More on time series

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Apr 2, 2019

Today's plan

1. Review reading topics
    1.1 Review Basics of Time Series Data
    1.2 Stationarity, Weak Dependence, and Strong Dependence
2. In-class activity: Work on project

Refresher on Time Series Data

Time Series Data

- Temporally ordered (earliest to latest)
- Example: US Quarterly GDP from 1901:Q1 until 2018:Q2
- Instead, need to worry about **serial correlation**
- Also need to worry about **trends**, **seasonality**, and **differencing**

Example of a time series regression

- Suppose we have the following model:

$$y_t = \beta_0 + \beta_1 x_t + \beta_2 x_{t-1} + \beta_3 z_t + \beta_4 z_{t-1} + \beta_5 y_{t-1} + u_t$$

  where y is interest rate, x is unemployment, z is trade deficit

- This model might perform better than simpler models at **forecasting** y

- y_{t-1} is a **lagged dependent variable**

Trends

- Many time series data sets exhibit trends—sometimes up, sometimes down
- Two series might appear “related” just because they are trending similarly
- This is known as **spurious correlation**
- A time trend can be included to fix this:

$$y_t = \alpha_0 + \alpha_1 t + e_t$$
$$E(e_t) = 0$$

- So the average value of y_t is a linear function of time: $E(y_t) = \alpha_0 + \alpha_1 t$

Trends

- Quadratic, Exponential, and other types of trends can also be useful

- Adding a time trend has a nice interpretation:

- If we include a time trend, the OLS coefficient on x_{tj} is the same ...

  ... as if we first remove a trend from y_t and each $x_{t1}, ..., x_{tk}$

- Note: R^2 is not correct when including trends

Seasonality

- When data are quarterly, monthly, weekly, or daily:

  May need to adjust for different seasonal patterns

- Best way to do this is to include **seasonal dummy variables**:

$$y_t = \beta_0 + \delta_1 feb_t + \delta_2 mar_t + \cdots + \delta_{11} dec_t + \\ \beta_1 x_{t1} + \cdots + \beta_k x_{tk} + u_t$$

- We can include both trends and seasonal dummies

- Note: R^2 is not correct when including either trends or seasonal dummies

Differencing

- Sometimes, we need to analyze **differenced** models:

$$\Delta y_t = \beta_0 + \beta_1 x_{t1} + \cdots + \beta_k x_{tk} + u_t$$

  where $\Delta y_t = y_t - y_{t-1}$

- The reason we might do this is to satisfy some properties required of OLS

Stationarity, Weak Dependence, and Strong Dependence

Inference in Time Series Models

- Usual inference methods (t and F stats) are more fragile in TS models
- Why? Because LLN and CLT don't always hold in the time series case
- The main reason is because we no longer have a random sample
- Instead assume **weak dependence** (rather than independence)
- Example: The AR(1) model

$$y_t = \rho y_{t-1} + e_t$$

provided $|\rho| < 1$

Weak dependence

- What is weak dependence?

- When correlations between obs get smaller as time between obs grows:

$$\lim_{h \to \infty} Corr\left(y_t, y_{t+h}\right) = 0$$

- This allows us to be able to separate cause from effect

- If this did not hold, we wouldn't be able to know if x causes y

- (Unless we could observe multiple draws of the same TS, which we can't)

Stationarity and Covariance Stationarity

- The strongest form of "weak dependence" is **stationarity**:
    1. $f(y_t, y_{t+1}, y_{t+2}, \ldots) = f(y_{t+h}, y_{t+h+1}, y_{t+h+2}, \ldots)$ for all $h \geq 1$
- Nowadays, assume **covariance stationarity**:
    1. $E(y_t)$ is constant
    2. $Var(y_t)$ is constant
    3. $Cov(y_t, y_{t+h})$ is only a function of h (and not t)

Back to the AR(1) model

- We can show that the AR(1) model is covariance stationary:

$$y_t = \rho y_{t-1} + e_t$$

- Can show the following:

1. $E(y_t) = 0$ (i.e. constant)
2. $Var(y_t) = \frac{\sigma_e^2}{1-\rho_1^2}$ (i.e. constant)
3. $Cov(y_t, y_{t+h}) = \rho^h \sigma_y^2$ (i.e. not fn. of t)

so long as $|\rho| < 1$

Moving Average [MA(1)] model

- Another covariance stationary model is the **MA(1) model**:

$$y_t = e_t + \alpha e_{t-1}$$

- No restrictions on the values α can take on
- Essentially, y is a weighted average of e_t and e_{t-1}
- This is a commonly used model, but can't be estimated with OLS
- Can show that MA(1) $\equiv$ AR(∞)

Strongly Dependent (Highly Persistent) Time Series

- Weak dependence is useful, but not always realistic
- Many economic time series are **strongly dependent** (or **highly persistent**)
- Best example: Random Walk

