Time series forecasting

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Today's plan

1. How to forecast using time series
    1.1 Forecasting
    1.2 Granger Causality
    1.3 Forecast Intervals
    1.4 Advanced Forecasting Methods
2. In-class activity: Practice forecasting

Forecasting

Brief refresher on purposes of time series

- One purpose of investigating time series is to find **causality**
- e.g. does inflation cause lower unemployment?
- Another purpose is **forecasting**
- e.g. what will tomorrow's stock price (or next quarter's GDP) be?
- or even what will our company's web server load be on Black Friday?

Elements of Forecasting

- Any forecast of y_{t+1} requires the following components:
- **Information set:** includes y_t, lags of y_t, and current/lags of other variables
- **Forecast error:** $e_{t+1} = y_{t+1} - f_t$
- **Loss function:** how we choose to weight forecast errors, typically min e_{t+1}^2
- Then, we choose our forecast to minimize the expected loss:

$$E\left(e_{t+1}^2 | I_t\right) = E\left[\left(y_{t+1} - f_t\right)^2 | I_t\right]$$

Example

- When forecasting, it's best to use only lagged y's and other variables

$$y_t = \delta_0 + \alpha_1 y_{t-1} + \gamma_1 z_{t-1} + u_t$$
$$E\left(u_t | I_{t-1}\right) = 0,$$

  for example

- Letting T be our sample size, the forecast is

$$\hat{f}_T = \hat{\delta}_0 + \hat{\alpha}_1 y_T + \hat{\gamma}_1 z_T$$

  and the forecast error is

$$\hat{e}_{T+1} = y_{T+1} - \hat{f}_T$$

How to know which variables to include

- Which variables should be included in the model? How many lags?
- Start with an $AR(2)$ model and see if lags are significant
- Number of lags depends on frequency (i.e. annual vs. monthly vs. daily)
- Once we choose an AR model for y, we can do the same for z, etc.

Model selection

- A forecasting problem is one of **model selection**
- This is quite different from causal inference
- In model selection, use **out-of-sample criteria** to measure performance
- Most common out-of-sample statistic is **Root Mean Squared Error (RMSE):**

$$RMSE = \sqrt{\frac{1}{m}\sum_{h=0}^{m-1} \hat{e}^2_{T+h+1}}$$

Note: This is computed on *future* forecast errors

Measuring forecast performance

- The model with the **lowest RMSE** has the best forecast performance
- This is the model that should be used
- Could also use the one with the lowest Mean Absolute Error (MAE):

$$MAE = \frac{1}{m}\sum_{h=0}^{m-1} |\hat{e}^2_{T+h+1}|$$

- There are many other metrics that could be used (AIC, BIC, HQC, ...)
- Forecasting is a key component of **machine learning**; take my other class!

Vector Autoregressive (VAR) Model

- We could also **jointly** forecast y_t and z_t:

$$\begin{cases} y_t = \delta_0 + \alpha_1 y_{t-1} + \gamma_1 z_{t-1} + u_t^y \\ z_t = \eta_0 + \beta_1 y_{t-1} + \rho_1 z_{t-1} + u_t^z \end{cases}$$

- This is known as a **Vector Autoregressive (VAR) model**

- (y, z) form a system of equations (or “vector” of equations)

- VARs are widely used in macroeconomics

- $y =$ AAPL stock price, $z =$ GOOGL stock price

Granger Causality

Granger Causality

- Recall our VAR model from before:

$$\begin{cases} y_t = \delta_0 + \alpha_1 y_{t-1} + \gamma_1 z_{t-1} + u_t^y \\ z_t = \eta_0 + \beta_1 y_{t-1} + \rho_1 z_{t-1} + u_t^z \end{cases}$$

- This allows us to test if lagged z's affect y **holding fixed** lagged y's

- We say z **Granger causes** y if

$$E\left(y_t | I_{t-1}\right) \neq E\left(y_t | J_{t-1}\right)$$

  where I_{t-1} contains past information on y and z but J_{t-1} only has y

- Note: Granger causality says nothing about *contemporaneous* causality!

