CENG 384 - Signals and Systems for Computer Engineers
Spring 2018-2019
Written Assignment 1

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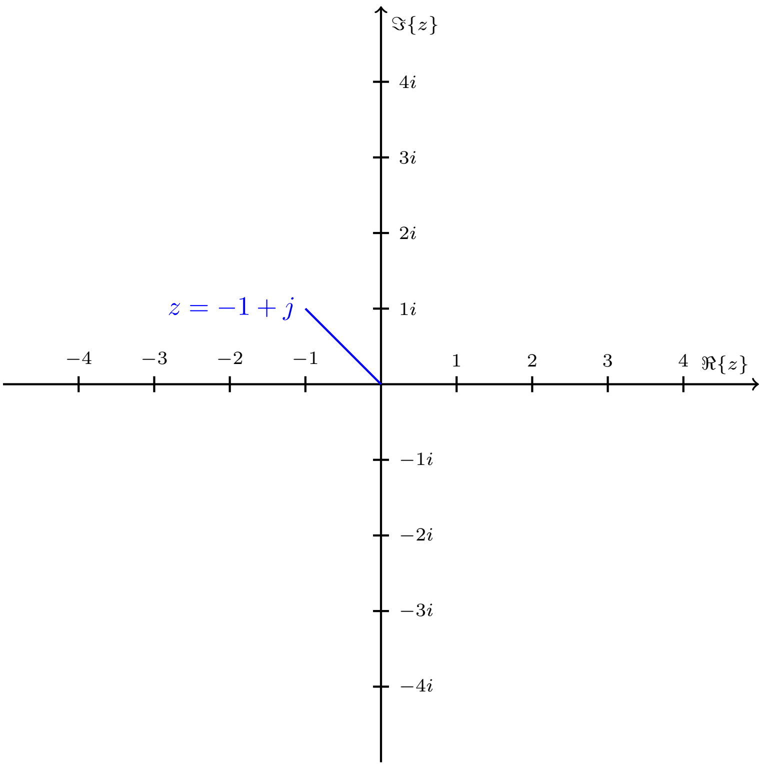
1. (a) Given $z = x + yj$ and $3z + 4 = 2j - \bar{z}$, finding $\bar{z}$ which is conjugate of z:
$\bar{z} = x - yj$
Applying the z and $\bar{z}$ to the given equation:
$3(x + yj) + 4 = 2j - (x - yj)$
$4x + 2yj = -4 + 2j$
$x = -1$
$y = 1$
Putting the values x and y in the z
$z = -1 + j$
$|z| = \sqrt{(-1)^2 + (1)^2} = \sqrt{2}$
$|z|^2 = 2$



(b) Given $z = re^{j\theta}$:
$z^3 = r^3 e^{j3\theta} = 64j$
This equation gives us:
$r = 4$,
$e^{j3\theta} = j$
Remembering equation ($e^{j\theta} = \cos\theta + j\sin\theta$):
$e^{j3\theta} = j = \cos\left(\frac{\pi}{2}\right) + j\sin\left(\frac{\pi}{2}\right)$
Therefore:
$3\theta = \frac{\pi}{2}$
$\theta = \frac{\pi}{6}$
z in polar form:
$z = 4(\cos\left(\frac{\pi}{6}\right) + j\sin\left(\frac{\pi}{6}\right))$

$z = 4e^{(j\frac{\pi}{6}+\frac{2\pi}{3}k)}$ where k=1,2,3,...

(c) Multiplying both numerator and denominator of given z with $(1-j)$:
$z = \frac{(1-j)^2(1+\sqrt{3}j)}{(1+j)(1-j)} = \frac{(1-j)^2(1+\sqrt{3}j)}{2} = \frac{(-2j)(1+\sqrt{3}j)}{2} = (-j)(1+\sqrt{3}j) = \sqrt{3} - j$

Finding magnitude r and angle θ of z:
$r = \sqrt{(\sqrt{3})^2 + (1)^2} = 2$
$\theta = 2\pi - \arctan\left(\frac{1}{\sqrt{3}}\right) = \frac{11\pi}{6}$

