Purely affine gravity

Aureliano Skirzewski[1,*] and Oscar Castillo-Felisola[2,3,†]

[1] *Centro de Física Fundamental, Universidad de los Andes, 5101 Mérida, Venezuela.*
[2] *Centro Científico Tecnológico de Valparaíso, Chile.*
[3] *Departamento de Física, Universidad Técnica Federico Santa María, Casilla 110-V, Valparaíso, Chile.*

We develop a topological theory of gravity with torsion where metric has a dynamical rather than a kinematical origin. This approach towards gravity resembles pre-geometrical approaches in which a fundamental metric is not assumed even though the affine connection gives place to a local inertial structure, which reminds us of Mach's principle that assumes the inertial forces, subject to locally Minkowskian metric structures, should have dynamical origin. Additionally a Newtonian like gravitational force is obtained in the perturbative limit of the theory.

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I. INTRODUCTION

Mach proposed that inertial forces should have a dynamical rather than kinematical origin. Think, for instance of an astronaut on space, simply floating, either rotating or not. Suppose for a second the possibility that there were no stars to be seen and no reference with respect to which the astronaut could say that he is rotating. Then we cannot argue that he is rotating unless inertial forces appear. But the question arises, should inertial forces not be in correspondence with the presence or matter elsewhere in the universe? We wish to call the attention of the reader to the fact that any locally Minkowskian metric in the kinematics of the description of spacetime will introduce a notion of inertial forces at a microscopic level. We will therefore explore the dynamical origin of inertial forces, by studying the appearance of a spacetime metric from a renormalizable model to describe the evolution of the Affine connection of a manifold with torsion. In this model we use the most general action that might be power counting renormalizable and includes only the d^3 components gauge connection associated with diffeomorphisms invariance in $d = 3$ and $d = 4$ dimensions.

On the other hand, General Relativity (GR) has proven to be the most successful theory of gravity. However, it is not as successful as we may wish. Part of the problem with GR is that the standard quantization procedure cannot be applied properly on it. Moreover, not only it is not renormalizable, but there are problems with the choice of variables to be quantized and the choice of the Hilbert space to be used.

Although there is nothing wrong about metric spacetimes, certainly it is an issue to sum over all possible field configurations of the metric, as this would imply summing Euclidean and Minkowski like contributions to the transition amplitudes on equal terms. Additionally, we might also consider the difficulties of quantizing non polynomial functions of the three-dimensional metric that appears in the Hamiltonian in an ADM formulation of GR, such as the square root of the metric's determinant or the Ricci scalar.

In order to address some of these issues, several approaches have been designed that use the connection as a fundamental field. For instance, in the context of Cartan formulations of gravity, letting the metric background become flat we can rewrite Einstein-Hilbert's Lagrangian as a function of the torsion field. This approach is known as Teleparallel Gravity and is equivalent to GR in spite of taking torsion as the fundamental gravitational field.

Furthermore, another alternative description of GR developed initially by Ashtekar uses the spin connection as the fundamental field and the frame field turns out to be its canonically conjugated momentum. In the context of Loop Quantum Gravity (LQG), using Ashtekar variables, a successful quantization program has been achieved. Some strengths of this quantization program lies within a theorem by Hanno Sahlman that states the only diffeomorphisms invariant Hilbert space that supports the Heisenberg algebra, for the connection and its associated momentum, is the one of Loop Quantum Gravity. In spite of its success, LQG has not advanced enough to conclude that its low energy effective description is GR. Currently, there is no clue about the LQG effective description at other scales, nor its continuum spacetime limit either. Therefore, we cannot conclude that the search for a fundamental theory of gravitational interactions has ended. On the contrary, there are many alternatives to the usual metric description of Gravity and they all must be tested against experiments and observations.

In this article we study a diffeomorphism invariant "toy" model consisting solely of an affine connection (with torsion), and the strict commitment to formulate a power counting renormalizable action. We find the condition on the background metric to be exactly zero to be appealing because the use of a background metric such as in teleparallel theories of gravity would in principle break background independence. We expect this strat-

*Corresponding Author: skirz@ula.ve
†o.castillo.felisola@gmail.com

egy may serve to overcome the uniqueness theorem about diffeomorphism invariant theories of connections, since we have no fundamental metric field to quantize. The earliest model (that we know of) that argues a description of gravitational interaction in terms of connections as fundamental fields was presented by Eddington [**?**], for an spacetime with positive cosmological constant. He proposed the square root of the determinant of the Ricci tensor as the gravitational Lagrangian. His aim was not to solve the problems of quantum gravity, others have emphasized the character of GR as a gauge theory in order to address the issues of quantization and regularization, as in LQG. Among other, the work by Krasnov studies a BF model that is conceived to **TO BE CONTINUED...**no se que decir al respecto

We aim to describe certain aspects of the gravitational interaction in four dimensions. To this end we have studied the non-relativistic limit of the parallel transport equations of motion of a particle moving on a flat, static, homogeneous and isotropic background.

