Note

Equations for Determining $1/H$ versus S Values in Computer Calculations of Interfacial Tension by the Pendent Drop Method

The pendent drop method of determining interfacial tensions is a very practical technique, since once the interface is formed, it is not subjected to any outside influences during the time measurements are being taken. There are various methods of calculating the boundary tensions from the pendent drop profiles, but the method of Andreas, Hauser, and Tucker (1) is the most commonly used. With this method, the equatorial diameter d_e and the diameter d_s in a selected plane, which is located by measuring vertically from the vortex a distance equal to d_e, are measured (see Fig. 1). The ratio of the two diameters, d_s/d_e, is designated as S, the drop shape; the quantity $1/H$ is a function of S. The interfacial tension γ is calculated by the equation:

$$\gamma = g\rho\, d_e^2/H,$$

where g is the acceleration due to gravity and ρ is the difference in density of the two phases.

Andreas and co-workers constructed a table for S versus $1/H$ from experimental data for water. The method was made independent of experimental calibration and placed on an absolute foundation when Fordham (2) calculated tables for S versus $1/H$ from $S = 0.660$ to 1.003. These have since been extended downward to $S = 0.460$ by Mills (3) and to $S = 0.300$ by Stauffer (4).

The combined tables of Fordham, Mills, and Stauffer are adequate for nearly all drop profiles encountered, and are in a very convenient form when the interfacial tension calculations are performed by hand. However, with the advent of computers, tables are no longer convenient owing to the loading time and storage space required for some 700 $1/H$ versus S values. Equations describing $1/H$ in terms of S would be much more desirable.

Stegemeier (5) derived an approximate equation, for slide rule calculations, by assuming that a plot of log $(1/H)$ versus log (S) is a straight line

$$1/H = 0.31270S^{-2.6444}. \qquad [1]$$

Althought this equation is convenient to use, it deviates considerably from the values of Fordham, Mills, and Stauffer, particularly for the very high and very low S values.

In order to calculate $1/H$ values that would give satisfactory agreement with those of Fordham, Mills, and Stauffer for S values from 0.401 to 1.00, it was necessary to divide the data into five groups and derive an equation that would best fit the data in each group. The equations were developed by assuming that log $(1/H)$ versus log (S) would plot a straight line. After the best log-log fit was obtained, the deviation between the values calculated from the equation and those from the tables was determined. A second equation was then written describing the deviation and subtracted from the first. In this manner, the five following equations were derived:

For $S = 0.401$ to $S = 0.46$

$$1/H = (0.32720/S^{2.56651}) - 0.97553S^2 + 0.84059S - 0.18069. \qquad [2]$$

For $S > 0.46$ to $S = 0.59$

$$1/H = (0.31968/S^{2.59725}) - 0.46898S^2 + 0.50059S - 0.13261. \qquad [3]$$

For $S > 0.59$ to $S = 0.68$

$$1/H = (.31522/S^{2.62435}) - 0.11714S^2 + 0.15756S - 0.05285. \qquad [4]$$

For $S > 0.68$ to $S = 0.90$

$$1/H = (.31345/S^{2.64267}) - 0.09155S^2 + 0.14701S - 0.05877. \qquad [5]$$

For $S > 0.90$ to $S = 1.00$

$$1/H = (0.30715/S^{2.84636}) - 0.69116S^3 + 1.08315S^2 - 0.18341S - 0.20970. \qquad [6]$$

The $1/H$ data were computed from the five equations for $S = 0.401$ to $S = 1.000$ in increments of 0.001 and compared to the values of Fordham, Mills, and Stauffer. The greatest deviation was ± 0.00033 at $S = 0.490$. In the region from $S = 0.601$ to $S = 0.893$, which are the values most commonly encountered, the maximum deviation was ± 0.00006. These equations are sufficiently

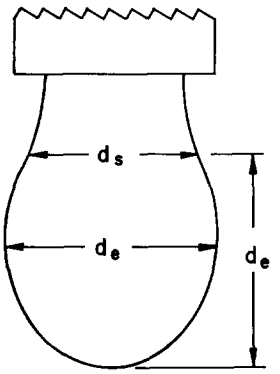

FIG. 1. Measurements performed on pendent drops.

accurate to be used in the most exacting interfacial tension calculations.

A computer program has been written, utilizing the above five equations, to calculate interfacial tension values from pendent drop measurements. A listing of the source program, written in Fortran IV for an IBM 1800, will be made available upon request.

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