(d) Given z, $-j$ can be rewritten as:
$-j = \cos\left(\frac{3\pi}{2}\right) + j\sin\left(\frac{3\pi}{2}\right) = e^{j\frac{3\pi}{2}}$

Therefore z can be rewritten as:
$z = e^{j\frac{3\pi}{2}}e^{j\frac{\pi}{2}} = e^{j2\pi}$

2. Below is the signal for $y(t) = x(\frac{1}{2}t + 1)$

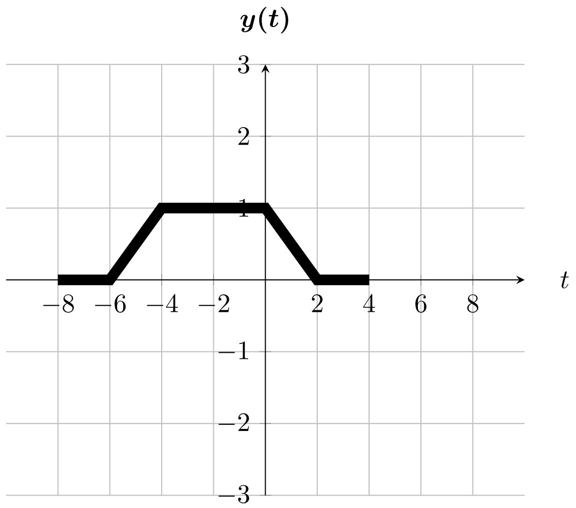


Figure 1: t vs. $y(t)$.

3. (a) Below is the signal for $x[-n] + x[2n+1]$

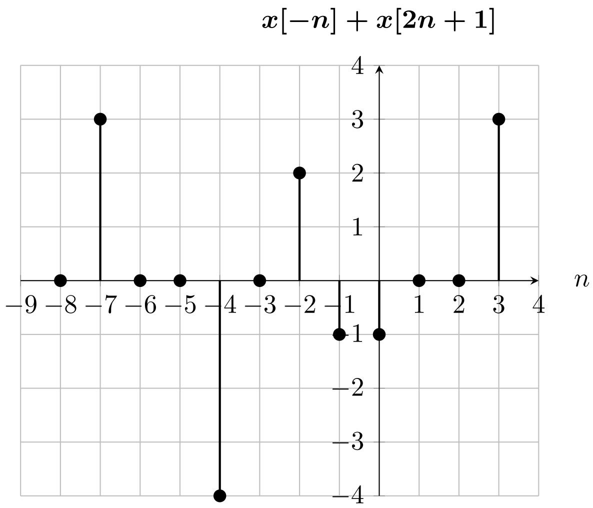


Figure 2: n vs. $x[-n] + x[2n+1]$.

(b) $x[-n] + x[2n+1]$ in terms of the unit impulse function is as follows:

$3\delta(n+7) - 4\delta(n+4) + 2\delta(n+2) - \delta(n+1) - \delta(n) + 3\delta(n-3)$

4. (a) To check if any signal is periodic or aperiodic we have the following:
$x(t)$ is periodic if $\exists T$ such that $x(t) = x(t+T)$ and $x(n)$ is periodic if $\exists N \epsilon I$ such that $x(n) = x(n+N)$.
In continious case, we can find fundamental period with $T = \frac{2\pi}{\omega}$ and in discrete case, we can find fundamental period with $N = \frac{2\pi}{\Omega}$. For parts a,b,c and d these information will be used.
We have:

$$x[n] = 3cos[\tfrac{13\pi}{10}n] + 5sin[\tfrac{7\pi}{3}n - \tfrac{2\pi}{3}]$$

This is a discrete signal, according to information mentioned above, fundamental period for $3cos[\frac{13\pi}{10}n]$ (Since this is a discrete signal we need an integer m to make N integer):

$$\begin{aligned} N_1 &= \frac{2\pi}{\Omega}m \\ N_1 &= \frac{2\pi}{\frac{13\pi}{10}}m \\ N_1 &= \frac{20}{13}m \qquad m = 13 \\ N_1 &= 20 \end{aligned}$$

Fundamental period for $5sin[\frac{7\pi}{3}n - \frac{2\pi}{3}]$ (Since this is a discrete signal we need an integer m to make N integer):

$$\begin{aligned} N_2 &= \frac{2\pi}{\Omega}m \\ N_2 &= \frac{2\pi}{\frac{7\pi}{3}}m \\ N_2 &= \frac{6}{7}m \qquad m = 7 \\ N_2 &= 6 \end{aligned}$$

To be able to find fundamental period for $x[n]$ we need to find $lcm(N_1, N_2)$:

$$\begin{aligned} N &= lcm(N_1, N_2) \\ N &= lcm(20, 6) \\ N &= 60 \end{aligned}$$

60 is the fundamental period of $x[n]$.

