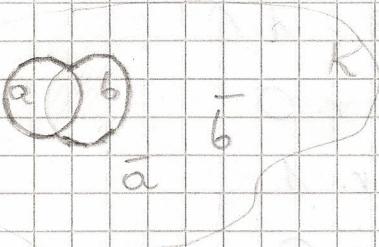


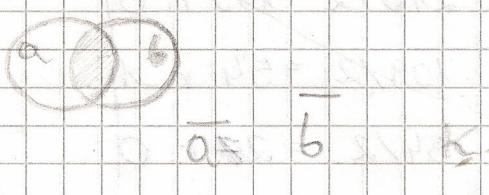
# PRACA DOMOWA 1 (PUL)

Zilustrować na zbiorach

$$9) \overline{a+b} = \overline{a} \cdot \overline{b}$$



$$10) \overline{a \cdot b} = \overline{a} + \overline{b}$$



Udowodnić:

$$13) a + ab = a$$

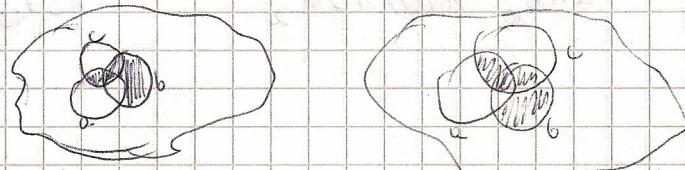
$$a + ab \stackrel{6.}{=} a(i + b) \stackrel{8+}{=} a \cdot i \stackrel{4.}{=} a$$

$$14) a(a+b) = a$$

$$a(a+b) = a \cdot a + ab \stackrel{6.}{=} a + ab \stackrel{13+}{=} a$$

Pokażć na zbiorach:

$$15) (a+b)(\bar{a}+c) = ac + \bar{a}b$$



Prelsztatać ilorzyń sumę na sumę ilorzyń:

$$(A+B+\bar{C})(A+\bar{B}+D)(A+B+E)(A+\bar{D}+E)(\bar{A}+C) =$$

$$(AA+AB+AD+\cancel{AB}+\cancel{BB}+\cancel{BD}+\cancel{AC}+\cancel{BC}+\cancel{CD})(AA+A\bar{D}+AE+\cancel{AB}+\cancel{BD}+\cancel{BE}+\cancel{AE}+\cancel{BD}+\cancel{EE})$$

$$\cdot (\bar{A}+C) = (A+AB+AD+\cancel{B}+\cancel{BD}+\cancel{AC}+\cancel{BC}+\cancel{CD})(A+A\bar{D}+\cancel{AB}+\cancel{BD}+\cancel{BE}+\cancel{AE}+\cancel{DE}+\cancel{E})(\bar{A}+C)$$

$$= (A+B+\bar{C}\bar{D})(A+E+\bar{B}\bar{D})(\bar{A}+C) = (A+\cancel{BE}+\cancel{BD}+\cancel{CDE}+\cancel{BCDD})(\bar{A}+C)$$

$$= (A+BE+\bar{B}\bar{D}+\bar{CDE})(A+C) = AA+AC+\bar{ABE}+\bar{BCE}+\bar{ABD}+\bar{BCD}+\bar{ACDE}+\bar{CCDE} =$$

$$= AC+\bar{ABE}+\bar{BCE}+\bar{ABD}+\bar{BCD}+\bar{ACDE}$$

Obliczyć makrymalne klasy zgodne dwoma sposobami, mając dane pary zgodne:

$$(1,2)(1,5)(1,6)(2,4)(2,5)(3,6)(4,5)(4,6)$$

2	✓
3	✗
4	✗
5	✓
6	✓
1	2
2	3
3	4
4	5
5	6

(1,2)	(1,2,5)
(1,5)	(2,4,5)
(1,6)	(1,6)
(2,4)	(3,6)
(2,5)	(4,6)
(3,6)	
(4,5)	
(4,6)	

$$(1+3)(1+4)(2+3)(2+6)(3+4)(3+5)(5+6) = (1+34)(2+36)(3+45)(5+6) =$$

$$= (12+136+234+346)(35+36+45+456) = 1235+1236+1245+1356+136+13456+2345+2346$$

$$+ \cancel{2345} + \cancel{3456} + 346 + \cancel{3456} = 1235 + 1245 + 136 + 2345 + 346$$

Dopełnienia:

