An Examination of UK Business Cycle Fluctuations: 1871-1997

Supplementary Paper

Jagjit S Chadha [a]
University of Cambridge

Norbert Janssen [b]
Bank of England

Charles Nolan [c]
University of Reading

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Abstract
This annex provides Tables 1-6, Appendices A-C and Annexes A-B of the paper.

a Clare College, Cambridge and Department of Applied Economics. Austin Robinson Building, Sidgwick Avenue, University of Cambridge, Cambridge CB3 9DE, United Kingdom. Tel: +44 1223 335280. E-mail: jagjit.chadha@econ.cam.ac.uk.

b Monetary Analysis Division, Bank of England, Threadneedle Street, London EC2R 8AH, London, United Kingdom. Tel: +44 20 7601 4523. E-mail: norbert.janssen@bankofengland.co.uk.

c Department of Economics, Reading University, Whiteknights, Reading, RG6 6AA, United Kingdom. Tel: +44 118 987 5123 x4068. E-mail: C.Nolan@reading.ac.uk.

TABLE 1: Average Length in months of UK Business Cycles: (from 1871 onwards)

Average, all cycles:	Expansion	Contraction	Peak to Peak	Trough to Trough
1871-1997	34.67	28.23	61.48	62.81
1871-1913	44.57	35.14	79.71	78.43
1946-1997	29.5	29.67	60.75	55.63
Average peacetime cycles:				
1871-1913	44.57	35.14	79.71	78.43
1919-1939	26.2	20.4	45.4	46.6
1946-1997	29.5	29.67	60.75	55.63
Standard error, all cycles:				
1871-1997	17.90	18.54	28.80	27.80
1871-1913	14.57	24.94	32.93	33.47
1946-1997	14.23	15.61	26.54	10.68
Standard error, peacetime cycles:				
1871-1913	14.57	24.94	32.93	33.47
1919-1939	23.74	9.76	20.94	32.75
1946-1997	14.23	15.61	26.54	10.68

Source: Artis *et al* (1995), Dow (1998) and Moore and Zarnowitz (1986).

TABLE 2A: Summary Statistics of per capita growth rates 1871-1997[(a)]

	μ	σ	μ/σ	Skewness	Kurtosis
Output	1.155**	3.901	0.296 (0.384)	-1.023**	5.682**†
Consumption	1.096**	3.126	0.351 (0.363)	-0.069	6.051**‡
Investment	1.730	12.514	0.138(0.445)	2.548**‡	22.430**‡
M0	3.464**	5.491	0.631 (0.264)	0.883**‡	1.526**‡
M3	4.669**	6.383	0.731 (0.232)	0.811**‡	0.446‡
Real M0	0.549	5.674	0.097 (0.461)	0.356†	1.511**‡
Real M3	1.754**	5.594	0.314 (0.377)	0.205	0.096‡
Short	4.513**	3.488	1.294 (0.098)	1.257**	1.000*‡
Long	5.203**	3.159	1.647 (0.050)	1.344**‡	0.839‡
Real Short	1.600**	5.430	0.295 (0.384)	0.077‡	4.668**‡
Real Long	2.183**	4.278	0.510 (0.305)	-0.472*‡	3.392**‡
RER	-1.801	9.991	-0.180 (0.429)	-0.393‡	4.351**‡
Real wages	1.584**	3.100	0.511 (0.305)	-0.496*†	1.128*‡
Price level	2.915**	6.196	0.470 (0.319)	0.375	2.381**‡
TFP[d]	0.975**	3.320	0.294 (0.386)	-0.900**	4.581**
CA	6.244	24.582	0.254 (0.401)	-0.747**‡	1.158**‡

Notes: (a) we use levels data for interest rates and current account balance relative to output; b) the numbers in parentheses in column four denote the probability of a negative draw ; c) * and ** indicates significant difference from zero at 5% and 1% respectively; † and ‡ indicate significance of the from zero at 5% and 1% level of the band pass filtered data (to be discussed later); d) .per employee.

TABLE 2B(1): Summary Statistics of per capita growth rates 1871-1939[(a)]

	μ	σ	μ/σ	Skewness	Kurtosis
Output	0.514	4.406	0.117 (0.452)	-1.190**	4.706**
Consumption	0.466	3.066	0.152 (0.440)	0.679*	10.459**
Investment	0.770	9.664	0.080 (0.468)	-0.279	1.460*
M0	1.430*	5.488	0.261 (0.397)	1.817**	4.970**
M3	1.737**	5.086	0.342 (0.367)	1.546**	5.322**
Real M0	0.911	5.639	0.162 (0.456)	0.154	1.286*
Real M3	1.218*	4.628	0.263 (0.397)	0.282	1.181
Short	2.884**	1.273	2.266 (0.012)	0.126	-0.079
Long	3.309**	0.804	4.116 (0.000)	0.706*	-0.366
Real Short	2.532**	6.237	0.406 (0.341)	0.144	4.191**
Real Long	2.687**	5.169	0.520 (0.302)	-0.659*	2.721**
RER	-0.770	9.174	-0.084 (0.468)	0.600*	6.860**
Real wages	0.910*	3.222	0.282 (0.390)	-0.331	1.689**
Price level	0.519	6.353	0.082 (0.468)	0.716*	3.967**
TFP[d]	0.243	3.740	0.065 (0.476)	-0.973**	3.217**
CA	18.669**	18.810	0.993 (0.161)	-0.288	-0.103

Notes: see Table 2A.

TABLE 2B(2): Summary Statistics of per capita growth rates 1871-1913[(a)]

	μ	σ	μ/σ	Skewness	Kurtosis
Output	0.638	3.572	0.179 (0.429)	0.455	0.445
Consumption	0.550**	1.154	0.363 (0.359)	0.316	-0.500
Investment	0.395	6.290	0.063 (0.476)	-0.623	0.894
M0	0.470	3.778	0.124 (0.452)	2.071**	9.484**
M3	0.794**	2.343	0.339 (0.345)	-0.120	1.258
Real M0	0.715	5.167	0.138 (0.444)	0.516	2.427**
Real M3	1.039**	3.475	0.299 (0.382)	-0.475	0.330
Short	2.772**	0.866	3.201 (0.000)	-0.011	-0.124
Long	2.827**	0.362	7.809 (0.000)	-0.623	0.188
Real Short	3.071**	3.277	0.937 (0.174)	0.086	0.541
Real Long	2.688**	3.007	0.894 (0.187)	-1.566**	5.084**
RER	-0.419	3.504	-0.120 (0.452)	-0.611	0.220
Real wages	0.990**	2.436	0.406 (0.341)	0.230	-1.133
Price level	-0.245	3.371	-0.073 (0.472)	0.236	0.809
TFP[d]	0.252	3.192	0.079 (0.468)	0.457	-0.498
CA	27.438**	13.758	1.994 (0.023)	0.263	-0.863

Notes: see Table 2A.

TABLE 2C: Summary Statistics of per capita growth rates 1946-1997[(a)]

	μ	σ	μ/σ	Skewness	Kurtosis
Output	1.955**	2.099	0.931 (0.176)	-0.517	0.413
Consumption	2.243**	2.366	0.948 (0.171)	0.643	0.794
Investment	5.214**	13.320	0.391 (0.348)	5.690**	3.792**
M0	4.990**	3.720	1.341 (0.090)	0.125	2.322**
M3	7.674**	6.227	1.232 (0.109)	0.644	-0.498
Real M0	-0.908	4.247	0.214 (0.417)	-0.906**	1.469*
Real M3	1.776*	6.232	0.285 (0.386)	0.116	-0.182
Short	7.046**	4.022	1.752 (0.040)	0.160	-0.717
Long	7.919**	3.285	2.411 (0.008)	0.244	-0.759
Real Short	0.914	3.709	0.246 (0.401)	-1.140**	1.336
Real Long	1.669**	2.860	0.584 (0.281)	-0.329	-0.712
RER	-3.399*	11.130	0.305 (0.378)	-0.981**	2.223**
Real wages	2.363**	2.631	0.898 (0.184)	-0.598	0.650
Price level	5.898**	4.523	1.304 (0.097)	1.503**	2.301**
Capital prod.[d]	0.490**	1.691	0.290 (0.386)	0.282	0.767
TFP[d]	1.947**	1.654	1.177 (0.119)	-0.761*	1.102
CA	-2.407	14.628	-0.165 (0.433)	-0.782*	0.378

Notes: see Table 2A.

TABLE 3A: Power Spectrum - full sample

Variable	Cycles Per Year					Durbin's
	0	<0.136	<0.5	<1	1	Peak Freq.
Output	4.248	8.726	47.027	35.292	4.708	0.589
	(2.140)				(2.373)	(5.334)
Cons.	7.278	8.425	55.768	25.801	2.729	1.669
	(3.668)				(1.375)	(1.882)
Invest.	2.390	9.075	61.933	23.593	3.010	1.227
	(1.204)				(1.517)	(2.560)
M0	16.251	32.347	41.552	9.293	0.557	0.589
	(8.189)				(0.281)	(5.334)
M3	27.037	32.552	35.605	4.295	0.510	0.295
	(13.63)				(0.257)	(10.668)
Real M0	6.068	9.857	50.672	31.433	1.970	1.669
	(3.058)				(0.993)	(1.882)
Real M3	6.818	14.916	56.950	19.714	1.603	0.884
	(3.436)				(0.808)	(3.555)
Short	49.555	35.649	12.427	2.255	0.115	0.147
	(24.875)				(0.058)	(21.328)
Long	54.374	36.416	8.004	1.130	0.077	0.147
	(27.294)				(0.039)	(21.328)
Real Short	8.984	27.197	46.536	16.481	0.803	0.884
	(4.510)				(0.403)	(3.555)
Real Long	11.281	37.003	48.793	2.696	0.227	0.736
	(5.663)				(0.114)	(4.267)
RER	1.192	4.501	47.032	45.495	1.780	0.589
	(0.601)				(0.897)	(5.334)
Real wages	7.200	10.278	58.169	23.744	0.608	1.129
	(3.629)				(0.307)	(2.783)
Prices	21.120	28.278	38.175	11.882	0.545	0.736
	(10.644)				(0.275)	(4.267)
TFP	5.922	5.837	36.868	45.783	5.589	0.0982
	(2.985)				(2.817)	(31.99)
CA	28.834	34.455	29.579	6.379	0.754	0.589
	(14.474)				(0.378)	(5.334)

Notes: a) the power spectra are estimated for each of the Bartlett, Tukey and Parzen approximations; b) we show the results for the Bartlett, the others are available on request but no significant differences were found; c) the figures in parentheses in columns one and five are the asymptotic standard errors in percentages of the point estimates for the trend and Nyquist frequency respectively, d) Durbin's (1969) test defines the peak frequency as the largest gap between the estimated power spectra and hypothetical white noise, e) the second figures in each row of the final column corresponds to the peak frequency in years.

TABLE 3B: Power Spectrum - Pre-War sample

Variable	Cycles Per Year					Durbin's
	0	<0.136	<0.5	<1	1	Peak Freq.
Output	3.668	4.821	39.825	43.230	8.456	0.736
	(2.054)				(4.736)	(4.268)
Cons.	3.404	3.638	43.260	43.475	6.223	1.865
	(1.907)				(3.485)	(1.685)
Invest.	4.267	6.323	49.469	32.345	7.586	1.276
	(2.390)				(4.255)	(2.462)
M0	14.772	19.839	48.158	16.047	1.184	0.785
	(8.274)				(0.663)	(4.002)
M3	12.503	18.725	58.519	9.357	0.896	0.884
	(7.003)				(0.502)	(3.554)
Real M0	2.819	2.962	31.618	58.250	4.351	1.276
	(1.379)				(2.437)	(2.462)
Real M3	5.257	4.937	41.579	45.702	2.525	1.718
	(2.945)				(1.414)	(1.829)
Short	25.080	22.683	38.488	13.195	0.553	0.196
	(14.047)				(0.310)	(16.029)
Long	51.251	30.673	15.523	2.329	0.224	0.196
	(28.706)				(0.125)	(16.029)
Real Short	9.114	14.156	51.374	24.081	1.275	0.785
	(5.105)				(0.714)	(4.002)
Real Long	17.598	22.484	53.981	5.526	0.411	0.785
	(9.857)				(0.230)	(4.002)
RER	1.181	2.008	42.579	52.734	1.499	2.356
	(0.662)				(0.840)	(1.333)
Real wages	6.034	5.291	57.339	29.937	1.400	1.374
	(3.380)				(0.784)	(2.286)
Prices	12.322	14.663	48.864	22.914	1.238	0.785
	(6.902)				(0.693)	(4.002)
TFP	0.731	2.661	36.606	52.588	7.415	1.816
	(0.501)				(5.086)	(1.730)
CA	21.902	21.598	42.068	13.827	0.606	0.442
	(12.267)				(0.339)	(7.108)

Notes: See Table 3A.

