Solution to Maxwell's Equations-Draft 2

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Abstract Solution to Maxwell's equations can be deduced analytically by a process of finding closed forms from summation series. From the solutions to Maxwell's equations, it reveals that 1). the speeds of most electromagnetic fields are not constant; 2). the speeds can vary in a range of $(-\infty, +\infty)$; 3). a constant speed electromagnetic field is a special case; 4). for a moving source, the light behaves in a way of pseudo-inertia: near the source the light looks inertial, as Michelson-Morley experiment showed; 5). far away from the source, the light speed does not follow inertial rule; 6). It is proved that the explanations of the result of Michelson-Morley experiment by the special theory of relativity is incorrect.

Contents

Solution to Maxwell's Equations

Maxwell's equations

Maxwell's equations are presented below

$\frac{\partial E(x,y,z,t)}{\partial t} = \frac{1}{\varepsilon}\nabla \times H - \frac{1}{\varepsilon}J(x,y,z,t)$	(1.1)
$\frac{\partial H(x,y,z,t)}{\partial t} = -\frac{1}{\mu}\nabla \times E$	(1.2)

$J(x,y,z,t)$ is a field source.

The solution to Maxwell's euqations is determined by following given values:

$J(x,y,z,t)\ is\ given$	(1.3)
$E(x,y,z,0)\ is\ given$	(1.4)
$H(x,y,z,0)\ is\ given$	(1.5)

To solve Maxwell's equation is to find 3D functions

$$E(x,y,z,t)$$

$$H(x,y,z,t)$$

which satisfy equations (1.1) to (1.5).

It is assumed that the values given in (1.3), (1.4) and (1.5) are indefinitely differentiable with respect to time and space. Also, it is required that

$$\nabla \cdot E(x,y,z,0) = 0$$

$$\nabla \cdot H(x,y,z,0) = 0$$

In deducing the solution, the above divergence requirements are not used. Actually, if

$$\nabla \cdot H(x,y,z,0) \neq 0$$

then the initial value can be seperated into two parts:

$$H(x,y,z,0) = H_0(x,y,z,0) + H_1(x,y,z,0)$$

$$\nabla \cdot H_0(x,y,z,0) = 0;\ \nabla \cdot H_1(x,y,z,0) \neq 0$$

Let

$$J(x,y,z,t) + \nabla \times H_1(x,y,z,0)$$

be the new field source in (1.1), the divergence requirement is satisfied. Similarly, if

$$\nabla \cdot E(x,y,z,0) \neq 0$$

it can be considered a new field source appears in (1.2). By linearity of Maxwell's equations, the solution will be to add more terms to the solutions for $\nabla \cdot E(x,y,z,0) = 0$. For simplicity, I am not including the case when field source presents in (1.2) because it does not change the solution deduction process, just that it involves more terms.

The above disscussion explains why the divergence requirements are not important in our process of finding generic solutions to Maxwell's equations.

Time-Space Theorem

I discovered the following Time-Space Theorem in 2017,

$\frac{\partial^{2n+1}H}{\partial t^{2n+1}} = \frac{1}{\mu}\frac{(-1)^{n+1}}{(\varepsilon\mu)^n}\nabla^{2n+1} \times E + \begin{cases} \vec{0}, n = 0 \\ \sum_{m=1}^{n}\frac{(-1)^{m+1}}{(\varepsilon\mu)^m}\nabla^{2m-1} \times \frac{\partial^{2(n-m)+1}J(t)}{\partial t^{2(n-m)+1}}, n > 0 \end{cases}$	(0.1)
$\frac{\partial^{2(n+1)}H}{\partial t^{2(n+1)}} = \frac{(-1)^{n+1}}{(\varepsilon\mu)^{n+1}}\nabla^{2(n+1)} \times H + \sum_{m=0}^{n}\frac{(-1)^m}{(\varepsilon\mu)^{m+1}}\nabla^{2m+1} \times \frac{\partial^{2(n-m)}J(t)}{\partial t^{2(n-m)}}$	(0.2)
$\frac{\partial^{2n+1}E}{\partial t^{2n+1}} = \frac{1}{\varepsilon}\frac{(-1)^n}{(\varepsilon\mu)^n}\nabla^{2n+1} \times H + \frac{1}{\varepsilon}\sum_{m=0}^{n}\frac{(-1)^{m+1}}{(\varepsilon\mu)^m}\nabla^{2m} \times \frac{\partial^{2(n-m)}J(t)}{\partial t^{2(n-m)}}$	(0.3)
$\frac{\partial^{2(n+1)}E}{\partial t^{2(n+1)}} = \frac{(-1)^{n+1}}{(\varepsilon\mu)^{n+1}}\nabla^{2(n+1)} \times E + \frac{1}{\varepsilon}\sum_{m=0}^{n}\frac{(-1)^{m+1}}{(\varepsilon\mu)^m}\nabla^{2m} \times \frac{\partial^{2(n-m)+1}J(t)}{\partial t^{2(n-m)+1}}$	(0.4)
$n = 0,1,2,\ldots$	

I am going to use the above formulas to deduce solution to Maxwell's equations.

Generic solution

For simplicity, denote

$$H(t) = H(x,y,z,t)$$

$$E(t) = E(x,y,z,t)$$

$$J(t) = J(x,y,z,t)$$

By Taylor's series,

$$H(t) = \sum_{h=0}^{\infty}\frac{\partial^h H(0)}{\partial t^h}\frac{t^h}{h!}$$

$$E(t) = \sum_{h=0}^{\infty}\frac{\partial^h E(0)}{\partial t^h}\frac{t^h}{h!}$$

Re-group the summations,

$$H(t) = H(0) + \sum_{n=0}^{\infty}\frac{\partial^{2(n+1)}H(0)}{\partial t^{2(n+1)}}\frac{t^{2(n+1)}}{(2(n+1))!} + \sum_{n=0}^{\infty}\frac{\partial^{2n+1}H(0)}{\partial t^{2n+1}}\frac{t^{2n+1}}{(2n+1)!}$$

$$E(t) = E(0) + \sum_{n=0}^{\infty} \frac{\partial^{2(n+1)}E(0)}{\partial t^{2(n+1)}} \frac{t^{2(n+1)}}{(2(n+1))!} + \sum_{n=0}^{\infty} \frac{\partial^{2n+1}E(0)}{\partial t^{2n+1}} \frac{t^{2n+1}}{(2n+1)!}$$

Substitute the time-space theorem into the summations:

$$H(t) = H(0) + \sum_{n=0}^{\infty}\left(\frac{(-1)^{n+1}}{(\varepsilon\mu)^{n+1}} \nabla^{2(n+1)} \times H(0) + \sum_{m=0}^{n} \frac{(-1)^m}{(\varepsilon\mu)^{m+1}} \nabla^{2m+1} \times \frac{\partial^{2(n-m)}J(0)}{\partial t^{2(n-m)}}\right)\frac{t^{2(n+1)}}{(2(n+1))!} + \sum_{n=0}^{\infty}\left(\frac{1}{\mu}\frac{(-1)^{n+1}}{(\varepsilon\mu)^{n}} \nabla^{2n+1} \times E(0) + \begin{cases} \vec{0}, n = 0 \\ \sum_{m=1}^{n} \frac{(-1)^{m+1}}{(\varepsilon\mu)^{m}} \nabla^{2m-1} \times \frac{\partial^{2(n-m)+1}J(0)}{\partial t^{2(n-m)+1}}, n > 0 \end{cases}\right)\frac{t^{2n+1}}{(2n+1)!}$$

$$E(t) = E(0) + \sum_{n=0}^{\infty}\left(\frac{(-1)^{n+1}}{(\varepsilon\mu)^{n+1}} \nabla^{2(n+1)} \times E(0) + \frac{1}{\varepsilon}\sum_{m=0}^{n} \frac{(-1)^{m+1}}{(\varepsilon\mu)^{m}} \nabla^{2m} \times \frac{\partial^{2(n-m)+1}J(0)}{\partial t^{2(n-m)+1}}\right)\frac{t^{2(n+1)}}{(2(n+1))!} + \sum_{n=0}^{\infty}\left(\frac{1}{\varepsilon}\frac{(-1)^{n}}{(\varepsilon\mu)^{n}} \nabla^{2n+1} \times H(0) + \frac{1}{\varepsilon}\sum_{m=0}^{n} \frac{(-1)^{m+1}}{(\varepsilon\mu)^{m}} \nabla^{2m} \times \frac{\partial^{2(n-m)}J(0)}{\partial t^{2(n-m)}}\right)\frac{t^{2n+1}}{(2n+1)!}$$