(b) We have:

$$x[n] = 5sin[3n - \tfrac{\pi}{4}]$$

This is also a discrete signal, according to information mentioned in part a, fundamental period can be found as:

$$\begin{aligned} N &= \frac{2\pi}{\Omega}m \\ N &= \frac{2\pi}{3}m \end{aligned}$$

There does not exist any integer m which can make $N = \frac{2\pi}{3}m$ integer, so we can safely say $x[n]$ is an aperiodic signal.

(c) We have:

$$x(t) = 2cos(3\pi t - \tfrac{2\pi}{5})$$

This is a continious signal, according to information mentioned in part a, fundamental period can be found as:

$$\begin{aligned} T &= \frac{2\pi}{\omega} \\ T &= \frac{2\pi}{3\pi} \\ T &= \frac{2}{3} \end{aligned}$$

$\frac{2}{3}$ is the fundamental period of $x(t)$.

(d) We have:

$$x(t) = -je^{j5t}$$

This is also a continious signal, according to information mentioned in part a, fundamental period can be found as:

$$T = \frac{2\pi}{\omega}$$
$$T = \frac{2\pi}{5}$$

$\frac{2\pi}{5}$ is the fundamental period of $x(t)$.

5. Let's first check whether the signal is even or odd. We see that $x[n]$ is neither even nor odd, since it does not have symmetry neither across y-axis nor across origin. To find even and odd decompositions of $x[n]$, we have:

$$x[n] = \text{Ev}\{\text{x[n]}\} + \text{Odd}\{\text{x[n]}\}$$
$$x[n] = \frac{1}{2}\{x[n] - x[-n]\} + \frac{1}{2}\{x[n] + x[-n]\}$$

So Ev{x[n]} can be drawn as:

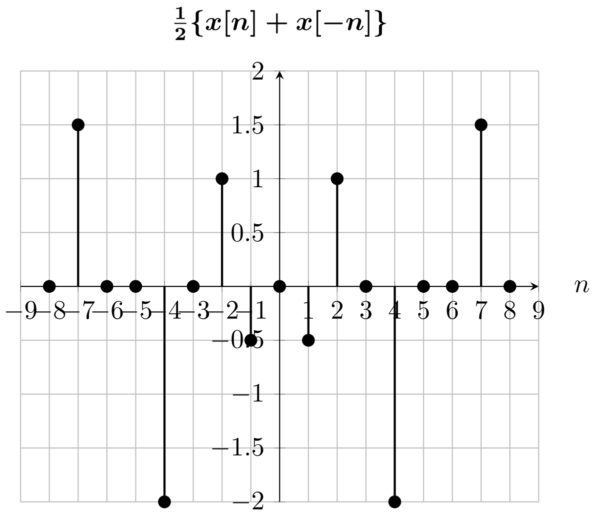


Figure 3: n vs. $\frac{1}{2}\{x[n] + x[-n]\}$.

and Odd{x[n]} can be drawn as:

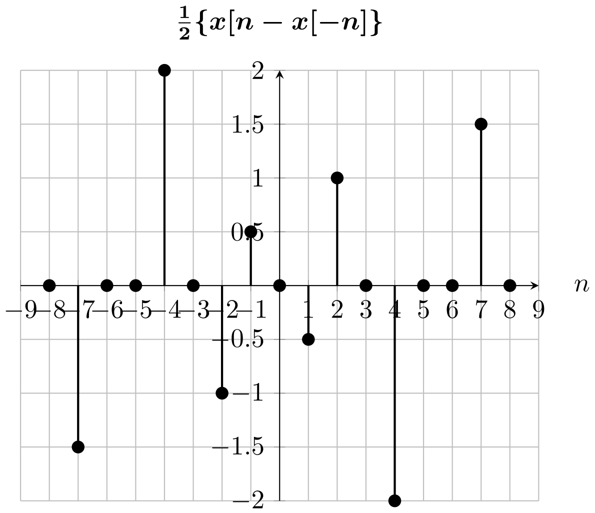


Figure 4: n vs. $\frac{1}{2}\{x[n] - x[-n]\}$.

