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Standard Model Fitting of Lepton Angular Distributions

In the Standard Model the energy dependence of the shape of the angular distributions, usually parametrized by the forward-backward asymmetry, is dominated by the interference of the s channel Z° exchange with a photon exchange. The effect can be easilly understood by considering the differential cross sections calculated from the sum of helicity amplitudes in lowest order.

$$\frac{d\sigma}{dcos\theta} = S * \sum_{i,j=1,2} \left| \frac{q_e q_f}{q^2} + \frac{C_e^i C_f^j}{S - M^2 + iS\frac{\Gamma}{M}} \right|^2 (1 \pm cos\theta)^2$$

where q^2 is the momentum transfer of the photon exchange, and the sum over helicities is denoted by the sum over i and j. The plus sign is taken when the helicities are the same, the minus sign when they are different. The interference, being the cross term, reduces to

$$\frac{d\sigma_{inter}}{dcos\theta} = 2S * \sum_{i,j=1,2} \left[\frac{q_e q_f}{q^2} * Re\left(\frac{C_e^i C_f^j}{S - M^2 + iS\frac{\Gamma}{M}} \right) (1 \pm cos\theta)^2 \right]$$

Taking just the term proportional to $cos\theta$ will isolate the weak axial coupling when the summation is made (assuming lepton universality for simplicity).

$$\frac{d\sigma_{inter}}{dcos\theta} = 2S * \frac{q_e q_f}{q^2} * \frac{S - M^2}{(S - M^2)^2 + S^2\frac{\Gamma^2}{M^2}} [C_l^2 + C_r^2 - 2C_l C_r] * cos\theta$$

which is just

$$\frac{d\sigma_{inter}}{dcos\theta} = 2S * \frac{q_e q_f}{q^2} * \frac{S - M^2}{(S - M^2)^2 + S^2\frac{\Gamma^2}{M^2}} * C_a^2 * cos\theta$$

Thus the energy dependence of the forward backward asymmetry can be fit to yield the axial coupling constant.

This can be done with the expostar fitting program by using the rescaling feature of the leptonic couplings that it provides[1]. These are incorporated through the following code

$$C_a = k_a * (C_l - C_r)$$

$$C_v = k_v * (C_l + C_r)$$

$$C_l = (C_v + C_a)/2$$

$$C_r = (C_v - C_a)/2$$

then the amplitudes are calculated with the new values of the helicity couplings, thus all the complex energy dependences of the running couplings and vertex corrections are included. A fit for the parameters k_a and k_v yields the optimal *deviations* from the standard model, so the exact standard model you are deviating from must first be established. This means selecting a value for the top and higgs masses and perhaps the Z^o mass as well.

When the data are fit in the manner described by the various memos discussing the standard model fits of the Aleph data[2, 3], the effect of the normalization and the partial widths are also included, obscuring just the result due the asymmetries. Isolating the asymmetry result in the case of the s channel muon and tau samples is straight forward. It requires that the fit luminosities simply be unconstrained from the LCAL measurement, thus floating the normalization at each energy.

In order to select an initial standard model a fit was performed to just line shape data for s channel processes. These included the hadronic number of events and the $\mu + \tau$ part of the rapidity cut common lepton selection[2, 3] but summed over angle. The event shape determination of the strong coupling constant was used to constrain the QCD radiative correction. The use of the rapidity cut sample allows a simpler calculation of the observed number of events, as has been previously discussed[1, 2]. The result of this fit yielded the following starting point with an assumed higgs mass of 200 GeV:

N_{had}	$N_{\mu+\tau}$	α_s	M_z	M_{top}	N_{col}	χ^2/NDF
X	$X \int_{-.9}^{.9}$	X	91.189 ± 0.010	200^{+33}_{-41}	3.008 ± 0.004	28.8/27

Fitting for the axial and vector multipliers, after seeding the standard model calculation to the above values, using the differential distribution of the $\mu + \tau$ part of the rapidity cut common leptons and the α_s constraint as before, yields an axial multiplier which is low by 3σ.

