Results of project-1

By Diego Sic

In the last week at the Computer Science class about voting theorem, I have been learning about Social Choice Function, tournaments, and preferences and pairwise majority. This project pretends to give answer to 4 main questions using an Impartial Culture to vote.

Before starting to answer those questions, I see prudent to determine a certain number of terms for the rest of the explanation. Condorcet is a social choice function defined like this:

A Condorcet winner for a profile P is an alternative x that defeats every other alternative in the strict pairwise majority sense: for all every Pairwise Majority Rule, henceforth PMR, declares the winning alternative to be the Condorcet winner and is undefined when a profile has no Condorcet winner.

And the relationship between Copeland's Social choice function and Condorcet is this:

The Copeland score of a Condorcet winner is (m being the number of alternatives) and uniquely highest, so Copeland is a Condorcet extension. Other well-known Condorcet extensions include.

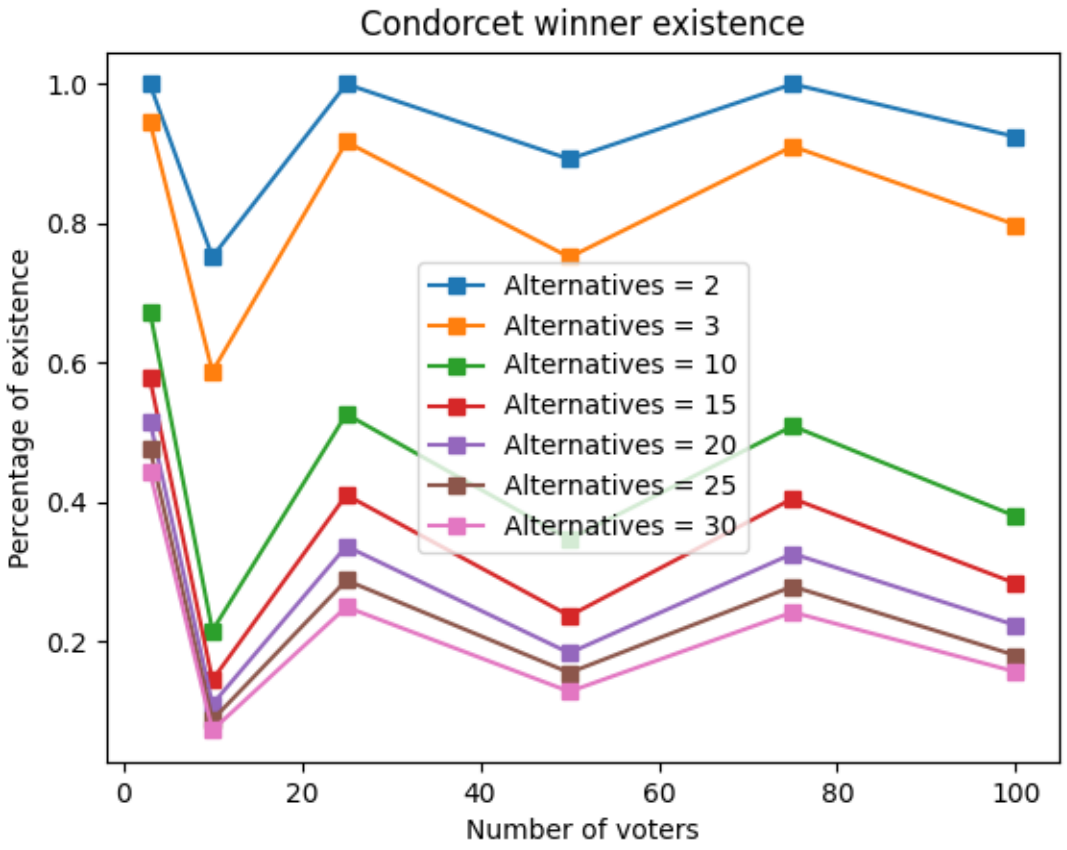
This can also be interpreted as saying that Copeland is Condorcet efficient, which means that if a Condorcet winner exists, Copeland is going to choose it.