(4,6), (3,6), (2,4,5), (1,6), (1,2,5) MKZ

Preliminaria do systemu dwójkowego:

$$218/2 = 109 \text{ r. } 0 \uparrow$$

$$109/2 = 54 \text{ r. } 1$$

$$54/2 = 27 \text{ r. } 0$$

$$27/2 = 13 \text{ r. } 1$$

$$13/2 = 6 \text{ r. } 1$$

$$6/2 = 3 \text{ r. } 0$$

$$3/2 = 1 \text{ r. } 1$$

$$\cancel{1}/2 = 0 \text{ r. } 1$$

$$218_{10} = 11011010_2$$

$$197/2 = 98 \text{ r. } 1 \uparrow$$

$$98/2 = 49 \text{ r. } 0$$

$$49/2 = 24 \text{ r. } 1$$

$$24/2 = 12 \text{ r. } 0$$

$$12/2 = 6 \text{ r. } 0$$

$$6/2 = 3 \text{ r. } 0$$

$$3/2 = 1 \text{ r. } 1$$

$$1/2 = 0 \text{ r. } 1$$

$$197_{10} = 11000101_2$$

# PRACA DOMOWA:

b)

$\begin{matrix} cd \\ ab \end{matrix}$	00	01	11	10
00	-	1	0	-
01	1	-	0	-
11	0	0	1	0
10	1	-	0	1

$$\begin{aligned}
 y &= (\bar{a}\bar{c}) \cdot (\bar{b}\bar{d}) + (\bar{a}\bar{b}\bar{d}) \cdot (\bar{b}\bar{c}\bar{d}) + (\bar{a}\bar{b}\bar{d}) \\
 &= (\bar{a} + (\bar{b} + \bar{d}))(\bar{b} + \bar{d}) \cdot (\bar{a} + \bar{c})(\bar{b} + \bar{c} + \bar{d}) \\
 &= (\bar{a} + \bar{b}\bar{b} + \bar{b}\bar{d} + \bar{d}\bar{d})(\bar{a}\bar{b} + \bar{a}\bar{c} + \bar{a}\bar{d} + \bar{b}\bar{c} + \bar{c}\bar{c} + \bar{c}\bar{d}) \\
 &= (\bar{a} + \bar{b}\bar{d} + \bar{b}\bar{d})(\bar{a}\bar{b} + \bar{a}\bar{c} + \bar{a}\bar{d} + \bar{b}\bar{c} + \bar{c}\bar{d}) = \\
 &= (\bar{a}\bar{a}\bar{b} + \bar{a}\bar{a}\bar{c} + \bar{a}\bar{a}\bar{d}) + \bar{a}\bar{b}\bar{c} + \bar{a}\bar{c}\bar{d} + \bar{a}\bar{b}\bar{d} + \bar{a}\bar{c}\bar{d} + \bar{b}\bar{c}\bar{d} \\
 &= \cancel{\bar{a}\bar{b}\bar{c}} + \bar{a}\bar{b} + \bar{a}\bar{c} + \bar{a}\bar{d} + \bar{b}\bar{c} + \bar{c}\bar{d}
 \end{aligned}$$

c)  $y = \bar{11}(3, 5, 7, 8, 9, 12, 13, 18, 20, 22, 24, 28) \cup (1, 2, 10, 14, 15, 16, 17, 21, 30, 31)$

$\begin{matrix} cde \\ ab \end{matrix}$	000	001	011	010	110	100	101	111
00	1	-	0	-	1	1	0	0
01	0	0	1	-	-	0	0	-
11	0	1	1	1	-	0	1	-
10	-	-	1	0	0	0	-	1

$$y = \frac{-000}{\bar{b}\bar{c}\bar{d}} + \frac{1-0-1}{a\bar{c}\bar{e}} + \frac{-101}{b\bar{c}\bar{d}} + \frac{001-0}{\bar{a}\bar{b}\bar{c}\bar{e}} + \frac{1-1-1}{a\bar{c}\bar{e}}$$

$\begin{matrix} cde \\ ab \end{matrix}$	000	001	011	010	110	100	101	111
00	1	-	0	-	1	1	0	0
01	0	0	1	-	-	0	0	-
11	0	1	1	1	-	0	1	-
10	-	-	1	0	0	0	-	1

$$\begin{aligned}
 y &= \frac{10-10}{(\bar{a} + b + \bar{d} + e)} \cdot \frac{1010-}{(\bar{a} + b + \bar{c} + d)} \cdot \frac{-1---}{\bar{b} \cdot (a + \bar{c} + \bar{e})} \\
 &\quad \cdot \frac{-11-0}{(\bar{b} + \bar{c} + e)} \cdot \frac{0-001}{(a + c + d + \bar{e})} \cdot \frac{0001-}{(a + b + \bar{c} + \bar{d})} \\
 &= (\bar{a} + (\bar{b} + \bar{d} + e))(\bar{b} + \bar{c} + d) \cdot (a + (\bar{c} + \bar{e}))(c + d + \bar{e})(\bar{b} + \bar{c} + \bar{d}) \\
 &\quad \cdot (\bar{b} + \bar{c} + e) \cdot \bar{b} = (\bar{a} + \bar{b}\bar{b} + \bar{b}\bar{c} + \bar{b}\bar{d} + \bar{b}\bar{d}\bar{d} + \bar{b}\bar{d}\bar{d}\bar{d} + \bar{b}\bar{c}\bar{e} + \bar{c}\bar{e}\bar{e}) \\
 &\quad \cdot (a + (\bar{c}\bar{c}) + \bar{c}\bar{d} + \bar{c}\bar{e} + \bar{c}\bar{e}\bar{e} + \bar{d}\bar{e} + \bar{e}\bar{e}) (\bar{b} + \bar{c} + \bar{d}) \cdot (\bar{b}\bar{b} + \bar{b}\bar{c}) \\
 &= (\bar{a} + b + \bar{c}\bar{d} + \bar{c}\bar{e} + de) (a + b\bar{c}\bar{d} + \bar{c}\bar{c}\bar{d} + \bar{c}\bar{d}\bar{d} + b\bar{e} + c\bar{e} + d\bar{e}) \cdot \bar{b} = (\bar{a}\bar{a}) + \bar{a}\bar{b}\bar{c}\bar{d} + \bar{a}\bar{b}\bar{e} + \bar{a}\bar{c}\bar{e} + \bar{a}\bar{d}\bar{e}
 \end{aligned}$$