$\frac{dN_{\mu+\tau}}{d\cos\theta'}$	α_s	k_a	k_v	N_c	$\chi^2/N.D.F.$
$X\|_{-.9}^{.9}$	X	0.89 ± 0.036	0.89 ± 0.14	3.008 ± 0.005	260/252

The connection between the energy dependence of the angular distribution and the axial coupling is very general. Consequently, any effect that shows up in the interference of the s channel photon exchange interfering with the s channel Z° exchange also has to occur in the interference of a *t channel* photon with the s channel Z°. In this case, however, there will be an amplification of the effect of $2/(1-cos\theta)$, due to the change from s to t in the momentum transfer (q^2) of the photon.

If the bhabha subsample of the common leptons is fit in exactly the same manner as the $\mu+\tau$ sample the following result is found

$\frac{dN_e}{d\cos\theta'}$	α_s	k_a	k_v	N_c	$\chi^2/N.D.F.$
$X\vert_{-.9}^{.9}$	X	1.006 ± 0.006	0.98 ± 0.29	3.008 ± 0.005	239/252

The error found on the axial coupling multiplier is 6 times smaller and the χ^2 is substantially better (by 20). The problem here is that the pure t channel exchange can only be affected by adjusting the luminosity. Consequently part of the result is due to the partial widths being determined from the normalized cross sections. In order to see only the effect of the angular distribution (interference) and not the normalization (widths) the differential number of events must be calculated slightly differently. In the previous fits a log likelihood for the number of events seen was constructed, where the expected number of events was essentially

$$N_{expect} = (Lum_{fit} * \frac{d\sigma}{dcos\theta} * efficiency) * binwidth + background$$

To isolate the effect of the angular distribution the pure t channel must be taken out and this can be done by calculating the number of events using 2 cross sections, the full bhabha cross section and a pure t channel cross section. The pure t channel cross section is then normalized with the fixed LCAL luminosity resulting in the expected number of events being calculated as

$$N_{expect} = \left[Lum_{fit}*(\frac{d\sigma_e}{dcos\theta}-\frac{d\sigma_t}{dcos\theta})+Lum_{lcal}*\frac{d\sigma_t}{dcos\theta}\right]*efficiency*binwidth+background$$

Where the t channel cross section is exactly what is in the bhabha cross section but with no s channel amplitudes, thus all running couplings, boxes and so on are included. The result of the angular fit then yields

$\frac{dN_e}{d\cos\theta'}$	α_s	k_a	k_v	N_c	$\chi^2/N.D.F.$
$X\vert_{-.9}^{.9}$	X	1.028 ± 0.036	1.033 ± 0.29	3.008 ± 0.005	241/252

A naive calculation of the relative statistical power of the bhabhas with respect to the $\mu+\tau$ sample (the ratio of the forward - backwards integrals of the terms proportional to the axial coupling squared) results in a factor of 3 and thus $\sqrt{3}$ in the expected errors. The errors are essentially equal for the two samples but the χ^2 for the bhabhas is substantially better. One would expect that the combined fit would yield approximately the average of the two fits as the errors are the same but in fact you find the following:

$\frac{dN_e}{d\cos\theta'}$	$\frac{dN_{\mu+\tau}}{d\cos\theta'}$	α_s	k_a	k_v	N_c	$\chi^2/N.D.F.$
$X\vert_{-.9}^{.9}$	$X\vert_{-.9}^{.9}$	X	0.99 ± 0.023	0.97 ± 0.14	3.008 ± 0.005	518/523

The result is striking in that the global fit prefers a minimum 1σ below the bhabha result but 2.5σ above the $\mu + \tau$ result. I believe that the reason for this is in the actual distribution of the contributions to χ^2. The four attached graphs are the distributions of the deviations from the fit in units of σ for the two data samples for the two individual fits and the combined fit. The $\mu + \tau$ distributions are clearly non gaussian which is reflected in the higher value of χ^2 that those data produce. In summation I conclude that we are dealing with a statistical fluctuation aggravated by non gaussian distributions.

