Arithmetical Procedure in Minoan Linear A and in Minoan-Greek Linear B*

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The study of how the Minoan numerical system could be used in arithmetical computations has not been attempted before, I believe. The study of how the Roman numerals could be used in arithmetic has recently been completed by the author.[1] The Minoan numerals follow the same general rules as Roman numerals. In this article the processes of addition, subtraction, multiplication, division, and extraction of square roots will be explained.

It will be convenient to use Linear B, since Linear A numerals were substantially the same (*SM* II 51 shows the slight differences). Almost the whole article applies to Linear A equally well.[2]

The computations, with one exception, will be theoretical and abstract, i.e. they will not be problems drawn from actual Minoan tablets. In the second half of the article, something will be said about the actual use of arithmetic by the users of Linear A and B.

Addition is extremely simple. It involves only the process of counting. If ooo=||| (323) is to be added to, say, oo==| (241), it is only necessary to count how many times each of the separate symbols is used in the two numbers, and the accumulated sum is the answer. To illustrate: there are a total of five hundreds (ooooo), six tens (≡≡), and four units (||||) in the above two numbers. Therefore the sum of these two numbers is ooooo≡≡|||| (564).

If a whole column of numbers is to be added, the process is the same.

Take, for example, an actual tablet, Cn04 (E. L. Bennett, *The Pylos Tablets*). In adding the column of symbols on the right, no intermediate steps need be written down. Any person, with a few minutes' training, can write down the answer directly from the tablet.

The procedure is as follows: First, count up all the units. There are 12. Write down ||. This is the units part of the answer. Now, count up the tens starting with two, since we have one ten from the units. We count 43 tens. Write down ≡. This is the tens part of the answer. Now count up the hundreds, beginning the counting with five since we have four from the tens. We count 10 hundreds. Therefore the answer is -ϕ-≡||.

With a little practice, this process can be accomplished much faster and more easily than one can add up a similar column of Hindu-Arabic numbers.

Subtraction involves the process of cancellation. The subtrahend (lower number) is cancelled, symbol by symbol, from the minuend (upper number). The symbols remaining in the minuend after cancellation constitute the answer. For example, if oo≡≡|| (262) is to be subtracted from ooo≡≡|||||||| (368), cancellation of the former from the latter leaves one hundred (o), no tens, and six units (||||||); or o|||||| (106).

If there are not sufficient symbols in the minuend for cancellation (i.e., if there are, say, seven tens in the subtrahend but only two tens in the minuend), then the next larger symbol in the minuend is written as ten of the smaller units, whereupon cancellation takes place as before. For example, if o≡≡≡ is to be subtracted from ooo=-, then one of the hundreds (o) from the ooo=- is written as

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[1] W. French Anderson, "Arithmetical Computations in Roman Numerals," CP 51 (1956) 145-150.

[2] The numerals are as follows:

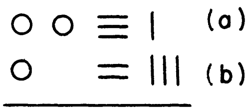
| one
— ten
o one hundred
ϕ one thousand
ϕ ten thousand

They are given in E. L. Bennett, Jr., *Minoan Linear B Index* (1953) p. 107; also in A. J. Evans-J. L. Myres, *Scripta Minoa* II, p. 51. Groupings of the numerals to make numbers are studied in S. Dow, *AJA* 58 (1954) 124-125. Cf. also note 3.

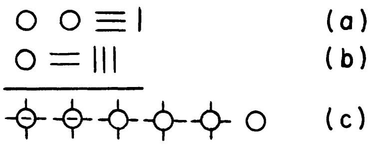
tens. Now the problem is ○○≡≡≡≡⁻ minus ○≡≡≡, which is ○≡⁻. If there are not sufficient symbols in the minuend for cancellation, then subtraction cannot take place (without involving negative numbers).