The aim of this work is to present a purely affine theory of gravity, where the symmetric tensor which acts like "metric" is a composite field defined in terms of the connection, **Is it still TRUE?**

$$g_{\mu\nu} = T^{\sigma}{}_{\rho\mu} T^{\rho}{}_{\sigma\nu}.$$

Not true...instead

$$\sqrt{g} g^{\mu\nu} = \frac{\delta}{\delta R_{(\mu\nu)}} S[\Gamma] \tag{1}$$

where g is the inverse of the determinant of $g^{\mu\nu}$. The structure of the paper ...

II. WARMING UP: THE THREE-DIMENSIONAL CASE

Formally, the curvature of a manifold is defined through the commutator of covariant derivatives under diffeomorphims, ∇_μ, but for general choice of the connection, $\Gamma^{\mu}{}_{\nu\lambda}$, there is an extra contribution given by its antisymmetric part in the lower indices, $T^{\mu}{}_{\nu\lambda} = 2\Gamma^{\mu}{}_{[\nu\lambda]}$. Therefore, the commutator of the covariant derivatives yields,

$$[\nabla_\mu, \nabla_\nu]\,\omega_\lambda = R_{\mu\nu}{}^{\rho}{}_{\lambda}\omega_\rho - T^{\rho}{}_{\mu\nu}\nabla_\rho\omega_\lambda. \tag{2}$$

aqui no falta un signo frente a R? Note that $T^{\rho}{}_{\mu\nu}$ is a $(d-1)d^2/2$ dimensional tensor representation under diffeomorphisms.

In order to build topological invariants of density one, one can use the skew-symmetric Levi-Čivita tensor $\epsilon^{\mu_1\mu_2\cdots\mu_n}$ in n-dimensional space(-time).

With these ingredients, in a three-dimensional space one can write an action

$$\begin{aligned} S[\Gamma] = \int \mathrm{d}^3x \,\Bigg\{ & R_{\mu_1\mu_2}{}^{\rho}{}_{\mu_3} T^{\sigma}{}_{\mu_4\mu_5} \sum_{\pi\in \mathrm{Z}_5} C_\pi \delta_\rho^{\mu_{\pi(1)}}\delta_\sigma^{\mu_{\pi(2)}}\epsilon^{\mu_{\pi(3)}\mu_{\pi(4)}\mu_{\pi(5)}} \\ & + T^{\rho}{}_{\mu_1\mu_2} T^{\sigma}{}_{\mu_3\mu_4} T^{\tau}{}_{\mu_5\mu_6} \sum_{\pi\in \mathrm{Z}_6} D_\pi \delta_\rho^{\mu_{\pi(1)}}\delta_\sigma^{\mu_{\pi(2)}}\delta_\tau^{\mu_{\pi(3)}}\epsilon^{\mu_{\pi(4)}\mu_{\pi(5)}\mu_{\pi(6)}} \\ & + T^{\rho}{}_{\mu_1\mu_2} \nabla_{\mu_3} T^{\sigma}{}_{\mu_4\mu_5} \sum_{\pi\in \mathrm{Z}_5} E_\pi \delta_\rho^{\mu_{\pi(1)}}\delta_\sigma^{\mu_{\pi(2)}}\epsilon^{\mu_{\pi(3)}\mu_{\pi(4)}\mu_{\pi(5)}} \Bigg\}, \end{aligned} \tag{3}$$

where all possible permutations of n elements $\pi \in \mathrm{Z}_n$ have been included in the sums with different constants C_π, D_π and E_π for permutation.

The torsion field can be decomposed into invariant tensors respecting the symmetry,

$$T^{\sigma}{}_{\mu\nu} = \epsilon_{\mu\nu\rho} T^{\sigma\rho} + A_{[\mu}\delta^{\sigma}{}_{\nu]}, \tag{4}$$

with a symmetric $T^{\sigma\rho}$ of density weight $w = 1$, and $A_\mu = T^{\nu}{}_{\mu\nu}$ is the trace part of the more arbitrary $T^{\sigma}{}_{\mu\nu}$.