TABLE 3C: Power Spectrum - Postwar sample

Variable	Cycles Per Year					Durbin's
	0	<0.136	<0.5	<1	1	Peak Freq.
Output	2.944	3.658	62.221	29.500	1.681	0.295
	(1.886)				(1.076)	(10.668)
Cons.	1.705	3.591	60.934	32.536	1.275	0.098
	(1.092)				(0.816)	(31.992)
Invest.	9.069	8.013	39.232	39.893	3.793	0.982
	(5.809)				(2.430)	(3.200)
M0	23.430	22.369	40.194	12.722	1.285	0.1963
	(15.008)				(0.823)	(16.004)
M3	35.910	23.222	34.282	5.760	0.826	0.1963
	(23.001)				(0.529)	(16.004)
Real M0	5.990	6.908	54.972	29.754	2.376	1.473
	(3.837)				(1.522)	(2.133)
Real M3	16.081	13.783	54.255	14.190	1.692	1.473
	(10.300)				(1.084)	(2.133)
Short	44.671	27.458	23.411	4.194	0.265	0.196
	(28.613)				(0.170)	(16.004)
Long	48.423	30.779	17.531	2.987	0.279	0.196
	(31.016)				(0.179)	(16.004)
Real Short	23.406	27.760	35.266	12.509	1.058	0.589
	(14.992)				(0.678)	(5.334)
Real Long	27.166	28.485	39.642	4.247	0.459	0.589
	(17.400)				(0.294)	(5.334)
RER	2.797	3.876	41.834	47.462	4.031	2.356
	(1.791)				(2.582)	(1.333)
Real wages	6.048	6.369	57.343	28.566	1.675	0.196
	(3.874)				(1.073)	(16.029)
Prices	27.797	26.448	35.586	9.562	0.607	0.393
	(17.805)				(0.389)	(7.994)
TFP	2.249	6.634	46.104	42.813	2.200	0.196
	(1.764)				(1.726)	(16.004)
CA	8.916	14.489	58.350	15.477	2.769	0.982
	(5.711)				(1.774)	(3.200)

Notes: See Table 3A.

TABLE 4A: Full Sample Correlation Matrix

	Output	Cons.	Invest.	M0	M3	Real M0	Real M3	Short	Long	Real Sh.	Real Lo.	RER	Wages	Prices	CA	TFP
Output	2.23															
Cons.	0.16	1.99														
Invest.	0.23**	0.48	8.01													
M0	0.01	0.51**	0.31**	2.28												
M3	-0.00	0.43**	0.60**	0.60**	2.46											
Real M0	0.09	0.52**	0.22**	0.62**	0.19**	3.39										
Real M3	0.09	0.54**	0.22**	0.42**	0.56**	0.79	3.11									
Short	-0.04	0.08	0.03	0.08	0.22**	-0.09	0.01	0.99								
Long	-0.22**	-0.17	-0.15	0.03	0.15	-0.16	-0.07	0.62**	0.43							
Real Sh.	-0.13	-0.03	0.09	-0.01	-0.05	0.48**	0.49**	-0.06	-0.10	3.31						
Real Lo.	-0.47**	-0.32**	-0.04	-0.05	0.15	-0.37**	-0.25**	0.03	0.22**	0.28**	1.49					
RER	0.20**	0.04	0.19**	0.10	0.05	0.31**	0.30**	0.08	0.04	0.19*	-0.17	6.43				
Wages	-0.05	0.42**	0.31**	0.44**	0.32**	0.37**	0.33**	0.28**	0.15	-0.01	-0.17	-0.15	2.07			
Prices	-0.11	-0.23**	-0.01	0.07	0.27**	-0.74**	-0.65**	0.19*	0.23**	-0.61**	0.43**	-0.31**	-0.09	2.67		
CA	-0.24**	-0.11	-0.04	-0.15	-0.18	-0.16	-0.21**	-0.03	0.03	0.15	0.33**	0.07	-0.06	0.08	8.92	
TFP	0.90**	-0.02	0.14	-0.14	-0.16	0.05	0.02	-0.23**	-0.27**	-0.01	-0.37**	0.15	-0.23**	-0.18	-0.22**	2.00

Notes: a) the data are band pass filtered, b) the diagonal corresponds to the variable's standard deviation, c) * indicates significant at 5% and ** at 1%, using a Student's t-distribution where $t = r\sqrt{n-2}\big/\sqrt{1-r^2}$, with *r* is the sample correlation coefficient and *n* the number of observations.

TABLE 4B(1): Pre-War Correlation Matrix 1871-1939

	Output	Cons.	Invest.	M0	M3	Real M0	Real M3	Short	Long	Real Sh.	Real Lo.	RER	Real wages	Prices	CA	TFP
Output	2.72															
Cons.	0.07	2.32														
Invest.	0.24	0.54**	6.52													
M0	-0.02	0.52**	0.39**	2.75												
M3	-0.12	0.46**	0.40**	0.71**	2.47											
Real M0	-0.01	0.45**	0.29*	0.58**	0.08	3.71										
Real M3	-0.11	0.50**	0.36**	0.42**	0.30*	0.85**	2.78									
Short	-0.23	0.09	-0.01	-0.08	0.28*	-0.42**	-0.24	0.62								
Long	-0.52**	0.07	-0.19	0.09	0.21	-0.23*	-0.22	0.61**	0.14							
Real Sh.	-0.23	-0.17	0.10	-0.09	-0.19	0.45**	0.53**	-0.14	-0.06	4.04						
Real Lo.	-0.46**	-0.22	0.02	0.03	0.34**	-0.29*	-0.12*	0.31*	0.50**	0.33**	1.72					
RER	0.31*	-0.08	0.21	0.05	-0.14	0.36**	0.32*	-0.38**	-0.45**	0.28*	-0.17	5.87				
Real wage	-0.18	0.41**	0.45**	0.44**	0.36**	0.29*	0.27*	0.08	0.19	-0.07	-0.03	-0.03	2.31			
Prices	-0.00	-0.08	-0.01	0.18	0.53**	-0.70**	-0.66**	0.44**	0.36**	-0.62**	0.38**	0.39**	0.04	3.12		
CA	-0.27*	-0.02	0.25*	-0.09	0.03	-0.04	0.07	0.37**	0.22	0.13	0.20	0.13	0.02	-0.03	7.77	
TFP	0.92**	-0.13	0.06	-0.14	-0.32**	0.01	-0.13	-0.39**	-0.51**	-0.05	-0.38**	0.29**	-0.32**	-0.13	-0.32**	2.43

Notes: a) the data are band pass filtered, b) the diagonal corresponds to the variable's standard deviation, c) * indicates significant at 5% and ** at 1%, using a Student's t-distribution where $t = r\sqrt{n-2}\big/\sqrt{1-r^2}$, with *r* is the sample correlation coefficient and *n* the number of observations.

TABLE 4B(2): Pre-War Correlation Matrix 1871-1913

	Output	Cons.	Invest.	M0	M3	Real M0	Real M3	Short	Long	Real Sh.	Real Lo.	RER	Real wages	Prices	CA	TFP
Output	2.19															
Cons.	0.49**	1.04														
Invest.	-0.02	0.50**	4.71													
M0	-0.11	0.10	-0.04	1.89												
M3	0.05	0.17	0.35**	0.20	1.13											
Real M0	-0.17	0.25	0.20	0.78**	-0.11	3.63										
Real M3	-0.15	0.38**	0.51	0.49**	0.15	0.86**	2.38									
Short	0.20	0.06	0.10	-0.48**	0.19	-0.37**	-0.10	0.52								
Long	0.09	-0.29	-0.53**	0.12	-0.22	-0.03	-0.24	0.20	0.06							
Real Sh.	-0.23	0.02	0.40**	0.25	-0.15	0.63**	0.69**	-0.00	-0.13	3..20						
Real Lo.	-0.35**	-0.36**	0.09	-0.10	-0.04	-0.20	-0.25	-0.10	0.02	0.27	1.15					
RER	-0.27	0.33**	0.50**	0.10	-0.04	0.55**	0.74**	-0.01	-0.21	0.49**	-0.13	2.73				
Real wage	-0.01	0.48**	0.39**	0.32**	0.15	0.43**	0.47**	-0.07	-0.12	0.18	-0.18	0.54**	2.05			
Prices	0.17	-0.29*	-0.33**	-0.37**	0.31**	-0.88**	-0.89**	0.18	0.13	-0.73**	0.23	-0.73**	-0.38**	2.47		
CA	-0.07	0.13	0.20	-0.15	-0.08	0.13	0.28	0.48**	-0.02	0.32**	-0.09	0.44**	0.37**	-0.31**	4.68	
TFP	0.91**	0.37**	-0.15	0.07	-0.15	-0.03	-0.16	-0.07	0.16	-0.12	-0.24	0.34**	-0.06	0.09	-0.18	1.99

Notes: a) the data are band pass filtered, b) the diagonal corresponds to the variable's standard deviation, c) * indicates significant at 5% and ** at 1%, using a Student's t-distribution where $t = r\sqrt{n-2}\Big/\sqrt{1-r^2}$, with *r* is the sample correlation coefficient and *n* the number of observations.

TABLE 4C: Postwar Correlation Matrix

	Output	Cons.	Invest.	M0	M3	Real M0	Real M3	Short	Long	Real Sh.	Real Lo.	RER	Real wages	Prices	CA	TFP
Output	1.41															
Cons.	0.56**	1.56														
Invest.	0.11	0.63**	7.31													
M0	0.18	0.45**	0.34*	1.56												
M3	0.25	0.41**	0.11	0.44**	2.57											
Real M0	0.57**	0.74**	0.34*	0.74**	0.42**	2.84										
Real M3	0.54**	0.67**	0.20	0.46**	0.85**	0.76**	3.63									
Short	0.15	0.10	0.10	0.31*	0.22	0.14	0.13	1.39								
Long	-0.27	-0.37*	-0.14	0.03**	0.20	-0.24	-0.06	0.64**	0.66							
Real Sh.	0.47**	0.47**	0.10	0.19	0.24	0.57**	0.52**	0.01	-0.26	1.82						
Real Lo.	-0.64**	-0.73**	-0.32*	-0.45	-0.30*	-0.70**	-0.57**	-0.22	0.28	-0.12	1.13					
RER	0.01	0.22	0.11	0.15	0.23	0.21	0.26	0.32*	0.17	-0.09	-0.28	7.26				
Real wage	0.39**	0.49**	0.23	0.44**	0.29	0.50**	0.41**	0.54**	0.22	0.00	-0.66**	0.33*	1.72			
Prices	-0.67**	-0.70**	-0.22	-0.28	-0.26	-0.85**	-0.73**	0.05	0.37*	-0.66**	0.65**	-0.18	-0.37*	1.98		
CA	-0.45**	-0.37*	-0.10	-0.51**	-0.54**	-0.57**	-0.62**	-0.23	0.00	-0.08	0.54**	-0.14	-0.44**	-0.43**	9.09	
TFP	0.83**	0.53**	0.14	-0.07	0.18	0.39**	0.46**	-0.22	-0.43**	0.53**	-.041**	-0.13	0.05	-0.61**	-0.19	1.14

Notes: a) the data are band pass filtered, b) the diagonal corresponds to the variable's standard deviation, c) * indicates significant at 5% and ** at 1%, using a Student's t-distribution where $t = r\sqrt{n-2}\big/\sqrt{1-r^2}$, with r is the sample correlation coefficient and n the number of observations.