Multiplication, taken step-by-step, is almost as simple as addition and subtraction. This process consists of multiplying each symbol of the multiplicand by each symbol of the multiplier. The multiplying proceeds from left to right, instead of from right to left as in our Hindu-Arabic system. When ○○≡| (231) is multiplied by ○=||| (123), the operation is as follows:

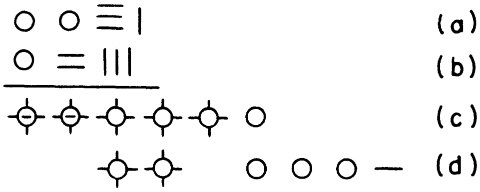
1) Set up the problem:



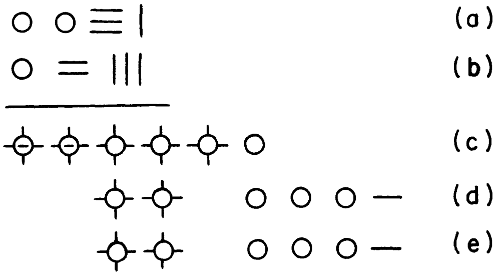
2) Multiply each symbol in (a) by ○, because it is the first symbol of (b):



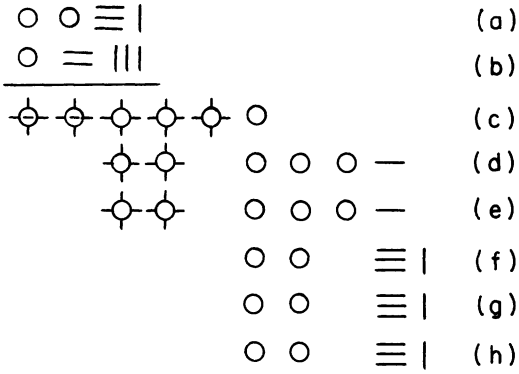
3) Multiply (a) by — because it is the second symbol of (b):



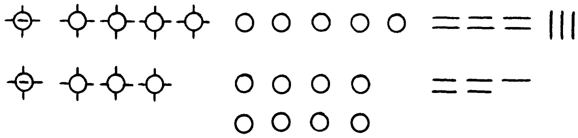
4) Multiply (a) by —, which is the third symbol of (b):



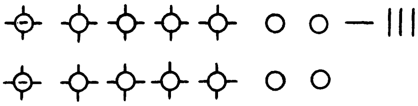
5),6),7) Multiply (a) three successive times by |, which occurs three times as the remaining symbols of (b):



8) Now add (c), (d), (e), (f), (g), and (h) in the way explained above.



When more than ten symbols of any denomination appear, ten of the smaller symbols are changed to the next larger unit. Therefore the final answer would be:



It might be interesting to compare the above computation with the similar computation of the same problem in our own system:

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  231
  123
  ---
  693
 462
231
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28413
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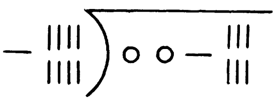
The basic operations in the multiplication of two numbers in Minoan are identical with the operations used in our system (or, for that matter, in any other system), namely, each symbol of the first number is multiplied individually by each symbol of the second number, then the individual results are added up to give the final answer.

Division, in Minoan numerals, has an advantage which is shared only by non-place-value numerical systems like itself. This advantage, over place-value systems, is that there need be no figuring-in-the-head to determine exactly how many times the divisor will go into the dividend. Contrast the procedure used by us in Arabic numerals. When we divide 216 by 18 (Arabic numerals) we have to use 1 (actually 10) as the first number of the quotient. Then, we determine that 2 is the second number of the quotient, thus finding 12 as the answer.

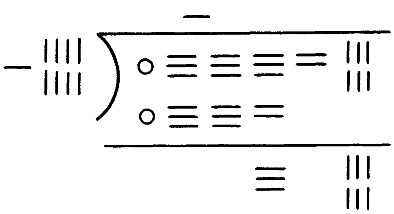
In Minoan, however, when dividing ○○—$\frac{|||}{|||}$ (216) by —$\frac{||||}{||||}$ (18), it is not necessary to make — the first number of the quotient. In this simple problem, the obvious number to use is —, but in a more complicated problem it cannot always be seen immediately which number would be the best to use as the first number of the quotient. In such a case, any first number is used, then repeated if necessary. The operation of successive cancellations merely continues until no more cancellation is possible. The answer is then complete. In the Hindu-Arabic system, as we customarily divide, such procedure is impossible, as pointed out above, for the exact value must be found for each place of the quotient. In the above problem, |, or $\frac{|||}{|||}$, or |||, etc., could be used. This characteristic will be illustrated by carrying out the division of the above problem in several different ways.

