Compile and Run TSS FDTD C++ Code for Linux and Windows

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Contents

Introduction

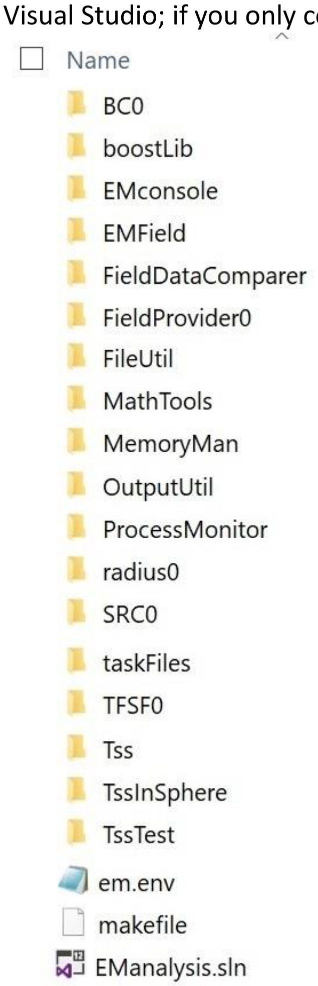
The TSS FDTD C++ code is provided as open source at GitHub (https://github.com/DavidGeUSA/TSS/tree/master/Source%20Code%20V2) , it can be compiled to run under Linux and Windows. This document provides compiling and running instructions. Unit tests and a few simulation results are provided for you to compare with your results, thus to confirm that the code may be compiled and run under your environments.

If you are not ready to compile and run the code and just want to know the benefits of the TSS FDTD algorithm then you may jump to section "Compare TSS algorithm with YEE algorithm".

Code Compilation

Code File Structure

The code was originally developed using Microsoft Visual Studio, and thus it uses the Visual Studio default code file structure. The code is divided into many modules. Each module takes one folder. Inside each folder, there are *.cpp and *.h source code files. In each folder there are also files generated by Visual Studio; if you only compile for Linux then you do not need those files.



All C++ source code files are in the listed folders. Above the folders there are 3 text files.

- makefile – this is the make file for compiling for Linux. If you only compile for Windows then you do not need it.
- EManalysis.sln – this is the solution file for Visual Studio to compile the code for Windows. If you only compile for Linux then you do not need it.
- em.env – this file is not for code compiling. It is for running the compiled program. Its usage will be explained later.

All files you download are text files. There are no binary files, and thus there is no any security concerns in downloading the TSS FDTD source files.

Compile for Windows

Before compiling, boost library (version 1.74.0) should be installed. See https://www.boost.org/users/history/version_1_74_0.html

Visual Studio Ultimate 2013 (Version 12.0.31101.00 Update 4) was used to open EManalysis.sln and built the solution. The target platform should be X64. EM.exe should be created by Visual Studio without errors and warnings.

Compile for Linux

Before you type the commands shown below, your <current folder> must be at where the makefile is, see the Code File Structure section above.

Use the following steps to prepare for a compiling the very first time:

1. Use following command to install the boost library:
   $ sudo apt-get install libboost-all-dev

2. Create a folder named lib. All compiled object files and library files will be saved in this folder
   $ mkdir lib
3. Set file permissions to the way you want, for example,
   $ sudo chmod 744 –R *

Once the above steps are through, you can compile the code:

$ make

There should be no errors and warnings during the compilation. A file named EM should be created at the current folder. You may try to run it:

$ ./EM

Execute the program

There are two ways of providing runtime parameters: 1) by command line; 2) by a text file named em.env sitting beside the executable (EM.exe for Windows, EM for Linux). These two methods are explained below.

Launch by command line

The following command line parameters are supported.

- /K<number>
  The <number> specifies the task to be carried out. For example, /K300 indicates a simulation task.
- /T<string>

The <string> specifies a text file path. The text file contains parameter values needed for the task or tasks. For example, /T./taskFiles/test.task

- /W<string>
  The <string> specifies a folder for generating temporary files. For example, /W./tmp
- /D<string>
  The <string> specifies a folder for generating simulation data files. For example, /D./data
- /E<string>
  The <string> specifies a folder if the task specified by /K needs it.
- /?
  Show help

Linux example:
$ ./EM /K300 /T./taskFiles/sim1.task /W./tmp /D./data
Windows example:
>EM /k300 /TC:\EMtest\TaskFiles\sim1.task /WC:\dat\tmp /DC:\dat\sim1

Launch by file em.env

Type a long command line repeatedly may be tedious. The EM program looks for a text file named em.env besides it and reads parameters from it. If em.env file is in the same folder of the executable (EM.exe for Windows, EM for Linux) then simply launch the executable as shown below.
Linux:
$ ./EM
Windows:
>EM
If you start a debug run from within the Visual Studio then the file em.env should be in the folder of EMconsole.
File em.env is a line based text file. You specify a parameter value in following format:
`<parameter name>=<parameter value>`
The supported <parameter name> are /K, /T, /W, /D and /E; the meaning of these names is the same as for the command line parameters explained previously.
For example

```
/K=300
/T=./taskFiles/task_gaussian.task
/W=./tmp
/D=./_data
```

If a line starts with "//" then the line is ignored.
If a line starts with "/*" then all contents following it are ignored until a line starts with "*/".
Using "/*" and "*/" you may put parameters of many executions into em.env and mark only one set of execution parameters to be enabled.
The following em.env file shows such usage:

```
/*
//  unit test
/K=301
/T=./taskFiles/UnitTest.task
*/

/*
/K=301
```

```
/T=./taskFiles/ricker21_nx120_NB.task
*/

/K=300
/T=./taskFiles/ricker21_nx120_PEMC.task


/*
/K=300,370,371
/T=.taskFiles/LTest1.task
*/

/*
/K=315,316,317
/T=./taskFiles/Heilder.task
*/

/*
/K=300
/T=./taskFiles/task_gaussian.task
*/

/*
/K=300
/T=./taskFiles/task_gaussianPML.task
*/

/W=./tmp
/D=./_data
```

In the above file, most lines are enclosed by "/*" and "*/", and thus are ignored. Some lines are not enclosed by "/*" and "*/" and will be used by the program, for example, the line /T=./taskFiles/ricker21_nx120_PEMC.task is enabled.

Because file paths are specified differently in Linux and Windows, it is likely the values of /T, /W and /D are specified differently for Linux and for Windows, and thus em.env is most likely different for Linux and Windows. The em.env file you downloaded is for Linux. If you play it under Windows then you need to modify it and copy it to X64\Release folder and/or the EMconsole folder.

Task File

As explained in the previous section, "/T" is used to specify a path to a task file. A task file is used to specify parameter values needed by a task to be carried out. A task file is in the same content format as em.env, but its supported parameter names are defined by various modules. In the taskFiles folder there are some sample task files which will be explained later in this document.

Unlike file em.env, contents of task files are the same for Linux and for Windows.

Sample contents of a task file:

```
TSS.DS=0.015
TSS.XMIN=-1.8
```

Before listing commonly supported parameter names, we need to list the FDTD simulation formula first.

The TSS FDTD software simulates the Maxwell's equations expressed as

$$\frac{\partial E}{\partial t} = \frac{1}{\varepsilon}\nabla \times H - \frac{\sigma}{\varepsilon}E - \frac{1}{\varepsilon}J_e(t)$$	(1)
$$\frac{\partial H}{\partial t} = -\frac{1}{\mu}\nabla \times E - \frac{\sigma_m}{\mu}H - \frac{1}{\mu}J_m(t)$$	(2)
$$\nabla \cdot E = \rho/\varepsilon$$	(3)
$$\nabla \cdot H = 0$$	(4)

Computing domain:

$$x = x_{min} + i\,\Delta_s; i = 0,1,2,\ldots,n_x$$ $$y = y_{min} + j\,\Delta_s; j = 0,1,2,\ldots,n_y$$ $$z = z_{min} + k\,\Delta_s; k = 0,1,2,\ldots,n_z$$	(5)

Time and space digitization:

$$\Delta_t = \frac{\Delta_s}{c_0}\frac{f_c}{\sqrt{3}}; c_0 = light\ speed\ in\ vacuum; 0 < f_c \le 1$$	(6)

Time advancement estimations:

Given initial fields $$E(q_0\Delta_t)$$ $$H(q_0\Delta_t)$$	(7)
Estimate fields $$E(q\Delta_t + q_0\Delta_t)$$ $$H(q\Delta_t + q_0\Delta_t)$$ $$q = 1,2,\ldots,q_{max}$$	(8)

FDTD algorithms are for calculating values of (8). For TSS algorithm, see https://github.com/DavidGeUSA/TSS/blob/master/GenericFdtdFullFormat.pdf and https://github.com/DavidGeUSA/TSS/blob/master/GenericFDTD.pdf .

Commonly used parameter names are listed below:

Parameter Name	C++ Type	Description
TSS.RHO	Double	ρ in Eq. (3)
TSS.EPS	Double	ε in Eq. (1)
TSS.MU	Double	μ in Eq. (2)
TSS.SIGMA_E	Double	σ in Eq. (1)
TSS.SIGMA_M	Double	σ_m in Eq. (2)
TSS.IS_VACUUM	Boolean	If its value is true then TSS.EPS and TSS.MU are ignored. ε_0 *and* μ_0 for vacuum are used.
SIM.MAXTIMES	unsigned int	The maximum time steps the simulation runs. Its value should be $q_0 + q_{max}$, see Eq. (7) and (8).
SIM.STARTTIME	unsigned int	Its value should be 0 or the time step the last simulation stopped at, see q_0 in Eq. (7) and (8). The next simulation will pick up from the last simulation and continue. For example, suppose the first simulation uses SIM.MAXTIMES=1000. The first simulation finishes on running 1000 time steps. Now suppose you want to run another 500 time steps. You can change the task file and set SIM.MAXTIMES=1500 and SIM.STARTTIME=1000. The next simulation will load the field data files generated by the last simulation and start

		simulation from time step 1001 and finish at time step 1500.
TSS.NX	unsigned int	x-axis domain: $i = 0,1,2, \dots, n_x$
TSS.NY	unsigned int	y-axis domain: $j = 0,1,2, \dots, n_y$
TSS.NZ	unsigned int	z-axis domain: $k = 0,1,2, \dots, n_z$
TSS.XMIN	Double	x-axis value corresponding to $i = 0, x = x_{min}$
TSS.YMIN	Double	y-axis value corresponding to $j = 0, y = y_{min}$
TSS.ZMIN	Double	z-axis value corresponding to $k = 0, z = z_{min}$
TSS.DS	Double	Space step size Δ_s
TSS.DT	Double	Time step size Δ_t
TSS.COURANT_FACTOR	Double	It specifies the value of f_c , see Eq. (6). If the value is 1 then a Courant number of $\frac{1}{\sqrt{3}}$ is used. If the value is less than 1 then a smaller Courant number is used. If this value is set then TSS.DT is ignored.
TSS.KMAX	unsigned int	Value of k_{max} . Time advancement estimation order=$2(k_{max} + 1)$
TSS.SMAX	unsigned int	Value of s_{max} . Space derivative estimation order=$2s_{max}$
TSS.MAKESTATISTICS	Boolean	If its value is true then a statistics file is generated on finishing a simulation. The statistics file contains field energy summaries and field divergences over time.
TSS.TIME.CLASS	char *	Its value is a C++ class name. The class is responsible for performing time advancement estimation. Currently supported values including • TimeTss – it runs TSS algorithm in a single thread • TimeTssMultiThreads – it runs TSS algorithm in multi-threads. Use SIM.USETHREADS to specify how many threads should be used • TimeYee – it runs Yee algorithm
TSS.BOUNDARY.CLASS	char*	Its value is a C++ class name. The class is responsible for applying boundary conditions. Currently supported values including • BoundaryTss – it applies PEMC conditions • BoundaryTssVoid – it applies no conditions
TSS.FIELDSETTER.CLASS	char*	Its value is a C++ class name. The class is responsible for setting initial fields values. Currently supported values including • FieldsNull – it sets fields to 0 • GaussianCurl – it sets fields by Gaussian function $$b\begin{bmatrix} \frac{yz}{2}e^{-k(x^2+y^2+z^2)} \\ -zxe^{-k(x^2+y^2+z^2)} \\ \frac{xy}{2}e^{-k(x^2+y^2+z^2)} \end{bmatrix}$$ b is specified using GAUSSIAN.MAGNITUDE

<table>
<tr><td></td><td></td><td colspan="2">k is specified using GAUSSIAN.EXPO
CurlXYZ – it sets fields by following function
$$a\begin{bmatrix}\frac{1}{2}yzx^2\\ -zxy^2\\ \frac{1}{2}xyz^2\end{bmatrix}$$
a is specified using CURLXYZ.MAGNITUDE
GaussianCurlYee – it sets fields the same way as GaussianCurl does, but it follows the space cell arrangement of the Yee algorithm</td></tr>
<tr><td>TSS.SOURCE.CLASS</td><td>char *</td><td colspan="2">Its value is a C++ class name. The class is responsible for applying a field source. Currently supported values including
FieldSourceTss – it applies no field source
RickerSource – it applies a Ricker field source
The pick frequency is specified by FS.PPW
HeidlerCurrent – it applies the Heidler Current</td></tr>
<tr><td>PML.DISABLE</td><td>Boolean</td><td colspan="2">If its value is true then no PML is applied</td></tr>
<tr><td>PML.THICK</td><td>unsigned int</td><td colspan="2">Its value indicates the thickness of the PML, in the number of space steps Δ_s. PML.THICK=4 means the PML thickness is $4\Delta_s$.</td></tr>
<tr><td>PML.ALPHA</td><td>Double</td><td>α_{max}</td><td rowspan="3">The PML is introduced by
$$\alpha+\frac{\beta}{j\omega}$$
$$\alpha=1+\alpha_{max}\left(\frac{s}{L}\right)^p$$
$$\beta=\beta_{max}\left(\frac{s}{L}\right)^p$$
s is the depth into the PML, L is the thickness of the PML</td></tr>
<tr><td>PML.BETA</td><td>Double</td><td>β_{max}</td></tr>
<tr><td>PML.POWER</td><td>unsigned int</td><td>p</td></tr>
<tr><td>PML.X.LOW</td><td>Boolean</td><td colspan="2">If its value is true then the PML is applied to the low end of the x-axis</td></tr>
<tr><td>PML.X.HIGH</td><td>Boolean</td><td colspan="2">If its value is true then the PML is applied to the high end of the x-axis</td></tr>
<tr><td>PML.Y.LOW</td><td>Boolean</td><td colspan="2">If its value is true then the PML is applied to the low end of the y-axis</td></tr>
<tr><td>PML.Y.HIGH</td><td>Boolean</td><td colspan="2">If its value is true then the PML is applied to the high end of the y-axis</td></tr>
<tr><td>PML.Z.LOW</td><td>Boolean</td><td colspan="2">If its value is true then the PML is applied to the low end of the z-axis</td></tr>
<tr><td>PML.Z.HIGH</td><td>Boolean</td><td colspan="2">If its value is true then the PML is applied to the high end of the z-axis</td></tr>
<tr><td>TSS.DATAFOLDER</td><td>char *</td><td colspan="2">Its value is a sub-folder name. The sub-folder is under the data folder specified by /D. The data files will be generated in the sub-folder by the simulation</td></tr>
<tr><td>TSS.REC_BASEFILE</td><td>char *</td><td colspan="2">Its value is a base file name for forming data file names.</td></tr>
</table>