With everything being said, we can start answering the questions.

Collection of Data

The number of simulations is 10,000 for each configuration of candidates an alternatives. The setup of alternatives is [2,3,10,15,20,25,30], and the number of voters is [3, 10, 25, 50, 75, 100].

How often does a Condorcet winner exist under Impartial Culture?

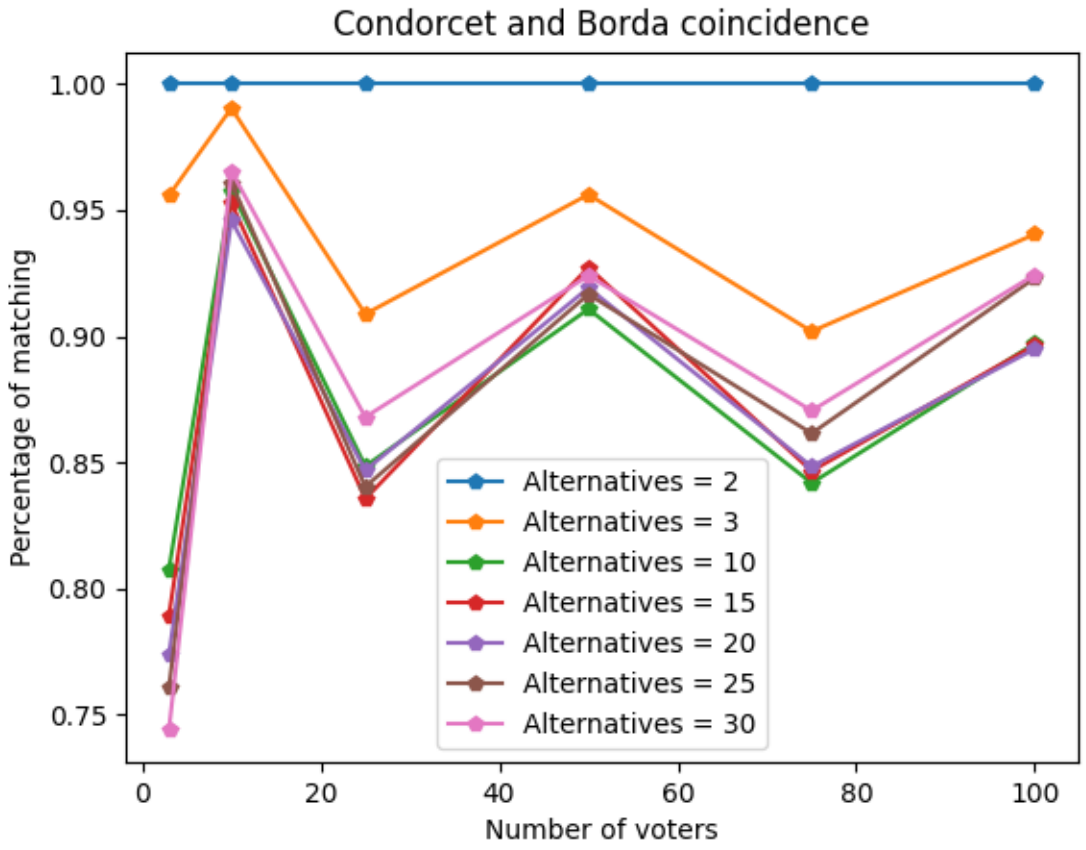


The first point to highlight here is what happens with Condorcet existence when we have 2 alternatives and an odd number of voters, Condorcet winner always exists. After proving this theorem, we can see how for each alternative, the Condorcet winner's existence slowly increases when the number of voters increases. We can also notice that the number of Condorcet winners increases when the number of voters is odd, as in the case of 3, 25, 75. Then we can notice that the amount of Condorcet winners decreases significantly when the number of alternatives increases.

Then we can say that the Condorcet winner percentage of existence, when the number of alternatives is fixed, will be directly proportional to the number of voters and will have some peaks if the number of voters is odd.

Then if we analyze the Condorcet winner percentage of existence when the number of voters is fixed, we can say that it will be inversely proportional to the number of alternatives.

