FDTD Tasks Test Results

Contents

Full source code is at GitHub at

https://github.com/Limnor/TSS/tree/master/Source%20Code

Yee FDTD Tasks Test Results

3D Demo From a book

This task uses parameters from a book "understanding FDTD method" by John B. Schneider. The boundary condition code and field source code are taken from the book and implemented as plugin modules. The code from the book can be found at https://github.com/john-b-schneider/uFDTD/tree/master/Code/Fdtd-3d

Task file:

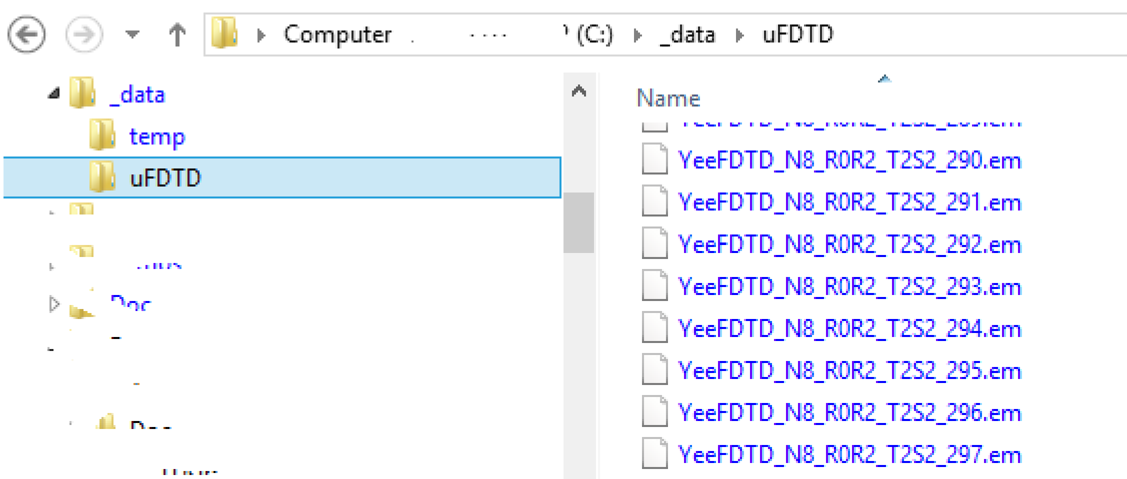
```
//this task file is for making a simulation as the book "understanding FDTD method" does.
//see https://github.com/john-b-schneider/uFDTD/tree/master/Code/Fdtd-3d
//and https://github.com/john-b-schneider/uFDTD/blob/master/Code/Fdtd-3d/3ddemo.c

//task number
SIM.TASK=100

//in grid3dhomo.c of the book, the grid size is 32, so, in one side of an axis the size is 16
//N is the number of double intervals, so, it is 8
FDTD.N=8

//half space range.
//the value can be any positive number because
//this simulation does not use Initial Value module, it does not use individual ds and dt,
//it only uses Courant number
FDTD.R=0.2

//use default base file name
SIM.BASENAME=DEF

//maximum time steps
FDTD.MAXTIMESTEP=300

//DLL file name for FDTD plugin module
SIM.FDTD_DLL=YeeFDTD.DLL

//class name for FDTD plugin module
SIM.FDTD_NAME=YeeFDTD

//DLL file for boundary condition plugin module
SIM.BC_DLL=BoundaryConditionA.dll

//class name for boundary condition plugin module
SIM.BC_NAME=AbcFirstOrder

//DLL file containing Initial Value plugin modules
```

```
SIM.IV_DLL=FieldProviders.dll

//class name of the Initial Value plugin module to be used
SIM.IV_NAME=ZeroFields

//DLL file for field source plugin module
SIM.FS_DLL=FieldSourceSamples.dll

//class name for field source plugin module, it is a Ricker source
SIM.FS_NAME=FieldSourceEz

//the points per wavelength for Ricker source
FS.PPW=2
```

A sample command line can be

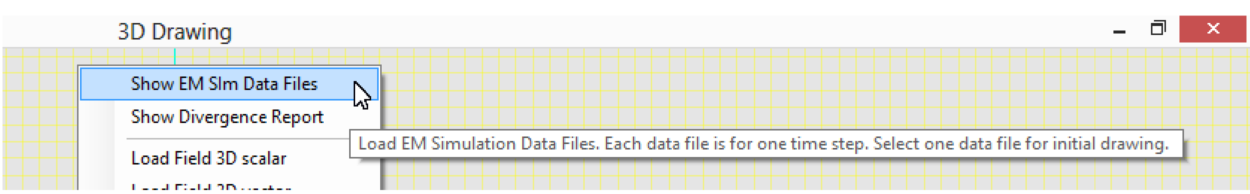
```
EM.EXE /TC:\EM\TSS\Doc\task100_uFDTD.task /WC:\_data\temp /DC:\_data\uFDTD
```

301 data files are created in the specified data folder:

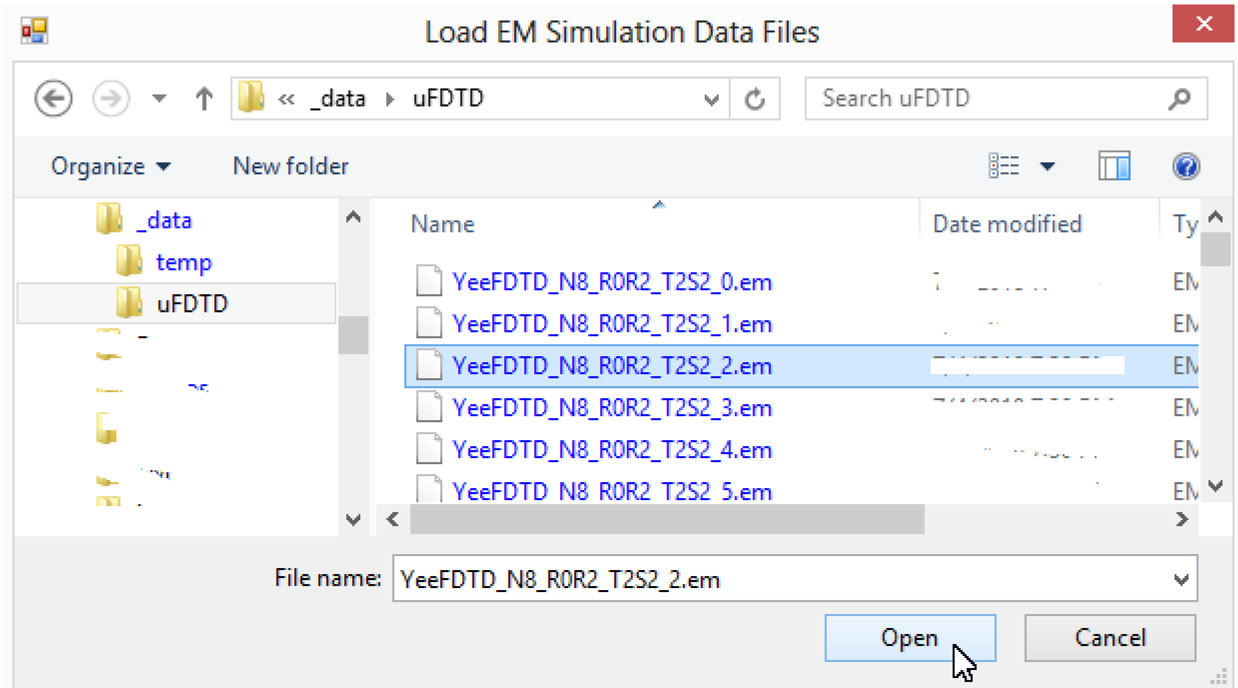


Now, run OpenGL3D.EXE to view the data files.

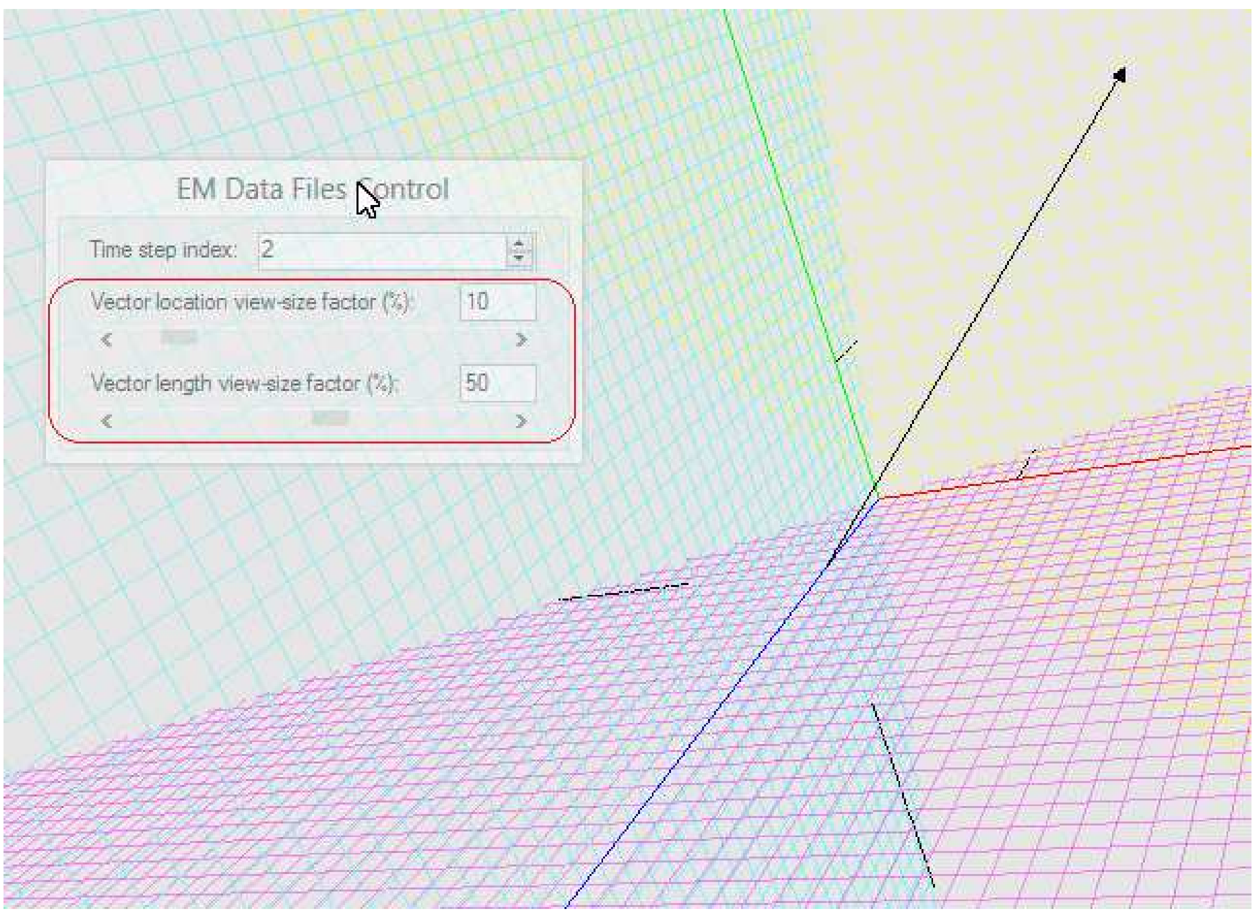
Right-click the view screen and choose “Show EM Sim Data Files”:



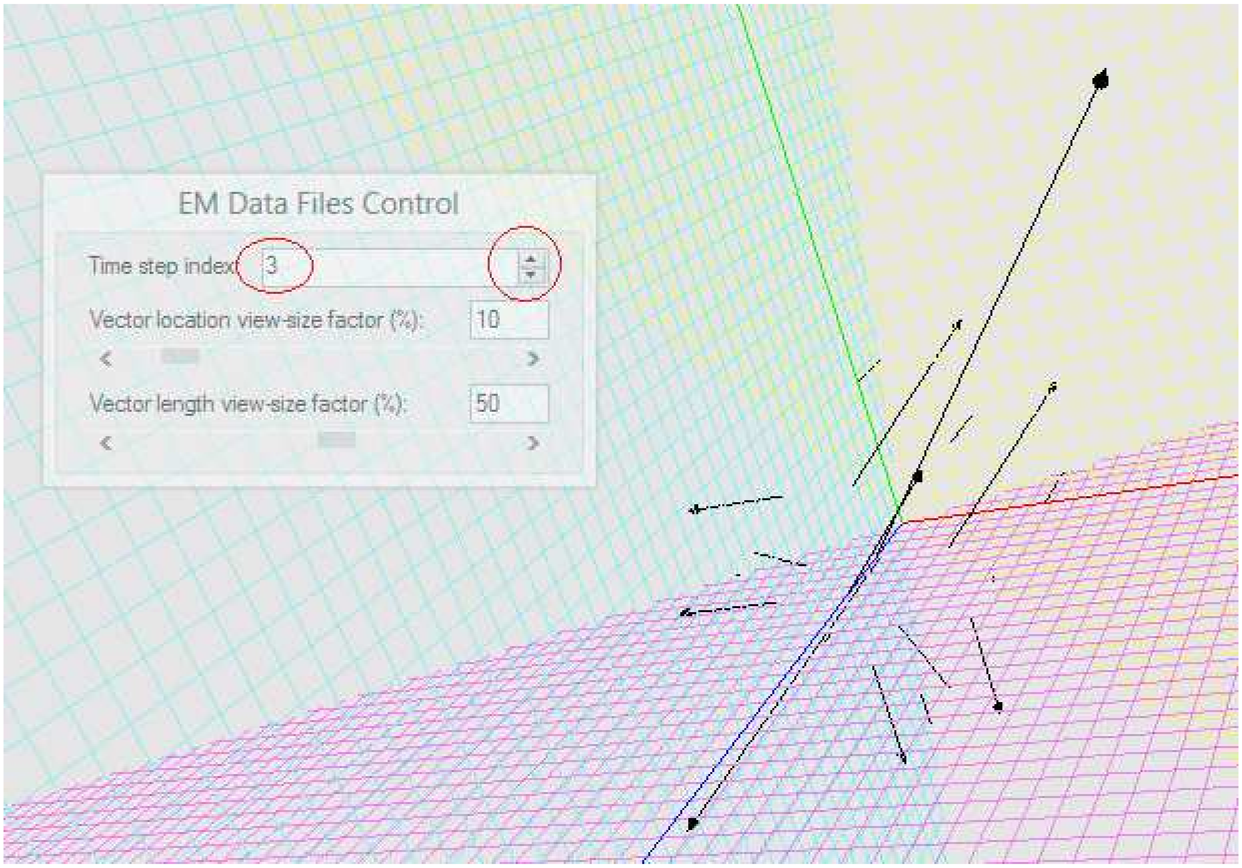
Select a data file:

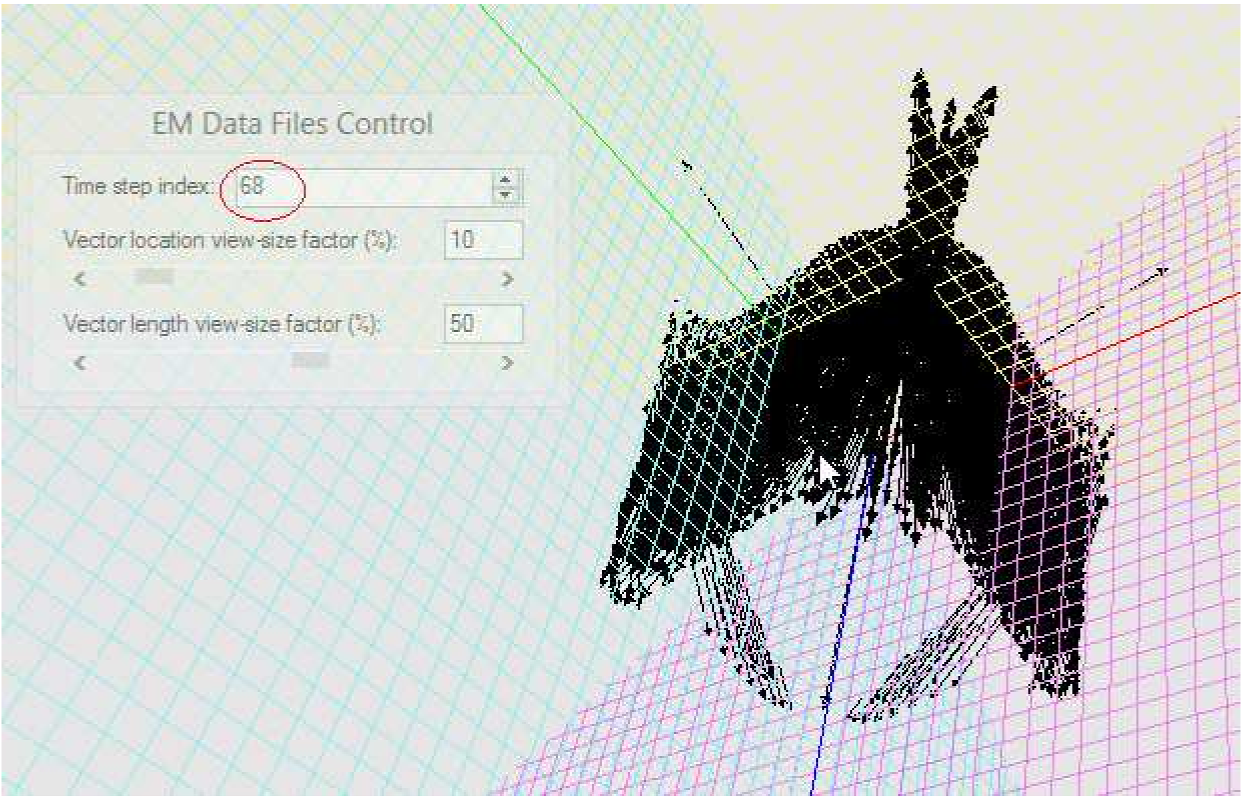


You may adjust zooming factors to get better 3D views of the field from different angles:



Changing the time step index to view field at different time:





Are these simulation results correct? Actually we don't know because we have not unit-tested each plugin modules. We don't know if the Yee FDTD module, the initial value module, the boundary condition module and the field source module are coded correctly and bug free.

Next, we will test the correctness of the Yee FDTD module.

Verify FDTD by Divergence

One way to verify any FDTD module can be to check field divergences. We may carry out a large domain simulation of a source-free field, as an open space initial value problem. Estimation errors occur from the boundary and propagate, in each simulation time step, from the boundary into the center region of the simulation space. Therefore, field divergence should be 0 in the center region of the space in a limited time steps before errors from the boundary arrive. We may use this fact to verify any FDTD algorithms.

But we cannot directly apply this idea to the Yee FDTD, because field data calculated by Yee FDTD cannot be used to calculate divergence. We will use 6 space-shifted Yee FDTD simulations to overcome this difficulty, as described below.

Note my new Time-Space-Synchronized FDTD algorithm does not have this problem, because my new algorithm estimates all 6 field components at the same space points and at the same time.

Verify Yee FDTD by Space-Shift

The Yee FDTD produces 6 field components at different space locations:

$$E_x \ is \ at \ \left(x + \frac{ds}{2}, y, z\right)$$

$$E_y \ is \ at \ \left(x, y + \frac{ds}{2}, z\right)$$

$$E_z \ is \ at \ \left(x, y, z + \frac{ds}{2}\right)$$

$$H_x \ is \ at \ \left(x, y + \frac{ds}{2}, z + \frac{ds}{2}\right)$$

$$H_y \ is \ at \ \left(x + \frac{ds}{2}, y, z + \frac{ds}{2}\right)$$

$$H_z \ is \ at \ (x + \frac{ds}{2}, y + \frac{ds}{2}, z)$$

We may use 6 Yee FDTD simulations, the first Yee FDTD shifts $-\frac{ds}{2}$ on x-axis so that its $E_x \ is \ at \ (x, y, z)$, the second Yee FDTD shifts $-\frac{ds}{2}$ on y-axis so that $E_y \ is \ at \ (x, y, z)$, and so on. Thus we get all 6 field components at the same space location (x, y, z) . Then, we can calculate field divergence at that space point.

An FDTD plugin module, YeeFDTDSpaceSynched, is developed for this purpose. We run a simulation task using this class. The task file uses YeeFDTDSpaceSynched for the FDTD module name; it uses an Initial Value module to provide fields at time 0.

```
//it uses 6 space-shifted YeeFDTD simulations to get a set of space-synchronized EM field

//task number
SIM.TASK=100

//the number of double intervals at one side of axis
// number of space points=(4N+3)^3=17373979
// memory size=833950992 bytes=0.8G
FDTD.N=64

//half space range
FDTD.R=0.2

//use default base file name
SIM.BASENAME=DEF

//maximum time steps
FDTD.MAXTIMESTEP=20

//DLL file for FDTD module
SIM.FDTD_DLL=YeeFDTD.DLL
```

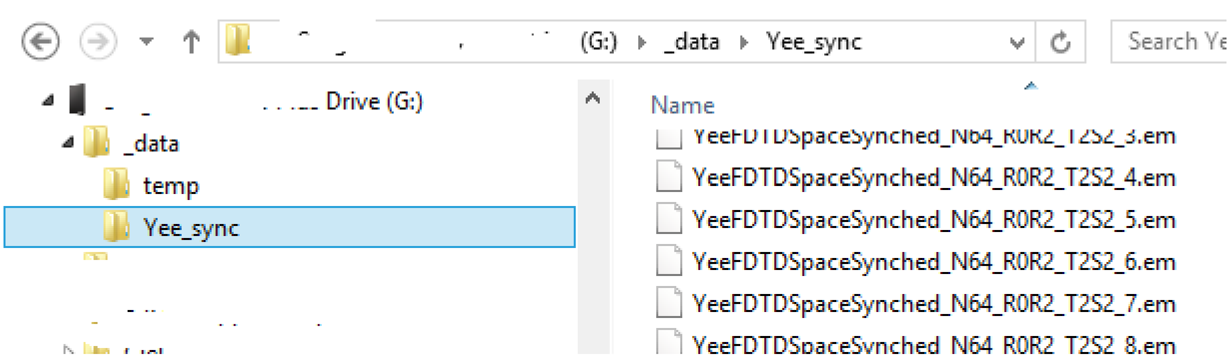
```
//class name for FDTD module
SIM.FDTD_NAME=YeeFDTDSpaceSynched

//DLL file for boundary condition module
SIM.BC_DLL=BoundaryConditionA.dll

//class name for boundary condition module
SIM.BC_NAME=VoidCondition

//DLL file containing Initial Value modules
SIM.IV_DLL=FieldProviders.dll

//class name of the Initial Value module to be used
SIM.IV_NAME=GaussianFields

//following task parameters are defined and used by class GaussianFields

//magnitude of field
IV.MAGNITUDE=120

//gaussian function width
IV.WIDTH=0.5
```

A sample command line can be

```
EM /TC:\EM\TSS\Doc\task100_Yee_sync.task /WG:\_data\temp /DG:\_data\Yee_sync
```

In this simulation, there are 17373979 space points; field memory is 0.8G. We are using 6 YeeFDTD simulations to produce one space synchronized field, totally 7 fields; so, we need 5.6G of memory. We allocate field memory in a G drive, which is USB storage.

A set of data files is created by the simulation task:



Divergence Reports

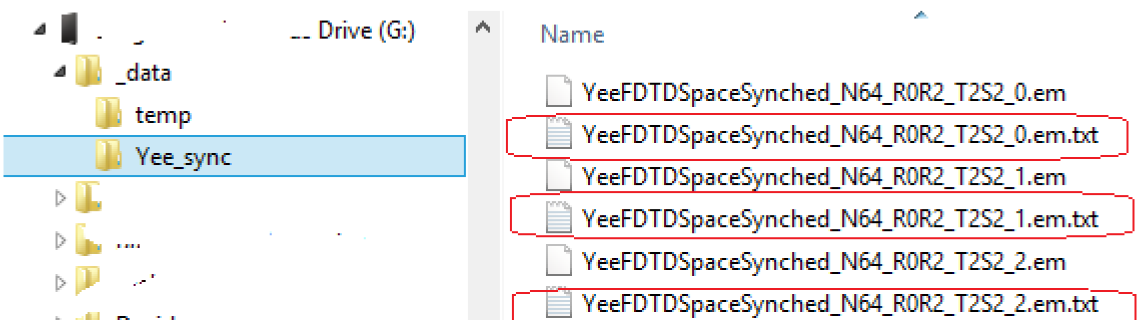
Task 120 can be used to create report files from field data files generated by task 100. The report files contain statistics on field divergences. We use following task file to do it:

```
//task number
SIM.TASK=120

//base file name
SIM.BASENAME=YeeFDTDSpaceSynched_N64_R0R2_T2S2_
```

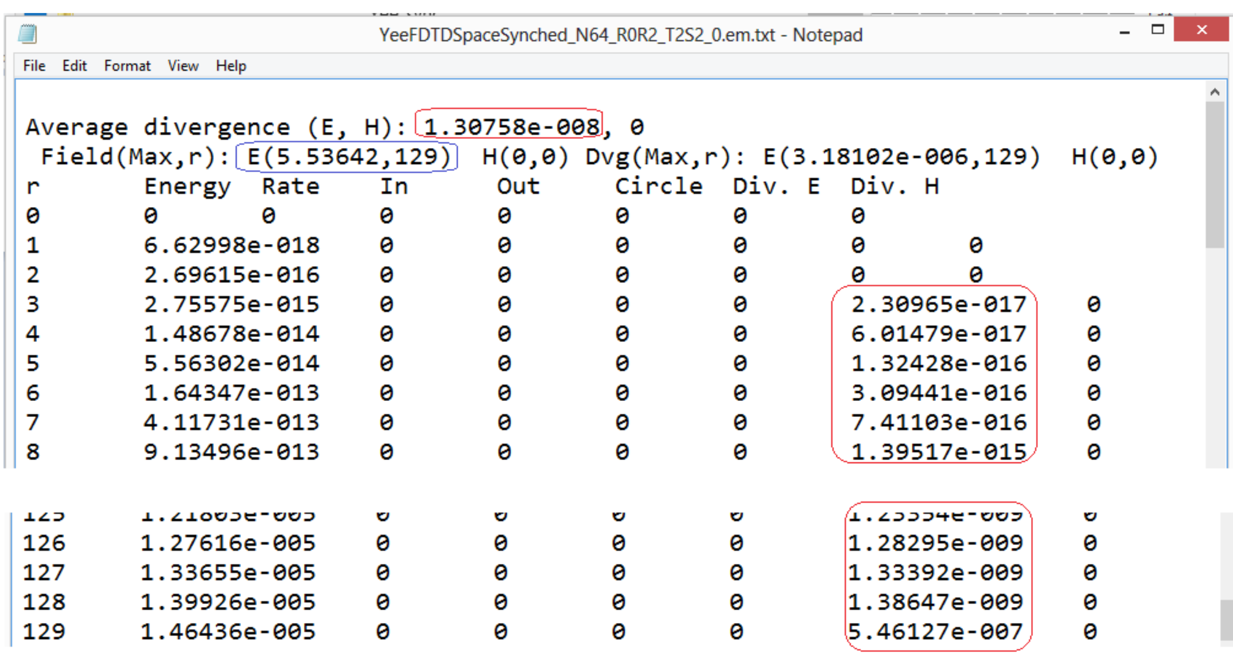
A sample command line can be:

```
EM /TC:\EM\TSS\Doc\task120_yee_sync.task /WG:\_data\temp /DG:\_data\Yee_sync
```

A report file is created for every data file by this task:



Let's examine the first report file YeeFDTDSpaceSynched_N64_R0R2_T2S2_0.em.txt:




```
Average divergence (E, H): 1.30758e-008, 0
 Field(Max,r): E(5.53642,129)  H(0,0) Dvg(Max,r): E(3.18102e-006,129)  H(0,0)
r       Energy  Rate    In      Out     Circle  Div. E  Div. H
0       0       0       0       0       0       0       0
1       6.62998e-018    0       0       0       0       0       0
2       2.69615e-016    0       0       0       0       0       0
3       2.75575e-015    0       0       0       0       2.30965e-017    0
4       1.48678e-014    0       0       0       0       6.01479e-017    0
5       5.56302e-014    0       0       0       0       1.32428e-016    0
6       1.64347e-013    0       0       0       0       3.09441e-016    0
7       4.11731e-013    0       0       0       0       7.41103e-016    0
8       9.13496e-013    0       0       0       0       1.39517e-015    0

126     1.27616e-005    0       0       0       0       1.28295e-009    0
127     1.33655e-005    0       0       0       0       1.33392e-009    0
128     1.39926e-005    0       0       0       0       1.38647e-009    0
129     1.46436e-005    0       0       0       0       5.46127e-007    0
```

The report file lists 8 pieces of data per row.

Radius	Sum of field energy	Energy transfer rate	% of inwards energy transfer	% of outwards energy transfer	% of circular energy transfer	Average divergence magnitude of electric field	Average divergence magnitude of magnetic field

We can see that the electric field divergence magnitude is in a range of $10^{-6}\sim10^{-16}$, the average divergence magnitude is 10^{-8}. Note that the electric field strength is 5.53642 at radius 129. Comparing the divergence magnitude value with the field strength value, it is safe to give a conclusion that at time 0 the field data is correct because field divergence is nearly 0.

The above field data file contains data produced by the Initial Value module. Therefore, we conclude that the Initial Value module is coded correctly; and the divergence estimation is coded correctly.

For time step index larger than 0, the data files are generated by the FDTD simulation. Let's review the report file for time step index 1:

```
YeeFDTDSpaceSynched_N64_R0R2_T2S2_1.em.txt - Notepad
File  Edit  Format  View  Help

Average divergence (E, H): 0.05936, 0.000407439
 Field(Max,r): E(6.36404,129)  H(0.0146413,129) Dvg(Max,r): E(2926.26,129)  H(0.0542797,129)
r       Energy  Rate    In      Out     Circle  Div. E  Div. H
0       0       0       0       0       0       0       0
1       1.65748e-017    4.49708e-009    100     0       0       7.17241e-018    4.27655e-016
2       3.97789e-016    1.05722e-007    100     0       0       2.82815e-017    1.96669e-015
3       3.37559e-015    7.47568e-007    100     0       4.90228e-017    7.56742e-017    4.53957e-015
4       1.67945e-014    3.06742e-006    100     0       1.59298e-017    1.02209e-016    8.14274e-015
5       6.02974e-014    9.24275e-006    100     0       0       1.75297e-016    1.2775e-014
6       1.73983e-013    2.28374e-005    100     0       0       3.4586e-016     1.84371e-014
7       4.29535e-013    4.91476e-005    100     0       0       7.446e-016      2.51278e-014
8       9.43812e-013    9.55474e-005    100     0       0       1.44325e-015    3.28474e-014

123     1.10841e-005    74.5148 100     0       0       1.13923e-009    7.46491e-012
124     1.16223e-005    77.4821 100     0       6.14153e-022    1.18563e-009    7.58153e-012
125     1.21817e-005    80.5404 100     0       5.95419e-022    1.23353e-009    7.69894e-012
126     1.27631e-005    83.6916 100     0       0       1.28294e-009    7.81711e-012
127     1.3367e-005     86.9378 100     0       0       1.33391e-009    7.93606e-012
128     1.39942e-005    90.2808 100     0       0       0.0127618       8.05577e-012
129     1.31494e-005    1706.61 2.71861 97.2814 2.89635e-023    2.5697  0.0177243
```

From this report file, we can see following observations:

- ✓ Comparing to the field strengths, both the electric field divergence and the magnetic field divergence are quite small, except at the boundary. This is an expected result.
- ✓ The boundary is at radius=129; at the boundary the electric field divergence is 2.5697; the magnetic field divergence is 0.0177243. These large magnitudes of field divergences indicate that the boundary condition acts as a field source. In this simulation, the electric and magnetic fields are assumed 0 outside of the simulation region. The large field divergences at the boundary are also expected results.

- ✓ One interesting phenomenon is that the energy transfer direction is 100% inwards. I do not know if this energy transfer direction was coded bug-free because I do not have other criteria to cross-verify the calculations.