TABLE 5A: Leads and Lags Correllogram with Output: Full Sample

Variable	Y_{t-4}	Y_{t-3}	Y_{t-2}	Y_{t-1}	Y_t	Y_{t+1}	Y_{t+2}	Y_{t+3}	Y_{t+4}	Ljung-Box Q Statistic		
										(-4,-1)	(1,4)	(-4,4)
Cons.	0.059	-0.105	-0.035	0.115	0.158	-0.369	-0.240	0.141	0.349	3.45	40.09***	46.45***
Invest.	-0.086	-0.137	-0.077	0.032	0.226	-0.110	-0.207	-0.021	0.196	3.99	11.26**	21.25**
M0	-0.065	0.046	0.164	0.364	0.012	-0.366	-0.395	0.055	0.407	19.60***	54.84***	74.46***
M3	-0.024	0.087	0.203	0.237	-0.005	-0.351	-0.293	-0.045	0.220	12.54**	30.91***	43.46***
Real M0	0.169	0.001	-0.221	0.153	0.093	-0.054	-0.270	-0.037	0.299	12.02**	20.02***	33.07***
Real M3	0.213	0.036	-0.200	0.087	0.089	-0.068	-0.236	-0.045	0.202	11.30**	12.38**	24.59***
Nom Sh.	-0.218	0.104	0.256	0.269	-0.043	-0.319	-0.143	0.163	0.166	23.39***	20.98***	44.58***
Nom Lo.	-0.141	0.041	0.295	0.170	-0.220	-0.291	-0.049	0.201	0.227	16.34***	21.39***	43.40***
Real Sh.	0.207	0.296	-0.188	-0.135	-0.127	0.119	0.066	-0.048	-0.088	22.11***	3.40	27.40***
Real Lo.	-0.080	0.395	0.315	-0.081	-0.470	-0.136	0.134	0.196	-0.055	32.13***	9.52**	67.53***
RER	0.002	-0.159	-0.184	0.102	0.204	0.107	-0.124	-0.168	-0.089	8.32*	7.53	20.72**
Real wage	-0.164	-0.031	0.275	0.248	-0.046	-0.289	-0.347	0.022	0.447	19.60***	48.47***	68.32***
Prices	-0.267	0.039	0.421	0.116	-0.108	-0.245	0.005	0.094	-0.032	31.66***	8.28*	41.32***
CA	0.014	0.047	0.070	-0.005	-0.235	-0.060	0.307	-0.05	-0.198	0.89	16.36***	23.73***
TFP	-0.045	-0.228	-0.400	-0.194	0.903	0.137	-0.125	-0.330	-0.246	30.00***	24.49***	49.88***

Notes: a) We calculate the correlation coefficient on the band pass filtered data and the b) Ljung-Box statistic is approximately χ^2.

TABLE 5B(1): Leads and Lags Correllogram with Output Pre-War Sample 1871-1939

Variable	Y_{t-4}	Y_{t-3}	Y_{t-2}	Y_{t-1}	Y_{t}	Y_{t+1}	Y_{t+2}	Y_{t+3}	Y_{t+4}	Ljung-Box Q Statistic		
										(-4,-1)	(1,4)	(-4,4)
Cons.	0.036	-0.070	-0.141	0.166	0.074	-0.551	-0.256	0.269	0.399	3.58	40.43***	44.38***
Invest.	-0.102	-0.012	0.142	0.188	0.235	-0.311	-0.440	-0.078	0.381	4.42	29.88***	37.89***
M0	-0.072	0.109	0.195	0.354	-0.015	-0.402	-0.387	0.076	0.395	12.00**	31.98***	44.00***
M3	-0.112	0.195	0.413	0.279	-0.119	-0.524	-0.293	0.186	0.326	20.07***	33.64***	54.63***
Real M0	0.228	0.073	-0.262	0.094	0.012	-0.089	-0.205	-0.050	0.286	9.15*	9.17*	18.32**
Real M3	0.280	0.164	-0.181	0.024	-0.107	-0.187	-0.154	0.159	0.285	9.50**	11.27**	21.51**
Nom Sh.	-0.118	-0.114	0.375	0.136	-0.226	-0.232	-0.069	0.126	-0.041	12.50**	5.08	20.89**
Nom Lo.	-0.113	0.104	0.502	-0.010	-0.520	-0.243	0.047	0.219	0.198	18.55***	10.03**	46.18***
Real Sh.	0.289	0.380	-0.213	-0.224	-0.228	0.092	0.128	0.022	-0.114	22.03***	2.60	28.02***
Real Lo.	-0.090	0.444	0.339	-0.093	-0.460	-0.171	0.049	0.191	-0.001	22.32***	4.58	40.65***
RER	0.083	-0.110	-0.395	0.009	0.307	0.264	-0.178	-0.303	-0.157	11.77**	14.70***	32.57***
Real wage	-0.025	0.094	0.282	0.154	-0.182	-0.303	-0.287	0.030	0.452	7.54	25.84***	35.54***
Prices	-0.338	0.009	0.487	0.199	0.001	-0.248	0.094	0.006	0.004	26.45***	4.65	31.11***
CA	0.196	0.038	-0.082	0.011	-0.270	-0.113	0.346	-0.078	-0.221	3.24	12.68**	20.66**
TFP	-0.076	-0.246	-0.321	-0.165	0.916	0.100	-0.090	-0.320	-0.210	13.22**	11.24**	78.99***

Notes: See Table 5A.

TABLE 5B(2): Leads and Lags Correllogram with Output Pre-War Sample 1871-1913

Variable	Y_{t-4}	Y_{t-3}	Y_{t-2}	Y_{t-1}	Y_t	Y_{t+1}	Y_{t+2}	Y_{t+3}	Y_{t+4}	Ljung-Box Q Statistic		
										(-4,-1)	(1,4)	(-4,4)
Cons.	-0.037	0.004	0.026	-0.311	0.491	-0.310	-0.098	0.230	0.040	4.076	6.718	20.451**
Invest.	-0.097	0.156	-0.146	-0.073	-0.020	-0.102	-0.021	-0.012	-0.078	2.600	0.724	3.340
M0	0.059	-0.052	-0.084	0.250	-0.107	-0.026	-0.116	-0.158	0.198	3.124	3.424	7.003
M3	-0.065	-0.056	0.153	0.317	0.051	-0.330	-0.128	0.004	-0.061	5.443	5.342	10.891
Real M0	0.088	-0.048	-0.257	0.178	-0.173	0.118	-0.083	-0.089	0.201	4.540	3.016	8.751
Real M3	0.056	-0.060	-0.254	0.225	-0.155	0.044	-0.095	-0.008	0.121	5.093	1.122	7.170
Nom Sh.	-0.095	-0.237	0.175	0.066	0.203	-0.191	0.005	-0.046	-0.098	4.320	2.024	7.990
Nom Lo.	-0.070	-0.236	0.278	-0.094	0.093	0.137	-0.159	-0.284	0.123	6.263	6.006	12.612
Real Sh.	0.137	0.112	-0.280	0.089	-0.227	0.122	0.033	-0.183	-0.017	5.022	2.123	9.211
Real Lo.	-0.116	0.326	0.013	-0.011	-0.351	0.064	-0.071	-0.030	-0.155	5.235	1.491	11.641
RER	0.060	0.026	-0.151	-0.010	-0.273	0.110	-0.041	0.157	0.036	1.165	1.702	5.841
Real wage	0.079	-0.061	-0.105	0.046	-0.005	-0.126	-0.014	-0.023	-0.002	0.996	0.683	1.680
Prices	-0.084	0.032	0.314	-0.071	0.172	-0.193	0.033	0.010	-0.144	4.731	2.521	8.434
CA	0.049	-0.151	-0.150	0.256	-0.068	-0.272	0.102	-0.111	0.153	4.738	5.049	9.973
TFP	0.057	0.034	-0.085	-0.421	0.906	-0.346	0.012	-0.082	0.003	7.774*	5.212	45.803***

Notes: See Table 5A.

TABLE 5C: Leads and Lags Correllogram with Output Postwar Sample

Variable	Y_{t-4}	Y_{t-3}	Y_{t-2}	Y_{t-1}	Y_t	Y_{t+1}	Y_{t+2}	Y_{t+3}	Y_{t+4}	Ljung-Box Q Statistic		
										(-4,-1)	(1,4)	(-4,4)
Cons.	0.061	-0.298	-0.359	0.011	0.564	0.186	-0.369	-0.352	0.094	11.23**	15.37***	41.90***
Invest.	0.023	-0.186	-0.212	-0.004	0.112	-0.094	-0.030	0.186	0.238	4.07	5.24	9.91
M0	-0.079	-0.392	-0.074	0.396	0.180	-0.256	-0.474	-0.096	0.392	16.15***	23.06***	40.76***
M3	0.203	-0.048	-0.208	0.036	0.248	-0.001	-0.377	-0.308	0.025	4.52	12.04**	19.51**
Real M0	0.022	-0.506	-0.390	0.311	0.568	0.086	-0.568	-0.388	0.238	25.53***	27.25***	68.29***
Real M3	0.194	-0.261	-0.420	0.098	0.542	0.176	-0.506	-0.480	0.035	14.82***	26.27***	55.20***
Nom Sh.	-0.492	-0.168	0.283	0.604	0.153	-0.634	-0.491	0.314	0.501	36.13***	50.08***	87.33***
Nom Lo.	-0.308	0.016	0.444	0.432	-0.269	-0.604	0.148	0.392	0.442	24.06***	37.13**	64.67***
Real Sh.	0.195	-0.044	-0.485	-0.149	0.469	0.375	-0.294	-0.325	-0.094	14.97***	17.12***	42.63***
Real Lo.	0.160	0.576	0.216	-0.393	-0.635	0.016	0.601	0.378	-0.216	28.28***	30.11***	77.77***
RER	-0.128	-0.135	0.145	0.204	0.014	-0.182	-0.020	0.089	0.030	4.89	2.10	7.00
Real wage	-0.483	-0.516	0.146	0.524	0.385	-0.310	-0.636	0.053	0.491	40.45***	37.85***	85.43***
Prices	-0.093	0.417	0.502	-0.134	-0.674	-0.325	0.441	0.481	-0.033	22.91***	26.89***	71.61***
CA	0.004	0.219	0.250	-0.285	-0.446	0.082	0.422	0.300	-0.214	9.56**	16.31***	35.44***
TFP	0.371	-0.201	-0.681	-0.171	0.825	0.395	-0.264	-0.500	-0.130	34.00***	24.89***	91.57***

Notes: See Table 5A.

TABLE 6: Cross-Spectra Analysis: Coherence, Gain and Phase with Output

Variable	Frequency Domain
Cons.	Significant coherence across the set of cycles; maximum gain at the shorter end of the business cycle; some evidence of lead consumption at the low frequencies.
Invest.	Significant coherence in the business cycle and high frequencies; maximum gain at short end of business cycle; some evidence of lead investment at low frequencies.
M0	Significant coherences at business cycle frequencies; gain decreasing with frequency; some evidence of lead output over business cycle.
M3	Significant coherences at longer business cycle frequencies; gain decreasing with frequency; some evidence of lead output over business cycle.
Real M0	Significant coherence at short end of business cycle; maximum gain at same frequency; unlagged relationship at the significant frequencies.
Real M3	As real M0 but more evidence of longer horizon business cycle coherence where output tends to lead.
Short	Significant coherences across the business cycle; gain suggests relatively low response and strong evidence of output lead.
Long	Marginal significance at the trend and short business cycle frequencies, strong gain only at the trend; little evidence of any important lags.
Real Short	Significant business cycle coherences; gain falling with frequency; evidence of real short rate lead over business cycle and Nyquist frequency.
Real Long	As short real, arguable clearer evidence of lead relationship from long real rates.
RER	Significant coherences at longer business cycles; gain peaks at around three years; relationship is essentially unlagged over the business cycle.
Wages	Significant coherences at shorter business cycles; gain increase with frequency up to three years; output leads at important frequencies.
Inflation	Significant coherences across the business cycle and at the Nyquist frequency; falling gain with frequency, evidence for lead output relationship over the business cycle.
CA	Significant coherences at 4-5 years with marginal significances at higher frequencies; gain falls rapidly with frequency; CA lead over longer business cycles.
TFP	Highly significant coherences across all business cycle frequencies – contemporaneous cross-spectra throughout.

Note: The three statistics are derived from a polar decomposition of the complex-valued spectral density: a) see Annex A for plots of data; b) the x-axis corresponds to the cycles per year, where the first vertical line represent the trend, the second vertical line two years and the third one year, c) coherence is the squared correlation coefficient at each frequency, d) the gain is the analogous to the regression coefficient and e) the phase is the fraction of the cycle by which one series leads (lags) the other.

