=1}^n X_{i1}$ and how \bar{y} is related to \bar{y}_T and \bar{y}_C

Solution: First, here's a proof that is not very concise but attempts to give intuition for the problem.

When we search for the function $\hat{f}(x) = \hat{\beta}_0 + \hat{\beta}_1 x$ minimizing the sum of squared residuals, \hat{f} will be evaluated only at the points $x = 0$ and $x = 1$, since those are the only values the X_{i1} s take. (A scatter plot of X_{i1} against y would look like 2 vertical slices at $x = 0$ and $x = 1$.) Since a line parametrized by an intercept and a slope can pass through any two points, our function \hat{f} can do the best possible job of minimizing the squared errors, by passing through the vertical “middles” of the slices at $x = 0$ and $x = 1$. More precisely, $\hat{f}(0)$ will be the value minimizing the sum of squared errors for those responses with $x = 0$, and similarly for $\hat{f}(1) = \hat{\beta}_0 + \hat{\beta}_1$. We have seen that the minimizer of the squared error between a number and a list of numbers is the mean of the list of numbers, so $\hat{f}(0) = \hat{\beta}_0 = \bar{y}_C$, and $\hat{f}(1) = \hat{\beta}_0 + \hat{\beta}_1 = \bar{y}_T$. Therefore, $\hat{\beta}_1 = \bar{y}_T - \bar{y}_C$.

Here is an alternative “brute force” proof by calculation. Let $m = \sum_{i=1}^n X_{i1}$, the number of treated individuals. Then:

$$\begin{bmatrix} \hat{\beta}_0 \\ \hat{\beta}_1 \end{bmatrix} = (X^T X)^{-1} X^T y \quad (25)$$

$$= \begin{bmatrix} n & m \\ m & m \end{bmatrix}^{-1} X^T y \quad (26)$$

$$= \frac{1}{m(n-m)} \begin{bmatrix} m & -m \\ -m & n \end{bmatrix} X^T y \quad (27)$$

$$= \frac{1}{n-m} \begin{bmatrix} 1 & -1 \\ -1 & \frac{n}{m} \end{bmatrix} \begin{bmatrix} \sum_i y_i \\ m\bar{y}_T \end{bmatrix} \quad (28)$$

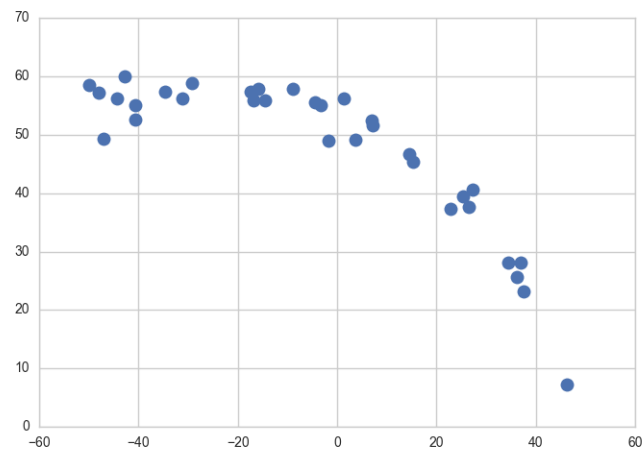
$\hat{\beta}_1$ is just the second element of this vector, which is:

$$\hat{\beta}_1 = \frac{1}{n-m} \left(-\sum y_i + \frac{n}{m} m\bar{y}_T \right) \quad (29)$$

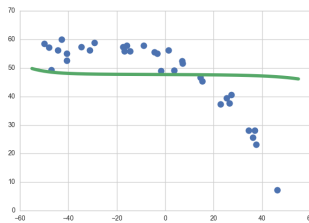
$$= \frac{1}{n-m} \left(-(m\bar{y}_T + (n-m)\bar{y}_C) + n\bar{y}_T \right) \quad (30)$$

$$= \bar{y}_T - \bar{y}_C \quad (31)$$

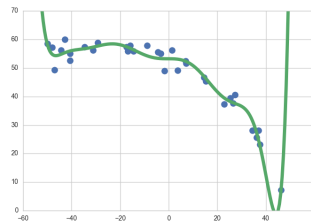
20. For this question we use the following toy dataset:



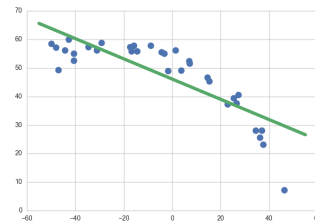
(1) [3 Pts.] We have fit several models depicted as curves in the following plots:



(a)



(b)



(c)

Select the plot that best matches each of the models below. **Each plot is used exactly once.**

1. Linear regression model

☐ (A) ☐ (B) ☒ (C)

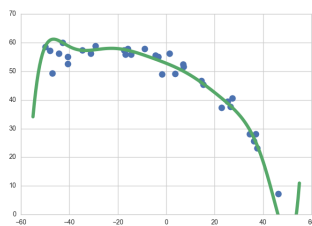
2. Linear regression with degree 10 polynomial features

☐ (A) ☒ (B) ☐ (C)

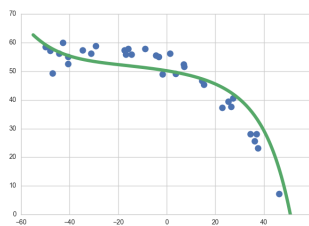
3. Ridge regression with degree 10 polynomial features and substantial regularization.

☒ (A) ☐ (B) ☐ (C)

- (2) [2 Pts.] We fit two more models to these data. Again, the solid curves display the predictions made by each model.