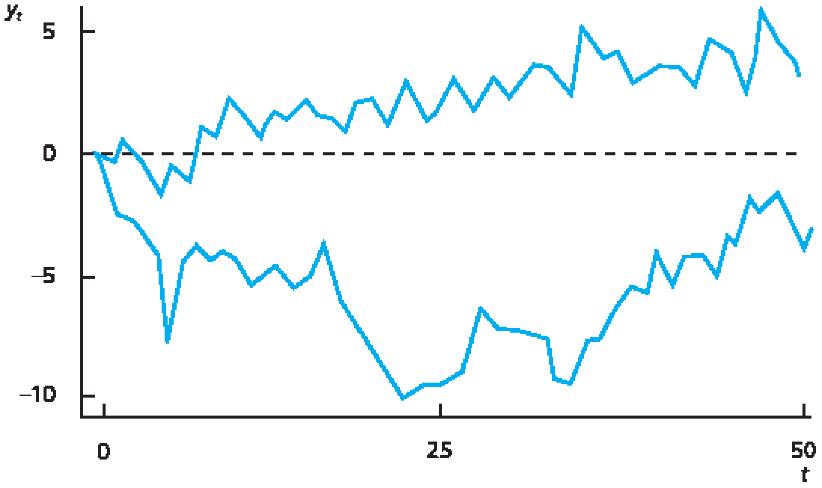
$$y_t = y_{t-1} + e_t$$

- Note: this is the AR(1) model with $\rho = 1$, a.k.a. a **unit root process**
- Can show that $Corr(y_t, y_{t+h}) = \sqrt{\frac{t}{t+h}}$

Trend vs. High Persistence

- It's possible to have a trending, but not highly persistent series
- Also possible to have a highly persistent, but not trending series
- Nevertheless, high persistence $\implies$ trend much of the time

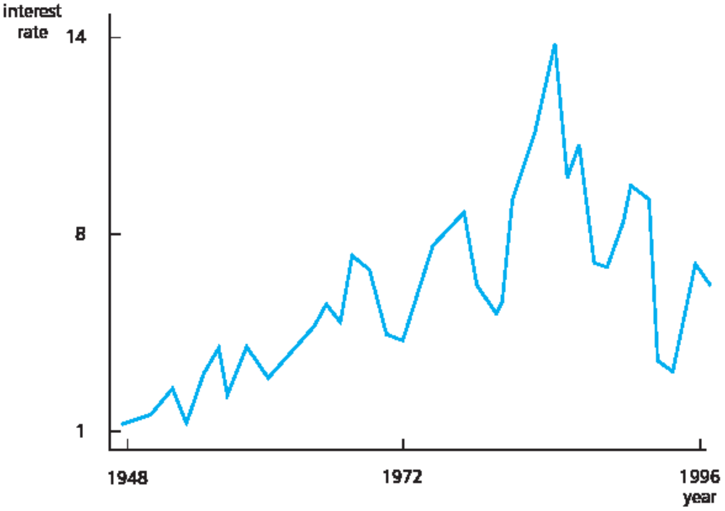
Visualizing a Random Walk



Wooldridge Fig. 11.1

- Two realizations of random walk
- both have $y_0 = 0$

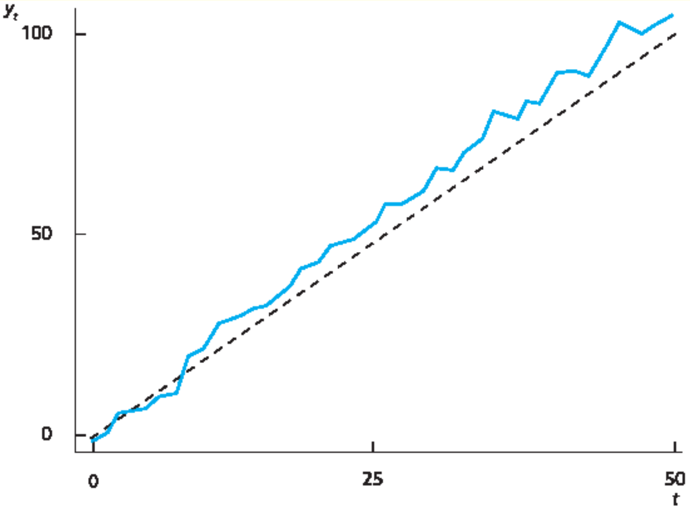
Visualizing a Random Walk (Interest rates)



Wooldridge Fig. 11.2

- 3-month T-Bill rate

Visualizing a Random Walk with Drift



Wooldridge Fig. 11.3

- A Random Walk with Drift
- $y_t = 2 + y_{t-1} + e_t$

How to Deal with High Persistence

- The easiest way to “fix” a highly persistent series is differencing
- Weakly dependent processes are known as **integrated of order zero** or **I(0)**
- Random walk, random walk with drift are **I(1)**
- This means that Δy_t is weakly dependent
- I(1) sometimes called **difference-stationary process**
- Note: Differencing also de-trends the time series

Why we care about high persistence

- It's really important to know if a series is $I(1)$
- A random walk $\implies$ policies have long-lasting effects
- e.g. GDP today highly correlated with GDP in 1980

Testing for a unit root

- If your time series has a unit root, you should difference it
- That way, you'll be working with a stationary time series
- Easiest way to test is to run an AR(1) model and test $H_0 : \rho = 1$
- Can also do the **Augmented Dickey-Fuller (ADF) test**
- In the `tseries` package, `adf.test(df$y)` will test if y has a unit root
- $H_0 : y$ **has a unit root!!**