Now let's review the data file for time step index 20:

YeeFDTDSpaceSynched_N64_R0R2_T2S2_20.em.txt - Notepad

File Edit Format View Help

```
Average divergence (E, H): 0.304578, 0.00119311
 Field(Max,r): E(13.3971,129)  H(0.0269738,120) Dvg(Max,r): E(1752.68,129)  H(0.582976,129)
r       Energy  Rate     In       Out      Circle  Div. E  Div. H
0       4.51506e-047    1.23585e-038    0       0       100     1.09559e-015    1.09267e-018
1       3.98162e-015    8.98027e-008    100     3.03161e-029    1.36859e-013    6.4763e-016     8.55016e-015
2       5.15014e-014    2.11118e-006    100     9.61524e-031    1.938e-014      1.12975e-015    3.93122e-014
3       2.50512e-013    1.49283e-005    100     0       2.85124e-015    1.34468e-015    9.07395e-014
4       7.85025e-013    6.12537e-005    100     0       1.57878e-015    1.60508e-015    1.62758e-013
5       1.92125e-012    0.00018457      100     1.10706e-032    5.51943e-016    2.2099e-015     2.55353e-013
6       4.01619e-012    0.000456043     100     5.89457e-033    2.31743e-016    2.45718e-015    3.68521e-013
7       7.52866e-012    0.000981434     100     0       3.51741e-016    2.99271e-015    5.0226e-013
8       1.30328e-011    0.001908        100     0       2.20778e-016    3.49554e-015    6.5656e-013

110     6.13677e-006    866.368 100     0       5.11558e-020    7.0955e-010     1.27951e-010
111     6.46474e-006    905.306 100     3.15236e-037    6.05381e-020    2.28103e-008    6.99613e-010
112     6.80705e-006    945.596 100     1.92397e-037    1.00527e-019    4.8361e-007     1.99965e-008
113     7.16407e-006    987.235 100     0       4.94865e-020    6.76577e-006    4.02863e-007
114     7.5346e-006     1029.75 100     0       7.93758e-020    6.54142e-005    5.33103e-006
115     7.90368e-006    1068.72 100     2.96051e-038    3.56328e-020    0.000445148     4.84709e-005
116     8.17795e-006    1079.19 100     9.05674e-038    8.23283e-020    0.00212467      0.000307698
117     8.03121e-006    1008.58 99.9998 0.000211092     5.50832e-020    0.00691774      0.00135375
118     7.25447e-006    1124.31 52.9055 47.0945 5.25417e-020    0.0143344       0.00398301
119     7.91062e-006    1309    46.3585 53.6415 2.53328e-020    0.015836        0.00715961
120     1.15598e-005    1862.38 84.113  15.887  2.0291e-020     0.0160993       0.00594671
121     1.11516e-005    1880.2  85.2065 14.7935 3.60669e-020    0.290278        0.00204513
122     9.04311e-006    1368.44 53.6349 46.3651 3.42177e-020    0.972888        0.00430136
123     1.05247e-005    1507.66 51.0901 48.9099 2.89382e-020    1.54775 0.00317386
124     1.12406e-005    1697.95 79.3997 20.6003 2.05247e-020    1.66865 0.00232039
125     1.01244e-005    1403.15 48.7183 51.2817 2.91539e-020    1.42927 0.00505379
126     1.14018e-005    1540.16 57.1512 42.8488 2.40572e-020    1.33207 0.00549572
127     1.10677e-005    1455.96 52.3705 47.6295 1.8451e-020     1.42778 0.00504342
128     1.17119e-005    1484.61 46.0047 53.9953 2.24942e-020    1.61685 0.00451828
129     1.21805e-005    1518.76 48.821  51.179  4.82159e-021    3.53939 0.00592452
```

From the report file for time step 20, we can see following observations:

- ✓ In the center region the field divergence magnitudes are quite small; it indicates accurate FDTD simulation. It is an expected result.
- ✓ The large field divergences have propagated from the boundary at radius 129 deep into a region of radius 115. It is also an expected result.
- ✓ One interesting phenomenon is that in the center region, where divergences remain 0, the energy transfer direction is 100% inwards; in the region near the boundary there are large portions of both inwards and outwards energy transfer. I need to find other criteria to verify the energy transfer direction calculations.

Viewing boundary error propagation

Since we are making a source-free simulation, magnitude of a field divergence can be considered as a magnitude of simulation error. From the report files we can see that the simulation error first occurs at the boundary, and then propagates into the center region. There is another report file which summarizes all the other report files so that we do not have to review all other report files one by one.

The task we used to generate the report files also generates a binary statistics file recording average simulation errors for each radius over time. Below is its file format:

4 bytes: integer, the maximum radius, let's denote it as {maxRadius}

((maxRadius+1) * 8) bytes: {maxRadius+1} double precision numbers, each number is an error magnitude for a radius, radius = 0, 1, …, {maxRadius}. It is for time step index =0

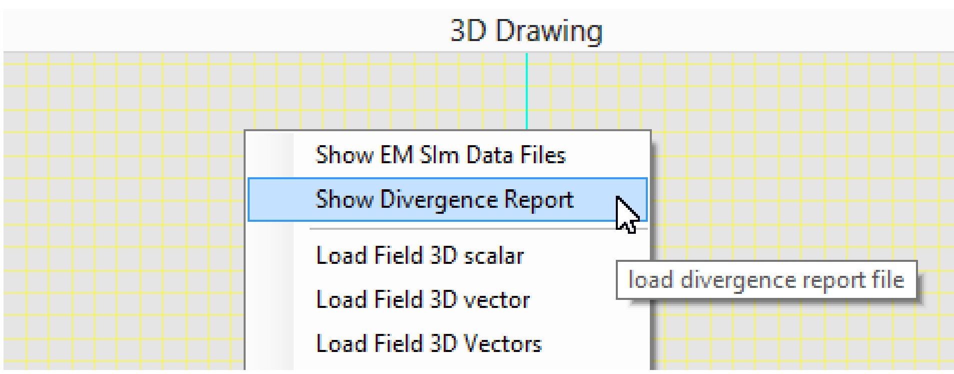
((maxRadius+1) * 8) bytes: for time step index = 1

((maxRadius+1) * 8) bytes: for time step index = 2

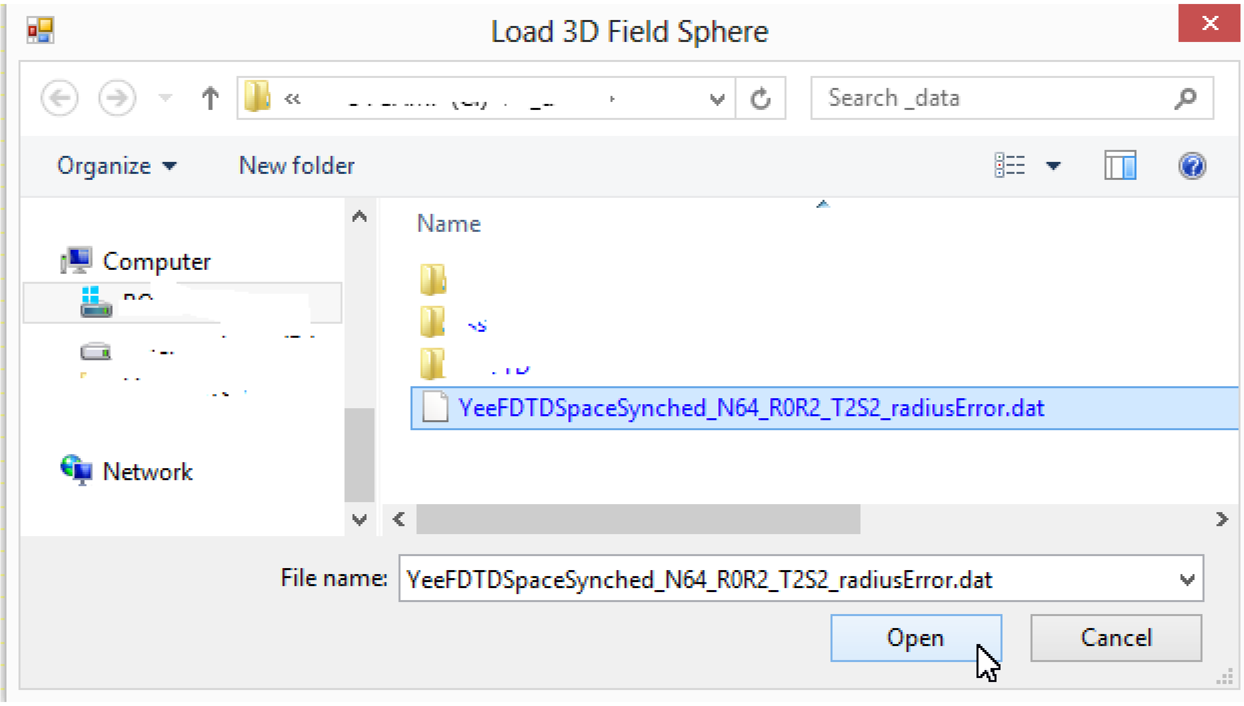
… and so on…

We can use the OpenGL3D program to view this file.

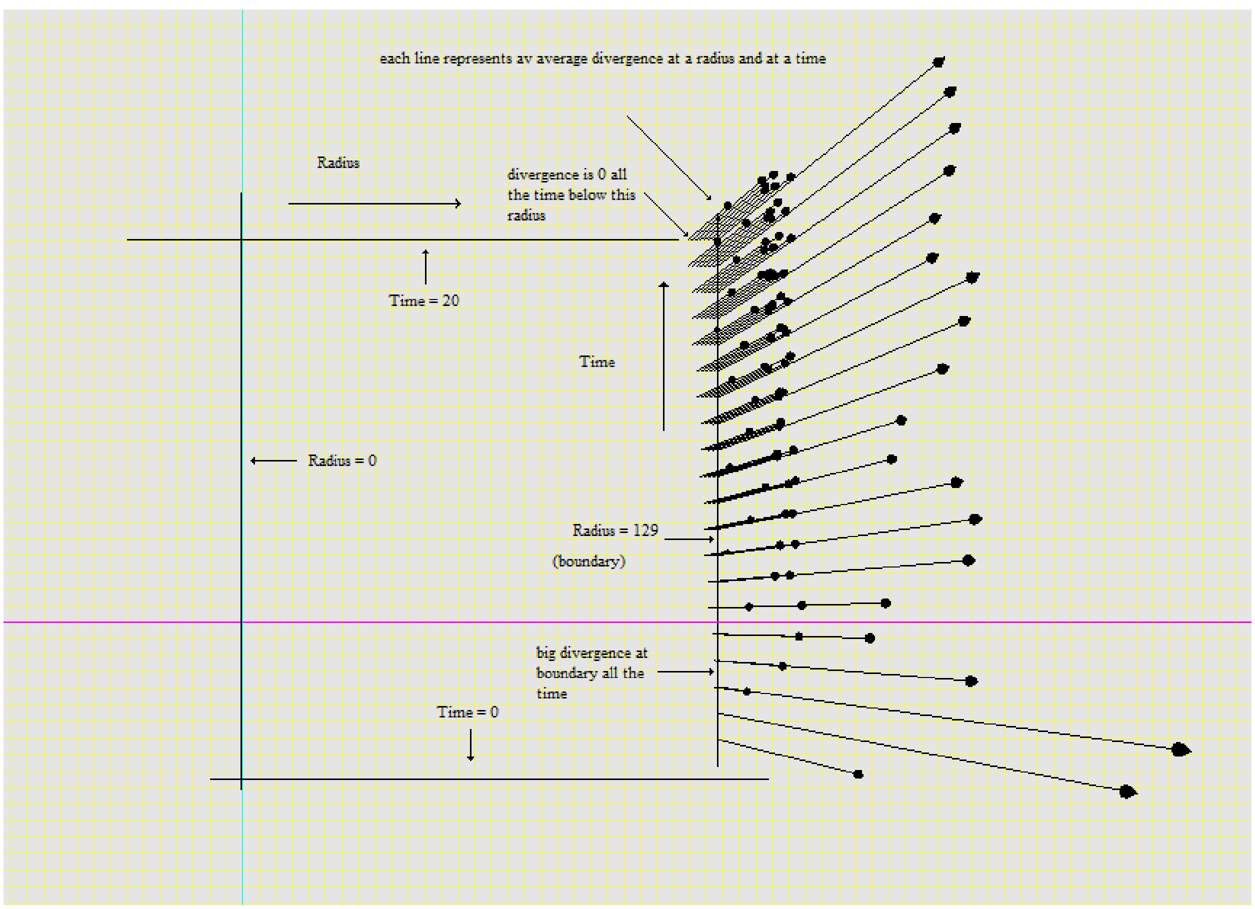
Right-click on the 3D view screen and choose "Show Divergence Report"



Select the report file:



Below is a 3D view of the report file,



This 3D drawing shows divergence statistics on radius and at all 20 time steps.

- At the boundary of radius = 129, the divergence is quite large for all 20 time steps. It shows constant simulation errors at the boundary.
- For most of smaller radius, the divergence remains 0 for all 20 time steps. It proves coding correctness of related modules, including the Initial Value module, the Yee FDTD module, the Space-shifted Yee FDTD module, and the simulation module.
- With time increasing, non-zero divergence appears gradually for radiuses away from the boundary. That is, the error at the boundary propagates into the center region.

View 3D field evolves over time

The report files confirm that the field simulation data are accurate away from the boundary. We can view 3D field simulation and see visually how fields evolve over time.

We cannot directly view the field like we did for the first sample. In this sample there are 17373979 space points contained in data file. The simulation program used file mapping to handle large size memories in our C++ code. But our OpenGL3D is a C# program, and C# types the program used cannot handle such big size. I did not try to overcome this memory size problem in C# because showing 17373979 space points on a 3D view will be too dense to see anything clearly. An approach I took was to remove most of space points and leave those points enough to show characteristics of the fields.

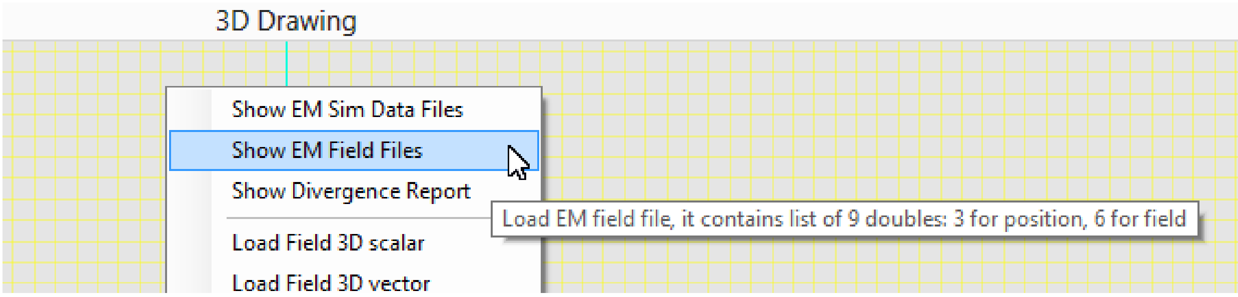
Which space points to remove is a big question. I choose to keep the space points with the largest field magnitudes. The approach is simple: given a desired number of space points to keep, go through all the space points and keep those with the largest field strengths.

A task is created to do the job. The task file is presented below.

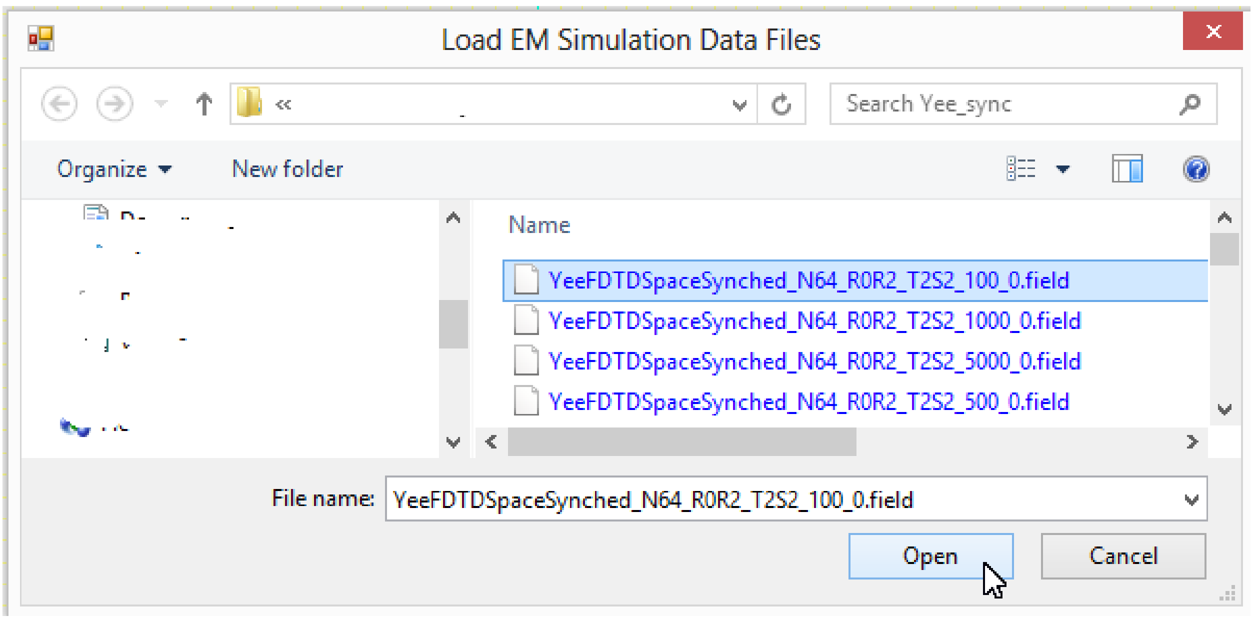
```
//task number
SIM.TASK=130

//half space range; it must match the data files
FDTD.R=0.2

//base file name
SIM.BASENAME=YeeFDTDSpaceSynched_N64_R0R2_T2S2_

//number of field points to pick
SIM.POINTS=100

//maximum number of data files to process, use 0 to process all data files
SIM.MAXTIMES=1
```

Use “SIM.MAXTIMES=1” to create only one field file for time =0 so taht we can try different values of “SIM.POINTS” to see how the field looks like. Use OpenGL3D to view files generated by this task:

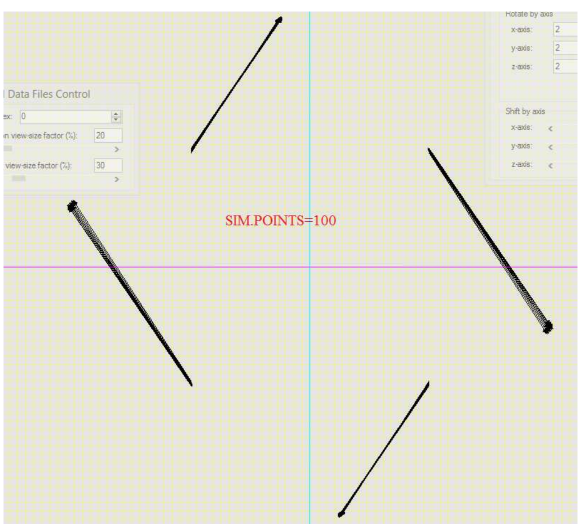
Run the OpenGL3D program; right-click the view screen, choose “Show EM Sim Data Files”

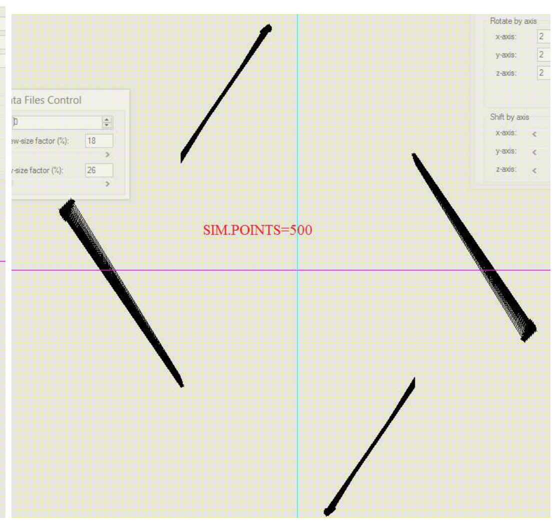


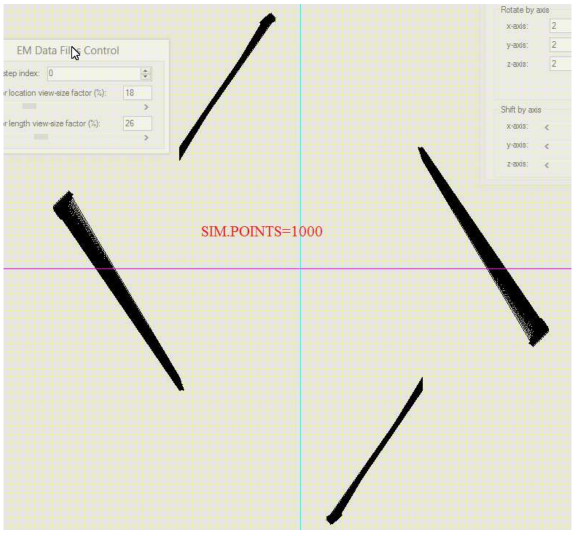
Select a field file:

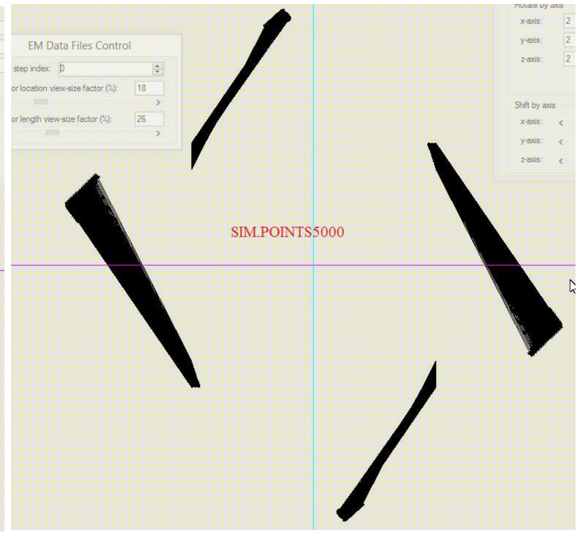


I tried 100, 500, 1000, and 5000; for these numbers of points, the field looks like:





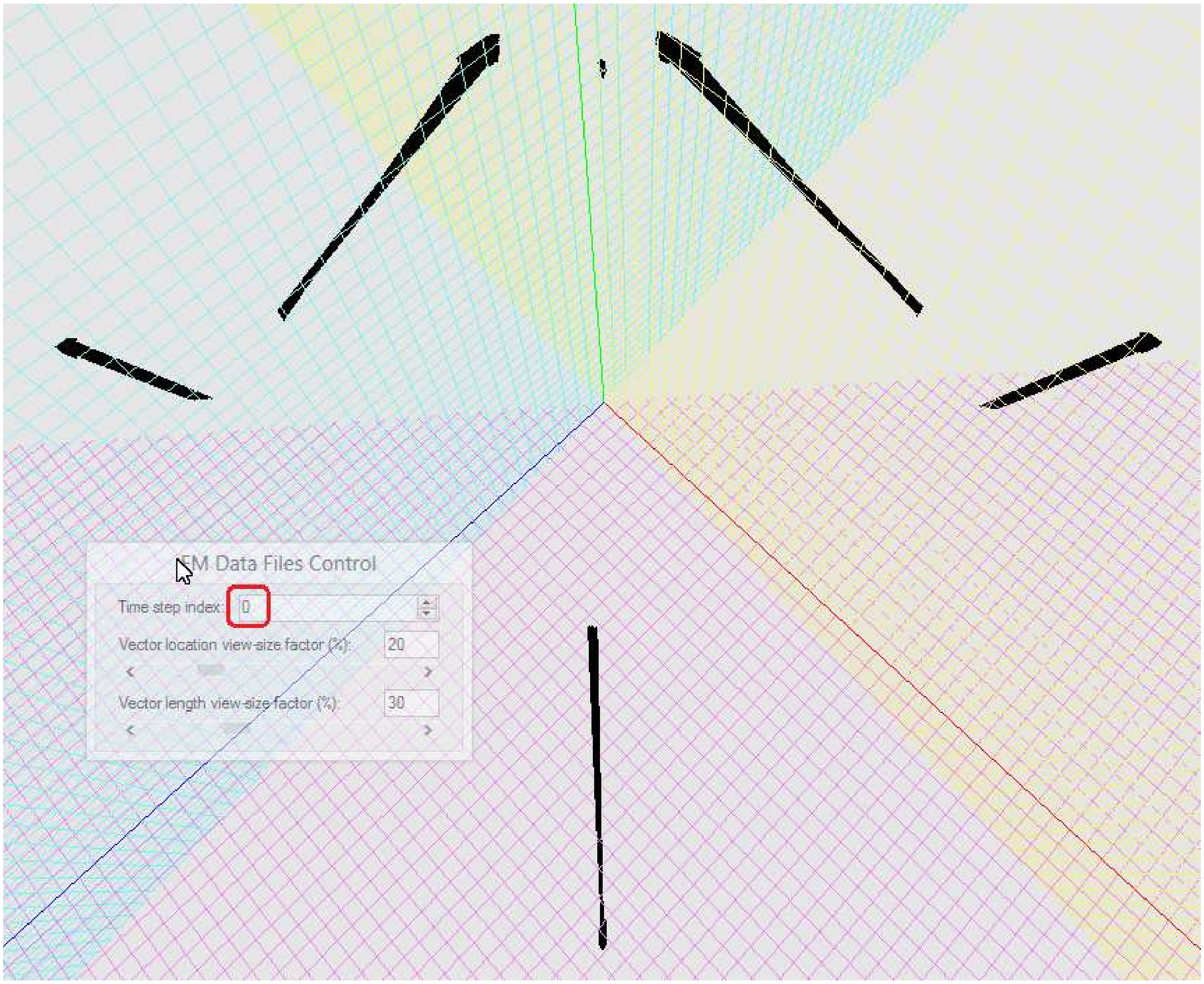


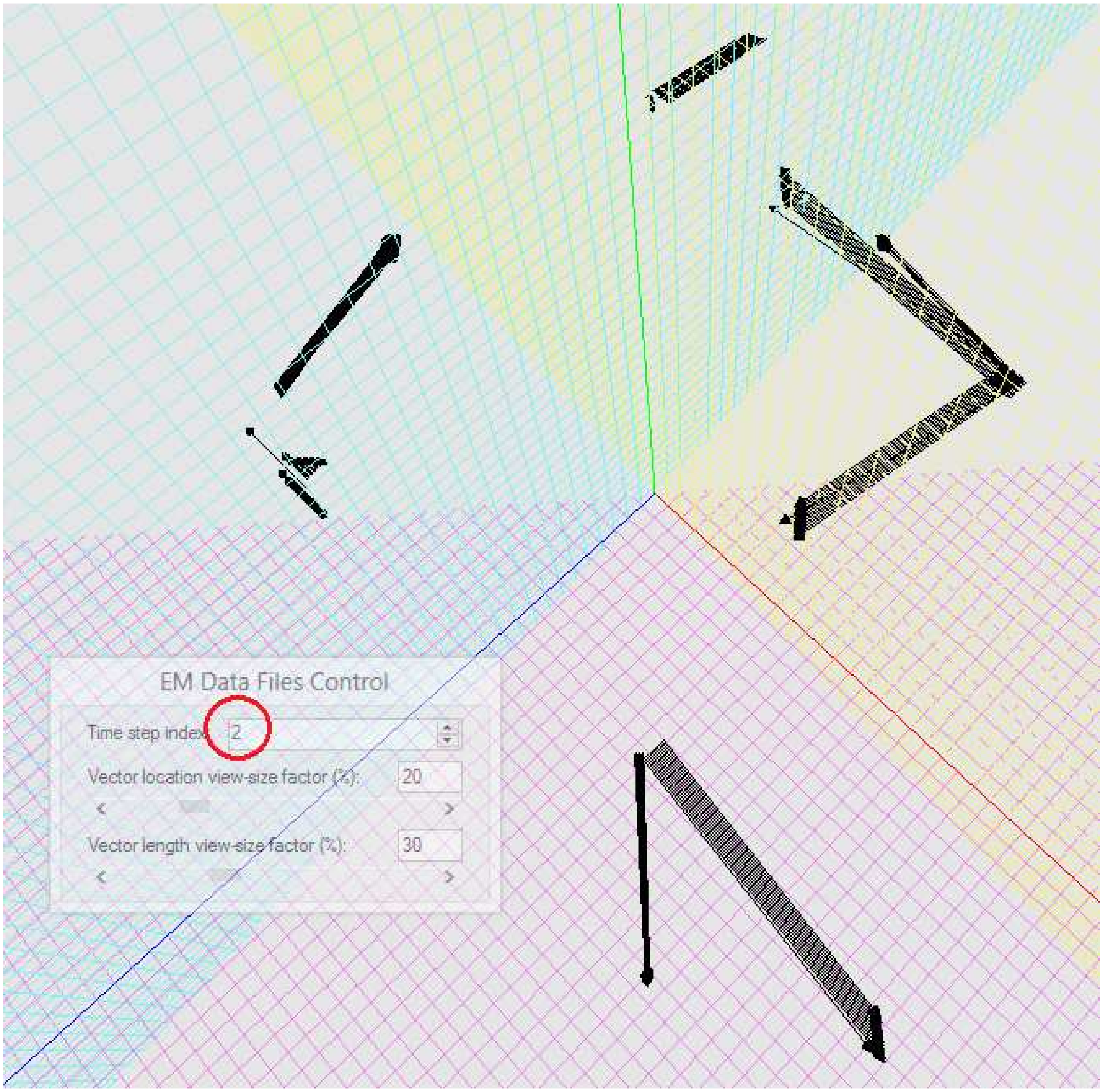


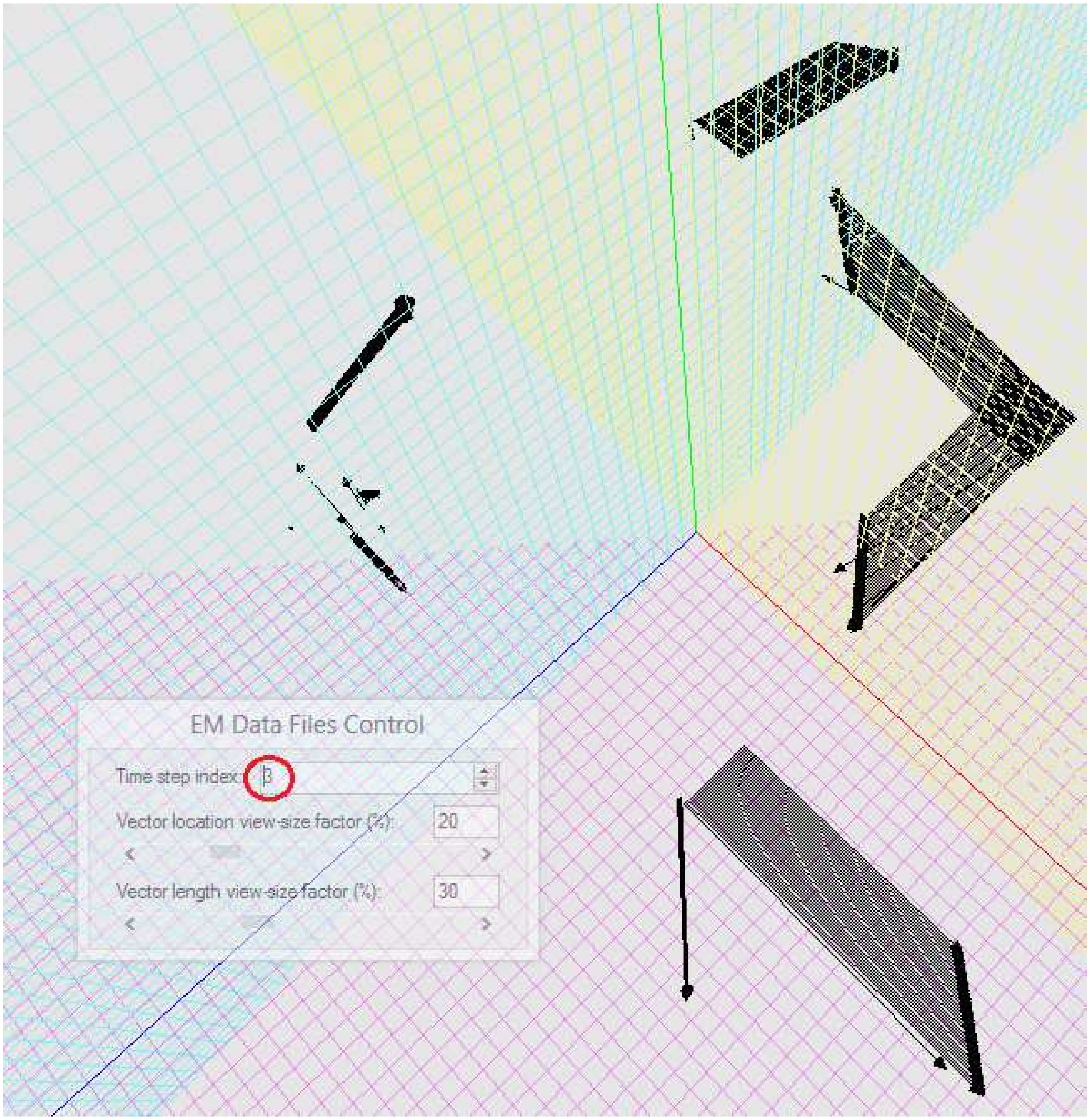
I choose 500 to run the task for all data files:

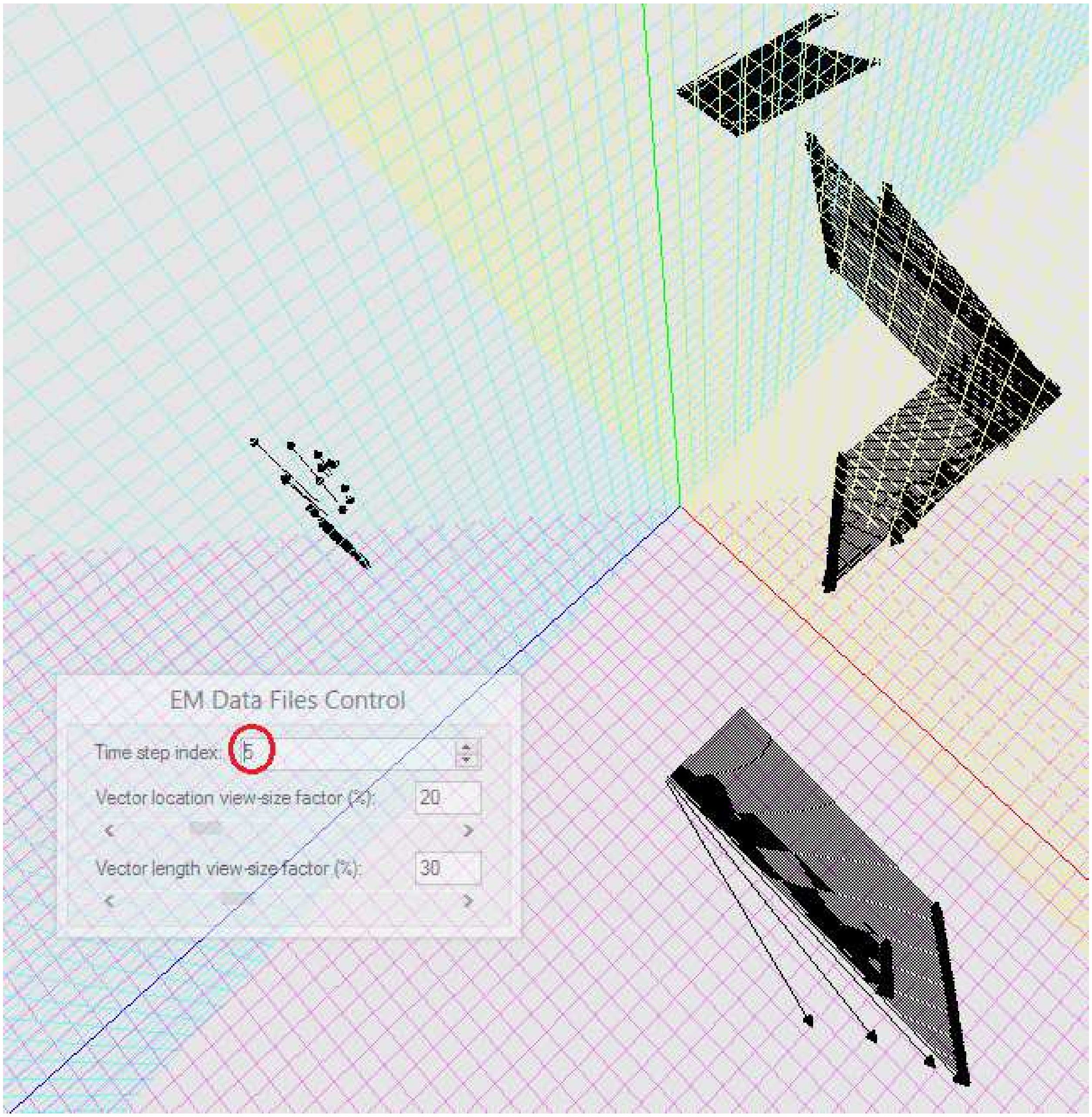
```
//task number
SIM.TASK=130

//half space range; it must match the data files
FDTD.R=0.2

//base file name
SIM.BASENAME=YeeFDTDSpaceSynched_N64_R0R2_T2S2_

//number of field points to pick
SIM.POINTS=500

//maximum number of data files to process, use 0 to process all data files
SIM.MAXTIMES=0
```

