MIDDLE EAST TECHNICAL UNIVERSITY
DEPARTMENT OF COMPUTER ENGINEERING

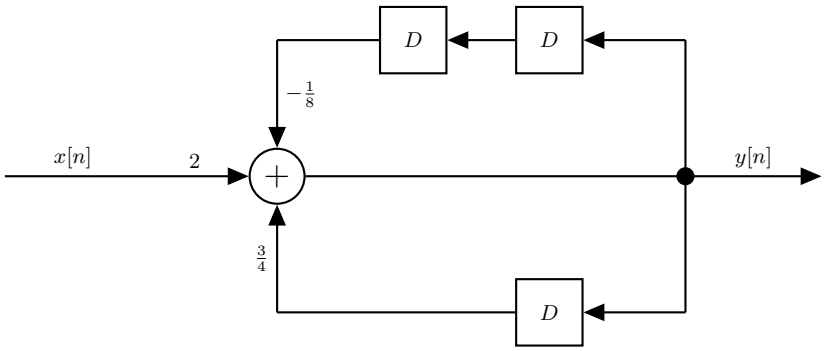
CENG 384

Spring 2019
Homework 4

Regulations:

- **Grouping:** You are allowed to work in pairs.
- **Submission:** We provide a latex template for your solutions. Use that template and create a hw4.tar.gz file that includes hw4.tex and all other related files. Tar.gz file should not contain any directories and should create a *hw4.pdf* file with the following commands, otherwise you will get zero;
  tar xvzf hw4.tar.gz
  pdflatex hw4.tex
  Submit hw4.tar.gz to the COW page of the course.
- **Deadline:** 23:55, 2 June, 2019 (Sunday).
- **Late Submission:** Not allowed.

1. (30 pts) Consider an LTI system given by the following block diagram:



   where D is the unit-delay operator.

   (a) (5 pts) Find the difference equation which represents this system.

   (b) (5 pts) Find the frequency response of this system.

   (c) (10 pts) Find the impulse response of this system from its frequency response.

   (d) (10 pts) Find the output $y[n]$ for the input $x[n] = (\frac{1}{4})^n u[n]$ using the frequency response.

2. (10 pts) Suppose that two discrete-time LTI systems with the impulse responses $h_1[n]$ and $h_2[n]$ are connected in parallel. We have the following information about this combined system:

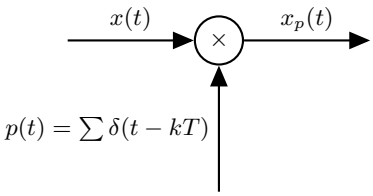
   i. The frequency response of the combined system is; $H(e^{j\omega}) = \frac{5e^{-j\omega}-12}{e^{-2j\omega}-7e^{-j\omega}+12}$.

   ii. The impulse response of the first system is: $h_1[n] = (\frac{1}{3})^n u[n]$.

   Find $h_2[n]$, the impulse response of the second system.

3. (30 pts) Consider the following sampling system where the input is

$$x(t) = \frac{\sin 2\pi t}{\pi t} + \cos 3\pi t$$

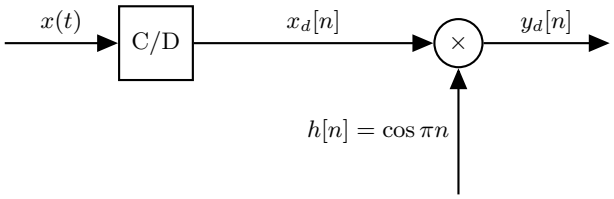


   (a) (10 pts) Find and plot the Fourier Transform of $x(t)$.

   (b) (10 pts) What is the Nyquist frequency and period for sampling?

   (c) (10 pts) Find and plot the Fourier Transform of $x_p(t)$ using the Nyquist rate.

4. (30 pts) Consider the following system with a C/D converter:



where the sampling frequency is $\omega_s = \pi$ and

$$X(j\omega) = \begin{cases} \frac{4}{\pi}\omega, & \text{if } |\omega| \leq \frac{\pi}{4} \\ 0, & \text{otherwise} \end{cases}$$

(a) (10 pts) Find $X_d(e^{j\omega})$.

(b) (10 pts) Find $H(e^{j\omega})$.

(c) (10 pts) Find $Y_d(e^{j\omega})$.