Power factor improvement

Introduction

Power factor improvement for an LR load (resistance and inductance) involves using capacitors to compensate for the inductive nature of the load, thereby enhancing the overall power factor. The power factor signifies the phase difference between voltage and current in an AC circuit. For a purely inductive load like LR, the current lags the voltage, resulting in a low power factor.

Series Capacitor Compensation:

- When a capacitor is connected in series with the load:
  - The total impedance of the circuit changes. The capacitive reactance $(-i/(C\omega))$ of the capacitor opposes the inductive reactance ($iL\omega$) of the load.

Parallel Capacitor Compensation:

- When a capacitor is connected in parallel with the load:
  - The capacitor provides a low-impedance path for higher frequency components, compensating for the lagging current due to inductance.
  - This reduces the overall impedance of the circuit and increases the total current. By doing so, it counteracts the phase shift caused by the inductive load, improving the power factor.

Power Factor Improvement:

- Benefits: Both series and parallel capacitor connections aim to counteract the lagging current caused by inductive loads.
- Drawbacks: However, these methods require careful calculation and consideration to prevent overcompensation, resonance issues, or excessive current flow.
- Selection Criteria: The choice between series and parallel connection depends on the specific load characteristics, the existing power factor, and the desired improvement. Engineering calculations and simulations help determine the optimal capacitor value and connection method for effective power factor correction without causing adverse effects.

It is crucial to perform thorough calculations, considering the inductance, resistance, operating frequency, and voltage levels to achieve efficient power factor improvement without introducing harmful effects or instability in the system. Consulting electrical engineering resources or software tools specialized in power factor correction can aid in accurate design and implementation.

For an impedance load $Z = R + iX$ (R and X stand for the real and imaginary parts respectively), connected to a voltage source $V(t) = V_0 \exp(i\omega t)$, where V_0 is a real amplitude, the current flowing through the load is given by $I(t) = V(t)/Z$:

$$I(t) = \frac{V_0 \exp(i\omega t)}{R+iX} = \frac{V_0 \exp(i\omega t)(R-iX)}{R^2+X^2} = \frac{V_0 \exp(i\omega t)(R-iX)\sqrt{R^2+X^2}}{(R^2+X^2)\sqrt{R^2+X^2}} = \\ = \frac{V_0 \exp(i\omega t)}{\sqrt{R^2+X^2}}\left(\frac{R}{\sqrt{R^2+X^2}} - i\frac{X}{\sqrt{R^2+X^2}}\right) = \frac{V_0 \exp(i\omega t)\times exp(i\varphi)}{\sqrt{R^2+X^2}} = \frac{V_0 \exp(i(\omega t+\varphi))}{\sqrt{R^2+X^2}} \tag{1}$$

where

$$\varphi = \tan^{-1}\left(-\frac{X}{R}\right) \tag{2}$$

is the current phase with respect to the voltage as the reference signal (zero phase).

The instantaneous power delivered to the load is calculated as follows:

$$P(t) = \mathrm{Re}[V(t)] \times \mathrm{Re}[I(t)] = \frac{V_0^2}{\sqrt{R^2+X^2}}\cos(\omega t)\cos(\omega t + \varphi) \tag{3}$$

where $\mathrm{Re}[\dots]$ signifies the real part. The power averaged over the period is:

$$\bar{P} = \frac{1}{T}\int_0^T P(t)dt = \frac{V_0^2}{\sqrt{R^2+X^2}} \times \frac{1}{T}\int_0^T \cos\left(\frac{2\pi}{T}t\right)\cos\left(\frac{2\pi}{T}t+\varphi\right)dt = \\ = \frac{V_0^2}{\sqrt{R^2+X^2}} \times \frac{1}{T}\int_0^T \cos\left(\frac{2\pi}{T}t\right)\left(\cos\left(\frac{2\pi}{T}t\right)\cos(\varphi) - \sin\left(\frac{2\pi}{T}t\right)\sin(\varphi)\right)dt = \\ = \frac{V_0^2\cos(\varphi)}{\sqrt{R^2+X^2}} \times \frac{1}{T}\int_0^T \cos^2\left(\frac{2\pi}{T}t\right)dt - \frac{V_0^2\sin(\varphi)}{\sqrt{R^2+X^2}}\frac{1}{T}\int_0^T \cos\left(\frac{2\pi}{T}t\right)\sin\left(\frac{2\pi}{T}t\right)dt = \\ = \frac{V_0^2\cos(\varphi)}{2\sqrt{R^2+X^2}} \tag{4}$$

where

$$\frac{1}{T}\int_0^T \cos^2\left(\frac{2\pi}{T}t\right)dt = \frac{1}{2}$$

$$\frac{1}{T}\int_0^T \cos\left(\frac{2\pi}{T}t\right)\sin\left(\frac{2\pi}{T}t\right)dt \equiv 0$$

Finally, we derive the expression for the average power:

$$\bar{P} = \frac{V_0^2\cos(\varphi)}{2\sqrt{R^2+X^2}} = V_{rms}I_{rms}\cos(\varphi) \tag{5}$$

