How to Understand Time-Space Synchronized FDTD Algorithm

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The Time-Space Synchronized (TSS) FDTD algorithm is given at https://github.com/DavidGeUSA/TSS/blob/master/TSS.pdf. It may not be easy to understand it if one is used to think in Yee FDTD algorithm. I'll try to provide some explanations to the TSS algorithm. The explanations are based on TSS.PDF.

What TSS does is that with E(t) and H(t) known (given or estimated), estimate $E(t+\Delta t)\ and\ H(t+\Delta t)$.

One innovation involved in TSS is to use curls and **higher order** curls to estimate $E(t+\Delta t)\ and\ H(t+\Delta t)$. Equations (10) and (11) describe this process.

Another innovation involved in TSS is to estimate space derivatives, which are needed for calculating curls, using more than two space points. I'll elaborate it. Suppose we have a function

$$v(s)$$

Suppose we know two values $v(s) and\ v(s+\Delta s)$, how do we estimate $\frac{dv(s)}{ds}$? It is easy, everybody knows it,

$$\frac{dv(s)}{ds} \approx \frac{v(s+\Delta s)-v(s)}{\Delta s}$$

Now, suppose we know 3 values

$$v(s-\Delta s), v(s), v(s+\Delta s)$$

Then, how do we estimate $\frac{dv(s)}{ds}$? Let's do it starting from the Taylor's series.

$$v(s+\Delta s) \approx v(s) + \Delta s\frac{dv(s)}{ds} + \frac{1}{2}\ (\Delta s)^2 \frac{d^2 v(s)}{ds^2}$$

$$v(s-\Delta s) \approx v(s) - \Delta s\frac{dv(s)}{ds} + \frac{1}{2}\ (\Delta s)^2 \frac{d^2 v(s)}{ds^2}$$

Or

$$\begin{bmatrix} v(s+\Delta s)-v(s) \\ v(s-\Delta s)-v(s) \end{bmatrix} \approx \begin{bmatrix} 1 & \frac{1}{2} \\ -1 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} \Delta s\frac{dv(s)}{ds} \\ (\Delta s)^2 \frac{d^2 v(s)}{ds^2} \end{bmatrix}$$

Now we get our derivative estimation by calculating the inverse matrix,

$$\begin{bmatrix} \Delta s \dfrac{dv(s)}{ds} \\ (\Delta s)^2 \dfrac{d^2 v(s)}{ds^2} \end{bmatrix} \approx \begin{bmatrix} 1 & \dfrac{1}{2} \\ -1 & \dfrac{1}{2} \end{bmatrix}^{-1} \begin{bmatrix} v(s+\Delta s) - v(s) \\ v(s-\Delta s) - v(s) \end{bmatrix}$$

Now, suppose we know two more values

$$v(s-2\Delta s), v(s-\Delta s), v(s), v(s+\Delta s), v(s+2\Delta s)$$

Then, how do we estimate $\frac{dv(s)}{ds}$? We use the same technique above, just that this time it is a 4x4 matrix and involves $\frac{(\Delta s)^3 d^3 v(s)}{ds^3}$ *and* $\frac{(\Delta s)^4 d^4 v(s)}{ds^4}$.

How about using as many number of available values as you want? Remember, we know all values of E and H in the domain. Page 19 of the TSS.PDF describes it.