TSS.TIMEMATRIXFILE	char *	Its value is a file name. Coefficients for time advancement estimation will be saved in this file.
TSS.SPACEMATRIXFILE	char *	Its value is a file name. Coefficients for space derivative estimation will be saved in this file.
SIM.TESTCASE	Integer	Its value indicates the unit tests to be carried out. Use value -1999 to carry out basic unit tests, including field initial values, space derivatives of the first, second and third order, space curls of the first, second and third order. A text file will be generated in the sub-folder, showing the test results.
TEST.EXCLUDE.BOUNDARY	Boolean	If its value is true then the estimation errors at the boundary will be excluded when doing unit testing.
TSS.MATRIXFILEFOLDER	char *	Its value is a sub-folder name. The sub-folder is under the data folder specified by /D. The folder is used for reading from and writing to a space-matrix-file. The space-matrix-file holds the coefficients for estimating space derivatives. These coefficients are only related to the estimation order, and they are independent of the simulation tasks. For the second order and the 4-th order estimations, calculating the estimation coefficients takes no time. For higher order estimation, calculating the estimation coefficients takes a long time; you may calculate it once and save it to the space-matrix-file. During the initialization of a simulation, the coefficients are loaded from the file, not re-calculated.
SIM.USE_RAM	Boolean	If its value is true then the fields memories are allocated using file mapping on RAM. If its value is false then the fields memories are allocated using file mapping on temporary files created in the folder specified via /W.
TSS.REC_INTERVAL	unsigned int	It is an integer value. If it is 0 or 1 then the fields are saved to disk on every simulation time step. If it is larger than 1 then the value indicates on how many time steps the simulation saves the fields to disk. SIM.MAXTIMES should be dividable by TSS.REC_INTERVAL so that field data files can be generated at the last time step.
SIM.OUTPUT.CSV	Array	It specifies an array to tell the simulator how to generate output files. Each array item specifies one output file and is enclosed in “(“ and “)” in a format of “I,j,k,Ex|Ey|Ez|Hx|Hy|Hz”, i.e. “(20,11,23,Ez)(42:13:21,Ex|Hy)”, this example tells the simulator to generate 2 CSV files, each line of the CSV file contains comma delimited values, the first value is time in seconds, followed by the specified field values, the sequence of the field values is the E field first, the H field next, in the order of x, y and z. If this parameter is not specified then at each time of saving fields one binary file is generated for E-field and another binary

		file is generated for H-field, each binary file contains 3D field values at every space point.

Tasks

As shown previously, /K is used to specify a task number. A task number indicates what task to be carried out. In file em.env, multiple task numbers can be used via a comma-delimited task number list, and the tasks will be executed one by one.

Task number	Description
300	Run a simulation
301	Run unit test
302	Verify TSS FDTD fields-updating coefficients
330	Generate coefficients without starting simulation. By running this task, you may examine the Courant number factor to see if you are happy with it.
370	From a set of simulation data files (*e_*.dat files or *h_*.dat files), generate one or more text files, each file for one space location, the file contains an array of 3 values of (Ex,Ey,Ez) or (Hx,Hy,Hz) at one given space location, the first set of the 3 field values is for time=0, the next sets for t=dt, 2dt, 3dt, ... Use task parameters POINT.GRIDNODES to specify space locations, see explanations below.
371	from a set of simulation data files (*e_*.dat files or *h_*.dat files), generate one bin file, the file contains a 2D array of values, each row for one space location, each column for one time, each value is a field value of Ex, Ey, Ez, Hx, Hy, or Hz. Use the same task file doing the simulation for this task. Use task parameters POINT.GRIDNODES=(i,j,k),(i,j,k)... to specify space locations i, j, and k; use POINT.EH=e|h to specify E-field or H-field; use FIELD.COMPONENT to specify field component, 1 for x, 2 for y and 3 for z. Task 370 and 371 only work if SIM.OUTPUT.CSV is not used when doing the simulation.

Unit Test

In the folder taskFiles, there is a file named UnitTest.task. It is used for unit test. Use following lines in em.env file to do unit test under Linux:

```
/K=301
/T=./taskFiles/UnitTest.task
/W=./tmp
/D=./_data
```

Under Windows, you need to modify it to point to the correct file path for UnitTest.task.
The contents of UnitTest.task are listed below

```
//material properties
TSS.IS_VACUUM=True

//maximum time steps
SIM.MAXTIMES=120

//time steps before saving fields to files
TSS.REC_INTERVAL=0

//generate a statistics file on finishing the simulation
```

```
TSS.MAKESTATISTICS=true

//plugins ------------------------
TSS.TIME.CLASS=TimeTssMultiThreads
SIM.USETHREADS=4
TSS.SOURCE.CLASS=FieldSourceTss
TSS.BOUNDARY.CLASS=BoundaryTss
TSS.FIELDSETTER.CLASS=GaussianCurl
GAUSSIAN.MAGNITUDE=120
GAUSSIAN.EXPO=0.5
//---------------------------------

//for task 301 - test space module
//use -1999 to test several cases
//SIM.TESTCASE=-120
//SIM.TESTCASE=-201
SIM.TESTCASE=-1999
//SIM.TESTCASE=3

TEST.EXCLUDE.BOUNDARY=true

//subfolder under /D for read/write space matrix file
TSS.MATRIXFILEFOLDER=Matrix

TSS.TIMEMATRIXFILE=timeMatrix.txt
TSS.SPACEMATRIXFILE=spacematrix.txt

//sub folder under folder specified by command line parameter /D
TSS.DATAFOLDER=unitTest

//time estimation order = 2*(kmax+1)
TSS.KMAX=1
//space estimation order = 2*smax
TSS.SMAX=2

//computation domain -------------
TSS.NX=42
TSS.NY=42
TSS.NZ=42
TSS.XMIN=-1
TSS.YMIN=-1
TSS.ZMIN=-1

//space step size, use -XMIN/(NX/2)
TSS.DS=0.047619047619047619
//-------------------------------


//forming E field file name: <BASEFILE>_TSS_smax<smax>_kmax<kmax>_e_<file index>.dat
//forming H field file name: <BASEFILE>_TSS_smax<smax>_kmax<kmax>_h_<file index>.dat
//<file index>=0,1,2,...
TSS.REC_BASEFILE=unitTest
```

The parameter SIM.TESTCASE indicates the test cases to be executed. Use -1999 to execute most important test cases, including various space derivative estimations.

Once you successfully compile the program, you should run the unit test and see if it runs without a problem.

1. Make sure in em.env /K=301 is used, /T points to UnitTest.task, other parts specifying /K and /T are disabled.
2. Create the two folders specified in the em.env file, if they do not exist already
   $ mkdir tmp
   $ mkdir _data

3. Run the program by
   $ ./EM

```
linux1@DESKTOP-QD88AGK:~/tss$ ./EM

.
  Electromagnetic Fields Analysis. Version 2.0.0.
 Created: 2017-02-08 for Microsoft Windows; updated 2021-03-11 to support Linux
 Author: David Ge (dge893@gmail.com)
.

Use em.env file or use command line parameters to execute tasks.



Current folder:/home/linux1/tss
The environment file to be used: /home/linux1/tss/em.env

Task file: ./taskFiles/UnitTest.task

   executing task 301 ...
Test case : -1999
Initializing simulator ...
Initialing, please wait ...
Initialization finished. Time used:14
Initializing simulator...done. time used:14

Task finished

Press a key to finish


Exiting, freeing memories...
linux1@DESKTOP-QD88AGK:~/tss$
```

In folder _data/unitTest 3 files are generated:

```
linux1@DESKTOP-QD88AGK:~/tss$ cd _data
linux1@DESKTOP-QD88AGK:~/tss/_data$ cd unitTest/
linux1@DESKTOP-QD88AGK:~/tss/_data/unitTest$ dir
GaussianCurl_smax2kmax1_1615844621137.txt  spacematrix.txt  timeMatrix.txt
linux1@DESKTOP-QD88AGK:~/tss/_data/unitTest$
```

File spacematrix.txt shows coefficients for space derivative estimations.
File timeMatrix.txt shows coefficients for time advancement estimations.
The file named as GaussianCurl_*.txt shows unit test results. Its contents are listed below. You should get the same results to prove that no problem is detected in your code compilation.

```
smax=2 kmax=1, dt=0.047619, exclude boundary:1, initial fields:GaussianCurl
Test case 0. Field initialization
max error:0, average error:0, average mag:22.0901

Test case -1. estimations of dEx/dx, dEy/dy, dEz/dz
max error:4.18408e-06, average error:2.7313e-06, average mag:9.2824

Test case -2. estimations of dEx/dy, dEy/dz, dEz/dx
max error:5.91449e-05, average error:2.91846e-06, average mag:35.3782

Test case -3. estimations of dEx/dz, dEy/dx, dEz/dy
```

```
max error:5.91449e-05, average error:2.91846e-06, average mag:35.3782

Test case -4. estimations of d2Ex/dz, d2Ey/dx, d2Ez/dy
max error:9.85978e-05, average error:6.01425e-06, average mag:55.2507

Test case -5. estimations of d2Ex/dy, d2Ey/dz, d2Ez/dx
max error:9.85978e-05, average error:6.01425e-06, average mag:55.2507

Test case -6. estimations of d2Ex/dx, d2Ey/dy, d2Ez/dz
max error:0.00108093, average error:5.1192e-06, average mag:17.3642

Test case -100. estimations of divergences
max error:3.60393e-07, average error:4.07054e-08, average mag:9.2824

Test case 1. Curl estimation
max error:5.1221e-05, average error:2.46501e-06, average mag:59.5505

Test case 2. Curl(Curl) estimation
max error:6.9111e-05, average error:1.14781e-05, average mag:127.137

Test case 3. Curl(Curl(Curl)) estimation
max error:0.202665, average error:0.000452972, average mag:326.234

Test case 101. Curl estimation (by higher order
max error:5.1221e-05, average error:2.46501e-06, average mag:59.5505

Test case 201. Curl estimation (by fast method)
max error:5.1221e-05, average error:2.46501e-06, average mag:59.5505

Test case 301. comapre the first curl calculated directly by optimized code and not-optimized code
max error:5.57499e-14, average error:3.02479e-16, average mag:2.83574

Test case 401. comapre the first curl calculated by the simulator and by optimized code
max error:0, average error:0, average mag:2.83574

Test case 102. Curl(Curl) estimation (by higher order)
max error:6.9111e-05, average error:1.14781e-05, average mag:127.137

Test case 103. Curl(Curl(Curl)) estimation (by higher order)
max error:0.202665, average error:0.000452972, average mag:326.234
```

For each test case, it shows the maximum error, the average error and the average magnitude. The average magnitude shows the field value magnitude to be compared with the errors. If the magnitude is 0 then 0 error does not mean a good result and there must be some coding problem.
If you get the same results as shown above then you can proceed to test run following simulations.

Examine and Adjust Courant Number

When you are designing a field simulation, you care about the Courant number to be used. Task 330 shows the space step size, time step size and the Courant number factor f_c.

$$\Delta_t = \frac{\Delta_s}{c_0}\frac{f_c}{\sqrt{3}}$$

If the Courant number factor is 1 then the Courant number used is

$$Courant\ Number = \frac{1}{\sqrt{3}}$$

If you want to use a smaller Courant number then you may reduce Δ_t. Task 330 helps you to decide how much adjustment you want to do. We may still use the task file UnitTest.Task. Change em.env to execute task 330:

```
/K=330
```

```
/T=./taskFiles/UnitTest.task
/W=./tmp
/D=./_data
```

Run the program

$ ./EM

The program generates a file timeMatrix.txt in folder _data/UnitTest. The first few lines of the file are shown below:

```
kmax=1, time estimation order = 2(kmax+1)=4
ds=0.047619, dt=9.17063e-11, courant factor=1, total time=1.10048e-08, data file interval=0
material parameters: eps=8.85419e-12, mu=1.25664e-06, sigma_e=0, sigma_h=0
```

Because in the task file UnitTest.Task, we only specified TSS.DS for Δ_s, the program assumes $f_c = 1$ and calculated Δ_t

If in the task file, TSS.DT is used to specify Δ_t then f_c is calculated. If TSS.COURANT_FACTOR is used to specify f_c then TSS.DT is ignored and Δ_t is calculated.

Simulations

Apply Boundary Conditions of Perfect Electromagnetic Conductor

For more information on applying PEMC boundary conditions to the TSS algorithm, see https://github.com/DavidGeUSA/TSS/blob/master/PemcForTss.pdf.

No Boundary Conditions

To see the effects of boundary conditions, let's first run a simulation without boundary conditions. Task file ricker21_nx120_NB.task was created for this purpose.

Modify the file em.env to make sure that only following lines are enabled:

```
/K=300
/T=./taskFiles/ricker21_nx120_NB.task
/W=./tmp
/D=./_data
```

Note that in the task file ricker21_nx120_NB.task the line for applying boundary conditions is disabled:

```
//use PEMC
//TSS.BOUNDARY.CLASS=BoundaryTss
```

Run the program:

$ ./EM

The program starts

```
linux1@DESKTOP-QD88AGK:~/tss$ ./EM

.
  Electromagnetic Fields Analysis. Version 2.0.0.
 Created: 2017-02-08 for Microsoft Windows; updated 2021-03-11 to support Linux
 Author: David Ge (dge893@gmail.com)
.

Use em.env file or use command line parameters to execute tasks.



Current folder:/home/linux1/tss
The environment file to be used: /home/linux1/tss/em.env

Task file: ./taskFiles/ricker21_nx120_NB.task

   executing task 300 ...

Starting electromagnetic field simulation. Press Ctrl-C to stop.
 Initialize and prepare fields at time 0 ...
```

It shows each time step simulated and the time used in milliseconds:

```
Initialing, please wait ...
Initialization finished. Time used:94
Cannot set process priority
Field source:RickerSource, boundary conditions:null
Preparing to start simulation, applying source and boundary...
Prepared fields. Time used:92
Start TSS simulation.
1:471
2:389
3:428
4:372
5:453
6:332
7:392
8:373
9:387
10:441
11:393
12:410
```

Once the simulation finishes, data files are generated in folder _data/R21NB:

```
linux1@DESKTOP-QD88AGK:~/tss$ cd _data
linux1@DESKTOP-QD88AGK:~/tss/_data$ cd R21NB
linux1@DESKTOP-QD88AGK:~/tss/_data/R21NB$ dir
R21NB_TSS_smax2_kmax1_e_0.dat   R21NB_TSS_smax2_kmax1_e_47.dat  R21NB_TSS_smax2_kmax1_h_3.dat
R21NB_TSS_smax2_kmax1_e_1.dat   R21NB_TSS_smax2_kmax1_e_48.dat  R21NB_TSS_smax2_kmax1_h_30.dat
R21NB_TSS_smax2_kmax1_e_10.dat  R21NB_TSS_smax2_kmax1_e_49.dat  R21NB_TSS_smax2_kmax1_h_31.dat
R21NB_TSS_smax2_kmax1_e_11.dat  R21NB_TSS_smax2_kmax1_e_5.dat   R21NB_TSS_smax2_kmax1_h_32.dat
R21NB_TSS_smax2_kmax1_e_12.dat  R21NB_TSS_smax2_kmax1_e_50.dat  R21NB_TSS_smax2_kmax1_h_33.dat
R21NB_TSS_smax2_kmax1_e_13.dat  R21NB_TSS_smax2_kmax1_e_51.dat  R21NB_TSS_smax2_kmax1_h_34.dat
R21NB_TSS_smax2_kmax1_e_14.dat  R21NB_TSS_smax2_kmax1_e_52.dat  R21NB_TSS_smax2_kmax1_h_35.dat
```

Files *_e_<n>.dat are data files for electric field; *_h_<n>.dat are data files for magnetic field. <n> is the time step number.
A statistics file is created named R21NB_TSS_smax2_kmax1_statistic0.txt, as shown below.