The most obvious way of working this problem would be as follows:

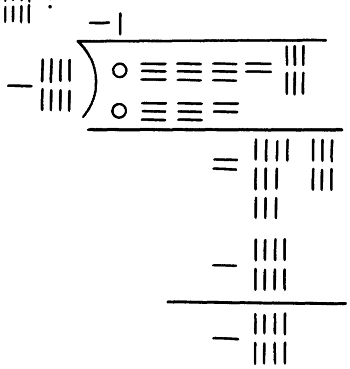
1) Set up the problem:



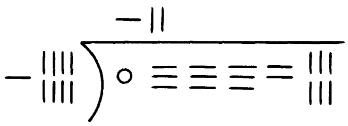
2) Find some number which, when multiplied by the divisor, can be cancelled from the dividend. Here — is the largest (and, therefore, best) such number. Multiply —$\frac{||||}{||||}$ by —, obtaining ○ ☰☰. Subtract this product from the dividend (○ is changed to ☰☰):



3) Repeat the process described in (2): | times —$\frac{||||}{||||}$ gives —$\frac{||||}{||||}$. Cancellation (subtraction) leaves —$\frac{||||}{||||}$:

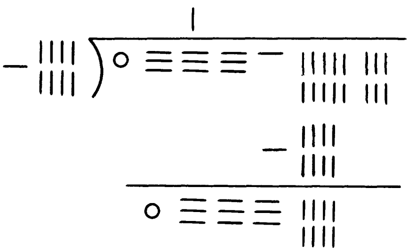


4) The final number of the quotient is |. Cancellation is complete:

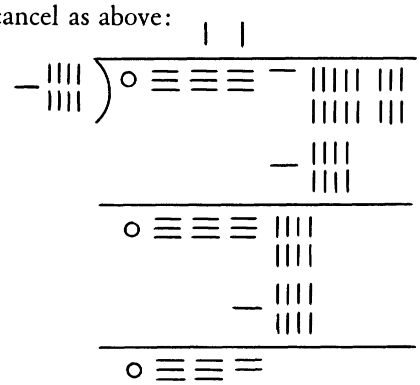


We could have used a different number as the first number of the quotient so long as it satisfied the requirement set forth in step (2) above. Let us use | as the first number of the quotient. Then, after setting up the problem, division could take place as follows:

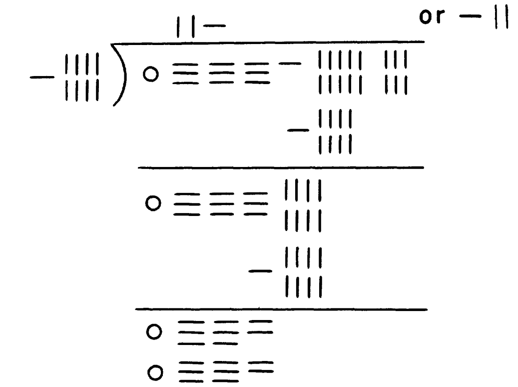
1) | is multiplied by the divisor and this product is cancelled from the dividend:



2) Again we add | to the quotient, and multiply and cancel as above:

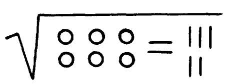


3) Now, seeing that the quotient is still much too small, we can try —. If this is too large we need only go back to the units. However, cancellation shows that — completes the answer exactly. The answer once again is —||. If there is a remainder, it is simply placed with the final quotient to show that the answer did not come out even. Notice that ||— is the same as —||. There is no subtractive principle in Minoan numerals as there is in Roman numerals (where IV is not the same as VI).

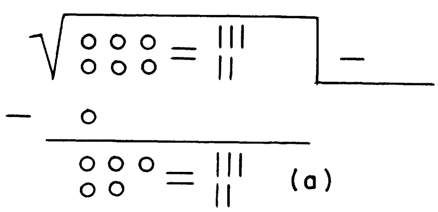


Even square roots can be taken in Minoan. The finding of the square root of ○○○○○○ = ||||| (625) is illustrated below. The operation is quite similar to the method of extracting square roots in our own Hindu-Arabic System.

1) Set up the problem:

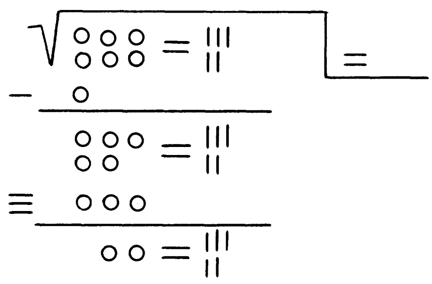


2) Find a number which, when multiplied by itself (i.e. squared), can be cancelled from the dividend. — is the largest such number. — times — gives ○. ○ subtracted from the dividend leaves ○○○○○ = |||||:

