Seminar 3

04.03.2024

Variabile Aleatoare Discrete

$$X : \begin{pmatrix} x_k \\ p_k \end{pmatrix}_{k \in I}, \quad \mathbb{P}\{X = x_k\} = p_k, \quad (\forall) k \in I$$

$I \overset{not}{=}$ mulțime de indici cel mult numărabilă

$$\sum_{k \in I} p_k = 1$$

Funcția de Repartiție a variabilei aleatoare X:

$$F_X : \mathbb{R} \to [0, 1], \quad F_X(x) = \sum_{\substack{x_k \leq x \\ k \in I}} p_k$$

Caracteristici numerice ale variabilei aleatoare:

1. Valoarea Medie

$$\boxed{\mathbb{E}(X) = \sum_{k \in I} x_k p_k}$$

2. Dispersia

$$\mathbb{V}(X) \overset{def.}{=} \sum_{k \in I} [x_k - \mathbb{E}(X)]^2 \cdot p_k$$

$$\boxed{\mathbb{V}(X) = \mathbb{E}(X^2) - [\mathbb{E}(X)]^2}$$

3. Corelația a 2 v.a. X și Y

$$\text{cov}(X, Y) \overset{def.}{=} \mathbb{E}[(X - \mathbb{E}(X)) \cdot (Y - \mathbb{E}(Y))]$$

$$\boxed{\text{cov}(X, Y) = \mathbb{E}(X \cdot Y) - \mathbb{E}(X) \cdot \mathbb{E}(Y)}$$

4. Coeficient de Corelație

$$\boxed{\rho_{X,Y} = \frac{cov(X,Y)}{\sqrt{\mathbb{V}(X)} \cdot \sqrt{\mathbb{V}(Y)}} \ , \ \mathbb{V}(X) \cdot \mathbb{V}(Y) \neq 0}$$

① SE DĂ VARIABILA ALEATOARE SIMPLĂ A AVÂND REPARTIȚIA:

$$A: \begin{pmatrix} -2 & 0 & 1 & 3 & 5 & 6 \\ 0.1 & 0.2 & 0.3 & 0.1 & 0.2 & 0.1 \end{pmatrix}$$

a) Să se determine funcția de repartiție

$$F_X(x) = \sum_{\substack{k \in I \\ x_k \leq x}} p_k = \begin{cases} 0, & x < -2 \\ 0.1, & -2 \leq x < 0 \\ 0.3, & 0 \leq x < 1 \\ 0.6, & 1 \leq x < 3 \\ 0.7, & 3 \leq x < 5 \\ 0.9, & 5 \leq x < 6 \\ 1, & x \geq 6 \end{cases}$$

b) $\mathbb{P} = \{ 0 \leq X < \underbrace{\frac{7}{2}}_{=3,5} \}$

Metoda I:

$$\underbrace{\mathbb{P}\{X=0\}}_{0.2} + \underbrace{\mathbb{P}\{X=1\}}_{0.3} + \underbrace{\mathbb{P}\{X=3\}}_{0.1} = 0,6$$

Metoda II:

$$F_X(x) = \mathbb{P}\{X \leq x\} = \mathbb{P}\{0 \leq X < \tfrac{7}{2}\} =$$

$$= \mathbb{P}\{(X < \tfrac{7}{2}) \setminus (X < 0)\} = \underbrace{F(\tfrac{7}{2})}_{0.7} - 0 - \underbrace{F(0)}_{0.3} + 0,2 = 0,6$$

Seminar 3

04.03.2024

c) $\mathbb{P}=\{1<x\le 6 \mid x>3\}$

$$\mathbb{P}=\{1<x\le 6 \mid x>3\}=\frac{\mathbb{P}\{(1<X\le 6)\cap(x>3)\}}{\mathbb{P}\{x>3\}}=$$

$$=\frac{\mathbb{P}\{3<X\le 6\}}{1-\mathbb{P}\{X\le 3\}}=\frac{F(6)-F(3)}{1-F(3)}=\frac{1-0,7}{1-0,7}=1$$

2) SE DĂ VARIABILA ALEATOARE X CU VALORILE $-1, 0, 1$ TOATE CU ACEEAȘI PROBABILITATE $=\frac{1}{3}$. SĂ SE DETERMINE DISTRIBUȚIA VARIABILEI $X+X^2$.

$$X:\begin{pmatrix}-1 & 0 & 1\\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3}\end{pmatrix}$$

$$X^2:\begin{pmatrix}1 & 0 & 1\\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3}\end{pmatrix} \Leftrightarrow X^2:\begin{pmatrix}0 & 1\\ \frac{1}{3} & \frac{2}{3}\end{pmatrix}$$

$$X+X^2:\begin{pmatrix}\cancel{-1+0} & \overbrace{-1+1}^{0} & \overbrace{0+0}^{0} & \cancel{0+1} & \cancel{1+0} & \overbrace{1+1}^{2}\\ \cancel{0} & \frac{1}{3} & \frac{1}{3} & \cancel{0} & \cancel{0} & \frac{1}{3}\end{pmatrix} \Leftrightarrow \begin{pmatrix}0 & 2\\ \frac{2}{3} & \frac{1}{3}\end{pmatrix}$$

3) FIE VARIABILE:

$$X:\begin{pmatrix}0 & 1 & 2\\ 0.3 & 0.5 & 0.2\end{pmatrix},\ Y:\begin{pmatrix}-1 & 1\\ 0.5 & 0.5\end{pmatrix}.$$

SĂ SE SCRIE REPARTIȚIA VARIABILEI $3X+Y$. (X, Y indep.)

$$3X:\begin{pmatrix}0 & 3 & 6\\ 0.3 & 0.5 & 0.2\end{pmatrix}$$

$$3X+Y:\begin{pmatrix}-1 & 2 & 5 & 1 & 4 & 7\\ 0.15 & 0.25 & 0.10 & 0.15 & 0.25 & 0.10\end{pmatrix}$$

$$\mathbb{P}\{3X+Y=-1\} = \mathbb{P}\{(3X=0)\cap(Y=-1)\}$$

$$\overset{ind}{=} \mathbb{P}\{3X=0\}\cdot\mathbb{P}\{Y=-1\}$$

$$= 0.3\cdot 0.5 = 0.15$$