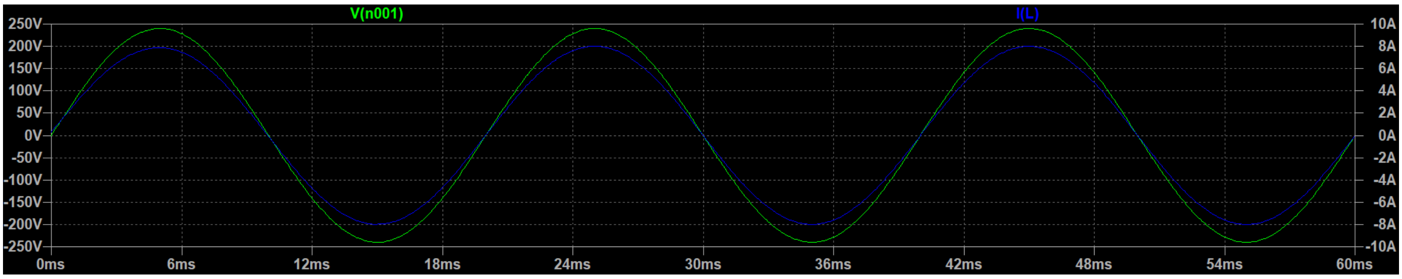
where $V_{rms} = \frac{V_0}{\sqrt{2}}$ and $I_{rms} = \frac{V_0}{\sqrt{2}\times\sqrt{R^2+X^2}} = \frac{V_{rms}}{\sqrt{R^2+X^2}}$. RMS amplitudes can be understood as constant voltage and current values that offer equivalent performance to an AC voltage source, given that the voltage and current are in phase. **The coefficient $\cos(\varphi)$ is known as the power factor. We aim to increase it.**

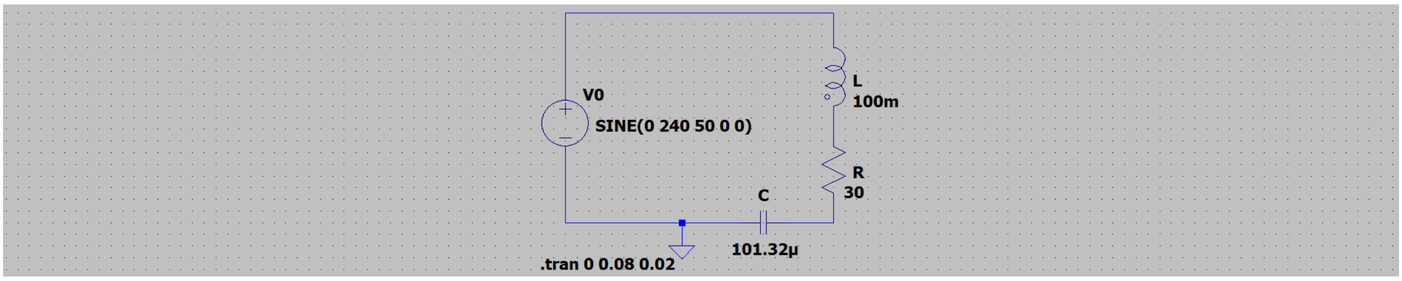
1-phase series capacitor compensation

When the compensation capacitor is connected in series with the RL load, the total impedance and the current phase become $Z = R + iX = R + i\left(L\omega - \frac{1}{C\omega}\right)$ and $\varphi = \tan^{-1}\left(-\frac{L\omega - \frac{1}{C\omega}}{R}\right) = \tan^{-1}\left(-\frac{LC\omega^2 - 1}{C\omega R}\right)$ respectively. The phase becomes zero, resulting in $\cos(\varphi) = 1$, for the specific value of the compensation capacitor:

$$C = \frac{1}{\omega^2 L} = \frac{1}{(2\pi f)^2 L} \quad (6)$$

The LTspice circuit simulation for $f = 50$ Hz ($\omega = 2\pi f$), $L = 100$ mH, $R = 30\ \Omega$, and $C = 101.32$ μF (calculated using Eq. (6)) is displayed below. Perfect phase alignment between the voltage and current sinusoids is observed. During simulation, click on a wire to display the voltage relative to the ground (reference point), and click on any element to view the current passing through it.