- Note: named for Sir Clive Granger, Nobel laureate

Forecast Intervals

Forecast Intervals

- From before, we used our model to obtain $\hat{f}_T$
- $\hat{f}_T$ is a **point forecast** of y_{T+1}
- $\hat{e}_{T+1} = y_{T+1} - \hat{f}_T$ is the forecast error
- We can compute the **forecast interval** as follows: (Why?)

$$se\left(\hat{e}_{T+1}\right) = \sqrt{\left[se\left(\hat{f}_T\right)\right]^2 + \hat{\sigma}^2}$$

$$\text{FI} \approx \hat{f}_T \pm 1.96 \cdot se\left(\hat{e}_{T+1}\right)$$

Properties of Forecast Intervals

- $se\left(\hat{f}_T\right)$ is usually much smaller than $\hat{\sigma}$
- Forecast intervals get increasingly wider with time (mostly because of $\hat{\sigma}$)
- Example: Yahoo (now Altaba) stock price tomorrow, next month, next year
- Example: Forecast ETA to OKC when driving from Norman vs. Stillwater

Advanced Forecasting Methods

Forecasting trending, seasonal, and integrated series

- It's easy to forecast with trending/seasonal/integrated processes
- Just plug in the values of the trend or seasonal dummies
- **Don't use a linear trend to forecast a random walk with drift**
- For I(1) processes, can forecast y_{t+1} through Δy_{t+1}
- Or just use a general AR or VAR model to forecast y_{t+1}

ARIMA models

- One well-known model for forecasting is the **ARIMA** model
- ARIMA = Auto Regressive Integrated Moving Average
- Hybrid of $AR(p)$, $I(d)$, and $MA(q)$ processes
- "$ARIMA(p, d, q)$ model" where p, d and q specify the time series process
- With just a few parameters, these models can forecast quite well

ARIMA equation

- The ARIMA model looks like:

$$E\left(y_t | I_t\right) = \alpha_0 + \sum_{j=1}^{p} \alpha_j y_{t-j} + \sum_{k=1}^{q} \beta_k e_{t-k}$$

- And if y is $I(1)$, then

$$E\left(\Delta y_t | I_t\right) = \alpha_0 + \sum_{j=1}^{p} \alpha_j \Delta y_{t-j} + \sum_{k=1}^{q} \beta_k e_{t-k}$$

GARCH models

- We talked about GARCH models back when we covered serial correlation

- GARCH = Generalized Auto Regressive Conditional Heteroskedasticity

- GARCH models are interested in $V\left(y_t|I_t\right)$

- Allow the variance of y_t and of e_t to differ with t

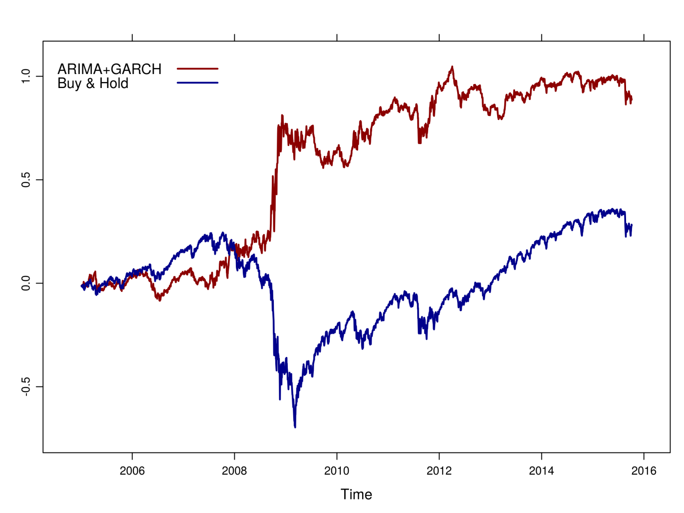
- Particularly useful when forecasting financial asset prices:

- Certain trading periods may have higher volatility

ARMA-GARCH models

- One can get an even better forecast by combining these two
- Use ARMA (or ARIMA) to forecast conditiona mean of y
- Use GARCH to forecast conditional variance of y

ARMA-GARCH vs. Buy-and-Hold since 2005



Source: Quantstart.com