$$H(t) = H(0) + \sum_{n=0}^{\infty}\left(\frac{(-1)^{n+1}}{(\varepsilon\mu)^{n+1}} \frac{t^{2(n+1)}}{(2(n+1))!} \nabla^{2(n+1)} \times H(0) + \sum_{m=0}^{n} \frac{(-1)^m}{(\varepsilon\mu)^{m+1}} \frac{t^{2(n+1)}}{(2(n+1))!} \nabla^{2m+1} \times \frac{\partial^{2(n-m)}J(0)}{\partial t^{2(n-m)}}\right)$$

$$+ \sum_{n=0}^{\infty}\left(\frac{1}{\mu}\frac{(-1)^{n+1}}{(\varepsilon\mu)^{n}} \frac{t^{2n+1}}{(2n+1)!} \nabla^{2n+1} \times E(0) + \begin{cases} \vec{0}, n = 0 \\ \sum_{m=1}^{n} \frac{(-1)^{m+1}}{(\varepsilon\mu)^{m}} \frac{t^{2n+1}}{(2n+1)!} \nabla^{2m-1} \times \frac{\partial^{2(n-m)+1}J(0)}{\partial t^{2(n-m)+1}}, n > 0 \end{cases}\right)$$

$$E(t) = E(0) + \sum_{n=0}^{\infty}\left(\frac{(-1)^{n+1}}{(\varepsilon\mu)^{n+1}} \frac{t^{2(n+1)}}{(2(n+1))!} \nabla^{2(n+1)} \times E(0) + \frac{1}{\varepsilon}\sum_{m=0}^{n} \frac{(-1)^{m+1}}{(\varepsilon\mu)^{m}} \frac{t^{2(n+1)}}{(2(n+1))!} \nabla^{2m} \times \frac{\partial^{2(n-m)+1}J(0)}{\partial t^{2(n-m)+1}}\right)$$

$$+ \sum_{n=0}^{\infty}\left(\frac{1}{\varepsilon}\frac{(-1)^{n}}{(\varepsilon\mu)^{n}} \frac{t^{2n+1}}{(2n+1)!} \nabla^{2n+1} \times H(0) + \frac{1}{\varepsilon}\sum_{m=0}^{n} \frac{(-1)^{m+1}}{(\varepsilon\mu)^{m}} \frac{t^{2n+1}}{(2n+1)!} \nabla^{2m} \times \frac{\partial^{2(n-m)}J(0)}{\partial t^{2(n-m)}}\right)$$

$$H(t) = H(0) + \sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{(\varepsilon\mu)^{n+1}} \frac{t^{2(n+1)}}{(2(n+1))!} \nabla^{2(n+1)} \times H(0) + \sum_{n=0}^{\infty} \frac{1}{\mu}\frac{(-1)^{n+1}}{(\varepsilon\mu)^{n}} \frac{t^{2n+1}}{(2n+1)!} \nabla^{2n+1} \times E(0) +$$

$$\sum_{n=0}^{\infty}\sum_{m=0}^{n} \frac{(-1)^m}{(\varepsilon\mu)^{m+1}} \frac{t^{2(n+1)}}{(2(n+1))!} \nabla^{2m+1} \times \frac{\partial^{2(n-m)}J(0)}{\partial t^{2(n-m)}} + \sum_{n=0}^{\infty} \begin{cases} \vec{0}, n = 0 \\ \sum_{m=1}^{n} \frac{(-1)^{m+1}}{(\varepsilon\mu)^{m}} \frac{t^{2n+1}}{(2n+1)!} \nabla^{2m-1} \times \frac{\partial^{2(n-m)+1}J(0)}{\partial t^{2(n-m)+1}}, n > 0 \end{cases}$$

$$E(t) = E(0) + \sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{(\varepsilon\mu)^{n+1}} \frac{t^{2(n+1)}}{(2(n+1))!} \nabla^{2(n+1)} \times E(0) + \sum_{n=0}^{\infty} \frac{1}{\varepsilon}\frac{(-1)^{n}}{(\varepsilon\mu)^{n}} \frac{t^{2n+1}}{(2n+1)!} \nabla^{2n+1} \times H(0) +$$

$$\sum_{n=0}^{\infty}\frac{1}{\varepsilon}\sum_{m=0}^{n} \frac{(-1)^{m+1}}{(\varepsilon\mu)^{m}} \frac{t^{2(n+1)}}{(2(n+1))!} \nabla^{2m} \times \frac{\partial^{2(n-m)+1}J(0)}{\partial t^{2(n-m)+1}} + \sum_{n=0}^{\infty}\frac{1}{\varepsilon}\sum_{m=0}^{n} \frac{(-1)^{m+1}}{(\varepsilon\mu)^{m}} \frac{t^{2n+1}}{(2n+1)!} \nabla^{2m} \times \frac{\partial^{2(n-m)}J(0)}{\partial t^{2(n-m)}}$$

Denote

$c = \frac{1}{\sqrt{\varepsilon\mu}}$	(1.6)
$\eta = \sqrt{\frac{\mu}{\varepsilon}}$	(1.7)

We have

$$\frac{1}{\mu} = \frac{1}{\mu}\frac{\sqrt{\varepsilon\mu}}{\sqrt{\varepsilon\mu}} = \frac{1}{\eta}c$$

$$\frac{1}{\varepsilon} = \frac{1}{\varepsilon}\frac{\sqrt{\varepsilon\mu}}{\sqrt{\varepsilon\mu}} = \eta c$$

Thus,

$$H(t) = H(0) + \sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{1} c^{2(n+1)} \frac{t^{2(n+1)}}{(2(n+1))!} \nabla^{2(n+1)} \times H(0) + \sum_{n=0}^{\infty} \frac{1}{\eta} c \frac{(-1)^{n+1}}{1} c^{2n} \frac{t^{2n+1}}{(2n+1)!} \nabla^{2n+1} \times E(0) +$$

$$\sum_{n=0}^{\infty}\sum_{m=0}^{n} \frac{(-1)^m}{1} c^{2(m+1)} \frac{t^{2(n+1)}}{(2(n+1))!} \nabla^{2m+1} \times \frac{\partial^{2(n-m)}J(0)}{\partial t^{2(n-m)}} + \sum_{n=0}^{\infty} \begin{cases} \vec{0}, n = 0 \\ \sum_{m=1}^{n} \frac{(-1)^{m+1}}{1} c^{2m} \frac{t^{2n+1}}{(2n+1)!} \nabla^{2m-1} \times \frac{\partial^{2(n-m)+1}J(0)}{\partial t^{2(n-m)+1}}, n > 0 \end{cases}$$

$$E(t) = E(0) + \sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{1} c^{2(n+1)} \frac{t^{2(n+1)}}{(2(n+1))!} \nabla^{2(n+1)} \times E(0) + \sum_{n=0}^{\infty} \eta c \frac{(-1)^{n}}{1} c^{2n} \frac{t^{2n+1}}{(2n+1)!} \nabla^{2n+1} \times H(0) +$$

$$\sum_{n=0}^{\infty}\eta c\sum_{m=0}^{n} \frac{(-1)^{m+1}}{1} c^{2m} \frac{t^{2(n+1)}}{(2(n+1))!} \nabla^{2m} \times \frac{\partial^{2(n-m)+1}J(0)}{\partial t^{2(n-m)+1}} + \sum_{n=0}^{\infty}\eta c\sum_{m=0}^{n} \frac{(-1)^{m+1}}{1} c^{2m} \frac{t^{2n+1}}{(2n+1)!} \nabla^{2m} \times \frac{\partial^{2(n-m)}J(0)}{\partial t^{2(n-m)}}$$