How often does Borda choose the Condorcet winner under I.C.?

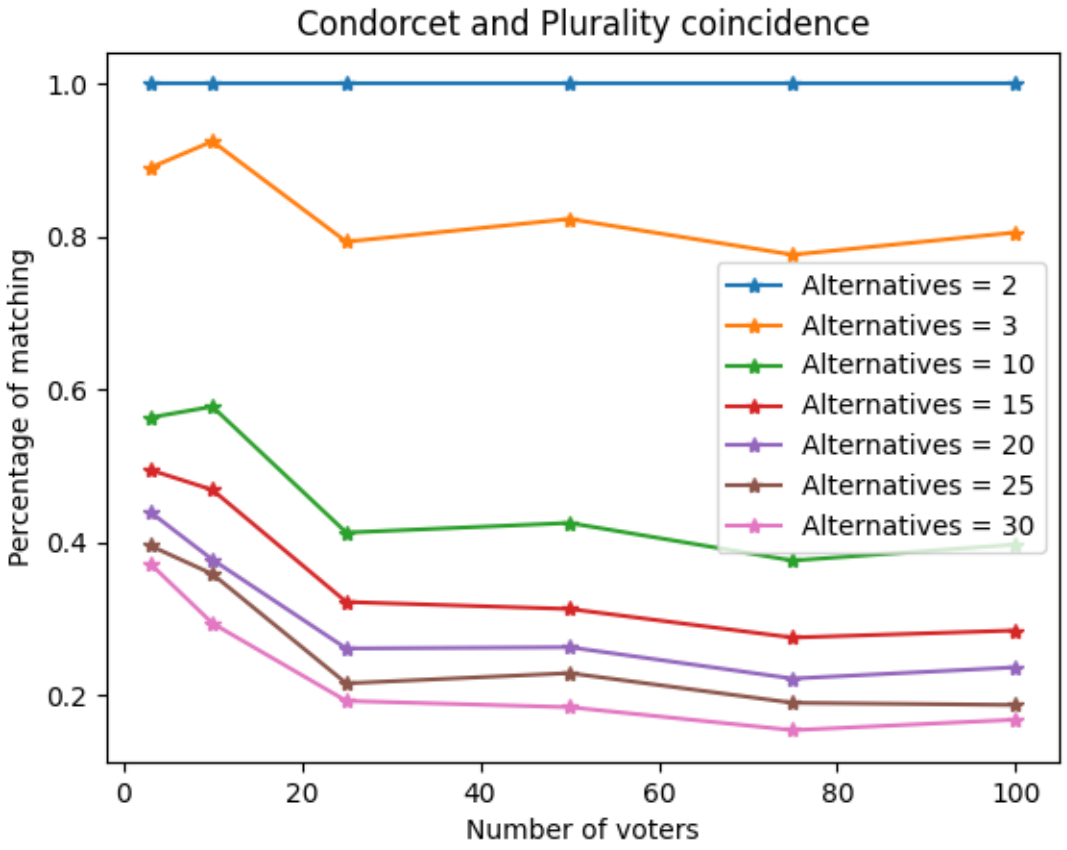


We can see that when we have 2 alternatives, it doesn't matter the number of voters, Borda will always choose the same as Condorcet. Then we can point out that if the number of voters is even or odd, Borda and Condorcet will agree more or less. If the number of voters is odd, Borda and Condorcet will agree less than if the number of voters is even. Finally, we can notice that when the number of alternatives increases, the number of matches between Condorcet and Borda decreases overall.

We can conclude that Condorcet percentage of matching with Borda, when the number of alternatives is fixed, will depend on if the number of voters is even or odd. If the number of voters is odd, Borda and Condorcet will agree less than if the number of voters is even. But overall, it will decrease as the number of voters increase.

Then if we analyze Condorcet percentage of matching with Borda when the number of voters is fixed, we can say that it will decrease when the number of alternatives increases.

How often does Plurality choose the Condorcet winner under I.C.?