$$\begin{aligned}
 &+ ab + b\bar{c}\bar{d} + b\bar{e} + b\bar{e}\bar{e} + b\bar{d}\bar{e} + a\bar{c}\bar{d} + (\bar{b}\bar{c}\bar{d}\bar{d}) + \bar{b}\bar{c}\bar{d}\bar{e} + (\bar{c}\bar{c}\bar{d}\bar{e}) + \bar{c}\bar{d}\bar{e} + ade + (\bar{b}\bar{c}\bar{d}\bar{d}\bar{e}) + (\bar{b}\bar{d}\bar{e}\bar{e}) + (c\bar{d}\bar{e}\bar{e}) + (\bar{d}\bar{d}\bar{e}\bar{e}) \cdot \bar{b} \\
 &= \bar{a}\bar{b}\bar{c}\bar{e} + \bar{a}\bar{b}\bar{d}\bar{e} + ab\bar{b} + b\bar{b}\bar{c}\bar{d} + b\bar{b}\bar{e} + b\bar{b}\bar{d}\bar{e} + ab\bar{c}\bar{d} + \bar{b}\bar{c}\bar{d}\bar{e} + ab\bar{d}\bar{e} = \bar{a}\bar{b}\bar{c}\bar{e} + \bar{a}\bar{b}\bar{d}\bar{e} + ab\bar{c}\bar{d} + \bar{b}\bar{c}\bar{d}\bar{e} + ab\bar{d}\bar{e}
 \end{aligned}$$

EKSPANSJA :

$$\text{II) } F = \begin{bmatrix} 100 & -0 \\ 010 & 00 \\ 010 & 10 \\ 111 & 10 \\ 011 & 01 \\ 110 & 11 \end{bmatrix}_{k_1}^{k_6} \quad R = \begin{bmatrix} 01100 \\ 00010 \\ 00110 \\ 10001 \\ 111-1 \end{bmatrix}$$

$$B(k_1, R) = \begin{bmatrix} 100 & -0 \\ 111 & 00 \\ 100 & 00 \\ 101 & 00 \\ 000 & 1 \\ 011 & 01 \end{bmatrix}_{k_1}^{k_6} \quad B(k_2, R) = \begin{bmatrix} 01000 \\ 00100 \\ 01010 \\ 01110 \\ 11001 \\ 10101 \end{bmatrix}_3^{k_6} \quad B(k_3, R) = \begin{bmatrix} 01010 \\ 00110 \\ 01010 \\ 01110 \\ 11001 \\ 10101 \end{bmatrix}_{3,4}^{k_6} \quad B(k_4, R) = \begin{bmatrix} 11110 \\ 10010 \\ 11100 \\ 11000 \\ 01111 \\ 00001 \end{bmatrix}_{1,4}^{k_6}$$