$$H(t) = H(0) + \sum_{n=0}^{\infty}(-1)^{n+1}\frac{(ct)^{2(n+1)}}{(2(n+1))!}\nabla^{2(n+1)} \times H(0) + \frac{1}{\eta}\sum_{n=0}^{\infty}(-1)^{n+1}\frac{(ct)^{2n+1}}{(2n+1)!}\nabla^{2n+1} \times E(0) +$$

$$\sum_{n=0}^{\infty}\sum_{m=0}^{n}(-1)^{m}c^{2(m+1)}\frac{t^{2(n+1)}}{(2(n+1))!}\nabla^{2m+1} \times \frac{\partial^{2(n-m)}J(0)}{\partial t^{2(n-m)}} + \sum_{n=0}^{\infty}\begin{cases}\vec{0}, n = 0\\ \sum_{m=1}^{n}(-1)^{m+1}c^{2m}\frac{t^{2n+1}}{(2n+1)!}\nabla^{2m-1} \times \frac{\partial^{2(n-m)+1}J(0)}{\partial t^{2(n-m)+1}}, n > 0\end{cases}$$

$$E(t) = E(0) + \sum_{n=0}^{\infty}(-1)^{n+1}\frac{(ct)^{2(n+1)}}{(2(n+1))!}\nabla^{2(n+1)} \times E(0) + \eta\sum_{n=0}^{\infty}(-1)^{n}\frac{(ct)^{2n+1}}{(2n+1)!}\nabla^{2n+1} \times H(0) +$$

$$\eta\sum_{n=0}^{\infty}\sum_{m=0}^{n}(-1)^{m+1}c^{2m+1}\frac{t^{2(n+1)}}{(2(n+1))!}\nabla^{2m} \times \frac{\partial^{2(n-m)+1}J(0)}{\partial t^{2(n-m)+1}} + \eta\sum_{n=0}^{\infty}\sum_{m=0}^{n}(-1)^{m+1}c^{2m+1}\frac{t^{2n+1}}{(2n+1)!}\nabla^{2m} \times \frac{\partial^{2(n-m)}J(0)}{\partial t^{2(n-m)}}$$

$$H(t) = H(0) + \sum_{n=0}^{\infty}(-1)^{n+1}\frac{(ct)^{2(n+1)}}{(2(n+1))!}\nabla^{2(n+1)} \times H(0) + \frac{1}{\eta}\sum_{n=0}^{\infty}(-1)^{n+1}\frac{(ct)^{2n+1}}{(2n+1)!}\nabla^{2n+1} \times E(0) +$$

$$\sum_{n=0}^{\infty}\sum_{m=0}^{n}(-1)^{m}c^{2(m+1)}\frac{c^{2(n+1)}}{c^{2(n+1)}}\frac{t^{2(n+1)}}{(2(n+1))!}\nabla^{2m+1} \times \frac{\partial^{2(n-m)}J(0)}{\partial t^{2(n-m)}} + \sum_{n=0}^{\infty}\begin{cases}\vec{0}, n = 0\\ \sum_{m=1}^{n}(-1)^{m+1}c^{2m}\frac{c^{2n+1}}{c^{2n+1}}\frac{t^{2n+1}}{(2n+1)!}\nabla^{2m-1} \times \frac{\partial^{2(n-m)+1}J(0)}{\partial t^{2(n-m)+1}}, n > 0\end{cases}$$

$$E(t) = E(0) + \sum_{n=0}^{\infty}(-1)^{n+1}\frac{(ct)^{2(n+1)}}{(2(n+1))!}\nabla^{2(n+1)} \times E(0) + \eta\sum_{n=0}^{\infty}(-1)^{n}\frac{(ct)^{2n+1}}{(2n+1)!}\nabla^{2n+1} \times H(0) +$$

$$\eta\sum_{n=0}^{\infty}\sum_{m=0}^{n}(-1)^{m+1}c^{2m+1}\frac{c^{2(n+1)}}{c^{2(n+1)}}\frac{t^{2(n+1)}}{(2(n+1))!}\nabla^{2m} \times \frac{\partial^{2(n-m)+1}J(0)}{\partial t^{2(n-m)+1}} + \eta\sum_{n=0}^{\infty}\sum_{m=0}^{n}(-1)^{m+1}c^{2m+1}\frac{c^{2n+1}}{c^{2n+1}}\frac{t^{2n+1}}{(2n+1)!}\nabla^{2m} \times \frac{\partial^{2(n-m)}J(0)}{\partial t^{2(n-m)}}$$

$$H(t) = H(0) + \sum_{n=0}^{\infty}(-1)^{n+1}\frac{(ct)^{2(n+1)}}{(2(n+1))!}\nabla^{2(n+1)} \times H(0) + \frac{1}{\eta}\sum_{n=0}^{\infty}(-1)^{n+1}\frac{(ct)^{2n+1}}{(2n+1)!}\nabla^{2n+1} \times E(0) +$$

$$\sum_{n=0}^{\infty}\sum_{m=0}^{n}(-1)^{m}\frac{1}{c^{2(n-m)}}\frac{(ct)^{2(n+1)}}{(2(n+1))!}\nabla^{2m+1} \times \frac{\partial^{2(n-m)}J(0)}{\partial t^{2(n-m)}} + \sum_{n=1}^{\infty}\sum_{m=1}^{n}(-1)^{m+1}\frac{1}{c^{2(n-m)+1}}\frac{(ct)^{2n+1}}{(2n+1)!}\nabla^{2m-1} \times \frac{\partial^{2(n-m)+1}J(0)}{\partial t^{2(n-m)+1}}$$

$$E(t) = E(0) + \sum_{n=0}^{\infty}(-1)^{n+1}\frac{(ct)^{2(n+1)}}{(2(n+1))!}\nabla^{2(n+1)} \times E(0) + \eta\sum_{n=0}^{\infty}(-1)^{n}\frac{(ct)^{2n+1}}{(2n+1)!}\nabla^{2n+1} \times H(0) +$$

$$\eta\sum_{n=0}^{\infty}\sum_{m=0}^{n}(-1)^{m+1}\frac{1}{c^{2(n-m)+1}}\frac{(ct)^{2(n+1)}}{(2(n+1))!}\nabla^{2m} \times \frac{\partial^{2(n-m)+1}J(0)}{\partial t^{2(n-m)+1}} + \eta\sum_{n=0}^{\infty}\sum_{m=0}^{n}(-1)^{m+1}\frac{1}{c^{2(n-m)}}\frac{(ct)^{2n+1}}{(2n+1)!}\nabla^{2m} \times \frac{\partial^{2(n-m)}J(0)}{\partial t^{2(n-m)}}$$

The following formulas are thus deduced.

$H(t) = H(0) + \sum_{n=0}^{\infty}\frac{(-1)^{n+1}(ct)^{2(n+1)}}{(2(n+1))!}\nabla^{2(n+1)} \times H(0) + \frac{1}{\eta}\sum_{n=0}^{\infty}\frac{(-1)^{n+1}(ct)^{2n+1}}{(2n+1)!}\nabla^{2n+1} \times E(0) + \sum_{n=0}^{\infty}\frac{(ct)^{2(n+1)}}{(2(n+1))!}\sum_{m=0}^{n}\frac{(-1)^{m}}{c^{2(n-m)}}\nabla^{2m+1} \times \frac{\partial^{2(n-m)}J(0)}{\partial t^{2(n-m)}} + \sum_{n=1}^{\infty}\frac{(ct)^{2n+1}}{(2n+1)!}\sum_{m=1}^{n}\frac{(-1)^{m+1}}{c^{2(n-m)+1}}\nabla^{2m-1} \times \frac{\partial^{2(n-m)+1}J(0)}{\partial t^{2(n-m)+1}}$	(1.8)
$E(t) = E(0) + \sum_{n=0}^{\infty}\frac{(-1)^{n+1}(ct)^{2(n+1)}}{(2(n+1))!}\nabla^{2(n+1)} \times E(0) + \eta\sum_{n=0}^{\infty}\frac{(-1)^{n}(ct)^{2n+1}}{(2n+1)!}\nabla^{2n+1} \times H(0) + \eta\sum_{n=0}^{\infty}\frac{(ct)^{2(n+1)}}{(2(n+1))!}\sum_{m=0}^{n}\frac{(-1)^{m+1}}{c^{2(n-m)+1}}\nabla^{2m} \times \frac{\partial^{2(n-m)+1}J(0)}{\partial t^{2(n-m)+1}} + \eta\sum_{n=0}^{\infty}\frac{(ct)^{2n+1}}{(2n+1)!}\sum_{m=0}^{n}\frac{(-1)^{m+1}}{c^{2(n-m)}}\nabla^{2m} \times \frac{\partial^{2(n-m)}J(0)}{\partial t^{2(n-m)}}$	(1.9)

Now we have a generic solution to Maxwell's equations expressed purely in given values. The next step is to find closed forms from the summations. There might not be a generic way of finding closed forms from summations, just like that there is not a generic way to find integral of an arbitrary function. I'll use several examples to show analytical closed form solutions obtained from the above summations.