The action in Eq. (3) can be rewritten as

$$\begin{aligned} S[\Gamma] = \int \mathrm{d}^3x \,\Bigg(& B_1 R_{\mu\nu}{}^{\mu}{}_{\rho} T^{\nu\rho} + B_2 \epsilon^{\mu\nu\rho} R_{\mu\nu}{}^{\sigma}{}_{\sigma} A_\rho \\ & + B_3 \epsilon^{\mu\nu\rho} R_{\mu\nu}{}^{\lambda}{}_{\rho} A_\lambda + B_4 \det(T^{\mu\nu}) \\ & + B_5 T^{\mu\nu} A_\mu A_\nu + B_6 T^{\mu\nu}\nabla_\mu A_\nu \\ & + B_7 \epsilon^{\mu\nu\rho} A_\mu \partial_\nu A_\rho + B_8 \epsilon^{\mu\nu\rho}\Gamma^{\sigma}{}_{\mu\sigma}\partial_\nu\Gamma^{\tau}{}_{\rho\tau} \\ & + B_9 \epsilon^{\mu\nu\lambda}\Big(\Gamma^{\sigma}{}_{\mu\rho}\partial_\nu\Gamma^{\rho}{}_{\lambda\sigma} + \frac{2}{3}\Gamma^{\tau}{}_{\mu\rho}\Gamma^{\rho}{}_{\nu\sigma}\Gamma^{\sigma}{}_{\lambda\tau}\Big)\Bigg). \end{aligned} \tag{5}$$

where the coefficients B_i are related with the original coefficients C_i, D_i and E_i, and the additional B_9 term can be added in three dimensions leaving the action invariant under diffeomorphisms. The affine connection can be decomposed into its symmetric and antisymmetric parts,

$$\Gamma^{\lambda}{}_{\mu\rho} = \hat{\Gamma}^{\lambda}{}_{(\mu\rho)} + \epsilon_{\mu\rho\sigma}T^{\lambda\sigma} + A_{[\mu}\delta^{\lambda}{}_{\rho]}, \tag{6}$$

where $\epsilon_{\mu\rho\sigma}$ has been introduced, and it is related to the skew symmetric $\epsilon^{\mu\rho\sigma}$ through the identity $\epsilon^{\lambda\mu\nu}\epsilon_{\rho\sigma\tau} = 3!\,\delta^{\lambda}{}_{[\rho}\delta^{\mu}{}_{\sigma}\delta^{\nu}{}_{\tau]}$. Therefore, the curvature tensor can be expressed as

$$\begin{aligned}R_{\mu\nu}{}^{\sigma}{}_{\rho} &= \hat{R}_{\mu\nu}{}^{\sigma}{}_{\rho} - 2\epsilon_{\rho\alpha[\mu}\hat{\nabla}_{\nu]}T^{\sigma\alpha} + \partial_{[\mu}A_{\nu]}\delta^{\sigma}_{\rho} \\ &+ \delta^{\sigma}_{[\mu}\hat{\nabla}_{\nu]}A_{\rho} + \epsilon_{\mu\nu\kappa}T^{\kappa\sigma}A_{\rho} - \delta^{\sigma}_{[\mu}\epsilon_{\nu]\rho\alpha}T^{\alpha\beta}A_{\beta} \\ &+ \frac{1}{2}\delta^{\sigma}_{[\mu}A_{\nu]}A_{\rho} - 2\epsilon_{\alpha\beta[\mu}\epsilon_{\nu]\rho\delta}T^{\sigma\alpha}T^{\beta\delta},\end{aligned} \tag{7}$$

where $\hat{\nabla}_{\rho}$ and $\hat{R}_{\mu\nu}{}^{\lambda}{}_{\rho}$ are the covariant derivative and curvature associated to the symmetric part of the connection. Notice that Bianchi identity, obtained as $\epsilon^{\mu\nu\lambda}\hat{R}_{\mu\nu}{}^{\rho}{}_{\lambda} = 0$, leads us to the following

$$\epsilon^{\mu\nu\rho}R_{\mu\nu}{}^{\lambda}{}_{\rho} = 4\hat{\nabla}_{\rho}T^{\rho\lambda} + 2\epsilon^{\mu\nu\lambda}\partial_{\mu}A_{\nu} - 4T^{\lambda\rho}A_{\rho}. \tag{8}$$