(a)



(b)

Select the plot that best matches each of the models below. **Each plot is used exactly once.**

1. Ridge regression with degree 10 polynomial features, $\lambda = 0.1$.
☒ (A) ☐ (B)
 2. Ridge regression with degree 10 polynomial features, $\lambda = 1.0$.
☐ (A) ☒ (B)
21. Suppose you are given a dataset $\{(x_i, y_i)\}_{i=1}^n$ where $x_i \in \mathbb{R}$ is a one dimensional feature and $y_i \in \mathbb{R}$ is a real-valued response. To model this data you choose a model characterized by the following objective function:

$$J(\theta) = \sum_{i=1}^n (y_i - \theta_0 - x_i\theta_1 - x_i^2\theta_2)^2 + \lambda \sum_{i=1}^2 |\theta_i| \quad (32)$$

- (1) [7 Pts.] **Select all the true statements** for the above objective function (Equation 32).

- A. This loss function likely corresponds to a classification problem.
- B. θ is the regularization parameter.

C. This is an example of L_1 regularization.

- D. This is not a linear model in θ .

E. This model includes a bias/intercept term.

F. This model incorporates a non-linear feature transformation.

G. Large values of λ would reduce the model to a constant θ_0 .

- H. None of the above are true.

- (2) [2 Pts.] Suppose in our implementation we accidentally forget to square the first term:

$$J(\theta) = \sum_{i=1}^n (y_i - \theta_0 - x_i\theta_1 - x_i^2\theta_2) + \lambda \sum_{i=1}^2 |\theta_i| \quad (33)$$

What would change if we tried to train a model using gradient descent on this objective function rather than the original objective function? **(Select only one)**

- A. The training code would raise an error due to a matrix/vector dimension problem.
 - B. The training process would diverge with $\theta_0 \rightarrow -\infty$
 - C. The training process would diverge with $\theta_0 \rightarrow \infty$**
 - D. The training process would converge to a different regression line.
 - E. Nothing; the training process would eventually converge to the same regression line.
22. [5 Pts.] Let X be a $n \times p$ design matrix with full column rank and y be a $n \times 1$ response vector. Let $\hat{\beta}$ be the optimal solution to the least squares problem and r be its associated error. In other words,

$$y = X\hat{\beta} + r \quad (34)$$

Consider X_2 the second column of X .

- (1) [1 Pt.] **True or False.** Without any additional assumptions,

$$r \cdot X_2 = 0$$

where \cdot denotes the usual dot product?

- (2) [4 Pts.] Provide a short proof or counter example.

You may use the following scratch space but we will only grade what you put on the answer sheet.

Solution: True. It suffices to show that r is orthogonal to the column space of X .

$$X^T r = X^T (y - X(X^T X)^{-1} X^T y) = (X^T - X^T X (X^T X)^{-1} X^T) y = (X^T - X^T) y = 0$$

6 Classification

23. For each of the following circle **T** for true or **F** for false.

- (1) [1 Pt.] Binary or multi-class **classification** techniques are most appropriate when making predictions about **continuous responses**.

Solution: False. Classification techniques are used in setting like spam *classification* where the response variable is one of potential many classes.

- (2) [1 Pt.] In a setting with extreme class imbalance in which 95% of the training data have the same label it is always possible to get at least 95% **training accuracy**.

Solution: True. In settings with class imbalance always predicting the most common label is guaranteed to get a training accuracy that matches the proportion of the most common label.

- (3) [1 Pt.] In a setting with extreme class imbalance in which 95% of the training data have the same label it is always possible to get at least 95% **test accuracy**.

Solution: False. The test accuracy could be much lower depending on the class imbalance in the test data.

- (4) [1 Pt.] In logistic regression, predictor variables (X) are continuous, with values from 0 to 1.

Solution: False. There is no such constraint on the values that predictor variables might take. They are continuous from $-\infty$ to ∞ .

- (5) [1 Pt.] In two-class logistic regression, the response variable (y) is continuous, with values from 0 to 1.

Solution: False. The response variable is categorical, only taking values 0 or 1.

- (6) [1 Pt.] In logistic regression, the outputs of the sigmoid function are continuous, with values from 0 to 1.

Solution: True. This is a simple property of the sigmoid function.

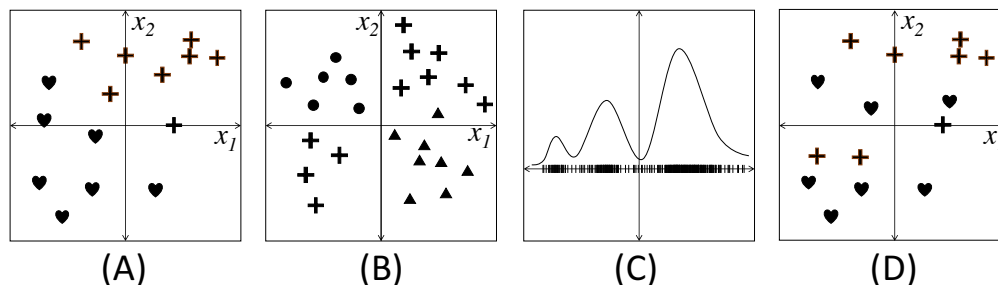
- (7) [1 Pt.] In two-class logistic regression, the output of the sigmoid function for each data point represents the category of that data point.

Solution: False. The outputs of the sigmoid function represent the probability that the data point falls into a certain class (0 or 1).

- (8) [1 Pt.] In logistic regression, we calculate the weights $\hat{\theta}$ as $\hat{\theta} = (X^T X)^{-1} X^T y$, and then fit responses as $\hat{y}_i = \sigma(x_i^T \hat{\theta})$.

Solution: False. We cannot analytically solve for $\hat{\theta}$ - we must use gradient descent. You can tell this wouldn't work because it would give us the same *decision boundary* (where $\sigma(x_i^T \hat{\theta}) = \frac{1}{2}$, or equivalently where $x_i^T \hat{\theta} = 0$) as linear regression.