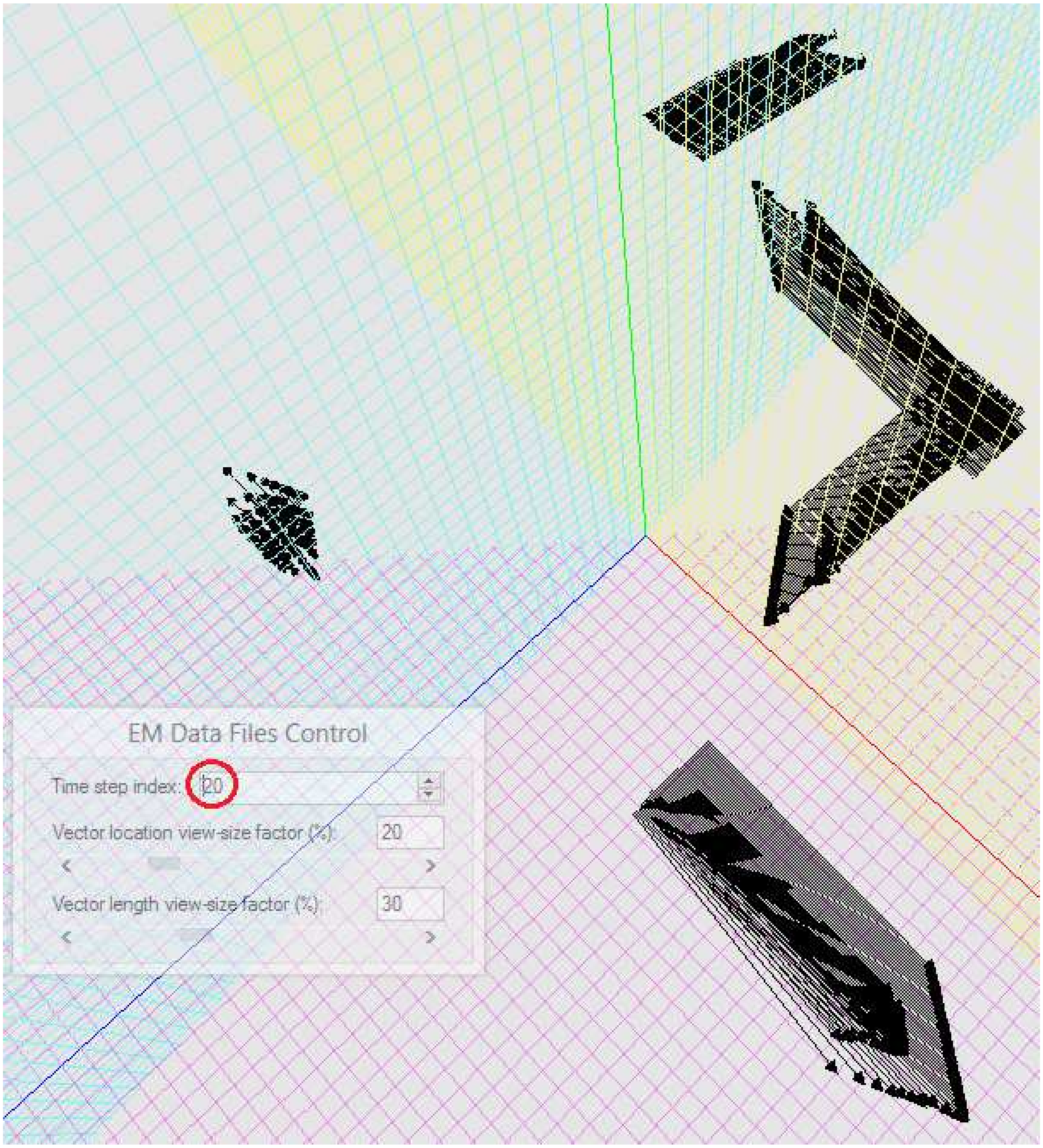
The OpenGL3D allows us to change time step index to view field evolve over time:


EM Data Files Control
Time step index:
0
Vector location view-size factor (%):
20
Vector length view-size factor (%):
30


EM Data Files Control
Time step index
2
Vector location view-size factor (%):
20
Vector length view-size factor (%):
30


EM Data Files Control
Time step index:
3
Vector location view-size factor (%):
20
Vector length view-size factor (%):
30


EM Data Files Control
Time step index:
5
Vector location view-size factor (%):
20
Vector length view-size factor (%):
30



TSS FDTD Tasks Test Results

Verify TSS Algorithm

First, we need to verify that the TSS algorithm is correct and the TSS module is coded bug-free. We do it in the same way as we did for the Yee FDTD module. We do a source-free simulation and examine the magnitudes of field divergences in the region away from the boundary.

Make a Source-Free TSS Simulation

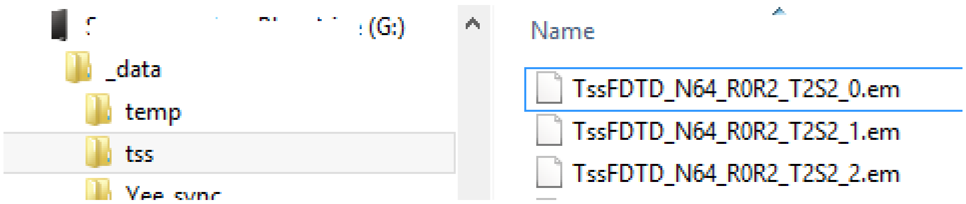
The same task file for the Yee FDTD test is used and only substitutes the FDTD plugin module with the TSS FDTD module:

```
//task number
SIM.TASK=100

//the number of double intervals at one side of axis
// number of space points=(4N+3)^3=17373979
// memory size=833950992 bytes=0.8G
FDTD.N=64

//half space range
FDTD.R=0.2

//use default base file name
SIM.BASENAME=DEF

//maximum time steps
FDTD.MAXTIMESTEP=20

//DLL file for FDTD module
SIM.FDTD_DLL=TssFDTD.DLL

//class name for FDTD module
SIM.FDTD_NAME=TssFDTD

//DLL file for boundary condition module
SIM.BC_DLL=BoundaryConditionA.dll

//class name for boundary condition module
SIM.BC_NAME=VoidCondition

//DLL file containing Initial Value modules
SIM.IV_DLL=FieldProviders.dll

//class name of the Initial Value module to be used
SIM.IV_NAME=GaussianFields

//following task parameters are defined and used by class GaussianFields

//magnitude of field
IV.MAGNITUDE=120

//gaussian function width
IV.WIDTH=0.5
```

A sample command line can be

```
EM /TC:\EM\TSS\Doc\task100_TSS.task /WG:\_data\temp /DG:\_data\tss
```

A set of data files are created:



Create Divergence Reports

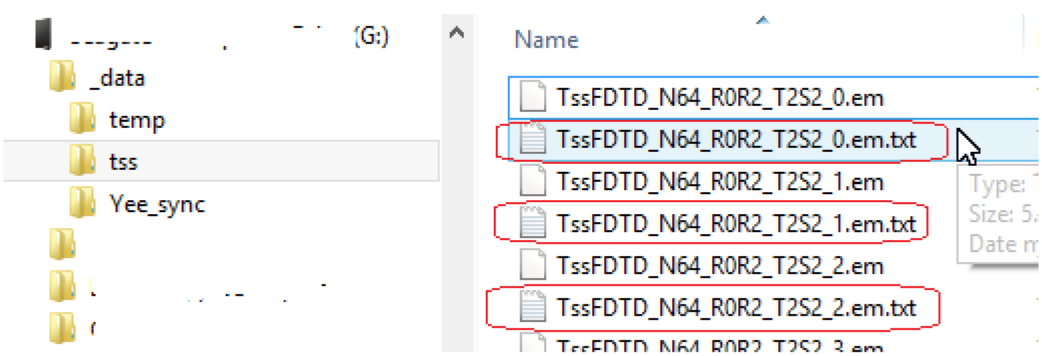
Run **task 120** to generate report files from the data files. The task file is

```
//task number
SIM.TASK=120

//base file name
SIM.BASENAME=TssFDTD_N64_R0R2_T2S2_

//half space range; it must match the data files
FDTD.R=0.2

//half estimation order for divergence estimations. Default value is 1
FDTD.HALF_ORDER_SPACE=1
```

Command line to execute the task file:

```
EM /TC:\EM\TSS\Doc\task120_TSS.task /WG:\_data\temp /DG:\_data\tss
```

A report file is created for each data file. A summary file is also created:



Examine Report files

I expected that these report files should be quite similar to the report files generated by Yee FDTD simulations. But by examining these report file, to my big surprise, TSS produces extraordinary results comparing to the Yee algorithm.

Report file for time 0

The report file for time 0 is identical to the report file for Yee FDTD simulation. This is an expected result because the field data is generated by the Initial Value module, not by Yee FDTD and not by TSS FDTD. Since the tasks used the same Initial Value module, the fields at time 0 for the two simulations must be identical.

This result just confirms that there is not a bug in the involved coding.

Report file for time 1

Fields at time 1 is generated by the TSS algorithm. Let's examine its divergence report file:

TssFDTD_N64_R0R2_T2S2_1.em.txt - Notepad

File Edit Format View Help

```
Average divergence (E, H): 1.30758e-008, 1.24083e-018
 Field(Max,r): E(5.53642,129)  H(0.000113125,129) Dvg(Max,r): E(3.18102e-006,129)  H(2.79997e-017,129)
r       Energy  Rate    In      Out     Circle  Div. E  Div. H
0       0       0       0       0       0       0       0
1       1.65749e-017    4.4971e-009     100     0       0       0       3.15194e-020
2       3.97791e-016    1.05723e-007    100     0       0       0       1.39371e-020
3       3.37562e-015    7.47571e-007    100     0       0       2.30965e-017    6.01471e-020
4       1.67947e-014    3.06743e-006    100     0       0       6.01479e-017    8.2092e-020
5       6.0298e-014     9.2428e-006     100     0       0       1.32428e-016    9.61987e-020
6       1.73985e-013    2.28375e-005    100     0       0       3.09441e-016    1.13557e-019
7       4.29539e-013    4.91478e-005    100     0       0       7.41103e-016    1.05974e-019
8       9.43822e-013    9.55478e-005    100     0       0       1.39517e-015    1.17757e-019

126     1.27632e-005    83.692  100     0       0       1.28295e-009    1.49979e-018
127     1.33672e-005    86.9382 100     0       0       1.33392e-009    1.50477e-018
128     1.39944e-005    90.2812 100     0       0       1.38647e-009    1.50876e-018
129     1.46455e-005    93.7232 100     0       0       5.46127e-007    4.01325e-018
```

The last row is the data for the boundary. Note that the divergence magnitudes at the boundary are very small, indicating very small errors at the boundary.

If you are not impressed by this result then let's recall the report file for the Yee FDTD:

YeeFDTDSpaceSynched_N64_R0R2_T2S2_1.em.txt - Notepad

File Edit Format View Help

```
Average divergence (E, H): 0.05936, 0.000407439
 Field(Max,r): E(6.36404,129)  H(0.0146413,129) Dvg(Max,r): E(2926.26,129)  H(0.0542797,129)
r       Energy  Rate    In      Out     Circle  Div. E  Div. H
0       0       0       0       0       0       0       0
1       1.65748e-017    4.49708e-009    100     0       0       7.17241e-018    4.27655e-016
2       3.97789e-016    1.05722e-007    100     0       0       2.82815e-017    1.96669e-015
3       3.37559e-015    7.47568e-007    100     0       4.90228e-017    7.56742e-017    4.53957e-015
4       1.67945e-014    3.06742e-006    100     0       1.59298e-017    1.02209e-016    8.14274e-015
5       6.02974e-014    9.24275e-006    100     0       0       1.75297e-016    1.2775e-014
6       1.73983e-013    2.28374e-005    100     0       0       3.4586e-016     1.84371e-014
7       4.29535e-013    4.91476e-005    100     0       0       7.446e-016      2.51278e-014
8       9.43817e-013    9.55474e-005    100     0       0       1.44325e-015    3.28474e-014

123     1.10841e-005    74.5148 100     0       0       1.13925e-009    7.46491e-012
124     1.16223e-005    77.4821 100     0       6.14153e-022    1.18563e-009    7.58153e-012
125     1.21817e-005    80.5404 100     0       5.95419e-022    1.23353e-009    7.69894e-012
126     1.27631e-005    83.6916 100     0       0       1.28294e-009    7.81711e-012
127     1.3367e-005     86.9378 100     0       0       1.33391e-009    7.93606e-012
128     1.39942e-005    90.2808 100     0       0       0.0127618       8.05577e-012
129     1.31494e-005    1706.61 2.71861 97.2814 2.89635e-023    2.5697  0.0177243
```

We can see that for Yee FDTD, the electric field divergence is **2.5697**, for TSS FDTD, the electric field divergence is $\mathbf{5.46127 * 10^{-7}}$. Because we are simulating source-free fields, the magnitudes of field divergences represent magnitudes of simulation errors. The report file shows that TSS algorithm is very accurate at the boundary.

Report file for time 20

Report file for time 20 is:

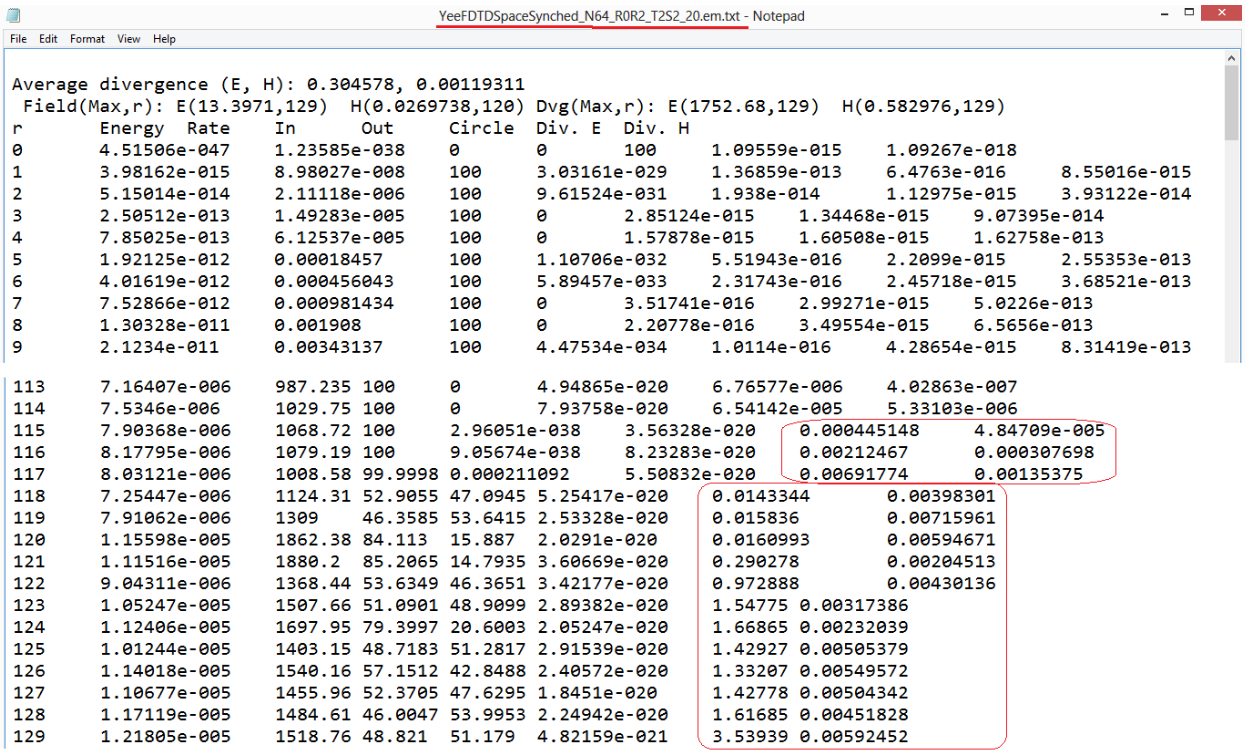
TssFDTD_N64_R0R2_T2S2_20.em.txt - Notepad

File Edit Format View Help

```
Average divergence (E, H): 1.30758e-008, 6.2468e-017
 Field(Max,r): E(5.5307,129)  H(0.00226189,129) Dvg(Max,r): E(3.18101e-006,129)  H(1.40518e-015,129)
r       Energy  Rate    In      Out     Circle  Div. E  Div. H
0       0       0       0       0       0       0       1.09267e-018
1       3.98204e-015    8.98174e-008    100     0       0       0       2.52155e-019
2       5.15068e-014    2.11153e-006    100     0       0       4.28149e-017    1.44946e-018
3       2.50538e-013    1.49307e-005    100     0       0       8.98197e-017    1.93473e-018
4       7.85109e-013    6.12637e-005    100     0       0       1.44935e-016    2.38916e-018
5       1.92146e-012    0.0001846       100     0       0       2.84719e-016    2.57014e-018
6       4.01663e-012    0.000456118     100     0       0       5.09221e-016    3.57831e-018
7       7.5295e-012     0.000981596     100     0       0       9.86989e-016    4.21486e-018
8       1.30343e-011    0.00190831      100     0       0       1.58222e-015    5.19197e-018

119     9.6497e-006     1268.63 100     0       0       9.67762e-010    7.26461e-017
120     1.01255e-005    1321    100     0       0       1.00857e-009    7.31291e-017
121     1.06205e-005    1375.04 100     0       0       1.05073e-009    7.29552e-017
122     1.11352e-005    1430.78 100     0       0       1.09428e-009    7.35204e-017
123     1.16703e-005    1488.27 100     0       0       1.13923e-009    7.37822e-017
124     1.22265e-005    1547.53 100     0       0       1.18564e-009    7.40001e-017
125     1.28042e-005    1608.62 100     0       0       1.23354e-009    7.43545e-017
126     1.34043e-005    1671.57 100     0       0       1.28295e-009    7.49528e-017
127     1.40273e-005    1736.41 100     0       0       1.33392e-009    7.53984e-017
128     1.4674e-005     1803.19 100     0       0       1.38647e-009    7.6165e-017
129     1.5345e-005     1871.95 100     0       0       5.46127e-007    1.9957e-016
```

Comparing to time 1, we can see that the magnitudes of divergences do not increase after 20 time steps. This is a quite surprising result because I expected that the errors at the boundary should have propagated to lower radius over time, like the Yee FDTD algorithm did.

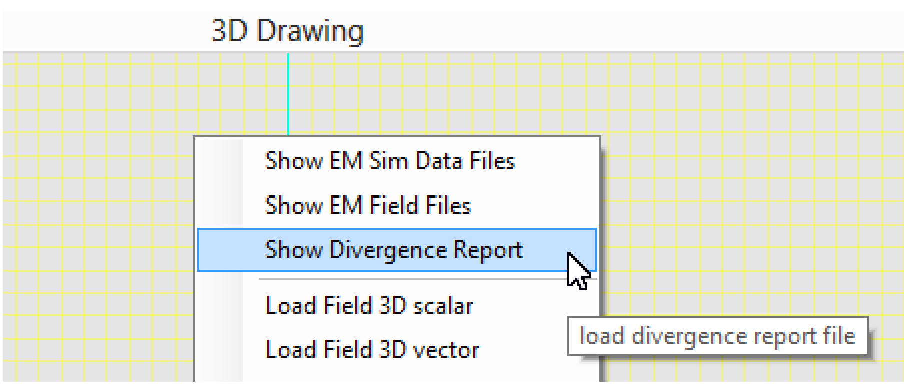
Recall the report file of the Yee FDTD:



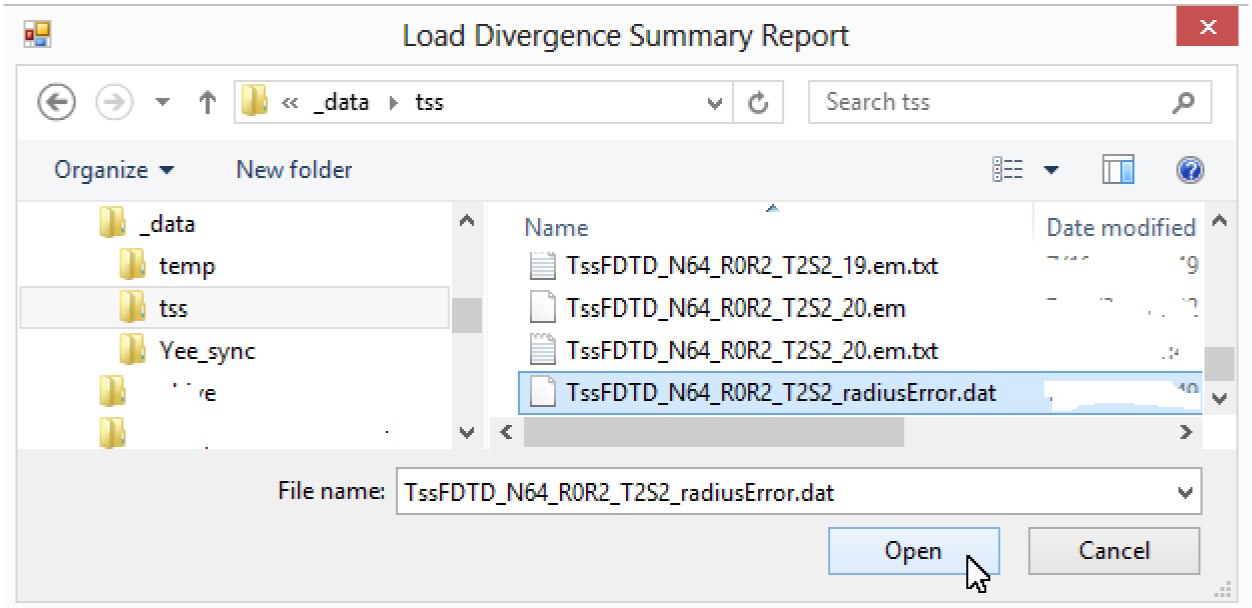
```
YeeFDTDSpaceSynched_N64_R0R2_T2S2_20.em.txt - Notepad
File Edit Format View Help

Average divergence (E, H): 0.304578, 0.00119311
 Field(Max,r): E(13.3971,129)  H(0.0269738,120) Dvg(Max,r): E(1752.68,129)  H(0.582976,129)
r       Energy  Rate     In       Out      Circle  Div. E  Div. H
0       4.51506e-047    1.23585e-038    0       0       100     1.09559e-015    1.09267e-018
1       3.98162e-015    8.98027e-008    100     3.03161e-029    1.36859e-013    6.4763e-016     8.55016e-015
2       5.15014e-014    2.11118e-006    100     9.61524e-031    1.938e-014      1.12975e-015    3.93122e-014
3       2.50512e-013    1.49283e-005    100     0       2.85124e-015    1.34468e-015    9.07395e-014
4       7.85025e-013    6.12537e-005    100     0       1.57878e-015    1.60508e-015    1.62758e-013
5       1.92125e-012    0.00018457      100     1.10706e-032    5.51943e-016    2.2099e-015     2.55353e-013
6       4.01619e-012    0.000456043     100     5.89457e-033    2.31743e-016    2.45718e-015    3.68521e-013
7       7.52866e-012    0.000981434     100     0       3.51741e-016    2.99271e-015    5.0226e-013
8       1.30328e-011    0.001908        100     0       2.20778e-016    3.49554e-015    6.5656e-013
9       2.1234e-011     0.00343137      100     4.47534e-034    1.0114e-016     4.28654e-015    8.31419e-013

113     7.16407e-006    987.235 100     0       4.94865e-020    6.76577e-006    4.02863e-007
114     7.5346e-006     1029.75 100     0       7.93758e-020    6.54142e-005    5.33103e-006
115     7.90368e-006    1068.72 100     2.96051e-038    3.56328e-020    0.000445148     4.84709e-005
116     8.17795e-006    1079.19 100     9.05674e-038    8.23283e-020    0.00212467      0.000307698
117     8.03121e-006    1008.58 99.9998 0.000211092     5.50832e-020    0.00691774      0.00135375
118     7.25447e-006    1124.31 52.9055 47.0945 5.25417e-020    0.0143344       0.00398301
119     7.91062e-006    1309    46.3585 53.6415 2.53328e-020    0.015836        0.00715961
120     1.15598e-005    1862.38 84.113  15.887  2.0291e-020     0.0160993       0.00594671
121     1.11516e-005    1880.2  85.2065 14.7935 3.60669e-020    0.290278        0.00204513
122     9.04311e-006    1368.44 53.6349 46.3651 3.42177e-020    0.972888        0.00430136
123     1.05247e-005    1507.66 51.0901 48.9099 2.89382e-020    1.54775 0.00317386
124     1.12406e-005    1697.95 79.3997 20.6003 2.05247e-020    1.66865 0.00232039
125     1.01244e-005    1403.15 48.7183 51.2817 2.91539e-020    1.42927 0.00505379
126     1.14018e-005    1540.16 57.1512 42.8488 2.40572e-020    1.33207 0.00549572
127     1.10677e-005    1455.96 52.3705 47.6295 1.8451e-020     1.42778 0.00504342
128     1.17119e-005    1484.61 46.0047 53.9953 2.24942e-020    1.61685 0.00451828
129     1.21805e-005    1518.76 48.821  51.179  4.82159e-021    3.53939 0.00592452
```

For the Yee FDTD algorithm, after 20 time steps, the big errors at the boundary of radius 129 propagate to radiuses as low as 115.

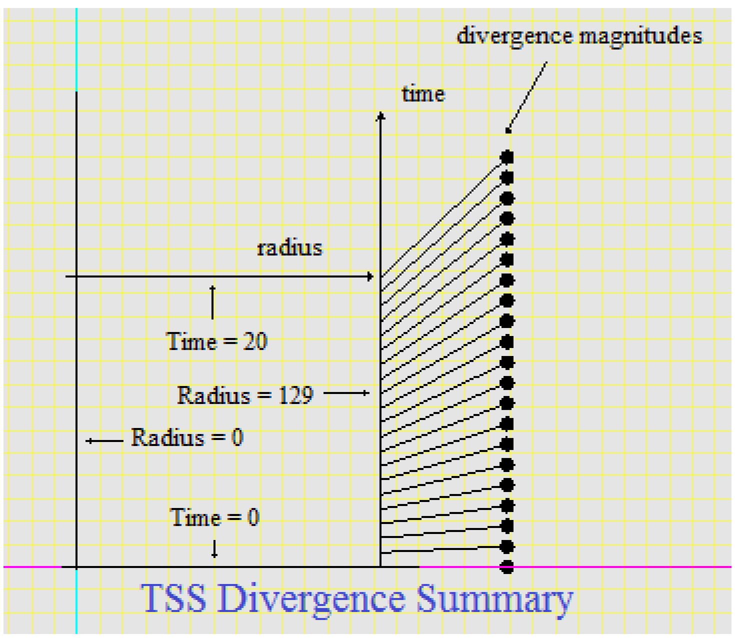
We may use OpenGL3D to view the summary report file. Right-click on the view screen, choose “Show Divergence Report”:



Select the summary file:

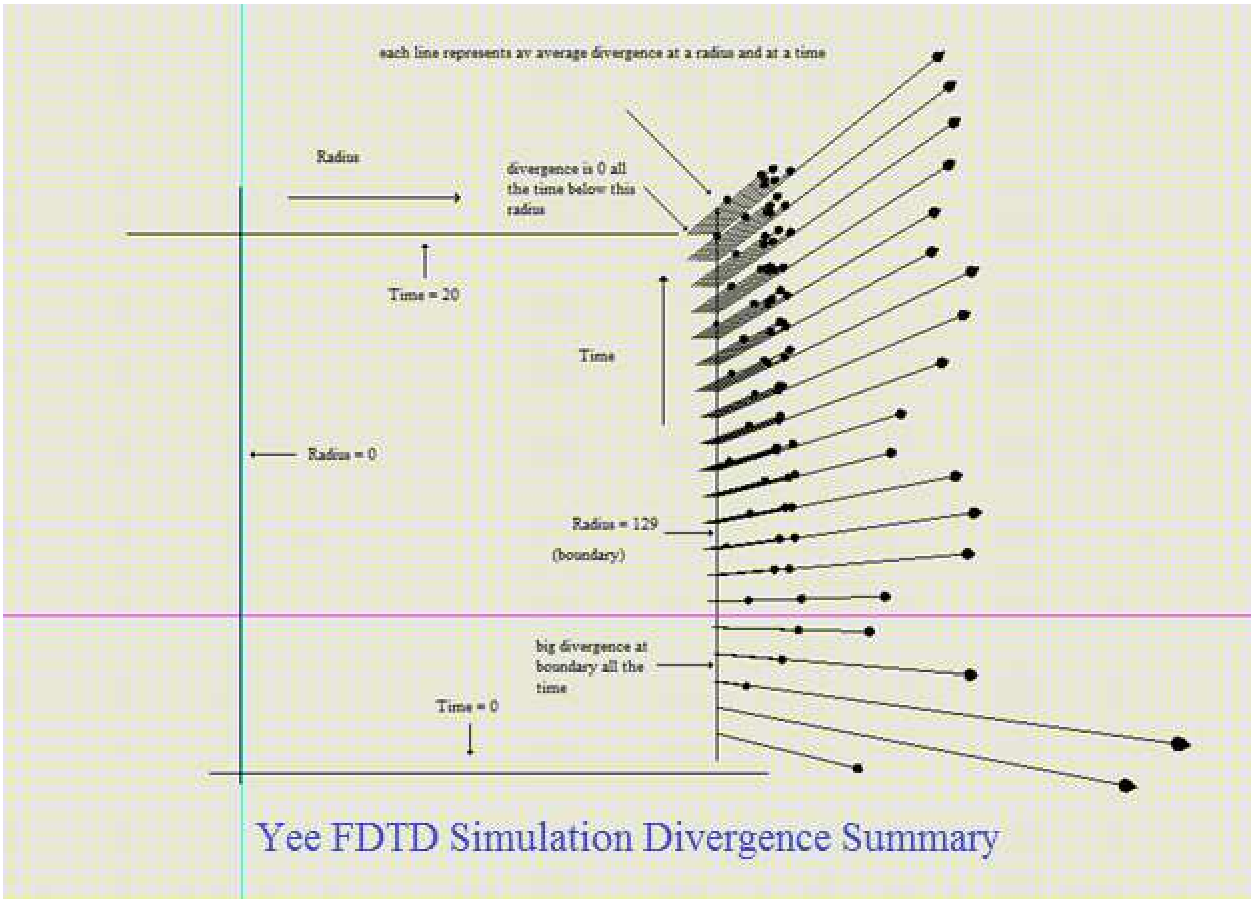


Some notes are added to a screenshot of the summary file:



We can see that in all the 20 time steps, non-zero divergences only appear at the boundary of radius=129. The error has not propagated into lower radiuses.

Comparing to the Yee FDTD simulation, the errors at the boundary propagate to lower radiuses over time:



Two TSS surprises shown in the simulation

In summary, the above TSS simulation gives us the following two surprises.

1. The boundary error of the TSS algorithm is 10^{-8} times smaller than the boundary error of the Yee FDTD algorithm
2. During 20 time steps of simulation, the boundary error of the TSS algorithm remains at the boundary of radius=129; even at radius=128 we do not see estimation error increases. For Yee FDTD algorithm, the boundary error starts propagating into lower radiuses right at the time step 1; at time step 20, large errors appear in all radiuses larger than 114.

Simulation Precisions Comparing with Yee FDTD

We have verified that the TSS algorithm and coding are correct. We'll evaluate TSS performance by comparing its accuracy with accuracy of Yee FDTD. We still use magnitude of field divergence as estimation error. The smaller divergence magnitude is considered more accurate.

One advantage of the TSS algorithm is that its estimation orders for time advancement and for space curls are arbitrary. In a task file, use "FDTD.HALF_ORDER_SPACE" and "FDTD.HALF_ORDER_TIME" to specify estimation orders for space curls and time advancement, respectively.

For Yee FDTD, changing estimation orders is a big deal. For a standard Yee FDTD, both estimation orders are second order. There are modified Yee FDTD algorithms using 4-th order for space curls estimation; time advancement estimation order is usually the second order.

We use accuracy of the standard Yee FDTD as a base for comparisons.