Using the Eqs. (7) and (8) one can rewrite the action in Eq. (5) as

$$\begin{aligned}S[\Gamma] = \int \mathrm{d}^3x \Bigg(&(B_1 + 2B_9)\hat{R}_{\mu\nu}{}^{\mu}{}_{\rho}T^{\nu\rho} \\ &+ (B_2 + B_9 + B_9)\epsilon^{\mu\nu\rho}\hat{R}_{\mu\nu}{}^{\sigma}{}_{\sigma}A_{\rho} \\ &+ (-6B_1 + B_4 - 4B_9)\det(T^{\mu\nu}) \\ &+ (\frac{1}{2}B_1 - 4B_3 + B_5 + B_9)T^{\mu\nu}A_{\mu}A_{\nu} \\ &+ (B_1 - 4B_3 + B_6 + 2B_9)T^{\mu\nu}\hat{\nabla}_{\mu}A_{\nu} \\ &+ (2B_2 + 2B_3 + B_7 + B_9)\epsilon^{\mu\nu\rho}A_{\mu}\partial_{\nu}A_{\rho} \\ &+ B_9\epsilon^{\mu\nu\lambda}\Big(\hat{\Gamma}^{\sigma}{}_{\mu\rho}\partial_{\nu}\hat{\Gamma}^{\rho}{}_{\lambda\sigma} + \frac{2}{3}\hat{\Gamma}^{\tau}{}_{\mu\rho}\hat{\Gamma}^{\rho}{}_{\nu\sigma}\hat{\Gamma}^{\sigma}{}_{\lambda\tau}\Big) \\ &+ B_8\epsilon^{\mu\nu\rho}\hat{\Gamma}^{\sigma}{}_{\mu\sigma}\partial_{\nu}\hat{\Gamma}^{\tau}{}_{\rho\tau} + \partial_{\alpha}\Big(4B_3T^{\alpha\mu}A_{\mu} \\ &+ B_9(2\Gamma^{[\delta}{}_{\delta\beta}T^{\alpha]\beta} - T^{\alpha\beta}A_{\beta} + \frac{1}{2}\epsilon^{\beta\alpha\eta}A_{\eta}\Gamma^{\delta}{}_{\delta\beta}) \\ &+ B_8\epsilon^{\beta\alpha\eta}A_{\eta}\hat{\Gamma}^{\delta}{}_{\delta\beta}\Big)\Bigg),\end{aligned} \tag{9}$$

or after dropping the boundary terms and rename the coefficients,

$$\begin{aligned}S[\hat{\Gamma}, T, A] = \int \mathrm{d}^3x \Big(&A_1\hat{R}_{\mu\nu}{}^{\mu}{}_{\rho}T^{\nu\rho} + A_2\epsilon^{\mu\nu\rho}\hat{R}_{\mu\nu}{}^{\sigma}{}_{\sigma}A_{\rho} \\ &+ A_3\epsilon^{\mu\nu\rho}A_{\mu}\partial_{\nu}A_{\rho} + A_4T^{\mu\nu}\hat{\nabla}_{\mu}A_{\nu} \\ &+ A_5T^{\mu\nu}A_{\mu}A_{\nu} + A_6\det(T^{\mu\nu}) \\ &+ A_7\epsilon^{\mu\nu\lambda}\Big(\hat{\Gamma}^{\sigma}{}_{\mu\rho}\partial_{\nu}\hat{\Gamma}^{\rho}{}_{\lambda\sigma} + \frac{2}{3}\hat{\Gamma}^{\tau}{}_{\mu\rho}\hat{\Gamma}^{\rho}{}_{\nu\sigma}\hat{\Gamma}^{\sigma}{}_{\lambda\tau}\Big) \\ &+ A_8\epsilon^{\mu\nu\rho}\hat{\Gamma}^{\sigma}{}_{\mu\sigma}\partial_{\nu}\hat{\Gamma}^{\tau}{}_{\rho\tau}\Big).\end{aligned} \tag{10}$$

Noticing that in the first term, the variation respect to the Ricci tensor yields to $T^{\mu\nu}$, it can be argued that in a standard theory of gravity this tensor density corresponds to $\sqrt{g}g^{\mu\nu}$. Therefore, Eq. (9) reveals a one to one correspondence with General Relativity non minimally coupled to the A_{μ} field, **I'd say that this result should be presented in terms of the action with coefficients C's... This result should be review!!!**ya?