24. Using the following figure to answer each of the following questions:



- (1) Which of the above plots represents a **linearly separable binary classification** task?
☒ (A) ☐ (B) ☐ (C) ☐ (D)
- (2) Which of the above plots represents a **binary classification** task that is **not linearly separable**?
☐ (A) ☐ (B) ☐ (C) ☒ (D)
- (3) Which of the above plots represents a **multi-class classification** task?
☐ (A) ☒ (B) ☐ (C) ☐ (D)
- (4) Which of the above plots depicts a **1-dimensional Gaussian mixture model**?
☐ (A) ☐ (B) ☒ (C) ☐ (D)

25. Consider the following buggy Python implementation of gradient descent.

```

1 def grad_descent(X, Y, theta0,
2                   grad_function, max_iter = 1000000):
3     """X: A 2D array, the feature matrix.
4     Y: A 1D array, the response vector.
5     theta0: A 1D array, the initial parameter vector.
6     grad_function: Maps a parameter vector, a feature matrix, and
7       a response vector to the gradient of some loss function at
8       the given parameter value. The return value is a 1D array."""
9     theta = theta0
10    for t in range(1, max_iter+1):
11        grad = grad_function(theta, X, Y)
12        theta = theta0 + 1/t * grad
13    return theta

```

Select all the issues with this Python implementation

- A. **Line 11** `theta` should be replaced by `theta0`.
- B. Line 12 `theta0` should be replaced by `theta`.**
- C. **Line 12** `1/t` should be replaced by `t`.
- D. Line 12 `+` should be replaced by `-`.**

26. Suppose we collect a binary classification dataset consisting of $\{(x_i, y_i)\}_{i=1}^n$ where $x_i \in \mathbb{R}$ and $y_i \in \{0, 1\}$. Recall that the probability mass function for a Bernoulli random variable $y \in \{0, 1\}$ is:

$$\mathbf{P}(y | \theta) = \theta^y (1 - \theta)^{(y-1)} \quad (35)$$

and the sigmoid function is given by:

$$\sigma(t) = \frac{1}{1 + e^{-t}} \quad (36)$$

Which of the following is the **loss function** for the logistic regression model with L_2 regularization?

- A. $J(\theta) = \sum_{i=1}^n \theta_i^y (1 - \theta)^{(y_i-1)} + \lambda |\theta|$
- B. $J(\theta) = - \sum_{i=1}^n (\theta x_i)^y (1 - \theta x_i)^{(y_i-1)} + \lambda \theta^2$
- C. $J(\theta) = \sum_{i=1}^n \sigma(\theta x_i)^y (1 - \sigma(\theta x_i))^{(y_i-1)}$
- D. $J(\theta) = \lambda \theta^2 - \sum_{i=1}^n [y_i \log \sigma(\theta x_i) + (y_i - 1) \log (1 - \sigma(\theta x_i))]$**
- E. $J(\theta) = \lambda \theta^2 + \sum_{i=1}^n [y_i \log \sigma(\theta x_i) + (y_i - 1) \log (1 - \sigma(\theta x_i))]$

27. [4 Pts.] Which of the following can help deal with overfitting in a logistic regression model?

- A. Adding additional features.
- B. Obtaining additional training data.**
- C. Performing regularization.**
- D. Removing data until your classes are linearly separable.

7 Classification 2

28. For each of the following select **T** for true or **F** for false on the answer sheet.

- (1) [1 Pt.] A binary or multi-class **classification** technique should be used whenever there are **categorical features**.

Solution: False. Categorical *features* may appear in both classification and regression settings and should be addressed using one-hot-encoding.

- (2) [1 Pt.] Logistic regression is actually used for classification.

Solution: True. Logistic regression is somewhat confusingly named as it applies to classifications tasks but builds on the linear models we introduced in least squares linear regression.

- (3) [1 Pt.] The logistic regression loss function was derived by modeling the observations as noisy observations with a Gaussian noise model.

Solution: False. Logistic regression was derived using the Bernoulli likelihood of function.

- (4) [1 Pt.] Class imbalance can be a serious problem in which the number of training data points from one class is much larger than another.

Solution: True. Class imbalance can be a serious problem and often occurs in settings like disease diagnosis where a large fraction of the population is healthy.

- (5) [1 Pt.] A broken *binary* classifier that *always* predicts 0 is likely to get a test accuracy around 50% on all prediction tasks.

Solution: False. In many case class imbalance could result in substantially higher or lower accuracy.

- (6) [1 Pt.] The root mean squared error is the correct metric for evaluating the prediction accuracy of a binary classifier.

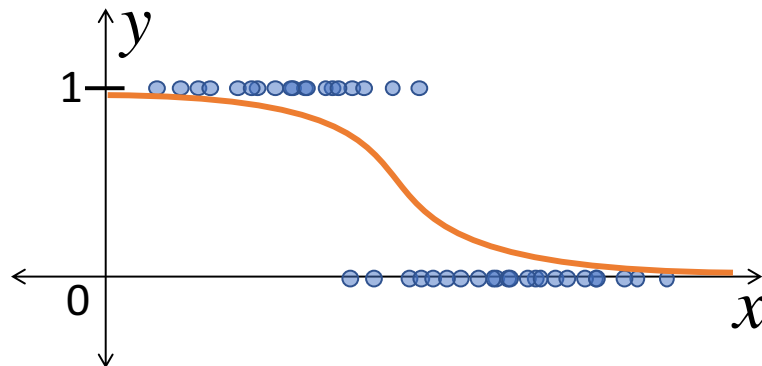
Solution: False. Root mean squared error is a standard measure of accuracy for regression. Logistic regression accuracy is often measured by the fraction of examples predicted correctly or in some cases the likelihood of the data under the model.

29. Consider the following binary classification dataset



- (1) [3 Pts.] Draw a reasonable approximation of the logistic regression probability estimates for $\mathbf{P}(Y = 1 \mid x)$ on top of the figure on the answersheet.