Before presenting closed form examples, I'd like to point out that the above generic solution already presents a generic numeric solution. Note that it is a "numeric solution", not a "numeric estimation" such as FDTD algorithms. Assume 0 initial values, the numeric solution formulas become

$$H(t + \Delta_t) = H(t) + \sum_{n=0}^{\infty}\frac{(c\Delta_t)^{2(n+1)}}{(2(n+1))!}\sum_{m=0}^{n}\frac{(-1)^{m}}{c^{2(n-m)}}\nabla^{2m+1} \times \frac{\partial^{2(n-m)}J(t)}{\partial t^{2(n-m)}} + \sum_{n=1}^{\infty}\frac{(c\Delta_t)^{2n+1}}{(2n+1)!}\sum_{m=1}^{n}\frac{(-1)^{m+1}}{c^{2(n-m)+1}}\nabla^{2m-1} \times \frac{\partial^{2(n-m)+1}J(t)}{\partial t^{2(n-m)+1}}$$

$$E(t + \Delta_t) = E(t) + \eta\sum_{n=0}^{\infty}\frac{(c\Delta_t)^{2(n+1)}}{(2(n+1))!}\sum_{m=0}^{n}\frac{(-1)^{m+1}}{c^{2(n-m)+1}}\nabla^{2m} \times \frac{\partial^{2(n-m)+1}J(t)}{\partial t^{2(n-m)+1}} + \eta\sum_{n=0}^{\infty}\frac{(c\Delta_t)^{2n+1}}{(2n+1)!}\sum_{m=0}^{n}\frac{(-1)^{m+1}}{c^{2(n-m)}}\nabla^{2m} \times \frac{\partial^{2(n-m)}J(t)}{\partial t^{2(n-m)}}$$

Note that in the numeic solution formulas, we need $J(t)$. If we can get a closed form solution from (1.8) and (1.9) then only $J(0)$ is needed because the summation terms already include information for $J(t)$.

The differences between a "numeric solution" and a "numeric estimation" are fundamental, as outlined below.

- the "numeric solution" gives precise solution which can never be achieved by a "numeric estimation". Its accuracy is at the same level as calculated using a closed form analytical solution.
- the "numeric solution" does not need a computing domain for estimations, and thus
  - it does not involve the boundary condition problem
  - it avoids huge number of unneccesary computing on uninterested space points
  - its memory requirements can be ignored.
- Because the factor $\frac{(c\Delta_t)^{2(n+1)}}{(2(n+1))!}$ decreases quickly for large Δ_t, large simulation time step can be used. Note that there is not a space step involved because the curls are calculated precisely at a space point, not estimated from nearby space points.

Still, a closed form analytical solution is much better than a numeric solution. Below, I am presenting some typical 3D electromanetic fields in closed anaytical forms.

Examples of closed form analytical solutions

Example 1: ever growing fields

The given values are

$$J(x,y,z,t) = 0$$

$$E(x,y,z,0) = \begin{bmatrix} x^2yz \\ -2xy^2z \\ xyz^2 \end{bmatrix}$$

$$H(x,y,z,0) = 0$$

Substitute the above values into the generic solution (1.8) and (1.9), we have the closed form solution

$$H(x,y,z,t) = -\frac{1}{\eta}ct\begin{bmatrix} x(z^2+2y^2) \\ y(x^2-z^2) \\ -z(x^2+2y^2) \end{bmatrix} + \frac{1}{\eta}(ct)^3\begin{bmatrix} -x \\ 0 \\ z \end{bmatrix}$$

$$E(x,y,z,t) = \begin{bmatrix} x^2yz \\ -2xy^2z \\ xyz^2 \end{bmatrix} - (ct)^2\begin{bmatrix} -yz \\ 2xz \\ -xy \end{bmatrix}$$

We can see that the fields are getting stronger and stronger over time.

Verify initial values:

$$E(x,y,z,t=0) = \begin{bmatrix} x^2yz \\ -2xy^2z \\ xyz^2 \end{bmatrix}$$

$$H(x,y,z,t=0) = 0$$

Verify Maxwell's equations:

$$\nabla \times H(x,y,z,t) = -\frac{1}{\eta}t\begin{bmatrix} -2zy \\ 4xz \\ -2xy \end{bmatrix} + \frac{1}{\eta}(t)^3\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\frac{1}{\varepsilon}\nabla \times H(x,y,z,t) = -\frac{1}{\varepsilon}\sqrt{\frac{\varepsilon}{\mu}}ct\begin{bmatrix} -2zy \\ 4xz \\ -2xy \end{bmatrix} = -\frac{2}{\sqrt{\varepsilon\mu}}ct\begin{bmatrix} -zy \\ 2xz \\ -xy \end{bmatrix} = -2c^2t\begin{bmatrix} -zy \\ 2xz \\ -xy \end{bmatrix}$$

$$\frac{\partial E(x,y,z,t)}{\partial t} = -2c^2t\begin{bmatrix} -yz \\ 2xz \\ -xy \end{bmatrix}$$

We have

$$\frac{\partial E(x,y,z,t)}{\partial t} = \frac{1}{\varepsilon}\nabla \times H(x,y,z,t)$$

Equation (1.1) is satisfied.

$$E(x,y,z,t)=\begin{bmatrix} x^2yz \\ -2xy^2z \\ xyz^2 \end{bmatrix}-(ct)^2\begin{bmatrix} -yz \\ 2xz \\ -xy \end{bmatrix}$$

$$\nabla\times E=\begin{bmatrix} x(z^2+2y^2) \\ y(x^2-z^2) \\ -z(2y^2+x^2) \end{bmatrix}-3(ct)^2\begin{bmatrix} -x \\ 0 \\ z \end{bmatrix}$$

$$\frac{\partial H(x,y,z,t)}{\partial t}=-\frac{1}{\eta}c\begin{bmatrix} x(z^2+2y^2) \\ y(x^2-z^2) \\ -z(x^2+2y^2) \end{bmatrix}+\frac{c^3}{\eta}3t^2\begin{bmatrix} -x \\ 0 \\ z \end{bmatrix}=-\frac{1}{\mu}\left(\begin{bmatrix} x(z^2+2y^2) \\ y(x^2-z^2) \\ -z(x^2+2y^2) \end{bmatrix}-3(ct)^2\begin{bmatrix} -x \\ 0 \\ z \end{bmatrix}\right)=-\frac{1}{\mu}\nabla\times E$$

Equation (1.2) is satisfied.

Example 2: sustained fields

The given values are

$$J(x,y,z,t)=0$$

$$E(x,y,z,0)=\begin{bmatrix} \cos(y)\cos(z)\sin(x) \\ -2\cos(z)\cos(x)\sin(y) \\ \cos(x)\cos(y)\sin(z) \end{bmatrix}$$

$$H(x,y,z,0)=0$$

Substitute the above values into the generic solution (1.8) and (1.9), we have the closed form solution

$$H(x,y,z,t)=-\frac{\sqrt{3}}{\eta}\sin(\sqrt{3}ct)\begin{bmatrix} -\cos(x)\sin(y)\sin(z) \\ 0 \\ \sin(x)\sin(y)\cos(z) \end{bmatrix}$$

$$E(x,y,z,t)=\cos(\sqrt{3}ct)\begin{bmatrix} \sin(x)\cos(y)\cos(z) \\ -2\cos(x)\sin(y)\cos(z) \\ \cos(x)\cos(y)\sin(z) \end{bmatrix}$$

We can see that the fields oscillate indefinitely over time.