$$\begin{aligned}S[g, \hat{\Gamma}, A] = \int \mathrm{d}^3x \Big(&\sqrt{g}\Big(A_1\hat{R} + A_4\hat{\nabla}^{\mu}A_{\mu} \\ &+ A_5A_{\mu}A^{\mu} + A_6\Big) + A_2\epsilon^{\mu\nu\rho}\hat{R}_{\mu\nu}{}^{\sigma}{}_{\sigma}A_{\rho} \\ &+ A_3\epsilon^{\mu\nu\rho}A_{\mu}\partial_{\nu}A_{\rho} + A_7\epsilon^{\mu\nu\lambda}\Big(\hat{\Gamma}^{\sigma}{}_{\mu\rho}\partial_{\nu}\hat{\Gamma}^{\rho}{}_{\lambda\sigma} \\ &+ \frac{2}{3}\hat{\Gamma}^{\tau}{}_{\mu\rho}\hat{\Gamma}^{\rho}{}_{\nu\sigma}\hat{\Gamma}^{\sigma}{}_{\lambda\tau}\Big) + A_8\epsilon^{\mu\nu\rho}\hat{\Gamma}^{\sigma}{}_{\mu\sigma}\partial_{\nu}\hat{\Gamma}^{\tau}{}_{\rho\tau}\Big)\end{aligned} \tag{11}$$

va pa fuera

$$\begin{aligned}S[g, \hat{\Gamma}, A] = \int \mathrm{d}^3x \Big\{ &\sqrt{g}\Big(B_1\hat{R} + \frac{1}{4}(4B_4 - 3B_1) \\ &- \frac{1}{4}(2B_2 - B_3)A_{\mu}A^{\mu}\Big) \\ &+ \frac{1}{2}(2B_2 - B_3)\epsilon^{\mu\nu\rho}A_{\mu}\hat{R}_{\nu\rho}{}^{\sigma}{}_{\sigma} \\ &+ \frac{1}{4}(B_1 + 4B_2 - 2B_3)\epsilon^{\mu\nu\rho}A_{\mu}\partial_{\nu}A_{\rho}\Big\}\end{aligned} \tag{12}$$

Thus, an interesting sector of the theory corresponds to the space of non-degenerated $T^{\mu\nu}$.

As a matter of fact, the vector field and the "gravitational" sector of the torsion disengage when $B_3 = 2B_2$

III. 4D METRICLESS (AND TORSIONFUL) ACTION

Following the precepts already learnt, one starts by defining an irreducible representation decomposition for the full connection field

$$\Gamma^{\mu}{}_{\rho\sigma} = \hat{\Gamma}^{\mu}{}_{\rho\sigma} + T^{\mu}{}_{\rho\sigma} = \hat{\Gamma}^{\mu}{}_{\rho\sigma} + \epsilon_{\rho\sigma\lambda\kappa}T^{\mu,\lambda\kappa} + A_{[\rho}\delta^{\mu}_{\nu]}, \tag{13}$$

where $\hat{\Gamma}^{\mu}{}_{\rho\sigma}$ denotes a $(d+1)d^2/2$ dimensional symmetric connection, A_μ is a d dimensional vector field that gives trace to the antisymmetric part of the full connection, and $T^{\mu,\lambda\kappa}$ is a $(d+1)d(d-2)/2$ dimensional (Curtright) field that is defined through the symmetry of its indices: antisymmetric in the last two indices, and it has a cyclic property $T^{\mu,\lambda\kappa}+T^{\lambda,\kappa\mu}+T^{\kappa,\mu\lambda}=0$. In other words that $T^{[\mu,\lambda]\kappa}=\frac{1}{2}T^{\kappa,\lambda\mu}$, just as for the Riemmann tensor $\hat{R}_{\mu[\nu}{}^{\alpha}{}_{\lambda]}=\frac{1}{2}\hat{R}_{\lambda\nu}{}^{\alpha}{}_{\mu}$. Notice that due to its symmetries, the contraction $\epsilon_{\rho\sigma\lambda\kappa}T^{\mu,\lambda\kappa}$ is traceless. Additionally, since no metric is present the epsilon symbols are not related by lowering or raising their indices, but instead one demands that $\epsilon^{\delta\eta\lambda\kappa}\epsilon_{\mu\nu\rho\sigma}=4!\,\delta^{\delta}{}_{[\mu}\delta^{\eta}{}_{\nu}\delta^{\lambda}{}_{\rho}\delta^{\kappa}{}_{\sigma]}$.