Solution: Anything close to the following would be acceptable:



It is important that:

1. the curve is higher for smaller values of x
2. the curve is smooth
3. the curve is a sigmoid

- (2) [1 Pt.] Are these data linearly separable?

A. Yes

B. No

30. [3 Pts.] Suppose you are given θ for the logistic regression model to predict whether a tumor is malignant ($y = 1$) or benign ($y = 0$) based on features of the tumor x . If you get a new patient x_* and find that $x_*^T \theta > 0$, what can you say about the tumor? **Select *only one*.**
- A. The tumor is benign
 - B. The tumor is more likely benign
 - C. The tumor is more likely to be malignant**
 - D. The tumor is malignant
31. [4 Pts.] Which of the following explanations that applying regularization to a logistic regression model? **Select *all that apply*.**
- A. The training error is too high.
 - B. The test error is too low.
 - C. The data are high-dimensional.**
 - D. There is a large class imbalance.
 - E. None of the above justify regularization for logistic regression.

8 Bias-Variance Tradeoff

32. For each of the following circle **T** for true or **F** for false.

- (1) Increasing the regularization penalty decreases bias.

Solution: False. Increasing the regularization penalty reduces variance but increases bias.

- (2) Without taking precautions, reducing bias often leads to increased variance.

Solution: True. This is the fundamental bias variance tradeoff.

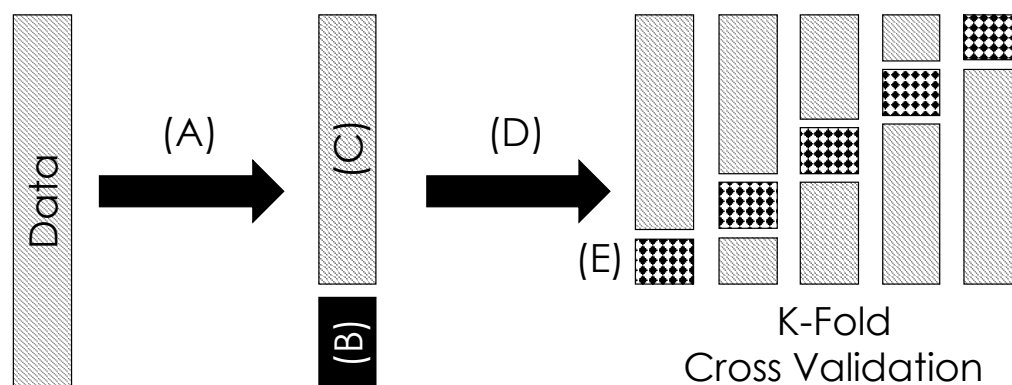
- (3) In the bias variance trade-off the variance refers to the variability in predictions across different training datasets.

Solution: True. In the bias variance trade-off the variance refers to variability in predictions due to variability in training data.

- (4) As we improve the model to reduce bias we often run the risk of **under-fitting**.

Solution: False. Improving the model by reducing bias often leads to *over-fitting*

33. In this question we complete the following figure describing the train-test split and k-fold cross validation. Note the data table with many records and a few columns is depicted on the left as a tall rectangle.



- (1) This part of the figure refers to the **validation** data.

☐ (A) ☐ (B) ☐ (C) ☐ (D) ☒ (E)

- (2) This part of the figure refers to the **testing** data

☐ (A) ☒ (B) ☐ (C) ☐ (D) ☐ (E)

- (3) This part of the figure refers to the process of constructing the *train-test* split.
☒ (A) ☐ (B) ☐ (C) ☐ (D) ☐ (E)
- (4) Select all the following statements that apply to the above figure.
- A. This figure illustrates 5-fold cross validation.
 - B. This figure illustrates 6-fold cross-validation.
 - C. Assuming all the data points are distinct each of the validation data sets are also distinct.
 - D. The test data should be used during cross-validation to fully evaluate the model.

34. For each of the following select **T** for true or **F** for false on the answer sheet.

- (1) [1 Pt.] Regularization can be used to manage the bias-variance trade-off.

Solution: True. Regularization encourages simpler models which can help to reduce variance but increase bias.

- (2) [1 Pt.] When conducting linear regression, adding polynomial features to your data often decreases the variance of your fitted model.

Solution: False. Adding more features tends to increase your model's variance since there are more parameters to fit.

- (3) [1 Pt.] When conducting linear regression, adding polynomial features to your data often decreases the bias of your fitted model.

Solution: True. Adding more features tends to decrease your model's bias since your model can fit more complicated patterns in the data.

- (4) [1 Pt.] Suppose your data are an i.i.d. sample from a population. Then collecting a larger sample for use as a *training set* can help reduce *bias*.

Solution: False. Increasing the dataset size without changing the modeling procedure can often reduce variance but is unlikely to address bias.

- (5) [1 Pt.] Suppose your data are an i.i.d. sample from a population. Then collecting a larger sample for use as a *training set* can help reduce *variance*.

Solution: True. More data often helps to reduce variance in the model fitting process.

- (6) [1 Pt.] Training error is typically larger than test error.

Solution: False. Training error often under-estimates the test error.

- (7) [1 Pt.] If you include the test set in your training data, your accuracy as measured on the test set will probably increase.

Solution: True. Training on the test data improves test accuracy but this improvement can be misleading due to over-fitting.

- (8) [1 Pt.] It is important to frequently evaluate models on the test data throughout the process of model development.

Solution: False. Nooooooooooooo. Once test data is used it is no longer test data. You should create validation datasets or use cross-validation procedures to evaluate models.

35. [2 Pts.] A colleague has been developing models all quarter and noticed recently that her *test* error has started to gradually increase while her training error *has been decreasing*. Which of the following is the most likely explanation for what is happening? **Select only one.**

- A. She is starting to over-fit to her training data.**
- B. She is starting to under-fit to her training data.
- C. The model is overly biased.
- D. None of the above.