APPENDIX A

UK business cycle expansions and contractions

Business cycle reference dates		**Duration in months**			
Month of peak	Month of trough	Expansion	Contraction	Full cycle (peak to peak)	Full cycle (trough to trough)
	Dec 1854				
Sep 1857	Mar 1858	33	6		39
Sep 1860	Dec 1862	30	27	36	57
Mar 1866	Mar 1868	39	24	66	63
Sep 1872	June 1879	54	81	66	134
Dec 1882	June 1886	42	42	123	84
Sep 1890	Feb 1895	51	53	93	104
Jun-1900	Sep-1901	64	15	117	79
Jun-1903	Nov-1904	21	17	36	38
Jun-1907	Nov-1908	31	17	48	48
Dec-1912	Sep-1914	49	21	66	70
Oct-1918	Apr-1919	49	6	70	55
Mar-1920	Jun-1921	11	15	17	26
Nov-1924	Jul-1926	41	20	56	61
Mar-1927	Sep-1928	8	18	28	26
Jul-1929	Aug-1932	10	37	28	47
Sep-1937	Sep-1938	61	12	98	73
Mar-1951	Aug-1952		17	n/a	n/a
Dec-1955	Nov-1958	40	35	57	75
Mar-1961	Jan-1963	28	22	63	50
Apr-1964	Apr-1967	15	36	37	51
Apr-1968	Jan-1971	12	33	48	45
Jan-1973	Mar-1975	24	26	57	50
Feb-1979	Apr-1982	47	38	73	85
Jan-1984	Mar-1984	21	2	59	23
Apr-1988	Feb-1992	49	46	51	95

These business cycle dates are compiled from the sources given in Table 1. Our reference cycle, the filtered series for output per head, is found to correspond closely to the dates documented here (See Appendix C). Annex A plots the filtered data with contractions – the period from peak to trough – shaded grey.

APPENDIX B

B.1. Data Sources

Mitchell (1988) collates most of the macroeconomic series that we use in this paper. This is generally regarded as the best available source, because it gathers together the most reliable data (or estimates) from primary sources and ends in 1980. We overwrite the Mitchell data with Office of National Statistics data where available to give us more up-to-date information.

Data on real GDP (at factor cost) are taken from Mitchell (p. 837) whose original source is Feinstein (1972) for the period 1855-1948. This GDP series is based on expenditure data, which makes it consistent with the consumers' expenditure and investment data used for this paper. Since 1920 data for the Republic of Ireland have been recorded separately, whereas before they were included in the UK data. Since this break affects all quantity series we do not adjust the various series for this break. From 1948 onwards we use ONS data and rebase the whole real GDP series to 1990.

The price index series is the RPI series (1987=100) provided by the Bank of England. The real exchange rate series is calculated using the US dollar/sterling exchange rate and US (source: Mitchell, 1993) and UK consumer price indices. Current account data are taken from Mitchell (1988) and supplemented by ONS data where available. For consistency real consumption and investment (real gross fixed domestic capital formation) data are taken from Mitchell (1988, p. 837) and are treated in a similar way as GDP data, although they have both been measured in 1995 prices.

Narrow and broad money series start in 1871 and are taken from Capie and Webber (1985). They define narrow money as the monetary base. From 1969 onwards we use data on M0 available from the Bank of England. We then use growth rates of the monetary base before 1969 to project M0 backwards to 1870. Broad money is defined as M3 from 1871 onwards. From 1969 we apply the growth rate of M4 to Capie and Webber's M3 series to give us up-to-date estimates of M3. We use M0 and M3 in order to enable comparison with other OECD countries. Nominal money data are deflated by the RPI series to obtain real variables.

The construction of a consistent series for real wages presented us with a number of difficulties, because in the past real wages were estimated from partial information available for some sectors of the economy only (source: Mitchell), whereas ONS data reflect total economy-wide earnings. We proceed as follows. First we take the average real wage rate series from Mitchell (p. 149), which allows for unemployment (indexed to 1850=100).[1] From 1880 onwards we use real wages from Mitchell (p. 150) (indexed to 1914=100).[2] From 1920 the real wage series is calculated as the basic weekly (nominal) wage rate from Mitchell (p. 151) (1956=100) deflated by the RPI series.[3] From 1946 onwards ONS data on total earnings are used and deflated by the RPI data. All component series are then reindexed to 1956=100.

Data on the employed labour force are from Feinstein (op. cit.), whereas Matthews *et al* (1982) provides data on the capital/labour split in total output.[4] We measure labour productivity as real GDP per employee, and total factor productivity (TFP) growth is then constructed as the difference between output growth and the weighted average of growth in factor inputs.

For long interest rates we use the Consol rate (which is the yield on 3% consols until 1888 and 2.5% consols thereafter) and for short rates we use the discount rate on prime bills.[5] Real interest rates are calculated ex-post using a four-year and one-year RPI inflation rate.

B.2. Data quality

The question temporal stability in sample moments in our dataset opens the question of the extent to which data problems, specifically measurement error might be driving differences in our results.[6] One possibility is that such measurement error biases the sample moments as: (i) the measurement error affects both output and the relevant macroeconomic time series and (ii) the measurement error might fall with time thus explaining part of any *fall* in sample second moments. Dealing with the second point first, although there is likely to be some degradation in the quality of data as the researcher travels to a time before statistical agencies released data[7] – we are fortunate in the UK, insofar as the work of Feinstein (1972) on the main national accounting aggregates and their principal components (note that we use a "compromise" estimate that averages across available estimates) and Capie and Weber (1985) on monetary series allows the construction of macroeconomic time series. We also expect to find no deterioration in the quality of financial prices. We therefore might expect some reduction in sample variances through time as income components are measured more accurately.

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[1] The original source for these data is Wood (1909).
[2] Mitchell's data are taken from Bowley (1937) who estimates economy-wide wages using partial information about wages in some industries.
[3] The Department of Employment and Productivity (1971) collected original data.
[4] These data are not annual, but only available as averages for six subperiods.
[5] Source: Homer and Sylla (1987).
[6] The US debate on the difference between pre and postwar cycles has been almost left incontestable following the convincing attack by Romer (1986) on the quality of data sources.
[7] See Table 1.9 in Feinstein (1972) for a description of the reliability of the component series.

Feinstein's (1972) income and expenditure estimates are cyclically similar and Sheffrin (1988) notes that the standard deviation of the two independent measures differ only in the third decimal place. Feinstein's estimates are given as being "continuous series, consistently defined and measures, over the whole period from 1855 to 1965". There is an extensive discussion in Chapter 1 of Feinstein to which we refer the reader, we interpret the findings as suggesting that the series are appropriate for analysing lower frequency fluctuations over a number of years – that is, of course, precisely our exercise.

Appendix C: The Construction of the Approximate Band-Pass Filter

The purpose of this appendix is to describe the construction of the band pass filters that we employ in the body of the paper. Although we will provide a fair amount of detail on this, our exposition proceeds at a somewhat informal level, and we deliberately side step several important technical issues. Hopefully this approach will aid intuition at minimal cost in terms of lack of rigour. In any case, more details can be found in the original papers by Baxter and King (1999) and Christiano and Fitzgerald (1999). A more rigorous exposition of some of the foundations of spectral analysis can be found in Cox and Miller (1965).

We first outline some important concepts from frequency domain analysis and show that the construction of the bandpass filter can be viewed as a building block in the construction of the spectral representation of an economic variable. Then, we outline the criterion that Baxter and King (BK) and Christiano and Fitzgerald (CF) use to evaluate their approximations to the ideal band-pass filter. We will present first the BK filter and then use this derivation of this filter to construct the recommended filter of CF. It is useful for expositional clarity to consider the issues in this order, although the reader should note that, strictly speaking, the BK filter is derived as a special case of the class of filters constructed by CF.

C.1. The Spectral Representation of an Economic Variable

Economists have long recognised the potential attraction of analysis in the frequency domain (Granger (1966), Granger and Hatanaka (1964), Granger and Morgenstern (1963), Sargent (1973), although it is probably fair to say that the vast majority of empirical work has taken place in the time domain.

A useful point of departure is to note a central result in time-series statistics that any stationary time series can be regarded as the sum of orthogonal sinusoidal components.[8] For example,

$$(1) \qquad Y_t = \int_0^{\pi} \cos\omega t du(\omega) + \int_0^{\pi} \sin\omega t dv(\omega)$$

where $\{Y_t\}_{t=0}^{\infty}$ represents a stationary stochastic real-valued process in discrete time, and $u(\omega)$ and $v(\omega)$ are orthogonal processes defined on the open interval $(0, \pi)$. Under certain fairly weak additional assumptions, the existence of the band-pass filter is implicit in (1) since it implies in effect that we can decompose our stationary time series

[8] We are here referring to the spectral representation theorem, which holds for all, complex and real-valued, functions. Cox and Miller (1965) derive the spectral representation theorem (Chapter 8).

into components indexed by frequency, ω. We can see this more easily if we re-write (1) in a more general form

(2) $$Y_t = \int_{-\pi}^{\pi} e^{i\omega k} dX(\omega)$$

where $X(\omega)$ is a stochastic process (more specifically a process with orthogonal increments) defined on $[-\pi,\ \pi)$.[9] We wish to isolate the fluctuations in Y_t which are due to fluctuations corresponding to frequencies in the range $\underline{\omega} < \omega < \bar{\omega}$. That is we wish to calculate $\hat{Y}_t$:

(5) $$\hat{Y}_t = \int_{\underline{\omega}}^{\bar{\omega}} e^{i\omega t} dX(\omega).$$

Canonically, this is calculated by linearly operating on Y_t in the following way:

(6) $$\hat{Y}_t = \sum_{h=-\infty}^{\infty} b_h Y_{t-h}$$

where b_k represent the correct weights for isolating the periodic components of interest. It follows that we need to calculate these weights, which are subject to the requirement that

(7) $$\beta(\omega) = \sum_k b_k e^{i\omega k} = \begin{matrix} 1 \ \ if \ \ (\underline{\omega} < \omega < \bar{\omega}) \\ \\ 0 \ \ otherwise \end{matrix}.$$

9 We note, as an aside, that because of these properties it is, in principle, straightforward to decompose the variance of our stationary time series by frequency. That is:

(3) $\lfloor \int_{-\pi} \quad \rfloor$

In other words, $F(\omega)$ is the spectral distribution function, the proportion of the variance produced by frequencies in the range $(0,\ \omega)$. The power spectral density function is then given by

(4) $f(\varpi) = dF(\varpi)/d\varpi$

Equation (4) forms the basis of our calculations of the power spectrum, which we discussed in the paper.

Applying the inverse Fourier transform to (7) recovers the optimal weights:

$$
(8) \qquad
\begin{aligned}
b_h &= \frac{1}{2\pi}\int_{\underline{\omega}}^{\bar{\omega}} e^{i\omega h}\, d\omega \\
&= \frac{1}{2\pi}\left[\frac{e^{i\bar{\omega}h} - e^{i\underline{\omega}h}}{ih}\right]
\end{aligned}.
$$

The second equation (8) is the optimal bandpass filter weights. It is easily demonstrated that it represents the difference between two low-pass filters. To see this define the ideal low-pass filter to be that which ensures $\beta(\omega) = 1$ for $|\omega| < \underline{\omega}$, and $\beta(\omega) = 0$ for $|\omega| > \underline{\omega}$ then using this in the first expression in (8) we get that at the zero frequency, $\omega = 0$

$$
(9) \qquad b_0 = \frac{1}{2\pi}\int_{-\underline{\omega}}^{\omega} e^{i\omega h}\, d\omega \Rightarrow \frac{1}{2\pi}\Big|_{-\underline{\omega}}^{\underline{\omega}} = \frac{\omega}{\pi}
$$

and for $\omega \neq 0$, the same steps give

$$
(10) \qquad b_h = \frac{1}{2\pi i h}\left(e^{i\underline{\omega}h} - e^{-i\underline{\omega}h}\right),
$$

and by applying the Euler relations we simplify to:

$$
b_h = \frac{\sin(\underline{\omega}h)}{\pi h}.
$$

It follows then that the optimal weights for the bandpass filter can be written as the following system of equations:

$$
(11) \qquad
\begin{aligned}
b_0 &= \frac{\underline{\omega} - \bar{\omega}}{\pi} \\
b_h &= \frac{\sin(\underline{\omega}h) - \sin(\bar{\omega}h)}{\pi h}, \quad \forall\ h \geq 1
\end{aligned}.
$$

The problem with equation (11) is that its construction employs an infinite-order moving average process (see equation (6)). As both Baxter and King and Christiano and Fitzgerald point out in practice some kind of an approximation is needed. In fact, our spectral density calculations are also approximations to the true density (see e.g., Priestley). In fact, frequency domain analysis more generally tends to be a data intensive exercise as we indicated in the text as it limited quite severely our ability to split our data into more than two sub-samples.

