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EXAMEN - FUNDAMENTELE ALGEBRICE ALE INFORMATICII
SEM. I

(3) $3^{2021}:100$ - ultimele două cifre este dat de 3^{2021} în $\mathbb{Z}_{100}$

$(3,100)=1 \xRightarrow{\text{T. Euler}} \hat{3}^{\varphi(100)}=\hat{1}$ în $\mathbb{Z}_{100}$

$\varphi(100)=\varphi(2^2\cdot 5^2)=(2^2-2)(5^2-5)=2\cdot 20=40$

$\hat{3}^{2021}=\hat{3}^{40\cdot 50+1}=(\hat{3}^{40})^{50}\cdot\hat{3}=\hat{1}\cdot\hat{3}=\hat{3} \Rightarrow$ ultimele două cifre sunt 03

(2) $p(abc^2)=(c^2ab)p$

$\left.\begin{array}{l} \ell(p)=10 \\ \ell(c^2ab)=4 \end{array}\right| \Rightarrow$ ~~cont~~ concatenăm cu $abc^2 \Big| \Rightarrow$

$\Rightarrow p(a^2b^2c^4)=(c^4a^2b^2)p$

$\left.\begin{array}{l} \ell(p)=10 \\ \ell(c^4a^2b^2)=8 \end{array}\right| \Rightarrow$ concatenăm cu $abc^2 \Big| \Rightarrow$

$\cancel{\Rightarrow p(a^4b^4c^6)=(c^6a^4b^4)p}$

$\cancel{\ell(p)=10}$

$\cancel{\ell(c^6a^4b^4)=14}$ $\cancel{\Rightarrow p=(c^4a^2b^2)ca \Rightarrow}$ $\cancel{\Rightarrow p=c^5a^3b^2}$

$\Rightarrow p(a^3b^3c^6)=(c^6a^3b^3)p$

$\left.\begin{array}{l} \ell(p)=10 \\ \ell(c^6a^3b^3)=12>10 \end{array}\right| \Rightarrow p=(c^4a^2b^2)c^2 \Rightarrow c^6a^2b^2$

④ $\sigma \in S_{2021}$

$$\left.\begin{array}{l}|S_{2021}:H| = 2020! \\ |S_{2021}| = 2021!\end{array}\right| \Rightarrow |H| = \frac{|S_{2021}|}{|S_{2021}:H|} = \frac{2021!}{2020!} = 2021 \Rightarrow$$

$\Rightarrow |\sigma| = 2021 \Rightarrow o(\sigma) = 2021$ c.m.m.m.c.-ul lungimilor ciclurilor în care s-a descompus

$2021 = 1 \cdot 2021 = 43 \cdot 47$

1. $\sigma = (a_1, a_2, \dots, a_{2021}) = (a_1, a_2)(a_2, a_3)(a_3, a_4)(a_4, a_5)\dots$

$\dots (a_{2020}, a_{2021}) \Rightarrow 2020$ transpoziții $\Rightarrow \varepsilon(\sigma) = (-1)^{2020} = 1 \Rightarrow \sigma$ pară

2. $\sigma = (a_{1_1}, a_{1_2}, a_{1_3}, \dots, a_{1_{47}})(a_{2_1}, a_{2_2}, \dots, a_{2_{47}})\dots(a_{43_1}, a_{43_2}, \dots, a_{43_{47}})$

$= \underbrace{(a_{1_1}, a_{1_2})(a_{1_2}, a_{1_3})\dots(a_{1_{46}}, a_{1_{47}})}_{46 \text{ transpoziții}}(a_{2_1}, a_{2_2})(a_{2_2}, a_{2_3})\dots(a_{2_{46}}, a_{2_{47}})\dots$

$\dots (a_{43_{46}}, a_{43_{47}}) \Rightarrow$ sunt $43 \cdot 46$ transpoziții $\Rightarrow \varepsilon(\sigma) = (-1)^{43 \cdot 46} = 1 \Rightarrow$
$\Rightarrow \sigma$ pară

3. $\sigma = (a_{1_1}, a_{1_2}, \dots, a_{1_{43}})(a_{2_1}, a_{2_2}, \dots, a_{2_{43}})\dots(a_{47_1}, a_{47_2}, \dots, a_{47_{43}}) =$

$= \underbrace{(a_{1_1}, a_{1_2})\dots(a_{1_{42}}, a_{1_{43}})}_{42 \text{ transpoziții}}\dots(a_{47_{42}}, a_{47_{43}}) \Rightarrow$