$$\begin{aligned}
 B(k_1, R) &= \begin{bmatrix} 10100 \\ 10100 \\ 10000 \\ 10100 \\ 00001 \\ 01101 \end{bmatrix}_{k_1}^{k_6} \quad B(k_2, R) = \begin{bmatrix} 01110 \\ 01110 \\ 01010 \\ 01110 \\ 11001 \\ 10101 \end{bmatrix}_{2,4}^{k_6} \quad B(k_3, R) = \begin{bmatrix} 01010 \\ 00110 \\ 01010 \\ 01110 \\ 11001 \\ 10101 \end{bmatrix}_{3,4}^{k_6} \quad B(k_4, R) = \begin{bmatrix} 11110 \\ 10010 \\ 11100 \\ 11000 \\ 01111 \\ 00001 \end{bmatrix}_{1,2}^{k_6} \\
 &B(k_1, R) = \begin{bmatrix} 3 \cdot (2k_4) \\ 3 \cdot (2k_4) + 1 \\ 3 \cdot (2k_4) + 2 \\ 3 \cdot (2k_4) + 3 \\ 3 \cdot (2k_4) + 4 \\ 3 \cdot (2k_4) + 5 \end{bmatrix}_{k_1}^{k_6} \quad B(k_2, R) = \begin{bmatrix} 3 \\ 3 + 2k_4 \\ 3 + 2k_4 + 1 \\ 3 + 2k_4 + 2 \\ 3 + 2k_4 + 3 \\ 3 + 2k_4 + 4 \end{bmatrix}_{k_1}^{k_6} \quad B(k_3, R) = \begin{bmatrix} 3 \\ 3 + 2k_4 \\ 3 + 2k_4 + 1 \\ 3 + 2k_4 + 2 \\ 3 + 2k_4 + 3 \\ 3 + 2k_4 + 4 \end{bmatrix}_{k_1}^{k_6} \quad B(k_4, R) = \begin{bmatrix} 3 \\ 3 + 2k_4 \\ 3 + 2k_4 + 1 \\ 3 + 2k_4 + 2 \\ 3 + 2k_4 + 3 \\ 3 + 2k_4 + 4 \end{bmatrix}_{k_1}^{k_6} \\
 &3 \cdot (2k_4) + 1 = 3 + 2k_4 + 1 \quad 3 \cdot (2k_4) + 2 = 3 + 2k_4 + 2 \quad 3 \cdot (2k_4) + 3 = 3 + 2k_4 + 3 \quad 3 \cdot (2k_4) + 4 = 3 + 2k_4 + 4 \\
 &3 \cdot (2k_4) + 5 = 3 + 2k_4 + 5 \quad 3 + 2k_4 + 1 = 3 + 2k_4 + 1 \quad 3 + 2k_4 + 2 = 3 + 2k_4 + 2 \quad 3 + 2k_4 + 3 = 3 + 2k_4 + 3 \\
 &3 + 2k_4 + 4 = 3 + 2k_4 + 4 \quad 3 + 2k_4 + 5 = 3 + 2k_4 + 5 \quad 3 + 2k_4 + 1 = 3 + 2k_4 + 1 \quad 3 + 2k_4 + 2 = 3 + 2k_4 + 2 \\
 &3 + 2k_4 + 3 = 3 + 2k_4 + 3 \quad 3 + 2k_4 + 4 = 3 + 2k_4 + 4 \quad 3 + 2k_4 + 5 = 3 + 2k_4 + 5 \quad 3 + 2k_4 + 1 = 3 + 2k_4 + 1 \\
 &3 + 2k_4 + 2 = 3 + 2k_4 + 2 \quad 3 + 2k_4 + 3 = 3 + 2k_4 + 3 \quad 3 + 2k_4 + 4 = 3 + 2k_4 + 4 \quad 3 + 2k_4 + 5 = 3 + 2k_4 + 5
 \end{aligned}$$

$$B(h_5, R) = \begin{bmatrix} 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 \end{bmatrix}_{15} \quad B(h_6, R) = \begin{bmatrix} 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}_{13+5}$$

$$3(2+4)(1+2+3) = 3(2+14+45)$$

$$= 23 + 134 + 345$$

III)

$$F = \begin{bmatrix} 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}_{15}^{h_1, h_2, h_3, h_4, h_5, h_6}$$

$$R = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

$$B(h_7, R) = \begin{bmatrix} 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}_{21+25+26}^{3+4+6, 1, 5, 6, 12+5, 6} \quad B(h_8, R) = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}_{16+26+46+56}^{12+5, 6, 12+5, 6, 12+5, 6, 12+5, 6}$$

$$B(h_9, R) = \begin{bmatrix} 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}_{12+25}^{3+5, 6, 12+5, 6, 12+5, 6, 12+5, 6, 12+5, 6}$$

$$B(h_{10}, R) = \begin{bmatrix} 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 \end{bmatrix}_{12+25}^{1, 2, 3, 5, 6, 12+5, 6}$$

$$B(h_{11}, R) = \begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}_{12+25}^{1, 2, 3, 5, 6, 12+5, 6}$$

$$B(h_{12}, R) = \begin{bmatrix} 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \end{bmatrix}_{24+26}^{1, 2, 3, 5, 6, 12+5, 6}$$

PUL - PRACA DOMOWA 3:

$$F = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix} \quad \begin{array}{l} l_1 \\ l_2 \\ l_3 \\ l_4 \\ l_5 \\ l_6 \end{array}$$

$$R = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & -1 & 1 \end{bmatrix}$$

$$B(l_1, R) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix} \quad \begin{array}{l} 1,2,3 \\ 1,3 \\ 1,5 \\ 2,3,5 \end{array}$$

$$B(l_2, R) = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 \end{bmatrix} \quad \begin{array}{l} 1,3 \\ 2,4 \\ 2,3,4 \\ 1,2,5 \\ 1,3,5 \end{array}$$

$$1 \cdot 5 = a\bar{e}$$

$$3 \cdot (2+4)(1+2+5) = 3(2+14+45) = 23+134+345 = \\ = b\bar{c} + \bar{a}\bar{c}\bar{d} + \bar{c}\bar{d}\bar{e}$$