Example 3: Decaying fields

The given values are

$$J(x,y,z,t)=0$$

$$E(x,y,z,0)=\begin{bmatrix} yz \\ -2zx \\ xy \end{bmatrix}e^{-ar^2}$$

$$H(x,y,z,0)=0$$

$$r^2=x^2+y^2+z^2$$

Substitute the above values into the generic solution (1.8) and (1.9), we have the closed form solution

$$\boxed{H(x,y,z,t)=-\frac{e^{-a(r^2+(ct)^2)}}{\eta}\frac{ct}{v}\left(\sum_{m=0}^{2}2^m p_m(\phi)u_{3m}(v)\begin{bmatrix} 3x \\ 0 \\ -3z \end{bmatrix}+2a\cdot\sum_{m=0}^{3}2^m p_m(\phi)u_{4m}(v)\begin{bmatrix} -x(y^2+2z^2) \\ y(x^2-z^2) \\ z(y^2+2x^2) \end{bmatrix}\right)}$$

$$\boxed{E(x,y,z,t)=e^{-a(r^2+(ct)^2)}\sum_{m=0}^{3}2^m p_m(\phi)\,s_m(v)\begin{bmatrix} yz \\ -2xz \\ xy \end{bmatrix}}$$

The symbols are given below

$$\boxed{c=\frac{1}{\sqrt{\varepsilon\mu}}}$$

$\eta = \sqrt{\frac{\mu}{\varepsilon}}$
$v = 2arct$
$\phi = -a(ct)^2$
$p_0(\phi) = 1$
$p_1(\phi) = \phi$
$p_2(\phi) = \phi + \phi^2$
$p_3(\phi) = \phi + 3\phi^2 + \phi^3$
$s_0(v) = \cosh v$
$s_1(v) = \frac{1}{v^5}((3v^4 + 14v^2 + 24)\sinh v - (6v^3 + 24v)\cosh v)$
$s_2(v) = \frac{1}{v^5}((3v^3 + 18v)\cosh v - (9v^2 + 18)\sinh v)$
$s_3(v) = \frac{1}{v^5}((v^2 + 3)\sinh v - 3v\cosh v)$
$u_{30}(v) = \sinh v$
$u_{31}(v) = \frac{1}{v^4}((2v^3 + 6v)\cosh v - (4v^2 + 6)\sinh v)$
$u_{32}(v) = \frac{1}{v^4}((v^2 + 3)\sinh v - 3v\cosh v)$
$u_{40}(v) = \sinh v$
$u_{41}(v) = \frac{1}{v^6}\left((3v^5 + 26v^3 + 120v)\cosh v - (9v^4 + 66v^2 + 120)\sinh v\right)$
$u_{42}(v) = \frac{1}{v^6}\left((3v^4 + 45v^2 + 90)\sinh v - (15v^3 + 90v)\cosh v\right)$
$u_{43}(v) = \frac{1}{v^6}\left((v^3 + 15v)\cosh v - (6v^2 + 15)\sinh v\right)$

Example 4: Sine source

The given values are

$$J(x,y,z,t) = \sin(ft)\,J_0$$

$$J_0 = \begin{bmatrix} 0 \\ 0 \\ e^{-ar^2} \end{bmatrix}$$

$$E(x,y,z,0) = 0$$

$$H(x,y,z,0) = 0$$

(I am still working on it …)

Speeds of electromanetic fields

From the examples of closed form analytical solutions, the speeds of electromagnetic fields vary; the values of speeds can be in $(-\infty, +\infty)$.

For over a hundred years, there have been a religious believe that the light speed is a constant, its value is $c = \frac{1}{\sqrt{\varepsilon\mu}}$. The special theory of relativity goes one more step forward and claims that when we are watching a moving light source, we still see its light speed is a constant of *c*, based on Lorentz Covariance of Maxwell's equations and the believe that any electromagnetic fields satisfying Maxwell's equations have a constant speed of *c*.

There have been two difficulties in questioning that the light speed is a constant,

1. it is almost impossible to measure one-way light speed accurately
2. Maxwell's equations have not been solved analytically except for a 1D special case of constant speed. This paper gives a way of solving Maxwell's equations analytically.

Stationary source

From the generic solution (1.8) and (1.9) we can see that most electromagnetic fields have varying speeds. A field with a constant speed is a specially formed field.

From the generic solution, we may calculate the speed. Suppose

$$E(0)=0;\ H(0)=0$$

$$E(t)=\eta\sum_{n=0}^{\infty}\frac{(ct)^{2(n+1)}}{(2(n+1))!}\sum_{m=0}^{n}\frac{(-1)^{m+1}}{c^{2(n-m)+1}}\nabla^{2m}\times\frac{\partial^{2(n-m)+1}J(0)}{\partial t^{2(n-m)+1}}+\eta\sum_{n=0}^{\infty}\frac{(ct)^{2n+1}}{(2n+1)!}\sum_{m=0}^{n}\frac{(-1)^{m+1}}{c^{2(n-m)}}\nabla^{2m}\times\frac{\partial^{2(n-m)}J(0)}{\partial t^{2(n-m)}}$$

$$\frac{\partial E(t)}{\partial t}=\eta c\sum_{n=0}^{\infty}\frac{(ct)^{2n+1}}{(2n+1)!}\sum_{m=0}^{n}\frac{(-1)^{m+1}}{c^{2(n-m)+1}}\nabla^{2m}\times\frac{\partial^{2(n-m)+1}J(0)}{\partial t^{2(n-m)+1}}+\eta c\sum_{n=0}^{\infty}\frac{(ct)^{2n}}{(2n)!}\sum_{m=0}^{n}\frac{(-1)^{m+1}}{c^{2(n-m)}}\nabla^{2m}\times\frac{\partial^{2(n-m)}J(0)}{\partial t^{2(n-m)}}+$$

$$\eta\sum_{n=0}^{\infty}\frac{(ct)^{2(n+1)}}{(2(n+1))!}\sum_{m=0}^{n}\frac{(-1)^{m+1}}{c^{2(n-m)+1}}\nabla^{2m}\times\frac{\partial^{2(n-m)+2}J(0)}{\partial t^{2(n-m)+2}}+\eta\sum_{n=0}^{\infty}\frac{(ct)^{2n+1}}{(2n+1)!}\sum_{m=0}^{n}\frac{(-1)^{m+1}}{c^{2(n-m)}}\nabla^{2m}\times\frac{\partial^{2(n-m)+1}J(0)}{\partial t^{2(n-m)+1}}$$

$$\frac{\partial E(t)}{\partial x}=\eta\sum_{n=0}^{\infty}\frac{(ct)^{2(n+1)}}{(2(n+1))!}\sum_{m=0}^{n}\frac{(-1)^{m+1}}{c^{2(n-m)+1}}\nabla^{2m}\times\frac{\partial^{2(n-m)+1}\partial J(0)/\partial x}{\partial t^{2(n-m)+1}}+\eta\sum_{n=0}^{\infty}\frac{(ct)^{2n+1}}{(2n+1)!}\sum_{m=0}^{n}\frac{(-1)^{m+1}}{c^{2(n-m)}}\nabla^{2m}\times\frac{\partial^{2(n-m)}\partial J(0)/\partial x}{\partial t^{2(n-m)}}$$

$$speed\ of\ E_x\ along\ x-axis=\frac{\partial E_x(t)}{\partial t}/\frac{\partial E_x(t)}{\partial x}$$

$$speed\ of\ E_y\ along\ x-axis=\frac{\partial E_y(t)}{\partial t}/\frac{\partial E_y(t)}{\partial x}$$

$$speed\ of\ E_z\ along\ x-axis=\frac{\partial E_z(t)}{\partial t}/\frac{\partial E_z(t)}{\partial x}$$

We can see that there is no way the above speeds can be constants for arbitrary initial values and field source.