Time	Energy	Max Divg E	Avg Divg E	Max Divg H	Avg Divg H
0	4.15903e-18	3.87215e-18	2.75612e-24	0	0
10	4.45926e-13	1.95104e-15	1.38871e-21	3.70826e-22	2.63947e-28
20	1.93422e-11	4.13524e-15	2.94338e-21	1.18392e-19	8.42689e-26
30	2.22299e-11	1.71903e-15	1.22357e-21	1.18536e-19	8.43717e-26
40	6.84223e-12	6.53272e-15	4.64986e-21	2.45523e-19	1.74758e-25
50	5.7674e-12	2.44414e-15	1.73969e-21	2.89908e-19	2.06351e-25
60	5.70703e-12	2.42514e-15	1.72617e-21	2.82926e-19	2.01381e-25
70	6.52135e-12	2.42403e-15	1.72538e-21	2.82963e-19	2.01408e-25
80	5.68692e-09	2.42522e-15	1.72623e-21	2.82903e-19	2.01365e-25
90	2.68261e-06	2.42237e-15	1.72419e-21	2.829e-19	2.01363e-25
100	0.000145436	2.4258e-15	1.72664e-21	2.82842e-19	2.01321e-25
110	0.0151445	2.42253e-15	1.72431e-21	2.82823e-19	2.01308e-25
120	98.5493	2.42561e-15	1.7265e-21	2.82809e-19	2.01298e-25
130	91426.1	2.42454e-15	1.72574e-21	2.82821e-19	2.01306e-25
140	4.294e+07	2.42283e-15	1.72452e-21	2.82852e-19	2.01329e-25
150	3.86587e+11	2.42532e-15	1.7263e-21	2.82872e-19	2.01343e-25
160	9.10469e+14	2.44021e-15	1.73689e-21	2.83933e-19	2.02098e-25
170	5.19209e+17	4.06405e-15	2.89271e-21	4.07879e-19	2.9032e-25
180	1.00125e+20	5.8589e-14	4.17025e-20	6.00391e-18	4.27346e-24
190	8.46779e+21	6.84458e-13	4.87183e-19	4.02549e-16	2.86527e-22
200	3.81116e+23	9.4268e-14	6.70981e-20	5.49798e-16	3.91335e-22
210	1.05124e+25	3.67492e-12	2.61573e-18	1.49648e-15	1.06516e-21
220	1.96964e+26	2.86874e-12	2.04191e-18	8.00037e-16	5.69451e-22
230	2.80426e+27	5.39251e-13	3.83828e-19	6.38154e-15	4.54225e-21
240	1.74317e+30	2.53765e-13	1.80625e-19	7.00306e-15	4.98464e-21
250	2.93196e+34	9.09495e-10	6.4736e-16	1.94699e-13	1.38583e-19
260	5.4336e+38	4.15769e-09	2.95936e-15	1.51276e-11	1.07675e-17
270	1.04826e+43	1.31383e-06	9.35158e-13	2.06965e-10	1.47314e-16
280	1.96324e+47	1.86265e-06	1.32579e-12	3.28213e-09	2.33616e-15
290	3.42703e+51	1.38368e-05	9.84876e-12	2.17866e-08	1.55073e-14
300	5.51217e+55	3.40598e-05	2.42431e-11	1.26629e-07	9.01321e-14
310	8.19735e+59	0	0	4.3834e-07	3.12002e-13
320	1.13817e+64	0	0	3.9407e-06	2.80491e-12
330	1.49467e+68	0	0	7.93095e-06	5.64509e-12
340	1.88537e+72	0	0	0.000377414	2.68636e-10
350	2.32548e+76	1714.29	0.00122019	0.094247	6.70831e-08
360	2.8585e+80	73142.9	0.0520616	1.55528	1.10702e-06
370	3.56121e+84	4.68114e+06	3.33195	1033.54	0.000735652
380	4.5425e+88	0	0	407387	0.28997
390	5.93575e+92	3.83479e+10	27295.3	1.47359e+07	10.4887
400	7.88563e+96	0	0	1.43941e+09	1024.54
410	1.05311e+101	6.28292e+14	4.47206e+08	1.73666e+10	12361.2
420	1.39872e+105	0	0	4.84693e+13	3.44995e+07
430	1.83287e+109	5.14697e+18	3.66351e+12	4.09277e+15	2.91315e+09
440	2.3577e+113	9.88218e+20	7.03394e+14	1.49541e+17	1.0644e+11
450	2.96931e+117	8.4328e+22	6.0023e+16	3.76257e+19	2.67812e+13
460	3.65776e+121	2.6985e+24	1.92074e+18	3.81001e+21	2.71189e+15
470	4.40763e+125	0	0	1.03791e+24	7.38766e+17
480	5.19935e+129	2.21061e+28	1.57347e+22	3.91532e+25	2.78685e+19
490	6.01089e+133	0	0	9.888e+26	7.03808e+20
500	6.81976e+137	9.05465e+32	6.44492e+26	2.01868e+29	1.43686e+23
510	7.60524e+141	4.63598e+34	3.2998e+28	1.29988e+31	9.25228e+24
520	8.35032e+145	1.18681e+37	8.44748e+30	1.98935e+33	1.41598e+27
530	9.04355e+149	1.8989e+38	1.3516e+32	4.89696e+35	3.48556e+29
540	9.68037e+153	2.43059e+40	1.73004e+34	1.84996e+37	1.31677e+31
550	1.02639e+158	1.55558e+42	1.10723e+36	1.97426e+39	1.40524e+33
560	1.08054e+162	3.98228e+44	2.83451e+38	2.13287e+41	1.51814e+35
570	1.13237e+166	5.09731e+46	3.62817e+40	2.89589e+41	2.06124e+35
580	1.18444e+170	1.1418e+49	8.12709e+42	5.15943e+45	3.67238e+39
590	1.23987e+174	4.17572e+50	2.97219e+44	3.03245e+47	2.15844e+41
600	1.30216e+178	4.00869e+52	2.85331e+46	9.12551e+49	6.49536e+43

The first column is the time step number; the second column is the summary of the field energy; the 3rd column is the maximum E-field divergence; the 4th column is the average E-field divergence; the 5th column is the maximum H-field divergence; the 6th field is the average H-field divergence.
The divergence statistics exclude the space points where field source is applied.
From the above data we can see that after 100 time steps, the field energy and field divergences grow quickly, showing huge impacts of the boundary errors.
Next, let’s add PEMC boundary conditions and run the simulation again.

Use PEMC boundary conditions

Modify the file em.env to make sure that only following lines are enabled:

```
/K=300
/T=./taskFiles/ricker21_nx120_PEMC.task
/W=./tmp
/D=./_data
```

Note that in the task file ricker21_nx120_PEMC.task the line for applying the boundary conditions is enabled:

```
//use PEMC
TSS.BOUNDARY.CLASS=BoundaryTss
```

Run the program:

```
$ ./EM
```

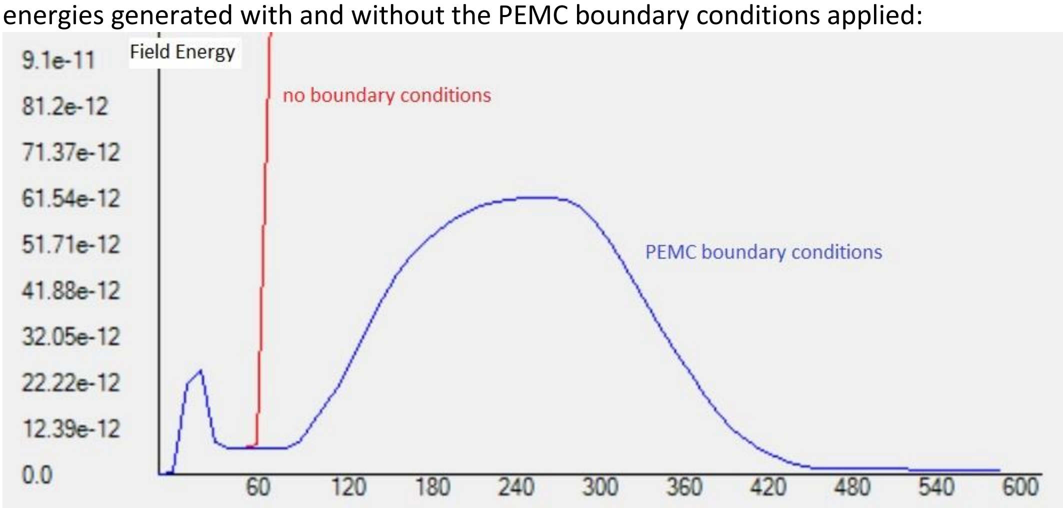
The program starts.
Once the simulation finishes, data files are generated in folder _data/R21PEMC.
A statistics file is also created named as R21PEMC_TSS_smax2_kmax1_statistic0.txt, as shown below.

Time	Energy	Max Divg E	Avg Divg E	Max Divg H	Avg Divg H
0	4.15903e-18	3.87215e-18	2.75612e-24	0	0
10	4.45926e-13	1.95104e-15	1.38871e-21	3.70826e-22	2.63947e-28
20	1.93422e-11	4.13524e-15	2.94338e-21	1.18392e-19	8.42689e-26
30	2.22299e-11	1.71903e-15	1.22357e-21	1.18536e-19	8.43717e-26
40	6.84223e-12	6.53272e-15	4.64986e-21	2.45523e-19	1.74758e-25
50	5.7674e-12	2.44414e-15	1.73969e-21	2.89908e-19	2.06351e-25
60	5.70703e-12	2.42514e-15	1.72617e-21	2.82926e-19	2.01381e-25
70	5.6457e-12	2.42763e-15	1.72794e-21	2.82971e-19	2.01413e-25
80	5.59516e-12	2.42322e-15	1.7248e-21	2.82996e-19	2.01431e-25
90	5.59224e-12	2.42384e-15	1.72524e-21	2.8299e-19	2.01427e-25
100	7.04886e-12	2.42763e-15	1.72794e-21	2.82963e-19	2.01407e-25
110	1.13093e-11	2.42431e-15	1.72557e-21	2.82978e-19	2.01418e-25
120	1.56427e-11	2.42093e-15	1.72317e-21	2.83005e-19	2.01437e-25
130	2.00574e-11	2.42859e-15	1.72862e-21	2.82964e-19	2.01408e-25
140	2.62096e-11	2.42534e-15	1.72631e-21	2.82897e-19	2.0136e-25
150	3.22936e-11	2.42563e-15	1.72652e-21	2.82918e-19	2.01375e-25
160	3.79062e-11	2.42756e-15	1.72789e-21	2.83034e-19	2.01458e-25
170	4.28462e-11	2.44446e-15	1.73992e-21	2.8292e-19	2.01377e-25
180	4.67729e-11	2.43763e-15	1.73505e-21	2.88257e-19	2.05176e-25
190	4.99088e-11	2.53545e-15	1.80468e-21	2.78254e-19	1.98056e-25
200	5.24764e-11	2.72658e-15	1.94073e-21	3.17445e-19	2.25951e-25
210	5.45065e-11	2.57256e-15	1.83109e-21	2.79862e-19	1.992e-25
220	5.60845e-11	2.48534e-15	1.76902e-21	2.93582e-19	2.08966e-25
230	5.72888e-11	2.56222e-15	1.82374e-21	2.73771e-19	1.94865e-25
240	5.81531e-11	2.55336e-15	1.81743e-21	2.70391e-19	1.92459e-25
250	5.8729e-11	2.24674e-15	1.59919e-21	2.97606e-19	2.1183e-25
260	5.90603e-11	2.38095e-15	1.69471e-21	2.81042e-19	2.0004e-25
270	5.919e-11	2.20502e-15	1.56949e-21	3.69873e-19	2.63268e-25
280	5.91671e-11	2.02754e-15	1.44316e-21	2.17243e-19	1.5463e-25
290	5.8702e-11	2.05119e-15	1.46e-21	2.37151e-19	1.688e-25

300	5.74888e-11	2.01903e-15	1.4371e-21	2.47512e-19	1.76174e-25
310	5.46592e-11	1.54897e-15	1.10252e-21	1.56079e-19	1.11094e-25
320	5.06301e-11	1.83883e-15	1.30885e-21	3.9729e-20	2.82783e-26
330	4.58529e-11	1.91532e-15	1.36329e-21	4.17365e-20	2.97072e-26
340	4.08637e-11	1.98633e-15	1.41383e-21	6.51658e-20	4.63837e-26
350	3.55517e-11	1.94846e-15	1.38688e-21	8.30598e-20	5.91203e-26
360	3.06758e-11	1.96409e-15	1.398e-21	7.44331e-20	5.298e-26
370	2.59706e-11	1.97536e-15	1.40602e-21	8.32782e-20	5.92758e-26
380	2.14373e-11	1.96492e-15	1.39859e-21	8.00822e-20	5.70009e-26
390	1.70366e-11	1.965e-15	1.39865e-21	7.77161e-20	5.53168e-26
400	1.30583e-11	1.97561e-15	1.4062e-21	8.6082e-20	6.12714e-26
410	9.76291e-12	2.0235e-15	1.44028e-21	8.45211e-20	6.01605e-26
420	7.29186e-12	2.00648e-15	1.42818e-21	7.96609e-20	5.67011e-26
430	5.34934e-12	2.04533e-15	1.45583e-21	8.23748e-20	5.86328e-26
440	3.86367e-12	1.97691e-15	1.40712e-21	6.96621e-20	4.95841e-26
450	2.80672e-12	2.04631e-15	1.45652e-21	5.44879e-20	3.87834e-26
460	2.09147e-12	2.05037e-15	1.45941e-21	5.46438e-20	3.88944e-26
470	1.69903e-12	2.06772e-15	1.47176e-21	5.20652e-20	3.7059e-26
480	1.49446e-12	2.06486e-15	1.46973e-21	5.22158e-20	3.71662e-26
490	1.41571e-12	2.05287e-15	1.46119e-21	4.85371e-20	3.45477e-26
500	1.38625e-12	2.08683e-15	1.48537e-21	4.89714e-20	3.48569e-26
510	1.36855e-12	2.06765e-15	1.47171e-21	4.6132e-20	3.28359e-26
520	1.34886e-12	2.07984e-15	1.48039e-21	4.72493e-20	3.36311e-26
530	1.321e-12	2.0778e-15	1.47893e-21	4.45997e-20	3.17452e-26
540	1.28005e-12	2.10037e-15	1.495e-21	4.8403e-20	3.44523e-26
550	1.22246e-12	2.09108e-15	1.48839e-21	4.73026e-20	3.3669e-26
560	1.15993e-12	2.03164e-15	1.44608e-21	7.02273e-20	4.99864e-26
570	1.10108e-12	2.04784e-15	1.45761e-21	6.14454e-20	4.37356e-26
580	1.03281e-12	2.00519e-15	1.42726e-21	7.78105e-20	5.5384e-26
590	9.6452e-13	2.04384e-15	1.45476e-21	7.05188e-20	5.01939e-26
600	9.03432e-13	2.13361e-15	1.51866e-21	5.512e-20	3.92333e-26

From the above data we can see that the field energy and field divergences are very small all the time. The extremely low divergences proved that our TSS algorithm and the coding are correct.

