Testing Hypotheses about a Single Population Parameter

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Today's plan

1. Review reading topics

   1.1 Sampling distribution of OLS

   1.2 Hypothesis testing of single population parameter

   - *p*-values
   - confidence intervals
   - statistical vs. practical significance

2. In-class activity: Practice conducting hypothesis tests of population parameters

Quick review

What we've done so far

- Given the population regression model

$$y = \beta_0 + \beta_1 x_1 + ... + \beta_k x_k + u,$$

- Show OLS is unbiased under assumptions 1-4
- Derive standard errors of $\hat{\beta}_j$ under homoskedasticity
- Assumptions 1-5 known as **Gauss-Markov Assumptions**
- Show that OLS is BLUE (Gauss-Markov Theorem)

What else we've done so far

- Compute direction of bias when $E(u|\mathbf{x}) = 0$ doesn't hold (omitted variable)
- Interpret meaning of $\hat{\beta}_j$ when x_j is transformed
- or when x_j contains qualitative information
- Interpret meaning of $\hat{\beta}_j$ when y is a dummy

Sampling distribution of OLS Estimators

Sampling distribution of OLS

- Now, we want to test hypotheses about the β_j's
- i.e. hypothesize a value of β_j and use data to see if it's likely to be false
- To do so, need more than $E(\hat{\beta}_j)$ or $Var(\hat{\beta}_j)$
- Need to know what the *whole distribution* of $\hat{\beta}_j$ looks like
- Must make one more assumption: u is distributed *iid* $N(0, \sigma^2)$
  - ***iid***: u's independent of x's; unit i's u is independent of unit k's u

The Classical Linear Model (CLM)

- This is our 6th assumption [$u \sim N(0, \sigma^2)$]
- Call the model under A1-A6 the **Classical Linear Model (CLM)**
- In other words,

CLM = Gauss-Markov + normality

- Normality is a crazy assumption
- But it is reasonable in large samples (see: Central Limit Theorem)

Normal Sampling Distributions

- Under CLM assumptions, we have

$$\hat{\beta}_j \sim N[\beta_j, Var(\hat{\beta}_j)]$$

  where $Var(\hat{\beta}_j)$ is our formula from before

- Can re-write this as a standard normal distribution:

$$\frac{\hat{\beta}_j - \beta_j}{sd(\hat{\beta}_j)} \sim Normal(0, 1)$$

- but $sd(\hat{\beta}_j)$ is a function of σ^2, which is unknown

t Distribution for Standardized Estimators

- Since we don't know σ^2 we estimate it as $\hat{\sigma}^2$

- Then, use $se(\hat{\beta}_j)$ instead of $sd(\hat{\beta}_j)$:

$$\frac{\hat{\beta}_j - \beta_j}{se(\hat{\beta}_j)} \sim t_{n-k-1} = t_{df}$$

- Using $\hat{\sigma}^2$ instead of σ^2 takes us to t distribution instead of Normal

- t with large df is indistinguishable from Normal

- In practice, will almost always have large enough df to use Normal

Hypothesis testing of single parameters

Example

- Consider a model of students' final grades:

$$final = \beta_0 + \beta_1 missed + \beta_2 priorGPA + \beta_3 ACT + u$$

  where *missed* is # of classes missed during the semester

- We want to test the following hypothesis:

$$H_0 : \beta_1 = 0$$

- This is a hypothesis that missing class does not affect final grade...
- once we control for prior performance (*priorGPA*) and cognition (*ACT*)

Example (cont'd)

- In R, we get the following table:

```
tidy(est)
# A tibble: 4 x 5
  term        estimate std.error
1 (Intercept)  12.4       1.17
2 missed       -0.0793    0.0352
3 priGPA        1.92      0.373
4 ACT           0.401     0.0532
```

- Which means that $\hat{\beta}_1 = -0.079$ and $se(\hat{\beta}_1) = 0.035$

Example (cont'd)

- Plug these values (-0.079 and 0.035) into our formula:

$$\frac{-0.079 - 0}{0.035} = t_{676}$$
$$= -2.25$$

  so our t-statistic is -2.25. Is that big enough to reject H_0?