$$B(l_3, R) = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \end{bmatrix} \quad \begin{array}{l} 3,4 \\ 2 \\ 2,3 \\ 1,2,4,5 \\ 1,3,5 \end{array}$$

$$B(l_4, R) = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad \begin{array}{l} 1,4 \\ 1,2,3 \\ 1,2 \\ 2,3,4,5 \\ 1,5 \end{array}$$

$$2 \cdot (3+4)(1+3+5) = 2(3+14+45) =$$

$$5(1+4)(1+2) = 5(1+24) = 15+245 = a\bar{e} + b\bar{d}\bar{e}$$

$$23+124+245 = b\bar{c} + \bar{a}\bar{b}\bar{d} + \bar{b}\bar{d}\bar{e}$$

$$B(l_5, R) = \begin{bmatrix} 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix} \quad \begin{array}{l} 5 \\ 23,4,5 \\ 2,4,5 \\ 1,2,3 \\ 1,5 \end{array}$$

$$B(l_6, R) = \begin{bmatrix} 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix} \quad \begin{array}{l} 1,3,4,5 \\ 1,2,5 \\ 1,2,3,5 \\ 2,4 \\ 3 \end{array}$$

$$1 \cdot 5 = \bar{a}e$$

$$3(2+4)(1+2+5) = 3(2+14+45) = 23+134+345 = \\ = b\bar{c} + \bar{a}\bar{c}\bar{d} + \bar{c}\bar{d}\bar{e}$$

$$k_1 1 - - 0 \quad I_1 \quad 1 - 0 1 - \quad I_8$$

$$k_2 - 1 0 - - \quad I_2 \quad - - 0 1 1 \quad I_9$$

$$0 - 0 0 - * \quad I_3$$

$$- - 0 0 0 \quad I_4$$

$$k_3 - 1 0 - -$$

$$0 1 - 1 - \quad I_5$$

$$- 1 \cancel{0} 1 0 \quad I_6$$

$$k_4 1 - - - 0$$

$$- 1 - 1 0$$

$$k_5 0 - - - 1 \quad I_7$$

$$k_6 - 1 0 - -$$

	$I_1$	$I_2$	$I_3$	$I_4$	$I_5$	$I_6$	$I_7$	$I_8$	$I_9$	
$k_1$	1	0	0	0	0	0	0	0	0	1

	$I_1$	$I_2$	$I_3$	$I_4$	$I_5$	$I_6$	$I_7$	$I_8$	$I_9$	
$k_2$	0	1	1	1	0	0	0	0	0	2,3,4

	$I_1$	$I_2$	$I_3$	$I_4$	$I_5$	$I_6$	$I_7$	$I_8$	$I_9$	
$k_3$	0	1	0	0	1	1	0	0	0	2,5,6

	$I_1$	$I_2$	$I_3$	$I_4$	$I_5$	$I_6$	$I_7$	$I_8$	$I_9$	
$k_4$	1	0	0	0	0	1	0	0	0	4,6

	$I_1$	$I_2$	$I_3$	$I_4$	$I_5$	$I_6$	$I_7$	$I_8$	$I_9$	
$k_5$	0	0	0	0	0	0	1	0	0	7

	$I_1$	$I_2$	$I_3$	$I_4$	$I_5$	$I_6$	$I_7$	$I_8$	$I_9$	
$k_6$	0	1	0	0	0	0	0	1	1	2,8,9

$$1 \cdot (2+3+4)(2+5+6) \cdot 7 \cdot (2+8+9) = 17(2+(3+4)(5+6)(8+9)) \\ = 17(2+(35+36+45+46)(8+9)) = 17(2+358+359+368+369 \\ +458+459+468+469)$$

$$= 127 + 13578 + 13579 + 13678 + 13679 + 14578 + 14579 + 14678 + 14679$$

Rozwiążmy:  $y = I_1 + I_2 + I_7$

lub  $y = I_1 + I_3 + I_5 + I_7 + I_8$

lub  $y = I_1 + I_3 + I_5 + I_7 + I_9$

lub  $y = I_1 + I_3 + I_6 + I_7 + I_8$

lub  $y = I_1 + I_3 + I_6 + I_7 + I_9$

lub  $y = I_1 + I_4 + I_5 + I_7 + I_8$

lub  $y = I_1 + I_4 + I_5 + I_7 + I_9$

lub  $y = I_1 + I_4 + I_6 + I_7 + I_8$

lub  $y = I_1 + I_4 + I_6 + I_7 + I_9$

III

$$F = \begin{array}{|c|c|} \hline & h_1 \\ \hline 0 & 1 & 1 & 0 & 1 & 0 \\ \hline 1 & 1 & 0 & 1 & 0 & 0 \\ \hline 0 & 0 & 1 & 1 & 0 & 1 \\ \hline 0 & 1 & 0 & 1 & 1 & 1 \\ \hline 1 & 1 & 0 & 0 & 1 & 1 \\ \hline 0 & 0 & 1 & 1 & 1 & 1 \\ \hline \end{array} \quad R = \begin{array}{|c|c|} \hline & h_1 \\ \hline 0 & 0 & 0 & 1 & 1 & 1 \\ \hline 1 & 1 & 1 & 0 & 0 & 1 \\ \hline 1 & 1 & 0 & 1 & 0 & 1 \\ \hline 0 & 0 & 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 1 & 0 & 1 & 0 \\ \hline \end{array}$$