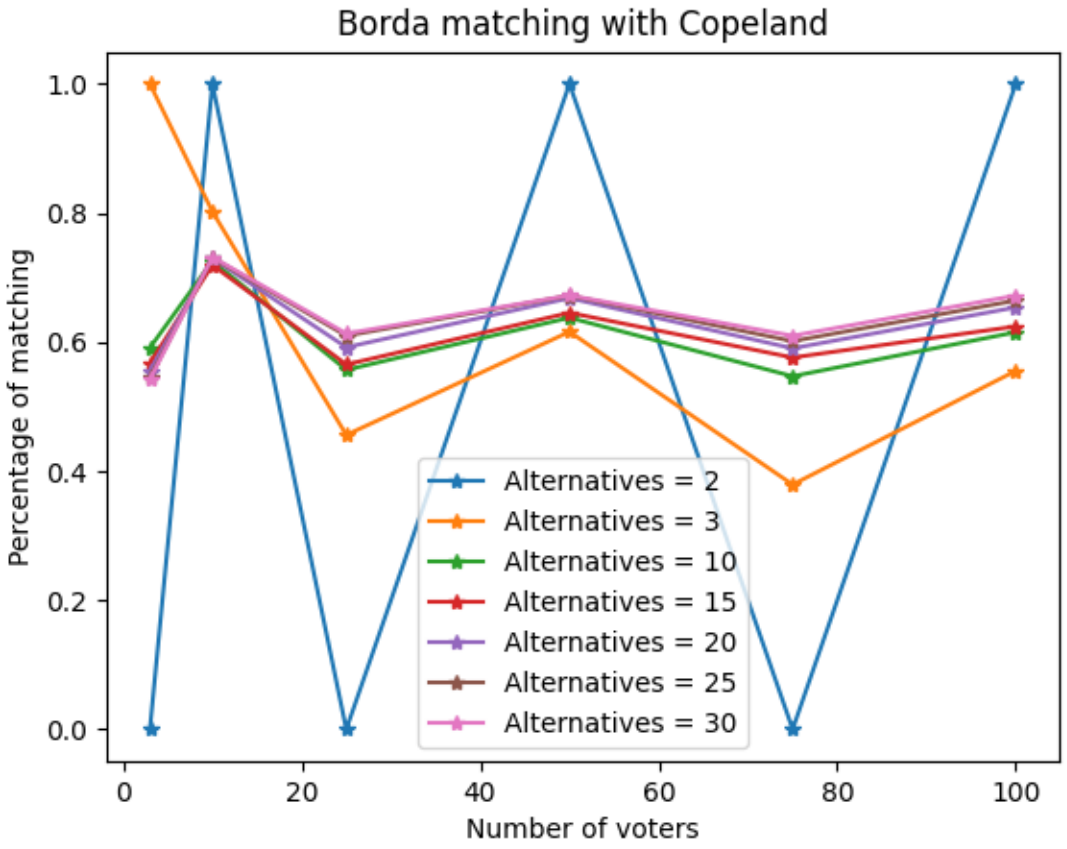
We can see that when we have 2 alternatives, it doesn't matter the number of voters, Plurality and Condorcet always match. Besides that outlier, we can notice that if the number of voters is even or odd affects the percentage of the match. If the number of voters is odd, the percentage of matching will be less than when the number of voters is even.

We can conclude that the Condorcet percentage of matching with plurality, when the number of alternatives is fixed, will depend on if the number of voters is even or odd. If the number of voters is odd plurality and Condorcet will agree less than if the number of voters is even. But overall, it will decrease as the number of voters increase.

Then if we analyze the Condorcet percentage of matching with plurality when the number of voters is fixed, we can say that it will decrease when the number of alternatives increases.