One can write all the combinations of fields that would presumably be renormalizable with these three independent fields

$$
\begin{aligned}
S[\hat{\Gamma},T,A]=\int \mathrm{d}^4x\Big(&\partial_\lambda\Big(A_1\hat{R}_{\mu(\nu}{}^{\mu}{}_{\rho)}T^{\nu,\rho\lambda}+A_2\epsilon^{\lambda\mu\nu\rho}\hat{R}_{\mu\nu}{}^{\sigma}{}_{\sigma}A_\rho+A_3\epsilon^{\lambda\mu\nu\rho}A_\mu\partial_\nu A_\rho+A_4T^{(\mu,\nu)\lambda}\hat{\nabla}_\mu A_\nu\\
&+A_5T^{(\mu,\nu)\lambda}A_\mu A_\nu+A_6\epsilon_{\mu\nu\rho\sigma}\epsilon_{\alpha\beta\gamma\delta}T^{\lambda,\mu\alpha}T^{\beta,\rho\sigma}T^{\nu,\gamma\delta}+A_7\epsilon^{\lambda\mu\nu\lambda}\Big(\hat{\Gamma}^{\sigma}{}_{\mu\rho}\partial_\nu\hat{\Gamma}^{\rho}{}_{\lambda\sigma}+\frac{2}{3}\hat{\Gamma}^{\tau}{}_{\mu\rho}\hat{\Gamma}^{\rho}{}_{\nu\sigma}\hat{\Gamma}^{\sigma}{}_{\lambda\tau}\Big)\\
&+A_8\epsilon^{\lambda\mu\nu\rho}\hat{\Gamma}^{\sigma}{}_{\mu\sigma}\partial_\nu\hat{\Gamma}^{\tau}{}_{\rho\tau}+A_9R_{\mu\nu}{}^{\lambda}{}_{\rho}T^{\rho,\mu\nu}+A_{10}T^{\lambda,\alpha\beta}T^{\kappa,\gamma\delta}A_\kappa\epsilon_{\alpha\beta\gamma\delta}\Big)+B_1\hat{R}_{\mu\nu}{}^{\mu}{}_{\rho}T^{\nu,\alpha\beta}T^{\rho,\gamma\delta}\epsilon_{\alpha\beta\gamma\delta}\\
&+B_2\Big(\hat{R}_{\mu\nu}{}^{\sigma}{}_{\rho}+\frac{2}{3}\delta^{\sigma}{}_{[\mu}\hat{R}_{\nu]\lambda}{}^{\lambda}{}_{\rho}\Big)T^{\beta,\mu\nu}T^{\rho,\gamma\delta}\epsilon_{\sigma\beta\gamma\delta}+B_3\hat{R}_{\mu\nu}{}^{\mu}{}_{\rho}T^{(\nu,\rho)\sigma}A_\sigma+B_4\Big(\hat{R}_{\mu\nu}{}^{\sigma}{}_{\rho}\\
&+\frac{2}{3}\delta^{\sigma}{}_{[\mu}\hat{R}_{\nu]\lambda}{}^{\lambda}{}_{\rho}\Big)\Big(T^{\rho,\mu\nu}A_\sigma-\frac{1}{4}\delta^{\rho}_{\sigma}T^{\kappa,\mu\nu}A_\kappa\Big)+B_5\hat{R}_{\mu\nu}{}^{\rho}{}_{\rho}T^{\sigma,\mu\nu}A_\sigma+C_1\hat{R}_{\mu\nu}{}^{\mu}{}_{\rho}\hat{\nabla}_\sigma T^{(\nu,\rho)\sigma}+C_2\Big(\hat{R}_{\mu\nu}{}^{\sigma}{}_{\rho}\\
&+\frac{2}{3}\delta^{\sigma}{}_{[\mu}\hat{R}_{\nu]\lambda}{}^{\lambda}{}_{\rho}\Big)\Big(\hat{\nabla}_\sigma T^{\rho,\mu\nu}-\frac{1}{4}\delta^{\rho}_{\sigma}\hat{\nabla}_\kappa T^{\kappa,\mu\nu}\Big)+C_3\hat{R}_{\mu\nu}{}^{\rho}{}_{\rho}\hat{\nabla}_\sigma T^{\sigma,\mu\nu}+D_1T^{\alpha,\mu\nu}T^{\beta,\rho\sigma}\hat{\nabla}_\gamma T^{(\lambda,\kappa)\gamma}\epsilon_{\beta\mu\nu\lambda}\epsilon_{\alpha\rho\sigma\kappa}\\
&+D_2T^{\alpha,\mu\nu}T^{\lambda,\beta\gamma}\hat{\nabla}_\lambda T^{\delta,\rho\sigma}\epsilon_{\alpha\beta\gamma\delta}\epsilon_{\mu\nu\rho\sigma}+D_3T^{\mu,\alpha\beta}T^{\lambda,\nu\gamma}\hat{\nabla}_\lambda T^{\delta,\rho\sigma}\epsilon_{\alpha\beta\gamma\delta}\epsilon_{\mu\nu\rho\sigma}+D_4T^{\lambda,\mu\nu}T^{\kappa,\rho\sigma}\hat{\nabla}_{(\lambda}A_{\kappa)}\epsilon_{\mu\nu\rho\sigma}\\
&+D_5T^{\lambda,\mu\nu}\hat{\nabla}_{[\lambda}T^{\kappa,\rho\sigma}A_{\kappa]}\epsilon_{\mu\nu\rho\sigma}+D_6T^{\lambda,\mu\nu}A_\nu\hat{\nabla}_{(\lambda}A_{\mu)}+D_7T^{\lambda,\mu\nu}A_\lambda\hat{\nabla}_{[\mu}A_{\nu]}\\
&+E_1\hat{\nabla}_{(\rho}T^{\rho,\mu\nu}\hat{\nabla}_{\sigma)}T^{\sigma,\lambda\kappa}\epsilon_{\mu\nu\lambda\kappa}+E_2\hat{\nabla}_{(\lambda}T^{\lambda,\mu\nu}\hat{\nabla}_{\mu)}A_\nu+T^{\alpha,\beta\gamma}T^{\delta,\eta\kappa}T^{\lambda,\mu\nu}T^{\rho,\sigma\tau}(\Lambda_1\epsilon_{\beta\gamma\eta\kappa}\epsilon_{\alpha\rho\mu\nu}\epsilon_{\delta\lambda\sigma\tau}\\
&+\Lambda_2\epsilon_{\beta\lambda\eta\kappa}\epsilon_{\gamma\rho\mu\nu}\epsilon_{\alpha\delta\sigma\tau})+\Lambda_3T^{\rho,\alpha\beta}T^{\gamma,\mu\nu}T^{\lambda,\sigma\tau}A_\tau\epsilon_{\alpha\beta\gamma\lambda}\epsilon_{\mu\nu\rho\sigma}+\Lambda_4T^{\eta,\alpha\beta}T^{\kappa,\gamma\delta}A_\eta A_\kappa\epsilon_{\alpha\beta\gamma\delta}\Big),
\end{aligned}
\tag{14}
$$