The issue now is to construct an approximate bandpass filter with desirable properties. There seems, as yet, little agreement as to what these additional properties with BK and

CF emphasising different properties as being desirable.[10] These differences can, in principle, result in substantial differences in filter construction (more specifically CF adopt, in effect, a different criterion, or objective, function). We outline the BK filter construction, and then indicate the filter recommended by CF as an extension.

BK adopt a quadratic criterion, which minimises the Euclidean distance between the optimal weights and the actual weights subject to the requirement that the filter return a stationary series. That is a side-constraint is imposed on the problem such that the filter weights partial out the zero frequency. Formally the Lagrangian for this problem can be written as

$$(12) \qquad L = \int_{-\pi}^{\pi} \left[\sum_{h=-\infty}^{\infty} b_h e^{-i\omega h} - \sum_{h=-K}^{K} a_h e^{-i\omega h} \right]^2 d\omega - \lambda \left[\sum_{h=-K}^{K} a_h \right].$$

Note that BK employ a symmetric moving average filter, a choice they justify on the ground that it avoids phase shift. The first summation term represents the optimal filter weights, with the second and third terms being our choice of weights. The problem proceeds by differentiation (12) with respect to a_h, evaluating the resulting first-order condition for each *h* and evaluating the relevant integral.

For *h*=0 we get

$$\frac{\partial L}{\partial a_0} = -2 \int [b_0 - a_0] d\omega = \lambda$$

which simplifies to

$$(13) \qquad (b_0 - a_0) = \lambda / 4\pi .$$

For *h*>0,

$$\frac{\partial L}{\partial a_h} = 2 \int [(b_K - a_K)(e^{i\omega K} + e^{-i\omega K})](e^{i\omega K} + e^{-i\omega K}) d\omega = 2\lambda$$

This is straightforward to simplify, and we get for all *h*>0 that

$$(14) \qquad (b_K - a_K) = \lambda / 4\pi .$$

The first-order conditions indicate that, in the absence of any constraint, it is optimal to set the actual weights of the filter equal to the optimal weights. However, for $\lambda \neq 0$ all the (2*K*+1) first-order conditions have to be altered to ensure a zero response at the zero frequency. It is easy to show that the adjustment factor for each equations results in the following sets of weights:

[10] BK and CF both contain detailed and important discussions as to these desirable properties. We do not cover these arguments.

$$a_0 = b_0 - \frac{\sum_{h=-K}^{K} b_h}{2K+1}$$

(15) $$a_1 = b_1 - \frac{\sum_{h=-K}^{K} b_h}{2K+1}$$

$$\vdots$$

$$\vdots$$

$$a_h = b_h - \frac{\sum_{h=-K}^{K} b_h}{2K+1}$$

C.2. The CF Filter

CF argue that the above filter fails to incorporate important information on the time series property of the raw underlying data. They derive formulas for optimal filter weights for a wide class of time series representations of the data. Their recommended filter, which we focus on here, assumes that the data are generated by a pure random walk. Although they note that this assumption is most likely false for most macro time series, they argue that it nevertheless produces a filter that works well in a wide range of circumstance. CF begin by adopting an alternative criterion which incorporates the assumed time series properties of the data:

$$L = \int_{-\pi}^{\pi} \left[\sum_{h=-\infty}^{\infty} b_h e^{-i\omega h} - \sum_{h=-k}^{K} a_h e^{-i\omega h} \right]^2 f_y(\omega) d\omega - \lambda \left[\sum_{h=-\infty}^{\infty} b_h \right]$$

We note that the filter is no longer symmetric. In general $k \neq K$, and indeed these lower and upper limits are not constant, so in fact each filtered observations uses all the data. The spectral density function, $f_y(\omega)$ plays a crucial role in raising the filter weights for frequencies where the data have higher spectral mass. As we noted above, the filter recommended by CF, and the filter that we employ in this paper, assumes that the data are generated by a pure random walk. The effects that this has on the calculations of the optimal weights can be seen intuitively by recalling the first-order conditions for the construction of the BK filter.

We note that we have a finite number (2*K*+1) of parameters to choose. However, we assume that the data are generated by a random walk. Therefore, let x_N denote the final observation in our raw data set, and x_1 denote the first observation. It follows then that:

$$E_t(x_{N+j}) = x_N \qquad \forall j \geq 0$$
$$E_t(x_{1-j}) = x_1 \qquad \forall j \geq 0$$

In effect, then, we have an infinite number of first-order conditions, where our weights on the first and last observations are calculated using the side constraint, and the weights on our other terms are as in (15), with $K \to \infty$. In other words, $a_i = b_i$ for all i except b_1 and b_N which represent the (time-varying) weight on the initial data point and the final data points, respectively. We then get that, using our first order condition with respect to the undetermined multiplier that, (i.e., $b_0 + 2\sum_{i=1}^{\infty} b_i = 0$),

$$b_N = -\frac{1}{2}b_0 - \sum_{i=1}^{N-1} b_i$$

and

$$b_1 = -(b_2 + b_3 + \ldots b_0 + \ldots b_N)$$

C.3. A brief comparison of the three filtering methods

The left hand side of Table 5 shows the correlations between the various series returned by the filters. The highest correlation in each case is printed in bold and notably corresponds to the correlation between the two versions of the bandpass filter. The right-hand side of the table assesses the similarities and differences in another dimension: how would the stylised facts (in terms of pro- or countercyclicality) stack up in the full sample and in the postwar period?[11] Finally, we report the extent to which the filters representation of the cyclical output series corresponds to the business cycle dates derived from the NBER, OECD and National Institute. We find that all three filters report a high Pearson's contingency coefficient in excess of 80.[12]

[11] Note that the postwar period here is defined as 1956-1990 in order to compare our results directly with Blackburn and Ravn (*op. cit.*) with quarterly data.

[12] The χ^2 statistic is corrected to lie between 0 and 100 and is found to be 88.55 in the case of the HP filter, 84.74 in the case of the BK filter and 85.62 in the case of CF.

TABLE C1: Correlations between Filters and correlations with output

Variable	BK-CF	BK-HP	CF-HP	BK Full	56-90	CF Full	56-90	HP Full	56-90
Output	**0.94**	0.85	0.83	2.358		2.180		2.431	
Cons.	**0.94**	0.90	0.82	0.19	0.74**	0.18	0.59**	0.05	0.73**
Invest.	**0.94**	0.89	0.82	0.24	0.74**	0.26	0.47**	0.08	0.76**
M0	**0.86**	0.70	0.68	0.03	0.48**	0.02	0.41**	0.00	0.47**
M3	**0.84**	0.76	0.76	0.01	0.33**	0.02	0.18	-0.06	0.24**
Real M0	**0.95**	0.88	0.84	0.11	0.75**	0.07	0.63**	-0.04	0.72**
Real M3	**0.91**	0.82	0.82	0.11	0.62**	0.08	0.42**	-0.09	0.49**
Short	**0.94**	0.92	**0.94**	-0.04	0.22**	-0.05	0.20**	0.02	0.25**
Long	**0.93**	**0.93**	0.76	-0.22	-0.28	-0.21	-0.30	-0.15	-0.26
Real Sh.	**0.96**	0.95	0.93	-0.13	0.59**	-0.09	0.48**	-0.21	0.45**
Real Lo.	**0.90**	0.87	0.76	-0.47	-0.66**	-0.38	-0.63**	-0.46	-0.68**
RER	**0.94**	0.91	0.85	0.20	0.21	0.26	0.21	0.13	0.29
Wages	**0.96**	0.89	0.90	-0.05	0.37**	-0.01	0.35**	-0.12	0.35**
Prices	**0.88**	0.79	0.74	-0.11	-0.71**	-0.08	-0.63**	0.04	-0.60**
CA	**0.96**	0.92	0.94	-0.24	-0.64**	-0.17	-0.54**	-0.31	-0.57**
TFP	**0.97**	0.92	0.88	0.90	0.81	0.73	-0.31**	0.90	0.78

Notes: a) the highest correlation in columns two-four is given in bold, b) in columns three and four, * denotes that the sample correlation coefficient is outside the 95% confidence limit of the BK-CF correlation coefficient; c) in columns six, eight and ten, ** denotes that the sample correlation coefficient is significantly different from the full sample under Fisher's Z transformation; and d) the numbers in the final panel of row one are the standard errors of output with each filter.

Again comparing across BK and CF, by and large there is no substantive difference for each set of correlations with output, especially not in terms of pro- or countercyclicality. However, and in contrast to the bandpass filtered data, the HP filter provides weak evidence of countercyclical nominal broad money, real narrow and broad money and procyclical short nominal interest rates and prices. We attach as an appendix the time-series of the cyclical components of the data (derived in the three ways detailed above) and compare these with cyclical output as a reference series. The shaded areas in the charts indicate business cycles measured from their peak to their respective trough.[13]

Table C2 presents summary statistics for the growth rates and the cyclical components of the data as a first test for structural breaks after WW II. It appears that the real quantities, except real broad money holdings and the current account balance, exhibit considerably less volatility postwar. Results for investment are mixed, in growth rates and for the Baxter/King filtered data pre-war volatility appears less than in the postwar period, whereas the HP and Christiano/Fitzgerald filters indicate the opposite. Relative prices also exhibit less volatility, except the real exchange rate. Nominal narrow money balances and the price level appear consistently more volatile pre-war than postwar.

[13] Appendix A details our precise business cycles dates.

TABLE C2(1) Ratio of variance of variables in pre- and post-World War II

	Per capita growth rate*	Hodrick-Prescott cyclical component	Baxter-King cyclical component	Christiano-Fitzger. cyclical component
Output	4.44	3.31	3.57	2.94
Cons.	1.7	1.54	2.16	2.37
Invest.	0.52	1.14	0.83	1.19
M0	2.21	2.29	3.27	2.68
M3	0.67	0.86	0.93	0.63
Real M0	1.79	1.51	1.78	1.86
Real M3	0.55	0.62	0.60	0.59
Short	0.30	0.23	0.21	0.23
Long	0.06	0.05	0.04	0.03
Real Sh.	5.37	5.80	5.01	4.92
Real Lo.	3.73	3.87	2.40	2.29
RER	0.64	0.60	0.68	0.63
Real wages	1.52	1.76	1.83	2.17
Prices	2.00	4.01	2.43	2.58
CA	0.61	0.89	0.77	0.79
TFP	4.86	5.13	4.52	3.90

Notes: a) * Real wages, interest rates, price level, current account and real exchange rate are not per capita calculations; b) # expressed as first difference.

TABLE C2(2) Ratio of variance of variables in pre- and post-World War II

	Per capita growth rate*	Hodrick-Prescott cyclical component	Baxter-King cyclical component	Christiano-Fitzger. cyclical component
Output		1.95	2.42	2.24
Cons.		0.40	0.44	0.65
Invest.		0.58	0.42	0.60
M0		0.86	1.48	1.39
M3		0.18	0.19	0.24
Real M0		1.42	1.64	1.78
Real M3		0.37	0.43	0.41
Short		0.17	0.14	0.15
Long		0.02	0.01	0.01
Real Sh.		3.28	3.08	3.23
Real Lo.		1.63	1.04	1.15
RER		0.15	0.14	0.12
Real wages		1.02	1.42	1.60
Prices		2.42	1.56	1.86
CA		0.30	0.27	0.30
TFP		2.89	3.01	2.74

Notes: a) * Real wages, interest rates, price level, current account and real exchange rate are not per capita calculations; b) # expressed as first difference.

ANNEX A The solid lines show the respective filtered series, whereas the dotted lines indicate real GDP filtered with the respective method.