The very small energy level shows excellent PEMC performance. We may graphically compare the field energies generated with and without the PEMC boundary conditions applied:



Time Step

From the above image we can see how the PEMC boundary conditions worked.

Specify the Number of Threads

In the above simulations, we specify that the number of threads is 4. The number of threads is specified in the task file

```
TSS.TIME.CLASS=TimeTssMultiThreads
```

```
SIM.USETHREADS=4
```

The minimum threads for TimeTssMultiThreads is 2. When you specify SIM.USETHREADS=1 it will use 2 threads. TimeTssMultiThreads always uses even number of threads. It will round SIM.USETHREADS down to an even number. For SIM.USETHREADS=5, it will use 4 threads.

If you want to use single thread then use TSS.TIME.CLASS=TimeTss
TimeTss always runs under a single thread.

Let's examine simulation speeds running different number of threads.
2-threads:

```
Prepared fields. Time used:87
Start TSS simulation.
1:622
2:548
3:480
4:496
5:676
6:725
7:699
8:604
9:463
10:460
```

4-threads:

```
Prepared fields. Time used:81
Start TSS simulation.
1:321
2:372
3:374
4:352
5:329
6:415
7:381
8:327
9:402
10:353
```

6-threads:

```
Prepared fields. Time used:81
Start TSS simulation.
1:391
2:382
3:396
4:381
5:376
6:389
7:404
8:401
9:372
10:380
11:361
```

We can see that increasing from 4 threads to 6 threads did not increase speed. This was due to the fact that a 4-core CPU computer was running the simulations.

Apply Perfect Match Layers

For more information on applying PML to TSS algorithm, see
https://github.com/DavidGeUSA/TSS/blob/master/PmlForTSS.pdf

In the taskFiles folder, there are 4 task files which were used to generate the data reported in the above document.

Here, I'll show 3 simulations, each with a different PML thickness. The task file is r21_84_NB_PML.task.

The PML thickness is specified by PML.THICK:

```
//PML
PML.DISABLE=false
PML.THICK=4
PML.ALPHA=0.01
PML.BETA=0.01
PML.X.LOW=true
PML.X.HIGH=true
PML.Y.LOW=true
PML.Y.HIGH=true
PML.Z.LOW=true
PML.Z.HIGH=true
PML.POWER=3
```

Set em.env to use

```
/K=300
/T=./taskFiles/r21_84_NB_PML.task
```

Layer thickness = 4

Set following lines in task file r21_84_NB_PML.task:

```
PML.THICK=4
TSS.DATAFOLDER=R21_84_NB_PML4_0_01
TSS.REC_BASEFILE=R21_84_NB_PML4_0_01
```

Run the simulation, data are generated in folder R21_84_NB_PML4_0_01.

```
linux1@DESKTOP-QD88AGK:~/tss$ cd _data
linux1@DESKTOP-QD88AGK:~/tss/_data$ cd R21_84_NB_PML4_0_01/
linux1@DESKTOP-QD88AGK:~/tss/_data/R21_84_NB_PML4_0_01$ dir
R21_84_NB_PML4_0_01_TSS_smax2_kmax1_e_0.dat    R21_84_NB_PML4_0_01_TSS_smax2_kmax1_h_10.dat
R21_84_NB_PML4_0_01_TSS_smax2_kmax1_e_1.dat    R21_84_NB_PML4_0_01_TSS_smax2_kmax1_h_100.dat
R21_84_NB_PML4_0_01_TSS_smax2_kmax1_e_10.dat   R21_84_NB_PML4_0_01_TSS_smax2_kmax1_h_101.dat
```

A statistics file, R21_84_NB_PML4_0_01_TSS_smax2_kmax1_statistic0.txt, is generated:

Time	Energy	Max Divg E	Avg Divg E	Max Divg H	Avg Divg H
0	4.15903e-18	1.35525e-18	3.0873e-24	0	0
10	4.48792e-13	8.44379e-19	1.92352e-24	3.73025e-24	8.49761e-30
20	1.93781e-11	9.04395e-16	2.06024e-21	2.57997e-20	5.87725e-26
30	2.24625e-11	5.7437e-16	1.30843e-21	4.60252e-20	1.04847e-25
40	7.10295e-12	1.25485e-15	2.85858e-21	7.23323e-20	1.64775e-25
50	6.01308e-12	6.35713e-16	1.44817e-21	6.23553e-20	1.42047e-25
60	5.92781e-12	6.25166e-16	1.42415e-21	6.35571e-20	1.44785e-25
70	5.594e-12	6.26244e-16	1.4266e-21	6.35599e-20	1.44791e-25
80	4.94324e-12	6.26806e-16	1.42788e-21	6.35488e-20	1.44766e-25
90	4.31918e-12	6.24934e-16	1.42362e-21	6.35463e-20	1.4476e-25
100	3.53981e-12	6.23972e-16	1.42143e-21	6.35424e-20	1.44751e-25

110	2.78459e-12	6.21664e-16	1.41617e-21	6.35476e-20	1.44763e-25
120	2.35491e-12	6.21473e-16	1.41573e-21	6.38572e-20	1.45469e-25
130	2.08797e-12	6.31055e-16	1.43756e-21	6.39474e-20	1.45674e-25
140	1.90739e-12	6.36757e-16	1.45055e-21	6.54019e-20	1.48987e-25
150	1.77903e-12	6.47646e-16	1.47536e-21	6.74328e-20	1.53614e-25
160	1.67911e-12	6.49233e-16	1.47897e-21	6.82994e-20	1.55588e-25
170	1.59485e-12	6.39638e-16	1.45711e-21	6.85071e-20	1.56061e-25
180	1.49876e-12	6.33171e-16	1.44238e-21	6.78219e-20	1.545e-25
190	1.35285e-12	6.32927e-16	1.44183e-21	6.97878e-20	1.58979e-25
200	1.19475e-12	6.51265e-16	1.4836e-21	7.0274e-20	1.60086e-25
210	1.04524e-12	6.57794e-16	1.49847e-21	7.04416e-20	1.60468e-25
220	8.93418e-13	6.34564e-16	1.44555e-21	7.07976e-20	1.61279e-25
230	7.77379e-13	6.41585e-16	1.46155e-21	7.13822e-20	1.62611e-25
240	7.00427e-13	6.46942e-16	1.47375e-21	7.1264e-20	1.62342e-25
250	6.32012e-13	6.3529e-16	1.44721e-21	7.19069e-20	1.63806e-25
260	5.76739e-13	6.35364e-16	1.44738e-21	7.2451e-20	1.65045e-25
270	5.33174e-13	6.61568e-16	1.50707e-21	7.21782e-20	1.64424e-25
280	4.97659e-13	6.39152e-16	1.45601e-21	7.47487e-20	1.7028e-25
290	4.6451e-13	6.58695e-16	1.50053e-21	7.47324e-20	1.70243e-25
300	4.2959e-13	6.48574e-16	1.47747e-21	7.5393e-20	1.71748e-25
310	3.92667e-13	6.38208e-16	1.45386e-21	7.58216e-20	1.72724e-25
320	3.54583e-13	6.4584e-16	1.47124e-21	7.66386e-20	1.74585e-25
330	3.07924e-13	6.43208e-16	1.46525e-21	7.65699e-20	1.74428e-25
340	2.64879e-13	6.44894e-16	1.46909e-21	7.6394e-20	1.74028e-25
350	2.34251e-13	6.57119e-16	1.49694e-21	7.52314e-20	1.71379e-25
360	2.08689e-13	6.23791e-16	1.42101e-21	7.4635e-20	1.70021e-25
370	1.86501e-13	6.41248e-16	1.46078e-21	7.54734e-20	1.7193e-25
380	1.68934e-13	6.60995e-16	1.50576e-21	7.43221e-20	1.69308e-25
390	1.55896e-13	6.38503e-16	1.45453e-21	7.46153e-20	1.69976e-25
400	1.43847e-13	6.3343e-16	1.44297e-21	7.41262e-20	1.68862e-25
410	1.32672e-13	6.42201e-16	1.46295e-21	7.39908e-20	1.68553e-25
420	1.21439e-13	6.36857e-16	1.45078e-21	7.40051e-20	1.68586e-25
430	1.0925e-13	6.34595e-16	1.44563e-21	7.38439e-20	1.68218e-25
440	9.61609e-14	6.39178e-16	1.45607e-21	7.40052e-20	1.68586e-25
450	8.35759e-14	6.41159e-16	1.46058e-21	7.43076e-20	1.69275e-25
460	7.3142e-14	6.3978e-16	1.45744e-21	7.42875e-20	1.69229e-25
470	6.49958e-14	6.46387e-16	1.47249e-21	7.42345e-20	1.69108e-25
480	5.74102e-14	6.43311e-16	1.46548e-21	7.43981e-20	1.69481e-25
490	5.23069e-14	6.42758e-16	1.46422e-21	7.48731e-20	1.70563e-25
500	4.79123e-14	6.4582e-16	1.4712e-21	7.49041e-20	1.70634e-25
510	4.37172e-14	6.40943e-16	1.46009e-21	7.47866e-20	1.70366e-25
520	4.01326e-14	6.38424e-16	1.45435e-21	7.46205e-20	1.69988e-25
530	3.7e-14	6.35572e-16	1.44785e-21	7.46271e-20	1.70003e-25
540	3.3932e-14	6.41408e-16	1.46115e-21	7.52403e-20	1.714e-25
550	3.06783e-14	6.50305e-16	1.48141e-21	7.57655e-20	1.72596e-25
560	2.7951e-14	6.48027e-16	1.47622e-21	7.56391e-20	1.72308e-25
570	2.44993e-14	6.49522e-16	1.47963e-21	7.57942e-20	1.72661e-25
580	2.18769e-14	6.54777e-16	1.4916e-21	7.58014e-20	1.72678e-25
590	1.9456e-14	6.64224e-16	1.51312e-21	7.5951e-20	1.73018e-25
600	1.74795e-14	6.6474e-16	1.5143e-21	7.61052e-20	1.7337e-25
610	1.57038e-14	6.63249e-16	1.5109e-21	7.62091e-20	1.73606e-25
620	1.44528e-14	6.60102e-16	1.50373e-21	7.61439e-20	1.73458e-25
630	1.33007e-14	6.63299e-16	1.51101e-21	7.64369e-20	1.74126e-25
640	1.24742e-14	6.58021e-16	1.49899e-21	7.60855e-20	1.73325e-25
650	1.15199e-14	6.60039e-16	1.50359e-21	7.59395e-20	1.72992e-25
660	1.08608e-14	6.63688e-16	1.5119e-21	7.59725e-20	1.73068e-25
670	1.07435e-14	6.64199e-16	1.51307e-21	7.62665e-20	1.73737e-25
680	1.12246e-14	6.64949e-16	1.51477e-21	7.63551e-20	1.73939e-25
690	1.2679e-14	6.65474e-16	1.51597e-21	7.67797e-20	1.74906e-25
700	1.58865e-14	6.64215e-16	1.5131e-21	7.65741e-20	1.74438e-25
710	2.25e-14	6.63298e-16	1.51101e-21	7.66796e-20	1.74678e-25
720	3.43116e-14	6.64317e-16	1.51333e-21	7.70399e-20	1.75499e-25
730	5.43833e-14	6.66755e-16	1.51889e-21	7.68835e-20	1.75143e-25
740	8.94826e-14	6.64039e-16	1.5127e-21	7.6672e-20	1.74661e-25
750	1.4502e-13	6.70641e-16	1.52774e-21	7.64072e-20	1.74058e-25
760	2.33831e-13	6.72499e-16	1.53197e-21	7.62591e-20	1.7372e-25