6. (a) i. System needs memory since it needs to remember both past and future inputs for some of the outputs.

ii. The system is stable since its amplitude is bounded by a constant value and does not vary due to input.

iii. The System does *not* depend on only past and present inputs, it has some future inputs too, so this system is not causal.

iv. For system to be linear, it needs to hold superposition property. So we need to check if this system holds superposition property. Let x_1 and x_2 be two input signals:

$$y_1(t) = x_1(2t-3)$$
$$y_2(t) = x_2(2t-3)$$

When we add these up and multiply by some scalars a_1 and a_2, we will have a y_3 as:

$$\begin{aligned} y_3(t) &= a_1 \times y_1(t) + a_2 \times y_2(t) \\ &= a_1 \times x_1(2t-3) + a_2 \times x_2(2t-3) \end{aligned}$$

and on the other hand when we first perform addition and multiplication then put the signal as input to the system we will have a $y_3^{'}$ such as:

$$\begin{aligned} x_3(t) &= a_1 \times x_1(t) + a_2 \times x_2(t) \\ y_3^{'}(t) &= x_3(2t-3) \\ &= a_1 \times x_1(2t-3) + a_2 \times x_2(2t-3) \end{aligned}$$

Since $y_3 = y_3'$ superposition property holds and system is linear.

v. The system is invertible since:

$$\begin{aligned} x(t) &= h^{-1}(y(t)) \\ &= y(\frac{t+3}{2}) \end{aligned}$$

vi. We will check time invariance as follows:

$$\text{Let } x_1(t) = x(t-t_0)$$
$$\text{We will have } y(t) = x_1(2t-3)$$
$$\text{So } y(t) = x(2t-t_0-3)$$

On the other hand we have:

$$\begin{aligned} y^{'}(t) &= y(t-t_0) \\ &= x(2(t-t_0)-3) \\ &= x(2t-2t_0-3) \end{aligned}$$

Since $y(t) \neq y^{'}(t)$ system is time variant.

(b) i. System does not need memory since it does not need to remember neither past nor future inputs. So this system is memoryless.

ii. The system is not stable since its amplitude is not bounded and it varies due to input.

iii. Since, system depends on only the present outputs, so this system is causal.

iv. For system to be linear, it needs to hold superposition property. So we need to check if this system holds superposition property. Let x_1 and x_2 be two input signals:

$$y_1(t) = t \times x_1(t)$$
$$y_2(t) = t \times x_2(t)$$

When we add these up and multiply by some constants a_1 and a_2, we will have a y_3 as:

$$\begin{aligned} y_3(t) &= a_1 \times y_1(t) + a_2 \times y_2(t) \\ &= a_1 \times t \times x_1(t) + a_2 \times t \times x_2(t) \end{aligned}$$

and on the other hand when we first perform addition and multiplication then put the signal as input to the system we will have a $y_3^{'}$ such as:

$$\begin{aligned} x_3(t) &= a_1 \times x_1(t) + a_2 \times x_2(t) \\ y_3^{'}(t) &= t \times x_3(t) \\ &= a_1 \times t \times x_1(t) + a_2 \times t \times x_2(t) \end{aligned}$$

Since $y_3 = y_3'$ superposition property holds and system is linear.

v. The system is invertible since:

$$x(t) = h^{-1}(y(t))$$
$$= \frac{1}{t} \times y(t)$$

vi. We will check time invariance as follows:

$$\text{Let } x_1(t) = x(t - t_0)$$
$$\text{We will have } y(t) = t \times x_1(t)$$
$$\text{So } y(t) = t \times x(t - t_0)$$

On the other hand we have:

$$y^{'}(t) = y(t - t_0)$$
$$= (t - t_0) \times x(t - t_0)$$

Since $y(t) \neq y^{'}(t)$ system is time variant.