We first use TSS field data of the second order for both time and space estimations in TSS. Then we will use TSS field data of higher estimation orders for time and space.

The Second Order Estimation

We already used Task 100 twice to do simulations, using YeeFDTD and TSS. Both simulations use the second order estimation. We also used Task 120 to generate report files for these two simulations. We may compare simulation accuracy using average divergence values from these report files. Task 140 can be used to help us to do the comparison.

Task 140 merges two summary files into one file for easier access. Recall that file format for a summary file is:

4 bytes: integer, the maximum radius, let's denote it as {maxRadius}

((maxRadius+1) * 8) bytes: {maxRadius+1} double precision numbers, each number is an average divergence magnitude for a radius, radius = 0, 1, …, {maxRadius}. It is for time step index =0

((maxRadius+1) * 8) bytes: for time step index = 1

((maxRadius+1) * 8) bytes: for time step index = 2

… and so on…

Task 140 merges two of above summary files into a file in following format:

4 bytes: integer, the maximum radius, let's denote it as {maxRadius}

2*((maxRadius+1) * 8) bytes: {maxRadius+1} double precision numbers, each number is an average divergence magnitude for a radius, radius = 0, 1, …, {maxRadius}, from the first summary file; followed by {maxRadius+1} double precision numbers from the second summary file,. It is for time step index =0

2*((maxRadius+1) * 8) bytes: for time step index = 1

2*((maxRadius+1) * 8) bytes: for time step index = 2

… and so on…

We use following task file to generate a merged file:

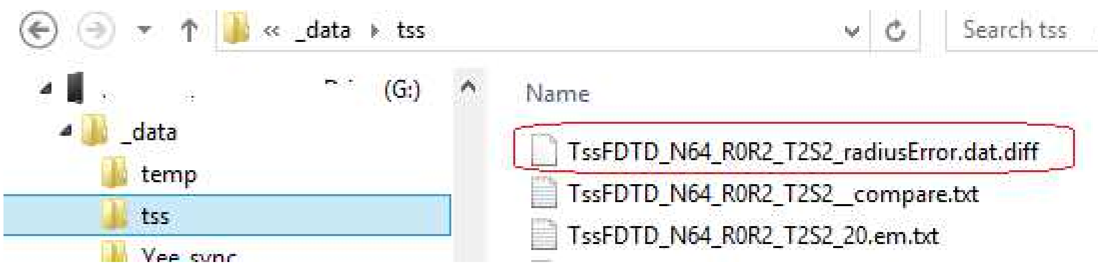
```
//task number
SIM.TASK=140
```

```
//the first summary file name, command line parameter /D specifies folder for it
SIM.FILE1=YeeFDTDSpaceSynched_N64_R0R2_T2S2_radiusError.dat

//the second summary file name, command line parameter /E specifies folder for it
SIM.FILE2=TssFDTD_N64_R0R2_T2S2_radiusError.dat

//boundary thickness to be excluded from the merged file
SIM.THICKNESS=0
```

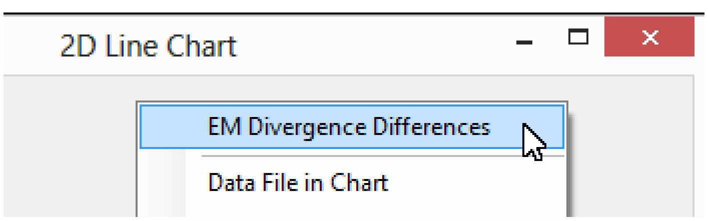
A sample command line can be

```
EM /TC:\EM\TSS\Doc\task140_Yee_Tss_T2R2.task /WG:\_data\temp /DG:\_data\Yee_sync
/EG:\_data\tss
```

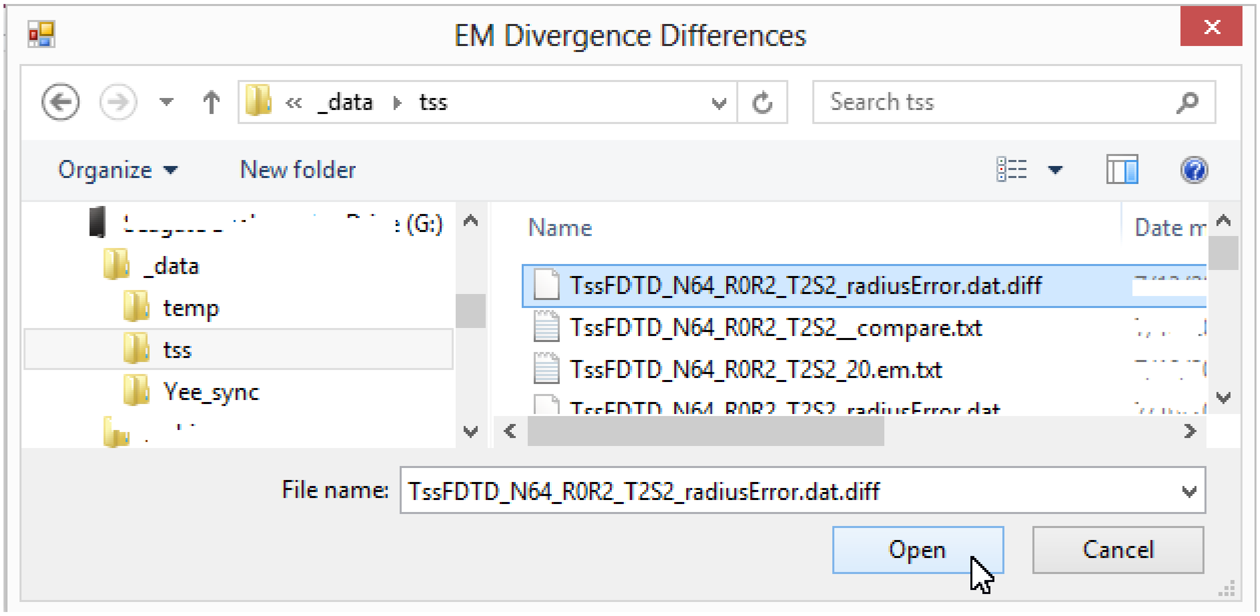
A merged file is created:



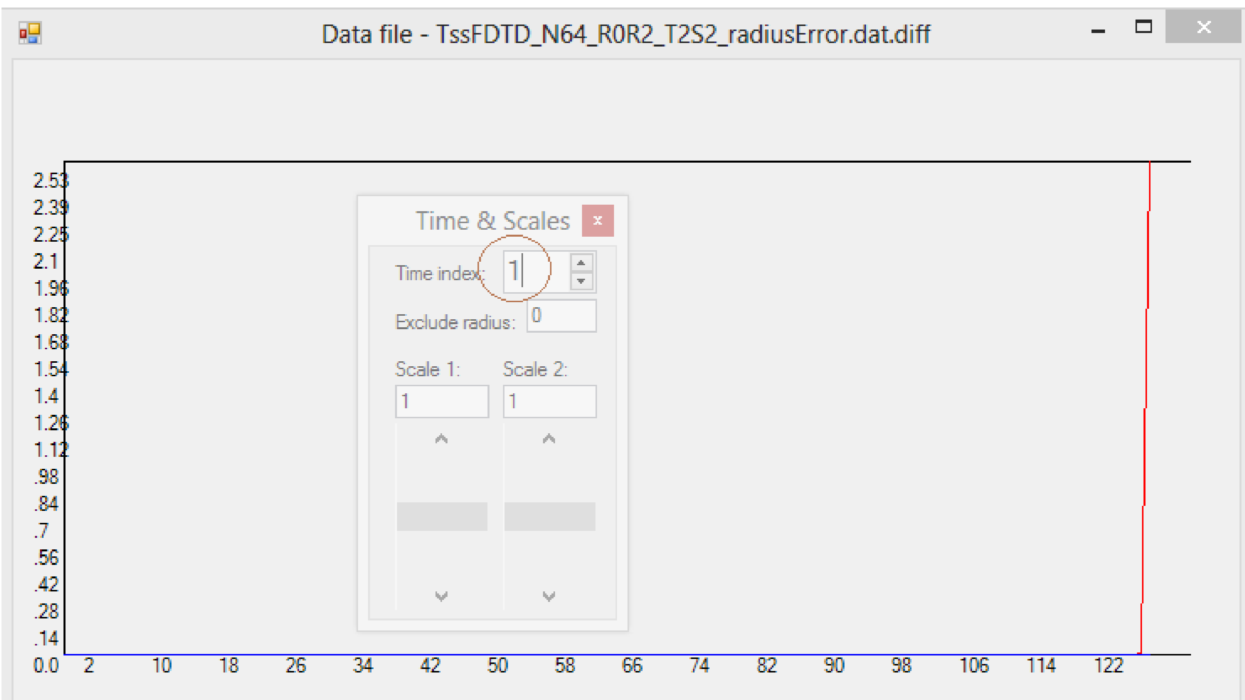
We may use Draw2D to view the merged file. Right-click on the view screen, choose “EM Divergence Differences”:



Choose the merged file:



Below is a 2D drawing for time index 1:

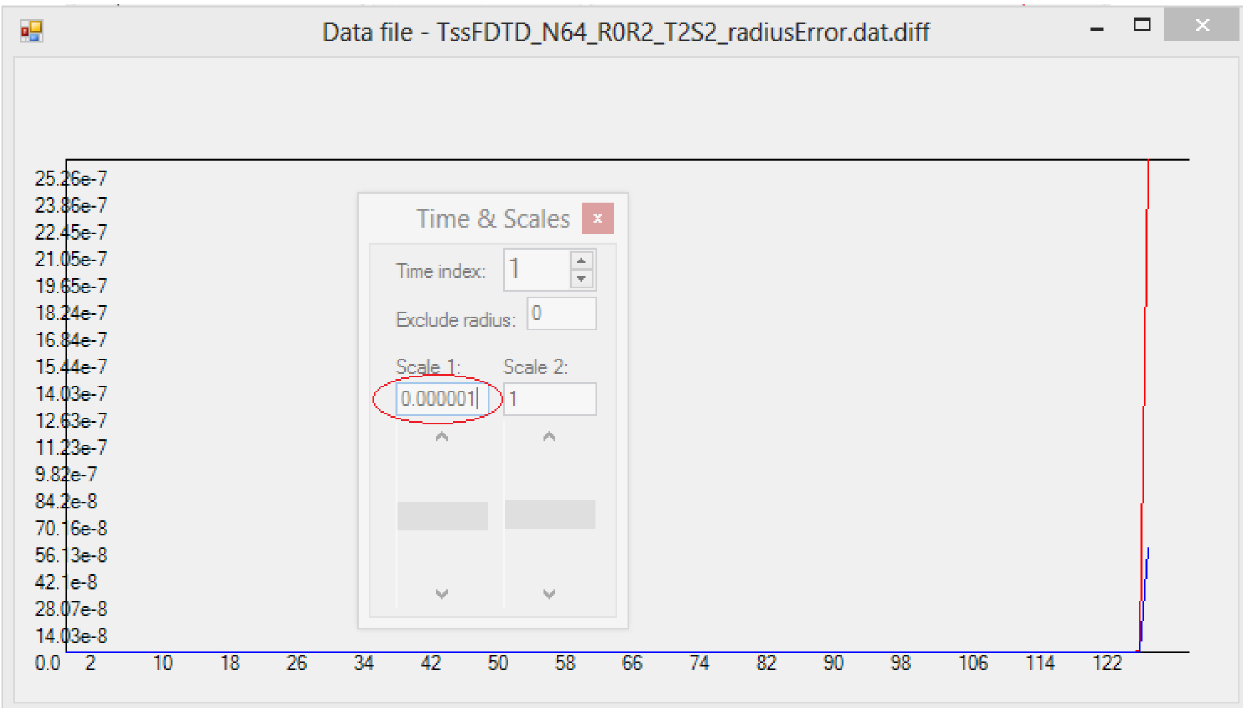


The horizontal axis is for radius; the vertical axis is for divergence magnitude.

The red line shows data from the first summary file; the blue line shows data from the second file. In our case, red line is for Yee FDTD data, blue line is for TSS data.

The two lines are at bottom, indicating very small divergences and thus proving the algorithms and coding.

The red line has a big jump at the last radius, showing a big divergence at the boundary. We can change the scale for the red line to make its magnitude comparable with the blue line:

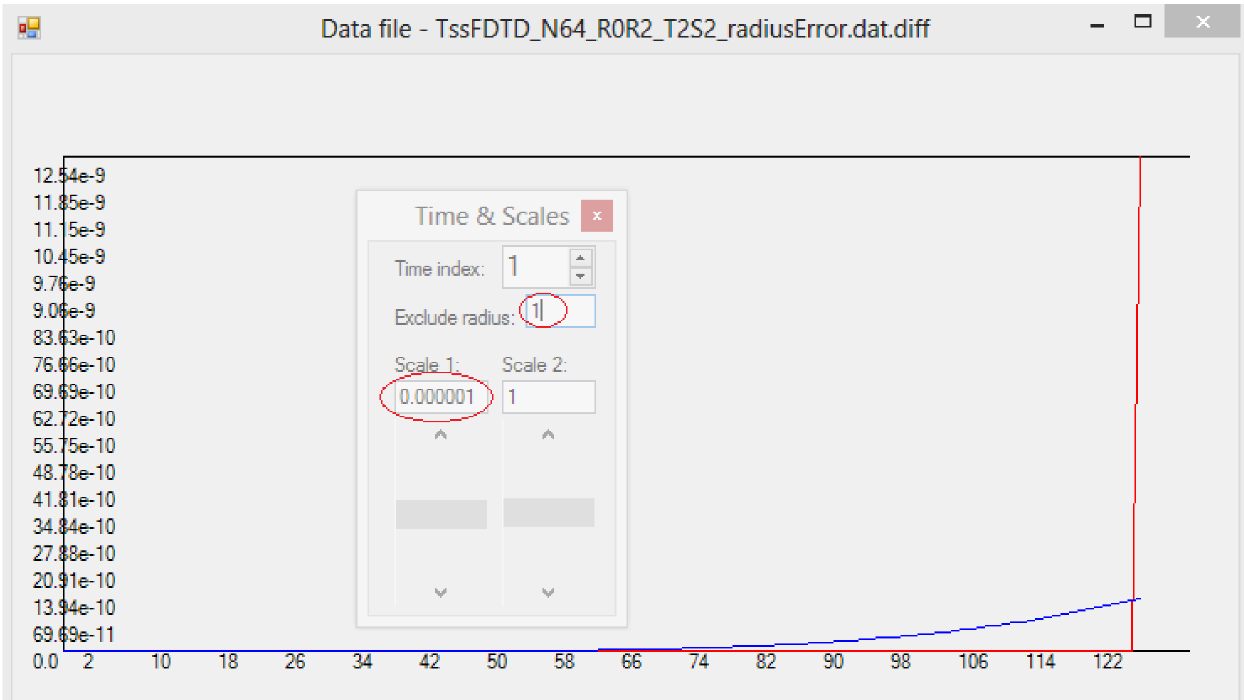


With a scale of 0.000001, the magnitude of the red line is reduced 10^5 times. Now the blue line also shows a jump at the boundary. It shows that TSS also generates non-zero divergence at the boundary, but its magnitude is 10^5 times smaller than that generated by Yee FDTD.

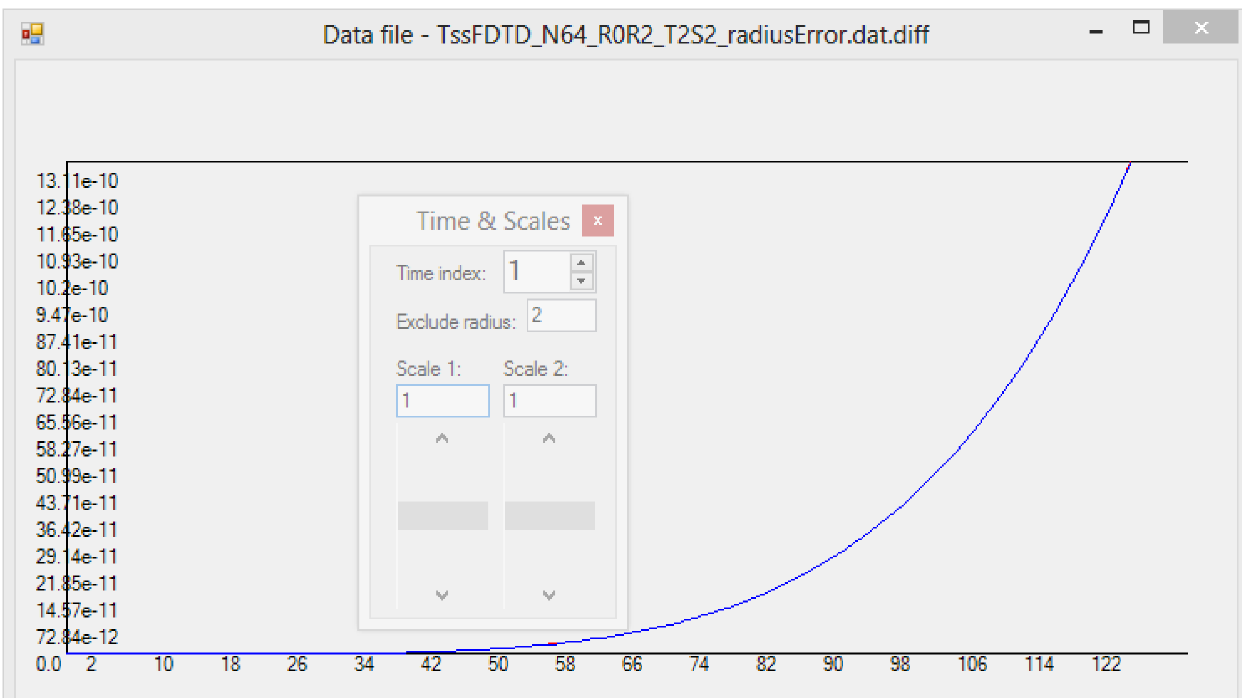
It visually displays that the TSS algorithm has a huge accuracy advantage over the Yee FDTD at and near the boundary.

To compare accuracy in the center region, we may exclude some radiuses from the display.

Excluding one radius from the boundary, the Yee data still show a big jump at the new boundary, the scale is still 0.000001:



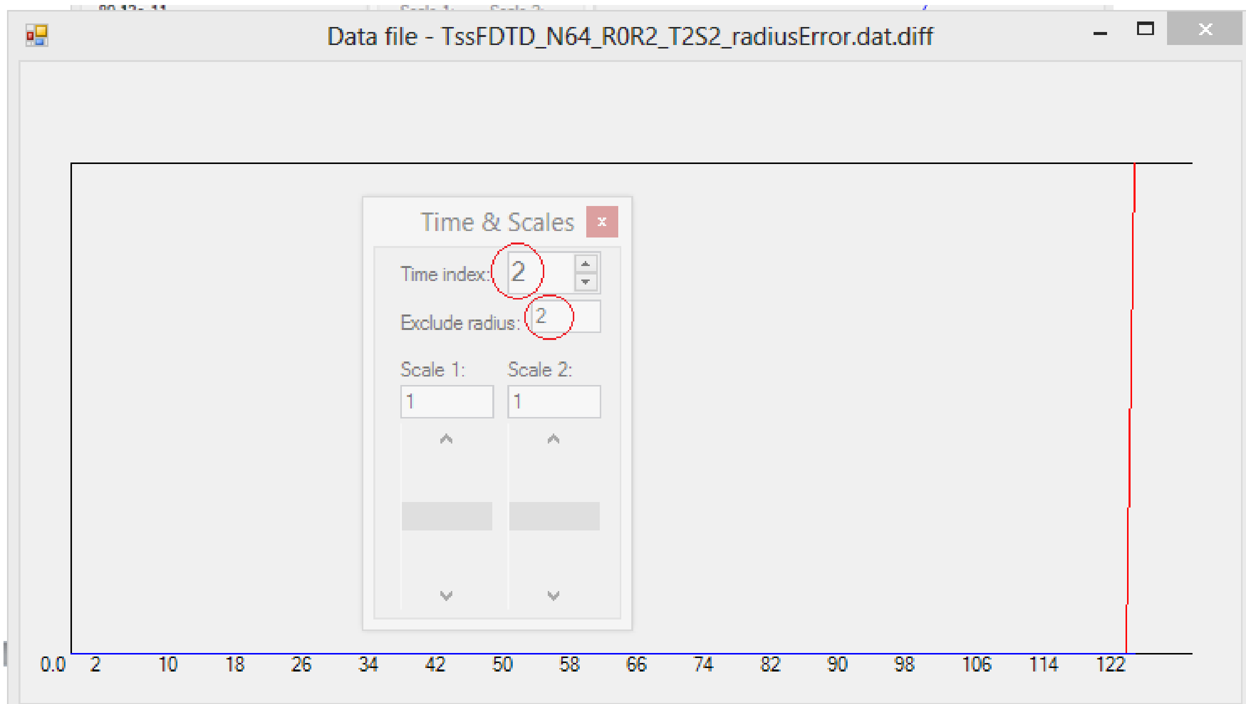
By excluding 2 radiuses from the boundary, the divergences look the same for Yee FDTD and TSS:



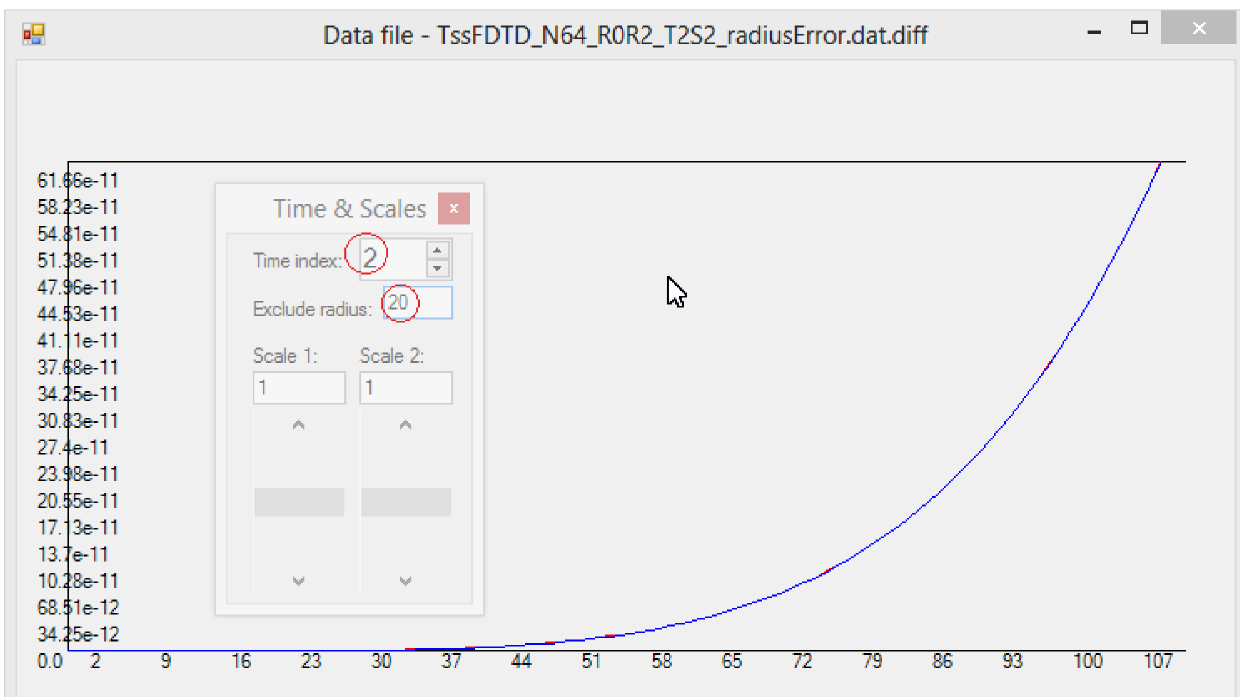
Note that the scales for both lines are 1.

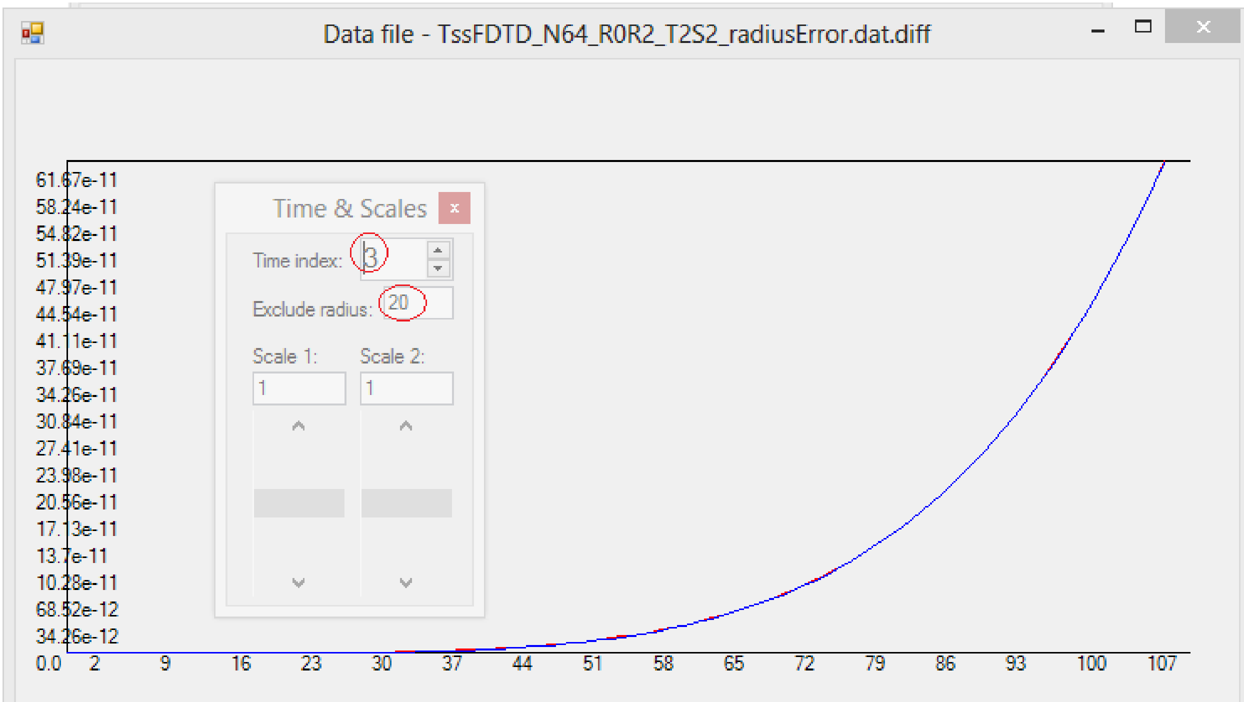
The almost identical lines show that accuracies of the second order TSS and Yee FDTD are almost the same.

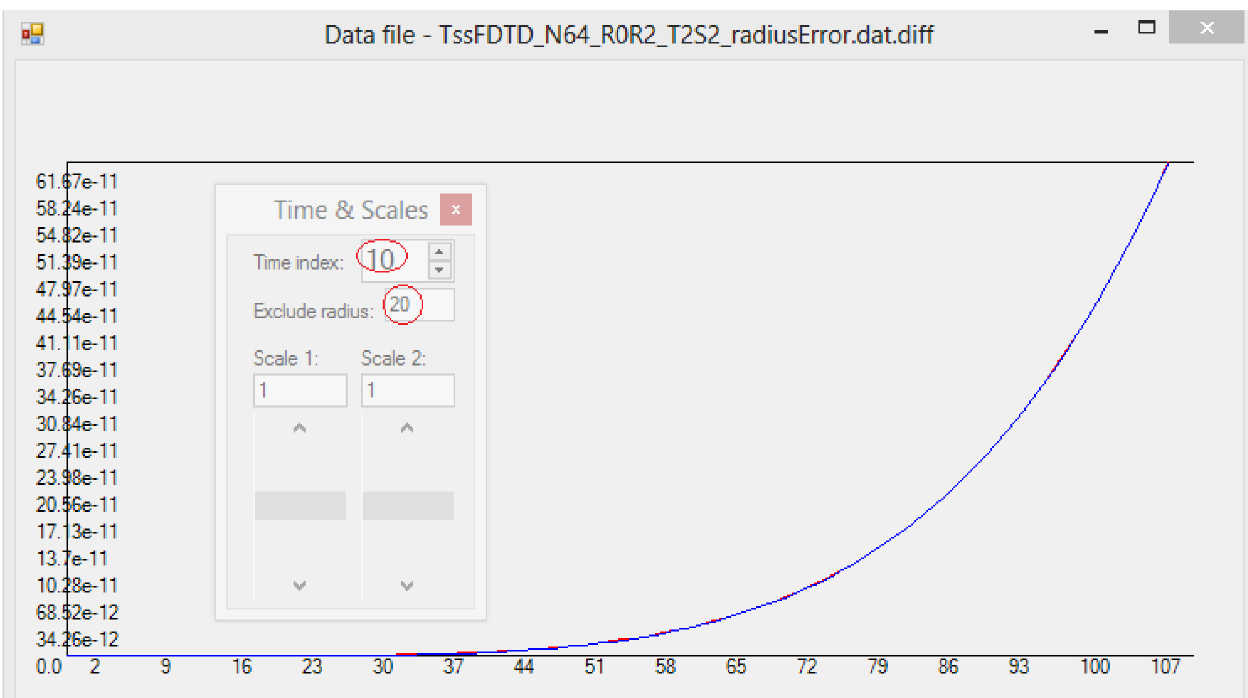
Increase the time index, the red line shows big jump again at the last radius, showing Yee FDTD error propagation from the boundary into the center region:

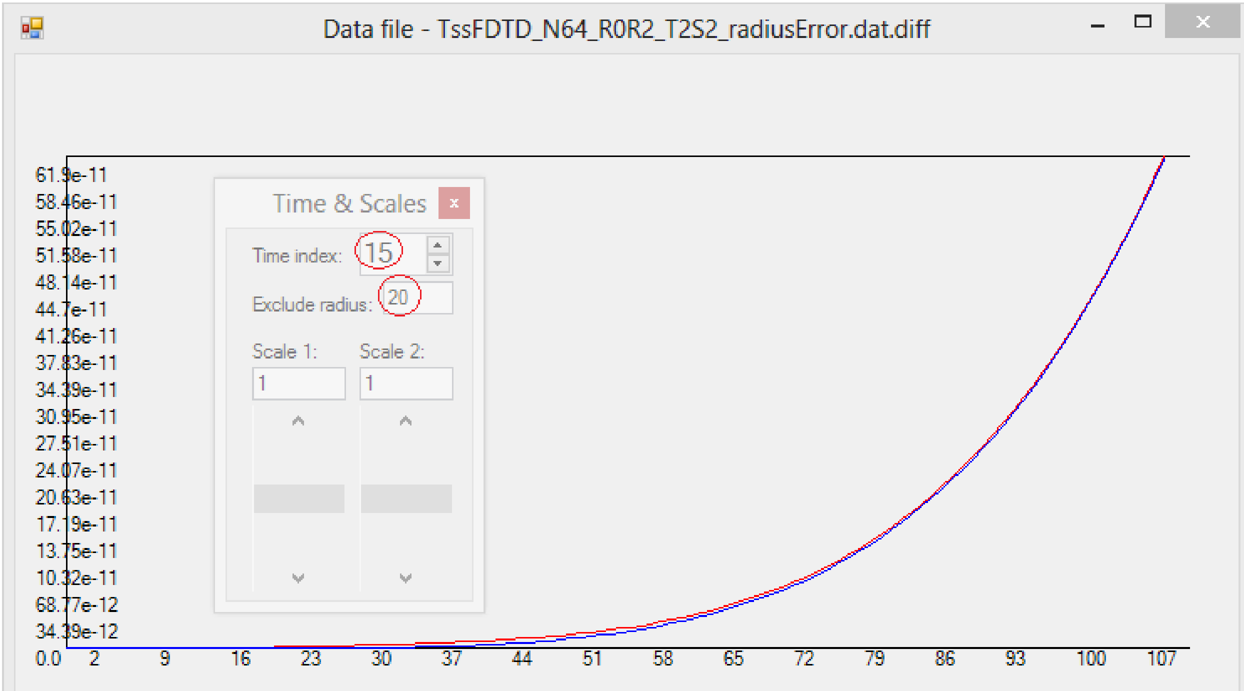


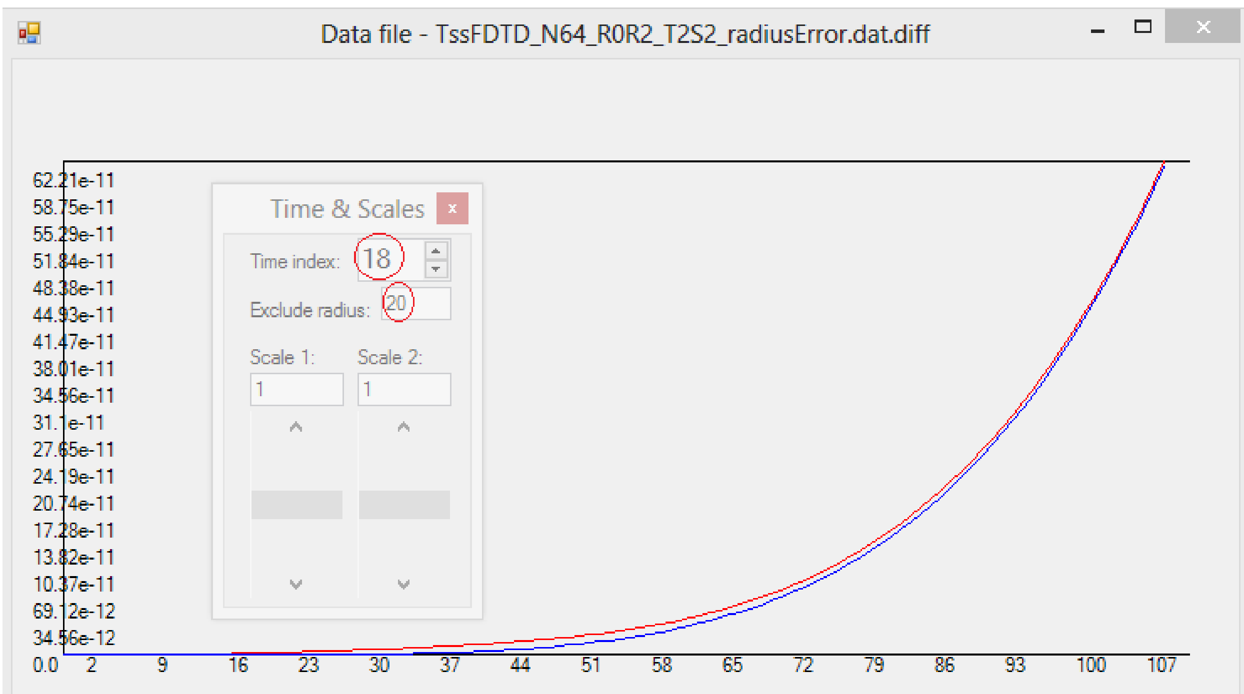
Now let's concentrate on the center region by exluding 20 radiuses from the boundary. For all the time indexes, the red line and the blue line are almost the same:

