

Nonlinear Systems and Control, Homework #3

Due May 19, 2023

Exercise #1: Controllability of a Tricycle

Consider the following system on $\mathcal{X} = \mathbb{R}^2 \times \mathbb{S}^1 \times \mathbb{S}^1$:

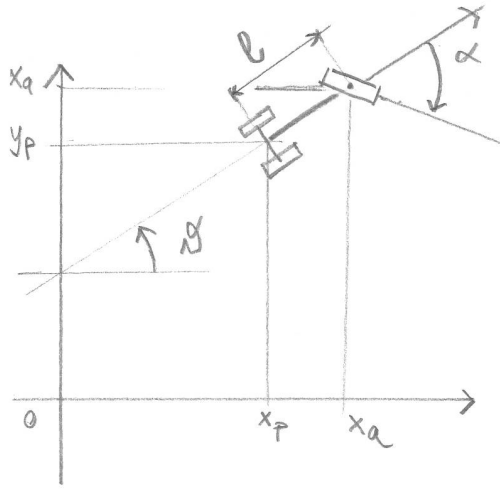


Figure 1: Depiction of the tricycle system.

$$\begin{aligned}\dot{x}_p &= v \cos(\vartheta) \\ \dot{y}_p &= v \sin(\vartheta) \\ \dot{\vartheta} &= \frac{1}{\ell} \tan(\alpha) v. \\ \dot{\alpha} &= \omega,\end{aligned}$$

where ℓ is the “length” of the tricycle, θ the angle of the body with respect to the x axis, α the steering angle. Assume we can control the linear speed v and the steering speed $\omega = \dot{\alpha}$, choosing their value in \mathbb{R} .

- What type of system is it: TI/TV? driftless or with drift? Control-affine?
- Find the linearized system around $(0, 0, 0, 0)$, for $v = 0, \omega = 0$ and discuss if any controllability property can be assessed via linearization.

- c. Discuss its accessibility, STLC at $(0, 0, 0, 0)$ and global controllability using the nonlinear results we have seen in class.

Exercise #2: Accessibility with broken actuators

Consider the following dynamical system on the open unit ball in \mathbb{R}^3 , namely $X = \mathcal{N}_1(0) \subsetneq \mathbb{R}^3$:

$$\begin{aligned}\dot{x}_1 &= a_1 x_2 x_3 + u_1 \\ \dot{x}_2 &= a_2 x_3 x_1 + u_2 \\ \dot{x}_3 &= a_3 x_1 x_2,\end{aligned}$$

where the controls are $u_{1,2} \in [-B, B]$ with $B > 2$, $0 < a_i < 1$.

- Prove that the system is globally accessible;
- If the actuator of the control u_2 breaks down, imposing a saturated constant control $u_2 = B$, is the system still accessible?

Rules and honor system: The instructor will evaluate one or more exercises of his choice. The total of the homeworks will account for 60% of your final evaluation. If you skip one (or more) homeworks, that fraction of the 60% will be evaluated zero. The agreement is the following: you can work on the homework until the beginning of class on the due day. HWs that have not been handed over to the instructor at the beginning of class or emailed to him before the beginning of class will not be evaluated. The students are required to work on the HWs alone. The course lecture notes or other material can be consulted, but the solution should not be copied from another source. In case of suspect violation of the honor system, the instructor can suspend grading using HWs either for the concerned students or the whole class, to his discretion.