Figure 1A Baxter-King

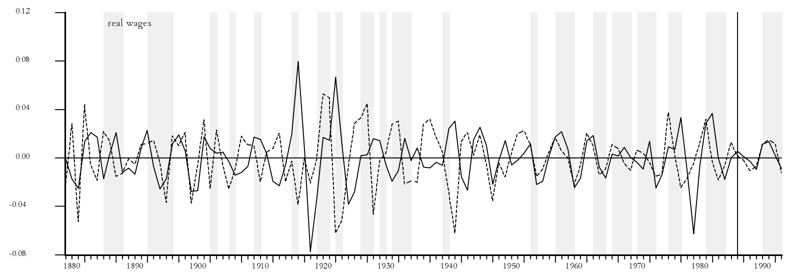

Figure 1B Christiano-Fitzgerald

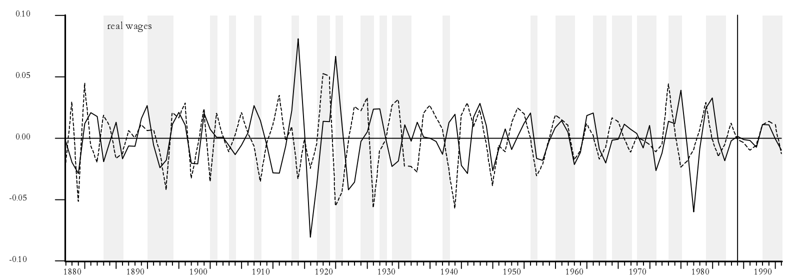

Figure 1C Hodrick-Prescott

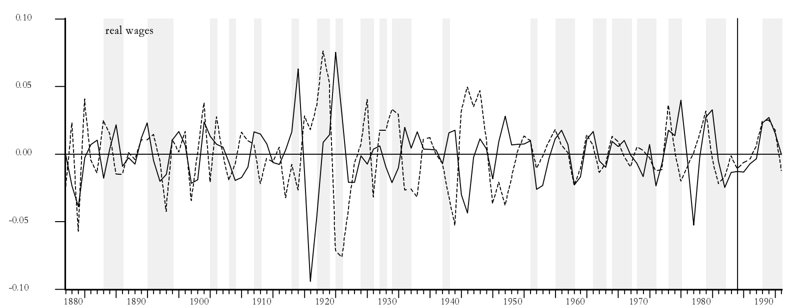

Figure 2A Baxter-King

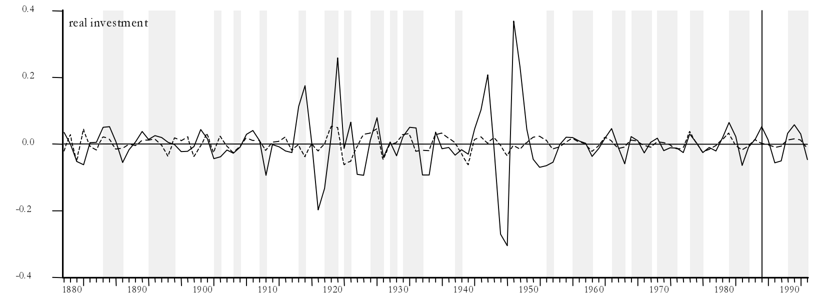

Figure 2B Christiano-Fitzgerald

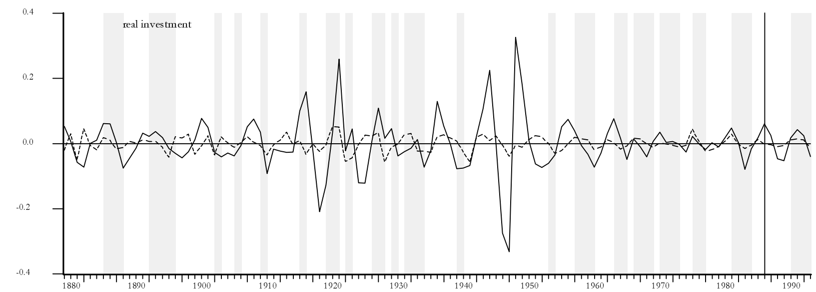

Figure 2C Hodrick-Prescott

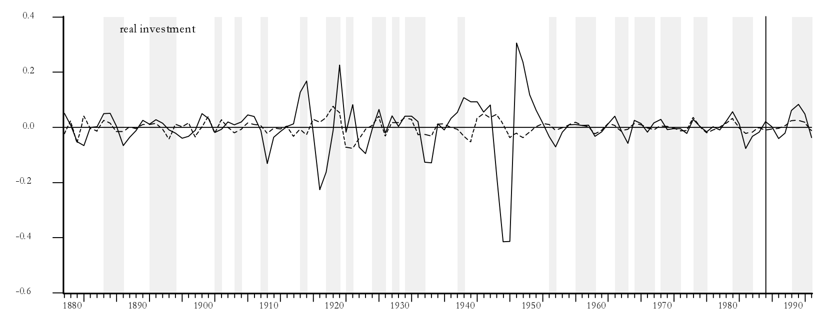

Figure 3A Baxter-King

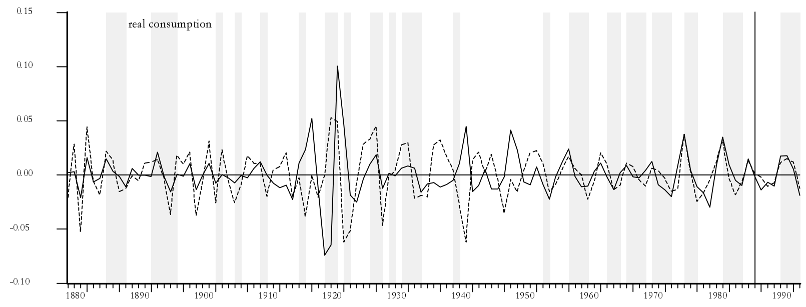

Figure 4A Baxter-King

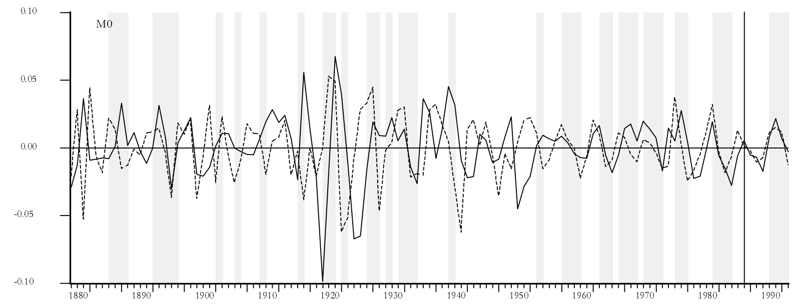

Figure 3B Christiano-Fitzgerald

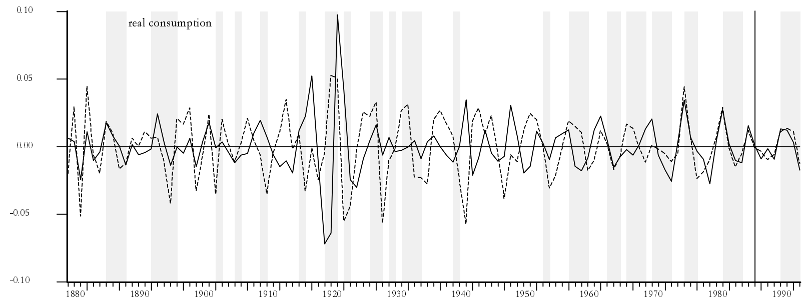

Figure 4B Christiano-Fitzgerald

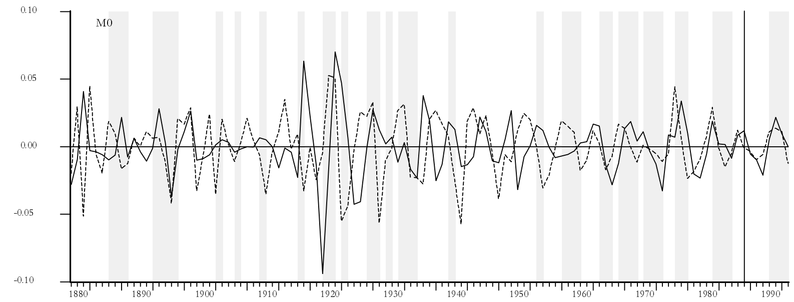

Figure 3C Hodrick-Prescott

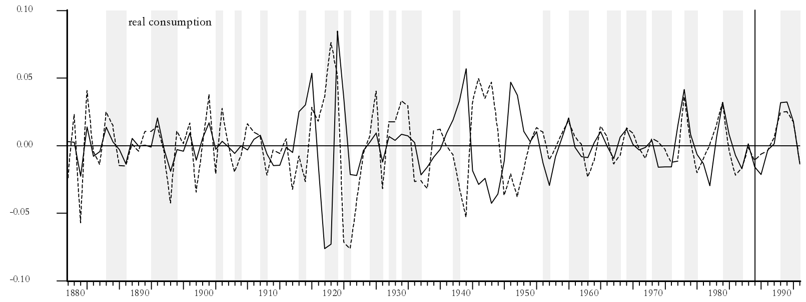

Figure 4C Hodrick-Prescott

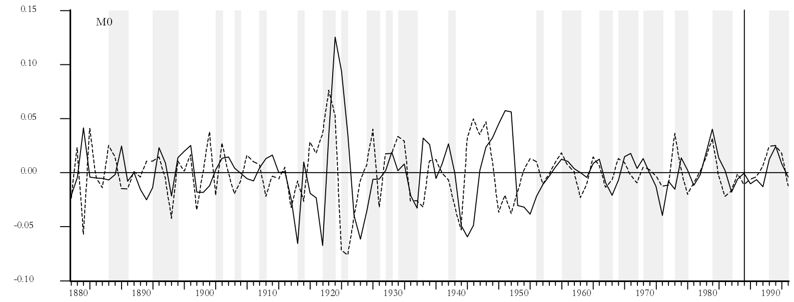

Figure 5A Baxter-King

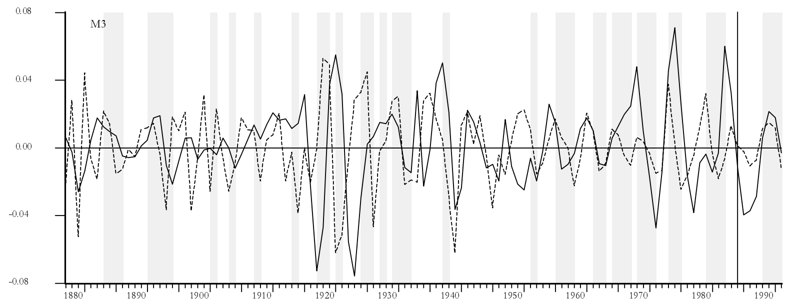

Figure 6A Baxter-King

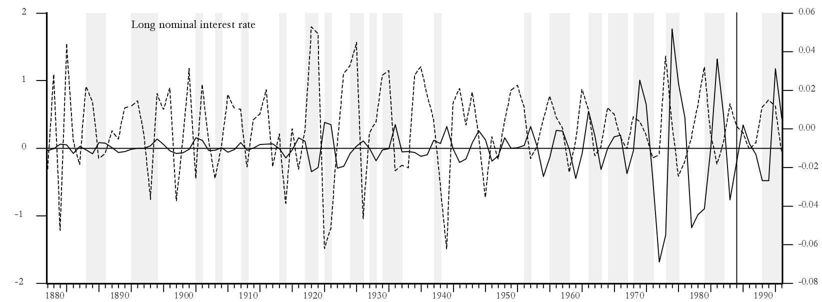

Figure 5B Christiano-Fitzgerald

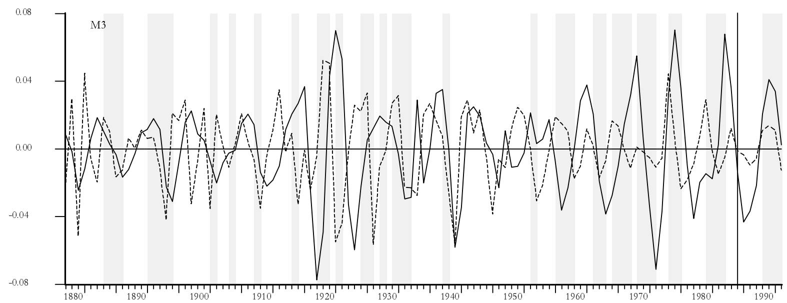