$\Rightarrow 42 \cdot 47$ transpoziții $\Rightarrow \varepsilon(\sigma) = (-1)^{42 \cdot 47} = 1 \Rightarrow \sigma$ pară

⑥ cod binar liniar $\iff C_{n,k} \leq \mathbb{Z}_2^n$

$\dim C_{n,k} = k \Rightarrow G \in M_{k,n}(\mathbb{Z}_2)$ și $H \in M_{n-k,k}(\mathbb{Z}_2)$

din enunț $G \in M_{3,6}(\mathbb{Z}_2) \Rightarrow \begin{cases} k=3 \\ n=6 \end{cases} \Rightarrow$ cod binar liniar $(6,3) \Rightarrow C_{6,3}$

$$C_{6,3}^{\perp}: G\cdot X = 0 \iff \begin{pmatrix} 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{pmatrix} = 0 \Rightarrow \begin{cases} x_3 + x_4 + x_5 + x_6 = 0 \\ x_2 + x_4 + x_5 = 0 \\ x_1 + x_4 + x_6 = 0 \end{cases}$$

nec. principale x_1, x_2, x_3

$$\begin{cases} x_4 = \alpha \\ x_5 = \beta \\ x_6 = \gamma \end{cases} \Rightarrow \begin{aligned} x_3 &= \alpha + \beta + \gamma \\ x_2 &= \alpha + \beta \\ x_1 &= \alpha + \gamma \end{aligned}$$

$$C_{6,3}^{\perp} = \{(\alpha + \gamma, \alpha+\beta, \alpha+\beta+\gamma, \alpha, \beta, \gamma) \mid \alpha, \beta, \gamma \in \mathbb{Z}_2\}$$

$$C_{6,3}^{\perp} = \{\alpha(1,1,1,1,0,0) + \beta(0,1,1,0,1,0) + \gamma(1,0,1,0,0,1) \mid \alpha, \beta, \gamma \in \mathbb{Z}_2\}$$

$$C_{6,3}^{\perp} = [\underbrace{111100}_{v_1}, \underbrace{011010}_{v_2}, \underbrace{101001}_{v_3}] \Rightarrow \{v_1, v_2, v_3\} \text{ s.g. pt. } C_{6,3}^{\perp}$$

Studiem l.i.

$$\text{rang} \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = 3 \Rightarrow \{v_1, v_2, v_3\} \text{ l.i.}$$

$\Rightarrow \{v_1, v_2, v_3\}$ bază în $C_{6,3}^{\perp}$

Matricea de control:

$$H = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} 1&1&1&1&0&0 \\ 0&1&1&0&1&0 \\ 1&0&1&0&0&1 \end{pmatrix}$$

Regulă de codificare/decodificare:

$$H\cdot y = 0 \iff \begin{pmatrix} 1&1&1&1&0&0 \\ 0&1&1&0&1&0 \\ 1&0&1&0&0&1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \end{pmatrix} = 0 \iff \begin{cases} y_1+y_2+y_3+y_4=0 \\ y_2+y_3+y_5=0 \\ y_1+y_3+y_6=0 \end{cases}$$

nec. principale y_4, y_5, y_6

$$\left.\begin{matrix} y_1=\alpha \\ y_2=\beta \\ y_3=\gamma \end{matrix}\right| \Rightarrow \begin{matrix} y_4=\alpha+\beta+\gamma \\ y_5=\beta+\gamma \\ y_6=\alpha+\gamma \end{matrix}$$

~~α, β, γ~~

$\alpha\beta\gamma \to \underline{\alpha}\ \underline{\beta}\ \underline{\gamma}\ \underline{\alpha+\beta+\gamma}\ \underline{\beta+\gamma}\ \underline{\alpha+\gamma}$

$111 \to 1\ 1\ 1\ 1\ 0\ 0$

TSD

$$H = \begin{pmatrix} 1&1&1&1&0&0 \\ 0&1&1&0&1&0 \\ 1&0&1&0&0&1 \end{pmatrix}$$

eroare	sindrom
000000	000
000001	001
000010	010
000100	100
000011	011
100000	101
010000	110
001000	111

Recepționez $v = 111111$

$$H\cdot v = \begin{pmatrix} 1&1&1&1&0&0 \\ 0&1&1&0&1&0 \\ 1&0&1&0&0&1 \end{pmatrix} \begin{pmatrix} 1\\1\\1\\1\\1\\1 \end{pmatrix} = \begin{pmatrix} 0\\1\\1 \end{pmatrix} \neq \begin{pmatrix} 0\\0\\0 \end{pmatrix} \Rightarrow$$