- Need to specify a **significance level** which corresponds to a **critical value**

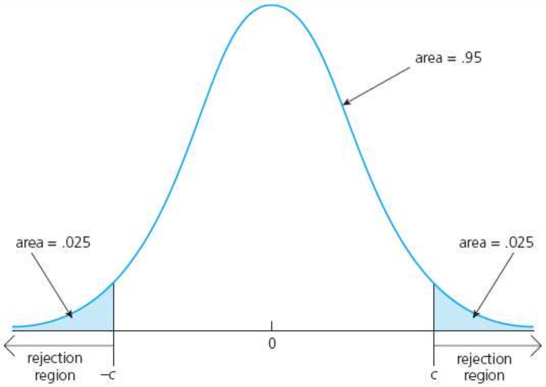
Review

- Need two things to conduct a hypothesis test:

  1. **Test statistic (T):** Some function of the sample of data

  2. **Critical value (c):** Value of T such that we reject H_0 if, e.g. $|T| > c$

- c is implicitly a function of the significance level α



A two-sided test with $\alpha = 0.05$ (Wooldridge Fig. C.6)

Back to the Example

- t statistic = -2.25, with 676 degrees of freedom
- For two-sided test ($\alpha = .05$), need the 2.5th and 97.5th percentiles
- $C = -1.96, 1.96$ (see `http://ttable.org`)
- $-2.25 < -1.96$, so reject H_0
- "We reject the null hypothesis at the 5% significance level"
- Critical values for other significance levels:

$$c_{.10} = 1.645$$
$$c_{.01} = 2.576$$

Computing t statistics automatically

- R automatically reports t stats for hypothesis tests with $H_0 = 0$:

```
tidy(est)
# A tibble: 4 x 5
  term        estimate std.error statistic
1 (Intercept)  12.4       1.17        10.6
2 missed       -0.0793    0.0352      -2.25
3 priGPA        1.92      0.373        5.14
4 ACT           0.401     0.0532       7.54
```

- We can then choose our critical value based on our preferred α

- Can also conduct one-sided instead of two-sided tests

Conducting tests with non-zero null values

- Suppose we want to instead test

$$H_0 : \beta_{missed} = -1$$
$$H_1 : \beta_{missed} \neq -1$$

- Need to plug it into the formula

$$t = \frac{\text{estimate} - \text{null}}{\text{std. err.}}$$
$$= \frac{-0.0793 - (-1)}{0.0352}$$
$$= 28.33$$

- Then compare with the appropriate critical value

p-values

- It's kind of a pain to look up a critical value and compare with test statistic
- Can instead use a p-value
- **p-value:** largest α level at which we could conduct test and fail to reject H_0
- if $\alpha = .05$, reject H_0 if $p < .05$
- if $\alpha = .01$, reject H_0 if $p < .01$
- $\vdots$

Computing p-values automatically

- R automatically reports p-values for **2-sided tests** with $H_0 = 0$:

```
tidy(est)
# A tibble: 4 x 5
  term        estimate std.error statistic  p.value
1 (Intercept)  12.4       1.17        10.6  3.19e-24
2 missed       -0.0793    0.0352     -2.25 2.47e- 2
3 priGPA        1.92      0.373       5.14 3.60e- 7
4 ACT           0.401     0.0532      7.54 1.57e-13
```

- β_{missed} has $p = .0247$, so reject at 5% (but not 1%) level

- For one-sided tests, halve the p-value

p-values with non-zero null values

- Suppose we want to compute p-value for test

$$H_0 : \beta_{missed} = -1$$
$$H_1 : \beta_{missed} \neq -1$$

- Need to plug it into the formula as before

$$t = \frac{\text{estimate} - \text{null}}{\text{std. err.}}$$
$$= 28.33$$

- Then need to compute area under the t-distribution curve
- In R: `2*pt(-abs(t), df)`, where `t` is the t-stat with `df` deg. of freedom

Confidence intervals

- It's also useful to construct **confidence intervals** around the parameters
- CI is supposed to give a "likely" range of values for the parameter
- CIs are of the form

$$\hat{\beta}_j \pm c \cdot se(\hat{\beta}_j)$$

  where $c > 0$ is chosen based on the **confidence level**

- Most common: 95% confidence level
- c comes from the 97.5 percentile of the t_{df} distribution

Computing CIs automatically

- R reports CIs using the `confint()` function:

```
confint(est)
                  2.5 %      97.5 %
(Intercept) 10.0719177 14.67415920
missed      -0.1485216 -0.01015563
priGPA       1.1836741  2.64691420
ACT          0.2965542  0.50557363
```

- Remember: c comes from the **two-sided** $(1 - \alpha)$ critical value

- Reject H_0 if the null value is **outside** the CI