Due: Apr. 11

Econ 4223 - Fall 2018 **Problem Set 5 - Solutions** **beginning of class**

Warm Up

1. Explain in detail why we care whether a time series process is covariance stationary.

> **Solution:** The main reason is because the typical Laws of Large Numbers and Central Limit Theorems do not apply to time series data (since the data set is not a random sample from a population). Thus, we can't assume that the error term is independent of the x's. Instead, we assume weak dependence between u and x, which is what covariance stationarity is.

Exercises

2. *Wooldridge*, 18.7

> **Solution:** If $unem_t$ follows a stable AR(1) process, then this is the null model used to test for Granger causality: under the null that gM_t does not Granger cause $unem_t$, we can write
>
> $$unem_t = \beta_0 + \beta_1 unem_{t-1} + u_t$$
> $$E\left(u_t | unem_{t-1}, gM_{t-1}, unem_{t-2}, gM_{t-2}, \ldots\right) = 0$$
>
> and $|\beta_1| < 1$. Now, it is up to us to choose how many lags of gM to add to this equation. The simplest approach is to add gM_{t-1} and to do a t test. But we could add a second or third lag (and probably not beyond this with annual data), and compute an F test for joint significance of all lags of gM_t.

3. *Wooldridge*, 18.9

> **Solution:** Let $\hat{e}_{n+1}$ be the forecast error for forecasting y_{n+1}, and let $\hat{a}_{n+1}$ be the forecast error for forecasting Δy_{n+1}. By definition,
>
> $$\begin{aligned} \hat{e}_{n+1} &= y_{n+1} - \hat{f}_n \\ &= y_{n+1} - (\hat{g}_n + y_n) \\ &= (y_{n+1} - y_n) - \hat{g}_n \\ &= \Delta y_{n+1} - \hat{g}_n \\ &= \hat{a}_{n+1} \end{aligned}$$

where the last equality follows by definition of the forecasting error for Δy_{n+1}.

Computer Exercises You should use `R` to complete these exercises. Any data set referred to in the question should be available in the `wooldridge` package in `R`. You do not need to turn in an R-script for these questions, but you are welcome to do so if you would like to.

4. *Wooldridge*, 11.C10 (Chapter 11, Computer Exercise 10)

Solution: **Part (i):** Using the data through 2003 gives

$$\Delta\widehat{inf}_t = \underset{(1.22)}{2.83} - \underset{(.209)}{.518}\, unem_t$$
$$n = 55, R^2 = .104$$

These estimates are similar to those obtained in equation (11.19), as we would hope. Both the intercept and slope have gotten a little smaller in magnitude.

Part (ii): The estimate of the natural rate is obtained as in Example 11.5. The new estimate is $2.83/.518 \approx 5.46$, which is slightly smaller than the 5.58 obtained using only the data through 1996.

Part (iii): The first order autocorrelation of *unem* is about .75. This is one of those tough cases: the correlation between $unem_t$ and $unem_{t-1}$ is large, but it is not especially close to one.

Part (iv): Just as when we use the data through 2003, the model with $\Delta unem_t$ as the explanatory variable fits somewhat better (and yields a more pronounced tradeoff between inflation and unemployment):

$$\Delta\widehat{inf}_t = -\underset{(.306)}{.072} - \underset{(.290)}{.833}\,\Delta unem_t$$
$$n = 55, R^2 = .135$$

5. *Wooldridge*, 18.C3

Solution: **Part (i):** The estimated AR(3) model for $pcip_t$ is

$$\widehat{pcip}_t = \underset{(0.55)}{1.80} + \underset{(.043)}{.349}\,pcip_{t-1} + \underset{(.045)}{.071}\,pcip_{t-2} + \underset{(.043)}{.067}\,pcip_{t-3}$$
$$n = 554, R^2 = .166, \hat{\sigma} = 12.15$$

When $pcip_{t-4}$ is added, its coefficient is .0043 with a t statistic of about .10.

Part (ii): In the model

$$pcip_t = \delta_0 + \alpha_1 pcip_{t-1} + \alpha_2 pcip_{t-2} + \alpha_3 pcip_{t-3} + \gamma_1 pcsp_{t-1} + \gamma_2 pcsp_{t-2} + \gamma_3 pcsp_{t-3} + u_t,$$

the null hypothesis is that $pcsp$ does not Granger cause $pcip$. This is stated as $H_0 : \gamma_1 = \gamma_2 = \gamma_3 = 0$. The F statistic for joint significance of the three lags of $pcspt$, with 3 and 547 df, is $F = 5.37$ and p-value = .0012. Therefore, we strongly reject H_0 and conclude that $pcsp$ does Granger cause $pcip$.

Part (iii): When we add $\Delta i3_{t-1}$, $\Delta i3_{t-2}$, and $\Delta i3_{t-3}$ to the regression from part (ii), and now test the joint significance of $pcsp_{t-1}$, $pcsp_{t-2}$, and $pcsp_{t-3}$, the F statistic is 5.08. With 3 and 544 df in the F distribution, this gives p-value = .0018, and so $pcsp$ Granger causes $pcip$ even conditional on past $\Delta i3$.

Cool Down

6. *Wooldridge*, 10.1.

Solution: **Part (i):** Disagree. Most time series processes are correlated over time, and many of them are strongly correlated. This means they cannot be independent across observations, which simply represent different time periods. Even series that do appear to be roughly uncorrelated—such as stock returns—do not appear to be independently distributed, as we saw in Chapter 12 under dynamic forms of heteroskedasticity (i.e. ARCH and GARCH models).

Part (ii): Agree. This follows immediately from Theorem 10.1. In particular, we do not need the homoskedasticity and no serial correlation assumptions.

Part (iii): Disagree. Trending variables are used all the time as dependent variables in a regression model. We do need to be careful in interpreting the results because we may simply find a spurious association between yt and trending explanatory variables. Including a trend in the regression is a good idea for trending dependent or independent variables. As discussed in Section 10.5, the usual R-squared can be misleading when the dependent variable is trending.

Part (iv): Agree. With annual data, each time period represents a year and is not associated with any season.

References