770	3.74998e-13	6.75167e-16	1.53805e-21	7.63583e-20	1.73946e-25
780	5.93886e-13	6.74805e-16	1.53722e-21	7.60568e-20	1.7326e-25
790	9.28721e-13	6.7535e-16	1.53847e-21	7.60231e-20	1.73183e-25
800	1.43613e-12	6.71379e-16	1.52942e-21	7.60137e-20	1.73162e-25
810	2.1932e-12	6.75551e-16	1.53892e-21	7.65216e-20	1.74318e-25
820	3.31625e-12	6.75096e-16	1.53789e-21	7.67581e-20	1.74857e-25
830	4.96342e-12	6.74625e-16	1.53681e-21	7.65383e-20	1.74356e-25
840	7.36108e-12	6.78198e-16	1.54495e-21	7.52715e-20	1.71471e-25
850	1.08191e-11	6.82772e-16	1.55537e-21	7.79867e-20	1.77656e-25
860	1.57721e-11	6.74926e-16	1.5375e-21	8.11519e-20	1.84866e-25
870	2.28109e-11	6.77817e-16	1.54409e-21	7.30847e-20	1.66489e-25
880	3.27564e-11	6.86118e-16	1.563e-21	6.81623e-20	1.55276e-25
890	4.67325e-11	6.92026e-16	1.57646e-21	8.0017e-20	1.82281e-25
900	6.62265e-11	6.99628e-16	1.59377e-21	7.25803e-20	1.6534e-25
910	9.32958e-11	6.9859e-16	1.59141e-21	7.98069e-20	1.81802e-25
920	1.30709e-10	6.30574e-16	1.43647e-21	8.68283e-20	1.97797e-25
930	1.8215e-10	6.51178e-16	1.4834e-21	6.20711e-20	1.414e-25
940	2.52575e-10	6.00864e-16	1.36879e-21	5.68902e-20	1.29597e-25
950	3.48577e-10	6.12447e-16	1.39517e-21	3.55211e-20	8.09181e-26
960	4.79021e-10	5.67512e-16	1.29281e-21	2.07739e-20	4.73236e-26
970	6.555e-10	6.15412e-16	1.40193e-21	5.01173e-20	1.14169e-25
980	8.93523e-10	6.65937e-16	1.51702e-21	6.45873e-20	1.47132e-25
990	1.21358e-09	7.32345e-16	1.6683e-21	3.28696e-20	7.48779e-26
1000	1.6426e-09	8.0483e-16	1.83342e-21	1.31773e-20	3.00182e-26
1010	2.21605e-09	5.18829e-16	1.18191e-21	1.80782e-20	4.11826e-26
1020	2.98066e-09	5.71684e-16	1.30231e-21	8.81737e-20	2.00862e-25
1030	3.9975e-09	5.45955e-16	1.2437e-21	9.50763e-20	2.16587e-25
1040	5.34679e-09	4.81919e-16	1.09783e-21	2.30077e-19	5.24122e-25
1050	7.13313e-09	6.96092e-16	1.58572e-21	7.51233e-20	1.71133e-25
1060	9.49338e-09	6.23332e-16	1.41997e-21	1.86916e-19	4.25799e-25
1070	1.26057e-08	5.32021e-16	1.21196e-21	2.27825e-19	5.18993e-25
1080	1.67021e-08	5.43287e-16	1.23762e-21	3.26258e-19	7.43224e-25
1090	2.20842e-08	1.77877e-16	4.05209e-22	3.17849e-19	7.2407e-25
1100	2.91438e-08	1.35525e-18	3.0873e-24	3.72546e-19	8.48672e-25
1110	3.83889e-08	2.14638e-16	4.88952e-22	7.1198e-19	1.62191e-24
1120	5.04776e-08	2.08031e-16	4.73901e-22	7.8721e-19	1.79329e-24
1130	6.62624e-08	2.97986e-16	6.78821e-22	1.19552e-18	2.72343e-24
1140	8.68452e-08	4.02849e-16	9.17701e-22	1.03248e-18	2.35202e-24
1150	1.13651e-07	1.01e-15	2.30081e-21	9.68041e-19	2.20522e-24
1160	1.48521e-07	1.6263e-17	3.70477e-23	1.35902e-18	3.09588e-24
1170	1.93835e-07	1.10114e-16	2.50844e-22	2.47326e-18	5.63415e-24
1180	2.52666e-07	7.02699e-16	1.60077e-21	1.85969e-18	4.23642e-24
1190	3.28987e-07	9.43256e-16	2.14876e-21	1.46401e-18	3.33505e-24
1200	4.2794e-07	3.10895e-15	7.08228e-21	1.79101e-18	4.07998e-24

Layer thickness = 8

Set following lines in task file r21_84_NB_PML.task:

```
PML.THICK=8
TSS.DATAFOLDER=R21_84_NB_PML8_0_01
TSS.REC_BASEFILE=R21_84_NB_PML8_0_01
```

Run the simulation, data are generated in folder R21_84_NB_PML8_0_01.

```
linux1@DESKTOP-QD88AGK:~/tss$ cd _data
linux1@DESKTOP-QD88AGK:~/tss/_data$ cd R21_84_NB_PML8_0_01
linux1@DESKTOP-QD88AGK:~/tss/_data/R21_84_NB_PML8_0_01$ dir
R21_84_NB_PML8_0_01_TSS_smax2_kmax1_e_0.dat    R21_84_NB_PML8_0_01_TSS_smax2_kmax1_h_10.dat
R21_84_NB_PML8_0_01_TSS_smax2_kmax1_e_1.dat    R21_84_NB_PML8_0_01_TSS_smax2_kmax1_h_100.dat
R21_84_NB_PML8_0_01_TSS_smax2_kmax1_e_10.dat   R21_84_NB_PML8_0_01_TSS_smax2_kmax1_h_101.dat
R21_84_NB_PML8_0_01_TSS_smax2_kmax1_e_100.dat  R21_84_NB_PML8_0_01_TSS_smax2_kmax1_h_102.dat
R21_84_NB_PML8_0_01_TSS_smax2_kmax1_e_101.dat  R21_84_NB_PML8_0_01_TSS_smax2_kmax1_h_103.dat
```

A statistics file, R21_84_NB_PML8_0_01_TSS_smax2_kmax1_statistic0.txt, is generated:

Time	Energy	Max Divg E	Avg Divg E	Max Divg H	Avg Divg H
0	4.15903e-18	1.35525e-18	3.0873e-24	0	0
10	4.48792e-13	8.44379e-19	1.92352e-24	3.73025e-24	8.49761e-30
20	1.93781e-11	9.04395e-16	2.06024e-21	2.57997e-20	5.87725e-26
30	2.24625e-11	5.7437e-16	1.30843e-21	4.60252e-20	1.04847e-25
40	7.10293e-12	6.27053e-16	1.42844e-21	6.38316e-20	1.4541e-25
50	6.00955e-12	6.76202e-16	1.54041e-21	5.76377e-20	1.313e-25
60	5.89192e-12	6.51658e-16	1.48449e-21	5.5933e-20	1.27417e-25
70	5.4869e-12	6.50001e-16	1.48072e-21	5.59227e-20	1.27394e-25
80	4.8233e-12	6.52886e-16	1.48729e-21	5.5924e-20	1.27396e-25
90	4.07415e-12	6.52846e-16	1.4872e-21	5.5937e-20	1.27426e-25
100	3.22924e-12	6.50865e-16	1.48269e-21	5.59484e-20	1.27452e-25
110	2.61796e-12	6.51656e-16	1.48449e-21	5.63416e-20	1.28348e-25
120	2.23513e-12	6.45002e-16	1.46933e-21	5.51961e-20	1.25738e-25
130	2.01437e-12	6.38067e-16	1.45354e-21	5.50533e-20	1.25413e-25
140	1.86335e-12	6.18849e-16	1.40976e-21	5.57908e-20	1.27093e-25
150	1.74715e-12	5.89642e-16	1.34322e-21	5.50168e-20	1.2533e-25
160	1.64546e-12	5.90982e-16	1.34627e-21	5.54224e-20	1.26254e-25
170	1.51519e-12	5.36429e-16	1.222e-21	5.5801e-20	1.27116e-25
180	1.33754e-12	5.73817e-16	1.30717e-21	5.54761e-20	1.26376e-25
190	1.16389e-12	5.79407e-16	1.31991e-21	5.56911e-20	1.26866e-25
200	1.00179e-12	5.7873e-16	1.31836e-21	5.5448e-20	1.26312e-25
210	8.49518e-13	5.62908e-16	1.28232e-21	5.43786e-20	1.23876e-25
220	7.45255e-13	5.57423e-16	1.26983e-21	5.41208e-20	1.23289e-25
230	6.64606e-13	5.53254e-16	1.26033e-21	5.43245e-20	1.23753e-25
240	5.97606e-13	5.64408e-16	1.28574e-21	5.76897e-20	1.31419e-25
250	5.46015e-13	5.59683e-16	1.27497e-21	5.95964e-20	1.35762e-25
260	5.03351e-13	5.60112e-16	1.27595e-21	5.76012e-20	1.31217e-25
270	4.64481e-13	5.42299e-16	1.23537e-21	5.89177e-20	1.34216e-25
280	4.21782e-13	5.31045e-16	1.20974e-21	6.10502e-20	1.39074e-25
290	3.78824e-13	5.49301e-16	1.25132e-21	6.10289e-20	1.39026e-25
300	3.31167e-13	5.53204e-16	1.26022e-21	6.16395e-20	1.40417e-25
310	2.81626e-13	5.52265e-16	1.25808e-21	6.02456e-20	1.37241e-25
320	2.44411e-13	5.60127e-16	1.27598e-21	6.01241e-20	1.36964e-25
330	2.15666e-13	5.5069e-16	1.25449e-21	5.93846e-20	1.3528e-25
340	1.89373e-13	5.49576e-16	1.25195e-21	5.76603e-20	1.31352e-25
350	1.68091e-13	5.51842e-16	1.25711e-21	5.80182e-20	1.32167e-25
360	1.52712e-13	5.4992e-16	1.25273e-21	5.75111e-20	1.31012e-25
370	1.39352e-13	5.54601e-16	1.2634e-21	5.764e-20	1.31305e-25
380	1.25839e-13	5.47812e-16	1.24793e-21	5.74158e-20	1.30795e-25
390	1.11877e-13	5.56291e-16	1.26725e-21	5.74904e-20	1.30965e-25
400	9.73934e-14	5.60523e-16	1.27689e-21	5.7277e-20	1.30479e-25
410	8.44698e-14	5.5599e-16	1.26656e-21	5.80245e-20	1.32181e-25
420	7.25487e-14	5.64416e-16	1.28576e-21	5.81428e-20	1.32451e-25
430	6.31908e-14	5.65002e-16	1.28709e-21	5.8314e-20	1.32841e-25
440	5.53135e-14	5.63037e-16	1.28261e-21	5.83713e-20	1.32972e-25
450	4.89964e-14	5.59288e-16	1.27408e-21	5.86919e-20	1.33702e-25
460	4.41809e-14	5.65292e-16	1.28775e-21	5.92279e-20	1.34923e-25
470	3.96272e-14	5.61523e-16	1.27917e-21	5.99831e-20	1.36643e-25
480	3.58554e-14	5.64415e-16	1.28575e-21	6.07448e-20	1.38378e-25
490	3.22731e-14	5.68782e-16	1.2957e-21	6.13229e-20	1.39695e-25
500	2.87553e-14	5.67068e-16	1.2918e-21	6.07761e-20	1.3845e-25
510	2.54712e-14	5.66093e-16	1.28958e-21	6.06466e-20	1.38155e-25
520	2.225e-14	5.66601e-16	1.29073e-21	6.05393e-20	1.3791e-25
530	1.95488e-14	5.66093e-16	1.28958e-21	6.04808e-20	1.37777e-25
540	1.7379e-14	5.68787e-16	1.29571e-21	6.00376e-20	1.36767e-25
550	1.52662e-14	5.75917e-16	1.31195e-21	6.04948e-20	1.37809e-25
560	1.33467e-14	5.82135e-16	1.32612e-21	6.04412e-20	1.37687e-25
570	1.18988e-14	5.84994e-16	1.33263e-21	6.06125e-20	1.38077e-25
580	1.0736e-14	5.77452e-16	1.31545e-21	6.05434e-20	1.3792e-25
590	9.5451e-15	5.81052e-16	1.32365e-21	6.08153e-20	1.38539e-25
600	8.51196e-15	5.80601e-16	1.32262e-21	6.07976e-20	1.38499e-25
610	7.6062e-15	5.78601e-16	1.31807e-21	6.06619e-20	1.3819e-25

620	6.77708e-15	5.82431e-16	1.32679e-21	6.06722e-20	1.38213e-25
630	6.07321e-15	5.83427e-16	1.32906e-21	6.09168e-20	1.3877e-25
640	5.40815e-15	5.82431e-16	1.32679e-21	6.08739e-20	1.38672e-25
650	4.78729e-15	5.82098e-16	1.32604e-21	6.09323e-20	1.38806e-25
660	4.27085e-15	5.77052e-16	1.31454e-21	5.99774e-20	1.3663e-25
670	3.81237e-15	5.72447e-16	1.30405e-21	5.97811e-20	1.36183e-25
680	3.44163e-15	5.73481e-16	1.30641e-21	6.00235e-20	1.36735e-25
690	3.13027e-15	5.74982e-16	1.30983e-21	5.99972e-20	1.36675e-25
700	2.84563e-15	5.74825e-16	1.30947e-21	5.98893e-20	1.36429e-25
710	2.56388e-15	5.71355e-16	1.30156e-21	5.96489e-20	1.35882e-25
720	2.32283e-15	5.76115e-16	1.31241e-21	6.01584e-20	1.37043e-25
730	2.11162e-15	5.75171e-16	1.31026e-21	5.9941e-20	1.36547e-25
740	1.91386e-15	5.74574e-16	1.3089e-21	5.97615e-20	1.36138e-25
750	1.74263e-15	5.75523e-16	1.31106e-21	6.00229e-20	1.36734e-25
760	1.59369e-15	5.75751e-16	1.31158e-21	5.9697e-20	1.35992e-25
770	1.43068e-15	5.76158e-16	1.3125e-21	5.96787e-20	1.3595e-25
780	1.264e-15	5.75285e-16	1.31052e-21	5.97209e-20	1.36046e-25
790	1.13559e-15	5.76291e-16	1.31281e-21	5.96296e-20	1.35838e-25
800	1.03896e-15	5.77035e-16	1.3145e-21	5.95951e-20	1.35759e-25
810	9.57327e-16	5.79954e-16	1.32115e-21	5.95844e-20	1.35735e-25
820	8.83655e-16	5.79122e-16	1.31926e-21	5.9637e-20	1.35855e-25
830	8.27453e-16	5.81935e-16	1.32566e-21	5.97986e-20	1.36223e-25
840	7.85636e-16	5.80622e-16	1.32267e-21	6.00213e-20	1.3673e-25
850	7.38796e-16	5.79144e-16	1.31931e-21	6.00317e-20	1.36754e-25
860	6.88572e-16	5.78318e-16	1.31743e-21	6.01793e-20	1.3709e-25
870	6.46505e-16	5.80305e-16	1.32195e-21	6.0208e-20	1.37156e-25
880	6.07441e-16	5.78727e-16	1.31836e-21	6.03553e-20	1.37491e-25
890	5.75026e-16	5.78781e-16	1.31848e-21	6.0418e-20	1.37634e-25
900	5.55041e-16	5.76698e-16	1.31373e-21	6.04002e-20	1.37593e-25
910	5.45292e-16	5.7796e-16	1.31661e-21	6.03894e-20	1.37569e-25
920	5.48661e-16	5.73671e-16	1.30684e-21	6.03931e-20	1.37577e-25
930	5.67363e-16	5.76943e-16	1.31429e-21	6.05175e-20	1.37861e-25
940	6.02966e-16	5.74468e-16	1.30865e-21	6.04084e-20	1.37612e-25
950	6.52028e-16	5.74387e-16	1.30847e-21	6.05498e-20	1.37934e-25
960	7.22386e-16	5.74523e-16	1.30878e-21	6.05479e-20	1.3793e-25
970	8.07568e-16	5.74417e-16	1.30854e-21	6.05564e-20	1.37949e-25
980	9.27211e-16	5.72652e-16	1.30452e-21	6.05924e-20	1.38031e-25
990	1.10809e-15	5.69141e-16	1.29652e-21	6.05659e-20	1.37971e-25
1000	1.36585e-15	5.70765e-16	1.30022e-21	6.05903e-20	1.38026e-25
1010	1.73866e-15	5.72239e-16	1.30358e-21	6.05666e-20	1.37972e-25
1020	2.29611e-15	5.70983e-16	1.30072e-21	6.06311e-20	1.38119e-25
1030	3.12405e-15	5.71424e-16	1.30172e-21	6.05781e-20	1.37999e-25
1040	4.34733e-15	5.69996e-16	1.29847e-21	6.05202e-20	1.37867e-25
1050	6.14477e-15	5.70073e-16	1.29864e-21	6.04346e-20	1.37672e-25
1060	8.7997e-15	5.69673e-16	1.29773e-21	6.04175e-20	1.37633e-25
1070	1.26848e-14	5.70555e-16	1.29974e-21	6.04588e-20	1.37727e-25
1080	1.83703e-14	5.69875e-16	1.29819e-21	6.04704e-20	1.37753e-25
1090	2.66526e-14	5.70899e-16	1.30052e-21	6.05722e-20	1.37985e-25
1100	3.86533e-14	5.69978e-16	1.29843e-21	6.05844e-20	1.38013e-25
1110	5.59241e-14	5.70836e-16	1.30038e-21	6.06063e-20	1.38063e-25
1120	8.07272e-14	5.68846e-16	1.29585e-21	6.06287e-20	1.38114e-25
1130	1.16074e-13	5.70532e-16	1.29969e-21	6.05953e-20	1.38038e-25
1140	1.66239e-13	5.71002e-16	1.30076e-21	6.07064e-20	1.38291e-25
1150	2.37007e-13	5.68226e-16	1.29443e-21	6.06749e-20	1.38219e-25
1160	3.36354e-13	5.67994e-16	1.29391e-21	6.07494e-20	1.38389e-25
1170	4.75071e-13	5.69013e-16	1.29623e-21	6.06686e-20	1.38205e-25
1180	6.67796e-13	5.67907e-16	1.29371e-21	6.08408e-20	1.38597e-25
1190	9.3424e-13	5.68963e-16	1.29611e-21	6.06244e-20	1.38104e-25
1200	1.30079e-12	5.70561e-16	1.29975e-21	6.071e-20	1.38299e-25