Along a similar line, the Z° mass can be determined from the angular distribution due to the different relative sign of the term proportional to the axial coupling in the t and s channel interference terms just discussed. Again doing a combined fit but not removing the pure t channel exchange in the bhabhas results in a standard model fit of

$\frac{dN_e}{d\cos\theta'}$	$\frac{dN_{\mu+\tau}}{d\cos\theta'}$	α_s	M_z	M_t	N_c	$\chi^2/N.D.F.$
$X\vert_{-.9}^{.9}$	$X\vert_{-.9}^{.9}$	X	91.186 ± 0.026	203^{-53}_{+78}	3.008 ± 0.005	517/523

Realizing again that the luminosity is being constrained by the pure t channel exchange, I perform the same manipulation to constrain the t channel part of the bhabha cross section to the LCAL luminosity. In that case the standard model fit produces the results below which now really only depend on the angular shape variations with energy.

$\frac{dN_e}{d\cos\theta'}$	$\frac{dN_{\mu+\tau}}{d\cos\theta'}$	α_s	M_z	M_t	N_c	$\chi^2/N.D.F.$
$X\vert_{-.9}^{.9}$	$X\vert_{-.9}^{.9}$	X	91.281 ± 0.048	232^{+48}_{-69}	3.008 ± 0.005	514/523

I suspect that the higher Z° mass is related to the low value of the axial and vector couplings found for the $\mu + \tau$ data, as this would cause a higher value for the intersection point of the "asymmetries". In any event it is only a two σ effect when the luminosity effect is removed. The agreement with the purely lepton based mass (the first fit) and the hadronic/LCAL value I find quite exceptional, and a good indication of the overall universal nature of the standard models description of weak interactions.

References

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Determination of the linear sum of the b quark charges

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Abstract

The energy dependence of the b quark forward-backward asymmetry is used to experimentally determine the linear sum over colors of the b quark charges. Using data from the KEK [1] and LEP [2] colliders and fitting with the expostar [3] electroweak fitting program the sum of the b quark charges is -0.86 ± 0.23.

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The energy dependence of forward-backward asymmetries is dominated by the $Z^{\circ} - \gamma$ interference. Due to this, even at the Born level the energy dependence of the b forward-backward asymmetry will be proportional to the linear sum over colors[4]. This is to be compared with the standard measurements of R and F_2 structure functions where the quadratic sum is measured. The pair of measures allows the exclusion of the quark charges depending on color[5]. Further the linear sum is the relevant measure to the required cancelation of the so called triangle anomoly[6].

The data on the b quark asymmetries were taken from the literature[1, 2] and corrected for $B - \bar{B}$ mixing where needed, using the value of ref[2]. The expostar[3] fitting program was modified to include the possibility of rescaling the b quark charge at the Born level, thus not affecting the loop calculations or the weak vector coupling. Such a procedure ensures that the fit is dominated by the energy dependence and not the small error bars of the points at the Z° peak. Fixing the top quark mass at 176.2 GeV, constraining the Z mass to $91.19 \pm .007$, and the strong coupling to $0.125 \pm .005$[7], yields the sum of the b quark charges to be -0.89 ± 0.24. Allowing the top quark mass to vary symmetrically by 29 GeV, thus varying the vector coupling yields, simultaneously, the sum of the b quark charges to be -0.86 ± 0.23 and a top quark mass of 151.9 ± 23. Note that the top quark mass error quoted is only the parabolic value. The inclusion of the $Z^{\circ} - \gamma$ boxes to the asymmetry calculations has a negligible effect on the fit results, which is reassuring as the data were all fit to a simple parabolic form.

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