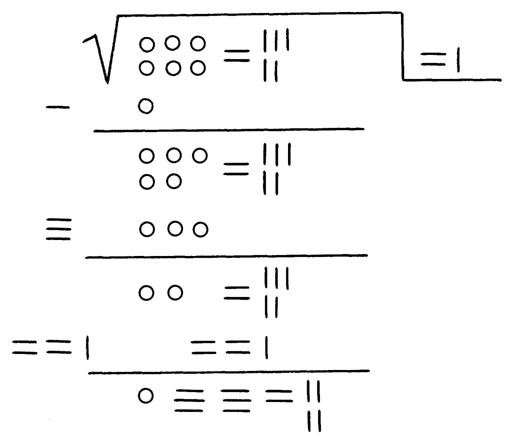


(a)

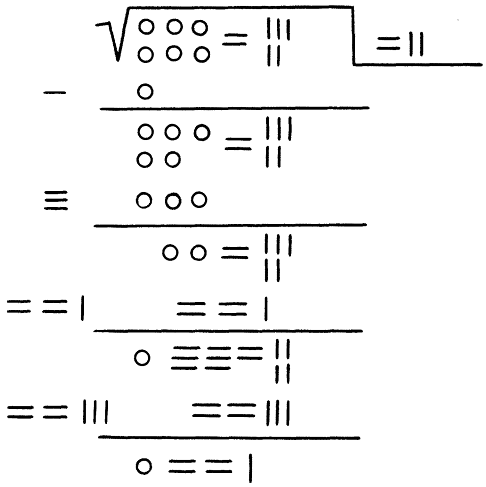
3) Find a number which, when multiplied by the quotient doubled plus the number, can be cancelled from the remaining dividend (a). (The temporary multiplicand is placed to the left of the problem.) — is such a number, for — times ☰ (the quotient doubled with the number, —, added to it) gives ○○○, which can be cancelled from the dividend to give ○○ = |||||



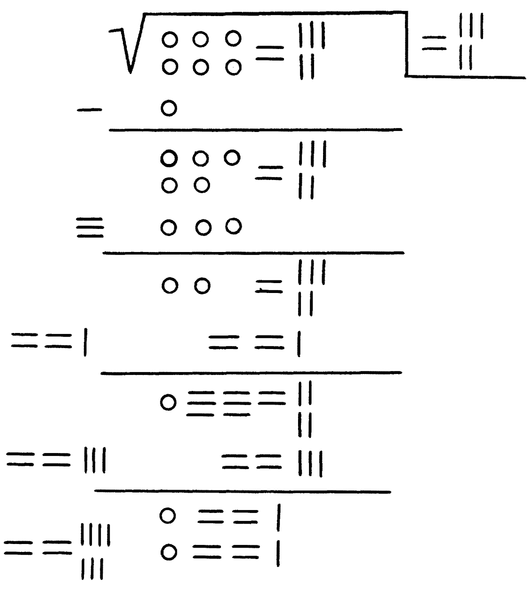
4) Repeat the above process using as many symbols as necessary in the quotient until no further cancellation is possible. | times = = | (quotient doubled, plus the number), gives = = |, which can be cancelled from the dividend:



5) Repeat 4) using | again. This time the quotient doubled, plus the number, is ==||| :



6) Step 4) could be repeated until cancellation could proceed no further. However, to hasten the solution, and by doing a little scratch work on the side, two or more symbols can be added to the quotient at one time. Here, by adding ||| to the quotient, the solution is obtained as follows: ||| times ==|||| gives o==| . This product cancels the dividend exactly:



It is obvious, of course, that the above problem could have been shortened by using ||||| in step 4) instead of proceeding with one | at a time as we did (for illustrative purposes).