where the A_i series of terms contribute solely to the boundary conditions, and the terms B_2, B_4 and C_2 contain a traceless contribution of the curvature.

In this case, the induced density 1 inverse metric (1) is

$$
\begin{aligned}
\sqrt{-g}g^{\mu\nu}=&B_1T^{\mu,\lambda\kappa}T^{\nu,\rho\sigma}\epsilon_{\lambda\kappa\rho\sigma}\\
&+B_3T^{(\mu,\nu)\lambda}A_\lambda+C_1\hat{\nabla}_\lambda T^{(\mu,\nu)\lambda},
\end{aligned}
\tag{15}
$$

3+1 Decomposition and Degrees of Freedom

The analysis of the degrees of freedom of the 2+1 action is pretty straightforward because it clearly corresponds to a gravitational action plus some extra stuff. In 3+1 dimensions there is not a clear equivalence between the model described by Eq. (**??**) to anything known, an deeper analysis should be done. Hence, one proposes a decomposition of the fields

$$A_\mu=\delta^0_\mu A_0+\delta^i_\mu A_i \tag{16}$$

and

$$T^{\mu,\nu\rho}=\delta^\mu_i\delta^{\nu\rho}_{jk}T^{i,jk}+(\delta^\mu_0\delta^{\nu\rho}_{ij}-\delta^\mu_i\delta^{\nu\rho}_{j0})a^{[ij]}+\delta^\mu_i\delta^{\nu\rho}_{j0}T^{(ij)}+\delta^\mu_0\delta^{\nu\rho}_{i0}c^i, \tag{17}$$

where $\delta^{\mu\nu}_{\lambda\kappa}=\delta^\mu_\lambda\delta^\nu_\kappa-\delta^\mu_\kappa\delta^\nu_\lambda$, $T^{i,jk}\epsilon_{ijk}=0$, a^{ij} is antisymmetric, T^{ij} is a symmetric tensor and c^i an arbitrary vector. In order to make perturbation theory we will expand around an isotropic and homogeneous solution, as these are characteristic of the observable universe. Thus, in order to find such solution we can substitute $A_\mu=\delta^0_\mu A_0$ and $T^{\mu,\nu\rho}=\delta^\mu_i\delta^{\nu\rho}_{j0}T^{ij}$; for a spacially flat metric type, with $k=0$, we would choose $T^{ij}=\delta^{ij}T(t)$. For this particular choice we wish to remark that the contraction of torsion terms