36. [5 Pts.] Given the following general loss formulation:

$$\arg \min_{\theta} \left[\sum_{i=1}^n (y_i - x_i^T \theta)^2 + \lambda \sum_{p=1}^d \theta_p^2 \right] \quad (37)$$

Which of the following statements are true? **Select *all* that apply.**

- A. There are d data points.
- B. There are n data points.**
- C. The data is d dimensional.**
- D. This is a classification problem.
- E. This is a linear model.**
- F. This problem has LASSO regularization.
- G. Larger values of λ imply increased regularization.**
- H. Larger values of λ will increase variance.
- I. Larger values of λ will likely increase bias.**
- J. None of the above are true.

37. [3 Pts.] In class we broke the least-squares error into three separate terms:

$$\mathbf{E} [(y - f_{\theta}(x))^2] = \mathbf{E} [(y - h(x))^2] + \mathbf{E} [(h(x) - f_{\theta}(x))^2] + \mathbf{E} [(f_{\theta}(x) - \mathbf{E}[f_{\theta}(x)])^2] \quad (38)$$

where $y = h(x) + \epsilon$, $h(x)$ is the true model and ϵ is zero-mean noise. For each of the following terms, indicate its usual interpretation in the bias variance trade-off:

1. $\mathbf{E} [(y - h(x))^2]$: A. Bias B. Variance **C. Noise**
2. $\mathbf{E} [(h(x) - f_{\theta}(x))^2]$: **A. Bias** B. Variance C. Noise
3. $\mathbf{E} [(f_{\theta}(x) - \mathbf{E}[f_{\theta}(x)])^2]$: A. Bias **B. Variance** C. Noise

9 Big Data

38. Which of the following are true:

A. Star schemas are designed to decrease redundancy.

B. A Data Warehouse is typically updated every time a change occurs in a related Operational Data Store.

Solution: False. That would be too resource-intensive. Instead, we run batch ETL periodically.

C. A typical Data Warehouse favors cleanliness over completeness: it rejects data that does not conform to the warehouse schema.

D. A typical Data Lake favors completeness over cleanliness: it allows you to store any data you like, without even requiring a schema.

E. The “T” in ETL involves many of the same tasks as Data Wrangling.

39. Consider a data warehouse of automobile sensor readings, which records information on sensors, readings, and vehicles where the sensors are placed. Which of the following are true:

A. Because a traditional ETL process only loads data into the warehouse periodically, it will lose sensor information recorded in the operational data store.

Solution: False. For this application, sensor readings would not be UPDATES to existing sensor readings; instead it makes more sense for each sensor reading to be timestamped and stored in the operational data store separately. So the ETL process would pick up the full history of sensor readings in this case.

B. Each sensor reading should be timestamped in the data warehouse.

C. There is no reason for the data warehouse to record timestamps for information on the vehicles.

Solution: False. It would be useful for the warehouse to record the history of changes to information about vehicles.

40. Which of the following features are typical of a distributed file system:

A. It can store large volumes of data.

B. It is optimized to store data as compactly as possible.

Solution: False. It in fact wastes storage by replicating data, to provide fault tolerance among other features.

C. It can keep serving files even after a certain number of machine failures.

D. After a crash, if any data can be recovered at all, then all the data can be recovered.

Solution: In general this is false. It is possible for some shards to be recoverable and others not so.

41. In class, we asserted that MapReduce is being used less and less in practice. It is being replaced by what other programming interfaces? Why?

Solution: SQL and Dataframe APIs. Both of those interfaces can achieve anything that MapReduce can do, and offer higher-level constructs that are convenient for tabular data: e.g. joins, built-in aggregates (reduce functions), etc.

42. Which of the following are true?

A. Because people can store any file in a Data Lake, it is harder to assess data quality in a Lake than in a Warehouse.

B. The raw data in a Data Lake will likely require more wrangling than the data in a well-governed Data Warehouse.

C. The lack of a unifying schema in a Data Lake makes it difficult to get a global view of information being captured.

D. Relative to traditional Data Warehouses, Data Lakes make it easier to secure data in a well-governed way.

Solution: The first 3 parts are true and self-explanatory. The last one is false ... data lakes encourage users to “land” data files that are unstructured and experimental. It is hard to know what confidential information might be in such files, and who should get access to them as a result.

43. Consider the following simple Data Warehouse schema from a Cellular Service Provider, which records activity on a cell phone network:

```
CREATE TABLE devices (  
  did integer, customer_id integer,  
  phone_number varchar(13),  
  firstname text, lastname text,
```



```
zip varchar(12), registered_on varchar(2),
PRIMARY KEY (did),
UNIQUE (customer_id) -- a 'candidate' key
);
```

```
CREATE TABLE billing (
  rate_code char PRIMARY KEY,
  description text, base_fee float, per_minute float,
  max_minutes integer, overage_fee float,
  PRIMARY KEY (rate_code));
```

```
CREATE TABLE calls (
  caller_handset_id integer, callee_handset_id integer,
  cell_tower_id integer, call_start datetime, call_end datetime,
  billing_code char,
  PRIMARY KEY (caller_handset_id, call_start),
  FOREIGN KEY (caller_handset_id) REFERENCES devices,
  FOREIGN KEY (billing_code) REFERENCES billing;
```

(1) [3 Pts.] Which of these tables is a dimension table? **Select *all* that apply.**

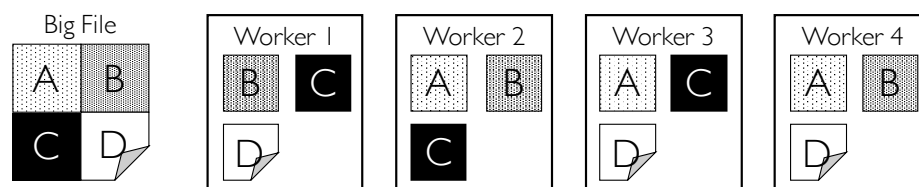
- A. devices
- B. calls
- C. billing
- D. None of the above.