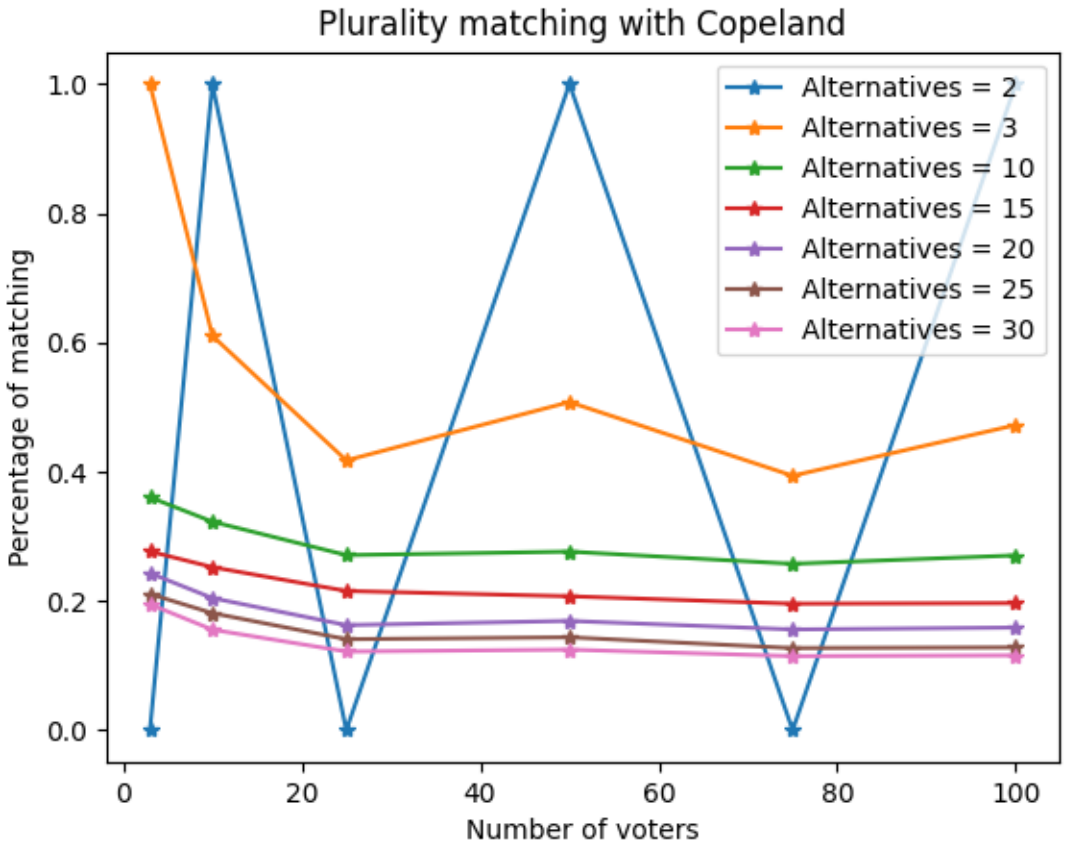
When the Condorcet winner does not exist, do Borda, plurality, and Copeland select the same alternative? How often do they coincide with each other?

4.1 How often Copeland and Borda chose the same alternative?



We can see an interesting behavior here. In the case where are 2 alternatives and the number of voters is odd will always have a Condorcet winner, that's why we can see some outliers in the blue line. We can see a very consistent behavior as the number of alternatives increases. In that consistent behavior, we can notice how when the number of voters is even, Borda and Copeland match more than when the number of voters is odd. The percentage of matching seems to be between 0.5 to 0.75.