$$B(h_1, R) = \begin{array}{|c|c|} \hline & h_1 \\ \hline 0 & 1 & 1 & 0 & 1 & 0 \\ \hline 0 & 1 & 1 & 1 & 0 & 1 \\ \hline 1 & 0 & 0 & 0 & 1 & 1 \\ \hline 1 & 0 & 1 & 1 & 1 & 1 \\ \hline 0 & 1 & 1 & 0 & 0 & 0 \\ \hline 0 & 1 & 0 & 0 & 0 & 0 \\ \hline \end{array} \quad B(h_2, R) = \begin{array}{|c|c|} \hline & h_2 \\ \hline 1 & 1 & 0 & 1 & 0 & 0 \\ \hline 1 & 1 & 0 & 0 & 1 & 1 \\ \hline 0 & 0 & 1 & 1 & 0 & 1 \\ \hline 0 & 0 & 0 & 0 & 1 & 0 \\ \hline 1 & 1 & 0 & 1 & 1 & 0 \\ \hline 1 & 1 & 1 & 1 & 1 & 0 \\ \hline \end{array}$$

$$2 \cdot (1+4+6) = 12 + 24 + 26$$

$$6 \cdot (1+2+4+5) = 16 + 26 + 46 + 56$$

$$B(h_3, R) = \begin{array}{|c|c|} \hline & h_3 \\ \hline 0 & 0 & 1 & 1 & 0 & 1 \\ \hline 0 & 0 & 1 & 0 & 1 & 0 \\ \hline 1 & 1 & 0 & 1 & 0 & 0 \\ \hline 1 & 1 & 1 & 0 & 0 & 0 \\ \hline 0 & 0 & 1 & 1 & 1 & 1 \\ \hline 0 & 0 & 0 & 1 & 1 & 1 \\ \hline \end{array}$$

$$B(h_4, R) = \begin{array}{|c|c|} \hline & h_4 \\ \hline 0 & 1 & 0 & 1 & 1 & 1 \\ \hline 0 & 1 & 0 & 0 & 0 & 0 \\ \hline 1 & 0 & 1 & 1 & 1 & 0 \\ \hline 1 & 0 & 0 & 0 & 1 & 0 \\ \hline 0 & 1 & 0 & 1 & 0 & 1 \\ \hline 0 & 1 & 1 & 0 & 1 & 0 \\ \hline \end{array}$$

$$(3+5)(1+2+4)(1+2+3)(4+5+6) =$$

$$2 \cdot (1+5) = 12 + 25$$

$$(1+2+34)(5+34+36) = 15 + 134 + 136$$

$$\begin{array}{|c|c|} \hline & h_5 \\ \hline 0 & 0 & 1 & 1 & 1 & 1 \\ \hline 0 & 0 & 1 & 0 & 0 & 0 \\ \hline 1 & 1 & 0 & 1 & 1 & 0 \\ \hline 1 & 1 & 1 & 0 & 1 & 0 \\ \hline 0 & 0 & 1 & 1 & 0 & 1 \\ \hline 0 & 0 & 0 & 1 & 0 & 1 \\ \hline \end{array}$$

$$+ 25 + 234 + 236 + 345 + 34 + 346 =$$

$$B(h_6, R) = \begin{array}{|c|c|} \hline & h_6 \\ \hline 1 & 1 & 0 & 1 & 0 & 0 \\ \hline 1 & 1 & 1 & 0 & 1 & 0 \\ \hline 0 & 0 & 1 & 1 & 0 & 1 \\ \hline 0 & 0 & 0 & 1 & 0 & 1 \\ \hline \end{array}$$

$$15 + 25 + 34 + 136 + 236$$

$$3 \cdot (4+6)(1+2+4+5) = 3(4+16+26+56) = 34 + 136 + 236 + 356$$

$$B(h_7, R) = \begin{array}{|c|c|} \hline & h_7 \\ \hline 1 & 1 & 0 & 0 & 1 & 1 \\ \hline 1 & 1 & 0 & 1 & 0 & 0 \\ \hline 0 & 0 & 1 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 & 1 & 0 \\ \hline 1 & 1 & 0 & 0 & 0 & 1 \\ \hline 1 & 1 & 1 & 0 & 0 & 1 \\ \hline \end{array}$$