Suppose we use a concise notation of

$$\frac{a}{b}=\frac{\begin{bmatrix}a_x\\a_y\\a_z\end{bmatrix}}{\begin{bmatrix}b_x\\b_y\\b_z\end{bmatrix}}=\begin{bmatrix}a_x/b_x\\a_y/b_y\\a_z/b_{bz}\end{bmatrix}$$

Then the speeds are

$$\frac{\frac{\partial E(t)}{\partial t}}{\frac{\partial E(t)}{\partial x}}=cS_0(t)+S_1(t)$$

where

$$S_0(t)=\frac{\sum_{n=0}^{\infty}\frac{(ct)^{2n+1}}{(2n+1)!}\sum_{m=0}^{n}\frac{(-1)^{m+1}}{c^{2(n-m)+1}}\nabla^{2m}\times\frac{\partial^{2(n-m)+1}J(0)}{\partial t^{2(n-m)+1}}+\sum_{n=0}^{\infty}\frac{(ct)^{2n}}{(2n)!}\sum_{m=0}^{n}\frac{(-1)^{m+1}}{c^{2(n-m)}}\nabla^{2m}\times\frac{\partial^{2(n-m)}J(0)}{\partial t^{2(n-m)}}}{\sum_{n=0}^{\infty}\frac{(ct)^{2(n+1)}}{(2(n+1))!}\sum_{m=0}^{n}\frac{(-1)^{m+1}}{c^{2(n-m)+1}}\nabla^{2m}\times\frac{\partial^{2(n-m)+1}\partial J(0)/\partial x}{\partial t^{2(n-m)+1}}+\sum_{n=0}^{\infty}\frac{(ct)^{2n+1}}{(2n+1)!}\sum_{m=0}^{n}\frac{(-1)^{m+1}}{c^{2(n-m)}}\nabla^{2m}\times\frac{\partial^{2(n-m)}\partial J(0)/\partial x}{\partial t^{2(n-m)}}}$$

$$S_1(t)=\frac{\sum_{n=0}^{\infty}\frac{(ct)^{2(n+1)}}{(2(n+1))!}\sum_{m=0}^{n}\frac{(-1)^{m+1}}{c^{2(n-m)+1}}\nabla^{2m}\times\frac{\partial^{2(n-m)+2}J(0)}{\partial t^{2(n-m)+2}}+\sum_{n=0}^{\infty}\frac{(ct)^{2n+1}}{(2n+1)!}\sum_{m=0}^{n}\frac{(-1)^{m+1}}{c^{2(n-m)}}\nabla^{2m}\times\frac{\partial^{2(n-m)+1}J(0)}{\partial t^{2(n-m)+1}}}{\sum_{n=0}^{\infty}\frac{(ct)^{2(n+1)}}{(2(n+1))!}\sum_{m=0}^{n}\frac{(-1)^{m+1}}{c^{2(n-m)+1}}\nabla^{2m}\times\frac{\partial^{2(n-m)+1}\partial J(0)/\partial x}{\partial t^{2(n-m)+1}}+\sum_{n=0}^{\infty}\frac{(ct)^{2n+1}}{(2n+1)!}\sum_{m=0}^{n}\frac{(-1)^{m+1}}{c^{2(n-m)}}\nabla^{2m}\times\frac{\partial^{2(n-m)}\partial J(0)/\partial x}{\partial t^{2(n-m)}}}$$

To make a constant speed field is to carefully choose $J(x,y,z,t)$ so that

$$S_0(t)=\frac{\sum_{n=0}^{\infty}\frac{(ct)^{2n+1}}{(2n+1)!}\sum_{m=0}^{n}\frac{(-1)^{m+1}}{c^{2(n-m)+1}}\nabla^{2m}\times\frac{\partial^{2(n-m)+1}J(0)}{\partial t^{2(n-m)+1}}+\sum_{n=0}^{\infty}\frac{(ct)^{2n}}{(2n)!}\sum_{m=0}^{n}\frac{(-1)^{m+1}}{c^{2(n-m)}}\nabla^{2m}\times\frac{\partial^{2(n-m)}J(0)}{\partial t^{2(n-m)}}}{\sum_{n=0}^{\infty}\frac{(ct)^{2(n+1)}}{(2(n+1))!}\sum_{m=0}^{n}\frac{(-1)^{m+1}}{c^{2(n-m)+1}}\nabla^{2m}\times\frac{\frac{\partial^{2(n-m)+1}\partial J(0)}{\partial x}}{\partial t^{2(n-m)+1}}+\sum_{n=0}^{\infty}\frac{(ct)^{2n+1}}{(2n+1)!}\sum_{m=0}^{n}\frac{(-1)^{m+1}}{c^{2(n-m)}}\nabla^{2m}\times\frac{\frac{\partial^{2(n-m)}\partial J(0)}{\partial x}}{\partial t^{2(n-m)}}}=\begin{bmatrix}1\\1\\1\end{bmatrix}$$

$$S_1(t)=\frac{\sum_{n=0}^{\infty}\frac{(ct)^{2(n+1)}}{(2(n+1))!}\sum_{m=0}^{n}\frac{(-1)^{m+1}}{c^{2(n-m)+1}}\nabla^{2m}\times\frac{\partial^{2(n-m)+2}J(0)}{\partial t^{2(n-m)+2}}+\sum_{n=0}^{\infty}\frac{(ct)^{2n+1}}{(2n+1)!}\sum_{m=0}^{n}\frac{(-1)^{m+1}}{c^{2(n-m)}}\nabla^{2m}\times\frac{\partial^{2(n-m)+1}J(0)}{\partial t^{2(n-m)+1}}}{\sum_{n=0}^{\infty}\frac{(ct)^{2(n+1)}}{(2(n+1))!}\sum_{m=0}^{n}\frac{(-1)^{m+1}}{c^{2(n-m)+1}}\nabla^{2m}\times\frac{\partial^{2(n-m)+1}\partial J(0)/\partial x}{\partial t^{2(n-m)+1}}+\sum_{n=0}^{\infty}\frac{(ct)^{2n+1}}{(2n+1)!}\sum_{m=0}^{n}\frac{(-1)^{m+1}}{c^{2(n-m)}}\nabla^{2m}\times\frac{\partial^{2(n-m)}\partial J(0)/\partial x}{\partial t^{2(n-m)}}}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

A $J(x,y,z,t)$ satisfying the above conditions must be quite special. But when $t\to 0$ the conditions may be easier to be satisfied.

when $t\to 0$ we may choose $J(x,y,z,t)$ such that

$$\lim_{t \to 0} S_0(t) = \vec{1}$$

$$\lim_{t \to 0} S_1(t) = \vec{0}$$

If the above conditions are satisfied then near $t = 0$ the speeds of electromagnetic fields are close to *c*.

Moving source

Suppose the source is moving along x-axis in a constant speed *v*.

$$J(x - vt, y, z, t)$$

$$\frac{d^n J(0)}{dt^n} = \sum_{m=0}^{n} (-1)^m \binom{n}{m} v^m \frac{\partial^n J(0)}{\partial t^{n-m} \partial x^m}$$

$$\binom{n}{m} \text{ is the binomial coefficient}$$

$$E(t) = \eta \sum_{n=0}^{\infty} \frac{(ct)^{2(n+1)}}{(2(n+1))!} \sum_{m=0}^{n} \frac{(-1)^{m+1}}{c^{2(n-m)+1}} \nabla^{2m} \times \sum_{k=0}^{2(n-m)+1} (-1)^k \binom{2(n-m)+1}{k} v^k \frac{\partial^{2(n-m)+1} J(0)}{\partial t^{2(n-m)+1-k} \partial x^k}$$

$$+\eta \sum_{n=0}^{\infty} \frac{(ct)^{2n+1}}{(2n+1)!} \sum_{m=0}^{n} \frac{(-1)^{m+1}}{c^{2(n-m)}} \nabla^{2m} \times \sum_{k=0}^{2(n-m)} (-1)^k \binom{2(n-m)}{k} v^k \frac{\partial^{2(n-m)} J(0)}{\partial t^{2(n-m)-k} \partial x^k}$$

$$\frac{\partial E(t)}{\partial t} = \eta c \sum_{n=0}^{\infty} \frac{(ct)^{2n+1}}{(2n+1)!} \sum_{m=0}^{n} \frac{(-1)^{m+1}}{c^{2(n-m)+1}} \nabla^{2m} \times \sum_{k=0}^{2(n-m)+1} (-1)^k \binom{2(n-m)+1}{k} v^k \frac{\partial^{2(n-m)+1} J(0)}{\partial t^{2(n-m)+1-k} \partial x^k} +$$