Solution: devices and billing

(2) [3 Pts.] Which of the following statements are true? **Select *all* that apply.**

- A. The `calls.billing_code` column violates star schema design because any update to a single billing fee requires updates to many call records.
- B. If we want to look for correlations between a device's average call length and the time since it was registered, we have to perform a join.**
- C. If the cell service provider implemented a Data Lake, it would make it easier for them to load audio recordings of calls for subsequent analysis.**
- D. None of the above statements are true.

44. [3 Pts.] The figure below depicts a distributed file system with one logical “big file” partitioned into 4 “shards” (A, B, C, D) and replicated across multiple worker machines (1, 2, 3, 4).



Suppose workers 1 AND 2 both fail. Which of the following statements are true? **Select *all* that apply.**

- A. The full file will remain available since worker 3 and worker 4 are both still running.**
- B. The system can tolerate one more worker failure without losing data.
- C. If every request requires all 4 shards of the file, then worker 3 and worker 4 can share the work evenly.**
- D. None of the above statements are true.

45. Consider only the mechanism of *partitioning* files into shards, and storing different shards on different machines. Which of the following statements are true? **Select *all* that apply.**

- A. Partitioning enhances the ability of the system to store large files.**
- B. Partitioning allows the system to tolerate machine failures without losing data.
- C. Partitioning allows the system to read files in parallel.**
- D. None of the above statements are true.

46. [2 Pts.] Recall the statistical query pattern discussed in class for computing on very large data sets. Which of the following statements are true? **Select *all* that apply.**

- A. It eliminates the need for the end-user device (e.g. a laptop) to acquire all the data.**
- B. It pushes the computational task closer to the large-scale data storage.**
- C. It is well suited to both MapReduce and SQL interfaces.**
- D. An alternative to the statistical query pattern for big data is to acquire a sample of the full dataset on the end-user device.**
- E. None of the above statements are true.

10 EDA and Visualization

47. [2 Pts.] Consider the following statistics for infant mortality rate. According to these statistics, which transformation would best symmetrize the distribution? (**Select only one.**)

Transformation	lower quartile	median	upper quartile
x	13	30	68
\sqrt{x}	3.5	5	8
$\log(x)$	1.15	1.5	1.8

- A. no transformation
- B. square root
- C. log**
- D. not possible to tell with this information
48. [5 Pts.] For each of the following scenarios, determine which plot type is *most* appropriate to reveal the distribution of and/or the relationships between the following variable(s). **For each scenario, select only one plot type. Some plot types may be used multiple times.**

- | | |
|--------------------------|------------------------|
| A. histogram | F. scatter plot |
| B. pie chart | G. stacked bar plot |
| C. bar plot | H. overlaid line plots |
| D. line plot | I. mosaic plot |
| E. side-by-side boxplots | |

- (1) [1 Pt.] sale price and number of bedrooms (assume integer) for houses sold in Berkeley in 2010.

Solution: E. Side-by-side Boxplots. We might imagine using a scatter plot since we are plotting the relationship between two numeric quantities. However because the number of bedrooms is an integer and most houses will only have a small number, we are likely to encounter *over-plotting* in the scatter plot. Therefore side-by-side boxplots are likely to be most informative.

- (2) [1 Pt.] sale price and date of sale for houses sold in Berkeley between 1995 and 2015.

Solution: F. Scatter Plot. Here we are plotting two numeric quantities with sufficient spread on each axis.

- (3) [1 Pt.] infant birth weight (grams) for babies born at Alta Bates hospital in 2016.

Solution: A. Histogram. Here we are plotting the distribution of a likely large number of observations and therefore a histogram would be most appropriate.

- (4) [1 Pt.] mother's education-level (highest degree held) for students admitted to UC Berkeley in 2016

Solution: C. Bar Plot. Here we want to visualize counts of a categorical variable.

- (5) [1 Pt.] SAT score and HS GPA of students admitted to UC Berkeley in 2016

Solution: F. Scatter Plot. Here we are visualizing the relationship between two continuous quantities.

- (6) [1 Pt.] race and gender of students admitted to UC Berkeley in 2016

Solution: I. mosaic plot Here we are visualizing the relationship between two categorical variables.

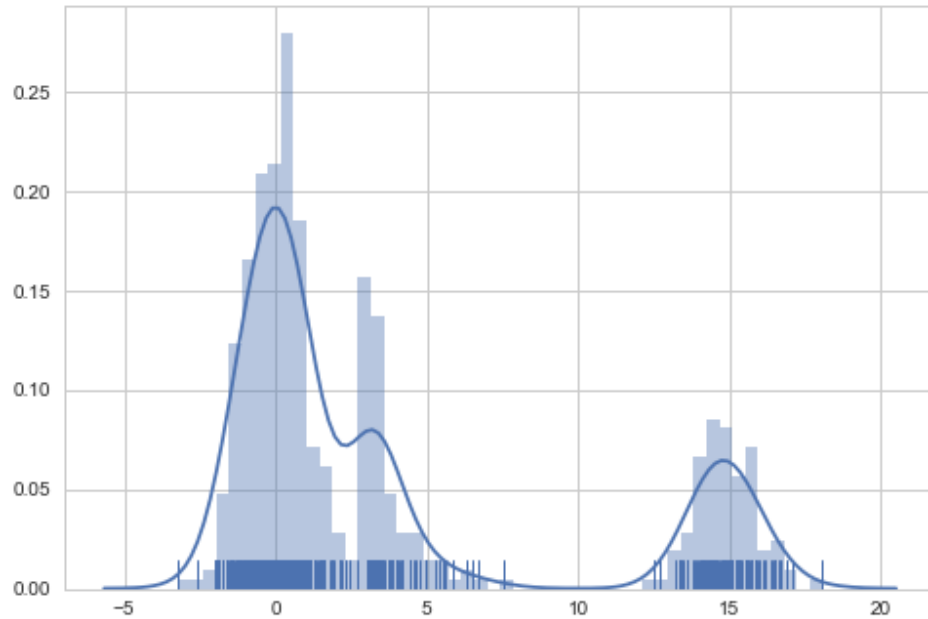
- (7) [1 Pt.] The percentage of female student admitted to UC Berkeley each year from 1950 to 2000.