4.2 How often Copeland and plurality chose the same alternative?

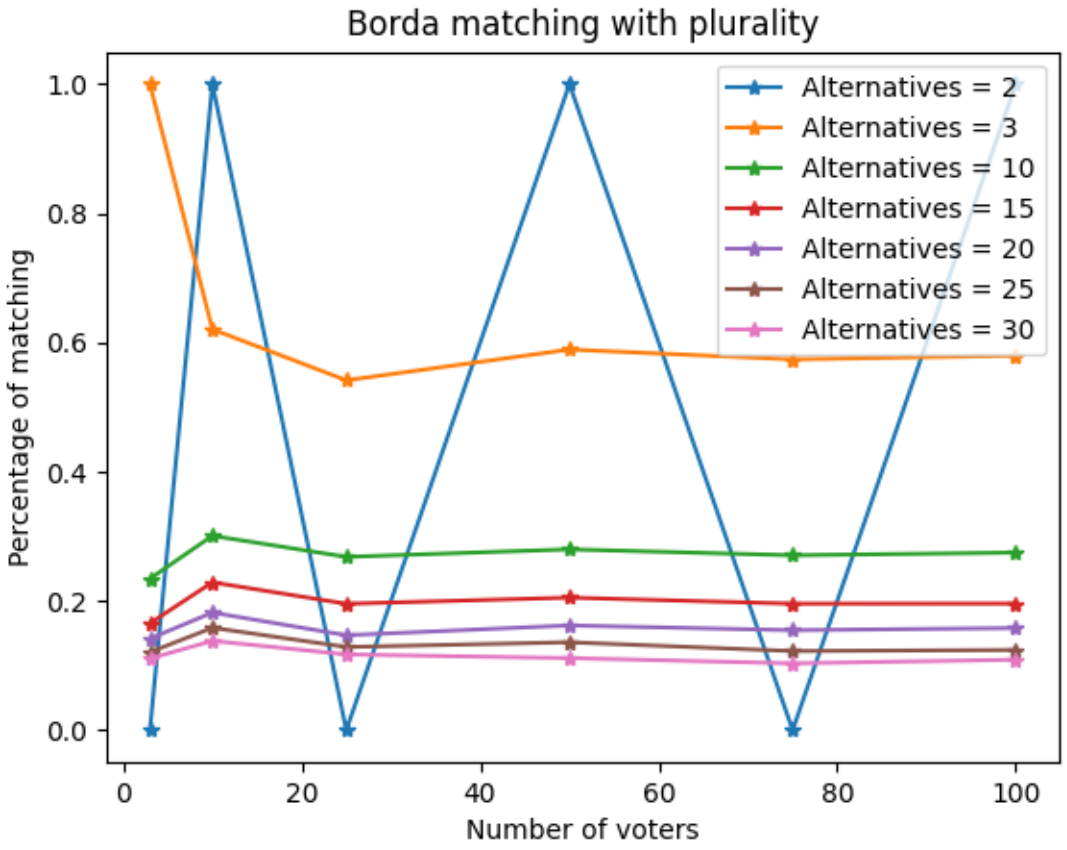


As in the previous case, the blue line (2 alternatives) has some outliers for the same reasons mentioned before. Besides the peculiar case of 2 alternatives, we can notice a pattern. If we take more and more alternatives, plurality and Copeland start to match less and less. The number of voters also affects the percentage of matching, if the number of voters is even, the percentage of matching is more than when the number of voters is odd.

Then we can say that Copeland's percentage of matching with plurality, when the number of alternatives is fixed, will depend on if the number of voters is even or odd. If the number of voters is odd, plurality and Copeland will agree less than if the number of voters is even. But overall, it will decrease as the number of voters increases.

Then if we analyze Copeland percentage of matching with plurality when the number of voters is fixed, it will decrease when the number of alternatives increases.