$$\eta c \sum_{n=0}^{\infty} \frac{(ct)^{2n}}{(2n)!} \sum_{m=0}^{n} \frac{(-1)^{m+1}}{c^{2(n-m)}} \nabla^{2m} \times \sum_{k=0}^{2(n-m)} (-1)^k \binom{2(n-m)}{k} v^k \frac{\partial^{2(n-m)} J(0)}{\partial t^{2(n-m)-k} \partial x^k} +$$

$$\eta \sum_{n=0}^{\infty} \frac{(ct)^{2(n+1)}}{(2(n+1))!} \sum_{m=0}^{n} \frac{(-1)^{m+1}}{c^{2(n-m)+1}} \nabla^{2m} \times \sum_{k=0}^{2(n-m)+2} (-1)^k \binom{2(n-m)+2}{k} v^k \frac{\partial^{2(n-m)+2} J(0)}{\partial t^{2(n-m)+2-k} \partial x^k} +$$

$$\eta \sum_{n=0}^{\infty} \frac{(ct)^{2n+1}}{(2n+1)!} \sum_{m=0}^{n} \frac{(-1)^{m+1}}{c^{2(n-m)}} \nabla^{2m} \times \sum_{k=0}^{2(n-m)+1} (-1)^k \binom{2(n-m)+1}{k} v^k \frac{\partial^{2(n-m)+1} J(0)}{\partial t^{2(n-m)+1-k} \partial x^k}$$

$$\frac{\partial E(t)}{\partial x} = \eta \sum_{n=0}^{\infty} \frac{(ct)^{2(n+1)}}{(2(n+1))!} \sum_{m=0}^{n} \frac{(-1)^{m+1}}{c^{2(n-m)+1}} \nabla^{2m} \times \sum_{k=0}^{2(n-m)+1} (-1)^k \binom{2(n-m)+1}{k} v^k \frac{\partial^{2(n-m)+1} \partial J(0)/\partial x}{\partial t^{2(n-m)+1-k} \partial x^k}$$

$$+\eta \sum_{n=0}^{\infty} \frac{(ct)^{2n+1}}{(2n+1)!} \sum_{m=0}^{n} \frac{(-1)^{m+1}}{c^{2(n-m)}} \nabla^{2m} \times \sum_{k=0}^{2(n-m)} (-1)^k \binom{2(n-m)}{k} v^k \frac{\partial^{2(n-m)} \partial J(0)/\partial x}{\partial t^{2(n-m)-k} \partial x^k}$$

The speeds become

$$\frac{\frac{\partial E(t)}{\partial t}}{\frac{\partial E(t)}{\partial x}} = c S_{v0}(t, v) + S_{v1}(t, v)$$

$$S_{v0}(t, v) = \frac{D_{11}(t, v) + D_{12}(t, v)}{M_1(t, v) + M_2(t, v)}$$

$$S_{v1}(t, v) = \frac{D_{21}(t, v) + D_{22}(t, v)}{M_1(t, v) + M_2(t, v)}$$

$$D_{11}(t, v) = \sum_{n=0}^{\infty} \frac{(ct)^{2n+1}}{(2n+1)!} \sum_{m=0}^{n} \frac{(-1)^{m+1}}{c^{2(n-m)+1}} \nabla^{2m} \times \sum_{k=0}^{2(n-m)+1} (-1)^k \binom{2(n-m)+1}{k} v^k \frac{\partial^{2(n-m)+1} J(0)}{\partial t^{2(n-m)+1-k} \partial x^k}$$

$$D_{12}(t, v) = \sum_{n=0}^{\infty} \frac{(ct)^{2n}}{(2n)!} \sum_{m=0}^{n} \frac{(-1)^{m+1}}{c^{2(n-m)}} \nabla^{2m} \times \sum_{k=0}^{2(n-m)} (-1)^k \binom{2(n-m)}{k} v^k \frac{\partial^{2(n-m)} J(0)}{\partial t^{2(n-m)-k} \partial x^k}$$

$$D_{21}(t, v) = \sum_{n=0}^{\infty} \frac{(ct)^{2(n+1)}}{(2(n+1))!} \sum_{m=0}^{n} \frac{(-1)^{m+1}}{c^{2(n-m)+1}} \nabla^{2m} \times \sum_{k=0}^{2(n-m)+2} (-1)^k \binom{2(n-m)+2}{k} v^k \frac{\partial^{2(n-m)+2} J(0)}{\partial t^{2(n-m)+2-k} \partial x^k}$$

$$D_{22}(t, v) = \sum_{n=0}^{\infty} \frac{(ct)^{2n+1}}{(2n+1)!} \sum_{m=0}^{n} \frac{(-1)^{m+1}}{c^{2(n-m)}} \nabla^{2m} \times \sum_{k=0}^{2(n-m)+1} (-1)^k \binom{2(n-m)+1}{k} v^k \frac{\partial^{2(n-m)+1} J(0)}{\partial t^{2(n-m)+1-k} \partial x^k}$$

$$M_1(t,v) = \sum_{n=0}^{\infty} \frac{(ct)^{2(n+1)}}{(2(n+1))!} \sum_{m=0}^{n} \frac{(-1)^{m+1}}{c^{2(n-m)+1}} \nabla^{2m} \times \sum_{k=0}^{2(n-m)+1} (-1)^k \binom{2(n-m)+1}{k} v^k \frac{\partial^{2(n-m)+1} \partial J(0)/\partial x}{\partial t^{2(n-m)+1-k} \partial x^k}$$

$$M_2(t,v) = \sum_{n=0}^{\infty} \frac{(ct)^{2n+1}}{(2n+1)!} \sum_{m=0}^{n} \frac{(-1)^{m+1}}{c^{2(n-m)}} \nabla^{2m} \times \sum_{k=0}^{2(n-m)} (-1)^k \binom{2(n-m)}{k} v^k \frac{\partial^{2(n-m)} \partial J(0)/\partial x}{\partial t^{2(n-m)-k} \partial x^k}$$

We can see that the light speed is affected by the speed of moving source.

Michelson-Morley Experiment

Now the solution to Maxwell's equations is available, the result of Michelson-Morley experiment can be explained. The solution to Maxwell's equations also indicates that the explanations given by Lorentz coordinate transformation or special theory of relativity are incorrect.

Seemingly inertial behaviour of light

We already know that it is incorrect to assume that light speed is a constant. There is another incorrect assumption regarding Galilean transformation. To explain it, let's just pretend that the assumption of constant light speed is true, because people use it with the Galilean transformation.

Michelson-Morley experiment assumed that for a source moving at speed *v*, the light speed is a constant *c* relative to the stationary reference frame, and therefore, at the moving source, the light speed is *c-v*. This speed difference was not detected, that is, this *-v* effect was not detected. It was as if the light has inertial behaviour.

To cancel this *-v* mathematically, Lorentz (or someone else) made a coordinate transformation between the stationary reference frame and the moving reference frame, which changes the time scale and space scale. This is called Lorentz coordindate transformation. The original coordinate transformation which everyone is used to is called Galilean transformation.

Einstein justified the Lorentz coordinate transformation by claiming that

1. the light uses an observer's coordinate system, as if the light is inertial;
2. use Lorentz coordinate transformation plus Einstein field transformation to build relationship between the moving refrence frame and the stationary reference frame, such that in each reference frame, the light follows Maxwell's equations

Here, Lorentz and Einstein took it for granted that if the Lorentz coordinate transformation was not used then mathematically the Galilean transformation was automatically assumed, that is, the light speeds beween the moving and stationary reference frames have a difference of $\pm v$. Let me call it "Galilean Assumption".

The "Galilean Assumption" can be true for the problems handled by Newton's laws. But it is not true for Maxwell's equations.

Sufficient condition for the "Galilean Assumption". For any field source

$$J_1(x,y,z,t)$$

genrating fields

$$E_1(x,y,z,t) \text{ and } H_1(x,y,z,t)$$

If the source

$$J_2(x,y,z,t) = J_1(x \pm vt, y, z, t)$$

generates fields

$$E_1(x \pm vt, y, z, t) \text{ and } H_1(x \pm vt, y, z, t)$$

Then then "Galilean Assumption" for Maxwell's equations is true.