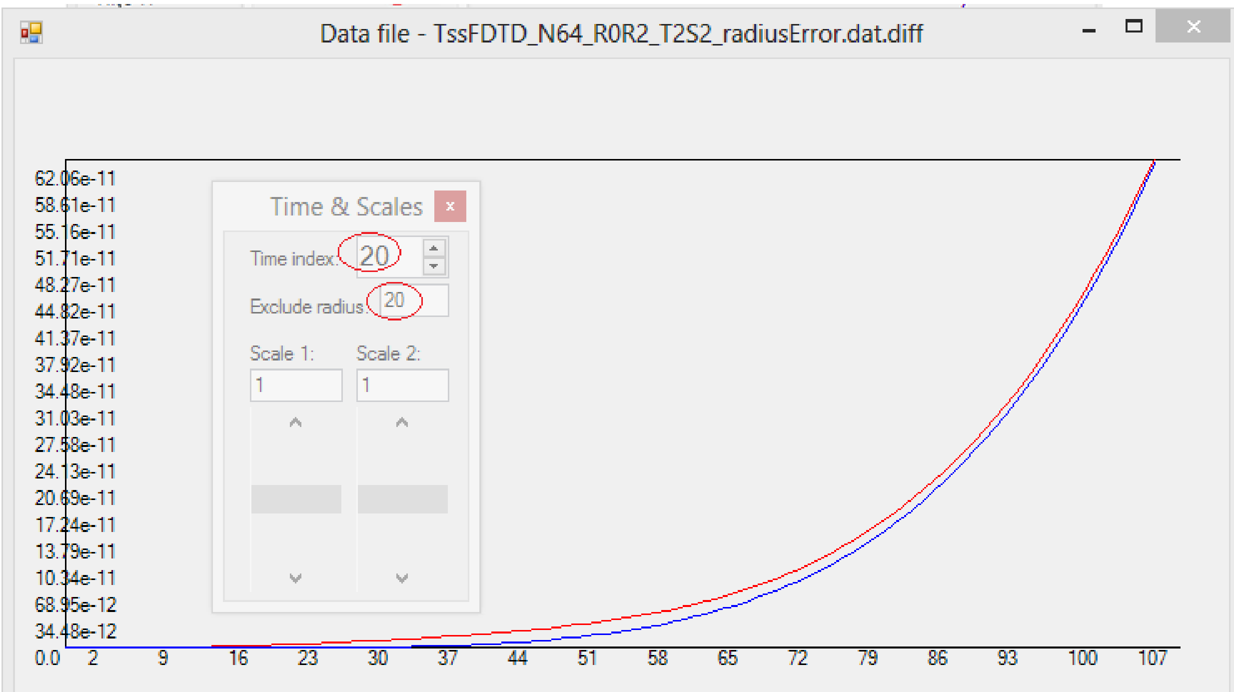



Data file - TssFDTD_N64_R0R2_T2S2_radiusError.dat.diff
Time & Scales
Time index: 3
Exclude radius: 20
Scale 1:
Scale 2:
1
1
61.67e-11
58.24e-11
54.82e-11
51.39e-11
47.97e-11
44.54e-11
41.11e-11
37.69e-11
34.26e-11
30.84e-11
27.41e-11
23.98e-11
20.56e-11
17.13e-11
13.7e-11
10.28e-11
68.52e-12
34.26e-12
0.0
2
9
16
23
30
37
44
51
58
65
72
79
86
93
100
107


Data file - TssFDTD_N64_R0R2_T2S2_radiusError.dat.diff
Time & Scales
Time index: 10
Exclude radius: 20
Scale 1:
Scale 2:
1
1
61.67e-11
58.24e-11
54.82e-11
51.39e-11
47.97e-11
44.54e-11
41.11e-11
37.69e-11
34.26e-11
30.84e-11
27.41e-11
23.98e-11
20.56e-11
17.13e-11
13.7e-11
10.28e-11
68.52e-12
34.26e-12
0.0
2
9
16
23
30
37
44
51
58
65
72
79
86
93
100
107


Data file - TssFDTD_N64_R0R2_T2S2_radiusError.dat.diff
Time & Scales
Time index: 15
Exclude radius: 20
Scale 1: 1
Scale 2: 1
61.9e-11
58.46e-11
55.02e-11
51.58e-11
48.14e-11
44.7e-11
41.26e-11
37.83e-11
34.39e-11
30.95e-11
27.51e-11
24.07e-11
20.63e-11
17.19e-11
13.75e-11
10.32e-11
68.77e-12
34.39e-12
0.0
2 9 16 23 30 37 44 51 58 65 72 79 86 93 100 107


Data file - TssFDTD_N64_R0R2_T2S2_radiusError.dat.diff
Time & Scales
Time index: 18
Exclude radius: 20
Scale 1: 1
Scale 2: 1
62.21e-11
58.75e-11
55.29e-11
51.84e-11
48.38e-11
44.93e-11
41.47e-11
38.01e-11
34.56e-11
31.1e-11
27.65e-11
24.19e-11
20.74e-11
17.28e-11
13.82e-11
10.37e-11
69.12e-12
34.56e-12
0.0
2 9 16 23 30 37 44 51 58 65 72 79 86 93 100 107



The red line is above the blue line, indicating that Yee FDTD is less accurate than TSS.

The above 2D drawings show following observations:

- TSS is 10^5 times more accurate than Yee FDTD at the boundary
- TSS does not show error propagation in these drawings. Yee FDTD shows error propagation for every time step.
- In the center region where boundary error has not arrived, the accuracy of the Yee FDTD is almost the same as the accuracy of TSS
- In the center region where boundary error has not arrived, the accuracy of the Yee FDTD is a little bit lower than the accuracy of TSS

The Fourth Order Estimation

Simulation

A simulation is made using following task file:

```
//task number
SIM.TASK=100

//the number of double intervals at one side of axis
// number of space points=(4N+3)^3=17373979
// memory size=833950992 bytes=0.8G
FDTD.N=64

//half space range
FDTD.R=0.2
```

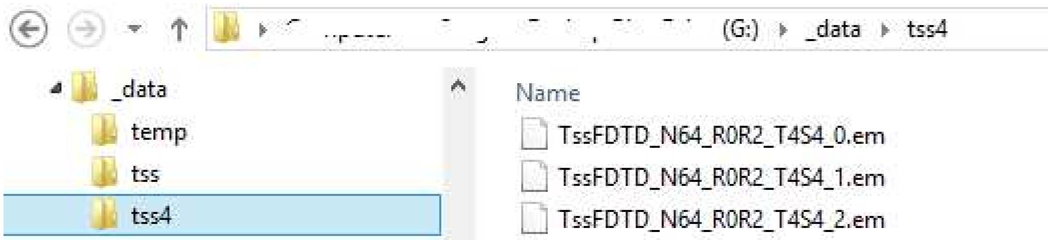
```
//half estimation order for divergence estimations. Default value is 1
FDTD.HALF_ORDER_SPACE=2

//half estimation order for time advance estimations. Default value is 1
FDTD.HALF_ORDER_TIME=2

//use default base file name
SIM.BASENAME=DEF

//maximum time steps
FDTD.MAXTIMESTEP=20

//DLL file for FDTD module
SIM.FDTD_DLL=TssFDTD.DLL

//class name for FDTD module
SIM.FDTD_NAME=TssFDTD

//DLL file for boundary condition module
SIM.BC_DLL=BoundaryConditionA.dll

//class name for boundary condition module
SIM.BC_NAME=VoidCondition

//DLL file containing Initial Value modules
SIM.IV_DLL=FieldProviders.dll

//class name of the Initial Value module to be used
SIM.IV_NAME=GaussianFields

//following task parameters are defined and used by class GaussianFields

//magnitude of field
IV.MAGNITUDE=120

//gaussian function width
IV.WIDTH=0.5
```

In the above file, the half estimation order for space derivative is 2; the half estimation order for time advancement is 2. Therefore, the estimation orders for space derivative and for time advancement are both fourth order.

A sample command line can be:

```
EM /TC:\EM\TSS\Doc\task100_TSS_T4R4.task /WG:\_data\temp /DG:\_data\tss4
```

A set of 21 data files is generated by the simulation:



Generate divergence reports

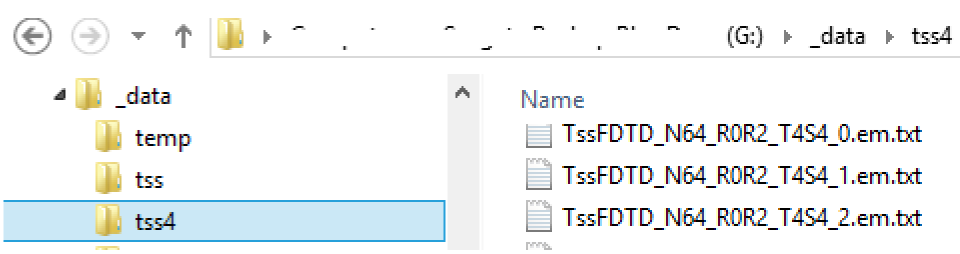
Task file:

```
//task number
SIM.TASK=120

//base file name
SIM.BASENAME=TssFDTD_N64_R0R2_T4S4_

//half space range; it must match the data files
FDTD.R=0.2

//half estimation order for divergence estimations. Default value is 1
FDTD.HALF_ORDER_SPACE=2
```

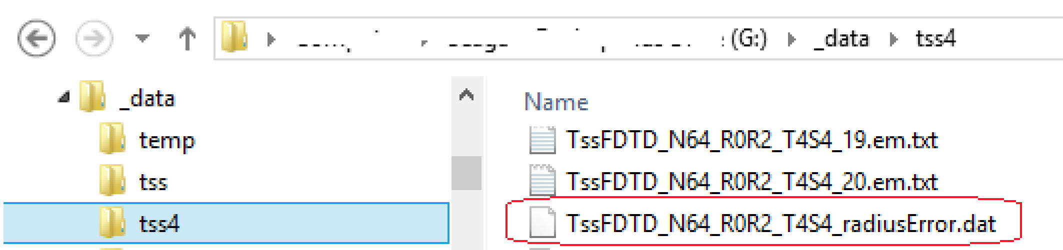
A sample command line:

```
EM /TC:\EM\TSS\DOC\task120_TSS_T4R4.task /WG:\_data\temp /DG:\_data\tss4
```

A set of report files is generated:



A summary file is also generated:



Compare divergence magnitudes with Yee FDTD

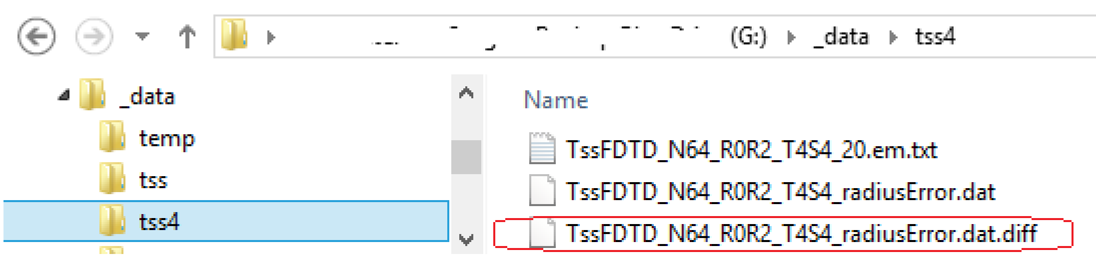
Task file:

```
//task number
SIM.TASK=140

//the first summary file name, command line parameter /D specifies folder for it
SIM.FILE1=YeeFDTDSpaceSynched_N64_R0R2_T2S2_radiusError.dat

//the second summary file name, command line parameter /E specifies folder for it
SIM.FILE2=TssFDTD_N64_R0R2_T4S4_radiusError.dat

//boundary thickness to be excluded from the merged file
SIM.THICKNESS=0
```

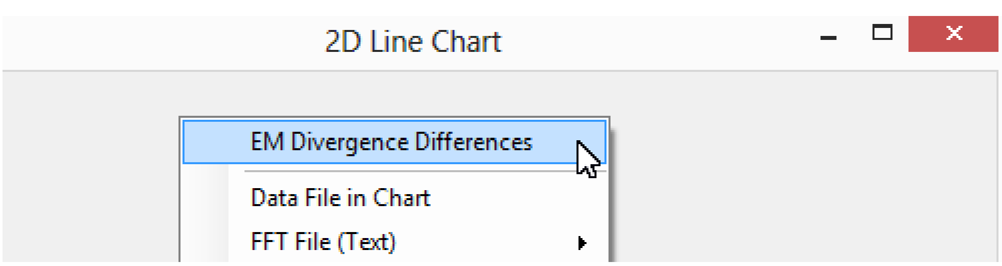
A sample command line:

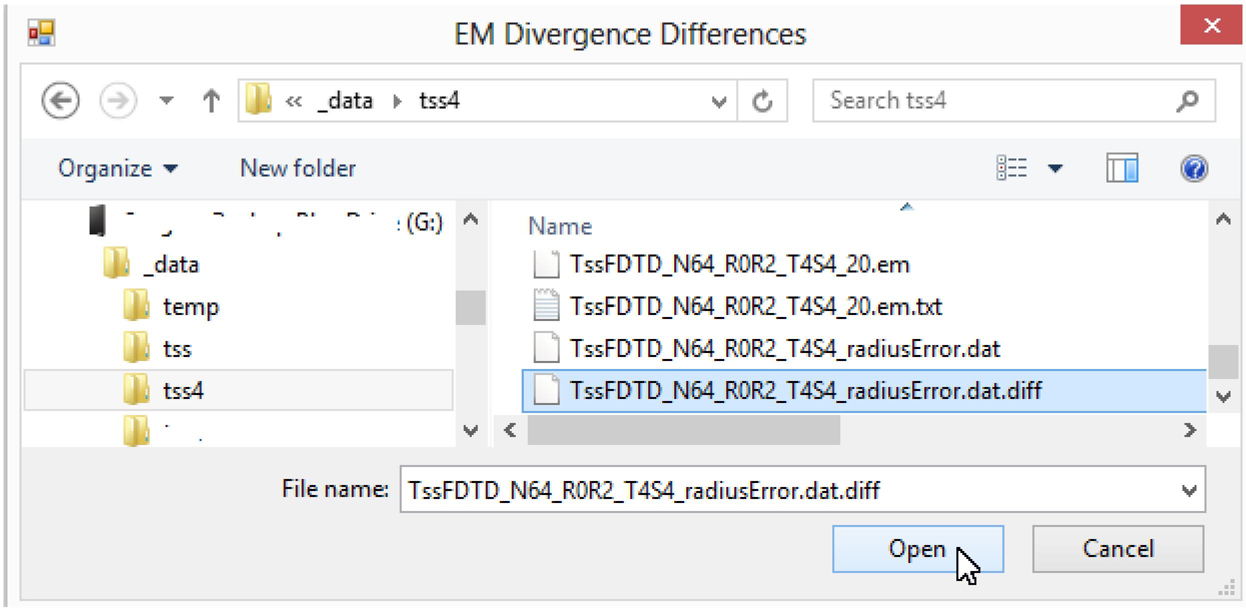
```
EM /TC:\EM\TSS\DOC\task140_Yee_Tss_T4R4.task /WG:\_data\temp /DG:\_data\Yee_sync
/EG:\_data\tss4
```

A merged file is generated:

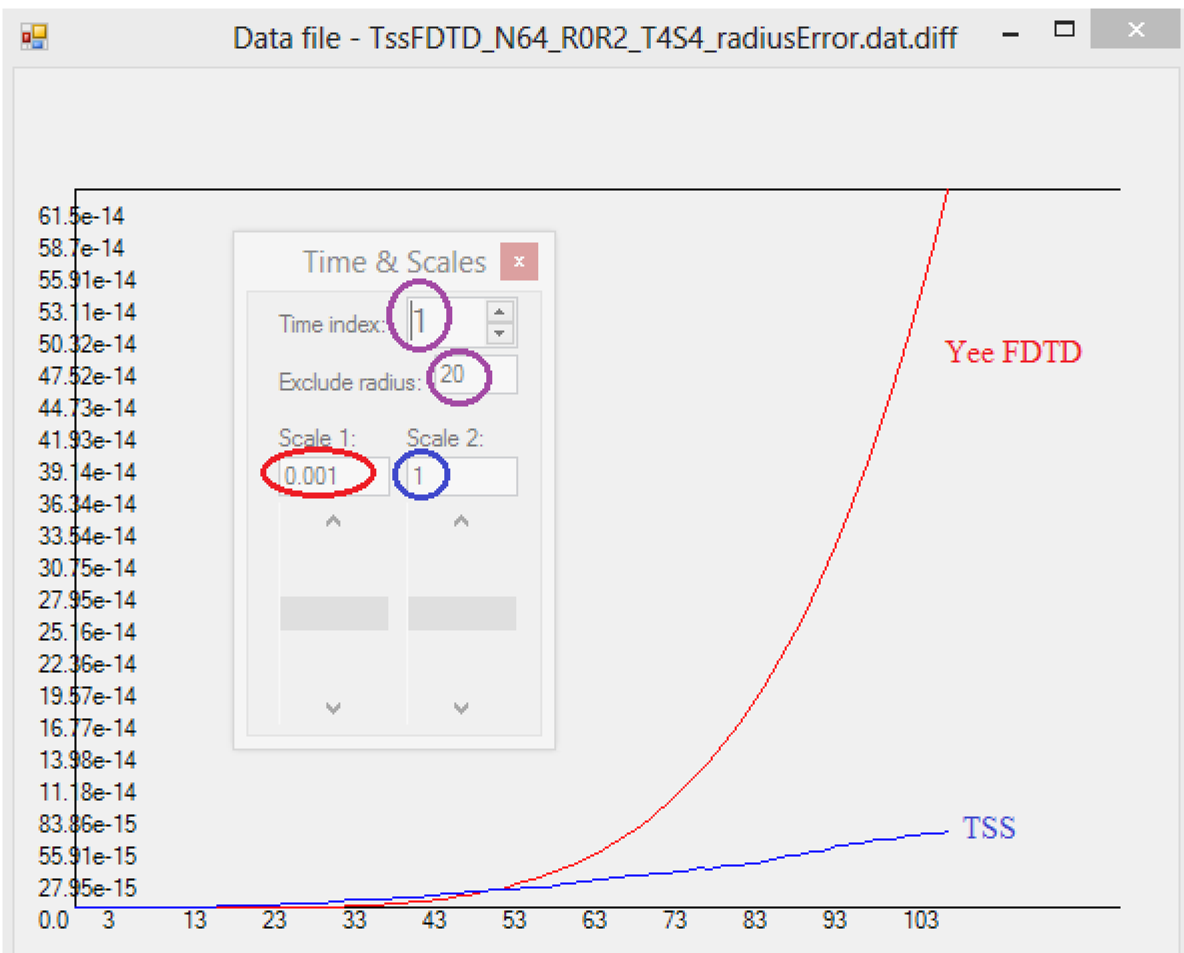


Using Draw2D to view the merged file:





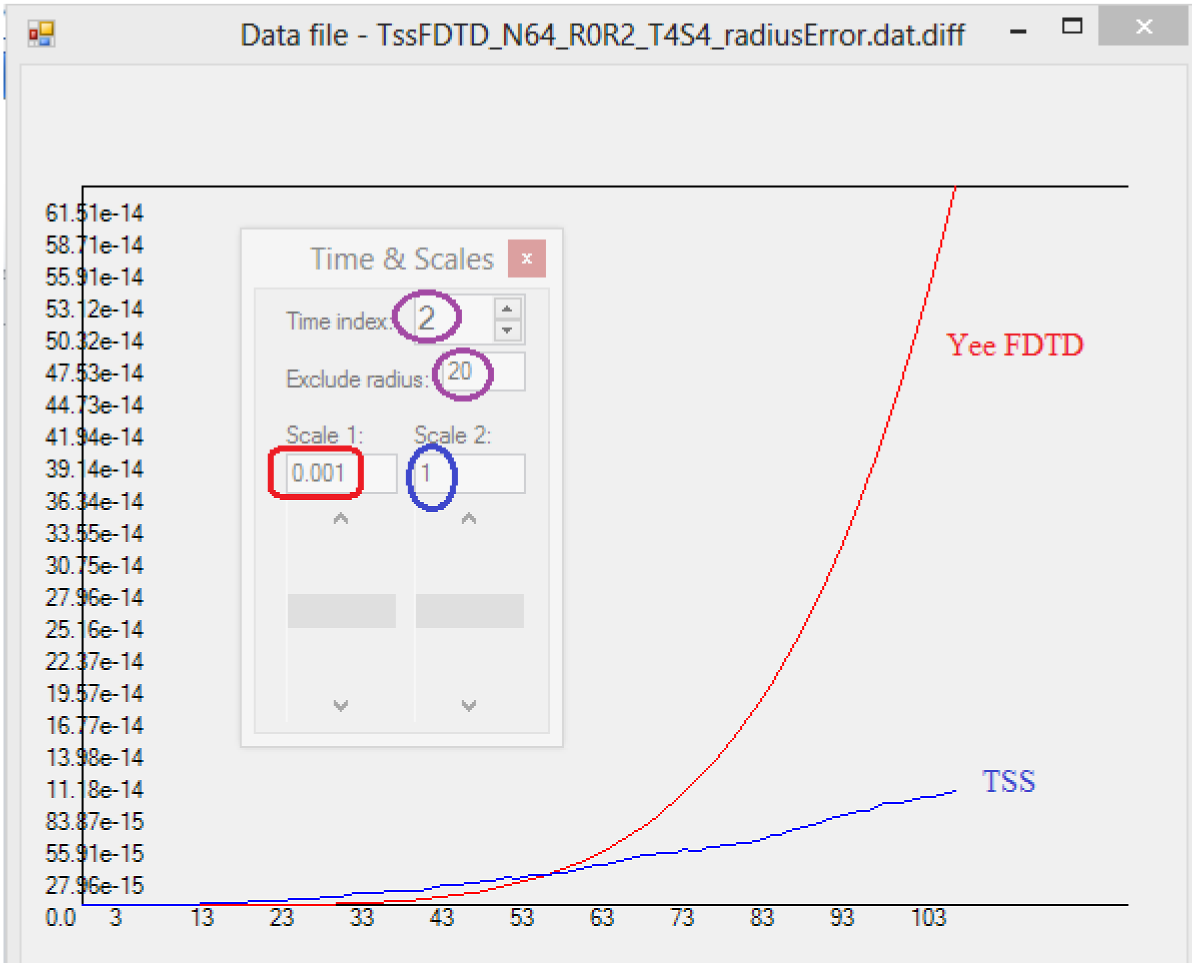
Display divergence comparisons for each time step:

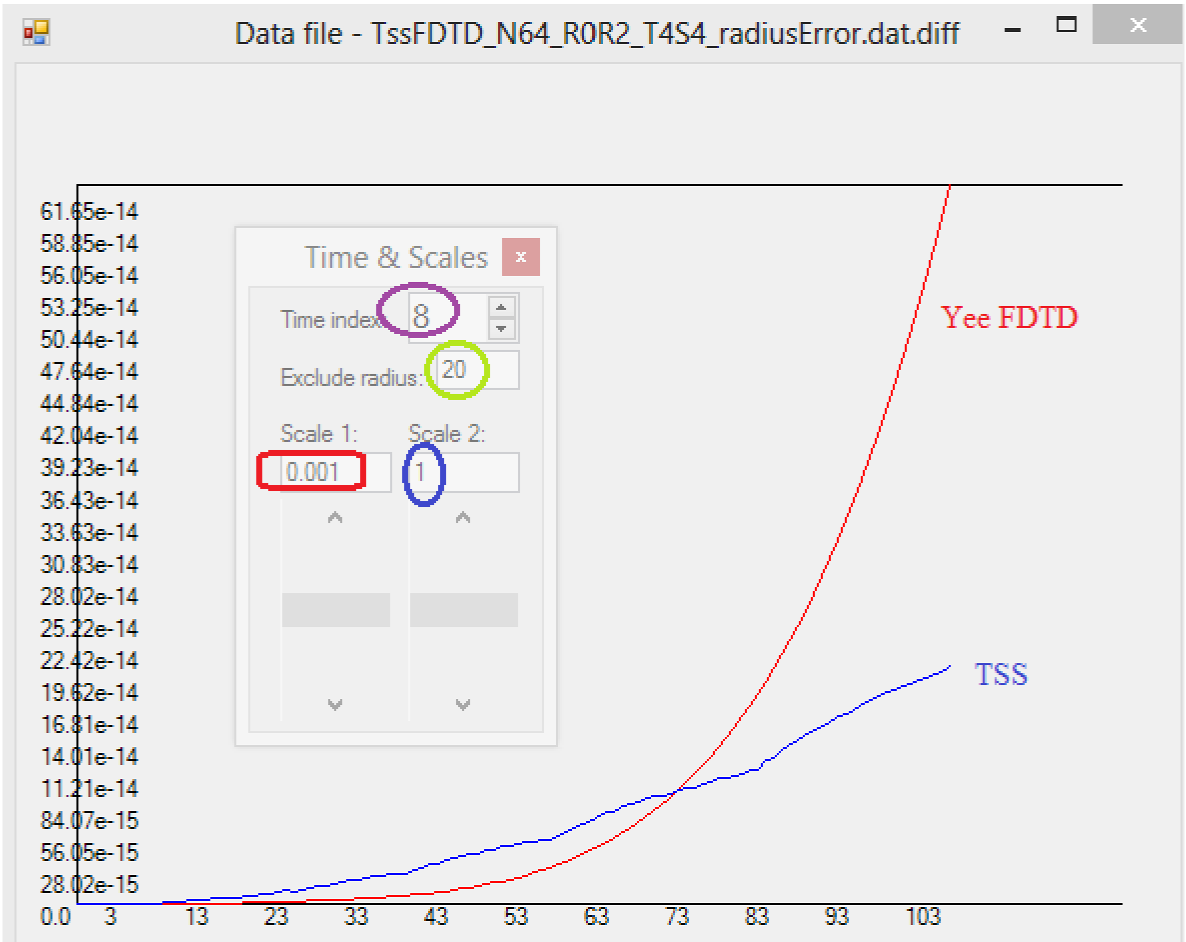


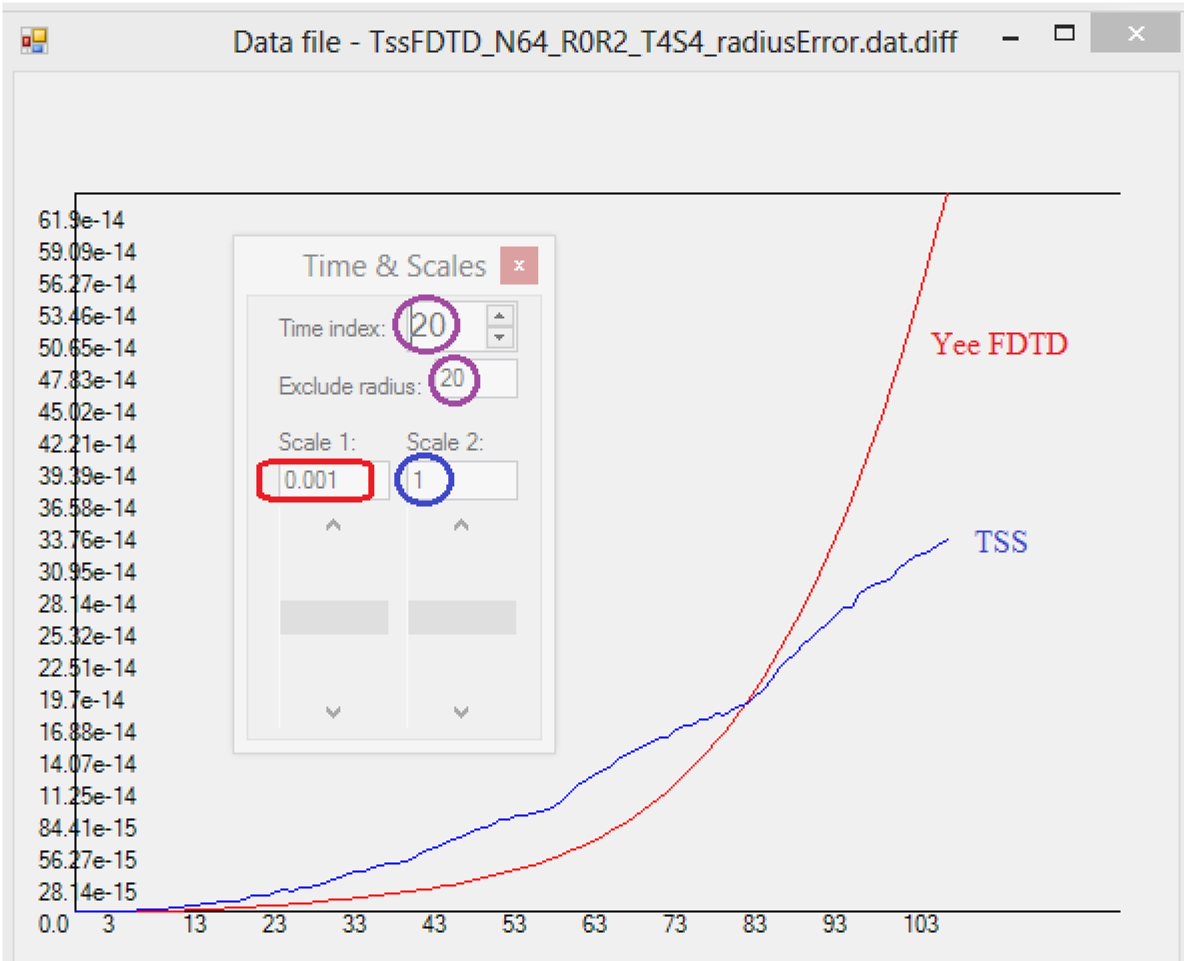
Because 20 radiuses are excluded from the boundary the comparison is focused in the center region.

The scale for Yee FDTD is 0.001. This chart shows that TSS in 4-th order is more than a thousand times more accurate than the standard Yee FDTD which is the second order.