Figure 6B Christiano-Fitzgerald

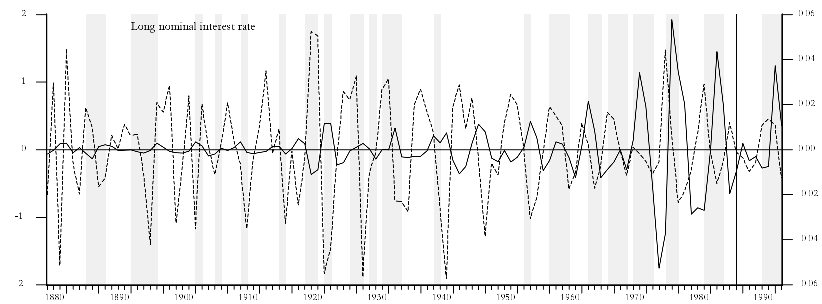

Figure 5C Hodrick-Prescott

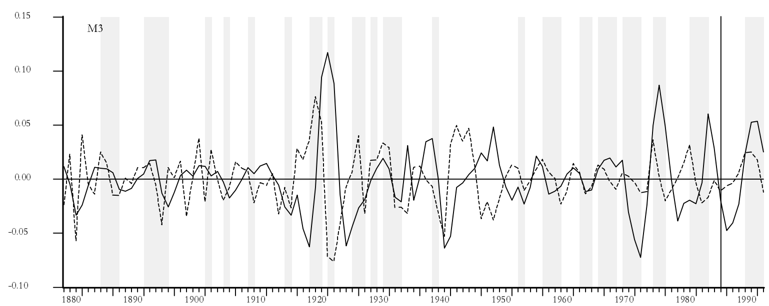

Figure 6C Hodrick-Prescott

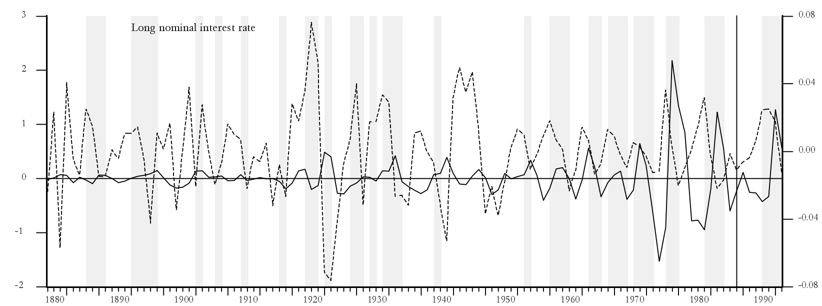

Figure 7A Baxter-King

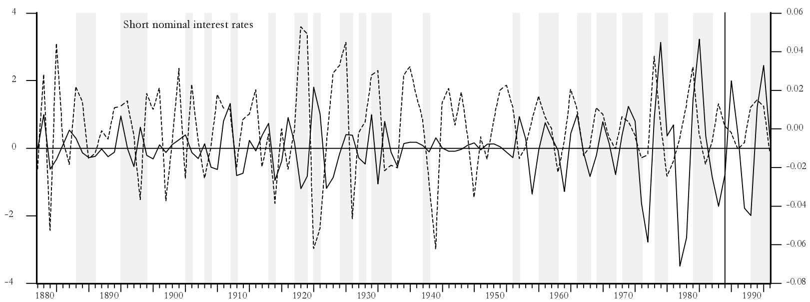

Figure 8A Baxter-King

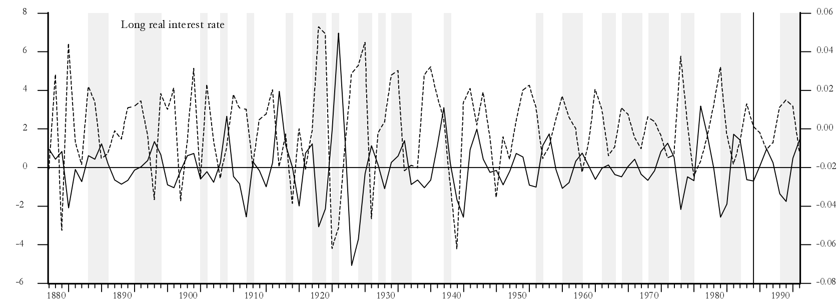

Figure 7B Christiano-Fitzgerald

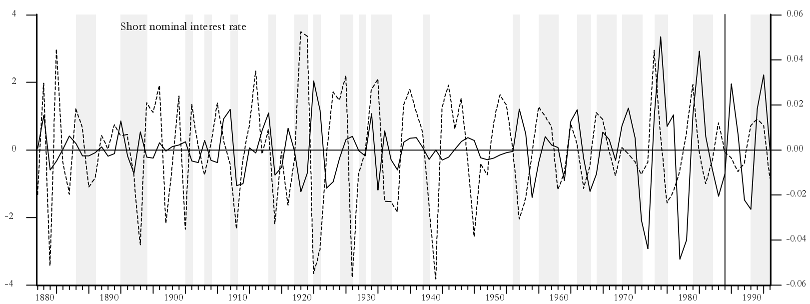

Figure 8B Christiano-Fitzgerald

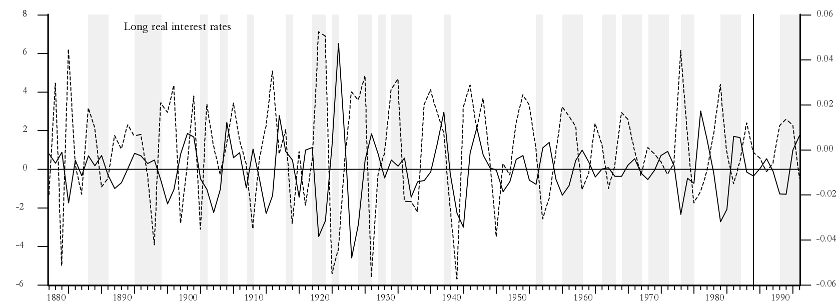

Figure 7C Hodrick-Prescott

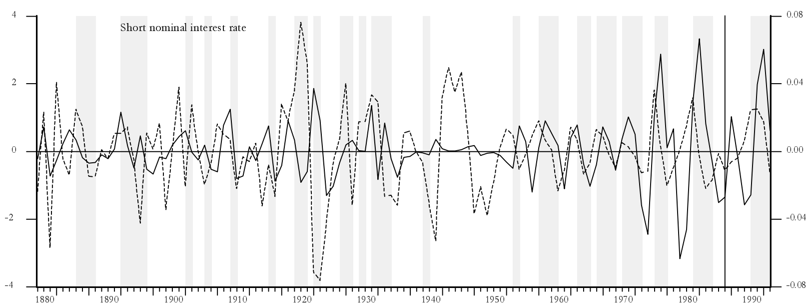

Figure 8C Hodrick-Prescott

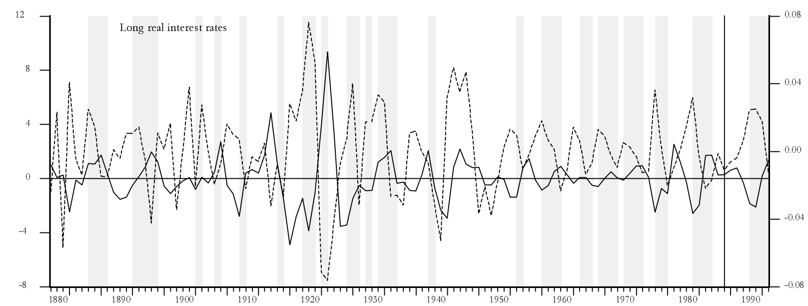

Figure 9A Baxter-King

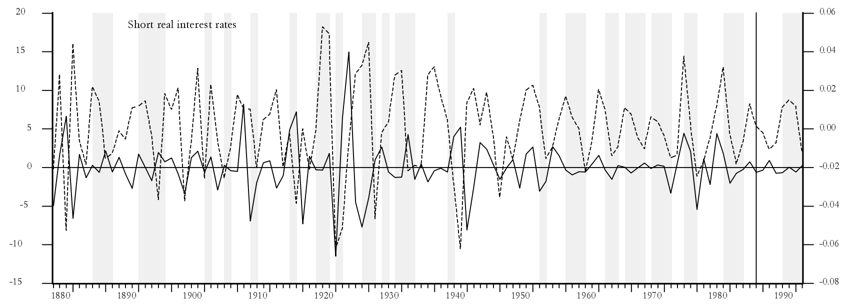

Figure 10A Baxter-King

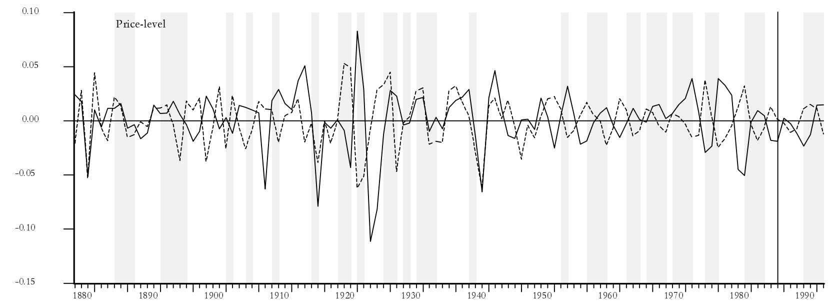

Figure 9B Christiano-Fitzgerald

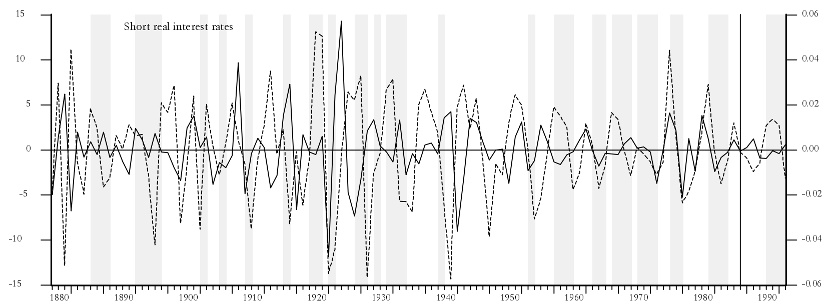

Figure 10B Christiano-Fitzgerald

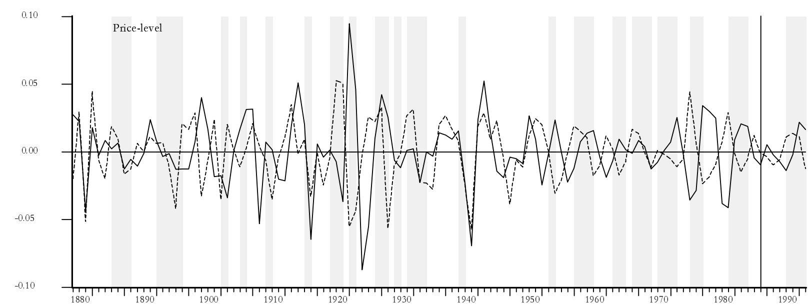

Figure 9C Hodrick-Prescott

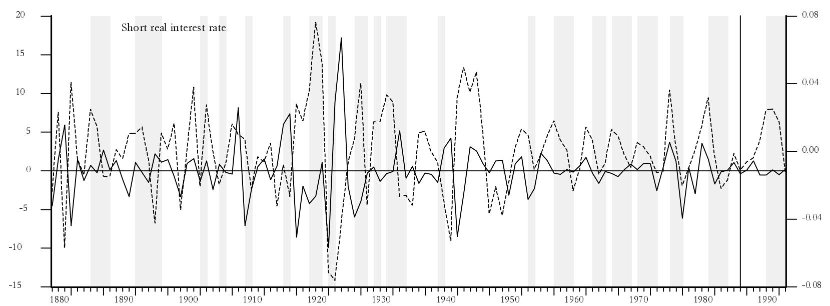

Figure 10C Hodrick-Prescott

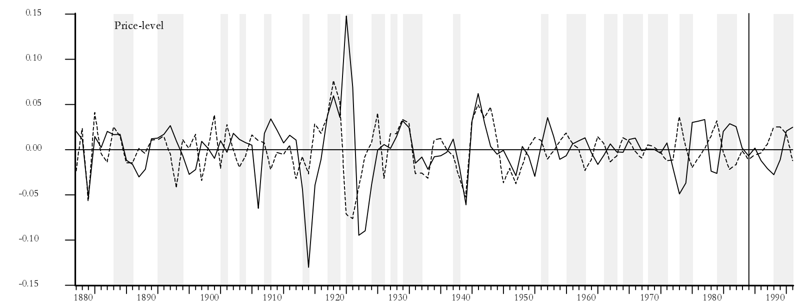