Layer thickness = 12

Set following lines in task file r21_84_NB_PML.task:

```
PML.THICK=12
TSS.DATAFOLDER=R21_84_NB_PML12_0_01
```

```
TSS.REC_BASEFILE=R21_84_NB_PML12_0_01
```

Run the simulation, data are generated in folder R21_84_NB_PML12_0_01.

```
linux1@DESKTOP-QD88AGK:~/tss$ cd _data
linux1@DESKTOP-QD88AGK:~/tss/_data$ cd R21_84_NB_PML12_0_01
linux1@DESKTOP-QD88AGK:~/tss/_data/R21_84_NB_PML12_0_01$ dir
R21_84_NB_PML12_0_01_TSS_smax2_kmax1_e_0.dat    R21_84_NB_PML12_0_01_TSS_smax2_kmax1_h_10.dat
R21_84_NB_PML12_0_01_TSS_smax2_kmax1_e_1.dat    R21_84_NB_PML12_0_01_TSS_smax2_kmax1_h_100.dat
R21_84_NB_PML12_0_01_TSS_smax2_kmax1_e_10.dat   R21_84_NB_PML12_0_01_TSS_smax2_kmax1_h_101.dat
R21_84_NB_PML12_0_01_TSS_smax2_kmax1_e_100.dat  R21_84_NB_PML12_0_01_TSS_smax2_kmax1_h_102.dat
R21_84_NB_PML12_0_01_TSS_smax2_kmax1_e_101.dat  R21_84_NB_PML12_0_01_TSS_smax2_kmax1_h_103.dat
```

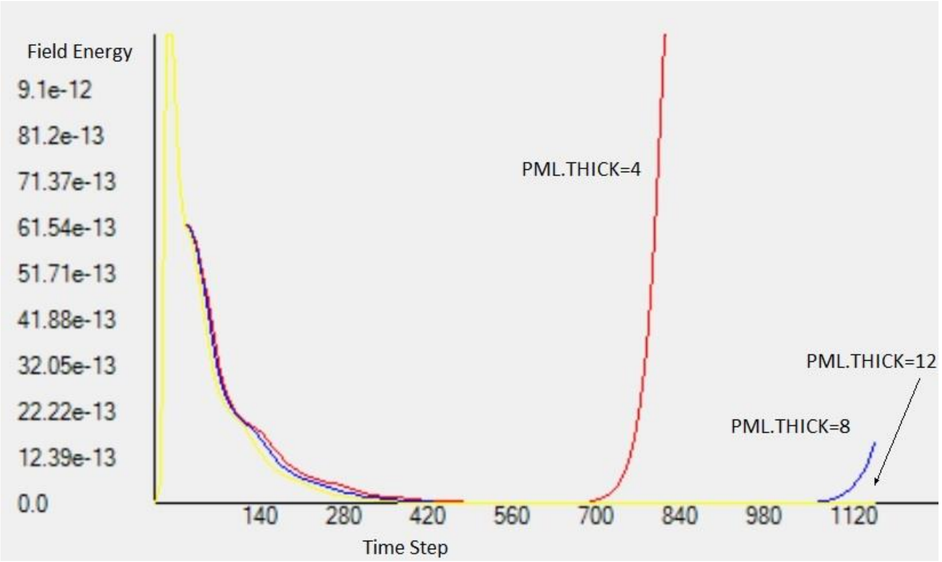
A statistics file, R21_84_NB_PML12_0_01_TSS_smax2_kmax1_statistic0.txt, is generated:

Time	Energy	Max Divg E	Avg Divg E	Max Divg H	Avg Divg H
0	4.15903e-18	1.35525e-18	3.0873e-24	0	0
10	4.48792e-13	8.44379e-19	1.92352e-24	3.73025e-24	8.49761e-30
20	1.93781e-11	9.04395e-16	2.06024e-21	2.57997e-20	5.87725e-26
30	2.24625e-11	5.7396e-16	1.3075e-21	4.52229e-20	1.03019e-25
40	7.1026e-12	1.22173e-16	2.78313e-22	1.52818e-20	3.48124e-26
50	5.99744e-12	2.09198e-16	4.76559e-22	2.90655e-20	6.62121e-26
60	5.73349e-12	2.14749e-16	4.89204e-22	2.83734e-20	6.46355e-26
70	5.11597e-12	2.14187e-16	4.87924e-22	2.8356e-20	6.45957e-26
80	4.30463e-12	2.14532e-16	4.8871e-22	2.8328e-20	6.4532e-26
90	3.42277e-12	2.14598e-16	4.8886e-22	2.83804e-20	6.46514e-26
100	2.71487e-12	2.2364e-16	5.09459e-22	2.87222e-20	6.543e-26
110	2.28894e-12	2.68348e-16	6.11305e-22	2.96308e-20	6.74998e-26
120	2.05222e-12	1.23529e-16	2.81403e-22	3.42059e-20	7.79221e-26
130	1.89184e-12	1.85955e-16	4.23612e-22	3.22627e-20	7.34953e-26
140	1.76311e-12	1.53751e-16	3.50249e-22	3.19374e-20	7.27544e-26
150	1.62619e-12	1.53253e-16	3.49115e-22	3.23352e-20	7.36604e-26
160	1.43878e-12	1.43249e-16	3.26325e-22	3.31913e-20	7.56108e-26
170	1.23108e-12	1.41255e-16	3.21784e-22	3.52797e-20	8.03682e-26
180	1.04165e-12	1.2893e-16	2.93706e-22	3.51997e-20	8.0186e-26
190	8.61203e-13	1.29694e-16	2.95446e-22	3.42958e-20	7.81269e-26
200	7.44162e-13	1.24242e-16	2.83028e-22	3.35691e-20	7.64714e-26
210	6.55679e-13	1.17846e-16	2.68457e-22	3.35989e-20	7.65392e-26
220	5.82173e-13	8.42804e-17	1.91993e-22	3.13312e-20	7.13734e-26
230	5.26925e-13	8.96647e-17	2.04259e-22	3.43866e-20	7.83336e-26
240	4.78972e-13	1.35555e-16	3.08799e-22	3.66705e-20	8.35365e-26
250	4.29601e-13	1.39414e-16	3.17589e-22	3.64997e-20	8.31473e-26
260	3.80007e-13	1.38075e-16	3.1454e-22	3.64822e-20	8.31076e-26
270	3.25491e-13	1.35688e-16	3.09101e-22	3.61851e-20	8.24308e-26
280	2.7287e-13	1.36893e-16	3.11845e-22	3.60733e-20	8.21759e-26
290	2.34684e-13	1.33027e-16	3.0304e-22	3.72851e-20	8.49365e-26
300	2.02234e-13	1.37932e-16	3.14213e-22	3.77056e-20	8.58945e-26
310	1.73939e-13	1.51993e-16	3.46245e-22	3.80887e-20	8.67671e-26
320	1.52899e-13	1.47291e-16	3.35534e-22	3.83202e-20	8.72946e-26
330	1.3627e-13	1.54093e-16	3.51028e-22	3.84137e-20	8.75075e-26
340	1.20614e-13	1.46778e-16	3.34364e-22	3.78275e-20	8.61722e-26
350	1.0502e-13	1.40615e-16	3.20326e-22	3.80223e-20	8.66159e-26
360	8.91908e-14	1.49909e-16	3.41496e-22	3.79307e-20	8.64073e-26
370	7.64813e-14	1.30644e-16	2.9761e-22	3.74556e-20	8.53248e-26
380	6.45888e-14	1.37673e-16	3.13624e-22	3.75229e-20	8.54782e-26
390	5.51815e-14	1.43151e-16	3.26102e-22	3.80078e-20	8.65828e-26
400	4.78547e-14	1.41683e-16	3.22758e-22	3.81985e-20	8.70174e-26
410	4.17053e-14	1.46341e-16	3.33369e-22	3.83604e-20	8.73861e-26
420	3.66006e-14	1.45831e-16	3.32208e-22	3.78666e-20	8.62612e-26
430	3.23113e-14	1.43596e-16	3.27115e-22	3.79398e-20	8.64279e-26
440	2.83637e-14	1.4801e-16	3.37171e-22	3.69004e-20	8.40602e-26
450	2.47426e-14	1.52976e-16	3.48484e-22	3.69258e-20	8.41181e-26
460	2.15018e-14	1.53385e-16	3.49415e-22	3.70448e-20	8.43891e-26
470	1.87411e-14	1.58154e-16	3.6028e-22	3.69572e-20	8.41896e-26
480	1.65e-14	1.51427e-16	3.44955e-22	3.70853e-20	8.44813e-26
490	1.45377e-14	1.56839e-16	3.57285e-22	3.67566e-20	8.37327e-26

500	1.26295e-14	1.67901e-16	3.82483e-22	3.67454e-20	8.3707e-26
510	1.09153e-14	1.73652e-16	3.95585e-22	3.65165e-20	8.31857e-26
520	9.73222e-15	1.65333e-16	3.76633e-22	3.63336e-20	8.2769e-26
530	8.54079e-15	1.71528e-16	3.90745e-22	3.6081e-20	8.21935e-26
540	7.38328e-15	1.65145e-16	3.76205e-22	3.61382e-20	8.2324e-26
550	6.53527e-15	1.65948e-16	3.78034e-22	3.59329e-20	8.18563e-26
560	5.80929e-15	1.66688e-16	3.7972e-22	3.5697e-20	8.13188e-26
570	5.1611e-15	1.57635e-16	3.59097e-22	3.53003e-20	8.0415e-26
580	4.55519e-15	1.63392e-16	3.72211e-22	3.53182e-20	8.04559e-26
590	3.99938e-15	1.56899e-16	3.5742e-22	3.5102e-20	7.99633e-26
600	3.56529e-15	1.58782e-16	3.6171e-22	3.53784e-20	8.05931e-26
610	3.1891e-15	1.56373e-16	3.56221e-22	3.52425e-20	8.02835e-26
620	2.85801e-15	1.59038e-16	3.62293e-22	3.53957e-20	8.06325e-26
630	2.53444e-15	1.53883e-16	3.5055e-22	3.53133e-20	8.04447e-26
640	2.25772e-15	1.58237e-16	3.6047e-22	3.53683e-20	8.057e-26
650	2.01885e-15	1.54659e-16	3.52318e-22	3.52471e-20	8.02939e-26
660	1.81423e-15	1.57342e-16	3.5843e-22	3.55667e-20	8.1022e-26
670	1.63827e-15	1.52122e-16	3.46538e-22	3.53499e-20	8.05281e-26
680	1.47892e-15	1.53083e-16	3.48727e-22	3.54079e-20	8.06602e-26
690	1.30369e-15	1.57408e-16	3.58581e-22	3.55961e-20	8.10889e-26
700	1.13232e-15	1.56056e-16	3.55499e-22	3.56542e-20	8.12213e-26
710	1.01098e-15	1.57606e-16	3.5903e-22	3.56803e-20	8.12807e-26
720	9.14347e-16	1.59369e-16	3.63047e-22	3.57201e-20	8.13715e-26
730	8.18755e-16	1.57475e-16	3.58734e-22	3.5709e-20	8.13462e-26
740	7.39249e-16	1.53392e-16	3.49431e-22	3.58997e-20	8.17805e-26
750	6.81577e-16	1.57297e-16	3.58327e-22	3.58483e-20	8.16635e-26
760	6.25483e-16	1.54224e-16	3.51326e-22	3.57099e-20	8.13481e-26
770	5.61739e-16	1.52441e-16	3.47266e-22	3.56875e-20	8.12972e-26
780	4.96665e-16	1.55168e-16	3.53477e-22	3.56275e-20	8.11604e-26
790	4.3468e-16	1.54302e-16	3.51504e-22	3.57765e-20	8.14999e-26
800	3.75321e-16	1.5304e-16	3.48631e-22	3.57124e-20	8.13539e-26
810	3.29081e-16	1.55667e-16	3.54613e-22	3.57607e-20	8.14639e-26
820	2.94325e-16	1.53484e-16	3.49642e-22	3.57558e-20	8.14527e-26
830	2.68947e-16	1.52667e-16	3.47781e-22	3.56998e-20	8.13252e-26
840	2.51258e-16	1.4993e-16	3.41545e-22	3.56605e-20	8.12357e-26
850	2.35682e-16	1.51037e-16	3.44067e-22	3.56677e-20	8.12521e-26
860	2.21477e-16	1.52317e-16	3.46983e-22	3.57302e-20	8.13945e-26
870	2.04562e-16	1.51601e-16	3.45351e-22	3.58157e-20	8.15893e-26
880	1.82676e-16	1.53939e-16	3.50676e-22	3.58064e-20	8.1568e-26
890	1.60086e-16	1.55837e-16	3.55002e-22	3.57469e-20	8.14325e-26
900	1.37886e-16	1.54731e-16	3.52482e-22	3.57246e-20	8.13818e-26
910	1.19071e-16	1.54134e-16	3.51122e-22	3.57445e-20	8.1427e-26
920	1.07612e-16	1.54085e-16	3.51011e-22	3.57949e-20	8.15417e-26
930	1.01089e-16	1.53474e-16	3.49618e-22	3.58164e-20	8.15908e-26
940	9.69533e-17	1.55129e-16	3.53388e-22	3.57854e-20	8.15202e-26
950	9.4119e-17	1.52924e-16	3.48365e-22	3.5812e-20	8.15807e-26
960	9.23427e-17	1.52708e-16	3.47874e-22	3.58183e-20	8.15951e-26
970	9.07734e-17	1.53145e-16	3.48869e-22	3.57951e-20	8.15422e-26
980	8.80356e-17	1.52512e-16	3.47427e-22	3.58112e-20	8.1579e-26
990	8.60238e-17	1.51816e-16	3.45842e-22	3.58156e-20	8.1589e-26
1000	8.5599e-17	1.53563e-16	3.49822e-22	3.5818e-20	8.15945e-26
1010	8.67098e-17	1.53427e-16	3.4951e-22	3.58178e-20	8.1594e-26
1020	9.0735e-17	1.50991e-16	3.43962e-22	3.58592e-20	8.16884e-26
1030	9.71329e-17	1.51926e-16	3.46093e-22	3.58559e-20	8.16808e-26
1040	1.05483e-16	1.55417e-16	3.54044e-22	3.58207e-20	8.16006e-26
1050	1.15685e-16	1.52447e-16	3.47279e-22	3.58065e-20	8.15683e-26
1060	1.27817e-16	1.53712e-16	3.50161e-22	3.58362e-20	8.1636e-26
1070	1.42141e-16	1.53797e-16	3.50354e-22	3.58314e-20	8.1625e-26
1080	1.58511e-16	1.53428e-16	3.49513e-22	3.58124e-20	8.15816e-26
1090	1.77166e-16	1.52322e-16	3.46994e-22	3.57675e-20	8.14794e-26
1100	1.98916e-16	1.52725e-16	3.47911e-22	3.57549e-20	8.14508e-26
1110	2.2388e-16	1.54143e-16	3.51141e-22	3.57353e-20	8.14061e-26
1120	2.52294e-16	1.52241e-16	3.46809e-22	3.57294e-20	8.13926e-26
1130	2.84632e-16	1.50179e-16	3.42113e-22	3.57238e-20	8.13798e-26
1140	3.21212e-16	1.54391e-16	3.51708e-22	3.57623e-20	8.14674e-26
1150	3.62639e-16	1.53311e-16	3.49246e-22	3.57533e-20	8.1447e-26