Let's show the chart for other time steps:




Data file - TssFDTD_N64_R0R2_T4S4_radiusError.dat.diff
Time & Scales
Time index
8
Exclude radius:
20
Scale 1:
0.001
Scale 2:
1
Yee FDTD
TSS
61.65e-14
58.85e-14
56.05e-14
53.25e-14
50.44e-14
47.64e-14
44.84e-14
42.04e-14
39.23e-14
36.43e-14
33.63e-14
30.83e-14
28.02e-14
25.22e-14
22.42e-14
19.62e-14
16.81e-14
14.01e-14
11.21e-14
84.07e-15
56.05e-15
28.02e-15
0.0
3
13
23
33
43
53
63
73
83
93
103



The 6-th Order Estimation

Simulation

Task file:

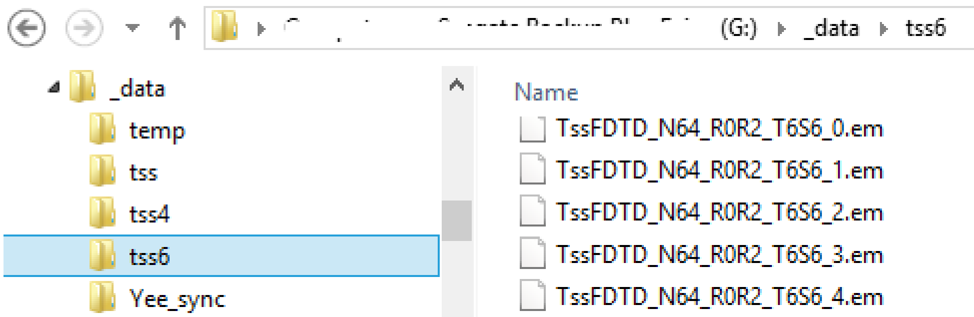
```
//task number
SIM.TASK=100

//the number of double intervals at one side of axis
// number of space points=(4N+3)^3=17373979
// memory size=833950992 bytes=0.8G
FDTD.N=64

//half space range
FDTD.R=0.2

//half estimation order for divergence estimations. Default value is 1
FDTD.HALF_ORDER_SPACE=3
```

```
//half estimation order for time advance estimations. Default value is 1
FDTD.HALF_ORDER_TIME=3

//use default base file name
SIM.BASENAME=DEF

//maximum time steps
FDTD.MAXTIMESTEP=20

//DLL file for FDTD module
SIM.FDTD_DLL=TssFDTD.DLL

//class name for FDTD module
SIM.FDTD_NAME=TssFDTD

//DLL file for boundary condition module
SIM.BC_DLL=BoundaryConditionA.dll

//class name for boundary condition module
SIM.BC_NAME=VoidCondition

//DLL file containing Initial Value modules
SIM.IV_DLL=FieldProviders.dll

//class name of the Initial Value module to be used
SIM.IV_NAME=GaussianFields

//following task parameters are defined and used by class GaussianFields

//magnitude of field
IV.MAGNITUDE=120

//gaussian function width
IV.WIDTH=0.5
```

Sample command line:

```
EM /TC:\EM\TSS\DOC\task100_TSS_T6R6.task /WG:\_data\temp /DG:\_data\tss6
```

A set of data files is generated:



Generate divergence reports

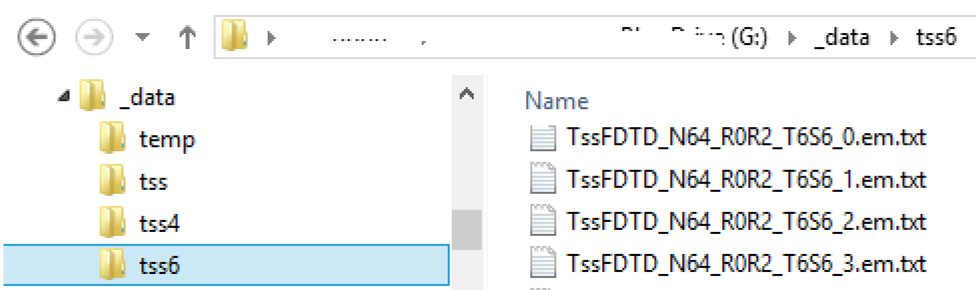
Task file:

```
//task number
SIM.TASK=120

//base file name
SIM.BASENAME=TssFDTD_N64_R0R2_T6S6_

//half space range; it must match the data files
FDTD.R=0.2

//half estimation order for divergence estimations. Default value is 1
FDTD.HALF_ORDER_SPACE=3
```

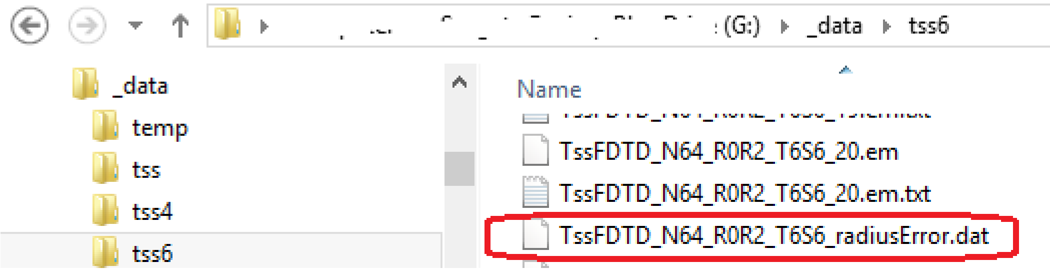
A sample command line:

```
EM /TC:\EM\TSS\DOC\task120_TSS_T6R6.task /WG:\_data\temp /DG:\_data\tss6
```

A set of report files is generated:



A summary file is also generated:



Compare divergence magnitudes with Yee FDTD

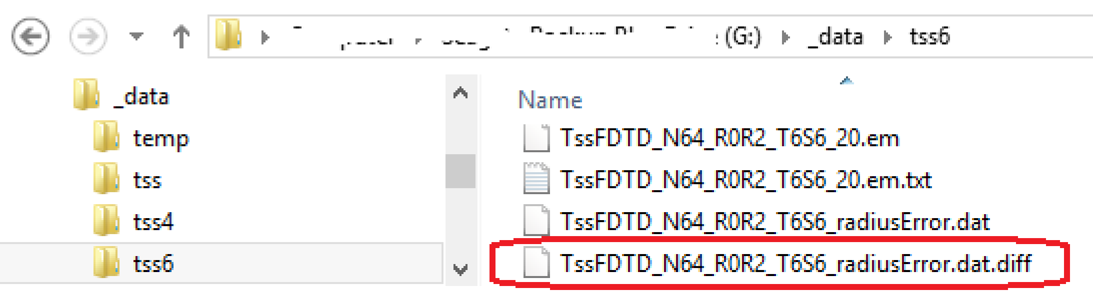
Task file:

```
//task number
SIM.TASK=140

//the first summary file name, command line parameter /D specifies folder for it
SIM.FILE1=YeeFDTDSpaceSynched_N64_R0R2_T2S2_radiusError.dat

//the second summary file name, command line parameter /E specifies folder for it
SIM.FILE2=TssFDTD_N64_R0R2_T6S6_radiusError.dat

//boundary thickness to be excluded from the merged file
SIM.THICKNESS=0
```

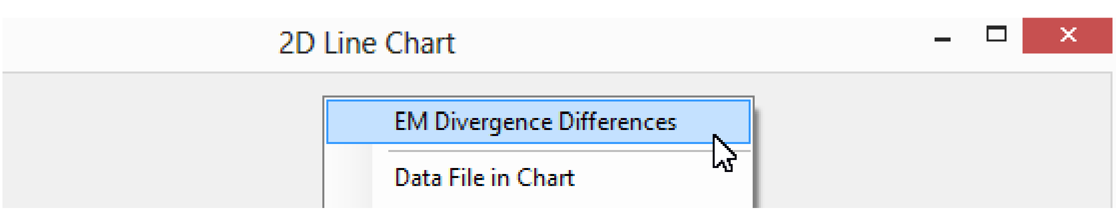
A sample command line:

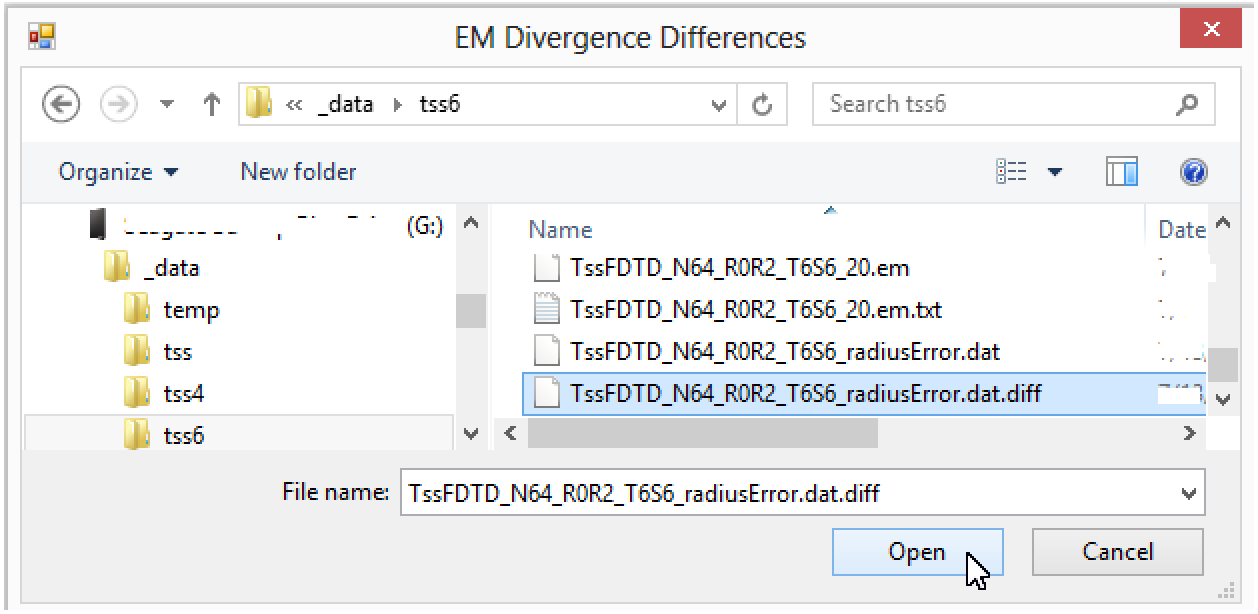
```
EM /TC:\EM\TSS\DOC\task140_Yee_Tss_T6R6.task /WG:\_data\temp /DG:\_data\Yee_sync
/EG:\_data\tss6
```

A merged file is generated:

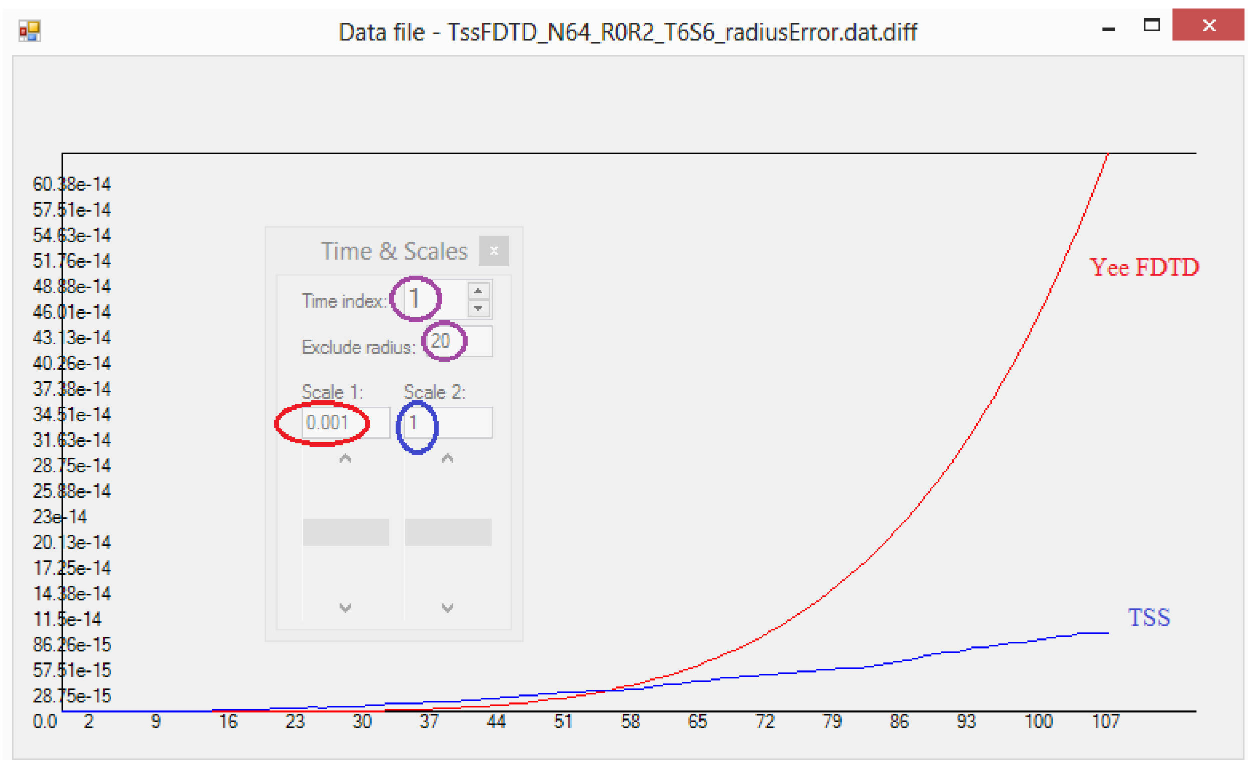


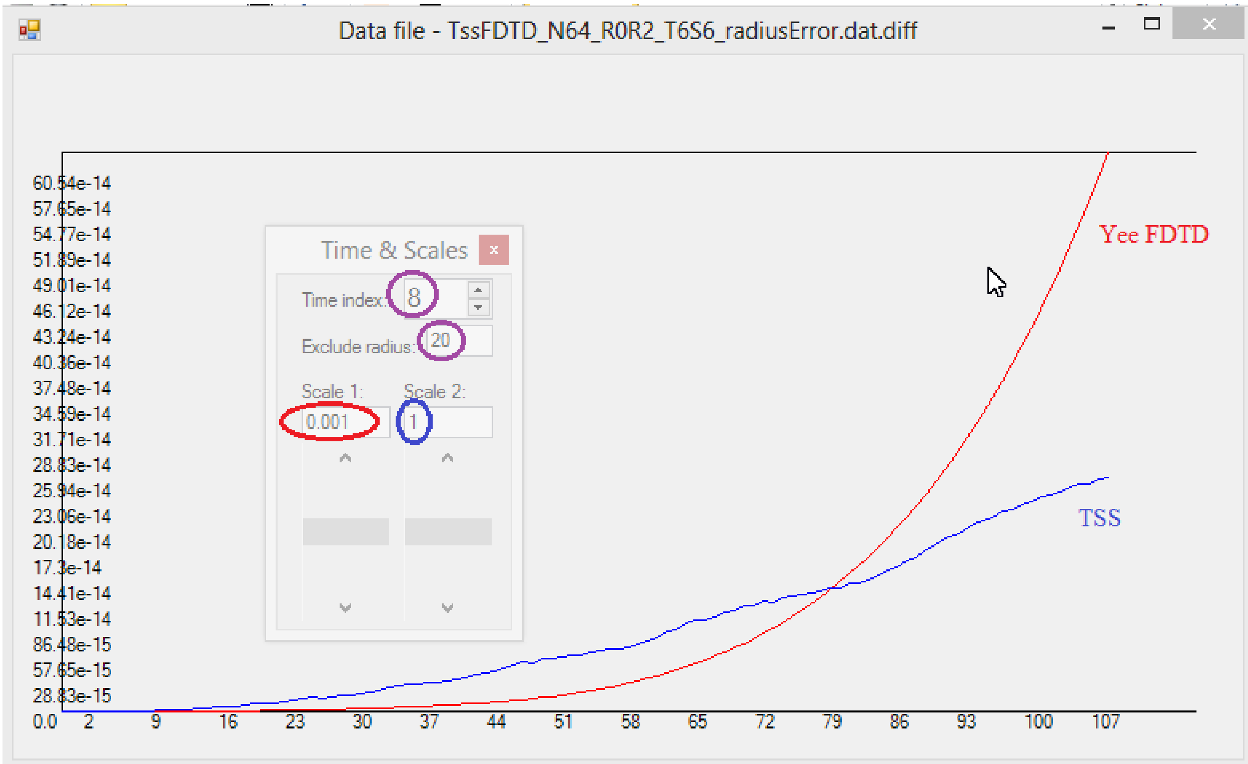
Use Draw2D to open this file:

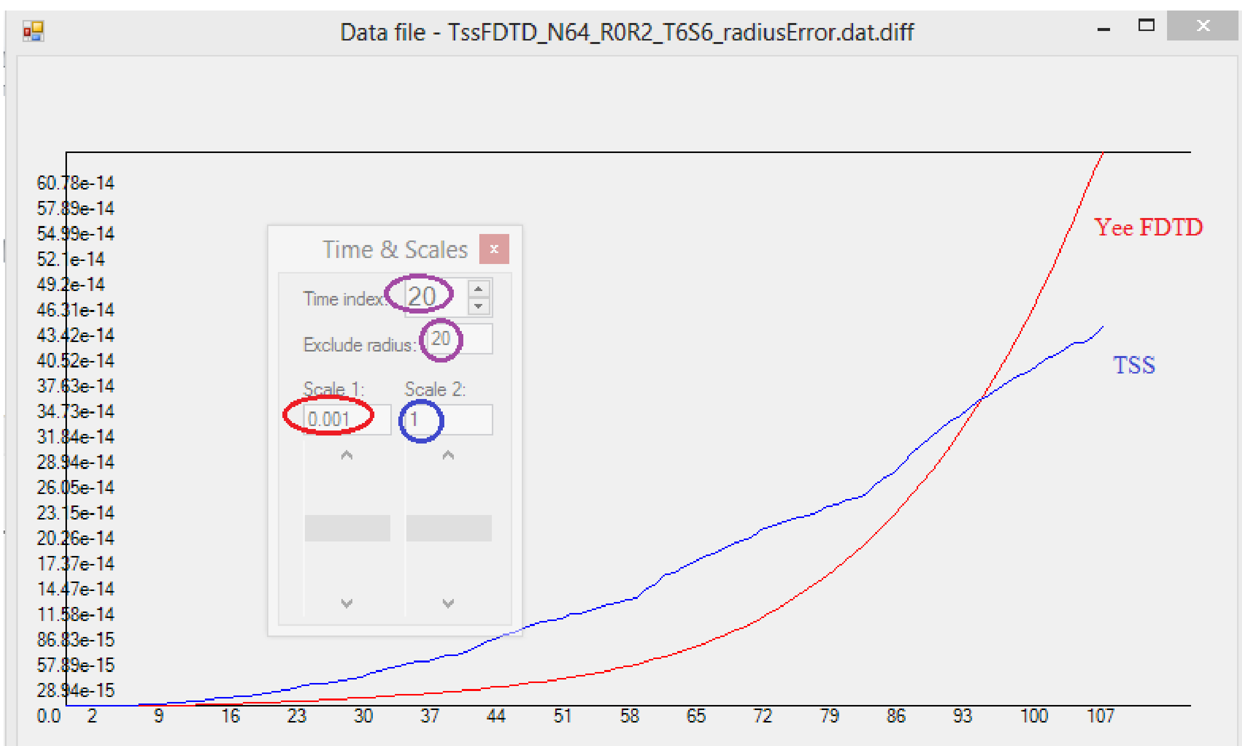




Draw2D displays divergence comparisons for each time step:







In these drawings, scale for Yee data is 0.001; it is the same scale used for comparing 4-th order TSS data with Yee data. Therefore, the 6-th order TSS estimation and the 4-th order TSS estimation produce similar accuracies. Why is that? Note that the divergences are in a range of 10^{-15}, numeric errors probably prevent higher orders estimations to get divergences lower than that.

If we cannot use higher order estimation to get higher accuracy then we can try to increase space step size and thus increase simulation speed.

The 6-th Order Simulation with Increased Space Step

Simulation

In the previous simulation, the parameters for space digitization is

$$N = 64$$

$$R = 0.2$$

$$maximum\ radius = 2N + 1 = 129$$

The space range is defined by

$$|x|, |y|, |z| \le R = 0.2$$

The space step is

$$ds = \frac{R}{maximum\ radius} = \frac{0.2}{129}$$

For this simulation, we want to increase the space step. We use

$$N_{new} = 32$$

$$\{maximu\ radius\}_{new} = 2N_{new} + 1 = 65$$

$$\{ds\}_{new} = \frac{R_{new}}{65}$$

To make it easy to compare with the previous simulations, we want following space relation holds:

$$\{radius\}\{ds\}_{new} = 2\{radius\}ds$$

Or,

$$\{ds\}_{new} = 2ds = 2 \cdot \frac{0.2}{129}$$

Thus,

$$R_{new} = (2N_{new} + 1)\{ds\}_{new} = 65 \cdot 2 \cdot \frac{0.2}{129} = 0.2015503875968992$$

Or

$$R_{new} = (N+1)2 \cdot ds$$

The task file is:

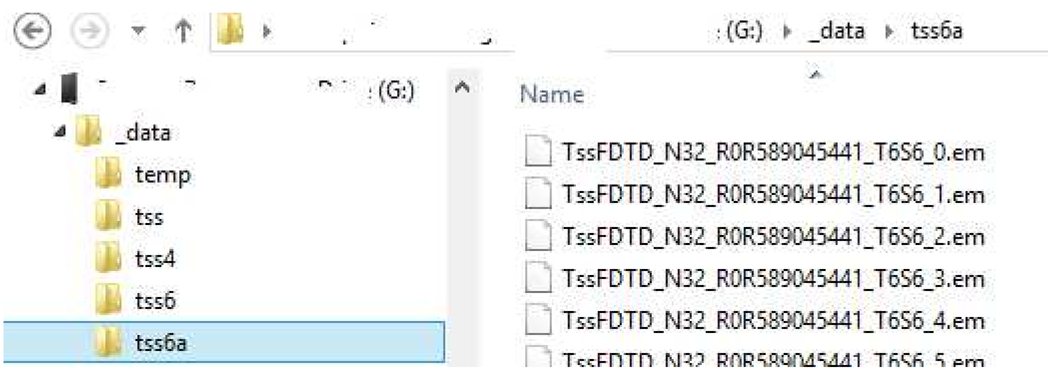
```
//task number
SIM.TASK=100

//the number of double intervals at one side of axis
FDTD.N=32

//half space range
//this value is chosen so that every space point matches a space point defined by (N=64, R=0.2)
//it is 65 * 2 * 0.2 / 129 = 26 / 129 = (about 0.2015503875968992)
FDTD.R=26/129

//half estimation order for divergence estimations. Default value is 1
FDTD.HALF_ORDER_SPACE=3

//half estimation order for time advance estimations. Default value is 1
FDTD.HALF_ORDER_TIME=3

//use default base file name
SIM.BASENAME=DEF

//maximum time steps
FDTD.MAXTIMESTEP=20

//DLL file for FDTD module
SIM.FDTD_DLL=TssFDTD.DLL

//class name for FDTD module
SIM.FDTD_NAME=TssFDTD

//DLL file for boundary condition module
SIM.BC_DLL=BoundaryConditionA.dll

//class name for boundary condition module
SIM.BC_NAME=VoidCondition

//DLL file containing Initial Value modules
SIM.IV_DLL=FieldProviders.dll

//class name of the Initial Value module to be used
SIM.IV_NAME=GaussianFields

//following task parameters are defined and used by class GaussianFields

//magnitude of field
```

```
IV.MAGNITUDE=120

//gaussian function width
IV.WIDTH=0.5
```

A sample command line can be:

```
EM /TC:\EM\TSS\DOC\task100_TSS_T6R6_2.task /WG:\_data\temp /DG:\_data\tss6a
```

A set of data files is generated:



Generate divergence magnitude report files

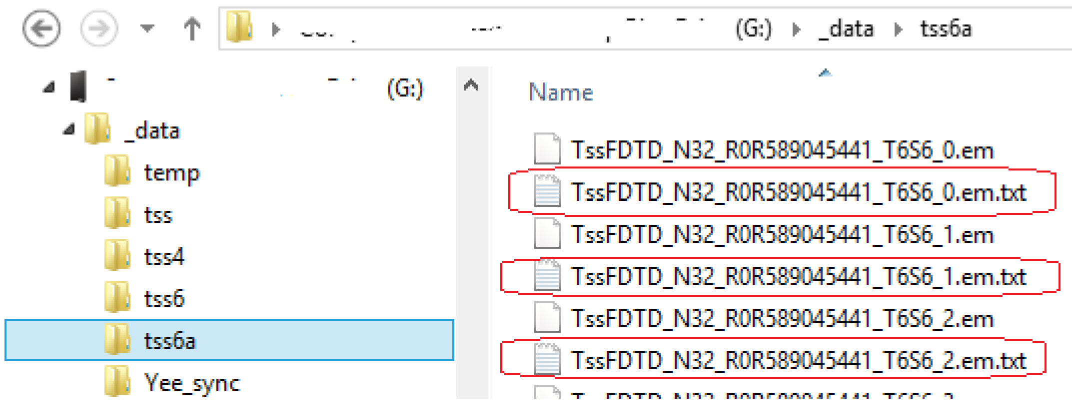
Task file:

```
//task number
SIM.TASK=120

//base file name
SIM.BASENAME=TssFDTD_N32_R0R589045441_T6S6_

//half space range; it must match the data files
FDTD.R=26/129

//half estimation order for divergence estimations. Default value is 1
FDTD.HALF_ORDER_SPACE=3
```

A sample command line can be:

```
EM /TC:\EM\TSS\Doc\task120_Tss_T6R6_2.task /WG:\_data\temp /DG:\_data\tss6a
```

A set of report files is generated:



Compare divergence magnitudes with Yee FDTD

We need to use Task 160 instead of Task 140 to merge summary files. Task 160 merges two summary files; the space step size of the first summary file is half the space size of the second summary file.

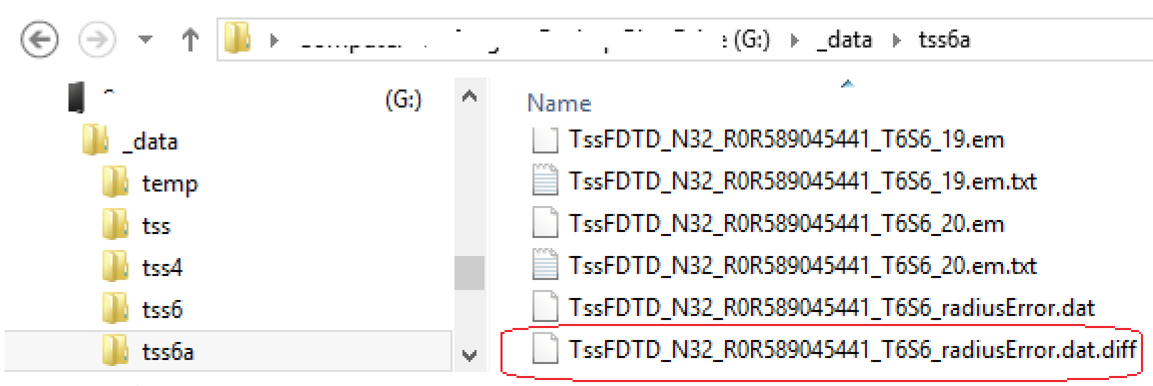
Task file:

```
//task number
SIM.TASK=160

//the first summary file name, command line parameter /D specifies folder for it
SIM.FILE1=YeeFDTDSpaceSynched_N64_R0R2_T2S2_radiusError.dat

//the second summary file name, command line parameter /E specifies folder for it
SIM.FILE2=TssFDTD_N32_R0R589045441_T6S6_radiusError.dat
```

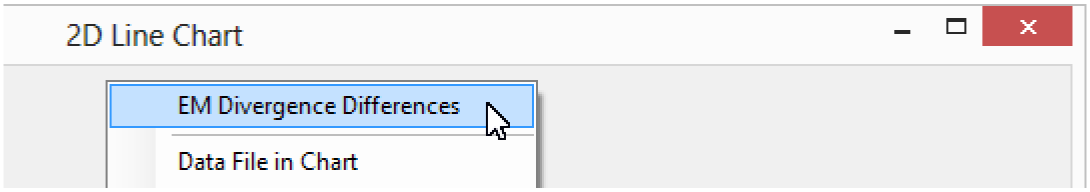
A sample command line can be:

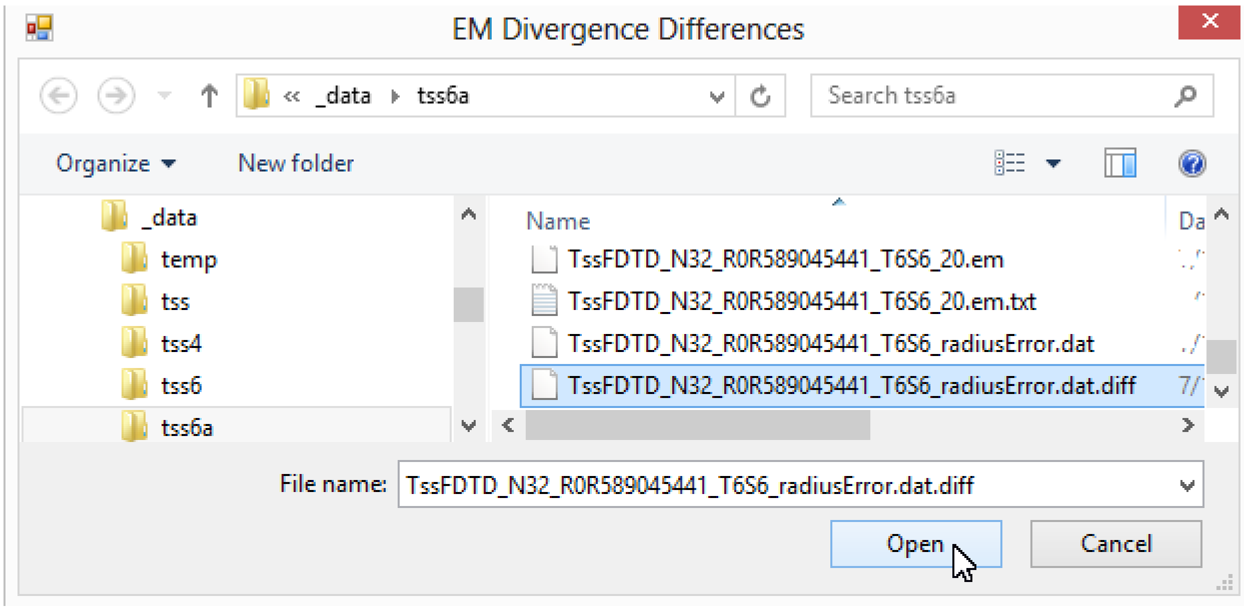
```
EM /TC:\EM\TSS\Doc\task160_Yee_Tss_T6R6.task /WG:\_data\temp /DG:\_data\Yee_sync
/EG:\_data\tss6a
```

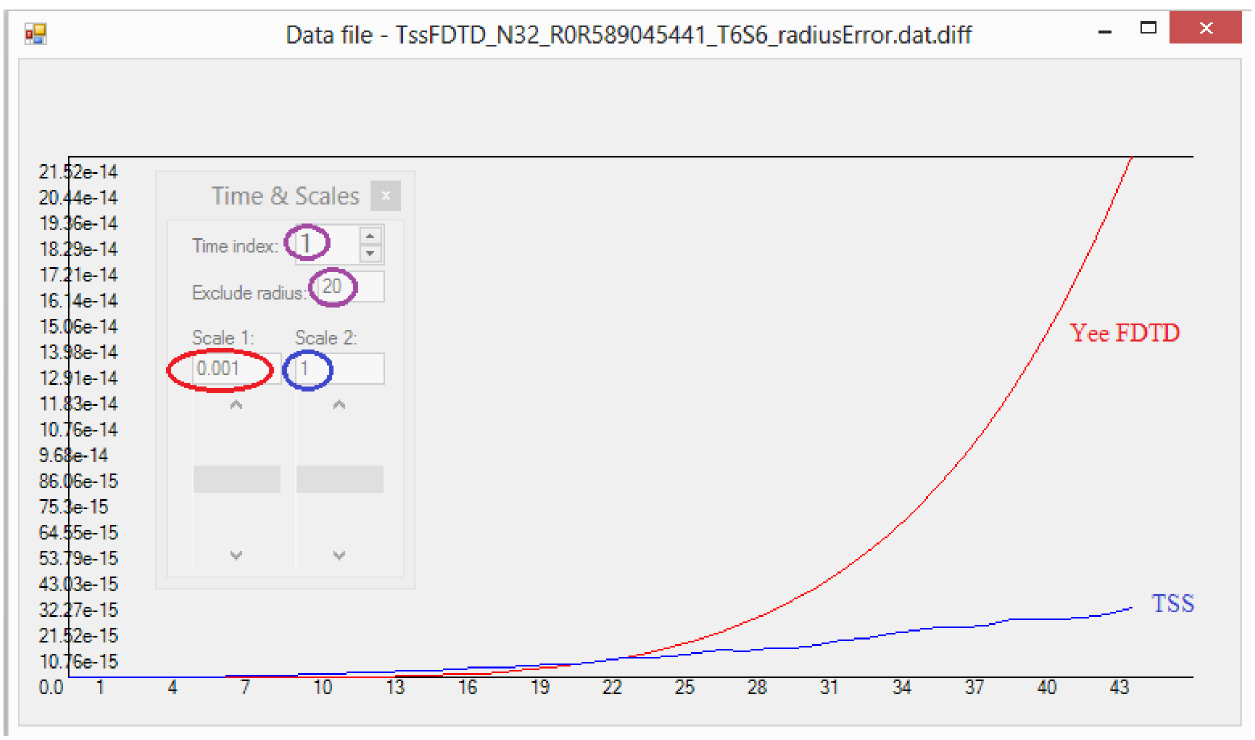
A merged file is generated:

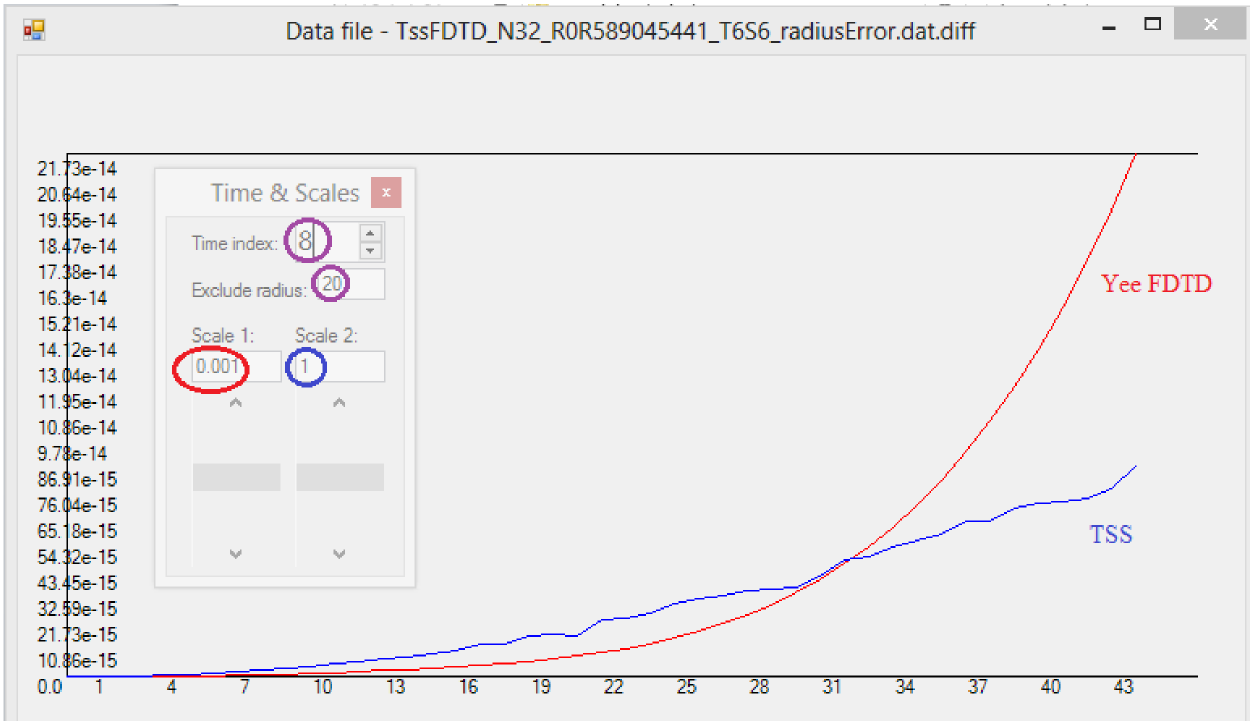


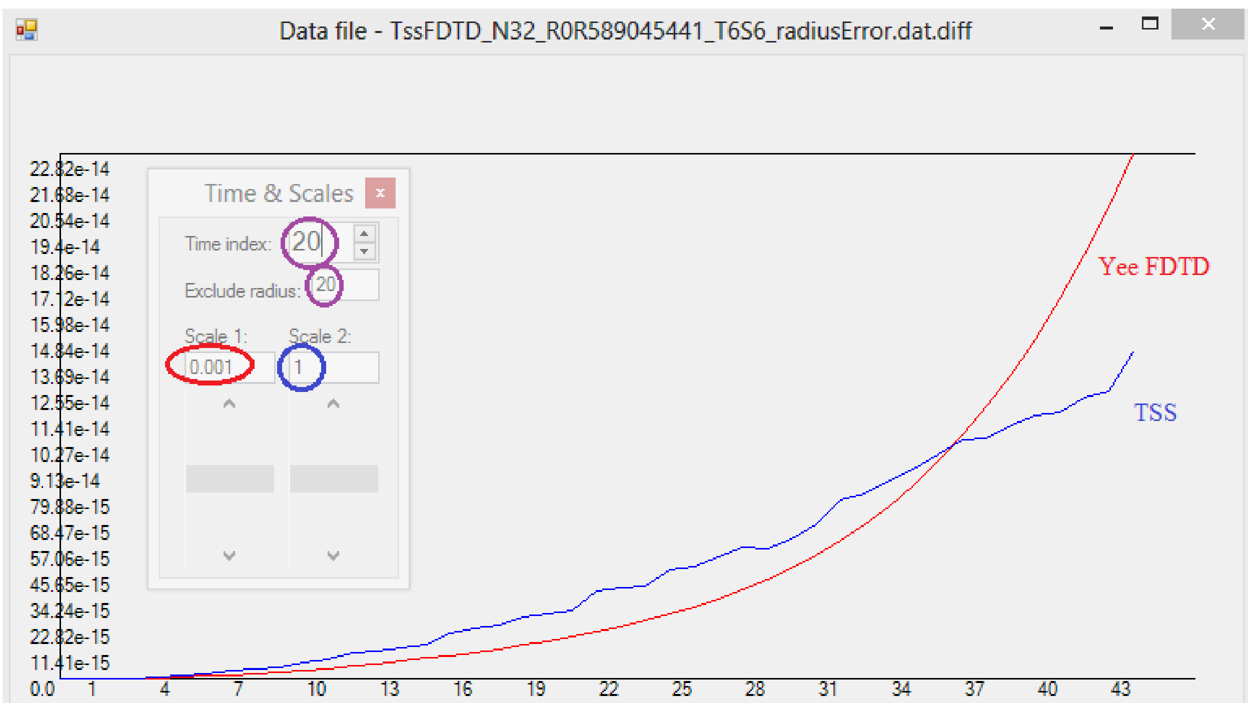
Use Draw2D to view the merged file:


2D Line Chart
EM Divergence Differences
Data File in Chart


EM Divergence Differences
« _data › tss6a
Search tss6a
Organize
New folder
_data
temp
tss
tss4
tss6
tss6a
Name
TssFDTD_N32_R0R589045441_T6S6_20.em
TssFDTD_N32_R0R589045441_T6S6_20.em.txt
TssFDTD_N32_R0R589045441_T6S6_radiusError.dat
TssFDTD_N32_R0R589045441_T6S6_radiusError.dat.diff
File name:
TssFDTD_N32_R0R589045441_T6S6_radiusError.dat.diff
Open
Cancel


Data file - TssFDTD_N32_R0R589045441_T6S6_radiusError.dat.diff
Time & Scales
Time index:
1
Exclude radius:
20
Scale 1:
Scale 2:
0.001
1
Yee FDTD
TSS
0.0
1
4
7
10
13
16
19
22
25
28
31
34
37
40
43





In these drawings, scale for Yee data is 0.001; it is the same scale used for comparing 4-th order TSS data with Yee data. Therefore, the 6-th order TSS estimation with double space step size and the 4-th order TSS estimation produce similar accuracies.