$$
\begin{aligned}
T_\mu{}^\sigma{}_\rho T_\nu{}^\rho{}_\sigma&=(\epsilon_{\mu\rho\lambda\kappa}T^{\sigma,\lambda\kappa}+A_{[\mu}\delta^\sigma_{\rho]})(\epsilon_{\nu\sigma\delta\eta}T^{\rho,\delta\eta}+A_{[\nu}\delta^\rho_{\sigma]})\\
&=\delta^0_\mu\delta^0_\nu\frac{3}{4}A_0^2-24\det(T^{ij})\delta^i_\mu\delta^j_\nu T_{ij},
\end{aligned}
\tag{18}
$$

where T_{ij} is defined as the inverse of T^{ij}. Being a covariant tensor, $T_\mu{}^\sigma{}_\rho T_\nu{}^\rho{}_\sigma$ can play the role of the metric of the *spacetime*, defined as a constructed field $g_{\mu\nu}:=T_\mu{}^\sigma{}_\rho T_\nu{}^\rho{}_\sigma$. Similarly, we can define another metric-like tensor,

$$\tilde{g}^{\lambda\kappa}:=T_\mu{}^\lambda{}_\nu T_\rho{}^\kappa{}_\sigma\epsilon^{\mu\nu\rho\sigma}=-8\delta^\lambda_i\delta^\kappa_j A_0T^{ij}, \tag{19}$$

but we can see that this would not be a four dimensional metric but only the space part of it.

Anyhow, we can now do perturbation theory expanding up to second order the fields around the solution $T^{\mu,\nu\lambda} = \delta^{\mu}_{i}\delta^{\nu\lambda}_{i0}T + t^{\mu,\nu\lambda}$ and $A_\mu = \delta^{0}_{\mu}A_0 + a_\mu$ and in the simplest case, where A_0 and T are constants in time we can also set $\hat{\Gamma}_{\mu}{}^{\lambda}{}_{\nu} = \gamma_{\mu}{}^{\lambda}{}_{\nu}$, where the lowcase fields are small perturbations. The expansion is required to be performed around a minimum of the action, therefore we will set linear terms to zero.

$$S_0[\hat{\Gamma}_{\mu}{}^{\nu}{}_{\rho}, T^{\lambda,\mu\nu}, A_\mu] = \int \mathrm{d}^4x \left(-3!\, A_0 T^3 \Lambda_3 \right), \qquad (20)$$

$$\begin{aligned} S_1[\hat{\Gamma}_{\mu}{}^{\nu}{}_{\rho}, T^{\lambda,\mu\nu}, A_\mu] = \int \mathrm{d}^4x \Bigg(& 2(-4B_2 + B_3)T^2 \partial_{[i}\gamma_{0]}{}^{k}{}_{j}\epsilon_{0ijk} \\ & + 2A_0T((B_4 - B_6)\partial_{[i}\gamma_{0]}{}^{0}{}_{i} + B_6\partial_{[i}\gamma_{j]}{}^{j}{}_{i}) \\ & + 8T^3(-3D_1\gamma_0{}^{0}{}_{0} + (-2D_1 + D_2 + D_3)\gamma_i{}^{i}{}_{0}) \\ & - TA_0^2(D_6 + D_7)\gamma_i{}^{0}{}_{i} - 12\Lambda_3(2t^{k,k0} + a_0) \Bigg). \end{aligned} \qquad (21)$$

Therefore, we must set $B_3 = 4B_2$, $B_4 = 0 = B_6$, $D_1 = 0$, $D_3 = -D_2$, $D_7 = -D_6$ and $\Lambda_3 = 0$ in order to be sure that the homogeneous and isotropic field configuration minimizes the action.

The second order perturbative expansion is

FALTA

- tal vez a^{ij} y c^i sean campo eléctrico y magnético
- study whether the spacetime solution can arise as a symmetry breaking from thermal fluctuations around $T = 0$
- estudiar la posibilidad de obtener Hořava-Lifshitz gravity en algun limite
- hacer teoria de perturbaciones alrededor de la solucion constante
- calcular el limite Newtoniano
- I am studying how to do the d.o.f. analysis.
- check the most general action in 4d.