1160	4.09492e-16	1.51521e-16	3.45169e-22	3.57345e-20	8.14043e-26
1170	4.62987e-16	1.51221e-16	3.44486e-22	3.57207e-20	8.13728e-26
1180	5.24729e-16	1.51792e-16	3.45786e-22	3.57505e-20	8.14407e-26
1190	5.96597e-16	1.51093e-16	3.44194e-22	3.57232e-20	8.13784e-26
1200	6.81759e-16	1.51308e-16	3.44683e-22	3.57299e-20	8.13939e-26

Graphically display the field energies shown in the above 3 statistics file, we can see clearly the effects of the absorbing layer thickness:



Compare TSS algorithm with YEE algorithm

Analytical Comparisons

Before comparing the simulation results of the Yee algorithm and the TSS algorithm, let's first compare the two algorithms analytically.

- Advantages of the Yee algorithm
  - It is simple
  - It achieves the second order estimation of half time step: $O\left(\left(\frac{\Delta_t}{2}\right)^2\right)$
- Advantage of the TSS algorithm
  - The fields data are synchronized both in time and in space
    - Easy to apply boundary conditions and absorbing layers
    - Easy to do field data analysis and statistics
  - It can be of arbitrary estimation order: $O\left(\Delta_t^{2h}\right); h = 1,2,3, \ldots$

Let's examine the 4-the order TSS algorithm, $h = 2$ the estimation order is $O(\Delta_t^4)$

For the 4-th order TSS algorithm to be more accurate than the Yee algorithm, we need to have

$$\Delta_t^4 < \left(\frac{\Delta_t}{2}\right)^2 \rightarrow \Delta_t^2 < \frac{\Delta_t}{2} \rightarrow \Delta_t < \frac{1}{2}$$

The 4-th order TSS needs about 10 times the calculations of the Yee algorithm. To reduce the calculations, we may increase the space step size. Let's examine it in details.

For simplicity, assume the computing domain is a cube, assume there are N_{yee} cells on each axis. There are totally N_{yee}^3 cells. Total simulation time is given below:

$$t_{yee} = N_{yee}^3 t_{yeeOneCell} t_{yeeTotalSteps}$$
$$t_{timeSimulated} = t_{yeeTotalSteps} \Delta_t$$

$$t_{yee}\ is\ the\ calculation\ time\ needed\ to\ simulate\ a\ real\ time\ of\ t_{timeSimulated}$$

For the TSS algorithm, we start with the same Δ_s:

$$N_{tss} = N_{yee}$$

Assuming both algorithms use the same Courant number and thus Δ_t is the same, so, we have

$$t_{tss} = N_{tss}^3 t_{tssOneCell} t_{tssTotalSteps}$$

$$t_{timeSimulated} = t_{tssTotalSteps}\Delta_t$$

$$t_{tss}\ is\ the\ calculation\ time\ needed\ to\ simulate\ a\ real\ time\ of\ t_{timeSimulated}$$

Because 4-th order TSS needs 10 times more calculations than the Yee algorithm

$$t_{tssOneCell} = 10t_{yeeOneCell}$$

We have

$$t_{tss} = N_{tss}^3 10t_{yeeOneCell} t_{tssTotalSteps}$$

$$t_{timeSimulated} = t_{tssTotalSteps}\Delta_t$$

Now let's increase the space step size by a factor n :

$$N_{tss} = \frac{1}{n} N_{yee}$$

The time step size is also increased by the same factor to keep the same Courant number:

$$t_{timeSimulated} = nt_{tssTotalSteps}\Delta_t = t_{yeeTotalSteps}\Delta_t$$

$$t_{tssTotalSteps} = \frac{1}{n}\ t_{yeeTotalSteps}$$

We get total simulation time for the 4-th order TSS algorithm:

$$t_{tss} = \frac{1}{n^3} N_{yee}^3 10t_{yeeOneCell} \frac{1}{n}\ t_{yeeTotalSteps} = \frac{10}{n^4} N_{yee}^3 t_{yeeOneCell} t_{yeeTotalSteps} = \frac{10}{n^4} t_{yee}$$

For the 4-th order TSS algorithm to use less time to finish the simulation, we need to have

$$\frac{10}{n^4} < 1$$

We can see that if we double the size of the space step size, the above condition is satisfied:

$$n = 2 \rightarrow \frac{10}{n^4} = \frac{10}{16} < 1$$

The estimation orders of the Yee algorithm and 4-th order TSS algorithm are given below.

$$Yee: O\left(\left(\frac{\Delta_t}{2}\right)^2\right)\ vs\ 4 - th\ order\ TSS: O((n\Delta_t)^4)$$

Or

$$\left(\frac{\Delta_t}{2}\right)^2\ vs\ (n\Delta_t)^4 \rightarrow 1\ vs\ 4n^4\Delta_t^2$$

For n=2, the estimation orders for the Yee algorithm and the 4-th order TSS are given by

$$n = 2 \rightarrow Yee: O\left(\left(\frac{\Delta_t}{2}\right)^2\right); 4 - th\ order\ TSS: O((2\Delta_t)^4)$$

$$4 - th\ order\ TSS: O((2\Delta_t)^4)\ vs\ Yee: O\left(\left(\frac{\Delta_t}{2}\right)^2\right) \rightarrow \frac{(2\Delta_t)^4}{\left(\frac{\Delta_t}{2}\right)^2} = 64\Delta_t^2$$

To make the 4-th order TSS more accurate than the Yee algorithm, we need to have

$$64\Delta_t^2 < 1 \rightarrow \Delta_t < \frac{1}{8} = 0.125$$

In electromagnetic field simulations, the time step size is much smaller than 0.125. That means, **the 4-th order TSS may get much more accurate simulations using shorter simulation time**.

Note that if we use larger n then we may get much shorter simulation time. Let's examine the cases of larger n.

Assume $\Delta_t = 10^{-9}$	Speed advantage of 4-th order TSS over Yee	Accuracy advantage of 4-th order TSS over Yee
$n = 2$	$\frac{10}{n^4} = \frac{1}{1.6} \rightarrow TSS\ is\ 1.6\ times\ faster$	$4n^4\Delta_t^2 = 6410^{-18}$ $\rightarrow TSS\ is\ 10^{17} times\ more\ accurate$
$n = 6$	$\frac{10}{n^4} = \frac{10}{1296}$ $\rightarrow TSS\ is\ 129.6\ times\ faster$	$4n^4\Delta_t^2 = 14410^{-18}$ $\rightarrow TSS\ is\ 10^{16} times\ more\ accurate$
$n = 10$	$\frac{10}{n^4} = \frac{1}{10^{-4}}$ $\rightarrow TSS\ is\ 10{,}000\ times\ faster$	$4n^4\Delta_t^2 = 410^{-14}$ $\rightarrow TSS\ is\ 10^{14} times\ more\ accurate$

We can see that for the case of $\Delta_t = 10^{-9}$, the speed and accuracy advantages of the 4-th order TSS over the Yee algorithm are enormous.
Are these advantages too good to be true? Well, next, we'll use simulation data to test.

Simulation using Yee algorithm

The task file task_gaussianYEE240.task is for this Yee algorithm simulation.

```
//set initial E-field to Gaussian function
//run Yee algorithm

//material properties
TSS.IS_VACUUM=True

//maximum time steps
SIM.MAXTIMES=120

//time steps before saving fields to files
TSS.REC_INTERVAL=2

//generate a statistics file on finishing the simulation
TSS.MAKESTATISTICS=true


//plugins ------------------------
TSS.TIME.CLASS=TimeYee

TSS.SOURCE.CLASS=FieldSourceTss

TSS.BOUNDARY.CLASS=BoundaryTssVoid

TSS.FIELDSETTER.CLASS=GaussianCurlYee
GAUSSIAN.MAGNITUDE=120
GAUSSIAN.EXPO=0.5
//---------------------------------

//subfolder under /D for read/write space matrix file
TSS.MATRIXFILEFOLDER=Matrix

TSS.TIMEMATRIXFILE=timeMatrix.txt
TSS.SPACEMATRIXFILE=spacematrix.txt

//sub folder under folder specified  by command line parameter /D
TSS.DATAFOLDER=G_YEE

//forming E field file name: <BASEFILE>_TSS_smax<smax>_kmax<kmax>_e_<file index>.dat
//forming H field file name: <BASEFILE>_TSS_smax<smax>_kmax<kmax>_h_<file index>.dat
//<file index>=0,1,2,...
TSS.REC_BASEFILE=G_YEE

//computation domain -------------
TSS.NX=240
TSS.NY=240
```

```
TSS.NZ=240
//XMIN = -DS*(NX/2)
TSS.XMIN=-1.8
TSS.YMIN=-1.8
TSS.ZMIN=-1.8
//--------------------------------
//space step size -XMIN/(NX/2)
TSS.DS=0.015

//time step size
TSS.DT=2.4e-11

//PML
PML.DISABLE=true

//for task 370-----------------------------------------------------------------
//from all the data files generate a file containing E and H at following point
POINT.GRIDNODES=(120,120,120),(140,140,140),(160,160,160)
//"e" for collecting Ex, Ey and Ez;"h" for collecting Hx, Hy and Hz
POINT.EH=e
//1 for x, 2 for y, 3 for z
FIELD.COMPONENT=1
//-----------------------------------------------------------------------------
```

Use TSS.TIME.CLASS=TimeYee to tell the simulator to use the Yee algorithm; use TSS.FIELDSETTER.CLASS=GaussianCurlYee to tell the simulator to initialize the field according to the Yee cell space arrangement.

Modify the file em.env to use the above task file:

```
/K=300
/T=./taskFiles/task_gaussianYEE240.task
```

It took 428350 milliseconds to finish the simulation.

A simulation statistics file, G_YEE_TSS_smax1_kmax0_statistic0.txt, is generated:

Time	Energy	Max Divg E	Avg Divg E	Max Divg H	Avg Divg H
0	0.0314235	0.00245088	0.000592815	0	0
2	0.0314052	0.0024481	0.000592088	3.53017e-07	1.06293e-07
4	0.0313767	0.240146	0.00171639	1.63696e-06	2.18452e-07
6	0.0313182	1.13689	0.0070805	2.96496e-05	4.68663e-07
8	0.031194	1.0427	0.0112336	0.00013093	1.09824e-06
10	0.0310948	2.75338	0.0168631	0.000648506	1.85963e-06
12	0.0310792	7.488	0.0248972	0.000910093	2.14237e-06
14	0.0310492	7.81628	0.0257887	0.00118323	2.68822e-06
16	0.0309952	9.79863	0.0298947	0.00218402	2.95631e-06
18	0.0309491	11.8299	0.0346508	0.00247368	3.42037e-06
20	0.0309147	12.4404	0.036908	0.00422472	3.69727e-06
22	0.0308729	15.6237	0.0436014	0.00419625	3.98689e-06
24	0.0308284	14.661	0.0447952	0.00476505	4.29861e-06
26	0.0307927	12.7566	0.0488574	0.00388489	4.47169e-06
28	0.0307562	11.4283	0.0514195	0.00477168	4.82051e-06
30	0.0307168	11.3627	0.0507364	0.00326922	5.14912e-06
32	0.0306828	10.214	0.0545633	0.00377339	5.42635e-06
34	0.0306508	9.85215	0.0573915	0.00308495	5.7642e-06
36	0.0306171	9.16181	0.0604458	0.0028754	5.96109e-06
38	0.0305866	8.6458	0.0640366	0.00295841	6.22084e-06
40	0.0305593	8.20951	0.0652262	0.00229686	6.46325e-06
42	0.0305315	8.37626	0.0677762	0.00258558	6.57755e-06
44	0.0305058	8.2847	0.0693918	0.00222122	6.80599e-06
46	0.0304836	8.068	0.0688504	0.00220241	7.06094e-06
48	0.0304621	8.02786	0.0705843	0.00220328	7.26022e-06
50	0.0304419	7.82039	0.0725289	0.00208674	7.5104e-06
52	0.0304252	7.78062	0.0755589	0.0021246	7.70888e-06
54	0.03041	7.43931	0.0780681	0.00186469	7.91477e-06
56	0.0303958	7.20889	0.0801873	0.00198945	8.09095e-06
58	0.030385	6.80223	0.0829834	0.00191451	8.19317e-06
60	0.030376	6.70542	0.0837447	0.00171939	8.35883e-06

62	0.030368	6.6415	0.0837781	0.00177974	8.52919e-06
64	0.030363	6.56604	0.0856515	0.00153181	8.66468e-06
66	0.0303602	6.43174	0.0869435	0.00166725	8.85891e-06
68	0.0303584	6.40664	0.0880006	0.00155458	9.04808e-06
70	0.0303591	6.21067	0.0910701	0.00148478	9.17071e-06
72	0.0303621	6.08754	0.0924246	0.00151988	9.28408e-06
74	0.0303661	5.95584	0.0940575	0.00128761	9.37915e-06
76	0.0303723	5.77678	0.0957194	0.00145247	9.49329e-06
78	0.0303806	5.63937	0.096044	0.00133402	9.61014e-06
80	0.03039	5.57734	0.0969776	0.00124911	9.69981e-06
82	0.0304011	5.51147	0.0977753	0.00130355	9.85026e-06
84	0.0304141	5.4894	0.0988591	0.00117874	1.00011e-05
86	0.0304282	5.3499	0.101837	0.00123614	1.00622e-05
88	0.0304435	5.25763	0.103188	0.001097	1.01915e-05
90	0.0304605	5.31171	0.104126	0.00109934	1.02909e-05
92	0.0304784	5.07699	0.106066	0.00105806	1.03232e-05
94	0.0304971	4.9907	0.106404	0.00100221	1.03732e-05
96	0.0305172	4.97377	0.106477	0.00102291	1.04445e-05
98	0.030538	4.92033	0.10734	0.000996505	1.05485e-05
100	0.0305592	4.85463	0.10869	0.000977612	1.06518e-05
102	0.0305814	4.78313	0.110226	0.000976263	1.07004e-05
104	0.0306041	4.69904	0.111686	0.000917419	1.07957e-05
106	0.0306268	4.61654	0.112854	0.000952324	1.0863e-05
108	0.0306501	4.52455	0.114954	0.000884466	1.08868e-05
110	0.0306735	4.52199	0.115262	0.000872744	1.09348e-05
112	0.0306966	4.48215	0.115376	0.000930041	1.10284e-05
114	0.0307199	4.46242	0.116252	0.000863584	1.10872e-05
116	0.0307431	4.41262	0.11681	0.000933551	1.11581e-05
118	0.0307656	4.34568	0.117602	0.000795025	1.1244e-05
120	0.0307878	4.48266	0.119119	0.000819122	1.13371e-05

The divergence values indicate how accurate the simulation is performed. We cannot directly use the field values to calculate the divergences because the field components are located at different space points due to the Yee algorithm cell arrangement. Field value averages are used to do the calculations. We will compare the simulation accuracy with the next simulation.