Figure 11A Baxter-King

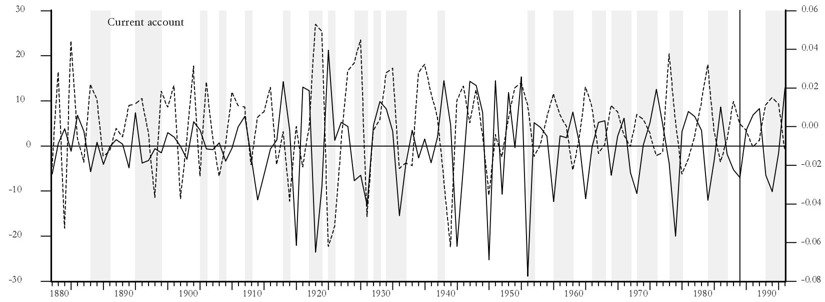

Figure 12A Baxter-King

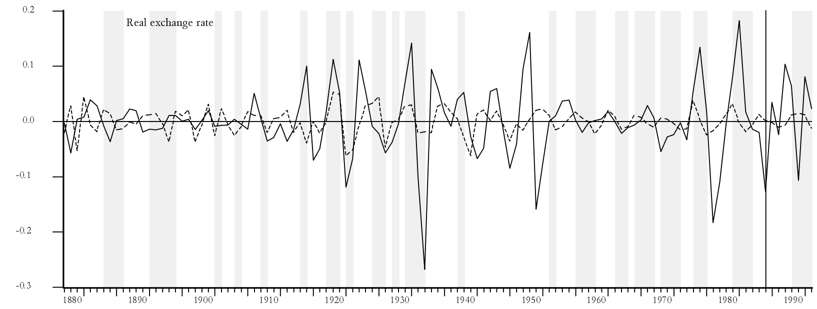

Figure 11B Christiano-Fitzgerald

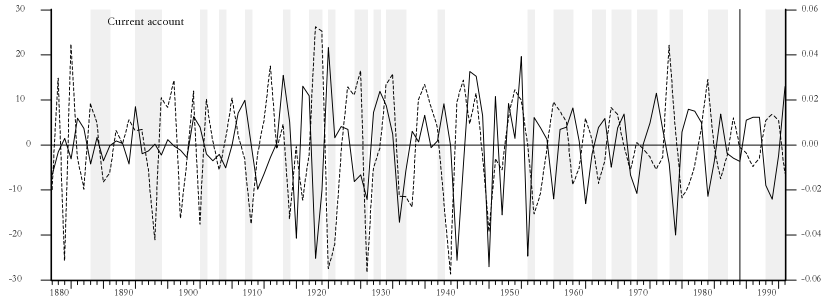

Figure 12B Christiano-Fitzgerald

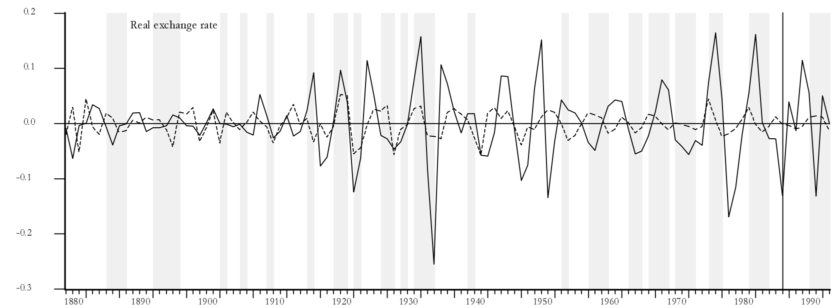

Figure 11C Hodrick-Prescott

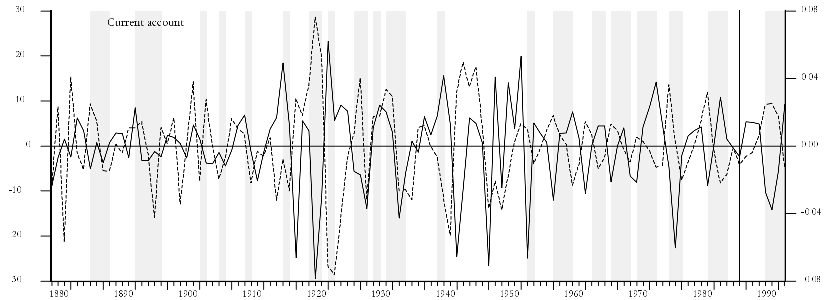

Figure 12C Hodrick-Prescott

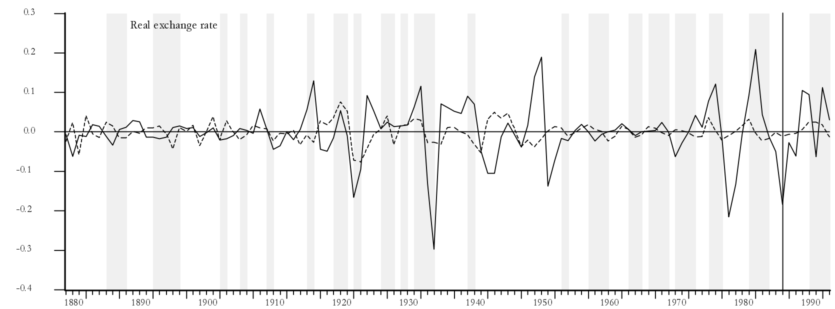

Figure 13A Baxter-King

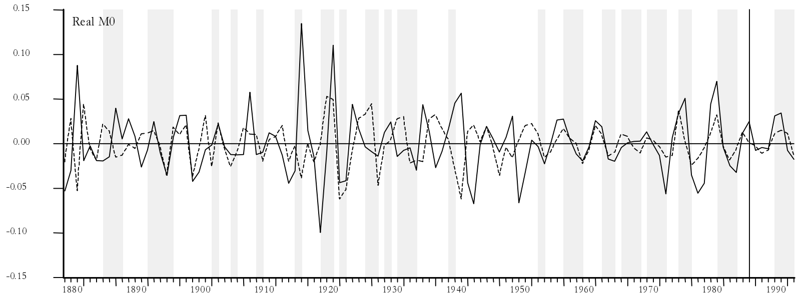

Figure 14A Baxter-King

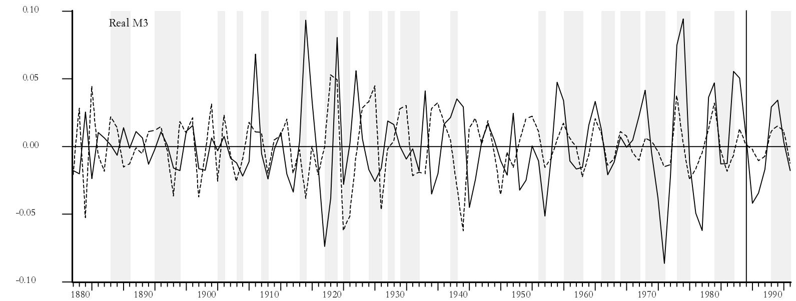

Figure 13B Christiano-Fitzgerald

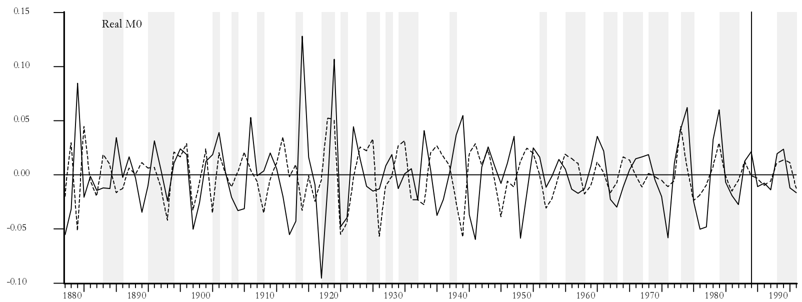

Figure 14B Christiano-Fitzgerald

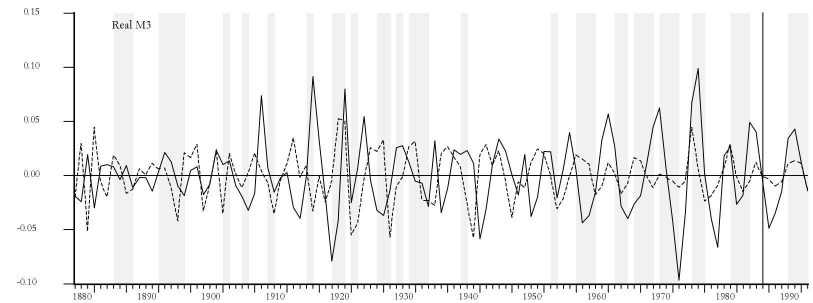

Figure 13C Hodrick-Prescott

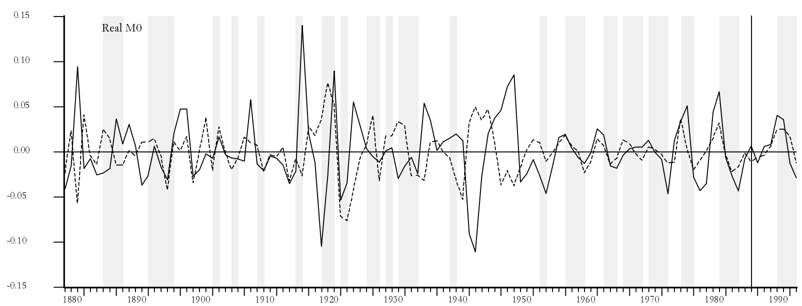

Figure 14C Hodrick-Prescott

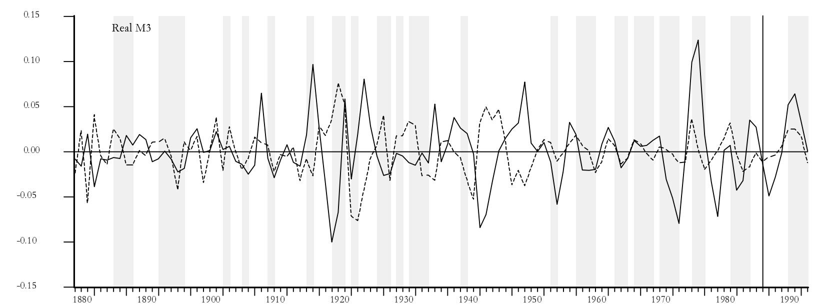

Figure 15A Baxter-King

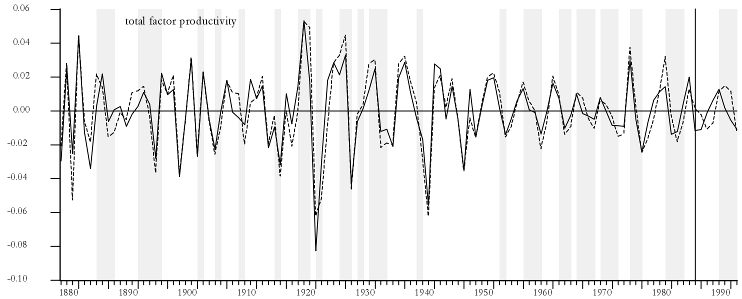

Figure 15B Christiano-Fitzgerald

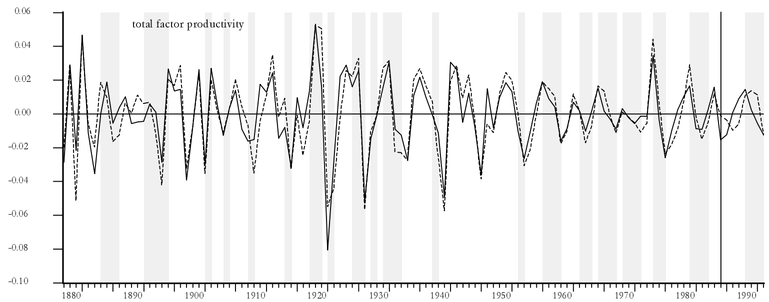

Figure 15C Hodrick-Prescott