Proof. Using a concise notation of

$$\frac{a}{b} = \frac{\begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix}}{\begin{bmatrix} b_x \\ b_y \\ b_z \end{bmatrix}} = \begin{bmatrix} a_x/b_x \\ a_y/b_y \\ a_z/b_{bz} \end{bmatrix}$$

Suppose the condition is true, then the speed along x-axis is

$$\frac{\frac{dE_1(x+vt,y,z,t)}{dt}}{\frac{\partial E_1(x+vt,y,z,t)}{\partial x}} = \frac{\frac{\partial E_1(x+vt,y,z,t)}{\partial x}v + \frac{\partial E_1(x+vt,y,z,t)}{\partial t}}{\frac{\partial E_1(x+vt,y,z,t)}{\partial x}} = v\begin{bmatrix}1\\1\\1\end{bmatrix} + \frac{\frac{\partial E_1(x+vt,y,z,t)}{\partial t}}{\frac{\partial E_1(x+vt,y,z,t)}{\partial x}}$$

The difference of the speeds between the two sources is v .

QED.

Later, I am going to use the above proof to explain the result of Michelson-Morley experiment. For now, notice that the above condition does not apply to Maxwell's equations. Look at the speed formula given in the "Moving source" section previously, we can see that the speed difference between the two reference frames is far from a constant v. That is, "Galilean Assumption" is not true.

Now the solution to Maxwell's equations tells us that Lorentz and Einstein made two wrong assumptions:

1. Light speed is a constant
2. Maxwell's equations follow "Galilean Assumption", that is, without Lorentz coordinate transformation, the difference of light speeds of the moving reference frame and the stationary reference frame is a constant

Thus, it is incorrect to explain the result of Michelson-Morley experiment by Lorentz coordinate transformation and Einstein special theory of relativity.

Pseudo inertial of light

Light looks like inertial from the result of Michelson-Morley experiment. The special theory of relativity effectively takes light as inertial. Previously we have proved that the special theory of relativity is incorrect. Then, how do we explain the result of Michelson-Morley experiment? Actually the solution to Maxwell's equations tells us that light behaves in a way of pseudo-inertial: it looks inertial, but not really inertial.

Recall that the generic solution to Maxwell's equations for a source

$$J(x-vt,y,z,t)$$

is

$$E(t) = \eta\sum_{n=0}^{\infty}\frac{(ct)^{2(n+1)}}{(2(n+1))!}\sum_{m=0}^{n}\frac{(-1)^{m+1}}{c^{2(n-m)+1}}\nabla^{2m}\times\sum_{k=0}^{2(n-m)+1}(-1)^k\binom{2(n-m)+1}{k}v^k\frac{\partial^{2(n-m)+1}J(0)}{\partial t^{2(n-m)+1-k}\partial x^k} + \eta\sum_{n=0}^{\infty}\frac{(ct)^{2n+1}}{(2n+1)!}\sum_{m=0}^{n}\frac{(-1)^{m+1}}{c^{2(n-m)}}\nabla^{2m}\times\sum_{k=0}^{2(n-m)}(-1)^k\binom{2(n-m)}{k}v^k\frac{\partial^{2(n-m)}J(0)}{\partial t^{2(n-m)-k}\partial x^k}$$

Where

$$J(0) = J(x-vt,y,z,t=0)$$

Consider the case when $t \to 0$

$$E(t)_{t\to 0} \approx E(t)_{n=0} =$$

$$\eta\frac{(ct)^2}{(2)!}\sum_{m=0}^{0}\frac{(-1)^1}{c^1}\sum_{k=0}^{1}(-1)^k\binom{1}{k}v^k\frac{\partial^1 J(0)}{\partial t^{1-k}\partial x^k} + \eta\frac{(ct)^1}{(1)!}\sum_{m=0}^{0}\frac{(-1)^1}{c^0}\sum_{k=0}^{0}(-1)^k\binom{0}{k}v^k\frac{\partial^0 J(0)}{\partial t^{0-k}\partial x^k} =$$

$$\eta\frac{(ct)^2}{(2)!}\frac{(-1)^1}{c^1}\sum_{k=0}^{1}(-1)^k\binom{1}{k}v^k\frac{\partial^1 J(0)}{\partial t^{1-k}\partial x^k} + \eta\frac{(ct)^1}{(1)!}\frac{(-1)^1}{c^0}\sum_{k=0}^{0}(-1)^0\binom{0}{0}v^0\frac{\partial^0 J(0)}{\partial t^{0-0}\partial x^0} =$$

$$\eta\frac{-(ct)^2}{2}\frac{1}{c}\left((-1)^0\binom{1}{0}v^0\frac{\partial^1 J(0)}{\partial t^{1-0}\partial x^0} + (-1)^1\binom{1}{1}v^1\frac{\partial^1 J(0)}{\partial t^{1-1}\partial x^1}\right) - \eta ctJ(0) =$$

$$\eta\frac{-(ct)^2}{2}\frac{1}{c}\left(\frac{\partial J(0)}{\partial t} - v\frac{\partial J(0)}{\partial x}\right) - \eta ctJ(0) \approx -\eta ctJ(x-vt,y,z,0)$$

$$H(t)_{t\to 0} \approx H(t)_{n=0} =$$

$$\sum_{n=0}^{0}\frac{(ct)^{2(n+1)}}{(2(n+1))!}\sum_{m=0}^{n}\frac{(-1)^m}{c^{2(n-m)}}\nabla^{2m+1}\times\sum_{k=0}^{2(n-m)}(-1)^k\binom{2(n-m)}{k}v^k\frac{\partial^{2(n-m)}J(0)}{\partial t^{2(n-m)-k}\partial x^k} =$$

$$\frac{(ct)^2}{(2)!}\sum_{m=0}^{0}\frac{(-1)^m}{c^{2(0-m)}}\nabla^{2m+1}\times\sum_{k=0}^{2(0-m)}(-1)^k\binom{2(0-m)}{k}v^k\frac{\partial^{2(0-m)}J(0)}{\partial t^{2(0-m)-k}\partial x^k} =$$

$$\frac{(ct)^2}{2}\frac{(-1)^0}{c^{2(0-0)}}\nabla^{0+1}\times\sum_{k=0}^{2(0-0)}(-1)^k\binom{2(0-0)}{k}v^k\frac{\partial^{2(0-0)}J(0)}{\partial t^{2(0-0)-k}\,\partial x^k}=$$

$$\frac{(ct)^2}{2}\frac{1}{1}\nabla\times\sum_{k=0}^{0}(-1)^0\binom{0}{0}v^0\frac{\partial^0 J(0)}{\partial t^{2(0-0)-0}\,\partial x^0}=$$

$$\frac{(ct)^2}{2}\frac{1}{1}\nabla\times(-1)^0\binom{0}{0}v^0\frac{\partial^0 J(0)}{\partial t^{2(0-0)-0}\,\partial x^0}=$$

$$\frac{(ct)^2}{2}\nabla\times J(0)=\frac{(ct)^2}{2}\nabla\times J(x-vt,y,z,0)$$

We have

$E(t)_{t\to 0}\approx E(t)_{n=0}\approx -\eta ctJ(x-vt,y,z,0)$
$H(t)_{t\to 0}\approx H(t)_{n=0}=\frac{(ct)^2}{2}\nabla\times J(x-vt,y,z,0)$

Note that the above fields satisfy the **Sufficient condition for the “Galilean Assumption”**. Thus, light’s behaviour is inertial.

Thus, the solution to Maxwell’s equations tells us that near a moving source the light is inertial; this behaviour explains the result of Michelson-Morley experiment. It also explains that Maxwell’s equations were deduced from physical experiements on earth, but its coordinate system can be in the absolute reference frame. In a short time and space, electromagnetic fields behave the same in the absolute reference frame and the earth’s moving reference frame, due to this pseudo-inertial behaviour.

The solution to Maxwell’s equations also tells us that if the light travel length of Michelson-Morley experiment is long enough then the expected result can be achieved. How long is enough? It can be calculated from the solution.