Simulation using 4-th order TSS algorithm

The task file task_gaussianTSS120.task is used for this simulation.

```
//set initial E-field to Gaussian function

//material properties
TSS.IS_VACUUM=True

//maximum time steps
SIM.MAXTIMES=60

//time steps before saving fields to files
TSS.REC_INTERVAL=1

//generate a statistics file on finishing the simulation
TSS.MAKESTATISTICS=true


//plugins ------------------------
//TSS.TIME.CLASS=TimeTssMultiThreads
//SIM.USETHREADS=4

//use single thread
TSS.TIME.CLASS=TimeTss

TSS.SOURCE.CLASS=FieldSourceTss

//TSS.BOUNDARY.CLASS=BoundaryTssVoid

//use PEMC
```

```
TSS.BOUNDARY.CLASS=BoundaryTss

TSS.FIELDSETTER.CLASS=GaussianCurl
GAUSSIAN.MAGNITUDE=120
GAUSSIAN.EXPO=0.5
//----------------------------------

//subfolder under /D for read/write space matrix file
TSS.MATRIXFILEFOLDER=Matrix

TSS.TIMEMATRIXFILE=timeMatrix.txt
TSS.SPACEMATRIXFILE=spacematrix.txt

//sub folder under folder specified by command line parameter /D
TSS.DATAFOLDER=G_TSS

//forming E field file name: <BASEFILE>_TSS_smax<smax>_kmax<kmax>_e_<file index>.dat
//forming H field file name: <BASEFILE>_TSS_smax<smax>_kmax<kmax>_h_<file index>.dat
//<file index>=0,1,2,...
TSS.REC_BASEFILE=G_TSS

//time estimation order = 2*(kmax+1)
TSS.KMAX=1
//space estimation order = 2*smax
TSS.SMAX=2

//computation domain --------------
TSS.NX=120
TSS.NY=120
TSS.NZ=120
//XMIN = -DS*(NX/2)
TSS.XMIN=-1.8
TSS.YMIN=-1.8
TSS.ZMIN=-1.8
//--------------------------------
//space step size -XMIN/(NX/2)
TSS.DS=0.03

//time step size
TSS.DT=4.8e-11

//use RAM instead of hard disk
SIM.USE_RAM=true

//PML
PML.DISABLE=true

//for task 370------------------------------------------------------------------
//from all the data files generate a file containing E and H at following point
POINT.GRIDNODES=(60,60,60),(70,70,70),(80,80,80)
//"e" for collecting Ex, Ey and Ez;"h" for collecting Hx, Hy and Hz
POINT.EH=e
//1 for x, 2 for y, 3 for z
FIELD.COMPONENT=1
//------------------------------------------------------------------------------
```

In the task file, TSS.TIME.CLASS=TimeTss is used to tell the simulator to use the TSS algorithm. The reason to use TimeTss instead of TimeTssMultiThreads is that TimeYee is a single thread implementation, but TimeTssMultiThreads runs multiple threads, it is not fair for Yee algorithm. TimeTss runs TSS algorithm under single thread.

Compare with the task file for the Yee algorithm, we can see that the number of grids on one axis is half the value, the space step size and the time step size are doubled. That is, $n = 2$

Modify the file em.env to use the above task file to run the simulation:

```
/K=300
```

/T=./taskFiles/task_gaussianTSS120.task

It took 147385 milliseconds to finish the simulation.

A simulation statistics file, G_TSS_TSS_smax1_kmax0_statistic0.txt, is generated:

Time	Energy	Max Divg E	Avg Divg E	Max Divg H	Avg Divg H
0	0.00366389	6.39282e-06	1.72655e-06	0	0
1	0.00366322	6.39282e-06	1.72655e-06	2.00634e-16	2.41368e-17
2	0.00366148	6.39282e-06	1.72655e-06	3.19162e-16	3.4708e-17
3	0.00365896	6.39282e-06	1.72655e-06	4.21032e-16	4.35929e-17
4	0.00365548	6.39282e-06	1.72655e-06	5.28774e-16	5.15498e-17
5	0.00365012	6.39282e-06	1.72655e-06	5.95408e-16	5.94949e-17
6	0.00364142	6.39282e-06	1.72655e-06	6.17995e-16	6.73427e-17
7	0.00362804	6.39282e-06	1.72655e-06	7.26415e-16	7.52343e-17
8	0.00360949	6.39282e-06	1.72655e-06	9.18184e-16	8.31073e-17
9	0.00358645	6.39282e-06	1.72655e-06	1.00244e-15	9.15917e-17
10	0.00356038	6.39282e-06	1.72655e-06	9.84817e-16	1.00598e-16
11	0.00353281	6.39282e-06	1.72655e-06	1.01734e-15	1.10244e-16
12	0.00350471	6.39282e-06	1.72655e-06	1.12757e-15	1.20051e-16
13	0.00347649	6.39282e-06	1.72655e-06	1.2902e-15	1.3029e-16
14	0.00344819	6.39282e-06	1.72655e-06	1.42573e-15	1.40725e-16
15	0.00341981	6.39282e-06	1.72655e-06	1.56306e-15	1.51157e-16
16	0.00339136	6.39282e-06	1.72655e-06	1.73834e-15	1.61641e-16
17	0.00336286	6.39282e-06	1.72655e-06	1.81604e-15	1.72154e-16
18	0.00333429	6.39282e-06	1.72655e-06	1.84495e-15	1.83175e-16
19	0.00330565	6.39282e-06	1.72655e-06	1.89735e-15	1.94523e-16
20	0.00327695	6.39282e-06	1.72655e-06	2.29851e-15	2.06018e-16
21	0.00324825	6.39282e-06	1.72655e-06	2.39247e-15	2.17897e-16
22	0.00321964	6.39282e-06	1.72655e-06	2.8117e-15	2.29768e-16
23	0.00319119	6.39282e-06	1.72655e-06	3.14419e-15	2.41721e-16
24	0.00316296	6.39282e-06	1.72655e-06	3.18755e-15	2.53877e-16
25	0.00313497	6.39282e-06	1.72655e-06	3.15141e-15	2.65937e-16
26	0.00310722	6.39282e-06	1.72655e-06	3.15141e-15	2.77918e-16
27	0.00307973	6.39282e-06	1.72655e-06	3.27068e-15	2.90148e-16
28	0.00305254	6.39282e-06	1.72655e-06	3.37548e-15	3.02468e-16
29	0.00302571	6.39282e-06	1.72655e-06	3.73237e-15	3.14676e-16
30	0.00299931	6.39282e-06	1.72655e-06	3.65738e-15	3.26748e-16
31	0.00297337	6.39282e-06	1.72655e-06	3.74411e-15	3.38539e-16
32	0.00294792	6.39282e-06	1.72655e-06	3.78748e-15	3.50365e-16
33	0.00292298	6.39282e-06	1.72655e-06	3.84214e-15	3.6189e-16
34	0.00289856	6.39282e-06	1.72655e-06	4.35126e-15	3.73775e-16
35	0.00287466	6.39282e-06	1.72655e-06	4.43213e-15	3.85267e-16
36	0.00285127	6.39282e-06	1.72655e-06	4.66207e-15	3.96417e-16
37	0.00282845	6.39282e-06	1.72655e-06	4.64039e-15	4.07585e-16
38	0.00280629	6.39282e-06	1.72655e-06	4.42354e-15	4.18594e-16
39	0.00278487	6.39282e-06	1.72655e-06	4.88614e-15	4.29365e-16
40	0.00276421	6.39282e-06	1.72655e-06	4.77049e-15	4.39797e-16
41	0.00274431	6.39282e-06	1.72655e-06	4.89337e-15	4.50024e-16
42	0.00272515	6.39282e-06	1.72655e-06	5.0307e-15	4.60054e-16
43	0.00270668	6.39282e-06	1.72655e-06	5.30536e-15	4.69643e-16
44	0.00268887	6.39282e-06	1.72655e-06	5.53666e-15	4.79024e-16
45	0.00267169	6.39282e-06	1.72655e-06	5.66676e-15	4.88117e-16
46	0.00265518	6.39282e-06	1.72655e-06	5.72459e-15	4.96909e-16
47	0.00263943	6.39282e-06	1.72655e-06	6.20164e-15	5.05949e-16
48	0.00262452	6.39282e-06	1.72655e-06	5.92697e-15	5.14798e-16
49	0.00261048	6.39282e-06	1.72655e-06	6.08599e-15	5.23326e-16
50	0.0025973	6.39282e-06	1.72655e-06	6.04262e-15	5.3169e-16
51	0.00258491	6.39282e-06	1.72655e-06	5.97034e-15	5.39926e-16
52	0.00257322	6.39282e-06	1.72655e-06	5.90529e-15	5.47832e-16
53	0.00256216	6.39282e-06	1.72655e-06	5.78241e-15	5.55589e-16
54	0.00255164	6.39282e-06	1.72655e-06	6.10044e-15	5.62936e-16
55	0.00254167	6.39282e-06	1.72655e-06	6.13658e-15	5.70413e-16
56	0.0025323	6.39282e-06	1.72655e-06	6.04262e-15	5.7735e-16
57	0.00252362	6.39282e-06	1.72655e-06	6.2703e-15	5.84279e-16
58	0.00251568	6.39282e-06	1.72655e-06	6.37511e-15	5.9089e-16

59	0.00250847	6.39282e-06	1.72655e-06	6.33264e-15	5.97186e-16
60	0.00250189	6.39282e-06	1.72655e-06	6.58472e-15	6.03569e-16

Comparison of the simulation results

First, let's examine the simulation speed. Yee used 428350 milliseconds and 4-th order TSS used 147385 milliseconds. The TSS was almost 3 times faster.

Let's compare the two statistics files. Each file contain 60 lines of statistics. Each line is for one time step. The first column is the time step value. The first column in Yee file should be twice the value of the TSS file because n=2.

The second column is the field energy summary. Note that the energy in the Yee file is much larger than the values in the TSS file. It does not mean that the Yee simulation produces larger energy values. The reason is that the Yee simulation uses many more space points. The energy value is a summation of the energy at all space points.

The next 4 columns are the maximum divergences and the average divergences for E and H fields respectively. We can see that the TSS produced much smaller divergences, meaning much more accurate simulations.

Let's summarize the comparison results.

Performance	Yee	4-th order TSS	Result
Simulation time	428350	147385	TSS is 3 times faster
Maximum E divergence	4.5	6.4e-6	TSS is 1,000,000 times more accurate
Average E divergence	0.12	1.7e-6	TSS is 100,000 times more accurate
Maximum H divergence	0.00082	6.6e-15	TSS is 10^{11} times more accurate
Average H divergence	1.1e-05	6e-16	TSS is 10^{11} times more accurate

The numeric results well confirmed the previous analytical comparisons.

We have seen that the TSS is so good in both speed and accuracy. What is the disadvantage of the TSS? Well, for one thing, it is much more complicated to code than the Yee algorithm. If you download my source code then you will still see some experimental coding. I still cannot say I optimized the code. Sorry about that. Hope you may help to optimize it to make it run faster, more clean, more robust and feature rich.

Higher order Yee style simulation

The above analytical and numerical results showed that higher order estimation can produce faster and more accurate simulations. The TSS algorithm is based on the time-space theorem I proved. I once thought that the Yee style cell arrangement blocks the higher order estimation. I have been thinking about it for sometimes, and now I believe that the Yee style cell arrangement can also benefit from the time-space theorem. I'll explain it.

By the time-space theorem, the time advancement estimation is given by (not including field sources)

$$H(t+\Delta_t)=\sum_{k=0}^{k_{max}}\left(g(k)\nabla^{\{2k\}}\times H(t)+f(k)\nabla^{\{2k+1\}}\times E(t)\right)$$

$$E(t+\Delta_t)=\sum_{k=0}^{k_{max}}\left(w(k)\nabla^{\{2k\}}\times E(t)+u(k)\nabla^{\{2k+1\}}\times H(t)\right)$$

If the Yee style cell arrangement is used then each time field-curl is calculated, the calculated curls are located at different space locations than the field. However the curl location follows the following rules:

$$\nabla^{\{1\}} \times E(t) \rightarrow located\ at\ H(t)$$
$$\nabla^{\{1\}} \times H(t) \rightarrow located\ at\ E(t)$$

The above rule is why the Yee algorithm can work. Now let's go higher orders:

$$\nabla^{\{2\}} \times E(t) \rightarrow located\ at\ E(t)$$
$$\nabla^{\{2\}} \times H(t) \rightarrow located\ at\ H(t)$$
$$\nabla^{\{3\}} \times E(t) \rightarrow located\ at\ H(t)$$
$$\nabla^{\{3\}} \times H(t) \rightarrow located\ at\ E(t)$$
$$\dots\dots$$
$$\nabla^{\{2k\}} \times E(t) \rightarrow located\ at\ E(t)$$
$$\nabla^{\{2k\}} \times H(t) \rightarrow located\ at\ H(t)$$
$$\nabla^{\{2k+1\}} \times E(t) \rightarrow located\ at\ H(t)$$
$$\nabla^{\{2k+1\}} \times H(t) \rightarrow located\ at\ E(t)$$

We can see that the time advancement estimation formulas derived from the time-space theorem still work for the Yee style cell arrangement due to the above curl location rules.

Question and suggestions?

Thank you for getting this far and reaching here. Please contact me if you have questions and suggestions.

For more information, see

https://github.com/DavidGeUSA/TSS/blob/master/GenericFdtdFullFormat.pdf

https://github.com/DavidGeUSA/TSS/blob/master/GenericFDTD.pdf

https://github.com/DavidGeUSA/TSS/tree/master/Source%20Code%20V2