Solution: D. Line plot. This allows us to see the trends over time.

- (8) [1 Pt.] SAT score for males and females of students admitted to UCB from 1950 to 2000

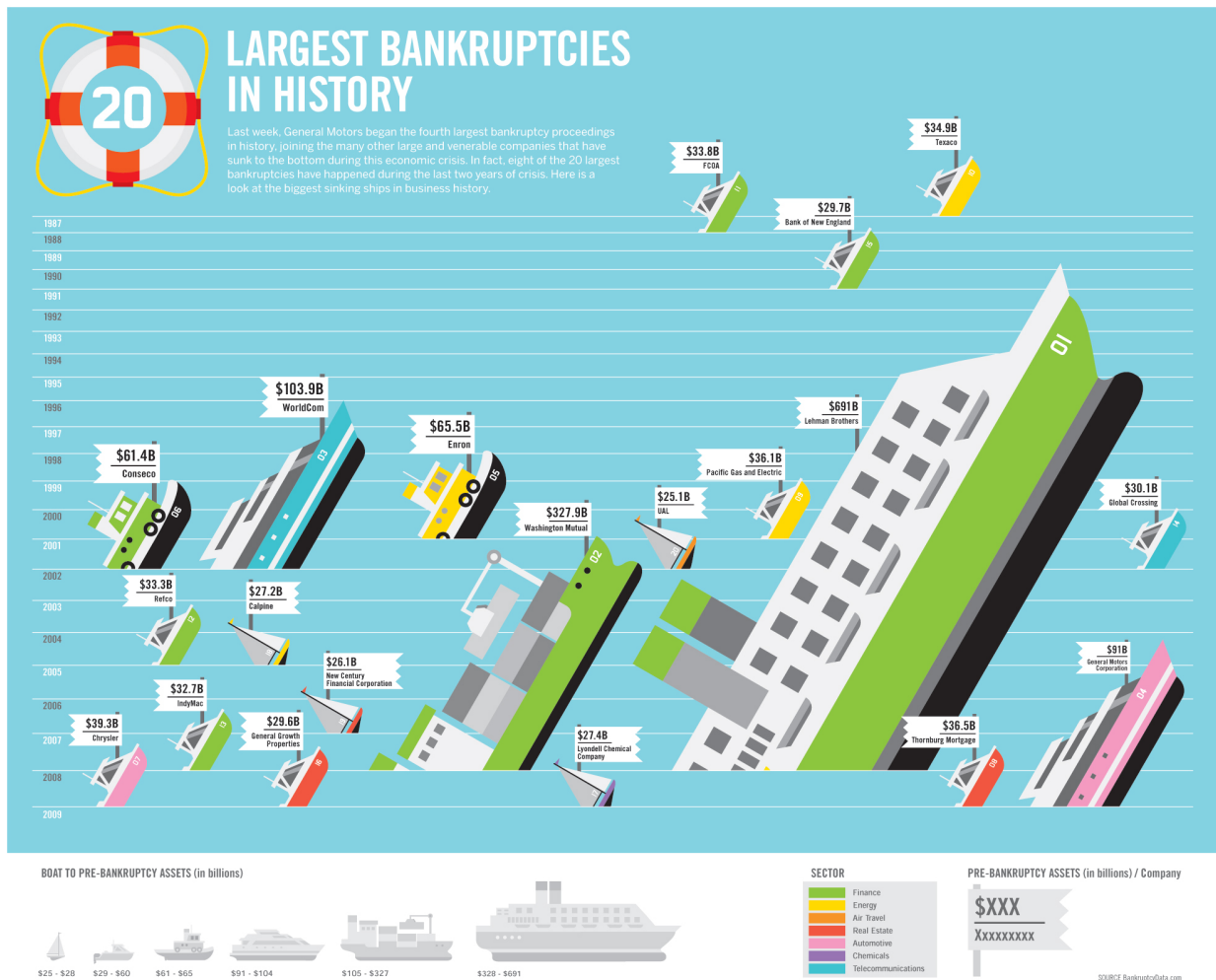
Solution: E. side-by-side boxplots. This allows us to see the distributions of SAT scores per gender and year.

49. [4 Pts.] Consider the following empirical distribution:



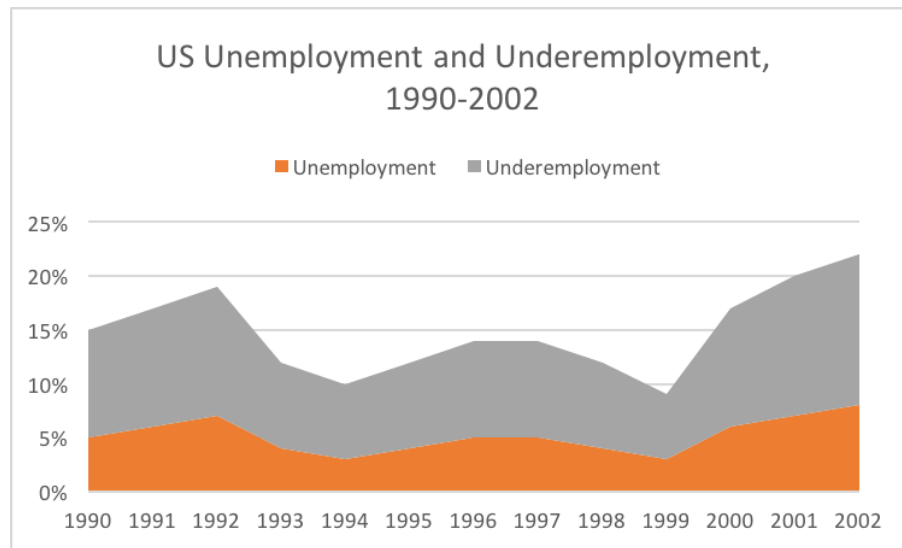
- (1) [1 Pt.] The distribution has _____ mode(s).
A. 1 B. 2 **C. 3** D. 4
- (2) [1 Pt.] The distribution is:
A. Skewed left
B. Symmetric
C. Skewed right
- (3) [2 Pts.] Select **all** of the following properties displayed by the distribution:
A. gaps
B. outliers
C. normal left tail
D. None of the above