If the reader is not acquainted with the process of extraction of square roots, he will probably have a great deal of difficulty in trying to follow the above operations. If, however, he is familiar with the Hindu-Arabic process, he will recognize the very great similarities between the above computation and that of the extraction of the square root of the same number in our system:

```
   √625   | 25
 2  4
   225
45 225
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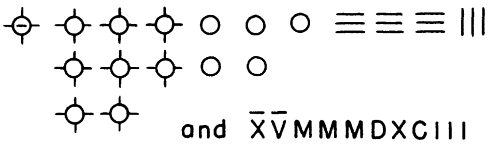
As can be seen, the Minoan system, like all other non-place systems, does not offer any real obstacle to performing the fundamental operations. The apparent difficulty lies only in the unfamiliarity of the symbols and in the few, but significant, differences involved in operating in a non-place-value numerical system rather than in our more familiar place-value system.

It is interesting to note several points of contrast between operations performed in Minoan and in our Hindu-Arabic system, and between Minoan and Roman numerals (the latter is also a non-place-value system like Linear B).

Minoan and Roman computations are practically identical. The major differences are as follows. First, the Minoan operation is a little more cumbersome, since in certain cases it works with up to nine symbols of the same denomination, whereas the Roman system provides symbols for 5, 50, etc., so that four is the largest number of the same type symbol used in any case, and the subtractive principle reduces this number to three. Secondly—and this is an advantage of the Minoan over the Roman numerals—the Minoan operations of addition and subtraction require no thinking whatsoever. They require only counting or cancelling. The subtractive principle in Roman numerals keeps these two operations from being entirely automatic in that system.

The major differences between Minoan and Hindu-Arabic stem from the already-noted fact that Minoan is a non-place-value system, whereas ours uses place to indicate value. Therefore, Hindu-Arabic has two distinct advantages. First, answers may be carried out to as great a degree of accuracy as desired, i.e. to as many decimal places as desired. Secondly, large numbers require comparatively few symbols to express them: compare 18,593 as expressed in our system with



and $\overline{\mathrm{X}}\overline{\mathrm{V}}$MMMDXCIII

The discussion of Minoan numerals and their use as given in *AJA* 58 (1954) 123-125 need not be repeated here. Linear A and B numerals do on the whole tend to indicate a limited degree of literacy, in the absence of any special numerals for 5, 50, 500, and 5000, and in other ways. The demonstrations given above of how actual arithmetical computations can be, and may have been, performed should cause our opinion of Linear B literacy in the area of arithmetic to rise appreciably. Addition is far easier in the Minoan Linear scripts than in our Hindu-Arabic system. Subtraction likewise is simpler, being largely mere cancellation. Division is easier with respect to finding the numbers for the quotient; this is also the case for the extraction of roots.

The purpose of the present paper is primarily to show that Linear B symbols *could* be used, without undue difficulty, for all the ordinary arithmetical computations. Just how the Minoans and Mycenean Greeks actually performed arithmetical computations is another matter. But although we know little, we can surmise something about their calculations.

The Egyptians had the abacus, and finger-counting has been widespread. We do not know whether either was used by the Minoans. Later, the Greeks had the abacus.

The vast majority of the preserved Linear B tablets were accounts—lists of items with quantities and sums indicated, and often totals shown. Addition was therefore used frequently. We have seen that addition with Minoan numerals is merely counting. If the counting is written out item by item it becomes what we call tallying. Actual tallying is preserved in one significant instance: EqO3rev (on five other instances see *AJA* 58 [1954] 123).[3] The Linear B users therefore knew this intermediate step in addition, i.e., recording so as to facilitate counting. In most instances, however, tallying would have been unnecessary; then no intermediate step was taken.

It should be borne in mind that although the preserved tablets give us mainly only simple additions (plus some proportions), so much preserved bookkeeping in the form of brief accounts almost certainly implies bookkeeping on a large scale. Totals for kinds of items, totals for periods, subtractions of losses from gains or vice versa, multiplications of items by a price-per-item factor, and divisions of items or income among interested parties, can all be reasonably imagined. It is not idle, therefore, to have shown the procedures by which these computations could be performed.

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[3] Since the present article was written, the important volume by M. Ventris and J. Chadwick, *Documents in Mycenaean Greek* (Cambridge, England, 1956), has been published. On pages 117-119 the authors present an excellent discussion of Mycenaean Arithmetic in which they have collected fully the instances which show that arithmetical computations of moderate complexity were in fact carried out by the scribes at Knossos and Pylos. The several instances of proportions show that the scribes were at least familiar with arithmetical operations more advanced than addition and subtraction. At Mykenai itself it is probable that engineering as well as commerce and government involved arithmetic fully as difficult. Ventris and Chadwick suggest no methods of computation. For comments on what they say of numbering systems, see S. Dow, *CP* forthcoming 1958.

Fractions are not considered in the present article: they would not clarify the operations, but would lead far afield. They are, however, important, and the prime article is E. L. Bennett, Jr., in *AJA* 54 (1950) 202-222; see also J. Sundwall, Soc. Sci. Fennica, *Com. Hum. Litt.*, 19, no. 2 (1953).