1-phase parallel capacitor compensation

When the compensation capacitor is connected in parallel with the RL load, the total impedance becomes:

$$Z = \frac{\left(-\frac{i}{C\omega}\right)(R + i\omega L)}{R + i\omega L - \frac{i}{C\omega}} = \frac{\omega L - iR}{CR\omega + i(LC\omega^2 - 1)} = \frac{(\omega L - iR)(CR\omega - i(LC\omega^2 - 1))}{(CR\omega)^2 + (LC\omega^2 - 1)^2} = \quad (7)$$
$$= \frac{R}{(CR\omega)^2 + (LC\omega^2 - 1)^2} - i\,\frac{CR^2\omega - \omega L(1 - LC\omega^2)}{(CR\omega)^2 + (LC\omega^2 - 1)^2}$$

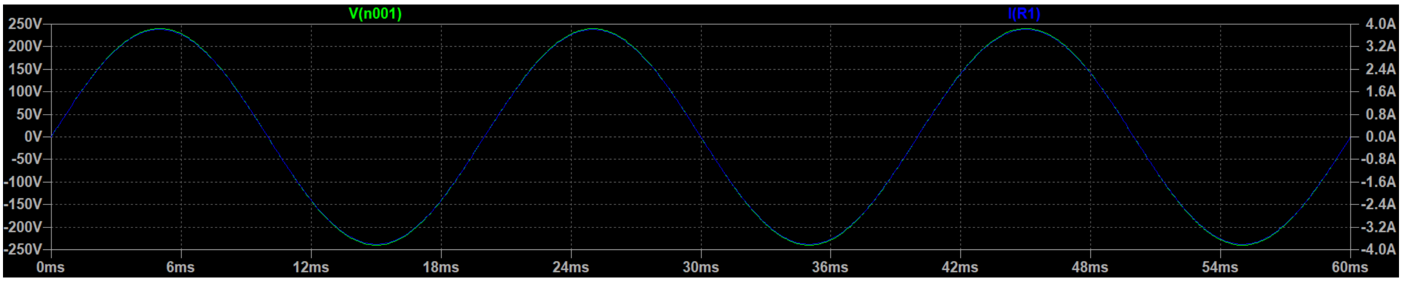
Here, $\frac{R}{(CR\omega)^2 + (LC\omega^2 - 1)^2}$ is the real part of the impedance and $X = -\frac{CR^2\omega - \omega L(1 - LC\omega^2)}{(CR\omega)^2 + (LC\omega^2 - 1)^2}$ is the imaginary part. Hence, based on Eq. (2), we derive the phase as follows:

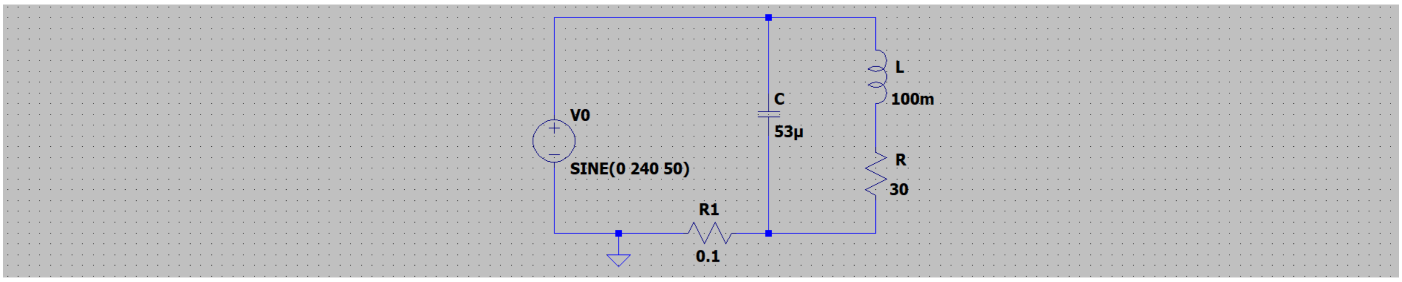
$$\varphi = \tan^{-1}\left(\frac{CR^2\omega - \omega L(1 - LC\omega^2)}{R}\right) \quad (8)$$

The phase becomes zero if $CR^2\omega - \omega L(1 - LC\omega^2) = 0$. From here, we derive the equation for the compensation capacitance:

$$C = \frac{L}{R^2 + L^2\omega^2} \quad (9)$$

The LTspice circuit simulation for $f = 50$ Hz, $L = 100$ mH, $R = 30\ \Omega$, and $C = 53\ \mu$F (computed using Eq. (9)) is displayed below. Perfect phase alignment between the voltage and current sinusoids is observed. During simulation, click on a wire to display the voltage relative to the ground (reference point). To measure the current outside the LCR network, we introduced a small "sensor resistor" $R_1 = 0.1\ \Omega$. This addition minimally impacts the current conditions but facilitates the probe's ability to measure the current by clicking on it. This method of current measurement is commonly employed in practical circuits. The sensor resistor typically possesses a very low value and high tolerance (e.g., 0.01%). By measuring the voltage across the resistor and dividing it by the resistance, one can calculate the current.





3-phase parallel capacitor compensation

Schematic diagrams of the parallel capacitor compensation in three-phase systems, where the load is star or triangular connected, are depicted in the Figures below. When creating such diagrams in LTspice, caution is advised. Unexpectedly, when components are rotated within the simulator, LTspice may display incorrect phase readings. To ensure accuracy, it is advisable to initially construct a separate phase and verify the compensation within that phase. Subsequently, replicate this circuit for the remaining phases and interconnect them.