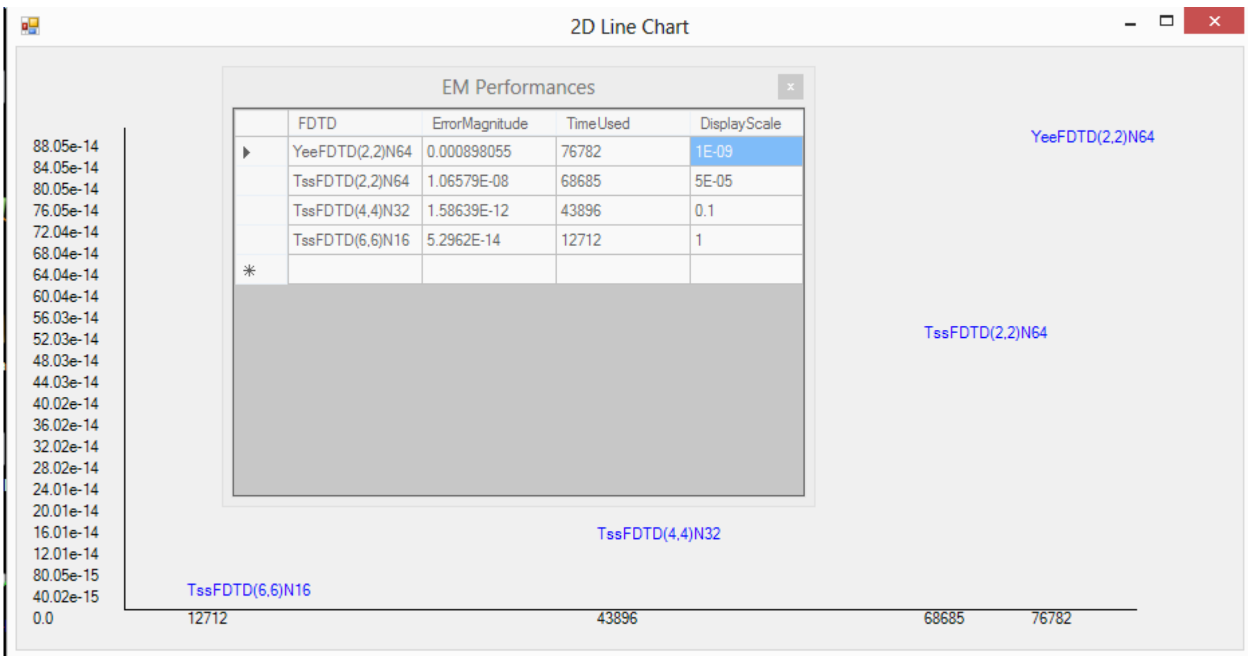
Doubling the space step size reduces number of space points from 17373979 to 2248091. It will speed up the simulation. Next, we will examine simulation speeds of different order of TSS, comparing with the standard Yee FDTD.

Simulation Speeds Comparing with Yee FDTD

Speeds and Precisions

Let's first show the test results and describe each test later.

4 simulations have been made; each simulation finished 20 time steps. The following chart shows speeds and precisions of these simulations.



The horizontal axis represents the time, in milliseconds, spent in FDTD calculations to finish 20 time steps.

The vertical axis represents average divergence magnitude.

Each simulation appears in the chart according to its total FDTD calculation time and its average divergence magnitude, excluding boundary radiuses to exclude boundary errors.

Each simulation appears in the chart by its FDTD module name, followed by time advance estimation order, space derivative estimation order, and the value of *N*, which determines the number of space points used:

$$the\ number\ of\ space\ points = (4N+1)^3$$

The larger the value of *N* the smaller the space step size and thus more calculations are needed, and thus more time is needed.

“YeeFDTD(2,2)N64” appears at far right and high up in the chart, showing that it used more time and was less accurate than other simulations. Note that a scale of 10^{-9} is used, indicating that it is 10^{-9} times less accurate. It used 76 seconds for FDTD calculations.

“TssFDTD(2,2)N64” appears near “YeeFDTD(2,2)N64”. It used a little less time than YeeFDTD did. Its divergence magnitude display scale is 5×10^{-5} , indicating that it is more accurate than YeeFDTD but it is still less accurate than the other two simulations. It used 68 seconds for FDTD calculations.

“TssFDTD(4,4)N32” appears in lower middle part of the chart, showing that it used half of the time “YeeFDTD(2,2)N64” used. So, its simulation speed is twice of YeeFDTD’s. Its scale is 0.1, indicating that it is 10^{8} times more accurate than YeeFDTD, and it is 10^{4} times more accurate than 2nd order TssFDTD.

“TssFDTD(4,4)N32” used 4-th order estimations to achieve much higher accuracy, but it also achieved much faster simulation speed by using large space step size. It used 43 seconds for FDTD calculations.

“TssFDTD(6,6)N16” appears in lower left corner of the chart, showing that it used 12 seconds for FDTD calculations. That is about 1/8 of the time YeeFDTD used. With is super faster speed, it achieved 10^{9} times more accurate than YeeFDTD.

“TssFDTD(6,6)N16” used 6-th order estimations to achieve high accuracy; it also allows much larger space step size to achieve very high simulation speed.

Record FDTD calculation time

The chart data presented previously were recorded by Task 100 through an optional task parameter

```
//enable FDTD time recording
FDTD.RECTIMESTEP=true
```

If this parameter is present in the task file then the FDTD calculation times will be recorded in a file in the folder specified by command line parameter “/W”.

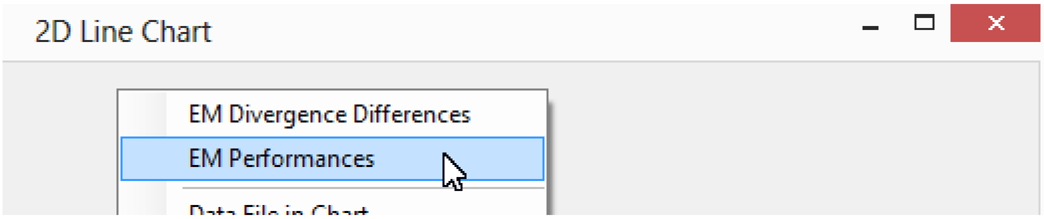
After finishing 4 simulations, the file contains 4 lines, each line records one simulation:

```
0.000898055,76782,YeeFDTD(2,2)N64
1.06579e-008,68685,TssFDTD(2,2)N64
1.58639e-012,43896,TssFDTD(4,4)N32
5.2962e-014,12712,TssFDTD(6,6)N16
```

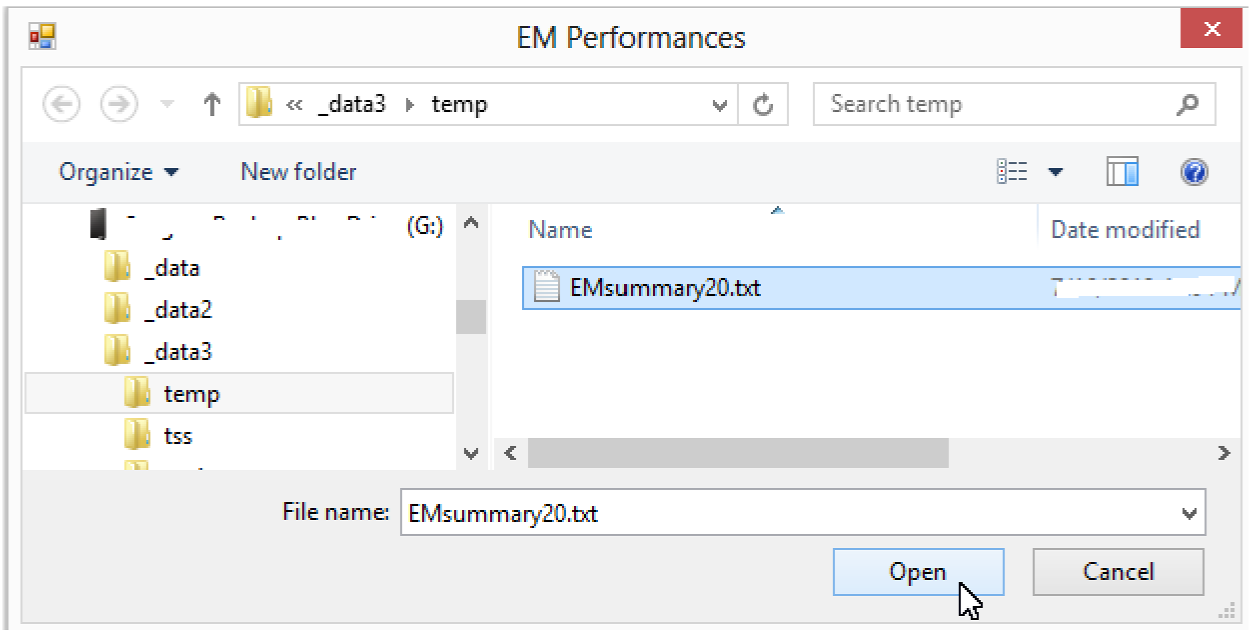
In each line, the first value is the average divergence magnitude; the second value is the total FDTD calculation time, in milliseconds; the third value is a string consists of FDTD module name, time advance estimation order, space derivative estimation order and the number of space points at one side of an axis.

The FDTD calculation time is the time used by FDTD algorithm. It can be much smaller than the whole simulation time. For example, managing large field memory can be quite time consuming, especially if the memory is allocated in a slow device, such as USB storage.

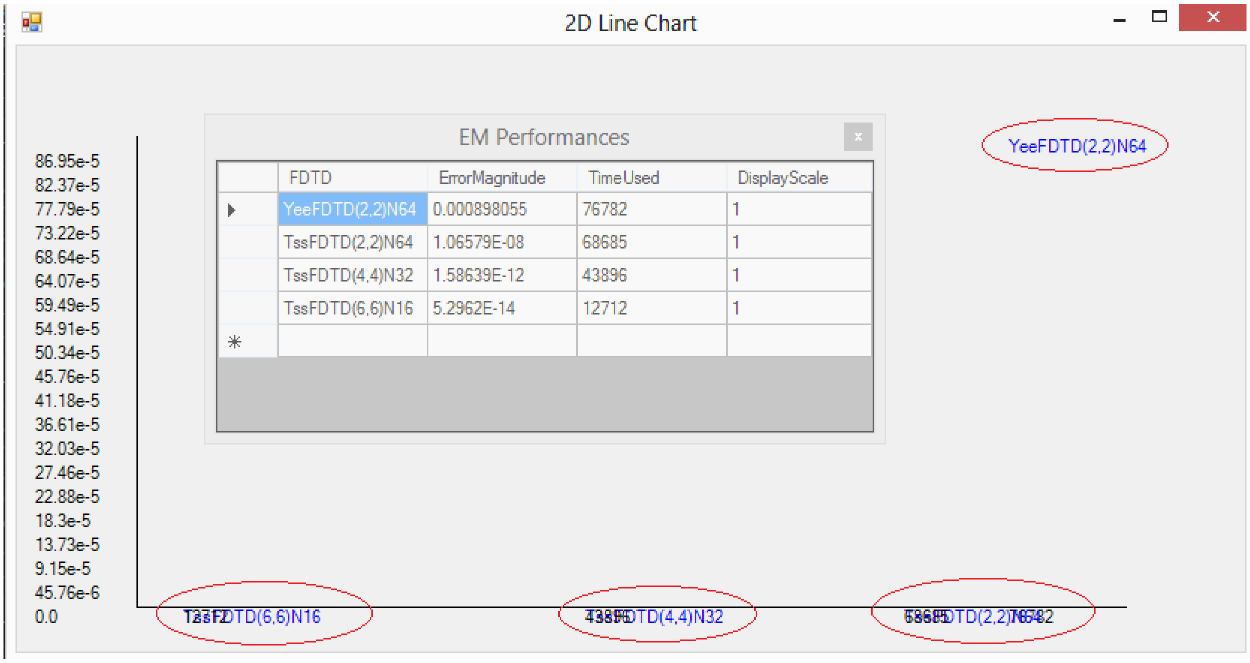
To use Draw2D to view the file visually, right-click the view screen, choose “EM Performances”:



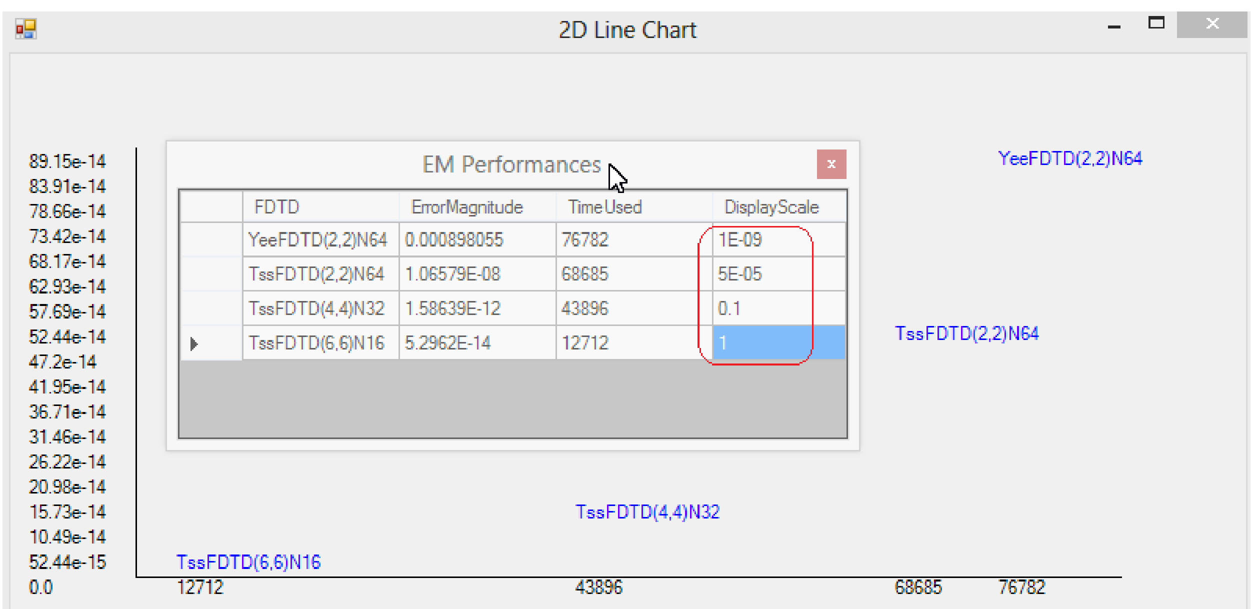
Choose the file:



The vertical axis is for divergence magnitude. The horizontal axis is for FDTD algorithm time. Each simulation appears in the chart by its average divergence magnitude and FDTD algorithm time:



Because the average divergence magnitude of the YeeFDTD is too large, it pushes all other 3 simulations to the bottom of the chart. We can adjust vertical position scales to raise those simulations up from the bottom:



Task files

Simulation - YeeFDTD(2,2)N64

//Task file name: **task100_Yee.task**

```
//task number
SIM.TASK=100

// the number of double intervals at one side of axis
FDTD.N=64

//space range at one side of an axis
FDTD.R=0.2

//enable FDTD time recording
FDTD.RECTIMESTEP=true

//use default base file name
SIM.BASENAME=DEF

//maximum time steps
FDTD.MAXTIMESTEP=20

//DLL file for FDTD module
SIM.FDTD_DLL=YeeFDTD.DLL

//class name for FDTD module
SIM.FDTD_NAME=YeeFDTD

//DLL file for boundary condition module
SIM.BC_DLL=BoundaryConditionA.dll

//class name for boundary condition module
SIM.BC_NAME=VoidCondition

//DLL file containing Initial Value modules
SIM.IV_DLL=FieldProviders.dll

//class name of the Initial Value module to be used
SIM.IV_NAME=GaussianFields

//following task parameters are defined and used by class GaussianFields

//magnitude of field
IV.MAGNITUDE=120

//gaussian function width
IV.WIDTH=0.5
```

A command line can be:

```
EM /TC:\EM\TSS\Doc\task100_Yee.task      /WG:\_data3\temp /DG:\_data3\Yee
```

Simulation – TssFDTD(2,2)N64

```
//Task file name: task100_Tss.task

//task number
SIM.TASK=100

//the number of double intervals at one side of axis
// number of space points=(4N+3)^3=17373979
// memory size=833950992 bytes=0.8G
FDTD.N=64

//half space range
FDTD.R=0.2

//enable FDTD time recording
FDTD.RECTIMESTEP=true

//use default base file name
SIM.BASENAME=DEF

//maximum time steps
FDTD.MAXTIMESTEP=20

//DLL file for FDTD module
SIM.FDTD_DLL=TssFDTD.DLL

//class name for FDTD module
SIM.FDTD_NAME=TssFDTD

//DLL file for boundary condition module
SIM.BC_DLL=BoundaryConditionA.dll

//class name for boundary condition module
SIM.BC_NAME=VoidCondition

//DLL file containing Initial Value modules
SIM.IV_DLL=FieldProviders.dll

//class name of the Initial Value module to be used
SIM.IV_NAME=GaussianFields

//following task parameters are defined and used by class GaussianFields

//magnitude of field
IV.MAGNITUDE=120

//gaussian function width
IV.WIDTH=0.5
```

A command line can be:

```
EM /TC:\EM\TSS\Doc\task100_Tss.task      /WG:\_data3\temp /DG:\_data3\tss
```

Simulation – TssFDTD(4,4)N32

//Task file name: **task100_Tss_T4R4a.task**

```
//task number
SIM.TASK=100

//the number of double intervals at one side of axis
// number of space points=(4N+3)^3=(131)^3=2248091
// memory size=6*sizeof(double)*(4N+3)^3=107908368 bytes=0.1G
FDTD.N=32

//space range at one side of an axis
//this value is chosen so that every space point matches a space point defined by (N=64, R=0.2)
//ds of (N, R) = R / (2N+1), ds of (64, 0.2) = 0.2 / 129
//range=(2N+1)2{ds of (64, 0.2)}=65 * 2 * 0.2 / 129  = 26 / 129
FDTD.R=26/129

//enable FDTD time recording
FDTD.RECTIMESTEP=true

//half estimation order for divergence estimations. Default value is 1
FDTD.HALF_ORDER_SPACE=2

//half estimation order for time advance estimations. Default value is 1
FDTD.HALF_ORDER_TIME=2

//use default base file name
SIM.BASENAME=DEF

//maximum time steps
FDTD.MAXTIMESTEP=20

//DLL file for FDTD module
SIM.FDTD_DLL=TssFDTD.DLL

//class name for FDTD module
SIM.FDTD_NAME=TssFDTD

//DLL file for boundary condition module
SIM.BC_DLL=BoundaryConditionA.dll

//class name for boundary condition module
SIM.BC_NAME=VoidCondition
```

```
//DLL file containing Initial Value modules
SIM.IV_DLL=FieldProviders.dll

//class name of the Initial Value module to be used
SIM.IV_NAME=GaussianFields

//following task parameters are defined and used by class GaussianFields

//magnitude of field
IV.MAGNITUDE=120

//gaussian function width
IV.WIDTH=0.5
```

A command line can be

```
EM /TC:\EM\TSS\Doc\task100_Tss_T4R4a.task /WG:\_data3\temp /DG:\_data3\tss4
```

Simulation – TssFDTD(6,6)N16

//task file name: **task100_Tss_T6R6a.task**

```
//task number
SIM.TASK=100

//the number of double intervals at one side of axis
// number of space points=(4N+3)^3=(67)^3=300763
// memory size=6*sizeof(double)*(4N+3)^3=14436624 bytes=14M
FDTD.N=16

//half space range
//this value is chosen so that every space point matches a space point defined by (N=32, R=26/129)
//ds of (N, R) = R / (2N+1), ds of (32, 26/129) = (26 / 129) / 65
//range=(2N+1)2{ds of (32, 26/129)} = 33 * 2 * (26 / 129) / 65 = 1716 / 8385
FDTD.R=1716/8385

//enable FDTD time recording
FDTD.RECTIMESTEP=true

//half estimation order for divergence estimations. Default value is 1
FDTD.HALF_ORDER_SPACE=3

//half estimation order for time advance estimations. Default value is 1
FDTD.HALF_ORDER_TIME=3

//use default base file name
SIM.BASENAME=DEF

//maximum time steps
FDTD.MAXTIMESTEP=20
```

```
//DLL file for FDTD module
SIM.FDTD_DLL=TssFDTD.DLL

//class name for FDTD module
SIM.FDTD_NAME=TssFDTD

//DLL file for boundary condition module
SIM.BC_DLL=BoundaryConditionA.dll

//class name for boundary condition module
SIM.BC_NAME=VoidCondition

//DLL file containing Initial Value modules
SIM.IV_DLL=FieldProviders.dll

//class name of the Initial Value module to be used
SIM.IV_NAME=GaussianFields

//following task parameters are defined and used by class GaussianFields

//magnitude of field
IV.MAGNITUDE=120

//gaussian function width
IV.WIDTH=0.5
```

A command line can be

```
EM /TC:\EM\TSS\Doc\task100_Tss_T6R6a.task /WG:\_data3\temp /DG:\_data3\tss6
```

Inhomogeneous Material Test

Sample inhomogeneous environment

Class **TssInhomogeneous** is an abstract class for applying TSS algorithm to any inhomogeneous environments.

For a specific inhomogeneous environment, a subclass of TssInhomogeneous should be developed to implement the specific inhomogeneous environment. The subclass should be implemented in a plugin DLL module.

Class **TssFDTDinhomo** is such a sample implementation of a kind of inhomogeneous environment. It is contained in TssFDTD.DLL. The sample inhomogeneous environment is presented in the following function:

```
/*
	a sample inhomogeneous space.
	this function defines what the inhomogeneous environments are.
```

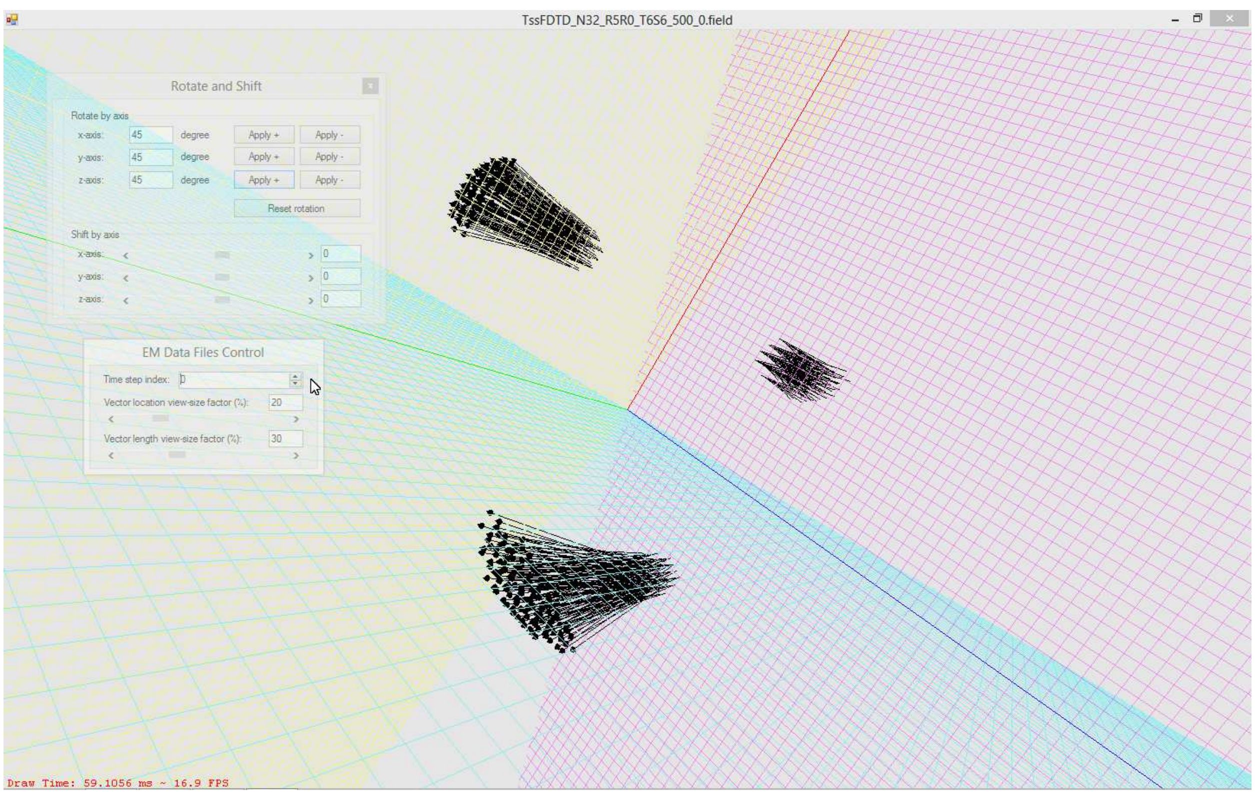
```
*/
void InitInhomogeneousParameters::handleData(int m, int n, int p)
{
	if(m > 0) //the space location is at positive x-axis
	{
		_mu[index] = mu0;
		_eps[index] = eps0 * 1.5; //this is where inhomogeneous presents
	}
	else //the space location is at 0 or negative x-axis
	{
		_mu[index] = mu0;
		_eps[index] = eps0;
	}
	index++;
}
```

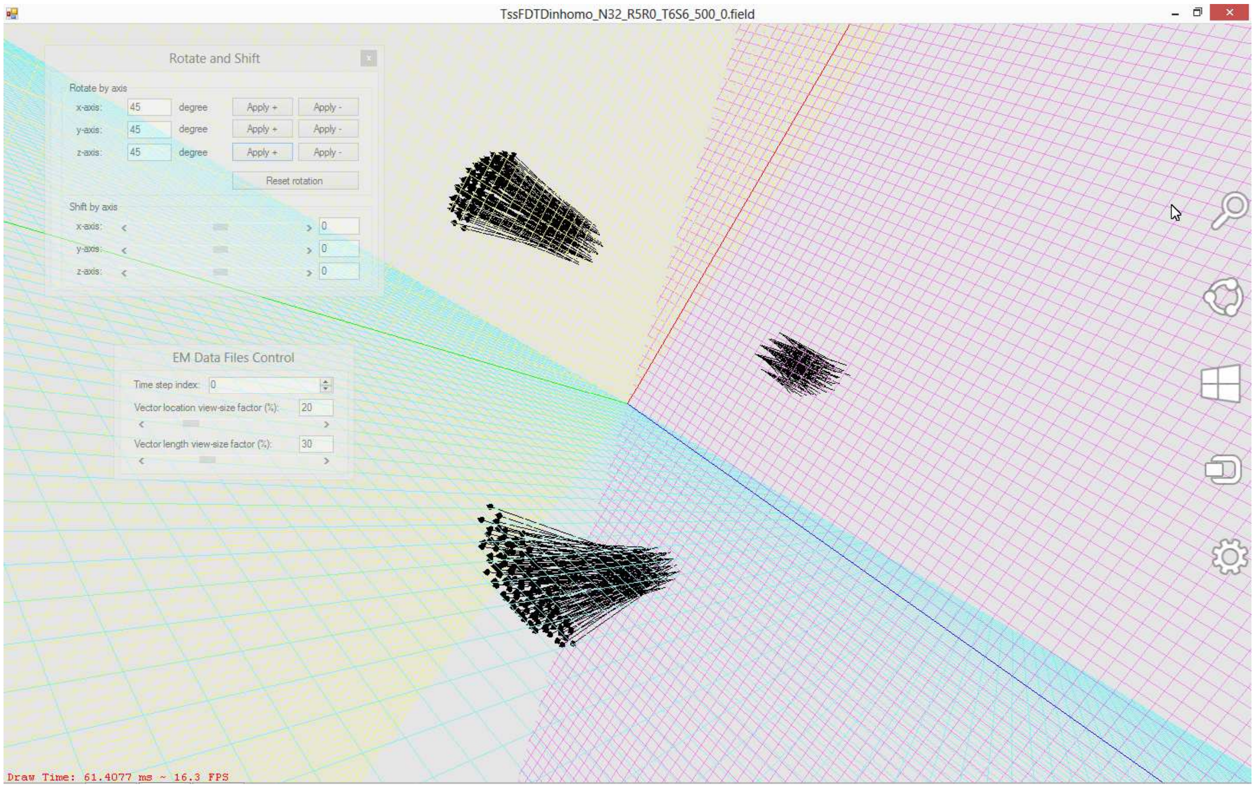
Simulation results comparing with homogeneous space

Using a Gaussian field as the initial value, TSS simulations are made for a homogeneous space and an inhomogeneous environment. Task files for these two simulations are given in the next section. Simulation results are compared here.

