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CSC226: Lab 8

1 Linear Select and Stack of Bricks Analysis

In class, we learned linear select with the median of medians pivot, where we took the median of the median of groups of 5 elements, and there are $n/5$ such medians. The runtime analysis of this algorithm led us to the following recurrence relation:

$$T(n) = T(n/5) + T(7n/10) + 5n$$

1.1 Stack of Bricks Analysis

For a general α (ratio of medians) and corresponding β (ratio of largest remaining group in the worst case), the recurrence relation has the form:

$$T(n) = T(n/5) + T(7n/10) + cn$$

where cn is some constant factor of n.

We will show another way to determine the runetime of the recurrence relation above by using the "Stack of Bricks" analysis (which will be shown in the lab). With appropriate α and corresponding β values, the selection process for medians of medians pivot takes $O(n)$ time.

2 Exercises

For the lab submission, please submit your solutions to the following two questions in a single PDF file.

1. Find the recurrence relation when you have groups of medians of size 3.
2. Show that the best upper bound you can find on the runtime complexity for the recurrence relation found in Exercise 1 is $O(n \log n)$

1. $T(n) = T(n/3) + T(2n/3) + 3n$

$n - \frac{(n/2)}{3} \cdot 2$

$n - \frac{2n}{6}$

2. $T(n) \le Cn$

$T(n) = 3n + C \cdot \frac{n}{3} + C \cdot \frac{2n}{3} \le Cn.$

since k a pivot

$T(n) = C(n \cdot k)$

$\frac{n}{3^k} = 1$

$k = \log_3 n$

$T_n = Cn \, C\log_3 n)$

$= O(n \log n)$

$T(n)$

$T(n/3) \quad T(2n/3) \quad 3n \quad \le cn.$

since we guess $T_n \le (cn)$

$T(n/3) + T(2n/3) + 3n \le cn$

$T(n/3) \quad T(2n/3)$

$T(\frac{n}{3^2}) \quad T(\frac{2n}{3^2}) \quad (n/3) \quad T(\frac{2n}{3}) \quad T(\frac{4n^2}{3}) \quad c(\frac{2n}{3}) \le cn.$