50. [4 Pts.] Select all of the problems associated with the following plot (there may be more than one problem):



- A. Over-plotting
- B. Use of chart junk**
- C. Vertical axis should be in log scale
- D. Missing vertical axis label**
- E. Poor use of the horizontal dimension**
- F. Graph elements interfere with data
- G. Stacking
- H. Use of angles to convey information
- I. None of the above are problems with this awesome plot.

51. In the odd questions, name the plot's type (for example, "scatter" or "box"). In the even questions, answer whether the plot is useful for answering the given query.

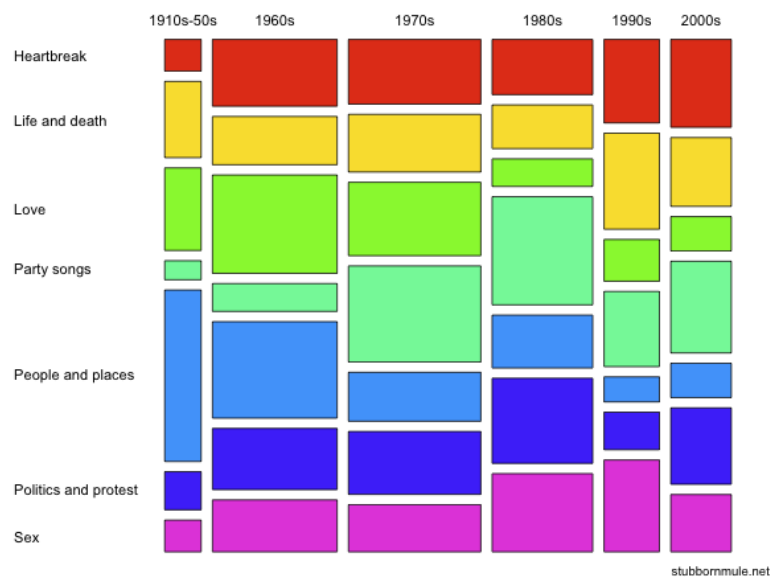


(1) [1 Pt.]

Solution: This is a stacked line plot.

- (2) [1 Pt.] *True or false:* This plot is useful for answering the question: "Did underemployment generally increase when unemployment increased?"

Solution: False. Because of the stacking, it's hard even to tell how much underemployment there was in each year, much less compare changes in underemployment with changes in unemployment. To answer this question, a good visualization would be a scatter plot of the change in unemployment and underemployment for each year.



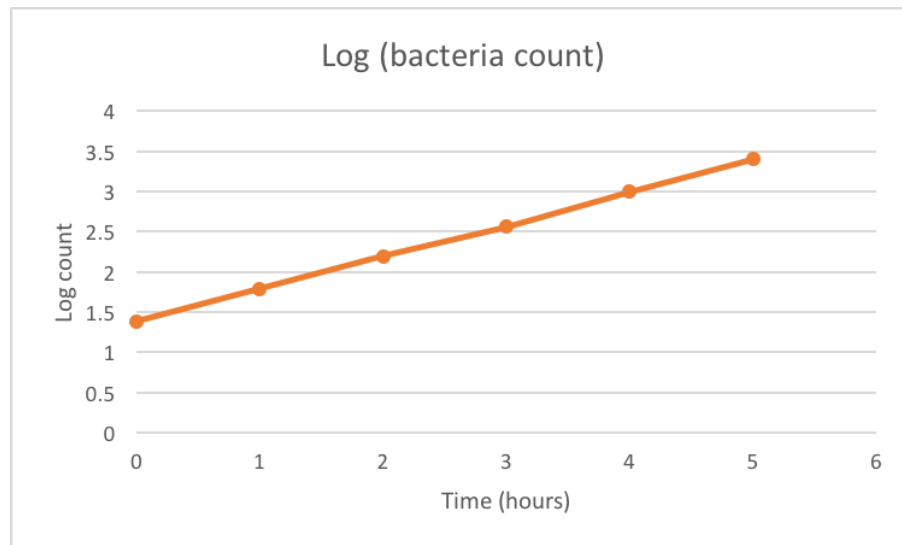
(3) [1 Pt.]

(The plot is from <http://www.stubbornmule.net/2009/07/love-old-fashioned/> and displays topics of selected popular music over time. Not all pop songs are represented in the dataset.)

Solution: This is a mosaic plot.

- (4) [1 Pt.] *True or false:* This plot is useful for answering the question: “Among the songs in this dataset, how many were released in each of the five decades from 1960 to 2010?”

Solution: True. The widths of the bars show us how many songs were released in each category, and the categories include the 1960s through 2000s.



- (5) [1 Pt.]

Solution: This is a line plot on a logarithmic scale.

- (6) [1 Pt.] *True or false:* This plot is useful for answering the question: “Assuming the bacteria population grew linearly over time, what was the rate of increase?”

Solution: False. Since the plot is on a log scale, the slope of the line is the *exponential* rate of growth, not the linear rate. (The question would have been clearer if it had read, “If we model the population growth as a linear function of time, what would be a good estimate of the linear growth rate?”)

End of Exam