$$(1+2+4)(3+5)(4+5)(1+2+6) = (5+34)(1+2+4+6) = 15 + 25 + 456 + 134 + 234 + 346$$

$k_1$	$I_1$	$I_2$	$I_3$	$I_4$	$I_5$	$I_6$	$I_7$	$I_8$	$I_9$	$I_{10}$	$I_{11}$	$I_{12}$	$I_{13}$	$I_{14}$	$I_{15}$	$I_{16}$
-1 - 0 - -	$I_2$	$k_1$	1 1	1 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0
-1 - - - 0	$I_3$	$k_2$	0 0	1 1	1 1	1 0	0 0	1 1	1 1	0 0	0 0	0 0	0 0	0 0	0 0	0 0
$k_2$	$I_4$	$k_3$	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0
-1 - - - 0		$k_4$	1 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0
- - - 1 - 0	$I_5$	$k_5$	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	1 1	1 1	1 1	1 1
- - - - 0 0	$I_6$	$k_6$	0 0	0 0	0 0	0 0	0 0	0 0	1 1	1 1	1 0	0 0	0 0	0 0	0 0	0 0

$k_3$  0 - - - 0 -  $I_7$

0 0 - - 0 -  $I_8$

- - 1 1 - -  $I_9$

0 - 1 - - 1  $I_{10}$

- 0 1 - - 1  $I_{11}$

$k_4$  0 1 - - -

0 1 - - 1 -  $I_{12}$

$I_{17}$  |  $I_{18}$  |  $I_{19}$

0 0 | 0 | 1, 2, 3

0 0 | 0 | 3, 4, 5, 6

0 0 | 0 | 7, 8, 9, 10, 11

0 0 | 0 | 1, 12

0 0 | 0 | 13, 14, 15, 16

0 0 | 1 | 9, 10, 11, 19

$k_5$  1 - - - 1 -  $I_{13}$

- 1 - - 1 -  $I_{14}$

- - - 0 1 1  $I_{15}$

1 - 0 0 - -  $I_{16}$

- 1 0 0 - -  $I_{17}$

- - 0 - 1 1 1  $I_{18}$

$k_6$  - - 1 1 - -

0 - 1 - - 1

- 0 1 - - 1

- - 1 - 1 1 1  $I_{19}$

$x_3 x_4$	00	01	11	10
00	0	0	$\boxed{1}$	1
01	-	$\boxed{1}$	1	0
11	$\boxed{1}$	-	-	-
10	-	1	$\boxed{1}$	1

$$\overline{x_4} \star_5 + x_4 x_6 + x_3$$

## PRZYGOTOWANIE DO KOLOKWIUM:

24.11.2010  
wytwarzanie PUC

### Zadanie 1:

	$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	$x_6$	$x_7$	F
1	0	1	1	0	1	0	0	1
2	1	1	1	0	0	1	1	1
3	1	0	0	1	0	1	0	1
4	1	1	0	1	1	0	0	0
5	1	0	1	0	0	1	1	1
6	0	1	1	1	0	0	0	1
7	1	0	0	0	0	1	0	0
8	1	0	0	0	1	0	1	1
9	1	1	0	1	1	1	0	1
10	1	0	0	0	0	0	1	0
11	0	1	1	0	1	1	0	1
12	0	1	1	0	0	1	0	1

$$P_4 = \left( \frac{0}{1, 2, 5, 7, 8, 10, 11, 12}, \frac{1}{3, 4, 6, 9} \right)$$

$$P_6 = \left( \frac{0}{1, 4, 6, 8, 10}, \frac{1}{2, 3, 5, 7, 9, 11, 12} \right)$$

$$P_4 \cdot P_6 = \left( \frac{00}{1,8,10}; \frac{01}{2,5,7,11,12}; \frac{10}{4,6}; \frac{11}{3,9} \right)$$

$$P_F = \overline{(1, 2, 3, 5, 6, 8, 9, 11, 12)}; \quad \overline{(4, 7, 10)}$$

$$P_4 \cdot P_6 \Big|_{P_F} = \left( \overline{(1,8)(10)}, \overline{(2,5,11,12)(7)}, \overline{(4)(6)}, \overline{(3,9)} \right)$$

<u>1, 10</u>	$x_1 \cup x_2 \cup x_3 \cup x_5 \cup x_7$
<u>8, 10</u>	$x_2 \cup x_5$
<u>2, 7</u>	$x_2 \cup x_3 \cup x_7$
<u>5, 7</u>	$x_3 \cup x_7$
<u>7, 11, *</u>	$x_1 \cup x_2 \cup x_3 \cup x_5$
<u>7, 12, *</u>	$x_1 \cup x_2 \cup x_3$
<u>4, 6</u>	$x_1 \cup x_3 \cup x_5$