(c) i. System needs memory since it needs to remember both past and future inputs for some of the outputs.

ii. The system is stable since its amplitude is bounded by a constant value and does not vary due to input.

iii. The System does *not* depend on only past and present inputs, it has some future inputs too, so this system is not causal.

iv. For system to be linear, it needs to hold superposition property. So we need to check if this system holds superposition property. Let x_1 and x_2 be two input signals:

$$y_1[n] = x_1[2n - 3]$$
$$y_2[n] = x_2[2n - 3]$$

When we add these up and multiply by some constants a_1 and a_2, we will have a y_3 as:

$$y_3[n] = a_1 \times y_1[n] + a_2 \times y_2[n]$$
$$= a_1 \times x_1[2n - 3] + a_2 \times x_2[2n - 3]$$

and on the other hand when we first perform addition and multiplication then put the signal as input to the system we will have a $y_3^{'}$ such as:

$$x_3[n] = a_1 \times x_1[n] + a_2 \times x_2[n]$$
$$y_3^{'}[n] = x_3[2n - 3]$$
$$= a_1 \times x_1[2n - 3] + a_2 \times x_2[2n - 3]$$

Since $y_3 = y_3'$ superposition property holds and system is linear.

v. The system is invertible since:

$$x[n] = h^{-1}(y[n])$$
$$= y[\frac{n+3}{2}]$$

vi. We will check time invariance as follows:

$$\text{Let } x_1[n] = x[n - n_0]$$
$$\text{We will have } y[n] = x_1[2n - 3]$$
$$\text{So } y[n] = x[2n - n_0 - 3]$$

On the other hand we have:

$$y^{'}[n] = y[n - n_0]$$
$$= x[2(n - n_0) - 3]$$
$$= x[2n - 2n_0 - 3]$$

Since $y[n] \neq y^{'}[n]$ system is time variant.

(d) i. System needs memory since its output depends on the sum of all its past values of input.

ii. The system is not stable since its amplitude is not bounded and it varies due to input. More specifically, even though each individual signal that makes up the sum is stable, their sum makes $y[n]$ unbounded since it goes up to ∞ .

iii. The system depends on only the past inputs, so this system is causal.

iv. For system to be linear, it needs to hold superposition property. So we need to check if this system holds superposition property. Let x_1 and x_2 be two input signals:

$$y_1[n] = \sum_{k=1}^{\infty} x_1[n-k]$$
$$y_2[n] = \sum_{k=1}^{\infty} x_2[n-k]$$

When we add these up and multiply by some constants a_1 and a_2, we will have a y_3 as:

$$\begin{aligned} y_3[n] &= a_1 \times y_1[n] + a_2 \times y_2[n] \\ &= a_1 \times \sum_{k=1}^{\infty} x_1[n-k] + a_2 \times \sum_{k=1}^{\infty} x_2[n-k] \end{aligned}$$

and on the other hand when we first perform addition and multiplication then put the signal as input to the system we will have a $y_3^{'}$ such as:

$$\begin{aligned} x_3[n] &= a_1 \times x_1[n] + a_2 \times x_2[n] \\ y_3^{'}[n] &= \sum_{k=1}^{\infty} x_3[n-k] \\ &= a_1 \times \sum_{k=1}^{\infty} x_1[n-k] + a_2 \times \sum_{k=1}^{\infty} x_2[n-k] \end{aligned}$$

Since $y_3 = y_3'$ superposition property holds and system is linear.

v. The system is invertible since:

$$\begin{aligned} x[n] &= h^{-1}(y[n]) \\ &= y[n+1] - y[n] \\ &= \{x[n] + x[n-1] + ...\} - \{x[n-1] + x[n-2] + ...\} \end{aligned}$$

vi. We will check time invariance as follows:

Let $x_1[n] = x[n-n_0]$
We will have $y[n] = x_1[n]$
So $y[n] = \sum_{k=1}^{\infty} x[n-n_0-k]$

On the other hand we have:

$$\begin{aligned} y^{'}[n] &= y[n-n_0] \\ &= \sum_{k=1}^{\infty} x[n-n_0-k] \end{aligned}$$

Since $y[n] = y^{'}[n]$ system is time invariant.