$\Rightarrow v$ nu e cuvânt cod $\Rightarrow v \notin C_{6,3} \Rightarrow$

$\Rightarrow v = v_{corect} + e$, sindrom 011 $\Rightarrow$

$\Rightarrow$ eroare $e = 000011 \Rightarrow$

$\Rightarrow$ $v_{corect} = v \dot{-} e$

$v \in \mathbb{Z}_2$ $\Rightarrow v_{corect} = v + e = 111111 + 000011 = 111100$

$\downarrow$

codificat 111

① $A = \{1, 2, 3, \dots, 10\}$

$3 \mid x - y \Rightarrow u.c.(x) = u.c.(y)$

- reflexivă: $3 \mid x - x, (\forall) x \in A$ este adevărat $\Rightarrow xRx, (\forall) x \in A$
- simetrică: Fie $xRy \Rightarrow 3 \mid x - y \Rightarrow 3 \mid -(x-y) \Rightarrow 3 \mid y - x \Rightarrow yRx$
- tranzitivă: Fie $xRy \Rightarrow 3 \mid x - y$, $yRz \Rightarrow 3 \mid y - z$ $\Rightarrow 3 \mid x - \cancel{y} + \cancel{y} - z \Rightarrow 3 \mid x - z \Rightarrow xRz$

$\Rightarrow$

$\Rightarrow R$ relație de echivalență

$3 \mid x - y \Rightarrow (x - y) \in \{3, 6, 9\}$

$x - y = 3 \Rightarrow x = y + 3$

$3 \mid 4-1, 3 \mid 5-2, 3 \mid 6-3, 3 \mid 7-4, 3 \mid 8-5, 3 \mid 9-6, 3 \mid 10-7 \Rightarrow$

$\Rightarrow card(x - y = 3) = 7$

$x - y = 6 \Rightarrow x = y + 6$

$3 \mid 7-1, 3 \mid 8-2, 3 \mid 9-3, 3 \mid 10-4 \Rightarrow card(x - y = 6) = 4$

$x - y = 9 \Rightarrow x = y + 9$

$3 \mid 10 - 1$ $\Rightarrow card(x - y = 9) = 1$

$7 + 4 + 1 = 12$ este cardinalul graficului

⑤ $T:\mathbb{R}^4 \to \mathbb{R}^4,\ T(x_1,x_2,x_3,x_4)=(x_1+x_2+x_3,\ x_1+x_2+x_4,\ x_2+x_3+x_4,\ x_1+x_3+x_4)$

$\operatorname{Im} T = \{ T(x) \mid x \in \mathbb{R}^4 \}$

$\operatorname{Im} T = \{ x_1+x_2+x_3,\ x_1+x_2+x_4,\ x_2+x_3+x_4,\ x_1+x_3+x_4 \mid x_1,x_2,x_3,x_4 \in \mathbb{R} \}$

$\operatorname{Im} T = \{ x_1(1+1+0+1)+x_2(1+1+1+0)+x_3(1+0+1+1)+x_4(0+1+1+1) \mid x_1,x_2,x_3,x_4 \in \mathbb{R} \}$

$\operatorname{Im} T = [\underbrace{(1101)}_{w_1},\underbrace{(1110)}_{w_2},\underbrace{(1011)}_{w_3},\underbrace{(0111)}_{w_4}] \Rightarrow$ s.g. pt. $\operatorname{Im} T$

$\operatorname{Rang}\begin{pmatrix}1&1&1&0\\1&1&0&1\\0&1&1&1\\1&0&1&1\end{pmatrix} = 4 \Rightarrow w_1,w_2,w_3,w_4$ l.i.

$\Rightarrow w_1,w_2,w_3,w_4$ bază în $\operatorname{Im} T \Rightarrow$

$\Rightarrow \dim(\operatorname{Im} T)=4$

$$\begin{pmatrix}1&1&1&0\\1&1&0&1\\0&1&1&1\\1&0&1&1\end{pmatrix} \underset{l_2-l_1}{\sim} \begin{pmatrix}1&1&1&0\\0&0&-1&1\\0&1&1&1\\1&0&1&1\end{pmatrix} \underset{l_4-l_1}{\sim} \begin{pmatrix}1&1&1&0\\0&0&-1&1\\0&1&1&1\\0&-1&0&1\end{pmatrix} \underset{l_3\leftrightarrow l_2}{\sim}$$

$$\sim \begin{pmatrix}1&1&1&0\\0&1&1&1\\0&0&1&-1\\0&-1&0&1\end{pmatrix}$$