$x_2 x_5$   
 $x_3 x_7$   
 $x_1 x_2 x_3$   
 $x_1 x_3 x_5$

$$(x_2 + x_5)(x_1 + x_3 + x_5)(x_3 + x_7)(x_1 + x_2 + x_3) =$$

$$(x_1 + x_3 + x_2)(x_1 + x_3 + x_5)(x_2 + x_5)(x_3 + x_7) =$$

$$(x_1 + x_3 + x_5)(x_2 x_3 + x_5 x_7) = \cancel{x_1 x_2 x_3} + x_1 x_5 x_7 + x_2 x_3 + \cancel{x_3 x_5} + x_2 x_5 + \cancel{x_2 x_7}$$

$$= x_1 x_5 x_7 + x_2 x_3 + \cancel{x_3 x_5} + x_2 x_5 x_7 + x_1 x_2 x_7 + \cancel{x_3 x_5}$$

## MINIMALNE ROZWIAZANIA

$$x_1 \quad x_5 \quad x_7$$

$x_2 x_3$

$$x_2 x_5 x_7$$

$x_1 \ x_2 \ x_7$

$X_3$   $X_5$

## O najmniejszej licności

$X_2 X_3$

$X_2 X_5 X_7$

$X_1 X_2 X_7$

$X_3 X_5$

$$x_2, x_3, x_4, x_6 \quad \checkmark$$

$x_3, x_4, x_5, x_6$

$X_1 X_2$	00	01	11	10
$X_1 X_2$	00	0	-	1
$X_1 X_2$	01	-	1	0
$X_1 X_2$	11	1	-	-
$X_1 X_2$	10	-	1	1

$$f = x_3 + x_4 x_6 + \overline{x_4} x_5$$

## Zadanie 2: Uproszczona metoda ekspansji

$$F = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix} \begin{matrix} k_1 \\ k_2 \\ k_3 \\ k_4 \end{matrix}$$

$$R = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$$

$$B(k_1, R) = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} x_3 & x_4 \\ x_1 & x_2 \\ x_2 & x_3 \\ x_1 \end{matrix}$$

$x_1 x_3$  - implicant to:  $\bar{x}_1 x_3$  0 - 1 -

$$F = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}_{k_2}^{k_1}$$

$$B(k_2, R) = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{matrix} x_1 x_3 \\ x_2 x_4 \\ x_1 x_2 x_3 x_4 \\ (x_4) \end{matrix}$$

$$\begin{matrix} x_1 x_4 & \vee x_3 x_4 \\ \overline{x_1 x_4} & \overline{x_3 x_4} \end{matrix}$$

# PRZYGOTOWANIE DO KONKURSU

19.01.2011  
wykłady PUL

$$P_3 \cdot P_5 = (\overline{1,5,10}, \overline{2,3}; \overline{4,7}, \overline{6}, \overline{8,9})$$

$$P_D = (\overline{1,2,3}, \overline{4,5,6}, \overline{7,8}; \overline{9,10})$$

$$P_{\text{poz}}$$

$$= (\overline{1,5,10}, \overline{2,3}, \overline{4,7}, \overline{6}, \overline{8,9})$$

$$\begin{array}{c|cc} 1,5 & \cancel{Q_1, Q_2, Q_6} \\ \hline 1,10 & \cancel{Q_2, Q_4} \\ \hline 5,10 & \cancel{Q_1, Q_4, Q_6} \\ \hline 4,7 & \cancel{Q_1, Q_2, Q_6} \end{array}$$

$$(2+4)(1+4+6)(1+2+6) = 12 \cdot 11 \cdot 9 = 1188$$

$$(4+12+26)(1+2+6) = 42 \cdot 9 = 378$$
~~$$+ 26$$~~

Równiania:  $Q_1, Q_3, Q_4, Q_5$

$Q_2, Q_3, Q_4, Q_5$

$Q_3, Q_4, Q_5, Q_6$

$Q_1, Q_2, Q_3, Q_5$

$Q_2, Q_3, Q_5, Q_6$

ZADANIE: Uogólnienie reguł decyzyjnych.

	<u>0</u>	<u>1</u>	<u>0</u>	<u>0</u>	<u>1</u>
$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	
1	0	0	0	0	T
2	0	0	0	1	0
3	0	1	0	1	0
4	0	1	1	1	1
5	1	1	0	1	1
6	0	0	1	1	0
7	0	1	0	0	N
8	0	1	1	0	N
9	1	0	0	0	1
10	1	0	1	1	N

Minimálne równanie: 1,5

$$1(3+4+5)(2+5) = 1(5+23+24) = 15 + 123 + 124$$

$$(x_1, 0) \wedge (x_5, 1) \rightarrow \text{TAK}$$

0 - - - 1

$$\begin{array}{c|ccccc} 0 & 0 & 1 & 1 & 1 & 3,4,5 \\ \hline 0 & 1 & 0 & 0 & 0 & 2,5 \\ \hline 0 & 1 & 1 & 0 & 0 & 2,2,5 \\ \hline 1 & 0 & 0 & 0 & 0 & 1 \\ \hline 1 & 0 & 1 & 1 & 0 & 1,4 \end{array}$$