How to Understand Time-Space Synchronized FDTD Algorithm

Part 3

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In part 2 I explained one method of handling computing domain boundary by estimating a function derivative with nearest function values. Equations (52), (53) and (54) in TSS.PDF express this method. The pseudo code on page 9 implements this method. It might not be easy to see how (52), (53) and (54) express the method and how the pseudo code implement the method. In this part I'll explain it by using actual data to show the meaning behind formulas.

Suppose we use follow data

$$r_{max} = 8$$

$$v(w\Delta s)\ are\ known, w = 0, \pm 1, \pm 2, \dots, \pm 8$$

$$choose\ M = 3$$

$$2M = 6 \quad this\ is\ the\ estimation\ order$$

We need to estimate

$$v'(w\Delta s), w = 0, \pm 1, \pm 2, \dots, \pm 8$$

using $v(w\Delta s)$ and other 6 values nearest $w\Delta s$. Below are some examples showing w and corresponding value set to be used.

$$w = 0: v(-3\Delta s), v(-2\Delta s), v(-\Delta s), v(0), v(\Delta s), v(2\Delta s), v(3\Delta s) \rightarrow v'(0)$$

$$w = 1: v(-2\Delta s), v(-\Delta s), v(0), v(\Delta s), v(2\Delta s), v(3\Delta s), v(4\Delta s) \rightarrow v'(\Delta s)$$

$$w = 7: v(2\Delta s), v(3\Delta s), v(4\Delta s), v(5\Delta s), v(6\Delta s), v(7\Delta s), v(8\Delta s) \rightarrow v'(7\Delta s)$$

$$w = -6: v(-8\Delta s), v(-7\Delta s), v(-6\Delta s), v(-5\Delta s), v(-4\Delta s), v(-3\Delta s), v(-2\Delta s) \rightarrow v'(-6\Delta s)$$

To express the values around $w\Delta s$, we can write the values to be

$$w = 0: v(0 - 3\Delta s), v(0 - 2\Delta s), v(0 - \Delta s), v(0), v(0 + \Delta s), v(0 + 2\Delta s), v(0 + 3\Delta s)$$

$$w = 1: v(\Delta s - 3\Delta s), v(\Delta s - 2\Delta s), v(\Delta s - \Delta s), v(\Delta s), v(\Delta s + \Delta s), v(\Delta s + 2\Delta s), v(\Delta s + 3\Delta s)$$

$$w = 7: v(7\Delta s - 5\Delta s), v(7\Delta s - 4\Delta s), v(7\Delta s - 3\Delta s), v(7\Delta s - 2\Delta s), v(7\Delta s - \Delta s), v(7\Delta s), v(7\Delta s + \Delta s)$$

$$w = -6: v(-6\Delta s - 2\Delta s), v(-6\Delta s - \Delta s), v(-6\Delta s), v(-6\Delta s + \Delta s), v(-6\Delta s + 2\Delta s), v(-6\Delta s + 3\Delta s), v(-6\Delta s + 4\Delta s)$$

We can see that on the right side of $v(w\Delta s)$, the values are at $w\Delta s + \Delta s, w\Delta s + 2\Delta s, \ldots, +P\Delta s$; on the left side of $v(w\Delta s)$, the values are at $w\Delta s - \Delta s, w\Delta s - 2\Delta s, \ldots, -(-N)\Delta s$.

For the above examples, we have

$$w = 0: P = 3, N = -3$$

$$w = 1: P = 3, N = -3$$

$$w = 7: P = 1, N = -5$$

$$w = -6: P = 4, N = -2$$

Equation (54) gives relationship between *P* and *w*. We can see that from (54) we can produce the above samples:

$$w = 0: P = 3, N = -3 : M - r_{max} < w < r_{max} - M \rightarrow P = r_{max} - M$$

$$w = 1: P = 3, N = -3 : M - r_{max} < w < r_{max} - M \rightarrow P = r_{max} - M$$

$$w = 7: P = 1, N = -5 : w \geq r_{max} - M \rightarrow P = r_{max} - w$$

$$w = -6: P = 4, N = -2 : w \leq M - r_{max} \rightarrow P = 2M - r_{max} - w$$

Once we get *P* we can get *N* by *N*=*P*-*2M*.

Once we get *P* and *N*, we can get derivative estimation by formula (17)

$$\begin{aligned} \Delta_s v^{\{1\}}(w\Delta_s) \approx & \sum_{k=1}^{P} Q_M[h, k-1]\big(v(w\Delta_s + k\Delta_s) - v(w\Delta_s)\big) \\ & + \sum_{k=1}^{-N} Q_M[h, k+P-1]\big(v(w\Delta_s - k\Delta_s) - v(w\Delta_s)\big) \end{aligned} \tag{1}$$

Where Q_M is a constant matrix and $Q_M[i, j]$ is the element at i-th row and j-th column.

We can see that only h-th row of the matrix is used. The row number h is determined by w. On page 9, step 1 of the pseudo code gives the relationship between w and h.

$$if\ w \geq 0\,\{$$

$$h = r_{max} - w + 1, P = h - 1, N = -2M + h - 1$$

$$if\ h > M\ then\ h = 0, P = M, N = -M$$

$$\}$$

$$if\ w < 0\,\{$$

$$h = r_{max} + w + 1 + M, P = 2M - h + 1, N = -h + 1$$

$$if\ h > 2M\ then\ h = 0, P = M, N = -M$$

$$\}$$

There is not a magic between w and h; it is the row sequence we used when forming the matrix Q_M, see page 11.