4.3 How often Copeland and plurality chose the same alternative?



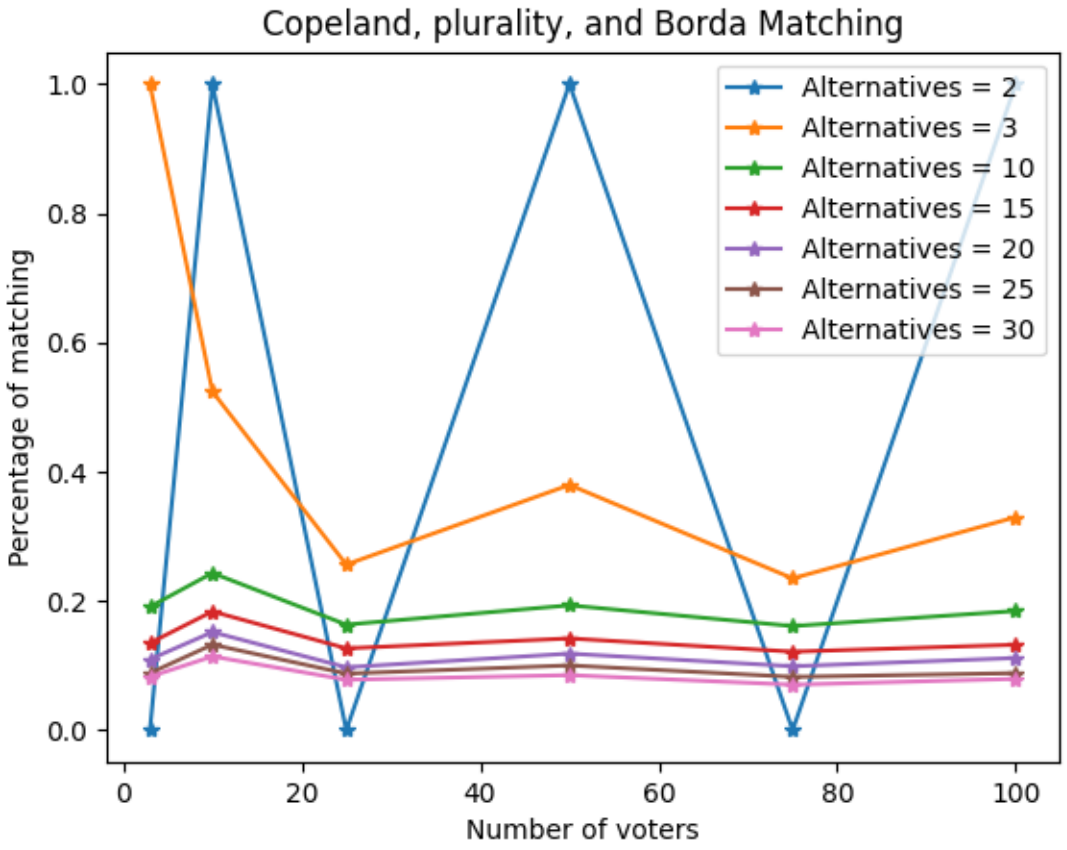
As in the previous 2 cases, the blue line has the same behavior. We can see that consistent behavior seems to appear. If we take more and more alternatives, plurality and Borda start to match less and less.

Also, we can notice that the number of voters seems to don't change the percentage of matching.

Then we can say that the Plurality percentage of matching with Borda, when the number of alternatives is fixed, doesn't have any extreme change as the number of voters increases if the number of alternatives is big.

Then if we analyze the plurality percentage of matching with Borda, when the number of voters is fixed, we can say that it will decrease when the number of alternatives increases.

4.4 How often Copeland, plurality and Borda chose the same alternative?



Ignoring the blue line, which is a special case, we can see a similar behavior as Borda-Plurality. Still, in this case, the matching percentage is less than Borda-Plurality, and an even and odd number of voters matters. If there's an even number of voters, the Borda-Plurality-Copeland seems to match more than when the number of voters is odd.

We can conclude that Borda-Plurality-Copeland, when the number of alternatives is fixed, will depend on if the number of voters is even or odd. If the number of voters is odd, Borda-Plurality-Copeland will agree less than if the number of voters is even. But overall, it will decrease as the number of voters increase.

Then if we analyze Copeland percentage of matching with plurality when the number of voters is fixed, we can say that it will decrease when the number of alternatives increases.

Conclusions

- Condorcet winner tends to exist more with less alternatives and an odd number of voters
- Condorcet and Borda agree more when there's an even quantity of voters.
- Condorcet and Plurality will agree less as the number of alternatives increases.
- When Condorcet do not exist, Copeland and Borda will agree more if the number of voters is even.
- When Condorcet do not exist, Plurality and Copeland will agree less as the number of alternatives increases.
- When Condorcet do not exist, Borda and Plurality will agree less as the number of alternatives increases.
- When Condorcet do not exist, Copeland, Plurality, and Borda will agree less as the number of alternatives increases, and will have their peaks of matching with an even amount of voters with a maximum of approximately 25 % of matching.