At time 0

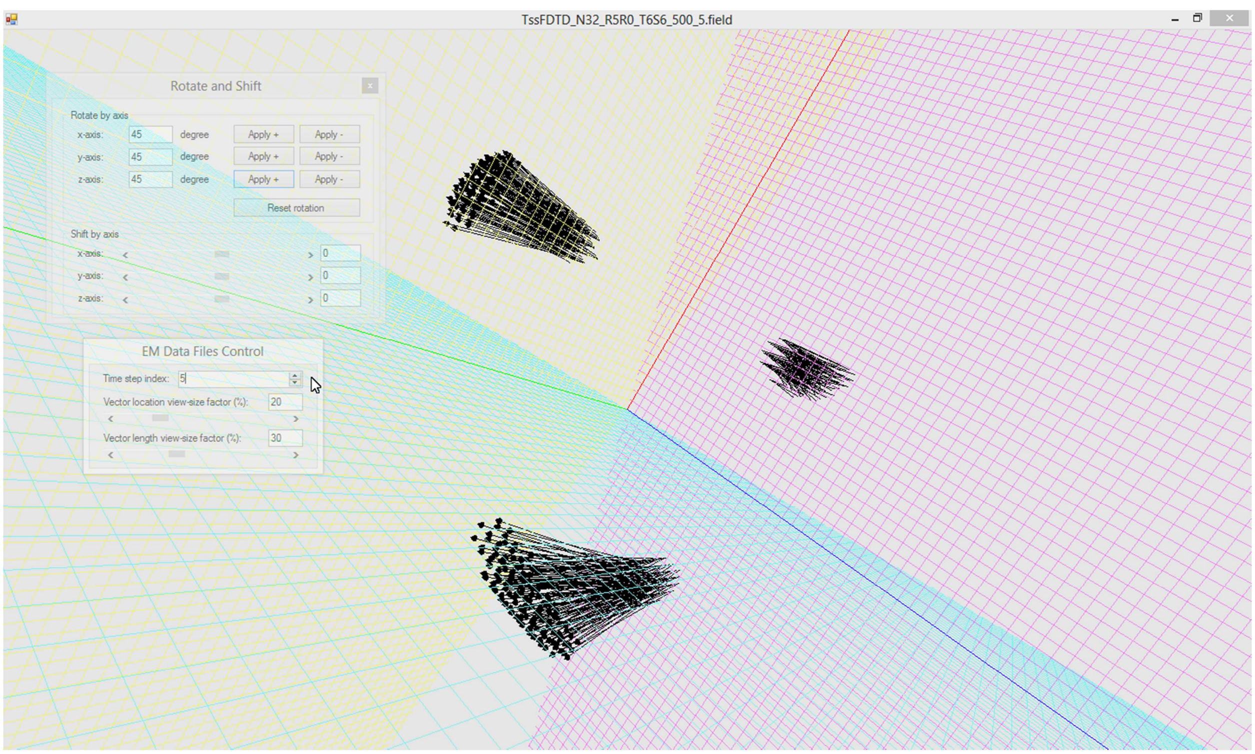
At the time 0, the two simulations are the same because the simulation has not started.



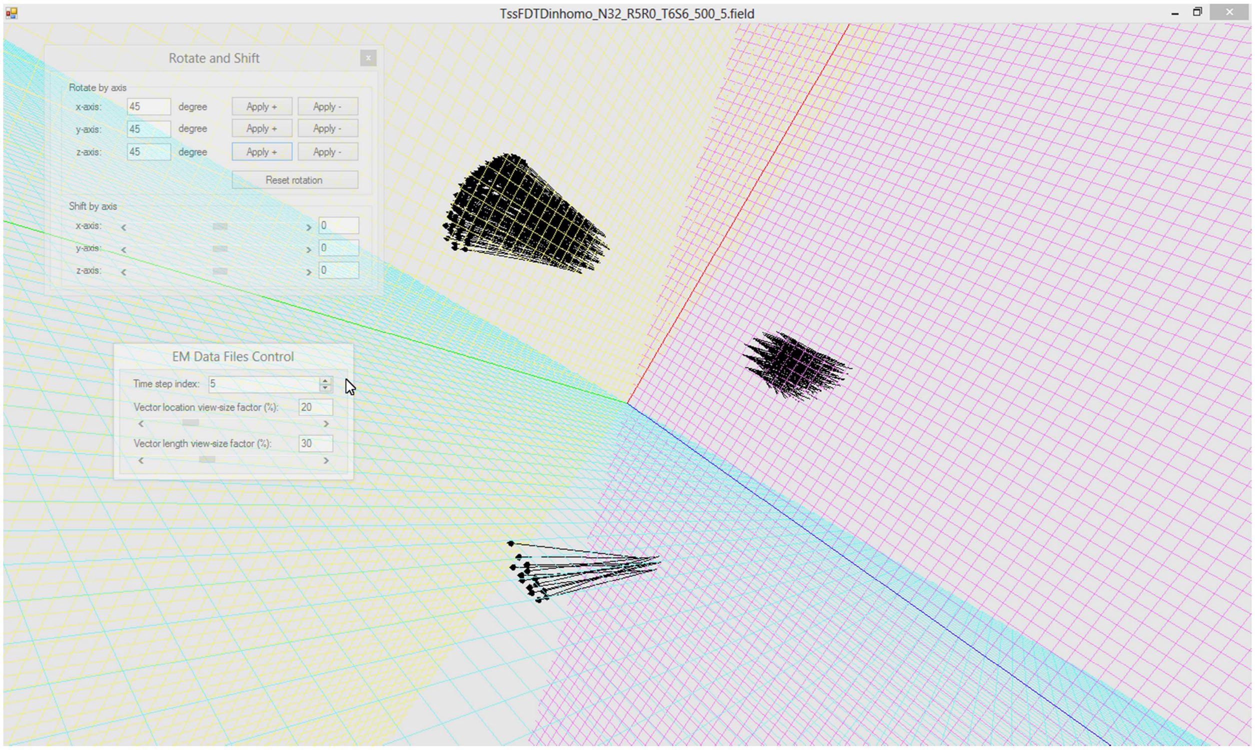


At Time step 5

In homogeneous space:



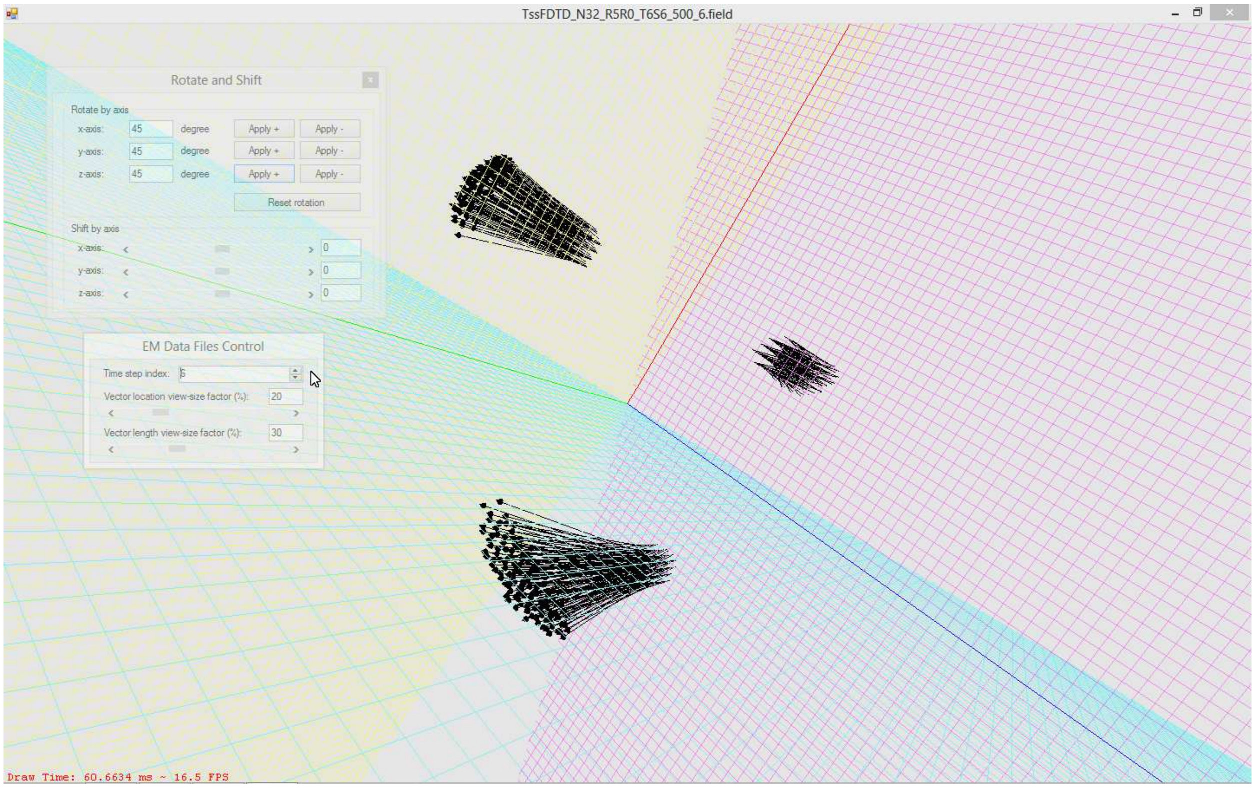
In inhomogeneous environment:



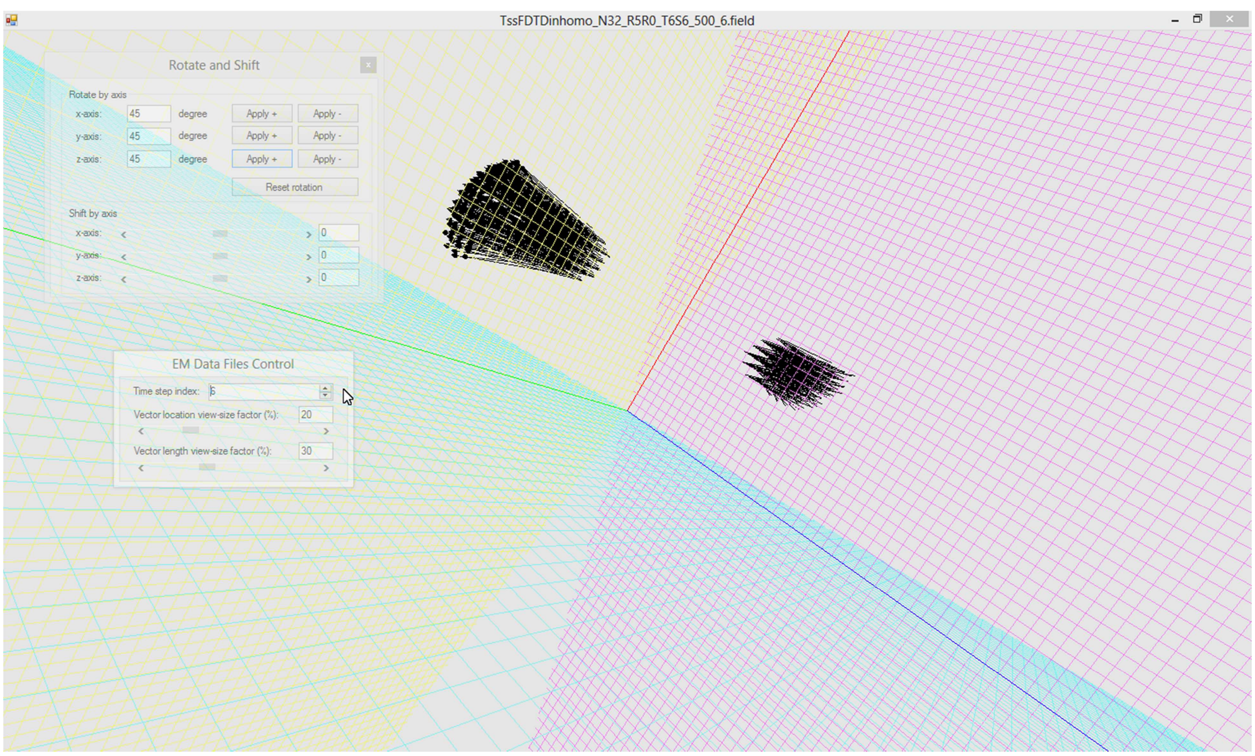
Comparing the two simulations, we can see that the inhomogeneous fields at the bottom part of the screen are far weaker than the homogeneous fields.

At time step 6

In homogeneous space:



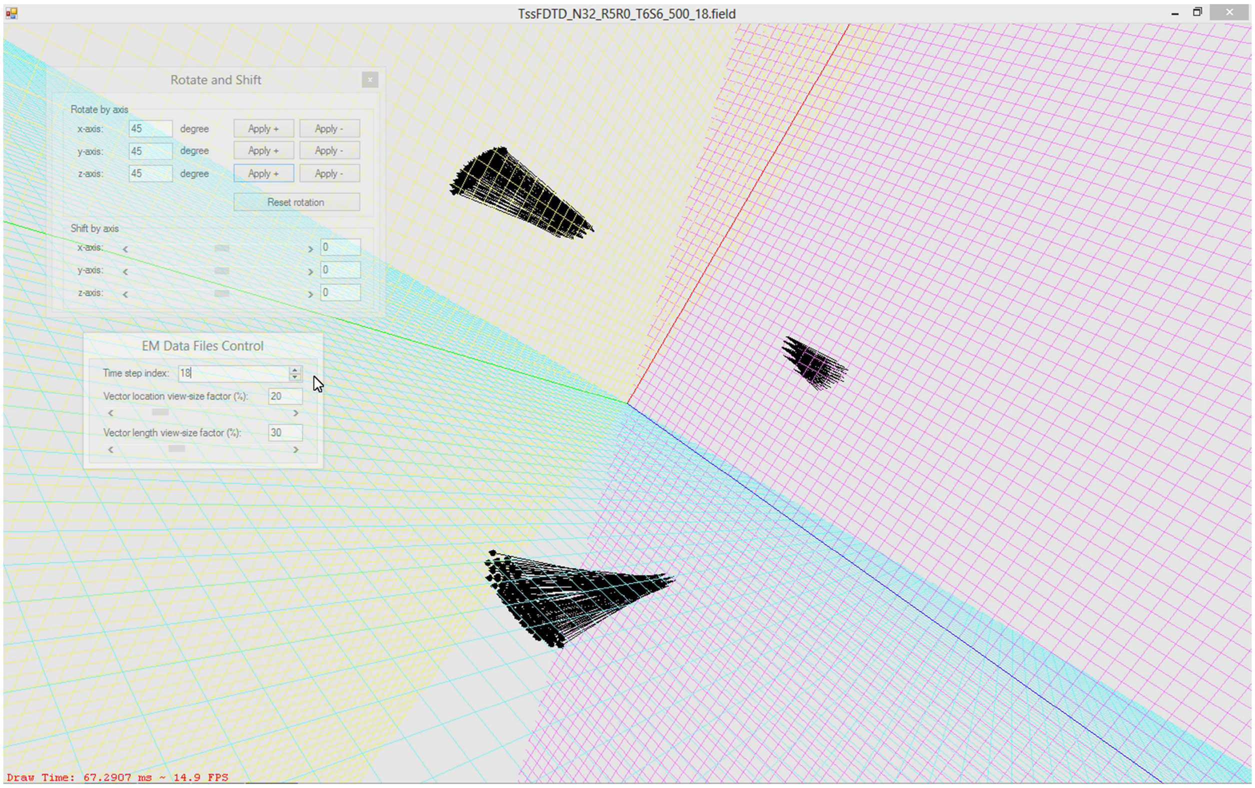
In inhomogeneous environment:



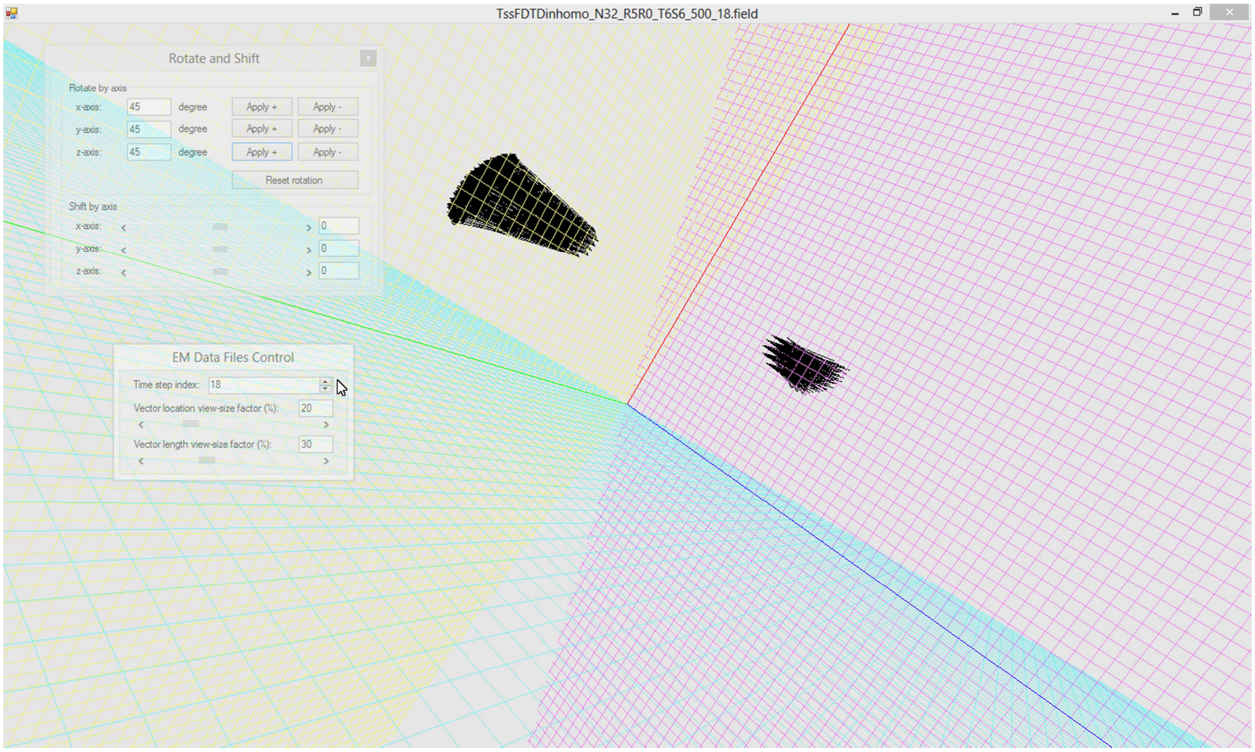
The inhomogeneous field becomes even weaker; we cannot see it at the bottom part of the screen. The homogeneous field remains strong.

At time step 18

In homogeneous space:



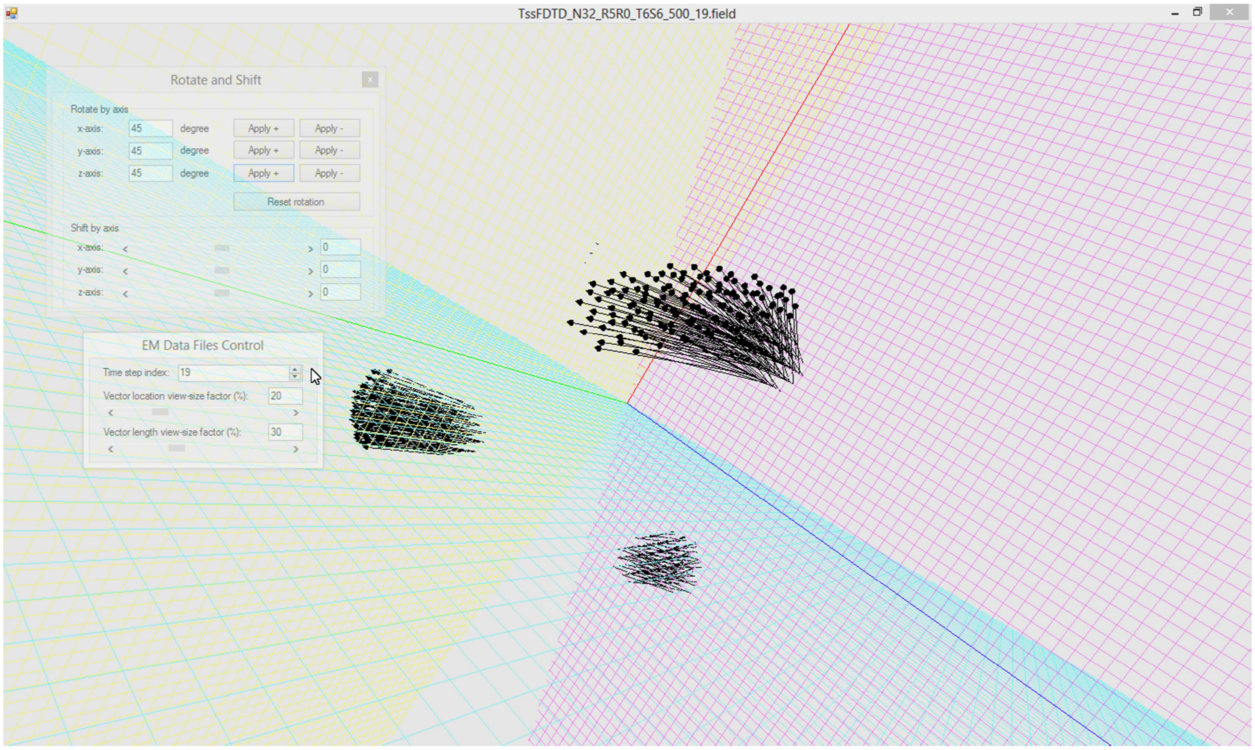
In inhomogeneous environment:



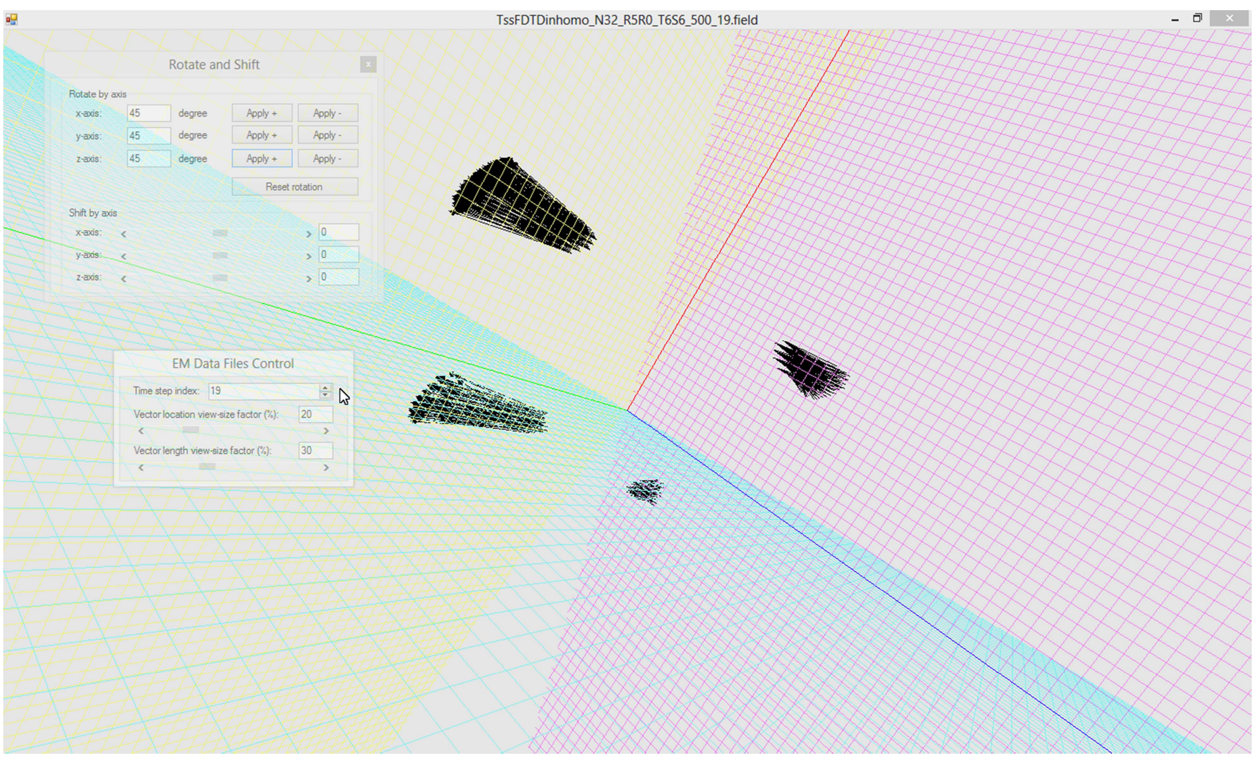
Comparing the homogeneous field, the inhomogeneous field is still missing at the bottom part of the screen.

At time step 19

In homogeneous space:



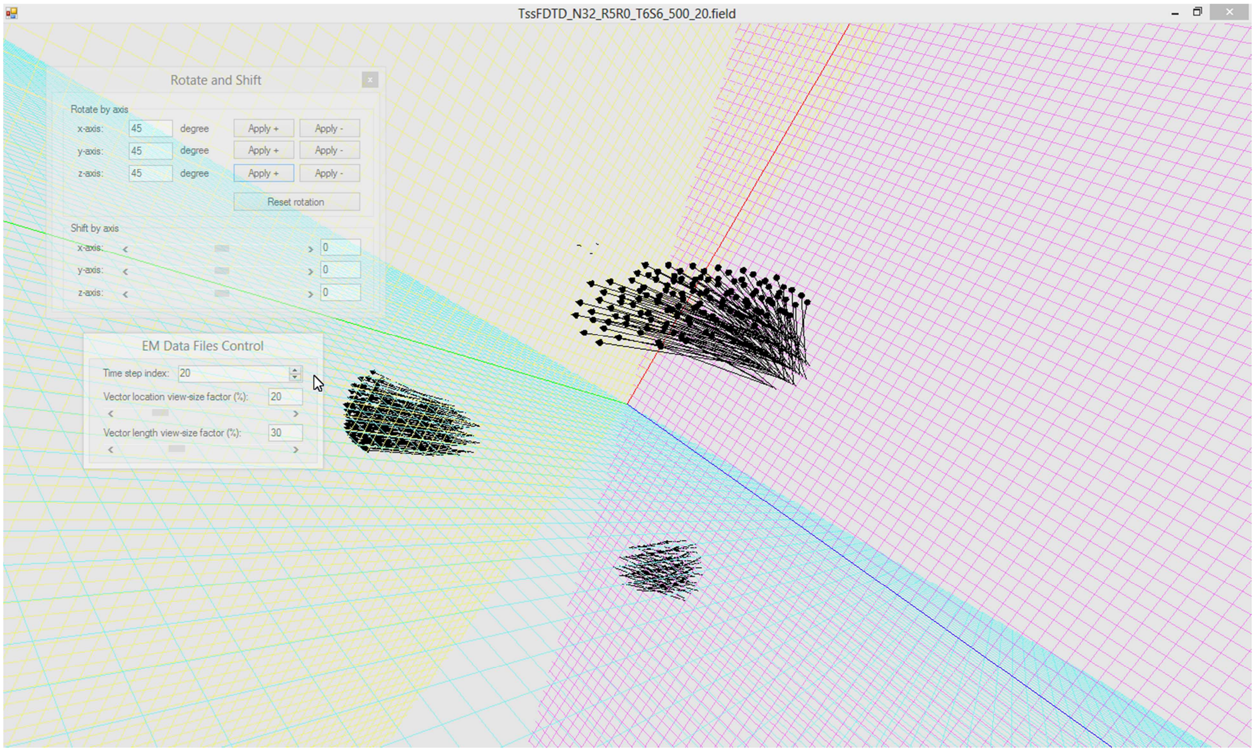
In inhomogeneous environment:



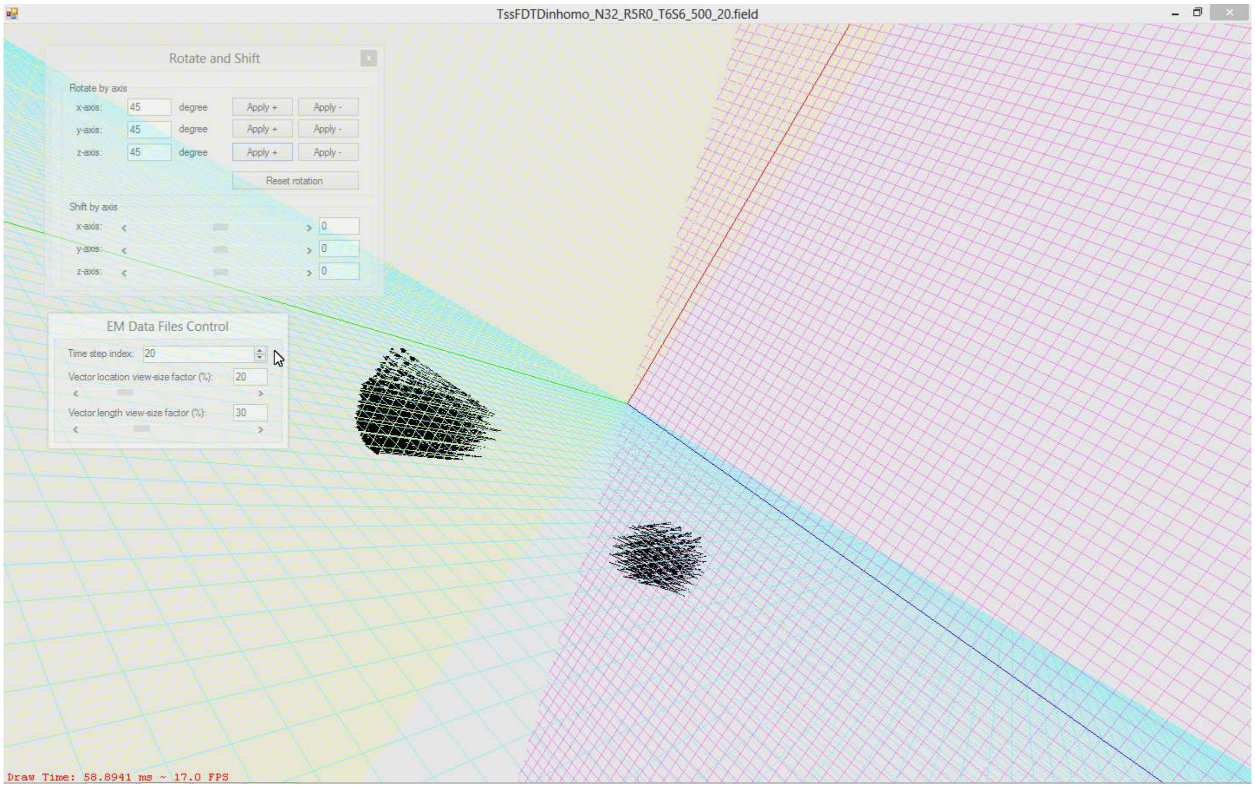
At this time step, the two fields look quite different.

At time step 20

In homogeneous space:



In inhomogeneous environment:



At this time step, the two fields look quite different.

Task files

Homogeneous

Task 100:

//task file name: **task100_Tss_T6R6N32.task**

//task number
SIM.TASK=100

//the number of double intervals at one side of axis
FDTD.N=32

//half space range
FDTD.R=5.0

//enable FDTD time recording
FDTD.RECTIMESTEP=false

//half estimation order for divergence estimations. Default value is 1
FDTD.HALF_ORDER_SPACE=3

//half estimation order for time advance estimations. Default value is 1

```
FDTD.HALF_ORDER_TIME=3

//use default base file name
SIM.BASENAME=DEF

//maximum time steps
FDTD.MAXTIMESTEP=30

//DLL file for FDTD module
SIM.FDTD_DLL=TssFDTD.DLL

//class name for FDTD module
SIM.FDTD_NAME=TssFDTD

//DLL file for boundary condition module
SIM.BC_DLL=BoundaryConditionA.dll

//class name for boundary condition module
SIM.BC_NAME=VoidCondition

//DLL file containing Initial Value modules
SIM.IV_DLL=FieldProviders.dll

//class name of the Initial Value module to be used
SIM.IV_NAME=GaussianFields

//following task parameters are defined and used by class GaussianFields

//magnitude of field
IV.MAGNITUDE=120

//gaussian function width
IV.WIDTH=0.5
```

A command line can be:

```
EM /TC:\EM\TSS\Doc\task100_Tss_T6R6N32.task /WG:\_data\temp /DG:\_data\tss6N32
```

Task 130:

```
//task file name: task130_TSS_T6S6N32.task

//task number
SIM.TASK=130

//half space range; it must match the data files
FDTD.R=5.0

//base file name
```

```
SIM.BASENAME=TssFDTD_N32_R5R0_T6S6_

//number of field points to pick
SIM.POINTS=500

//maximum number of data files to process, use 0 to process all data files
SIM.MAXTIMES=0
```

A command line can be:

```
EM /TC:\EM\TSS\DOC\task130_TSS_T6S6N32.task /WG:\_data\temp /DG:\_data\tss6N32
```

These two tasks generate homogeneous field files displayed in the 3D views.

Inhomogeneous

Task 100:

```
//task file name: task100_TssInhomo_T6R6N32.task

//task number
SIM.TASK=100

//the number of double intervals at one side of axis
FDTD.N=32

//half space range
FDTD.R=5.0

//enable FDTD time recording
FDTD.RECTIMESTEP=false

//half estimation order for divergence estimations. Default value is 1
FDTD.HALF_ORDER_SPACE=3

//half estimation order for time advance estimations. Default value is 1
FDTD.HALF_ORDER_TIME=3

//use default base file name
SIM.BASENAME=DEF

//maximum time steps
FDTD.MAXTIMESTEP=30

//DLL file for FDTD module
SIM.FDTD_DLL=TssFDTD.DLL

//class name for FDTD module
SIM.FDTD_NAME=TssFDTDinhomo
```

```
//DLL file for boundary condition module
SIM.BC_DLL=BoundaryConditionA.dll

//class name for boundary condition module
SIM.BC_NAME=VoidCondition

//DLL file containing Initial Value modules
SIM.IV_DLL=FieldProviders.dll

//class name of the Initial Value module to be used
SIM.IV_NAME=GaussianFields

//following task parameters are defined and used by class GaussianFields

//magnitude of field
IV.MAGNITUDE=120

//gaussian function width
IV.WIDTH=0.5
```

A command line can be:

```
EM /TC:\EM\TSS\Doc\task100_TssInhomo_T6R6N32.task /WG:\_data\temp
/DG:\_data\tss6N32Inhomo
```

Task 130:

```
//task file name: task130_TSSInhomo_T6S6N32.task

//task number
SIM.TASK=130

//half space range; it must match the data files
FDTD.R=5.0

//base file name
SIM.BASENAME=TssFDTDinhomo_N32_R5R0_T6S6_

//number of field points to pick
SIM.POINTS=500

//maximum number of data files to process, use 0 to process all data files
SIM.MAXTIMES=0
```

A command line can be:

```
EM /TC:\EM\TSS\DOC\task130_TSSInhomo_T6S6N32.task /WG:\_data\temp
/DG:\_data\tss6N32Inhomo
```

These two tasks generate inhomogeneous field files displayed in the 3D views.